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## Lagrangian scheme for scalar advection

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## PROBLEM

Solve advection–diffusion equation, for low diffusivity ( $\Gamma$ ) or high Péclet numbers values. **Applications:** pollutant transport, two phase-flow, ...

## SPLITTING APPROACH

$$\frac{\Phi^* - \Phi^n}{\Delta t} + \nabla \cdot (\mathbf{u}^n \Phi^n) = 0 \Rightarrow \text{QUICK, MUSCL, WENO and VSM [1] schemes}$$

$$\frac{\Phi^{n+1} - \Phi^*}{\Delta t} = \nabla \cdot (\Gamma \nabla \Phi^{n+1}) \Rightarrow \text{centered scheme, implicit, direct solver (MUMPS)}$$

## LAGRANGIAN SCHEME

1.  $M$  markers (Lagrangian particles) of positions  $\mathbf{X}_m$  and volumes  $\delta V_m$  carry the local information  $\phi_m$  of the Eulerian field  $\Phi$ . At initial time,  $\phi_m^0 = \Phi^0(\mathbf{X}_m)$ ,  $m = 1, \dots, M$ .
2. The markers are advected with the fluid velocity:  $\frac{d\mathbf{X}_m^{n+1}}{dt} = \mathbf{u}^n$ ,  $m = 1, \dots, M$ .
3. Post advection value of  $\Phi$  is evaluated with averages on each Eulerian cell  $\Omega_{i,j}$ .

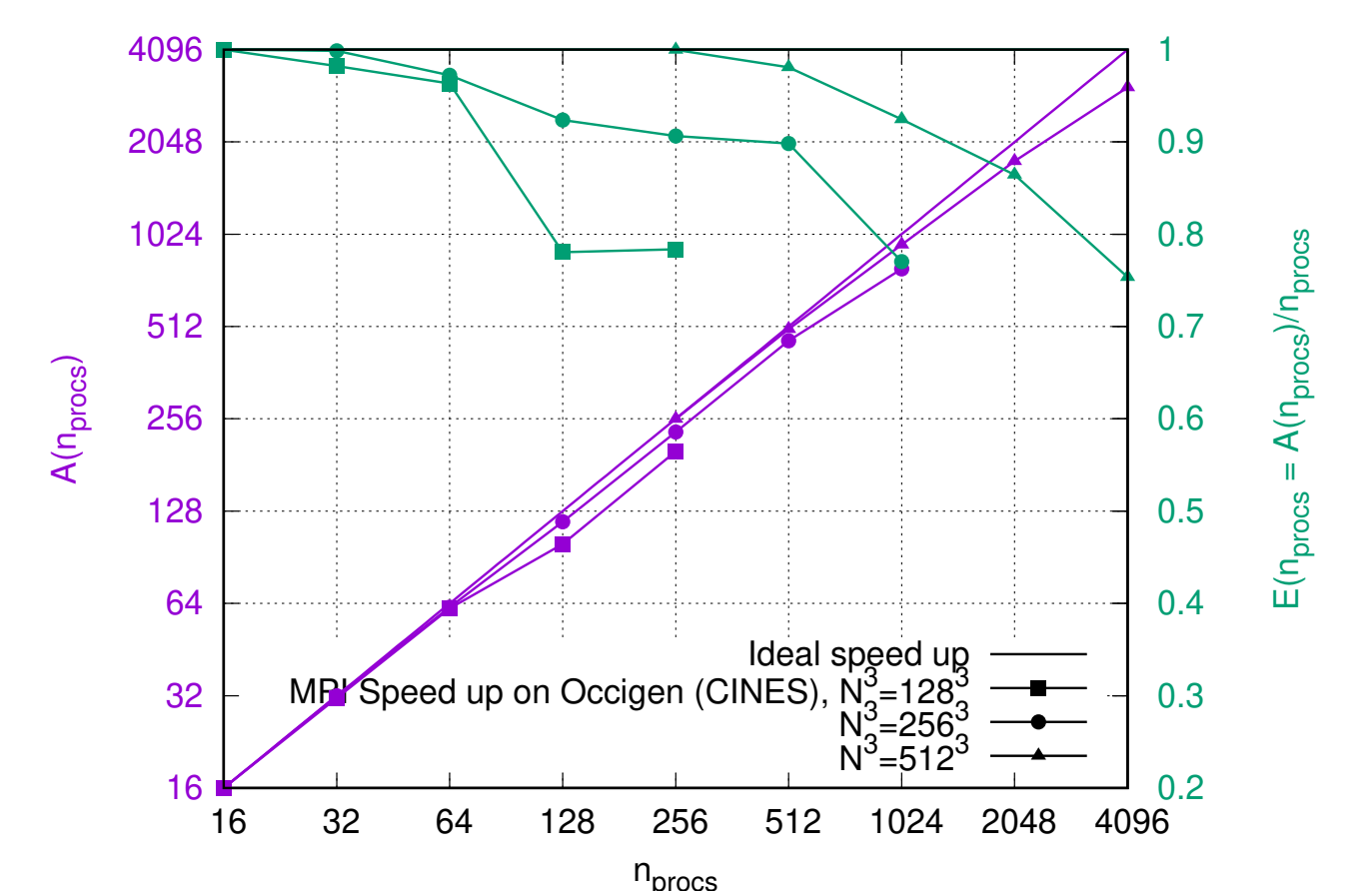
$$\Phi_{i,j}^* = \frac{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \phi_m^n \delta V_m'}{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \delta V_m'} \quad \text{with} \quad \delta V_m' = \delta V_m \cap \Omega_{i,j}$$

4.  $\Phi^{n+1}$  is then obtained solving the unsteady diffusion equation.
5. The local (Lagrangian) information is updated according the variation of  $\Phi$  at the particle position:  $\frac{\partial \phi_m}{\partial t} \Big|_{\mathbf{X}_m^{n+1}} = \frac{\partial \Phi}{\partial t} \Big|_{\mathbf{X}_m^{n+1}}$ . For a first order integration scheme,

$$\phi_m^{n+1} = \phi_m^n + \Phi^{n+1}(\mathbf{X}_m^{n+1}) - \Phi^*(\mathbf{X}_m^{n+1}), \quad m = 1, \dots, M.$$

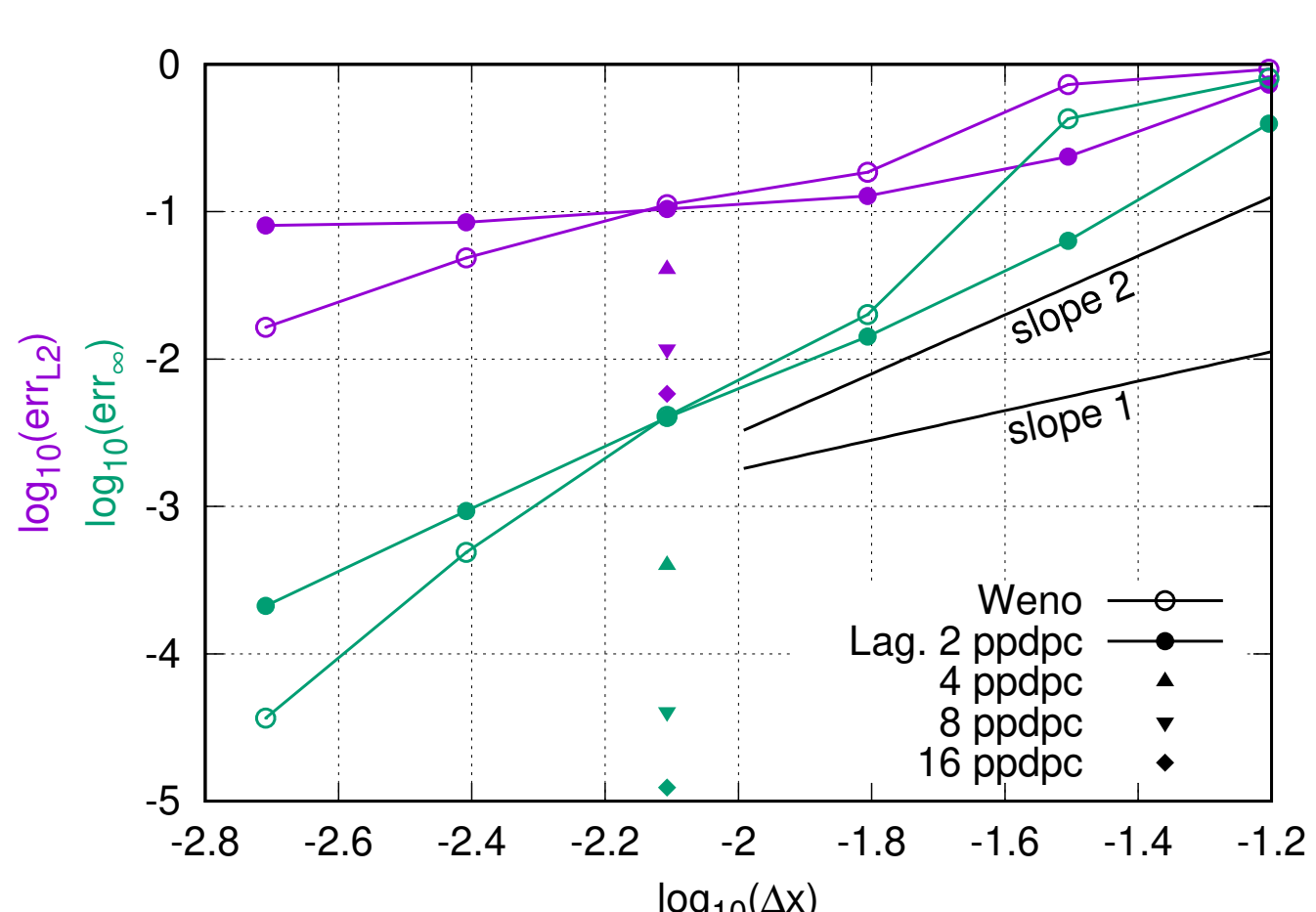
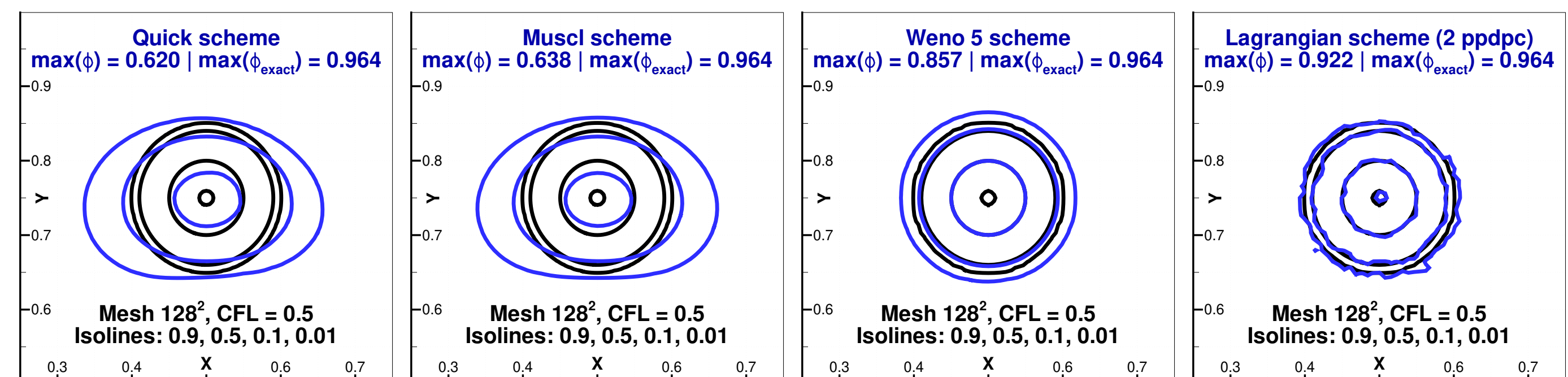
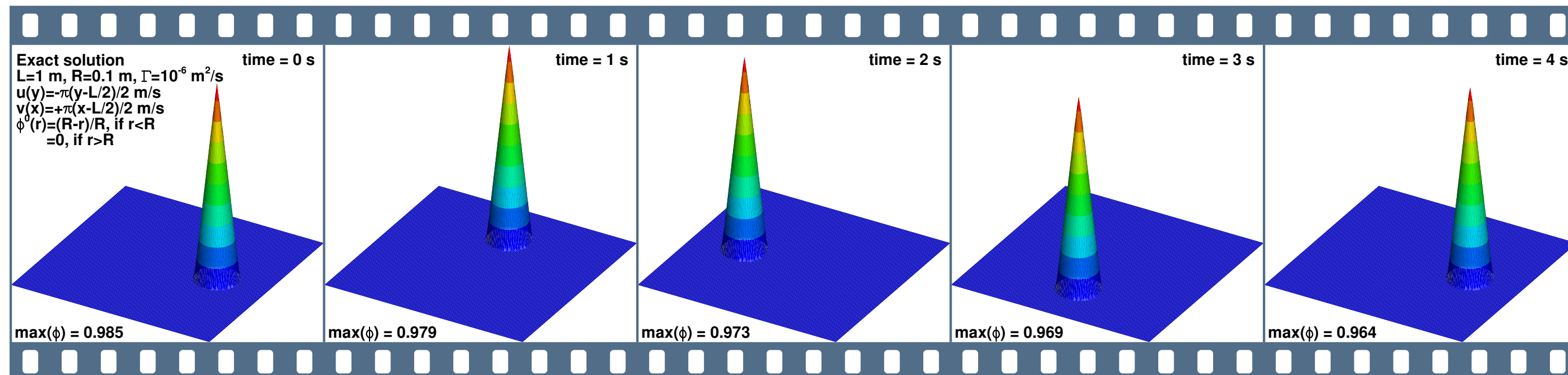
## CODE DESCRIPTION

- Finite-Volumes on staggered grids.
- Augmented Lagrangian or KSP [2] for Pressure / Velocity coupling.
- VOF, Level Set, Front Traking.
- Lagrangian particle tracking.
- DNS and LES turbulence modelling.
- Penalty methods.



## RESULTS

### Advection/Diffusion of a pollutant peak

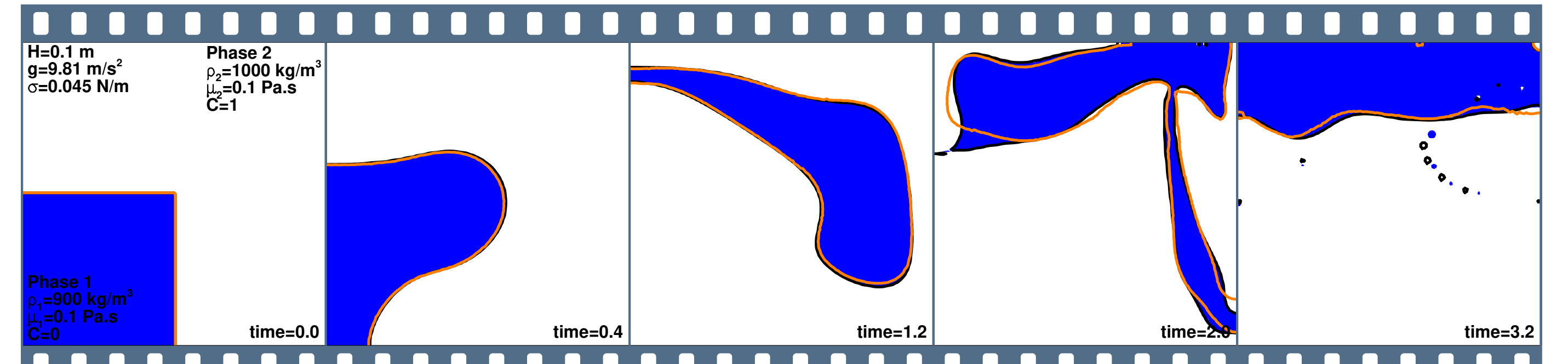


- Same conclusion for pure transport ( $\Gamma = 0$ ).
- Oscillations are reduced increasing  $M$  (ppdpc).

### Computational cost in % of the Ref. (Weno, 512<sup>2</sup> (13h))

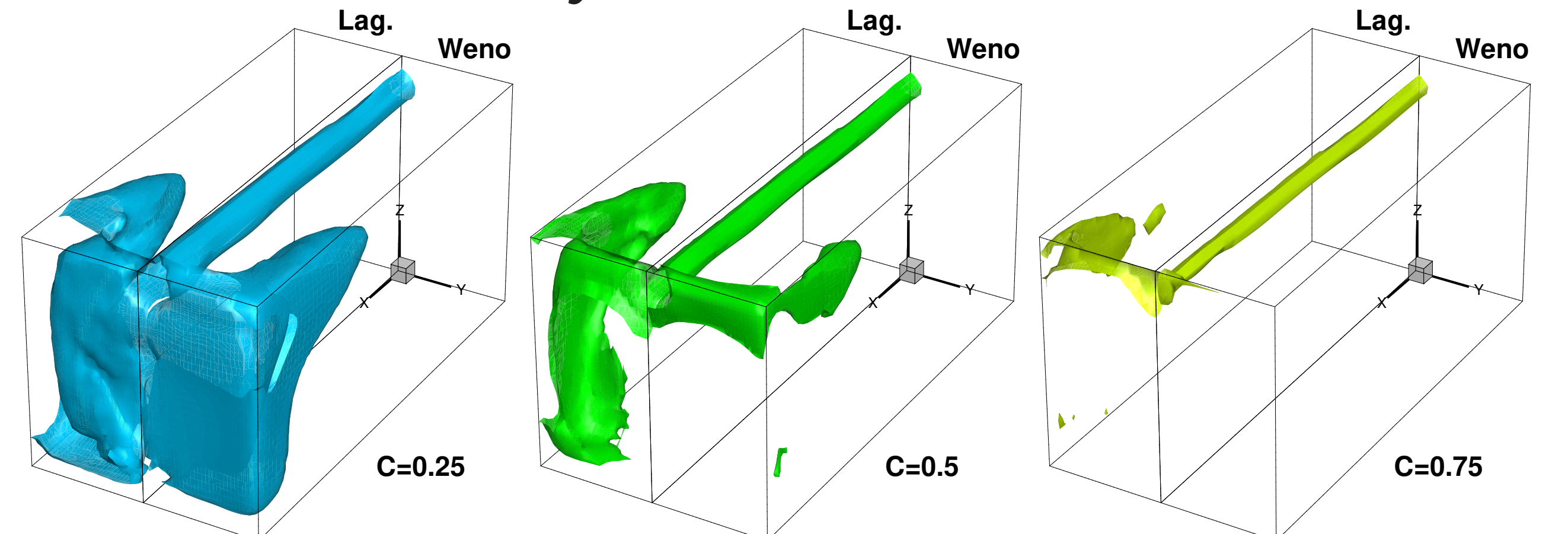
Weno, 128 <sup>2</sup>	1.2%	Lag., 128 <sup>2</sup> , 2–4 pddpc	~ 2%
Weno, 256 <sup>2</sup>	19.2%	Lag., 128 <sup>2</sup> , 8 pddpc	6.1%
Weno, 512 <sup>2</sup>	<b>Ref.</b>	Lag., 128 <sup>2</sup> , 16 pddpc	48%

### Phase inversion problem with KSP method [2]



Snapshots of the color function over time. Mesh 128<sup>2</sup>. **Blue** is KSP, VOF-PLIC, **black line** is  $C = 0.5$  for the **Augmented Lagrangian**, VOF-PLIC method and **orange** line stands for the **Lagrangian advection scheme** with 4 pddpc and KSP.

### Ventilated cavity (collaboration with INRS)



Simulation of air flow at  $Re = 1500$  (injection) in a cuboid cavity. Air flow carries a tracer gas with air properties ( $\Gamma \approx 10^{-5} \text{ m}^2/\text{s}$ ). Iso-surfaces 0.25, 0.5 and 0.75 of concentration  $C$  for Lagrangian scheme with 4 pddpc (left) and Weno 5 scheme (right) are plotted for  $t = 1.5$  second. The non diffusive character of the Lagrangian scheme is observed. (Mesh:  $64 \times 32 \times 32$ ,  $\Delta t = 5 \times 10^{-3} \text{ s}$ .)

## ACKNOWLEDGMENT

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- [1] S. Vincent et al. Eulerian-Lagrangian multiscale methods for solving scalar equations. Application to incompressible two-phase flows. In *Journal of Computational Physics* (2010)
- [2] J.-P. Caltagirone & S. Vincent. A Kinematic Scalar Projection method (KSP) for incompressible flows with variable density. In *Open Journal of Fluid Dynamics* (2015)