



**HAL**  
open science

## Lagrangian scheme for scalar advection

Benoît Trouette, Georges Halim Atallah, Stéphane Vincent

► **To cite this version:**

Benoît Trouette, Georges Halim Atallah, Stéphane Vincent. Lagrangian scheme for scalar advection. Turbulence and Interactions TI2018, Jun 2018, Trois Ilets, France. hal-01821098

**HAL Id: hal-01821098**

**<https://hal.science/hal-01821098v1>**

Submitted on 22 Jun 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## PROBLEM

Solve advection–diffusion equation, for low diffusivity ( $\Gamma$ ) or high Péclet numbers values. **Applications:** pollutant transport, two phase-flow, ...

## SPLITTING APPROACH

$$\frac{\Phi^* - \Phi^n}{\Delta t} + \nabla \cdot (\mathbf{u}^n \Phi^n) = 0 \Rightarrow \text{QUICK, MUSCL, WENO and VSM [1] schemes}$$

$$\frac{\Phi^{n+1} - \Phi^*}{\Delta t} = \nabla \cdot (\Gamma \nabla \Phi^{n+1}) \Rightarrow \text{centered scheme, implicit, direct solver (MUMPS)}$$

## LAGRANGIAN SCHEME

1.  $M$  markers (Lagrangian particles) of positions  $\mathbf{X}_m$  and volumes  $\delta V_m$  carry the local information  $\phi_m$  of the Eulerian field  $\Phi$ . At initial time,  $\phi_m^0 = \Phi^0(\mathbf{X}_m)$ ,  $m = 1, \dots, M$ .
2. The markers are advected with the fluid velocity:  $\frac{d\mathbf{X}_m^{n+1}}{dt} = \mathbf{u}^n$ ,  $m = 1, \dots, M$ .
3. Post advection value of  $\Phi$  is evaluated with averages on each Eulerian cell  $\Omega_{i,j}$ .

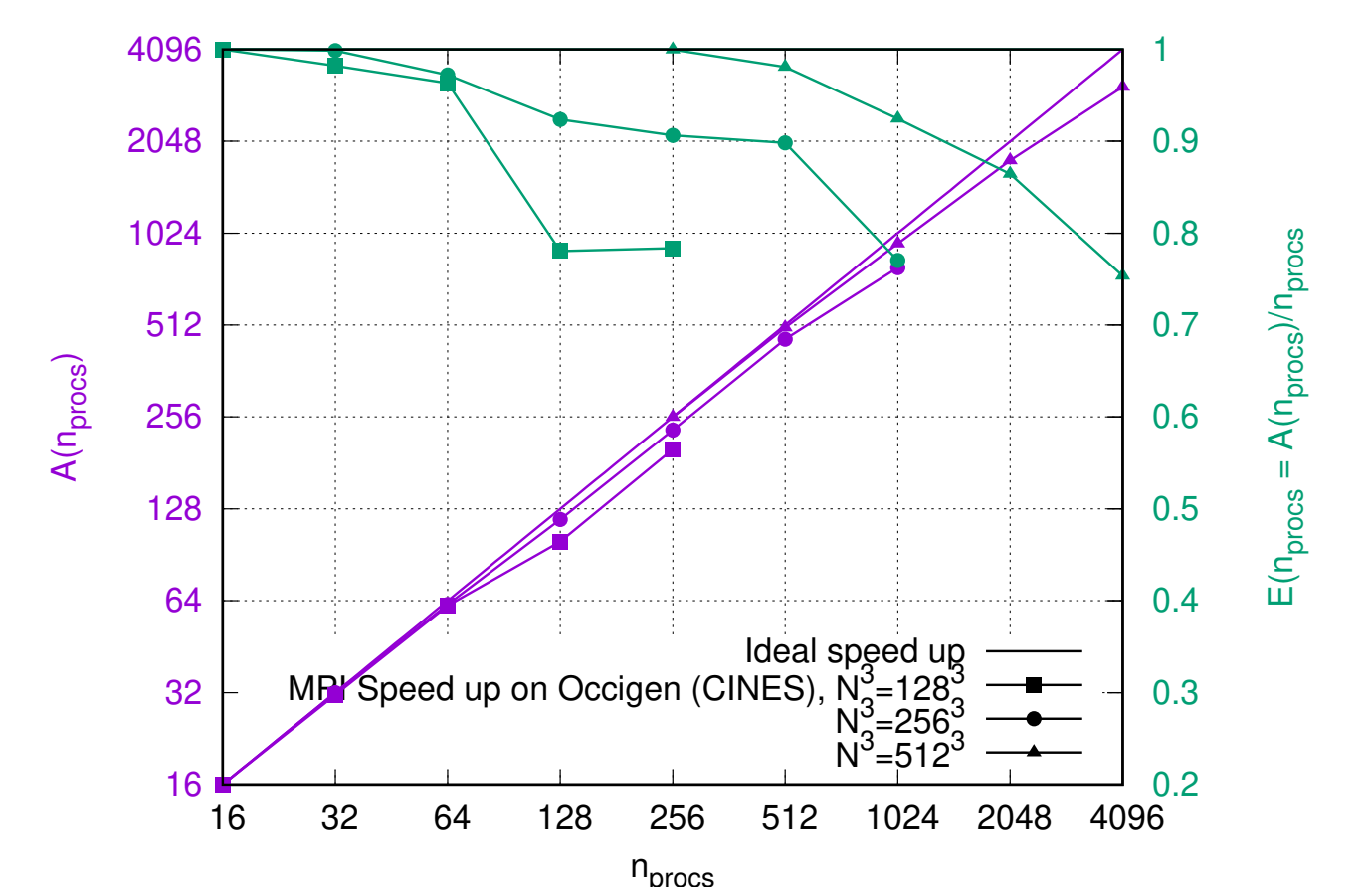
$$\Phi_{i,j}^* = \frac{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \phi_m^n \delta V_m'}{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \delta V_m'} \quad \text{with} \quad \delta V_m' = \delta V_m \cap \Omega_{i,j}$$

4.  $\Phi^{n+1}$  is then obtained solving the unsteady diffusion equation.
5. The local (Lagrangian) information is updated according the variation of  $\Phi$  at the particle position:  $\frac{\partial \phi_m}{\partial t} \Big|_{\mathbf{X}_m^{n+1}} = \frac{\partial \Phi}{\partial t} \Big|_{\mathbf{X}_m^{n+1}}$ . For a first order integration scheme,

$$\phi_m^{n+1} = \phi_m^n + \Phi^{n+1}(\mathbf{X}_m^{n+1}) - \Phi^*(\mathbf{X}_m^{n+1}), \quad m = 1, \dots, M.$$

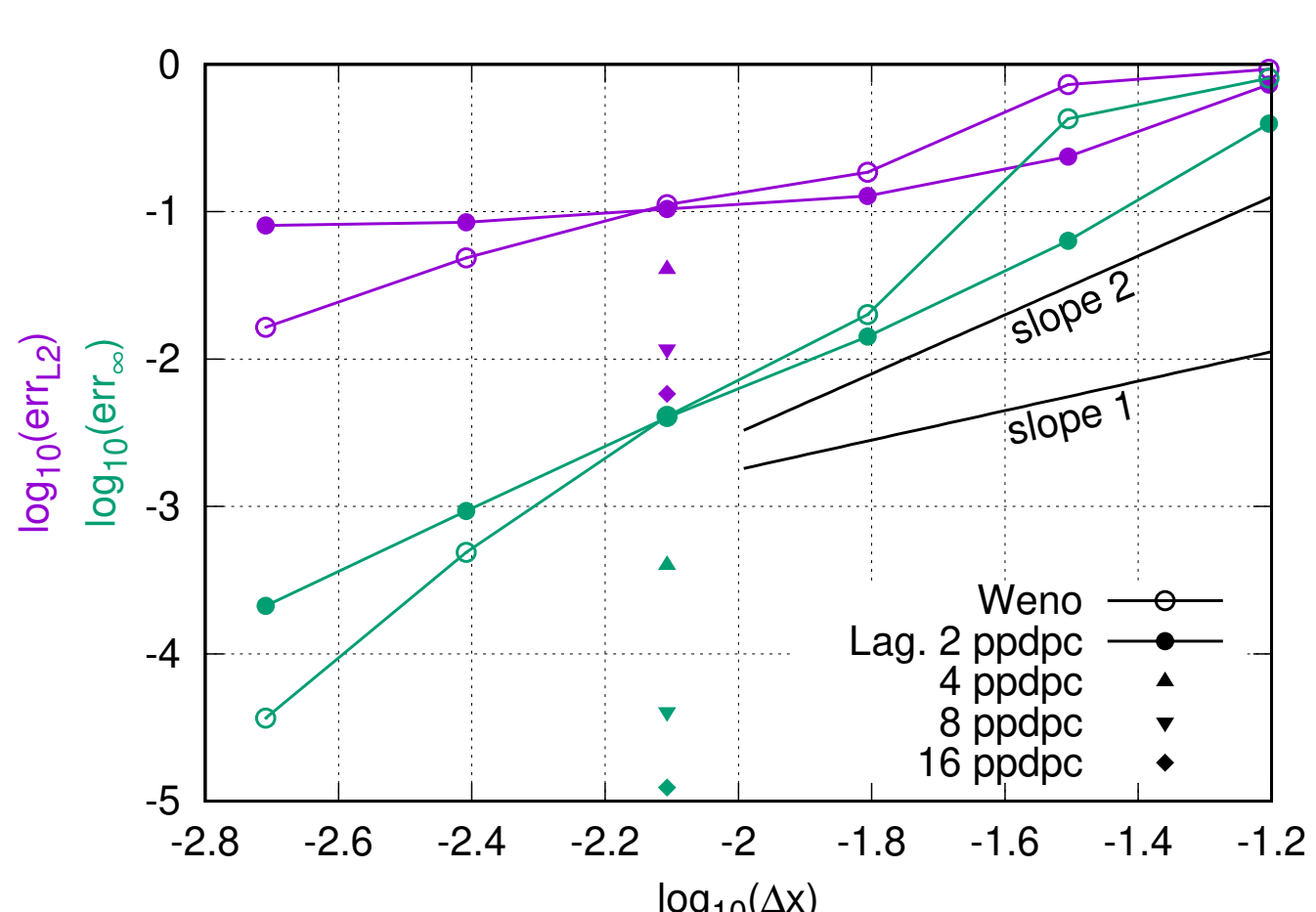
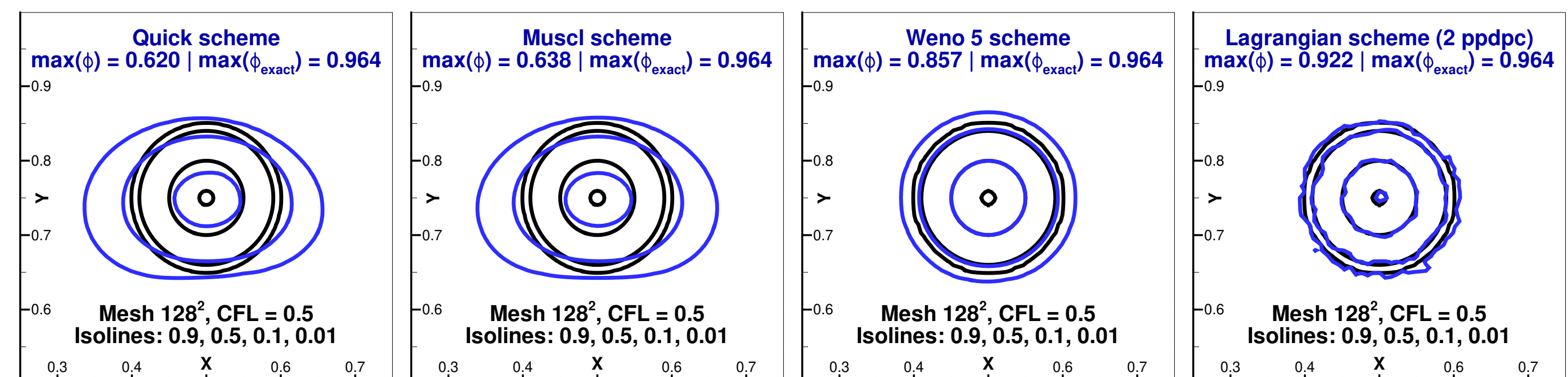
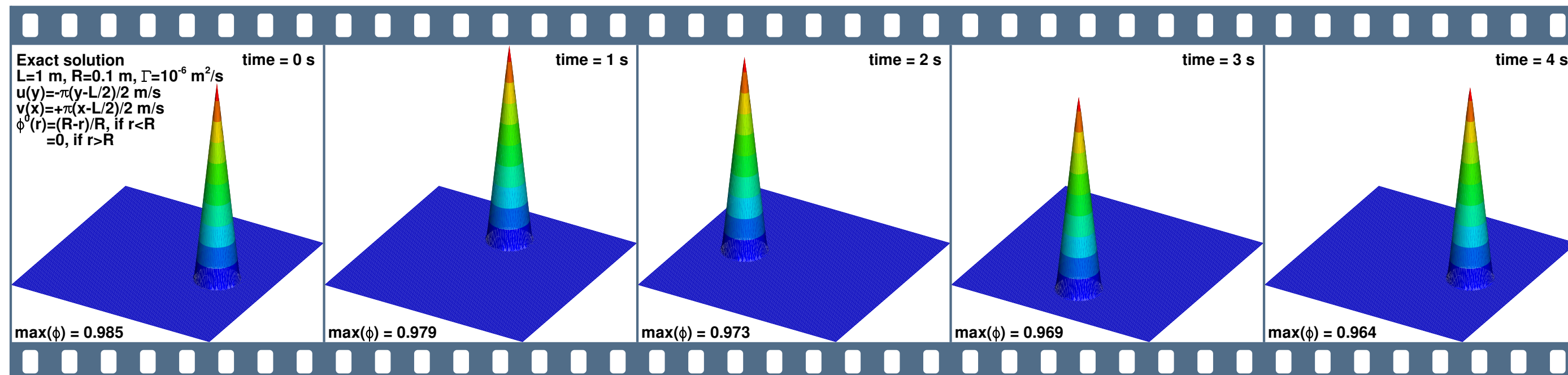
## CODE DESCRIPTION

- Finite-Volumes on staggered grids.
- Augmented Lagrangian or KSP [2] for Pressure / Velocity coupling.
- VOF, Level Set, Front Traking.
- Lagrangian particle tracking.
- DNS and LES turbulence modelling.
- Penalty methods.



## RESULTS

### Advection/Diffusion of a pollutant peak

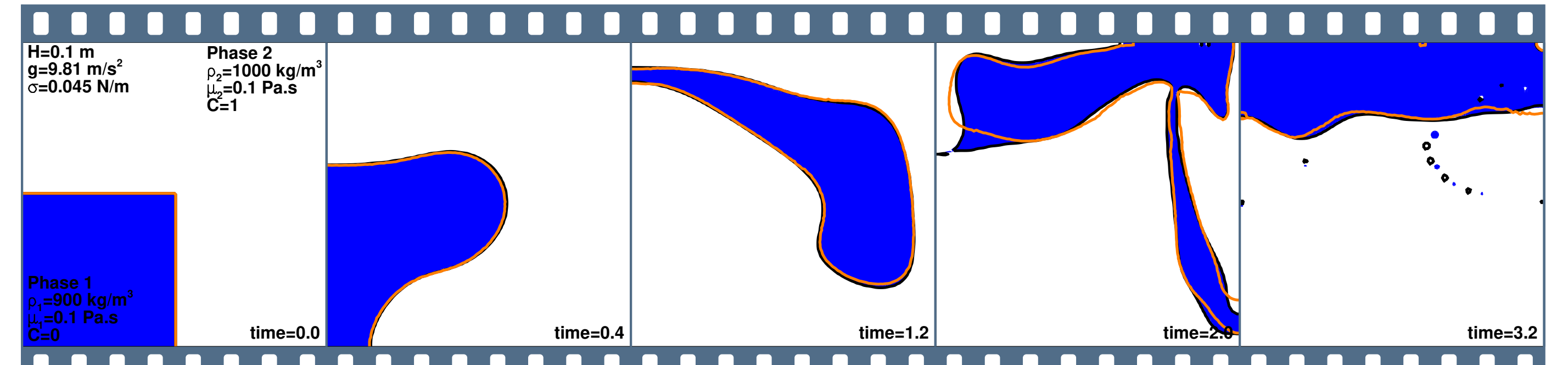


- Same conclusion for pure transport ( $\Gamma = 0$ ).
- Oscillations are reduced increasing  $M$  (ppdpc).

### Computational cost in % of the Ref. (Weno, 512<sup>2</sup> (13h))

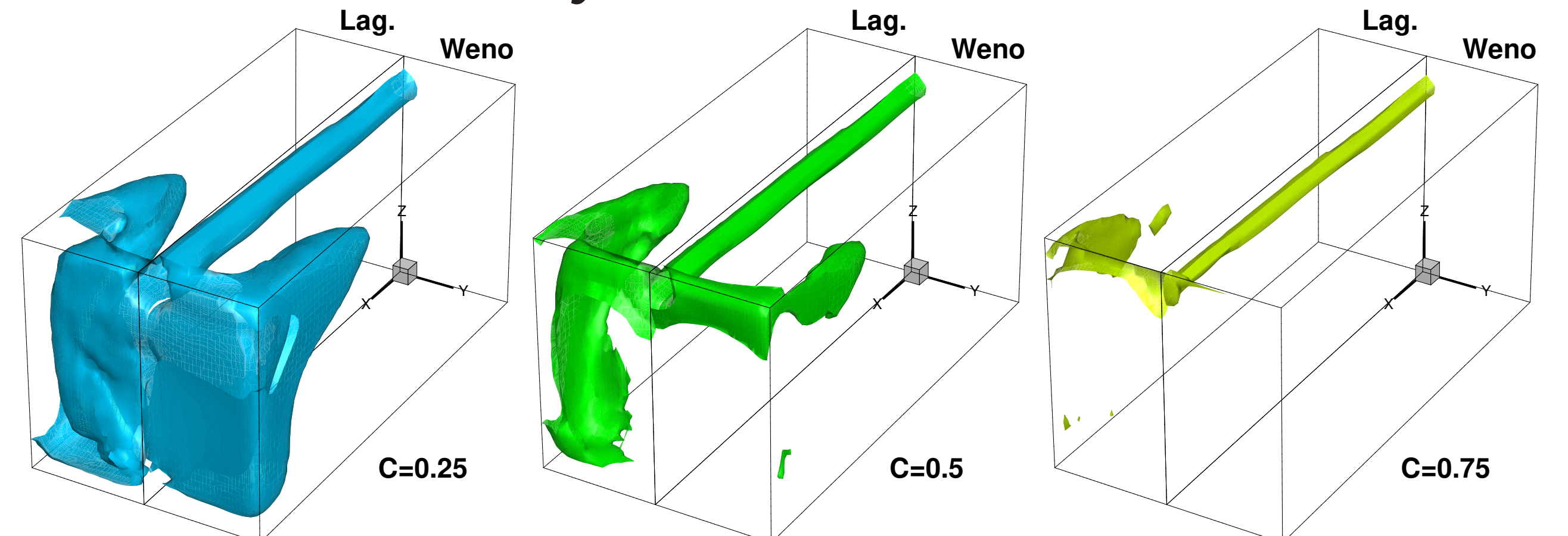
Weno, 128 <sup>2</sup>	1.2%	Lag., 128 <sup>2</sup> , 2–4 pddpc	~ 2%
Weno, 256 <sup>2</sup>	19.2%	Lag., 128 <sup>2</sup> , 8 pddpc	6.1%
Weno, 512 <sup>2</sup>	<b>Ref.</b>	Lag., 128 <sup>2</sup> , 16 pddpc	48%

### Phase inversion problem with KSP method [2]



Snapshots of the color function over time. Mesh 128<sup>2</sup>. **Blue** is KSP, VOF-PLIC, black line is  $C = 0.5$  for the **Augmented Lagrangian**, VOF-PLIC method and **orange** line stands for the **Lagrangian advection scheme** with 4 pddpc and KSP.

### Ventilated cavity (collaboration with INRS)



Simulation of air flow at  $Re = 1500$  (injection) in a cuboid cavity. Air flow carries a tracer gas with air properties ( $\Gamma \approx 10^{-5} \text{ m}^2/\text{s}$ ). Iso-surfaces 0.25, 0.5 and 0.75 of concentration  $C$  for Lagrangian scheme with 4 pddpc (left) and Weno 5 scheme (right) are plotted for  $t = 1.5$  second. The non diffusive character of the Lagrangian scheme is observed. (Mesh:  $64 \times 32 \times 32$ ,  $\Delta t = 5 \times 10^{-3}$  s.)

## ACKNOWLEDGMENT

The authors are grateful for the computational facilities of GENCI under project n°A0032B06115 and to M. El Ouafa, M. Mbaye and to E. Belut & S. Lechêne (INRS).

## REFERENCES

- [1] S. Vincent et al. Eulerian-Lagrangian multiscale methods for solving scalar equations. Application to incompressible two-phase flows. In *Journal of Computational Physics* (2010)
- [2] J.-P. Caltagirone & S. Vincent. A Kinematic Scalar Projection method (KSP) for incompressible flows with variable density. In *Open Journal of Fluid Dynamics* (2015)