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STOCHASTIC DIFFERENTIAL EQUATION FOR MODELING GLOBAL SOLAR RADIATION SEQUENCES

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ABSTRACT

A potential function estimated from solar radiation measurement data is combined with a stochastic differential equation and used for simulating short sequences of solar radiation fluctuations.

This potential function depends on observed meteorological conditions.

This work is a first step in reconstructing a daily solar radiation signal.

KEY WORDS

Solar irradiation model; potential function, stochastic differential equation, solar irradiation prediction.

1. Introduction

Under tropical climate, solar radiation is a very fluctuating data, notably because of numerous clouds. Fast changes in the local meteorological conditions, as observed in tropical climate can provoke large variation of solar radiation. The amplitude of these variations can reach 700 W/m² and occur within a short time interval, from few seconds to few minutes according to the geographical location. These variations depend on the clouds size, speed and number.

Therefore in power distribution grids with high density of PV, rapid fluctuations of the produced electrical power can appear, leading to unpredictable variations of node voltage and power in electric networks. In small grids as they exist on islands (such as in Guadeloupe, FWI) such fluctuations can cause instabilities.

Hence management of the electrical network and of the alternative power sources requires a better identification of these small time scales variations. This stresses the need for power system operators to develop real time estimation tools for such disturbances in order to anticipate power shortages and power surges.

Climatic variable, such as solar radiation, can be decomposed into two components, a deterministic one and a stochastic one. The deterministic component can be easily calculated, since it is derived from the earth revolution. The stochastic component comprises fluctuations about the deterministic component. This has led to the development of stochastic model.

More generally nonlinear complex systems can be represented by Stochastic Differential Equations (SDE). Initially used for trajectory or path simulation they are now used in a very large area field: electrical circuit (Friedrich 2000,[1]), elephant seals (Brilinger 2010, [2]), social sciences (Cob, [5])...

We have applied this powerful tool in the simulation of solar radiation measured in Guadeloupe (FWI).

Our global solar radiations measurements were performed in Guadeloupe, an island in the West Indies which is located at 16°15 N latitude and 60° 30 W longitudes. In such a tropical zone, solar radiation is an important climatic data to be taken into account as the daily average for the solar load for a horizontal surface is around 5 kWh/m². A constant sunshine combined with the thermal inertia of the ocean makes the air temperature variation quite weak, between 17°C and 33°C with an average of 25°C to 26°C. Relative humidity ranges from 70% to 80% and the trade winds are relatively constant all along the year.

Our measurements sampled at 1 Hz are performed since 2005. The measurements made with two pyranometers from KIPP&ZONEN were recorded by a CAMPBELL SCIENTIFIC data logger.

On Figure 1 below, is presented an example of measurement for one day.

2. Stochastic Gradient system

In this work we use of stochastic gradient systems in the modeling of solar radiation time series, estimating a potential function from experimental data sampled every second.

A stochastic gradient system G (Brillinger et al. (2001); Cobb et Watson (1981)) is a solution of the following stochastic differential equation:

$$dG(t) = -\nabla V(G(t))dt + \sigma G(t)dB(t) \quad (1)$$

The function V is called a potential function can be estimated from real data. Here $G(t)$ is the global solar radiation at time t , B denotes a standard Brownian motion and σ a positive diffusion factor.

The integration of this equation will allow generating sequences of solar radiation signals. We first need to estimate the potential function V .

2.1 Estimation of the potential function

In his paper “learning a potential from a trajectory”, Brillinger [3] presents a new estimation method to determine potential functions from experimental data. We apply this method to solar radiation signals. General forms may be considered for the potential. Suppose

$$V(r) = \varphi(G)^T \beta \quad (2)$$

where φ is an L by 1 vector of known functions and β an L by 1 unknown parameter. The gradient of V is the p by 1 vector $\nabla \varphi(G)^T \beta$.

Equation (1) can be approximated by:

$$y_{i+1} = -\varphi(G_i)^T \beta \sqrt{\bar{r}_{i+1} - \bar{r}_i} + \sigma Z_{i+1}, \quad i = 1, \dots, n \quad (3)$$

$$\text{with } y_{i+1} = \frac{\sigma_{i+1} - \sigma_i}{\sqrt{\bar{r}_{i+1} - \bar{r}_i}}$$

Stack the n values $y_{i+1}, i=1, \dots, n$ vertically to vector Y_n . Stack the n matrices $-\nabla \varphi(G_i)^T \sqrt{\bar{r}_{i+1} - \bar{r}_i}$ to form the matrix X_n . Let x_i denote the i -th row of X_n so $X_n^T X_n = \sum x_i x_i^T$. Stack the n values σZ_{i+1} to form ε_n .

Then one has the regression model $Y_n = X_n \beta + \varepsilon_n$ with the difference from ordinary regression that X_n is random.

Supposing the matrix $X_n^T X_n$ to be non singular one can compute the ordinary least squares estimate $b = (X_n^T X_n)^{-1} X_n^T Y_n$ of β and then $\varphi(G)^T b$ as an estimate of $V(G)$.

3. Application to solar radiation regimes

Solar radiation is a very fluctuating data, especially under tropical climate. This is shown on figure 1. Within a same day we can encounter different regimes of solar radiation: regime of cloudy sky, regime of clear sky and a high frequency fluctuations regime.

3.1 Estimation of a potential function for each solar radiation regime

At this stage of the study we have arbitrarily selected a regime of solar radiation. For each identified regime we will apply the estimation algorithm to determine the potential function derived from the experimental data. Once the potential function is obtained we use equation (1) to simulate a time series of solar radiation signal for a given regime.

We have identified three main regimes [6]:

1. High frequency fluctuation regimes:

These sequences of solar radiation are characterized by rapid fluctuations occurring within a few seconds (from 10 to 60 seconds). These sequences correspond to meteorological conditions with “clouds train”.

On figure 2 is shown an example of such sequence of 600 s lengths (10 minutes). On this short time period we have about 8 clouds.

2. Clear sky regimes:

These sequences are characteristic of clear sky conditions,

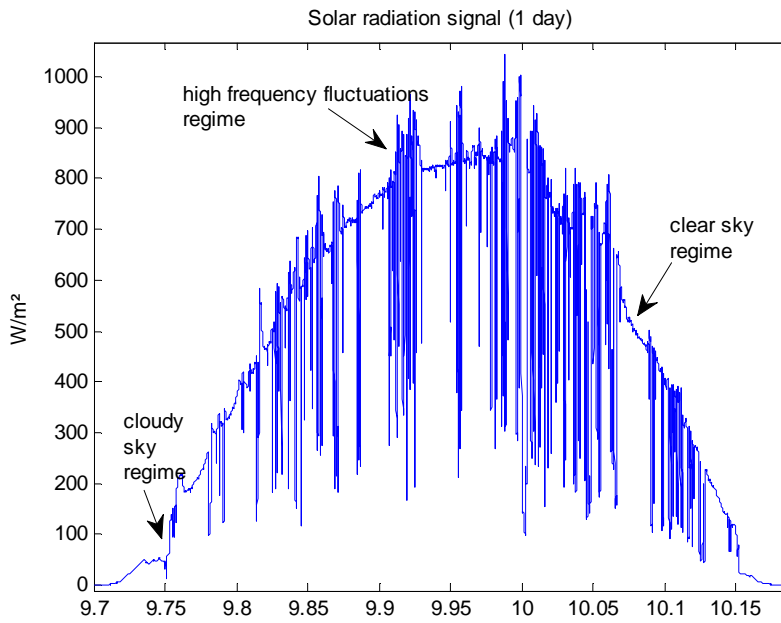


Figure 1. Example of solar radiation signal (1 day) $\times 10^5$

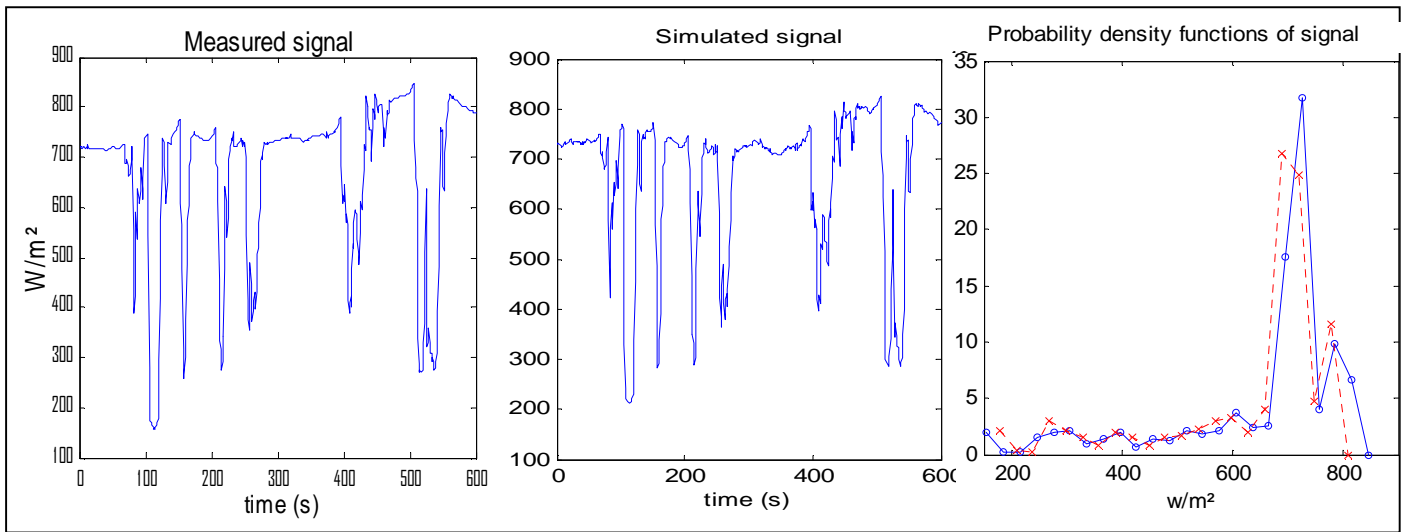


figure 2: a) measured sequence of high fluctuations b) simulated signal c) comparison of pdf of both signals

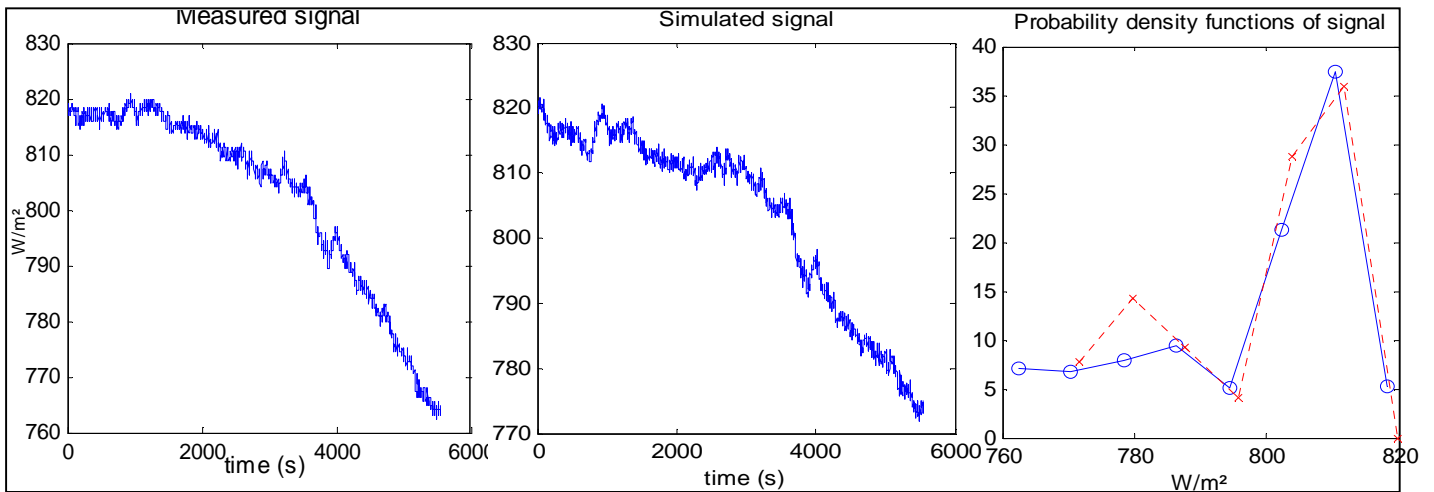


figure 3: a) measured clear sky sequence b) simulated signal c) comparison of pdf of both signals

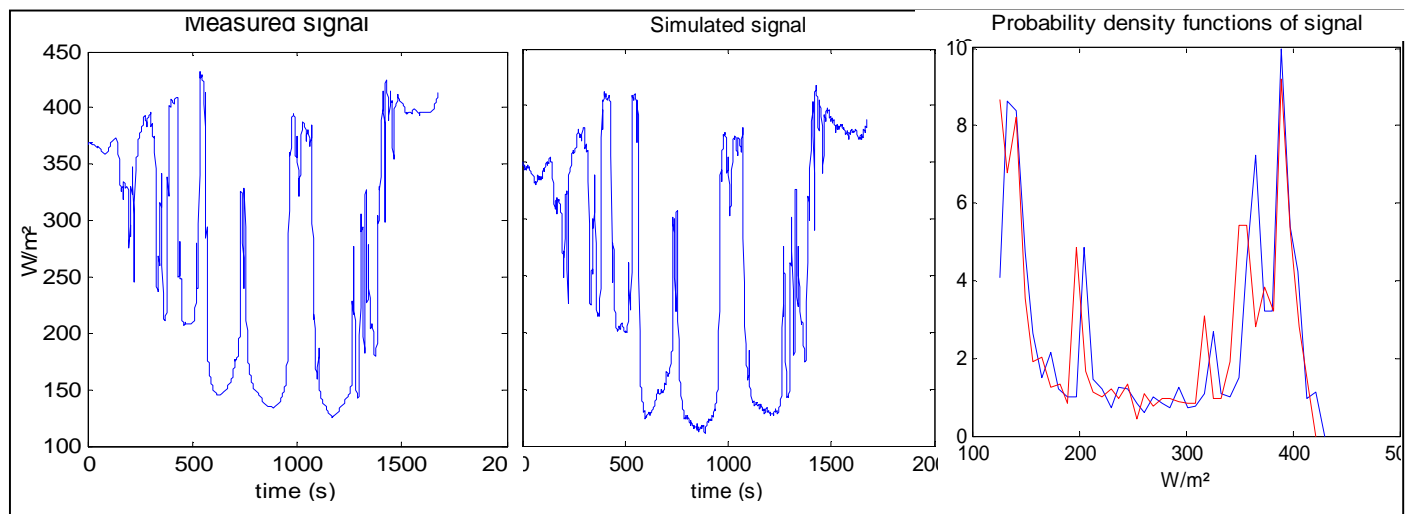


figure 4: a) measured clear sky sequence b) simulated signal c) comparison of pdf of both signals

there are no fluctuations due to clouds passage and the variation observed is only due to solar path during the day.

On the example shown on figure 3 we have a 6000 s length signal (about 1h40 minutes) of clear sky.

1. Cloudy sky regimes:

The cloudy sky regime is due to “big” clouds passage s occurring on time from one minute to 10 or more minutes. This regime is characterized by low dynamic fluctuations On figure 4 is shown a sequence of 1800 s (30 minutes).

4. Simulation results

The simulation results are shown in figures 2, 3, 4 for the fluctuations regime, clear sky regime and cloudy sky regime respectively.

For each sequence we have plotted the experimental sequence, the simulated signal and a comparison of the PDF of both signals. To complete this PDF comparison a Kolmogorov-Smirnov Test was completed.

As shown on the figures, the K-S test shows that PDF are from the same continuous distribution.

High fluctuations regime :

p- value = 0.971
k = 0.0746

Clear sky regime :

p- value = 0.472
k = 0.400

Cloudy sky regime :

p- value = 0.939
k = 0.1174

To achieve such a result the initial value of the simulated sequence of G was taken as equal to the initial value of the observed sequence. The diffusion coefficient σ in equation (1) was estimated so as to minimize the mean difference between the observed and the simulated sequence.

5. Conclusion

Our work is a first contribution in the stochastic modeling of solar radiation sequences sampled every second. We used a stochastic gradient system that is a stochastic differential equation based on a potential function estimated from measured data. The application of this method to solar radiation domain seems to be new.

The obtained results indicate that this method is suitable for simulation in various regimes of solar radiation signals.

Some improvements are still needed and we think that the method should provide some nice perspectives for further works dedicated to solar radiation prediction on short time scale.

References

- [1] R. Friedrich, S. Siegert, J. Peinke, St. Luck , M. Siefert, M. Lindemann, J. Raethjen, G. Deuschl, G. Pfister, Extracting model equations from experimental data, *Physics Letters A* 271 (2000) 217–222
- [2] David R.Brillinger , Brent.Stewart, Stochastic modeling of particle movement with application to marine biology and oceanography, *Journal of Statistical Planning and Inference*, 140 (2010) 3597–3607.
- [3] Brillinger, D.R., Stewart, B.S., 1998. Elephant seal movements: modelling migration. *Can. J. Stat.* 26, 431–443.
- [4] Brillinger, D.R., Stewart, B., Littnan, C., 2008. Three months journeying of a Hawaiian monk seal. *Probability and statistics: essays in honor of David A. Freedman*. Institute of Mathematical Statistics Collections. 2, pp. 246–264.
- [5] Cob, L, Stochastic differential equation for social sciences, 1981, *Mathematical frontiers of the social and policy sciences*.
- [6] T. Soubdhan, R. Emilion, R. Calif, Classification of daily solar radiation distributions using a mixture of Dirichlet distributions, *Solar Energy Journal*, 83 (7), pp. 1056-1063, 2009