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The gravitational wave event GW170817 together with its electromagnetic counterparts constrains the speed of gravity to be extremely close to that of light. We first show, on the example of an exact Schwarzschild–de Sitter solution of a specific beyond-Horndeski theory, that imposing the strict equality of these speeds in the asymptotic homogeneous Universe suffices to guarantee so even in the vicinity of the black hole, where large curvature and scalar-field gradients are present. We also find that the solution is stable in a range of the model parameters. We finally show that an infinite class of beyond-Horndeski models satisfying the equality of gravity and light speeds still provides an elegant self-tuning: the very large bare cosmological constant entering the Lagrangian is almost perfectly counterbalanced by the energy-momentum tensor of the scalar field, yielding a tiny observable effective cosmological constant.

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The simultaneous detection of the gravitational wave event GW170817 and its electromagnetic counterparts [1,2] constrains the speeds of light and gravity to differ by no more than a few parts in 10^{15}. References [3,4] (see also [5]) have characterized which Horndeski theories [6–12] and their generalizations [13–25] satisfy exactly $c_{\rm grav} = c_{\rm light}$ in a homogeneous universe. The aim of the present Letter is twofold. First, we check that this speed equality remains satisfied even in a very inhomogeneous situation, namely, in the vicinity of a black hole, where gradients are large and where the separation of spin-2 and spin-0 degrees of freedom is difficult. This will be done for an exact Schwarzschild–de Sitter solution of a specific model [26] (see also [27,28]). We also report that this solution is ghost free and has no gradient instability for some ranges of the parameters defining the theory. We will refer to such solutions as being "stable" throughout this Letter. We then show that self-tuning cosmological models [29–37] still exist while taking into account the $c_{\rm grav} = c_{\rm light}$ constraint. In such models, the energy-momentum tensor of the scalar field almost perfectly counterbalances the very large bare cosmological constant assumed to be present in the Lagrangian, so that the observable accelerated expansion of the Universe is consistent with a tiny effective cosmological constant.

In the present Letter, we focus on the subclass of shift-symmetric beyond-Horndeski theories, i.e., which do not involve any undifferentiated scalar field φ. (In the following, we use the sign conventions of Ref. [38] and, notably, the mostly plus signature.) Their Lagrangian reads

$$\begin{aligned}\mathcal{L} &= G_2(\varphi_\lambda^2) + G_3(\varphi_\lambda^2)\Box\varphi + G_4(\varphi_\lambda^2)R \\ &\quad - 2G_4'(\varphi_\lambda^2)\left[(\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu}\right] \\ &\quad + F_4(\varphi_\lambda^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_\sigma\varphi_\mu\varphi_\alpha\varphi_{\nu\beta}\varphi_{\rho\gamma} \\ &\quad + \mathcal{L}_{\rm matter}[\psi, g_{\mu\nu}],\end{aligned} \tag{1}$$

where $\epsilon^{\mu\nu\rho\sigma}$ denotes the fully antisymmetric Levi-Civita tensor, $\varphi_\mu \equiv \partial_\mu\varphi$ and $\varphi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\varphi$ are the first and second covariant derivatives of the scalar field, G_2, G_3, G_4, and F_4 are functions of the standard scalar kinetic term $\varphi_\lambda^2 = g^{\mu\nu}\varphi_\mu\varphi_\nu$ [We do not denote φ_λ^2 as X, because this letter will be used for a dimensionless variant in Eq. (18) below.] G_4' is the derivative of G_4 with respect to its argument (φ_λ^2), and ψ denotes globally all matter fields (including gauge bosons) assumed to be minimally coupled to the metric $g_{\mu\nu}$. In order to ensure $c_{\rm grav} = c_{\rm light}$, no quintic beyond-Horndeski term is allowed in this shift-symmetric subclass, and the function F_4 must be related to G_4 by [3,4,39]

$$F_4(\varphi_\lambda^2) = -\frac{2G_4'(\varphi_\lambda^2)}{\varphi_\lambda^2}. \tag{2}$$

This condition (2) ensures that the speeds of gravitational and electromagnetic waves coincide at least in a homogeneous cosmological background. However, the waves of the GW170817 event did pass nearby massive bodies during their 40 Mpc journey, and if their speeds slightly differed in such inhomogeneous situations, this would

a priori suffice to increase the delay between their detections. It is thus important to check that these speeds remain equal even in inhomogeneous backgrounds. Actually, Ref. [4] claims that condition (2) also suffices around arbitrary backgrounds, and we will confirm so below for a specific exact solution—see Eq. (13). But this Ref. [4] uses the results of [40], which needed to neglect scalar-graviton mixing terms in order to extract the spin-2 excitations. Generically, the separation of the spin-2 and spin-0 degrees of freedom is background dependent and highly nontrivial. The same argument applies to the Arnowitt-Deser-Misner (ADM) decomposition of the Lagrangian (1) under condition (2). In the unitary gauge, one recovers the same decomposition as in general relativity [3,41]. However, this does not necessarily mean that the helicity-2 perturbations propagate at $c_{\rm grav} = c_{\rm light}$, because the Lagrangian contains mixing terms proportional to $\dot{h}_{ij} D_i N_j$, in usual ADM notation; the shift N_i cannot be eliminated as in general relativity, because the gauge is already fixed. It remains thus important to check whether this speed equality is also satisfied in very curved backgrounds, with large scalar-field gradients, for instance, in the neighborhood of a black hole horizon. This is what we do now for the particular case of an exact Schwarzschild–de Sitter solution of the model

$$\mathcal{L} = \zeta(R - 2\Lambda_{\rm bare}) - \eta\varphi_\lambda^2 + \beta G^{\mu\nu}\varphi_\mu\varphi_\nu, \tag{3}$$

where $G^{\mu\nu}$ denotes the Einstein tensor, $\zeta = \frac{1}{2}M_{\rm Pl}^2 > 0$, and $M_{\rm Pl} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. (The $G^{\mu\nu}\varphi_\mu\varphi_\nu$ term has been nicknamed "John" in the "Fab-Four" model [29,30].) In terms of the notation of Eq. (1), this corresponds to

$$G_2(\varphi_\lambda^2) = -2\zeta\Lambda_{\rm bare} - \eta\varphi_\lambda^2, \tag{4}$$

$$G_4(\varphi_\lambda^2) = \zeta - \frac{\beta}{2}\varphi_\lambda^2, \tag{5}$$

and $G_3 = F_4 = 0$. Since this vanishing of F_4 is in contradiction with Eqs. (2) and (5), we can immediately conclude that this model does not satisfy the $c_{\rm grav} = c_{\rm light}$ constraint, if matter (and thereby light) is assumed to be minimally coupled to $g_{\mu\nu}$ as in Eq. (1). However, as already underlined in [3,4], it suffices to couple matter to a different metric $\tilde{g}_{\mu\nu}$, related to $g_{\mu\nu}$ by a disformal transformation, to change the matter causal cone so that $c_{\rm grav} = c_{\rm light}$ is ensured, at least in a homogeneous universe. In the present model, the disformal transformations given in [13,14,22,23] or the gravity speed derived in [4,40] allow us to prove that this physical metric must read (or be proportional to)

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}\varphi_\lambda^2}\varphi_\mu\varphi_\nu. \tag{6}$$

One may also rewrite Lagrangian (3) in terms of this $\tilde{g}_{\mu\nu}$, and one finds that it becomes of the form (1), with rather complicated functions $\tilde{G}_4(\tilde{\varphi}_\lambda^2)$ and $\tilde{F}_4(\tilde{\varphi}_\lambda^2)$ (involving nested square roots), which now do satisfy the constraint (2) in terms of the variable $\tilde{\varphi}_\lambda^2 \equiv \tilde{g}^{\mu\nu}\varphi_\mu\varphi_\nu$. This guarantees that the speeds of light and gravity coincide at least in the asymptotic homogeneous universe, far away from any local massive body.

We can go beyond this result by studying the speed of spin-2 perturbations around a spherical black hole. An exact Schwarzschild–de Sitter solution has indeed been found in [26] for model (3), assuming linear time dependence of the scalar field [42]

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{7}$$

$$A(r) = 1 - \frac{2Gm}{r} - \frac{\Lambda_{\rm eff}}{3}r^2, \tag{8}$$

$$\Lambda_{\rm eff} = -\frac{\eta}{\beta}, \tag{9}$$

$$\varphi = q\left(t - \int \frac{\sqrt{1 - A(r)}}{A(r)}dr\right), \tag{10}$$

$$q^2 = \frac{\eta + \beta\Lambda_{\rm bare}}{\eta\beta}\zeta, \tag{11}$$

where this last equality (11) forces its right-hand side to be positive. Equation (9) defines the effective cosmological constant $\Lambda_{\rm eff}$ entering the line element (7), and one can note that it may be as small as one wishes, independent of the magnitude of $\Lambda_{\rm bare}$ (it does not even depend at all on $\Lambda_{\rm bare}$, in the present model). This is a particularly simple example of self-tuning. However, the observer, made of matter, is now assumed to be coupled to the physical metric (6), and this changes her perception of the Universe. A straightforward calculation shows that $\tilde{g}_{\mu\nu}$ remains of the exact Schwarzschild–de Sitter form, with a scalar field of the form (10) in the relevant transformed coordinate system, but the observable cosmological constant now reads

$$\tilde{\Lambda}_{\rm eff} = \left(\frac{\Lambda_{\rm eff} + \Lambda_{\rm bare}}{3\Lambda_{\rm eff} - \Lambda_{\rm bare}}\right)\Lambda_{\rm eff}. \tag{12}$$

At this stage, it thus seems that a very small $\tilde{\Lambda}_{\rm eff}$ remains possible, for instance, if $\Lambda_{\rm eff} = -\eta/\beta$ is chosen to almost compensate $\Lambda_{\rm bare}$. However, the field equations written in the physical frame $\tilde{g}_{\mu\nu}$ actually always imply $\tilde{\Lambda}_{\rm eff} \sim \Lambda_{\rm bare}$ [43]. Moreover, we will see below that the stability of the

solution forces both $\Lambda_{\rm eff}$ and the observable $\tilde{\Lambda}_{\rm eff}$ to be of the same order of magnitude as $\Lambda_{\rm bare}$ (or even larger). Therefore, in this simple model (3), the small observed cosmological constant cannot be explained by the self-tuning mechanism, and some other reason must be invoked, like in standard general relativity. It remains that this model is observationally consistent if the constant $\Lambda_{\rm bare}$ entering (3) is small enough.

The odd-parity perturbations of solution (7)–(11) have been analyzed in [44], and they define the effective metric, say $\mathcal{G}_{\mu\nu}$, in which spin-2 perturbations propagate. We can thus compare it with the metric $\tilde{g}_{\mu\nu}$, Eq. (6), to which matter (including photons) is assumed to be coupled, and we find

$$\mathcal{G}_{\mu\nu} = \left(\frac{\Lambda_{\rm eff}}{\Lambda_{\rm eff}+\Lambda_{\rm bare}}\right)\tilde{g}_{\mu\nu}. \tag{13}$$

Therefore, even close to the black hole, their causal cones exactly coincide. In other words, the universal coupling of matter to the disformal metric (6) suffices to ensure $c_{\rm grav} = c_{\rm light}$ even in a very inhomogeneous configuration. Details will be given in [43].

The perturbative analysis of Ref. [44] was actually performed to claim that the above Schwarzschild-de Sitter solution is always unstable, but this claim is incorrect. The argument was that the Hamiltonian of these perturbations is unbounded by below, close enough to the black hole horizon. However, although a bounded Hamiltonian does guarantee the stability of the lowest-energy state, the converse theorem does not exist. Indeed, a Hamiltonian is not a scalar quantity, and it may take large negative values in a very boosted frame even if it was bounded by below initially. It gets mixed with other conserved quantities that are not bounded by below, corresponding to the three momentum of the system. A correct stability criterion may thus be formulated as follows: if the Hamiltonian is bounded by below in at least one coordinate system, then the solution is stable. As we shall detail in [43], when focusing on kinetic terms, it suffices that the causal cones of all interacting degrees of freedom share a common interior timelike [45] direction (which will become the time coordinate of the "safe" frame in which the Hamiltonian can be proven to be bounded by below) and also a common exterior spacelike hypersurface, on which initial data may be set to define the Cauchy problem (see [46–50] for related discussions).

In the present solution (7)–(11) for model (3), we saw that the graviton and matter causal cones coincide everywhere, because of the disformal metric (6) to which we universally couple matter. There remains, however, to check that both interiors are indeed timelike, otherwise matter and gravitons would have opposite kinetic energies. This means that the factor entering Eq. (13) must be positive. Moreover, the scalar field φ itself has a different causal cone, that we study in [43] by analyzing the $\ell = 0$ even-parity perturbations. Therefore, stability can be ensured only if the scalar causal cone shares a common interior direction and a common exterior hypersurface with that of $\tilde{g}_{\mu\nu}$ (and gravitons). We found that it is possible provided the parameters of Lagrangian (3) satisfy the following inequalities. We write them here in terms of the ratio $-\eta/\beta$, denoted as $\Lambda_{\rm eff}$ in Eq. (9), and we assume that the observed $\tilde{\Lambda}_{\rm eff} = 3\tilde{H}^2$ is positive (which implies that $\Lambda_{\rm bare}$ and $\Lambda_{\rm eff}$ are positive too, when taking into account these very inequalities),

$$\text{either } \eta>0,\ \beta<0,\ \text{and } \frac{1}{3}\Lambda_{\rm bare} < -\frac{\eta}{\beta} < \Lambda_{\rm bare}, \tag{14}$$

$$\text{or } \eta<0,\ \beta>0,\ \text{and } \Lambda_{\rm bare} < -\frac{\eta}{\beta} < 3\Lambda_{\rm bare}. \tag{15}$$

It is straightforward to prove analytically that these conditions suffice for the consistency of the causal cones in the asymptotic de Sitter universe. Close to the black hole, the analytic expressions are so long that we checked instead specific examples in a numerical way, by following the relative positions of the scalar and graviton (or matter) causal cones while varying the distance r to the center of the black hole. Our conclusion is that, for the above ranges of the ratio $-\eta/\beta$, Eq. (14) or (15), the perturbation Hamiltonian is bounded by below in a well-chosen frame, at any spacetime point, and the stability criterion we established is satisfied.

Note that, when setting $\Lambda_{\rm bare} = 0$, the interval of stability disappears, in agreement with the perturbation analysis of [51] around a cosmological background. In other words, it is the presence of vacuum energy that allows for a window of stability for the black hole solution.

In terms of the observed $\tilde{\Lambda}_{\rm eff}$, Eq. (12), conditions (14) and (15) imply

$$\text{either } \eta>0,\quad \beta<0,\quad \text{and}\quad \Lambda_{\rm bare} < \tilde{\Lambda}_{\rm eff}, \tag{16}$$

$$\text{or } \eta<0,\ \beta>0,\ \text{and } \Lambda_{\rm bare} < \tilde{\Lambda}_{\rm eff} < \frac{3}{2}\Lambda_{\rm bare}. \tag{17}$$

As stressed below Eq. (12), this means that self-tuning is impossible in this specific model, since the observed cosmological constant must always be larger than the bare one.

But there exists an infinite class of other beyond-Horndeski models that do provide self-tuning, as shown in Ref. [37], and we prove below that a subclass of them also satisfies the $c_{\rm grav} = c_{\rm light}$ constraint. From now on, we assume that matter is minimally coupled to $g_{\mu\nu}$, as in Lagrangian (1), and we no longer consider any disformal transformation such as (6).

To avoid hiding several different scales in the functions of φ_λ^2, it is convenient to work with the dimensionless quantity

$$X \equiv \frac{-\varphi_\lambda^2}{M^2}, \tag{18}$$

M being the only mass scale entering the Lagrangian of the scalar field φ, itself chosen dimensionless (beware not to confuse M with the Planck mass $M_{\rm Pl}$). All the coefficients entering dimensionless functions of X will also be assumed to be of order $\mathcal{O}(1)$. Up to a total derivative, action (1) may then be rewritten as

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}(R - 2\Lambda_{\rm bare}) - M^4 X f_2(X) - 4 s_4(X) G^{\mu\nu}\varphi_\mu\varphi_\nu - \frac{f_4(X)}{M^2}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma}{}_\sigma \varphi_\mu\varphi_\alpha\varphi_{\nu\beta}\varphi_{\rho\gamma} + \mathcal{L}_{\rm matter}, \tag{19}$$

where we do not include the G_3 term because it must anyway be passive for the self-tuning solutions derived in [37] [see this reference for the explicit translation between (1) and (19), as well as other notation used in the literature]. (The above model (3) corresponds, for instance, to constant values $\zeta = \frac{1}{2}M_{\rm Pl}^2$, $f_2 = -\eta/M^2$, $s_4 = -\frac{1}{4}\beta$, and $f_4 = 0$.)

The $c_{\rm grav} = c_{\rm light}$ constraint (2) becomes then

$$f_4(X) = -\frac{4 s_4(X)}{X}, \tag{20}$$

while $f_2(X)$ and $s_4(X)$ are arbitrary. For monomials, this means that we need

$$f_2 = k_2 X^\alpha, \qquad s_4 = \kappa_4 X^\gamma, \qquad f_4 = -4\kappa_4 X^{\gamma-1}. \tag{21}$$

where k_2, κ_4, α, and γ are dimensionless constants of order $\mathcal{O}(1)$. Note that negative exponents α and γ are perfectly allowed and consistent in this cosmological context, where the background solution corresponds to a strictly positive value of X. Perturbations are thus well defined around such a background.

Particular self-tuning models respecting $c_{\rm grav} = c_{\rm light}$ are thus easily obtained from (21). However, it should be stressed that it is not enough to find a theory with $\Lambda_{\rm eff} \ll \Lambda_{\rm bare}$, since Newton's constant also generically gets renormalized, giving $(M_{\rm Pl}^2\Lambda)_{\rm eff} \sim (M_{\rm Pl}^2\Lambda)_{\rm bare}$ [37]. But, luckily, a subclass of models (21) is such that $M_{\rm Pl}$ remains unrenormalized, and it is thus possible to get $M_{\rm Pl}^2\Lambda_{\rm eff} \ll M_{\rm Pl}^2\Lambda_{\rm bare}$ by choosing an appropriate value of M. This subclass corresponds to the exponent $\gamma = -\frac{3}{2}$; i.e., $s_4 = \kappa_4 X^{-3/2}$ and $f_4 = -8\kappa_4 X^{-5/2}$. In terms of the G_i notation of Eq. (1), this reads

$$G_2(\varphi_\lambda^2) = -M_{\rm Pl}^2\Lambda_{\rm bare} - k_2 M^4\left(\frac{-\varphi_\lambda^2}{M^2}\right)^{\alpha+1}, \tag{22}$$

$$G_4(\varphi_\lambda^2) = \frac{1}{2}M_{\rm Pl}^2 - 2\kappa_4 M^3(-\varphi_\lambda^2)^{-1/2}, \tag{23}$$

while F_4 is given by Eq. (2). One then finds that the Schwarzschild–de Sitter equations of Ref. [37] can be solved provided $\alpha \neq -1$ and $\alpha \neq -\frac{1}{2}$, and they imply

$$(H^2)^{\alpha+1}(M^2)^{\alpha+2} \propto (M_{\rm Pl}^2\Lambda_{\rm bare})^{\alpha+3/2}. \tag{24}$$

The proportionality factor depends on the $\mathcal{O}(1)$-dimensionless constants k_2, κ_4, and α, and is thus itself of order $\mathcal{O}(1)$. Therefore, if $\alpha \neq -2$, it suffices to choose M appropriately to get H equal to the observed value, whatever the large $\Lambda_{\rm bare}$ entering the action. Note that *all* these models (with $\alpha \notin \{-2, -1, -\frac{1}{2}\}$ and $\gamma = -\frac{3}{2}$) do admit exact Schwarzschild–de Sitter solutions such that $M_{\rm Pl}^2\Lambda_{\rm eff}$ is consistent with its small observed value, and they also satisfy $c_{\rm grav} = c_{\rm light}$ at least in the asymptotic homogeneous universe.

As underlined at the very end of [37], if the bare cosmological constant happens to take the huge value $\Lambda_{\rm bare} \sim M_{\rm Pl}^2$, then the particular case $\alpha = -\frac{5}{4}$ needs a rather natural value of the scale $M \sim 100$ MeV, similar to usual elementary particle masses.

Another interesting particular case is $\alpha = -\frac{3}{2}$, for instance, $f_2 = s_4 = -X^{-3/2}$ and $f_4 = 4X^{-5/2}$ (choosing here $k_2 = \kappa_4 = -1$ to simplify, the signs being imposed by the field equations). This corresponds to

$$G_2(\varphi_\lambda^2) = -M_{\rm Pl}^2\Lambda_{\rm bare} + M^5(-\varphi_\lambda^2)^{-1/2}, \tag{25}$$

$$G_4(\varphi_\lambda^2) = \frac{1}{2}M_{\rm Pl}^2 + 2M^3(-\varphi_\lambda^2)^{-1/2}, \tag{26}$$

$$F_4(\varphi_\lambda^2) = 2M^3(-\varphi_\lambda^2)^{-5/2}. \tag{27}$$

Then the exact version of Eq. (24) implies that one must choose $M = 2\sqrt{3}H$; i.e., the very small observed Hubble expansion rate H must actually be put by hand in the action via the scale M. But this drawback comes with the great bonus that this observed H now depends only on M and no longer on the bare vacuum energy density $M_{\rm Pl}^2\Lambda_{\rm bare}$. Therefore, even if $\Lambda_{\rm bare}$ happens to change because of a phase transition during the cosmological evolution of the Universe, the effective $\Lambda_{\rm eff} = (M/2)^2 = 3H^2$ remains constant and small.

The important conclusion is that elegant self-tuning cosmological models are still allowed, even when taking into account the experimental constraint $c_{\rm grav} = c_{\rm light}$. Note that, for these models, we did not prove that the speed equality remains valid in the vicinity of massive bodies. However, our result above for the simple model (3) and the argument of Refs. [3,4,41] show that it may remain true, at least for Schwarzschild-de Sitter black hole solutions. The stability of these self-tuning models should also be analyzed, as we did above for model (3). Aside of this, it would be of great interest to study a more realistic cosmological evolution for these self-tuning models, as in [36], where

certain branches of solutions were shown to screen matter as well as the cosmological constant.

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