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Lagrangian scheme for scalar advection

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PROBLEM

Solve advection–diffusion equation, for low diffusivity (Γ) or high Péclet numbers values. **Applications:** pollutant transport, two phase-flow, ...

SPLITTING APPROACH

$$\frac{\Phi^* - \Phi^n}{\Delta t} + \nabla \cdot (\mathbf{u}^n \Phi^n) = 0 \Rightarrow \text{QUICK, MUSCL, WENO and VSM [1] schemes}$$

$$\frac{\Phi^{n+1} - \Phi^*}{\Delta t} = \nabla \cdot (\Gamma \nabla \Phi^{n+1}) \Rightarrow \text{centered scheme, implicit, direct solver (MUMPS)}$$

LAGRANGIAN SCHEME

1. M markers (Lagrangian particles) of positions \mathbf{X}_m and volumes δV_m carry the local information ϕ_m of the Eulerian field Φ . At initial time, $\phi_m^0 = \Phi^0(\mathbf{X}_m)$, $m = 1, \dots, M$.
2. The markers are advected with the fluid velocity: $\frac{d\mathbf{X}_m^{n+1}}{dt} = \mathbf{u}^n$, $m = 1, \dots, M$.
3. Post advection value of Φ is evaluated with averages on each Eulerian cell $\Omega_{i,j}$.

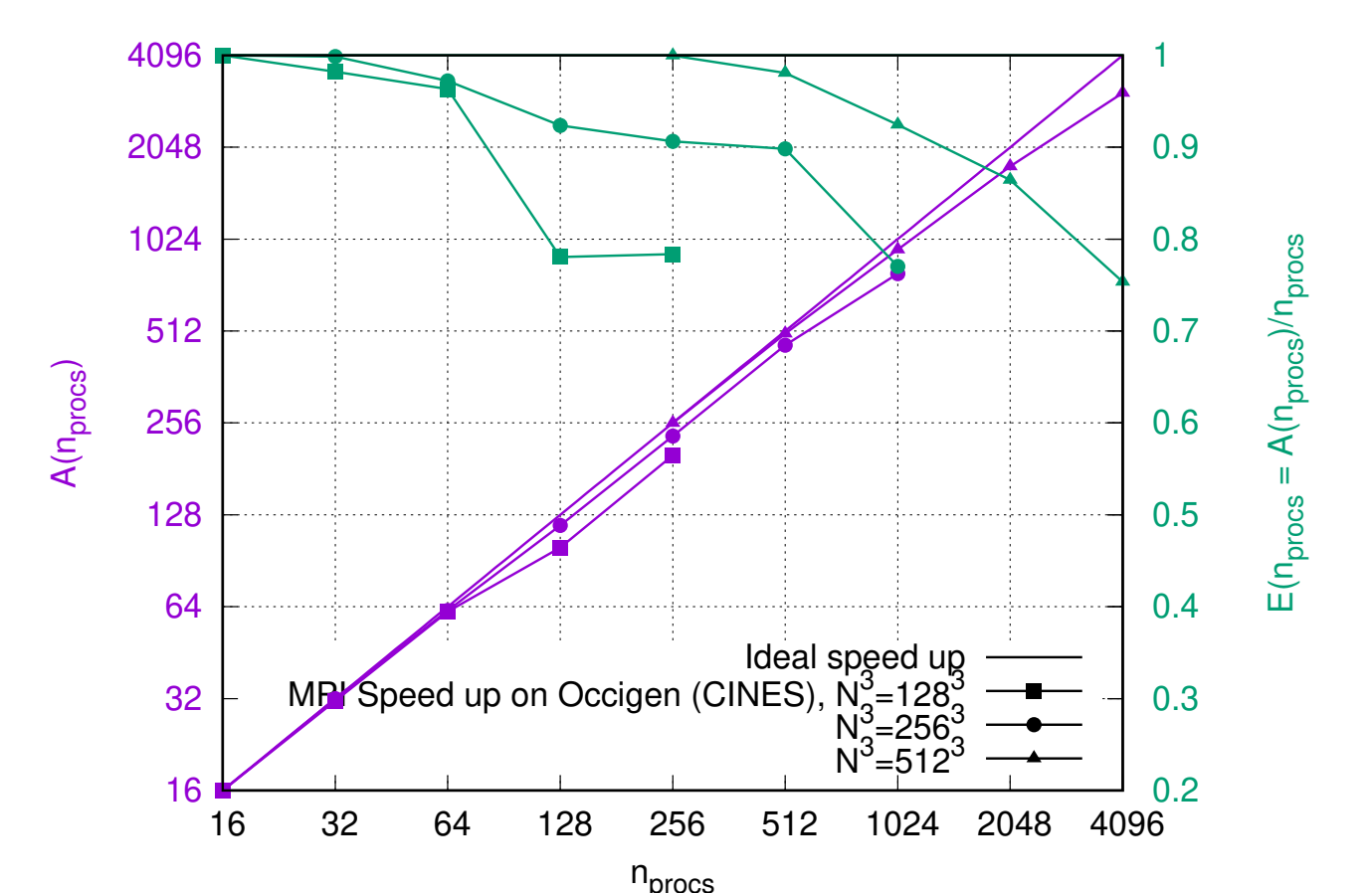
$$\Phi_{i,j}^* = \frac{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \phi_m^n \delta V_m'}{\sum_{m: \mathbf{X}_m^{n+1} \in \Omega_{i,j}} \delta V_m'} \quad \text{with} \quad \delta V_m' = \delta V_m \cap \Omega_{i,j}$$

4. Φ^{n+1} is then obtained solving the unsteady diffusion equation.
5. The local (Lagrangian) information is updated according the variation of Φ at the particle position: $\frac{\partial \phi_m}{\partial t} \Big|_{\mathbf{X}_m^{n+1}} = \frac{\partial \Phi}{\partial t} \Big|_{\mathbf{X}_m^{n+1}}$. For a first order integration scheme,

$$\phi_m^{n+1} = \phi_m^n + \Phi^{n+1}(\mathbf{X}_m^{n+1}) - \Phi^*(\mathbf{X}_m^{n+1}), \quad m = 1, \dots, M.$$

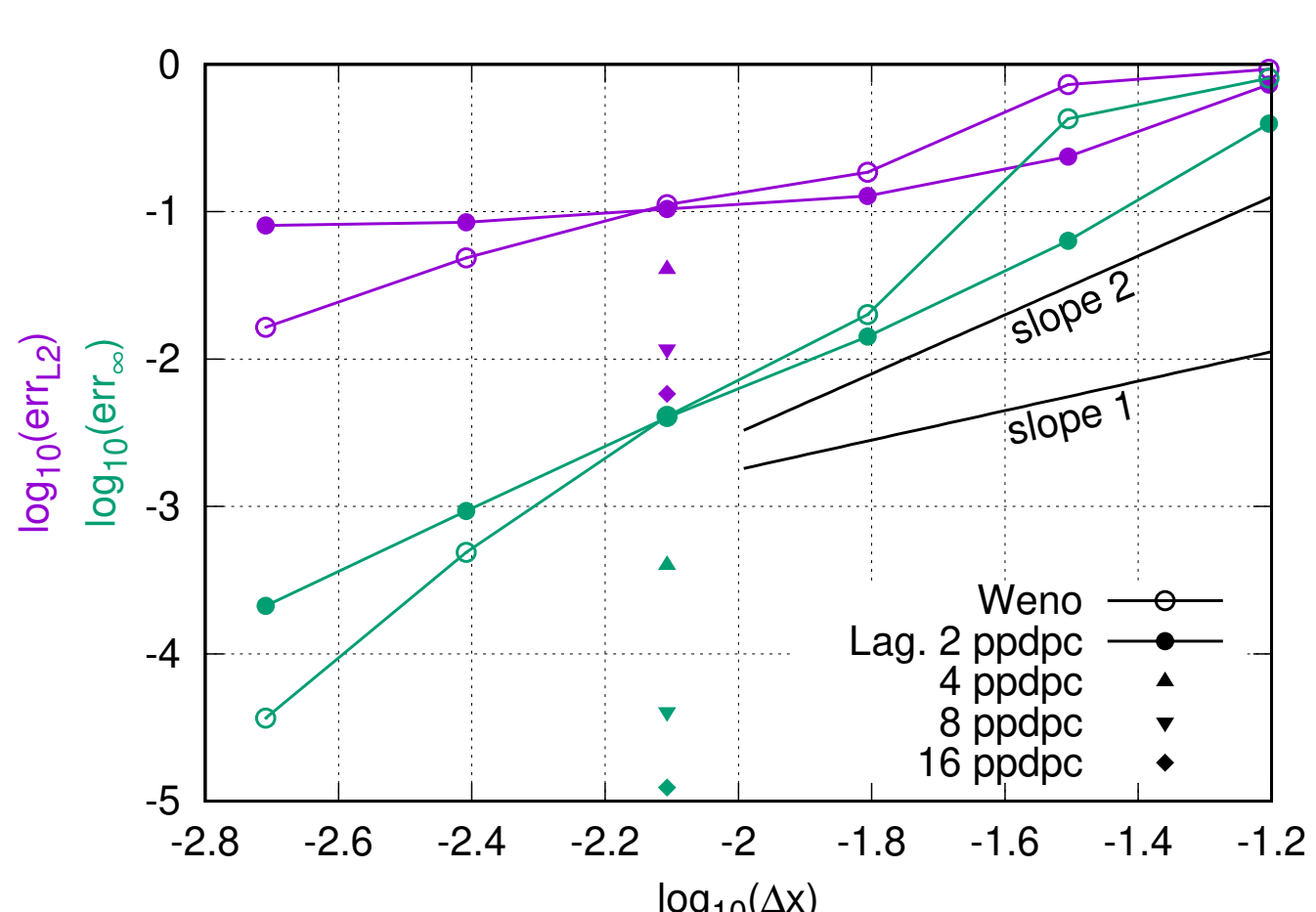
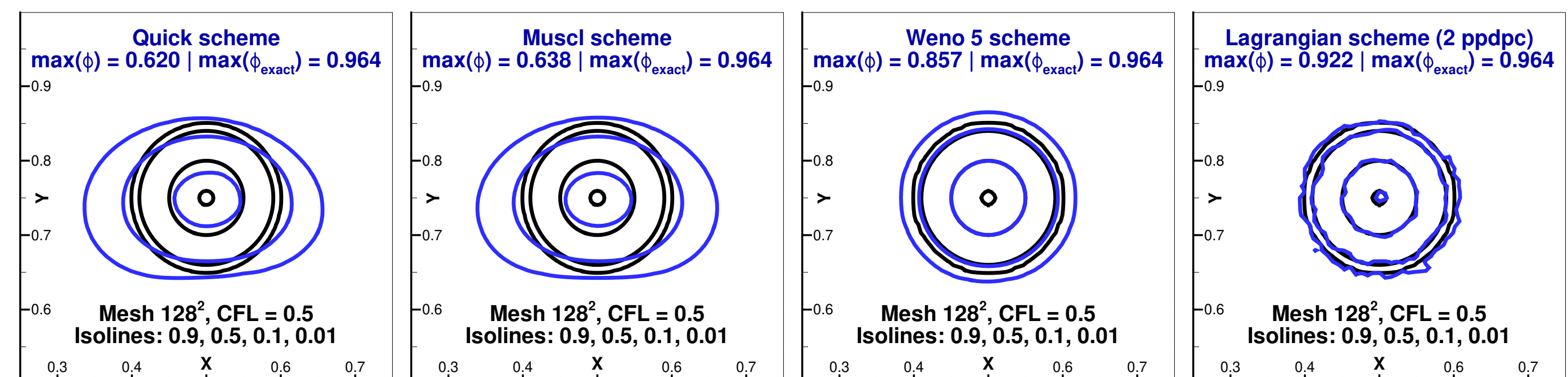
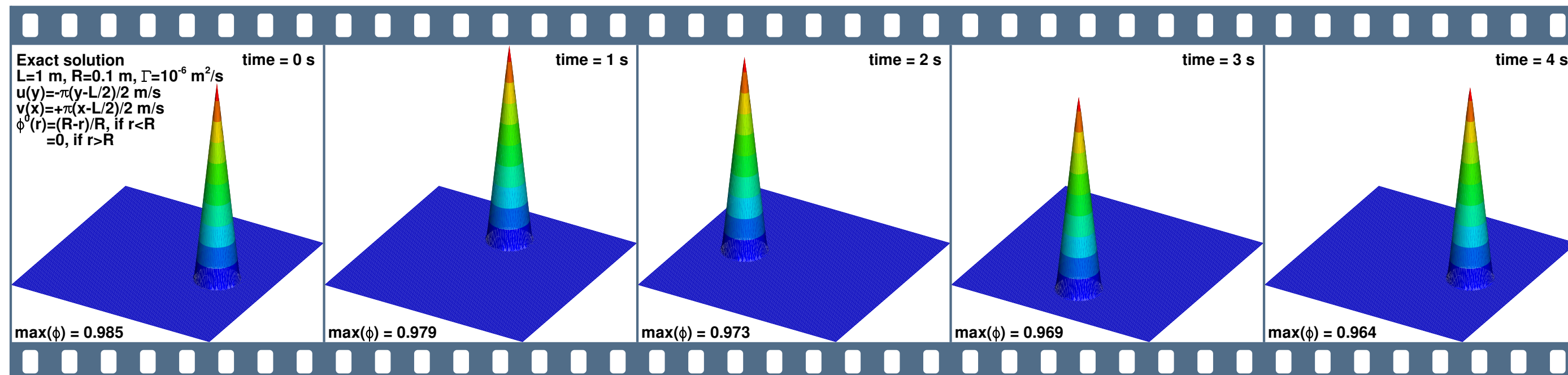
CODE DESCRIPTION

- Finite-Volumes on staggered grids.
- Augmented Lagrangian or KSP [2] for Pressure / Velocity coupling.
- VOF, Level Set, Front Traking.
- Lagrangian particle tracking.
- DNS and LES turbulence modelling.
- Penalty methods.



RESULTS

Advection/Diffusion of a pollutant peak

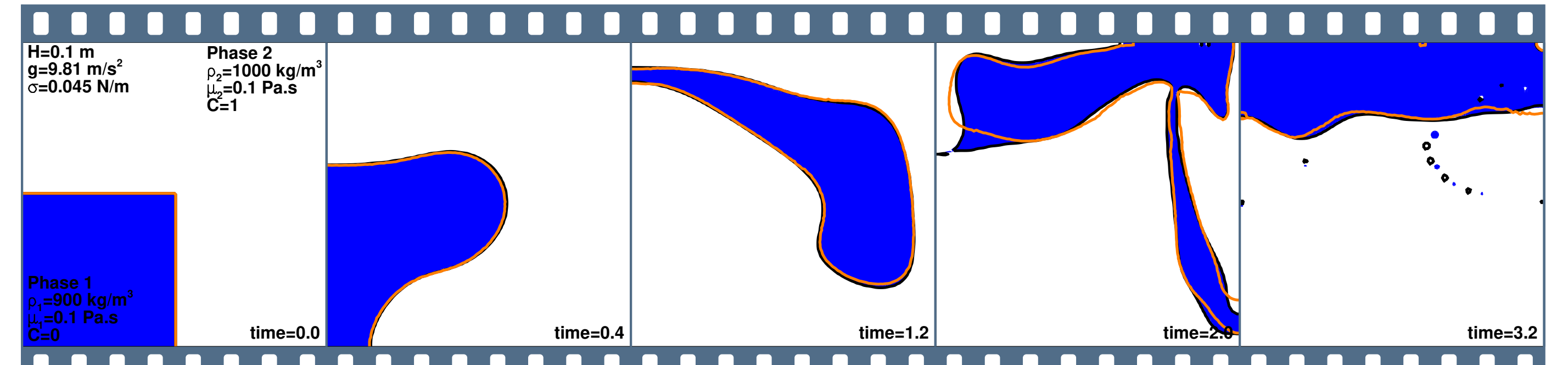


- Same conclusion for pure transport ($\Gamma = 0$).
- Oscillations are reduced increasing M (ppdpc).

Computational cost in % of the Ref. (Weno, 512² (13h))

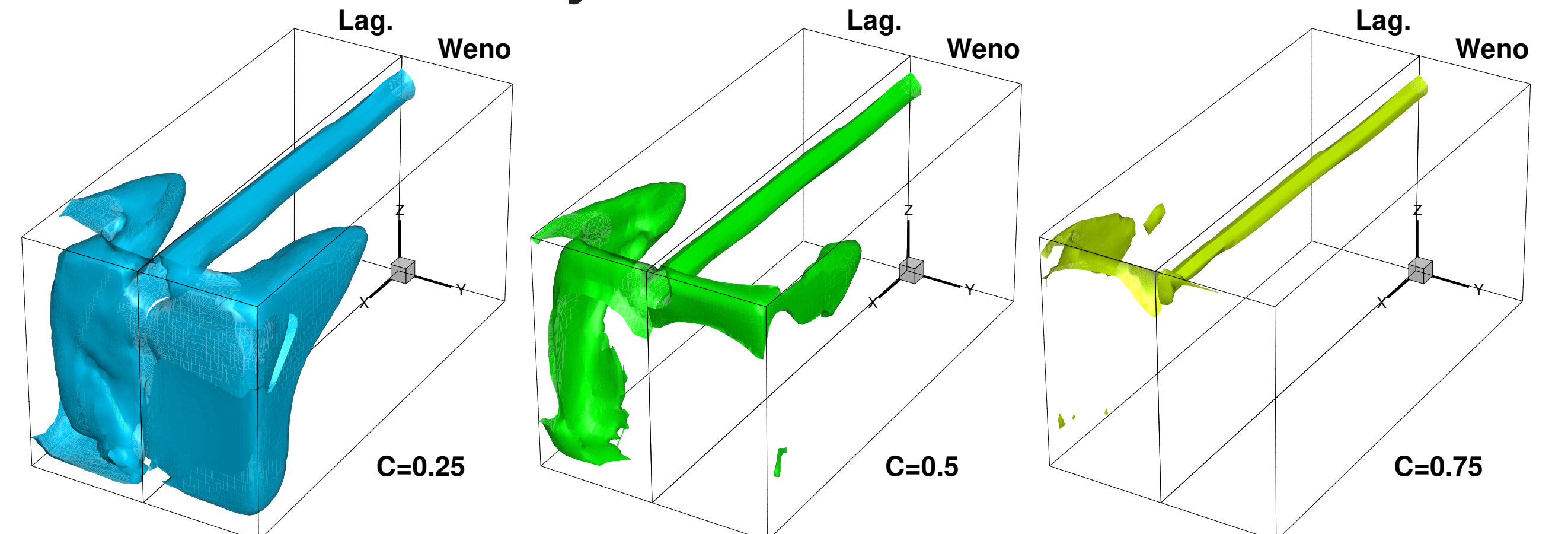
| | | | |
|------------------------|-------------|------------------------------------|------|
| Weno, 128 ² | 1.2% | Lag., 128 ² , 2–4 pddpc | ~ 2% |
| Weno, 256 ² | 19.2% | Lag., 128 ² , 8 pddpc | 6.1% |
| Weno, 512 ² | Ref. | Lag., 128 ² , 16 pddpc | 48% |

Phase inversion problem with KSP method [2]



Snapshots of the color function over time. Mesh 128². **Blue** is KSP, VOF-PLIC, **black line** is $C = 0.5$ for the **Augmented Lagrangian**, VOF-PLIC method and **orange** line stands for the **Lagrangian advection scheme** with 4 pddpc and KSP.

Ventilated cavity (collaboration with INRS)



Simulation of air flow at $Re = 1500$ (injection) in a cuboid cavity. Air flow carries a tracer gas with air properties ($\Gamma \approx 10^{-5} \text{ m}^2/\text{s}$). Iso-surfaces 0.25, 0.5 and 0.75 of concentration C for Lagrangian scheme with 4 pddpc (left) and Weno 5 scheme (right) are plotted for $t = 1.5$ second. The non diffusive character of the Lagrangian scheme is observed. (Mesh: $64 \times 32 \times 32$, $\Delta t = 5 \times 10^{-3}$ s.)

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REFERENCES

- [1] S. Vincent et al. Eulerian-Lagrangian multiscale methods for solving scalar equations. Application to incompressible two-phase flows. In *Journal of Computational Physics* (2010)
- [2] J.-P. Caltagirone & S. Vincent. A Kinematic Scalar Projection method (KSP) for incompressible flows with variable density. In *Open Journal of Fluid Dynamics* (2015)