Employee stock ownership in the toolbox of economics

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2018 Beyster symposium

Three papers or three episodes in the toolbox

The motivation is to answer some criticism targeting employee ownership (portfolio diversification, management entrenchment, behavioral economics) with the toolbox of economics

1. The optimal employee ownership contract with information asymmetry and employee's portfolio diversification

2. The first solution extended to the case of management entrenchment

3. The second model with ambiguity aversion

Is employee ownership so senseless?

Published in Finance the French Finance association journal in 2009

Aubert N.
Grand B.
Lapied A.
Rousseau P.

1. Introduction

Starting point:

- Enron and the ruin of its employee owners

Academic answers: ESO is senseless!

Questions:

- Discrepancy between theory and practice
- Incentive effect less important within retirement plan

Outcomes of the paper:

- Shows that ESO is not senseless (Perfect Nash equilibrium in sub-game)
- Illustrates the properties of the solution

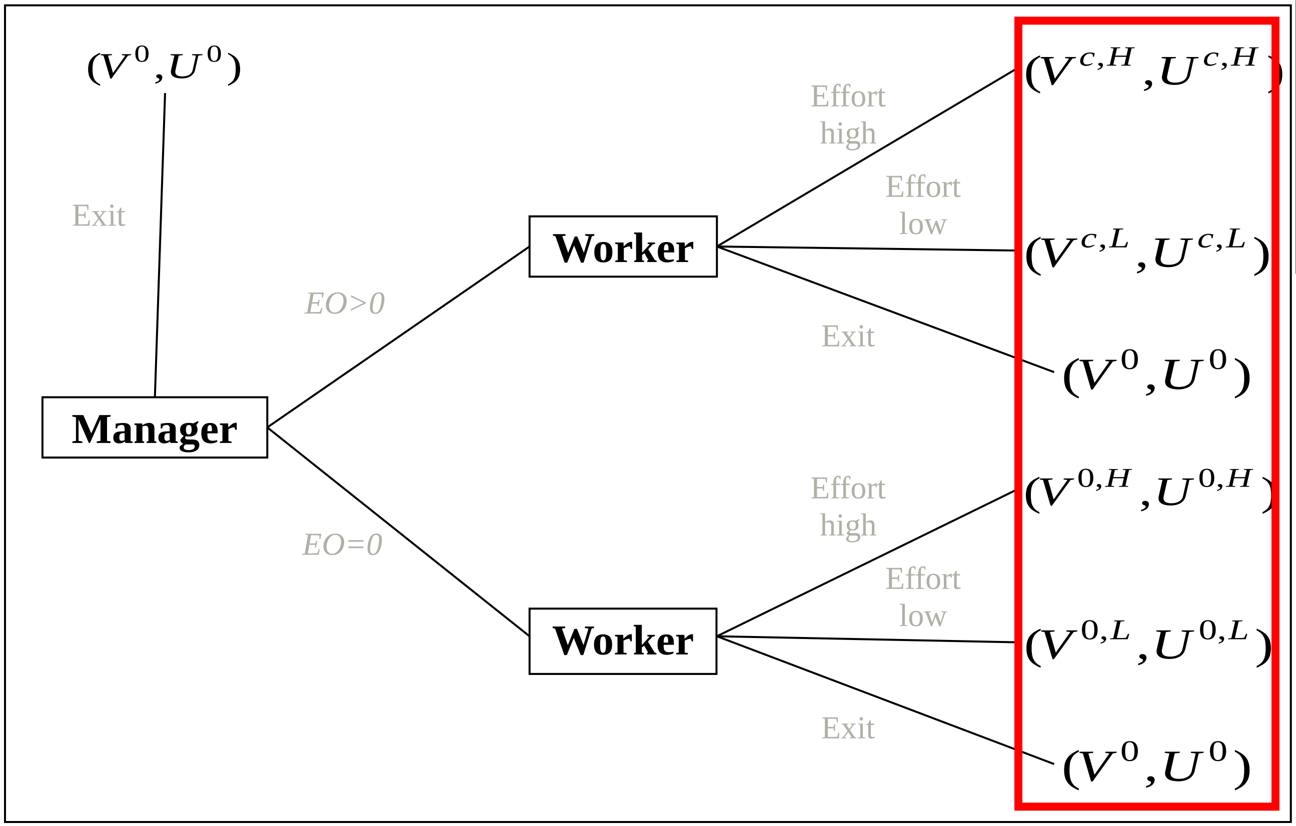
2. Literature

- Employee ownership literature:
  - No answer

- Portfolio theory
  - Employees do not hold private information
  - Under diversification costs are too high

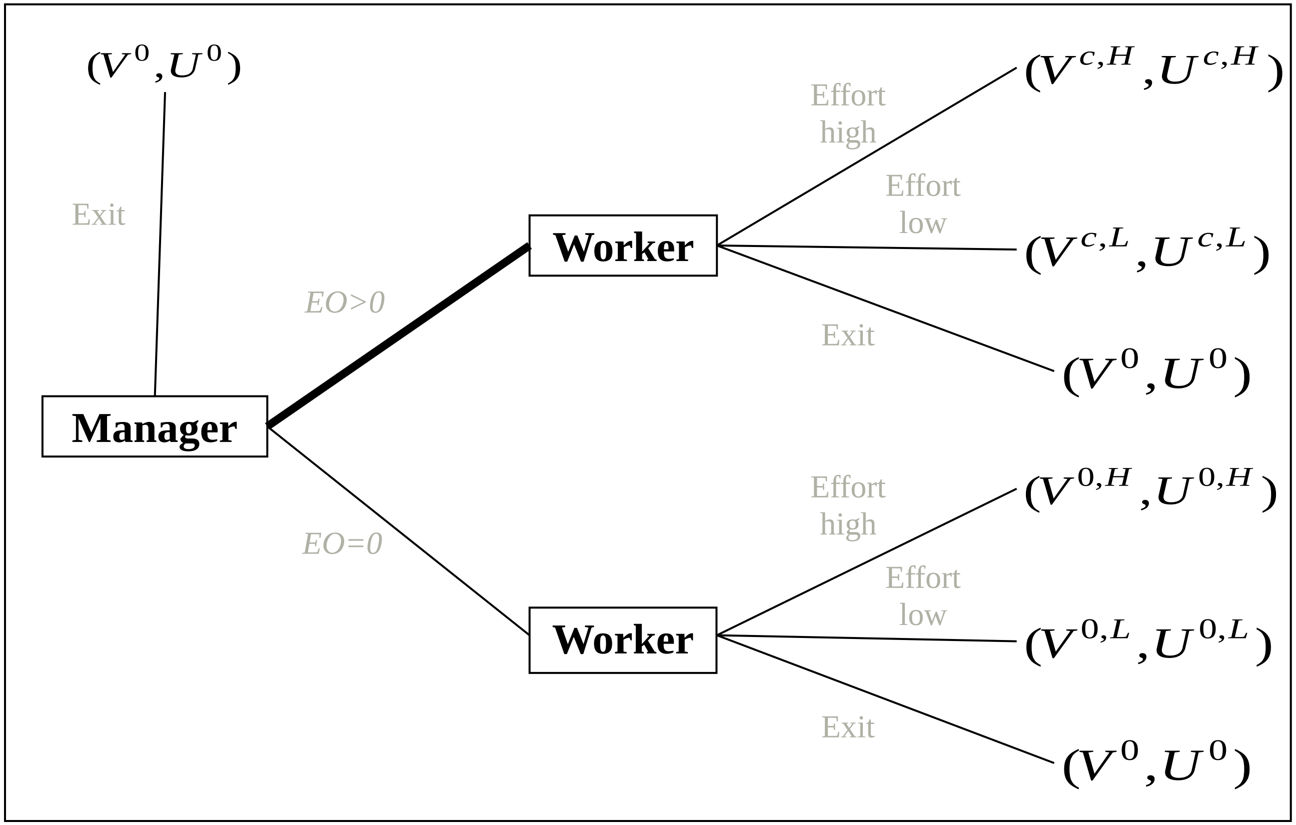
- Behavioral finance
  - Cognitive biases

- No attempt to include information asymmetry and portfolio diversification in the same model



Example:
$(V^{c,H}, U^{c,H})$
High effort (H) with employee ownership granted (c>0)

V: Employer's risk neutral utility function
U: Employee's risk averse utility function
H/L: High (H) or low (L) level of effort exerted by the employee
0/c: positive contribution in employee ownership (c) or no (0)



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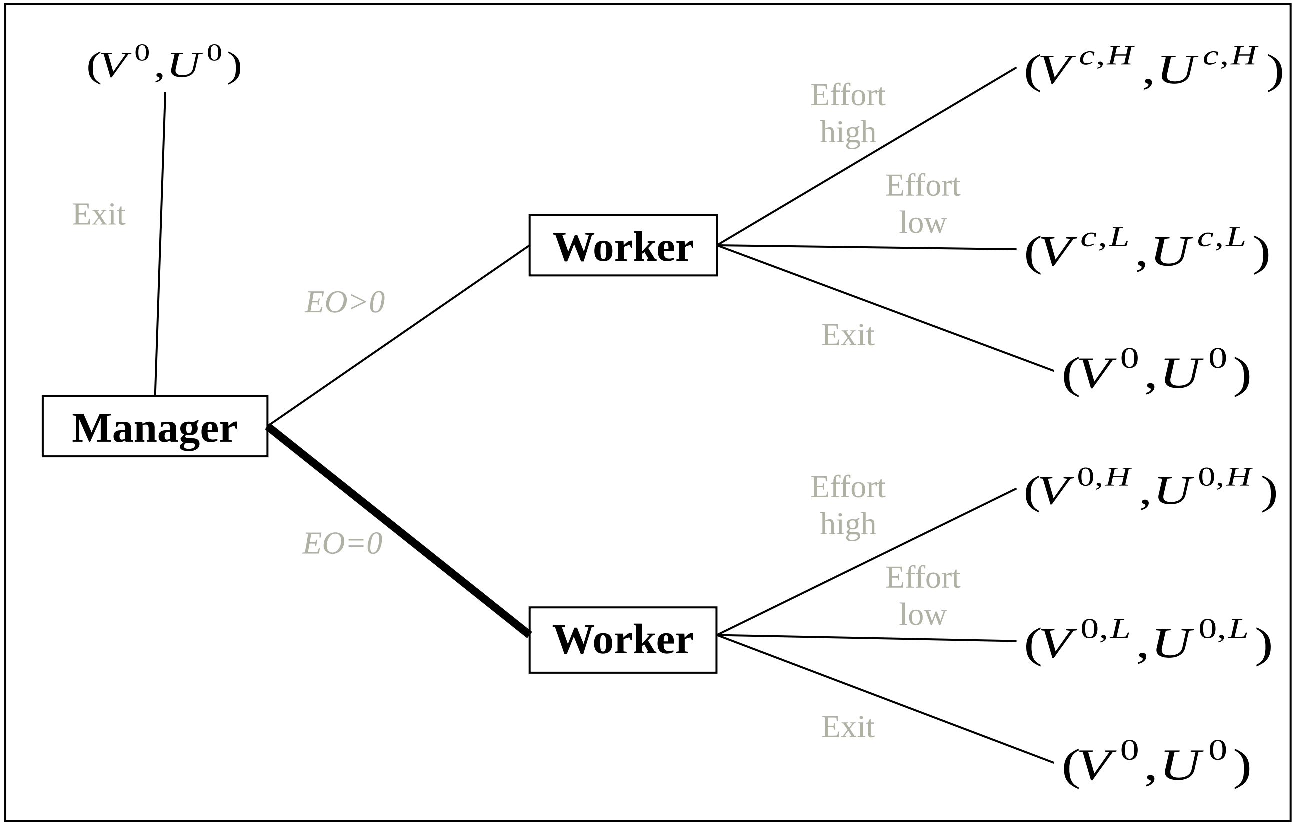

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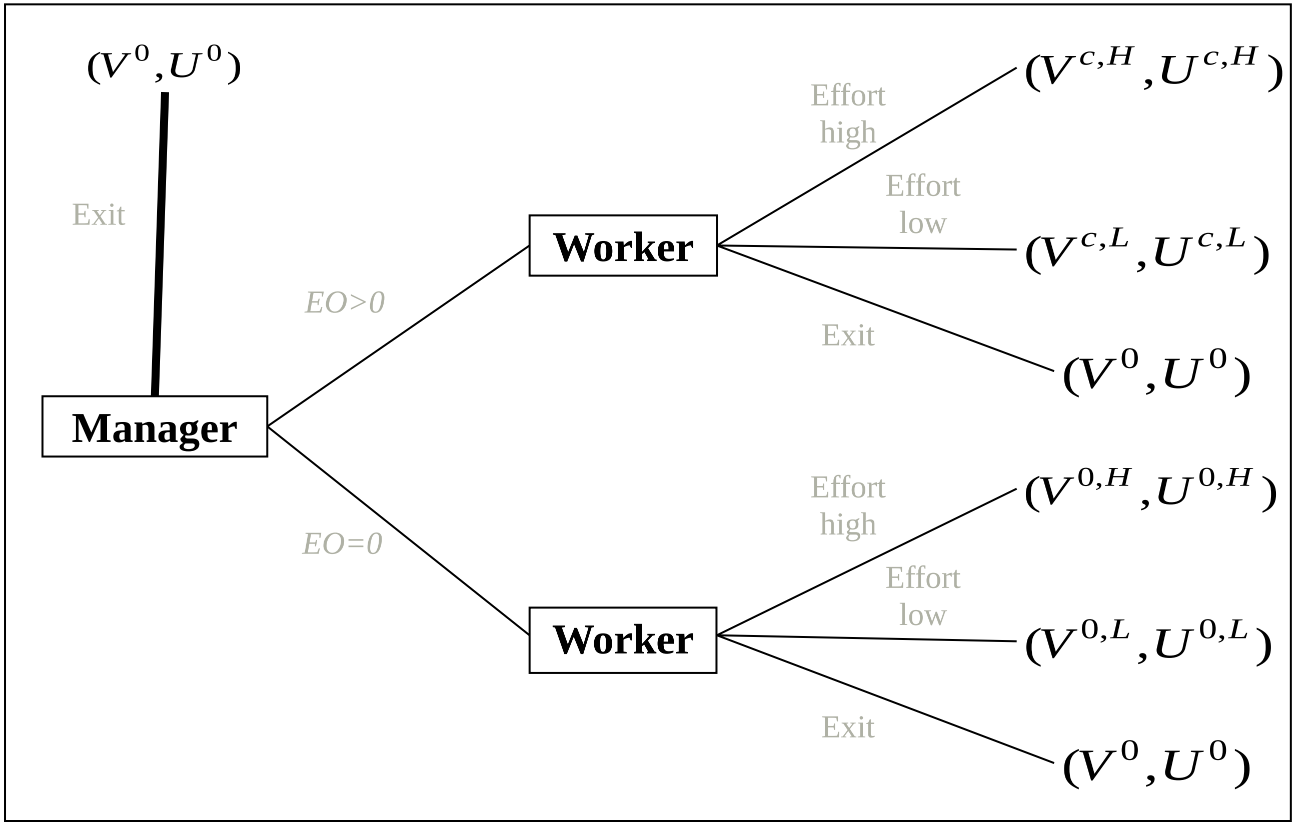

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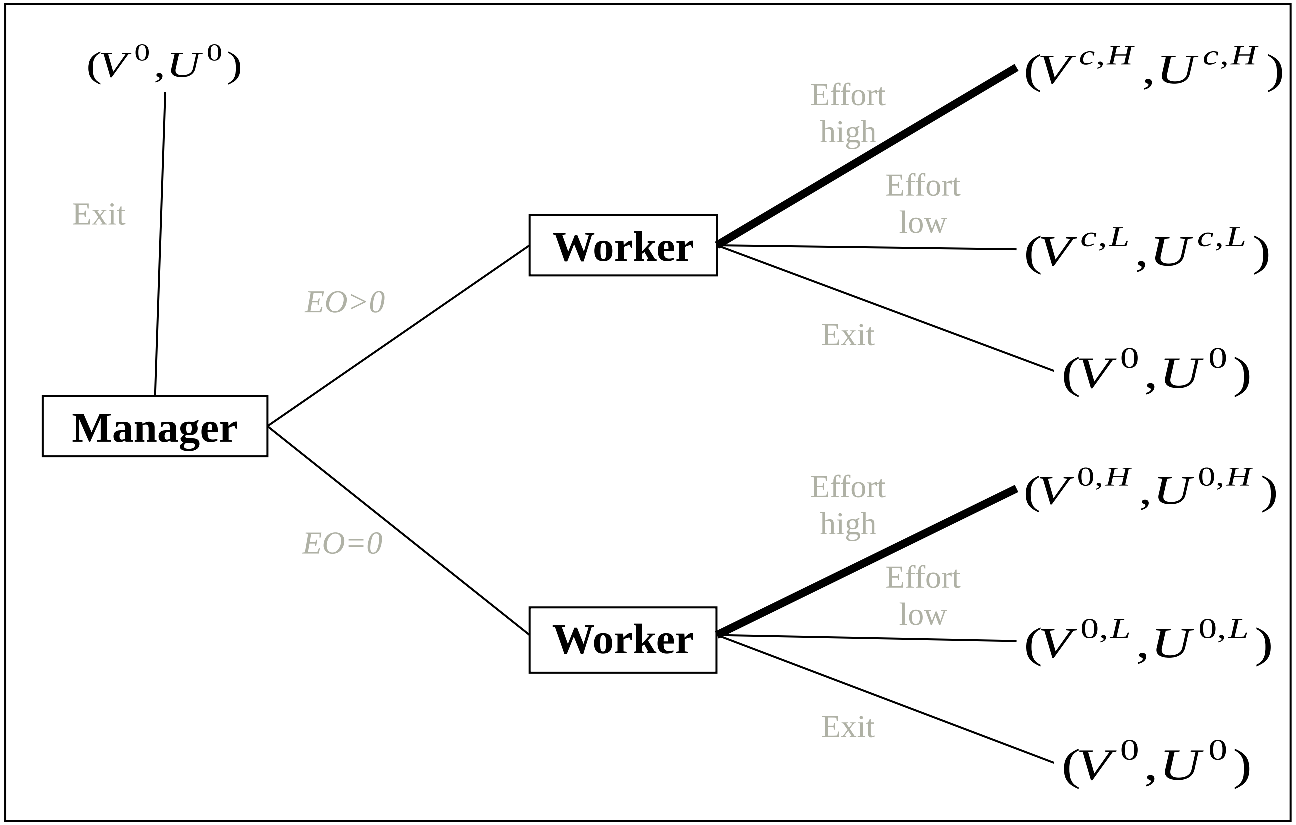

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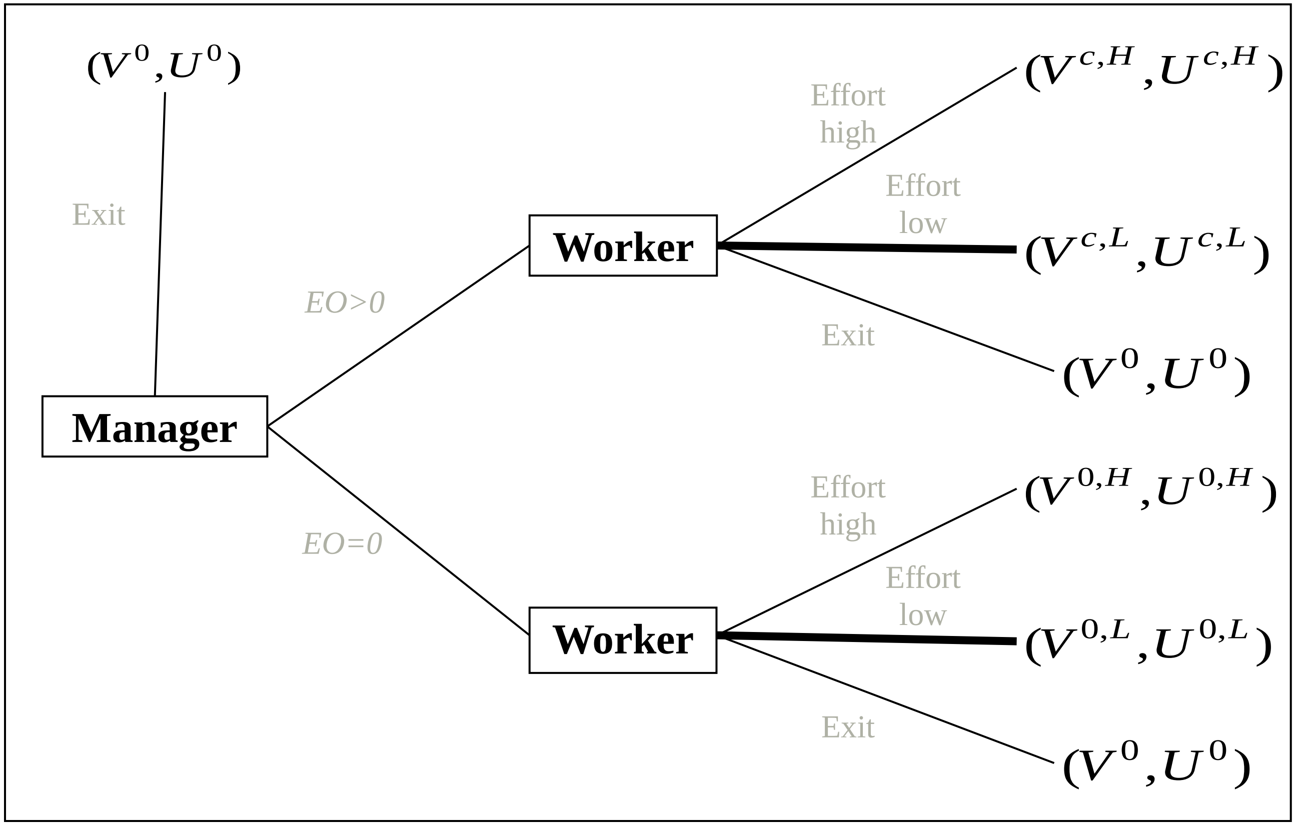

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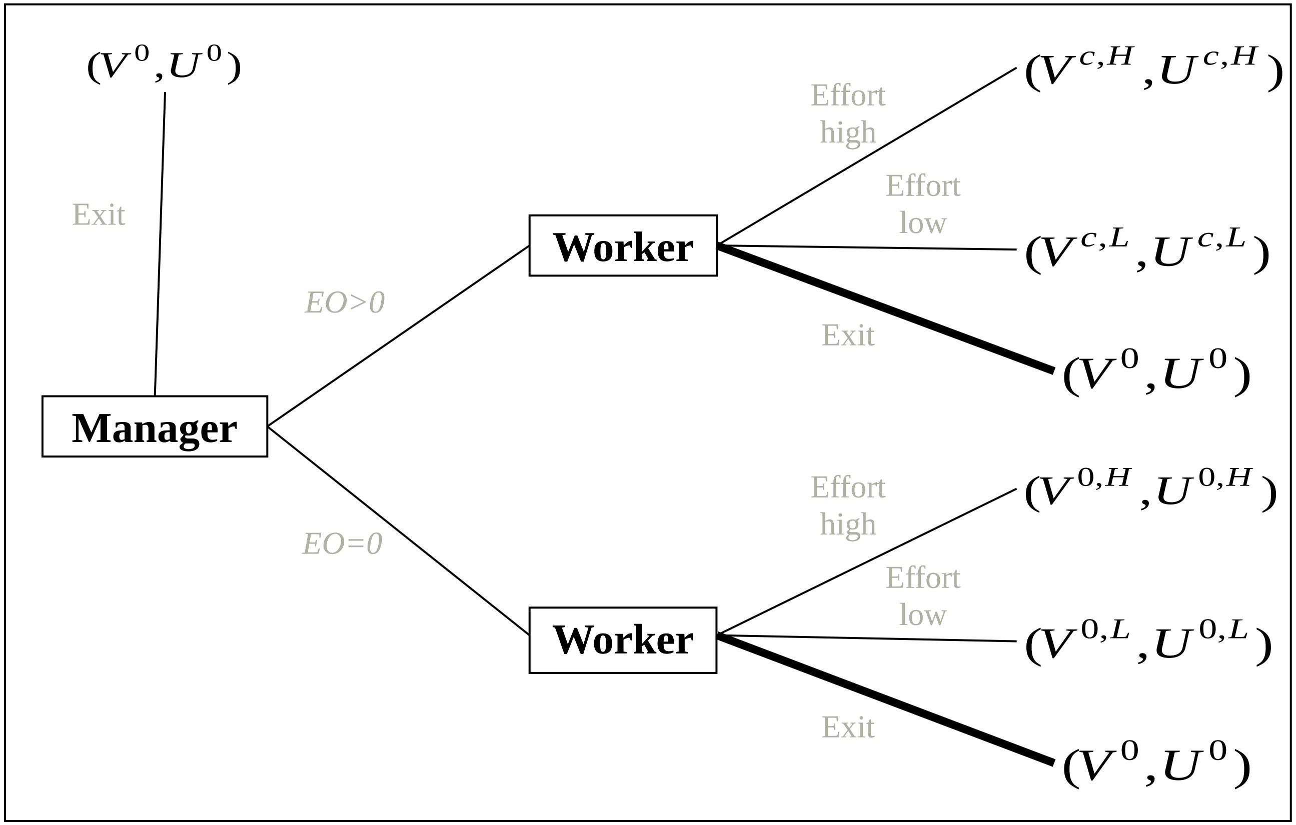

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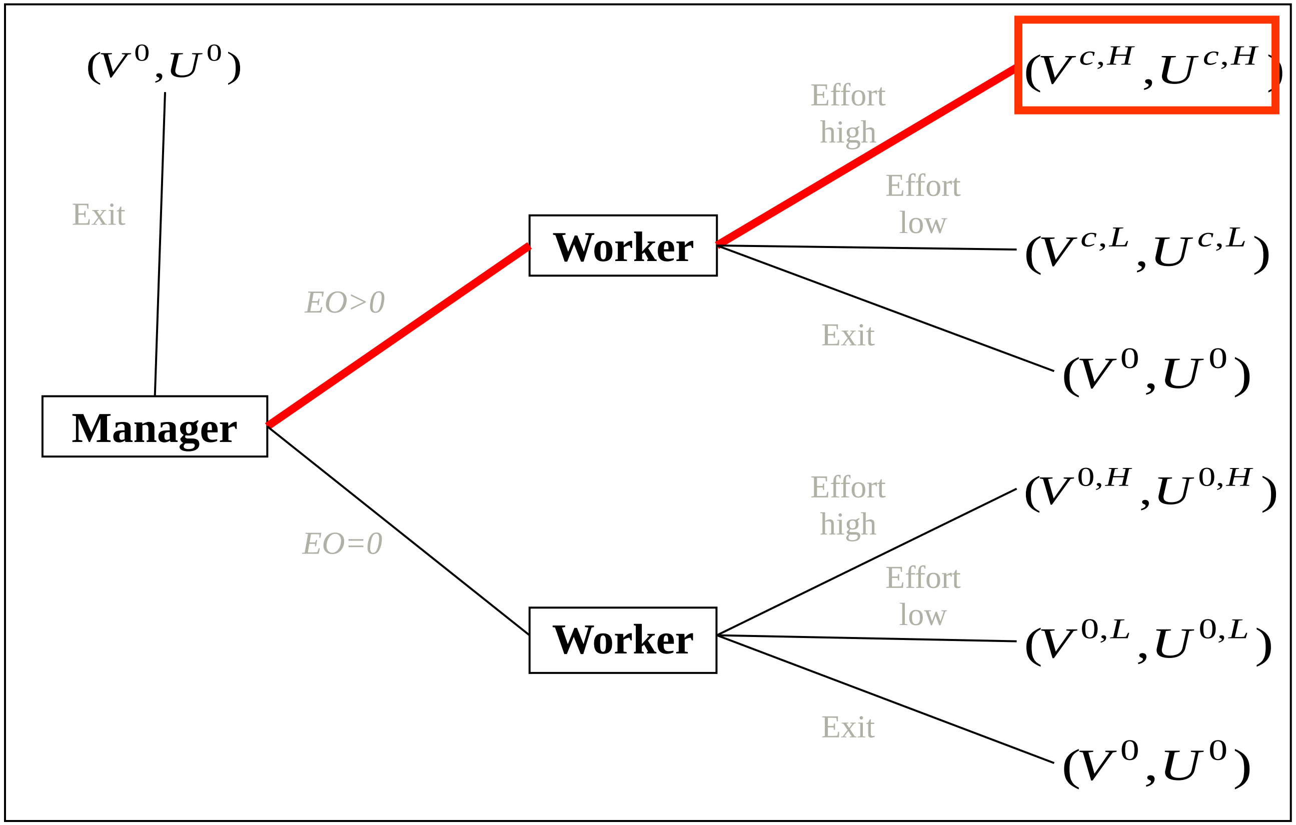
We are looking for the equilibrium where:
1) the employee exerts the high level of effort
2) the manager offers employee ownership



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H/L: High (H) or low (L) level of effort exerted by the employee
0/c: positive contribution in employee ownership (c) or no (0)

3. Model (2)

- Risk neutral manager's utility function
  - Without

$$V^{0,j} = W_d[1+\mu_e^j]$$

  - With contribution in company's stocks

$$V^{c,j} = (W_d - cW_s)[1+\mu_e^j]$$

With j={*H*,*L*}: H=high effort; L=low effort

- Risk averse worker's utility function
  - Without

$$U^{0,j} = \int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)]f(r_m)dr_m - \psi(e^j)$$

  - With contribution in company's stocks

$$U^{c,j} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)+cW_s(1+r_e+\mu_e^j)]f(r_e,r_m)dr_e dr_m - \psi(e^j)$$

3. Model (3)

Assumptions:

Assumption A1: The employee's expected utility is required to be positive even when company stock is not granted.

$$U^{0,L} = \int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)]f(r_m)dr_m - \psi(e^L) > 0 \quad \text{(A1)}$$

Assumption A2: The employee's expected utility is higher with employee ownership for a given level of effort. $\Rightarrow U^{c,j} \geq U^{0,j}$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)+cW_s(1+r_e+\mu_e^j)]f(r_e,r_m)dr_e dr_m - \psi(e^j) \geq \int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)]f(r_m)dr_m - \psi(e^j) \quad \text{(A2)}$$

Assumption A3: The employee is cautious. His relative risk aversion is smaller than one.

$$\forall x : \alpha = -x\frac{u''(x)}{u'(x)} \leq 1 \quad \text{(A3)}$$

Assumption 4: The market portfolio return's distributions are symmetric. (A4)

4. Solution

PROPOSITION: Under assumptions A1 to A4, given a small disutility of effort difference $\psi(e^H)-\psi(e^L)^4$, an optimal transfer of company stock $c^* \in]0,\bar{c}]$ exists. It satisfies the conditions of a perfect Nash equilibrium in sub-game (Selten, 1965) given:

$$\omega(c^*) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)+c^* W_s(1+r_e+\mu_e^H)]f(r_e,r_m)dr_e dr_m$$

$$-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u[W_s(1+r_m+\mu_m)+c^* W_s(1+r_e+\mu_e^L)]f(r_e,r_m)dr_e dr_m = \psi(e^H)-\psi(e^L)$$

with $\bar{c} = \dfrac{W_d}{W_s}\dfrac{\mu_e^H-\mu_e^L}{1+\mu_e^H}$

According to:

LEMMA 1: When the entrepreneur does not grant company stock (*c=0*), the employee exerts low level of effort.

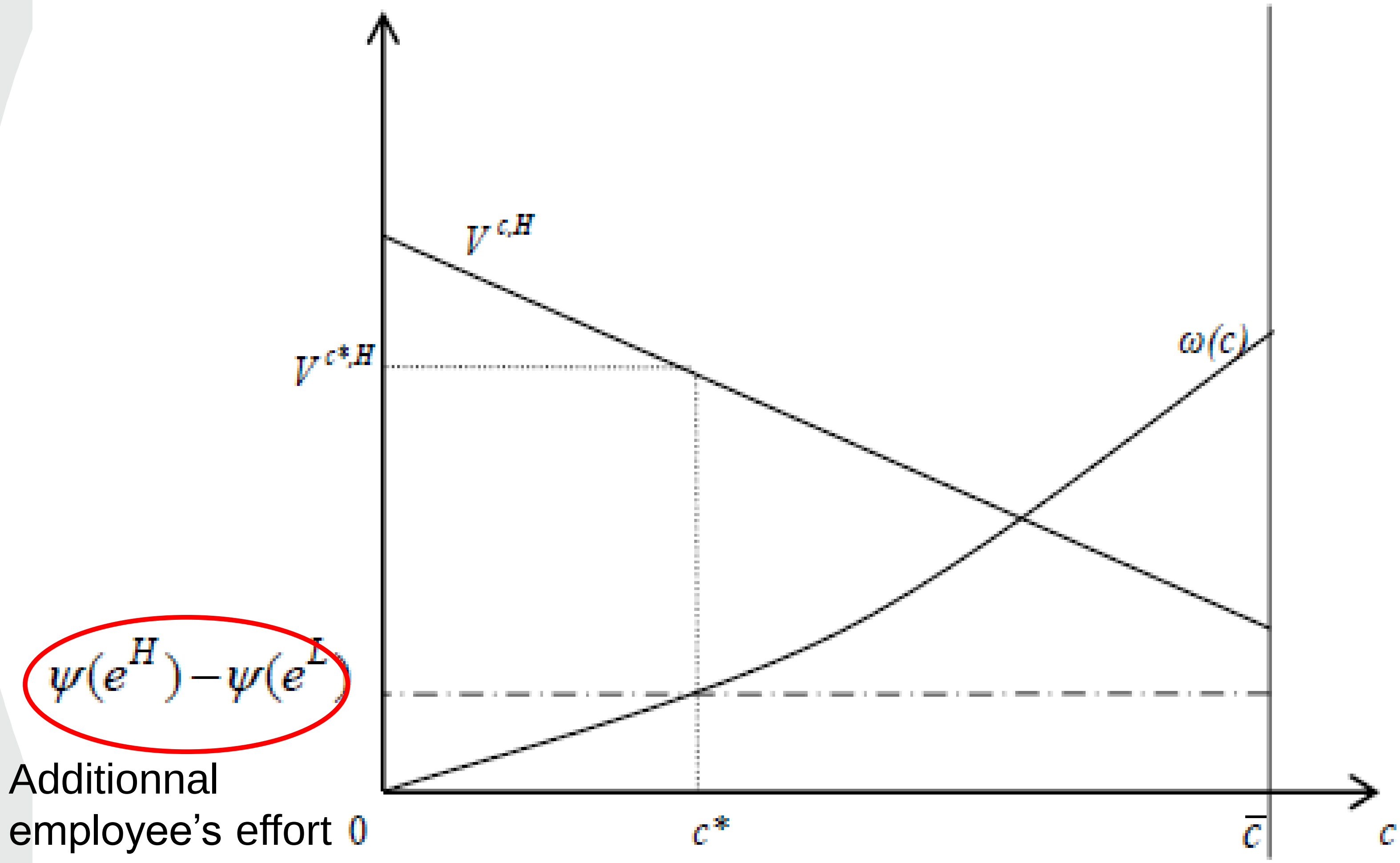
LEMMA 2: Entrepreneur's exit does not lead to a perfect Nash equilibrium in sub-game.

LEMMA 3: The condition $V^{c,H} \geq V^{0,L}$ implies that the condition $V^{c,H} \geq V^0$ is satisfied.

LEMMA 4: The condition $U^{c,H} \geq U^{c,L}$ implies that the condition $U^{c,H} \geq U^0$ is satisfied.

LEMMA 5: $\omega(c)$ is an increasing function from 0.

4. Solution (2): Determination of the optimal c*



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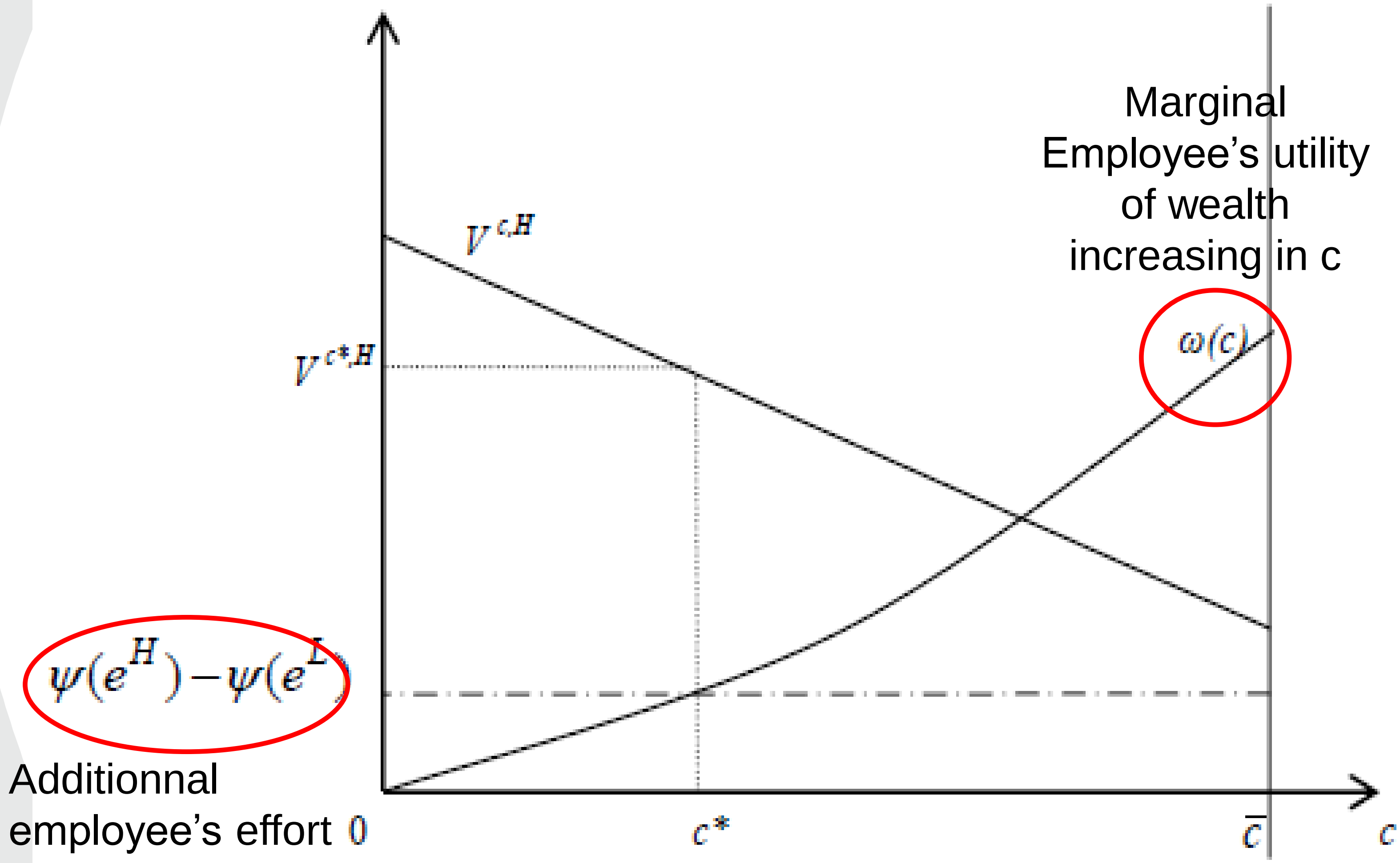

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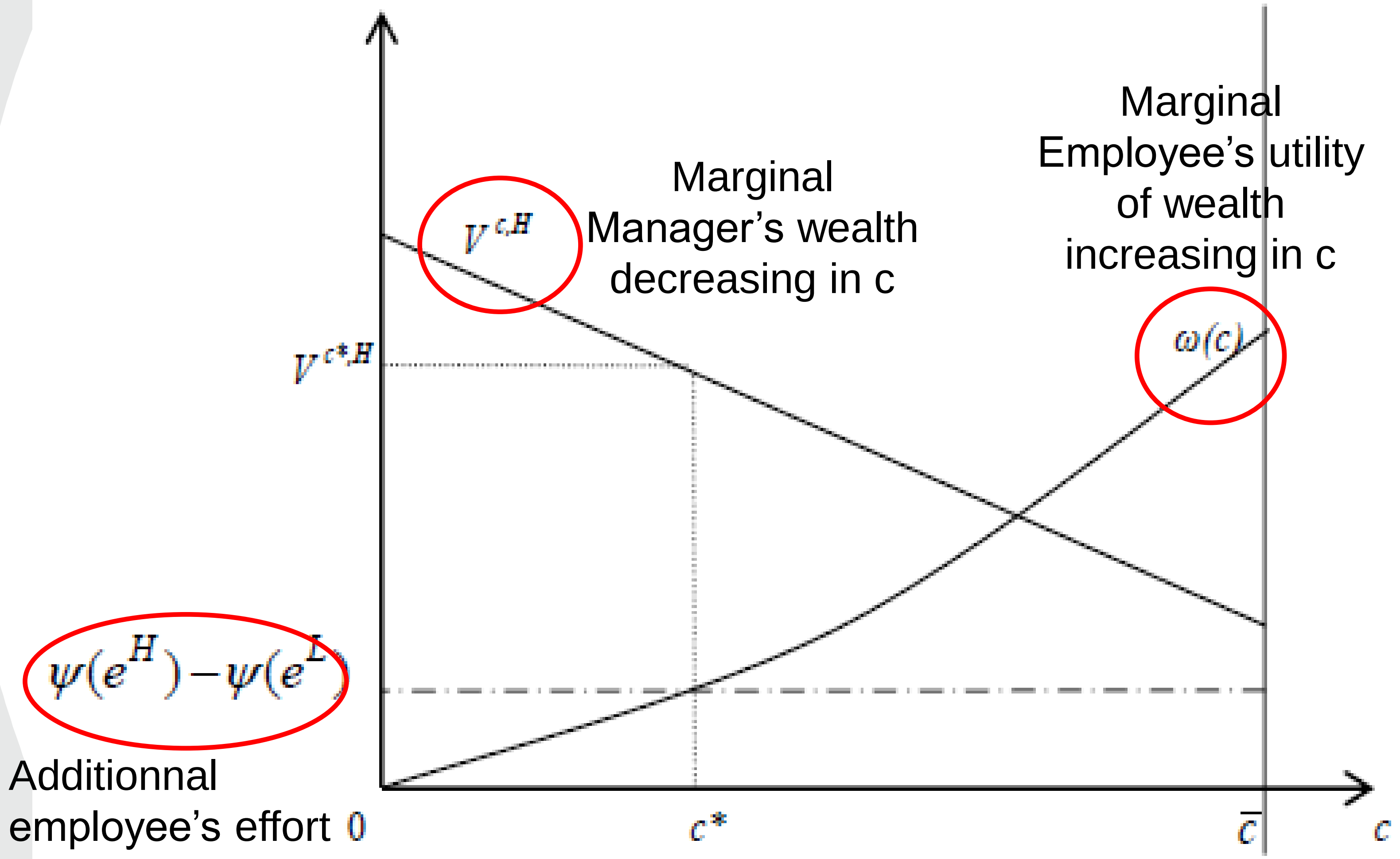

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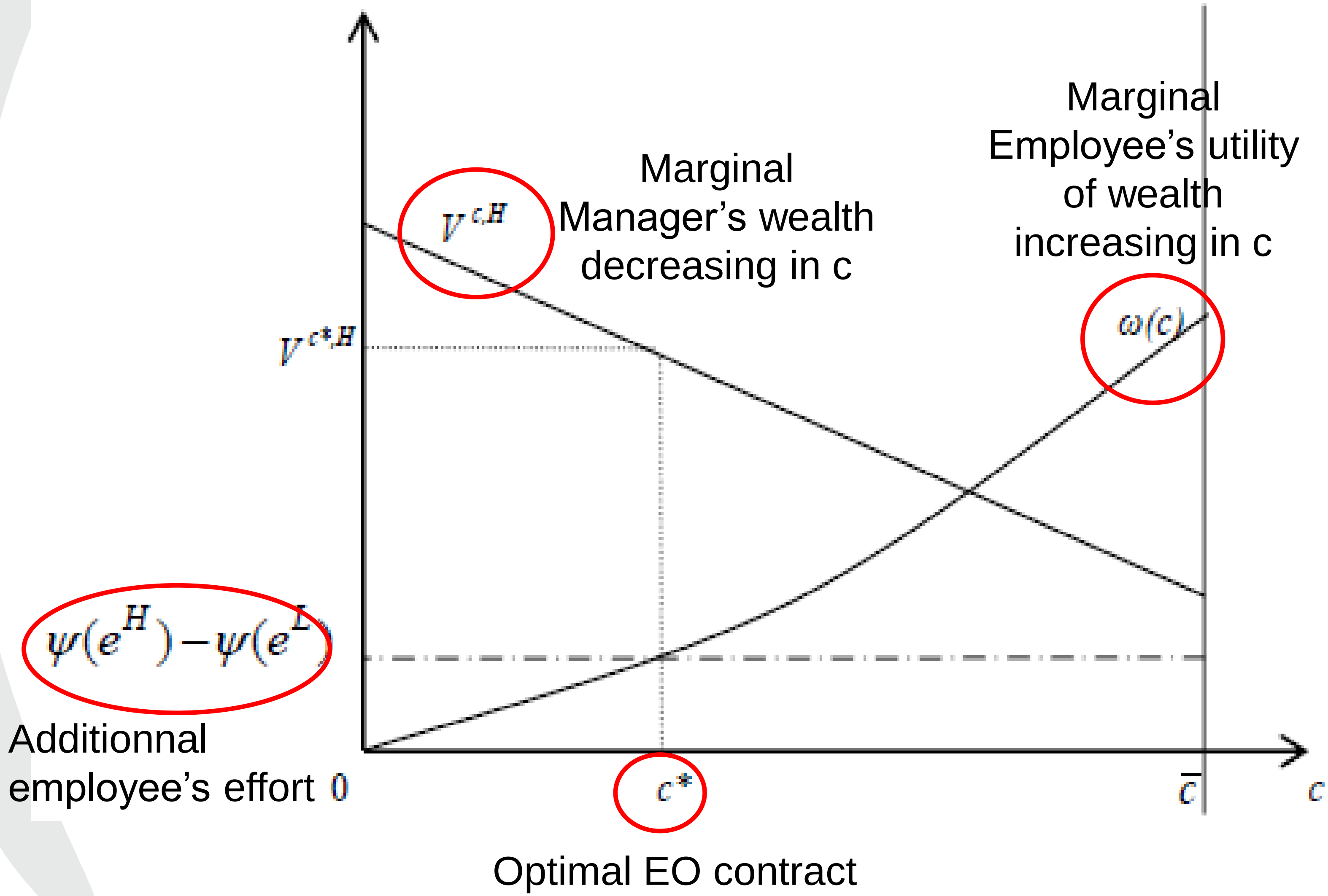

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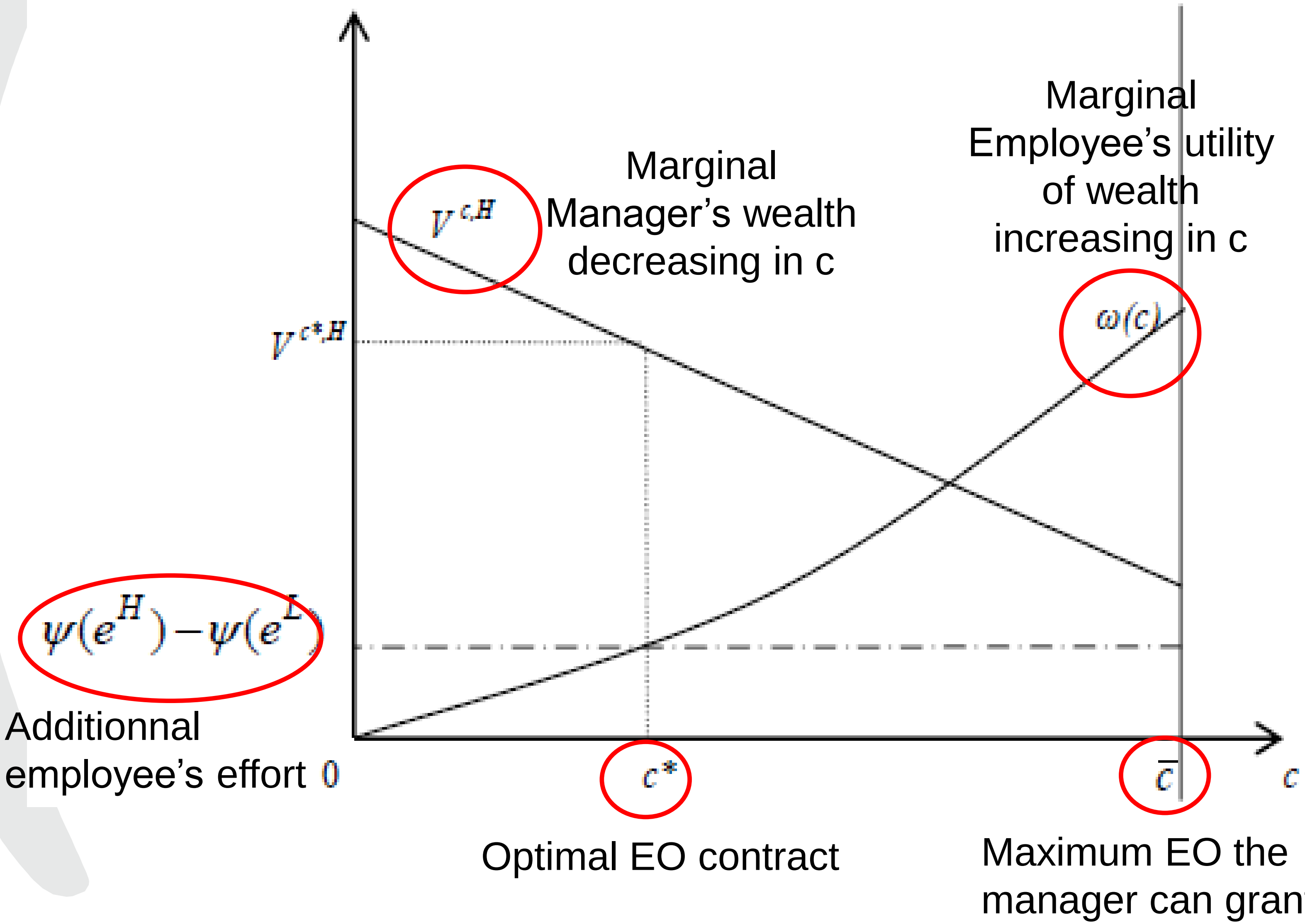
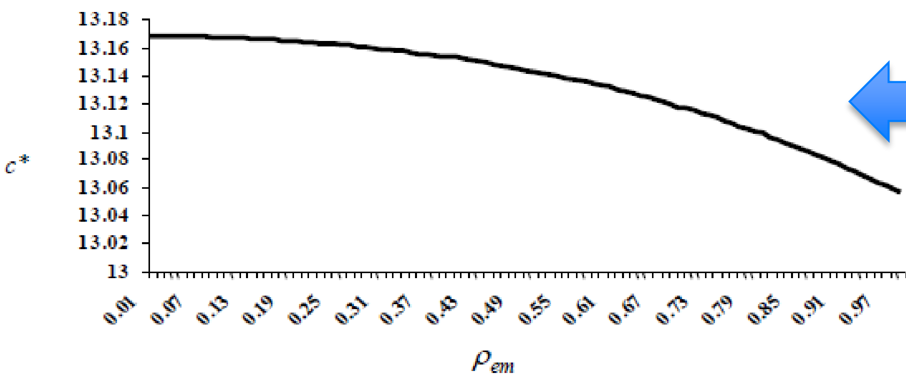


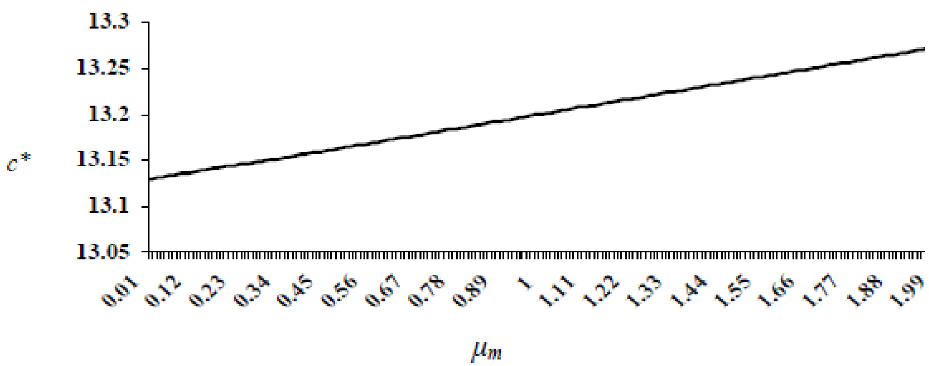
Figure 3: Relationships between c^* and the variables of the model

(a)

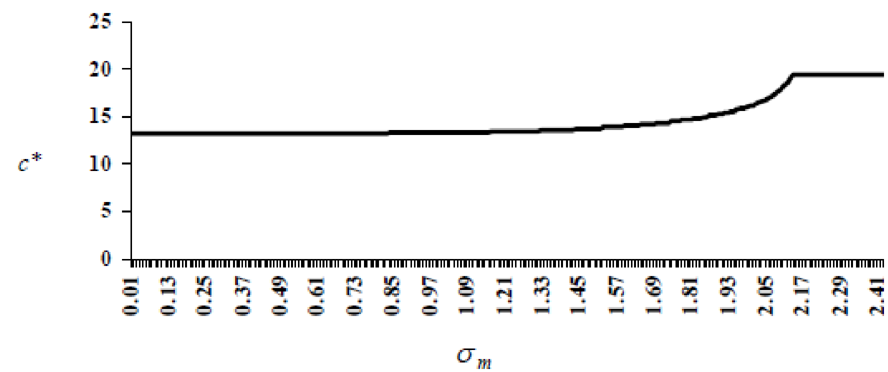


As far as the return on company stock is highly correlated with the market, incentive effect associated with employee ownership is obtained for a lower level of c^*.

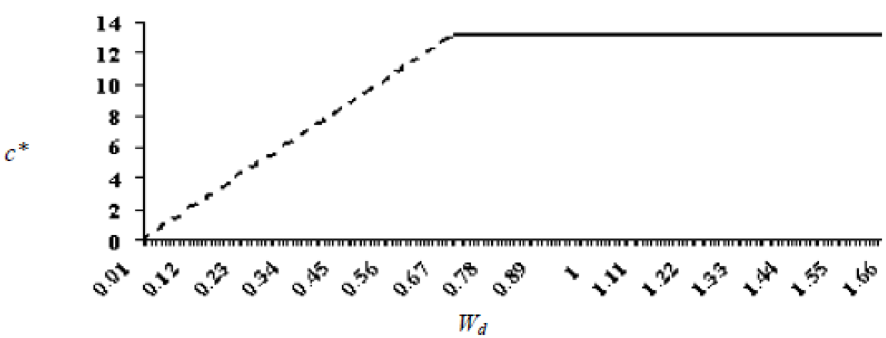
(c)



(d)



(e)



(f)

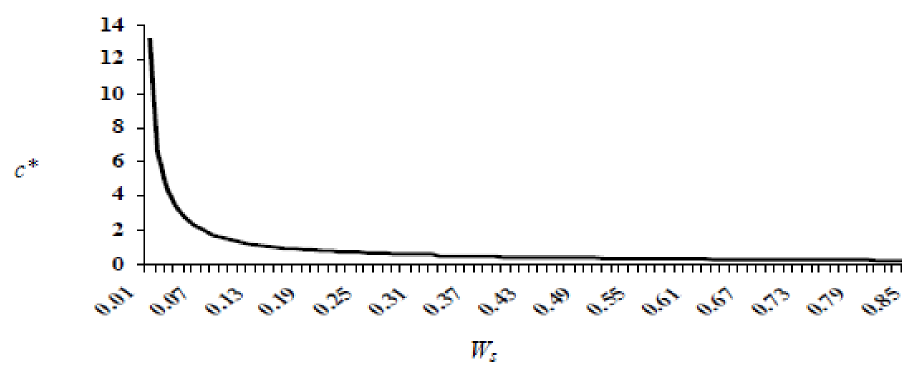
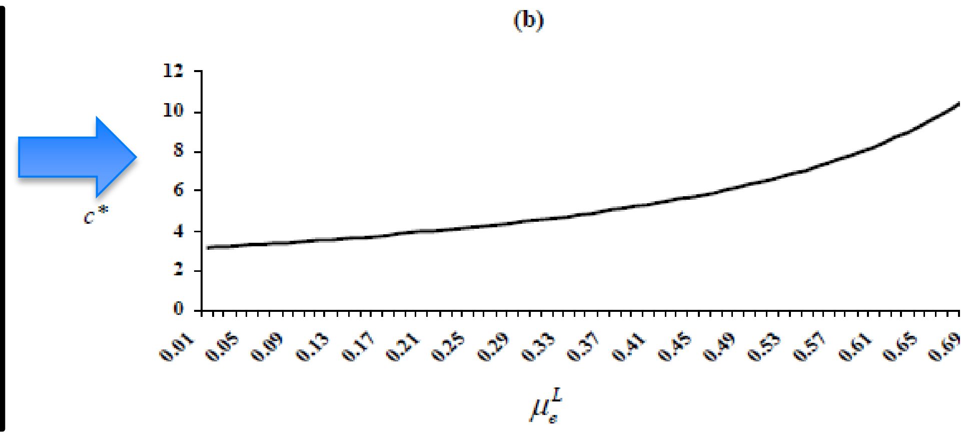
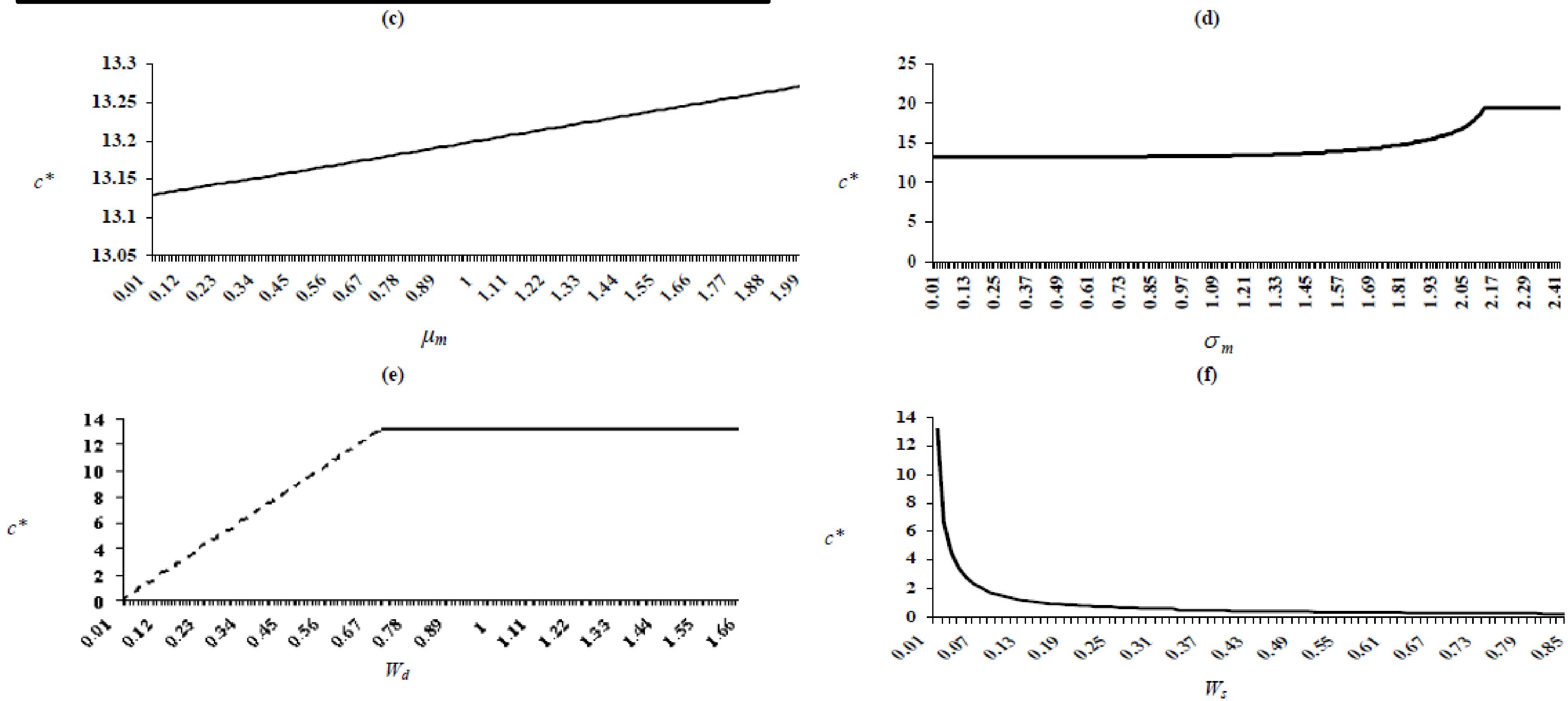


Figure 3: Relationships between c^* and the variables of the model

Ceteris paribus, the marginal efficiency of one dollar distributed to the employee is more important for a low level of productivity than for a higher level. The compensation plan will be more costly for a highly productive firm than for a lower one.





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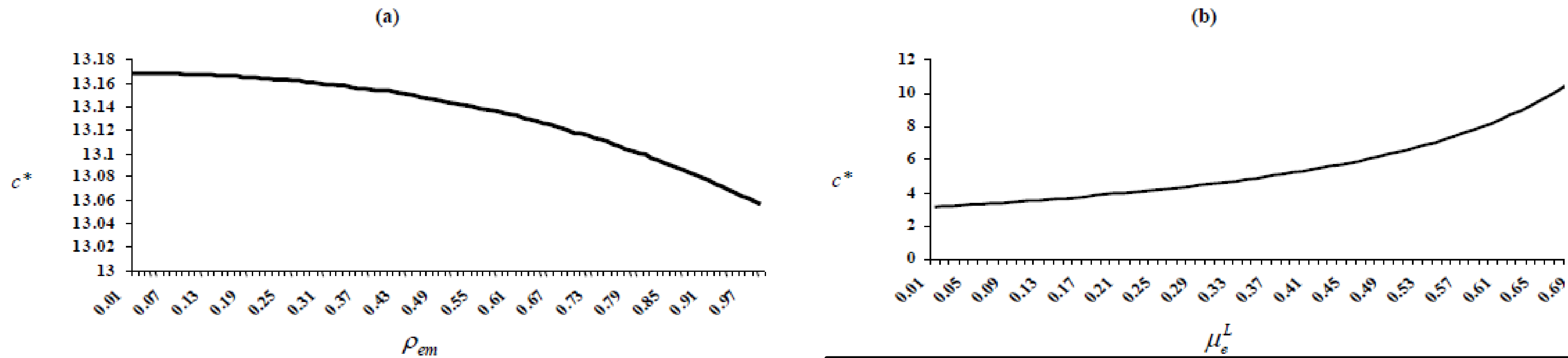
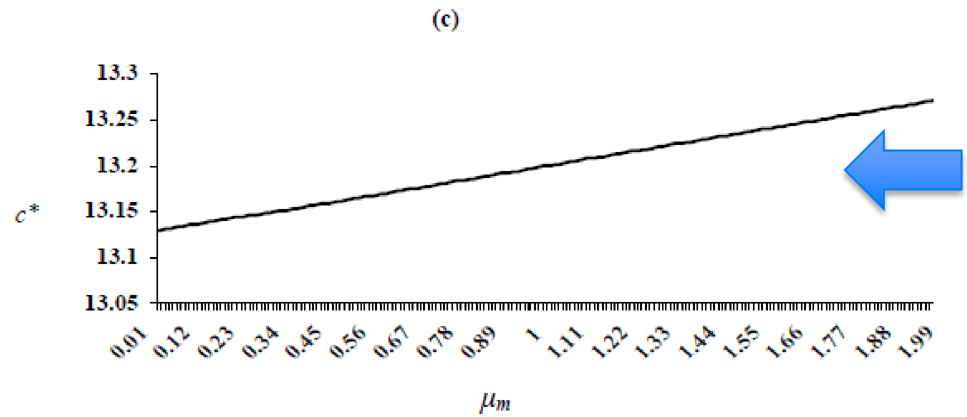
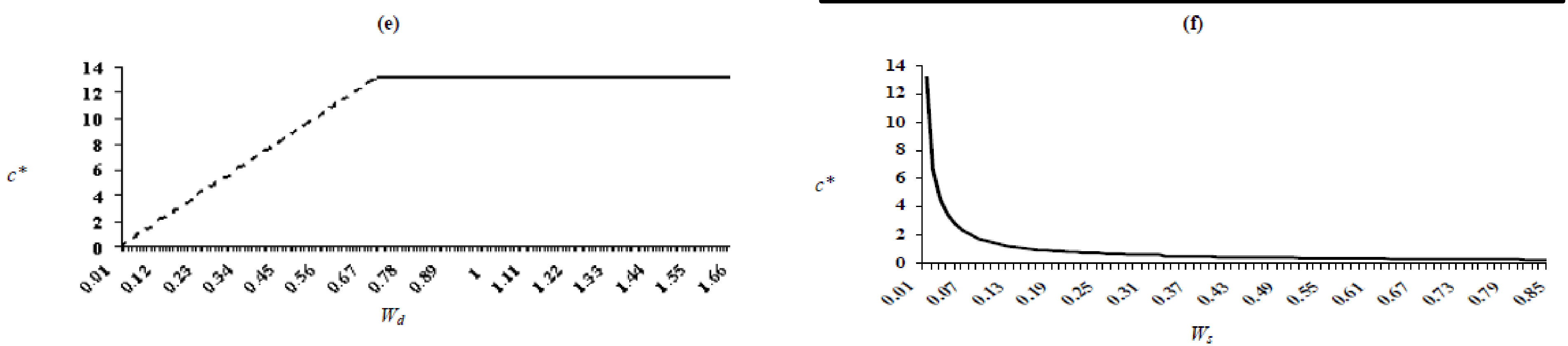




As the market return is higher, the opportunity cost of investing in company stock whose return remains constant is higher.



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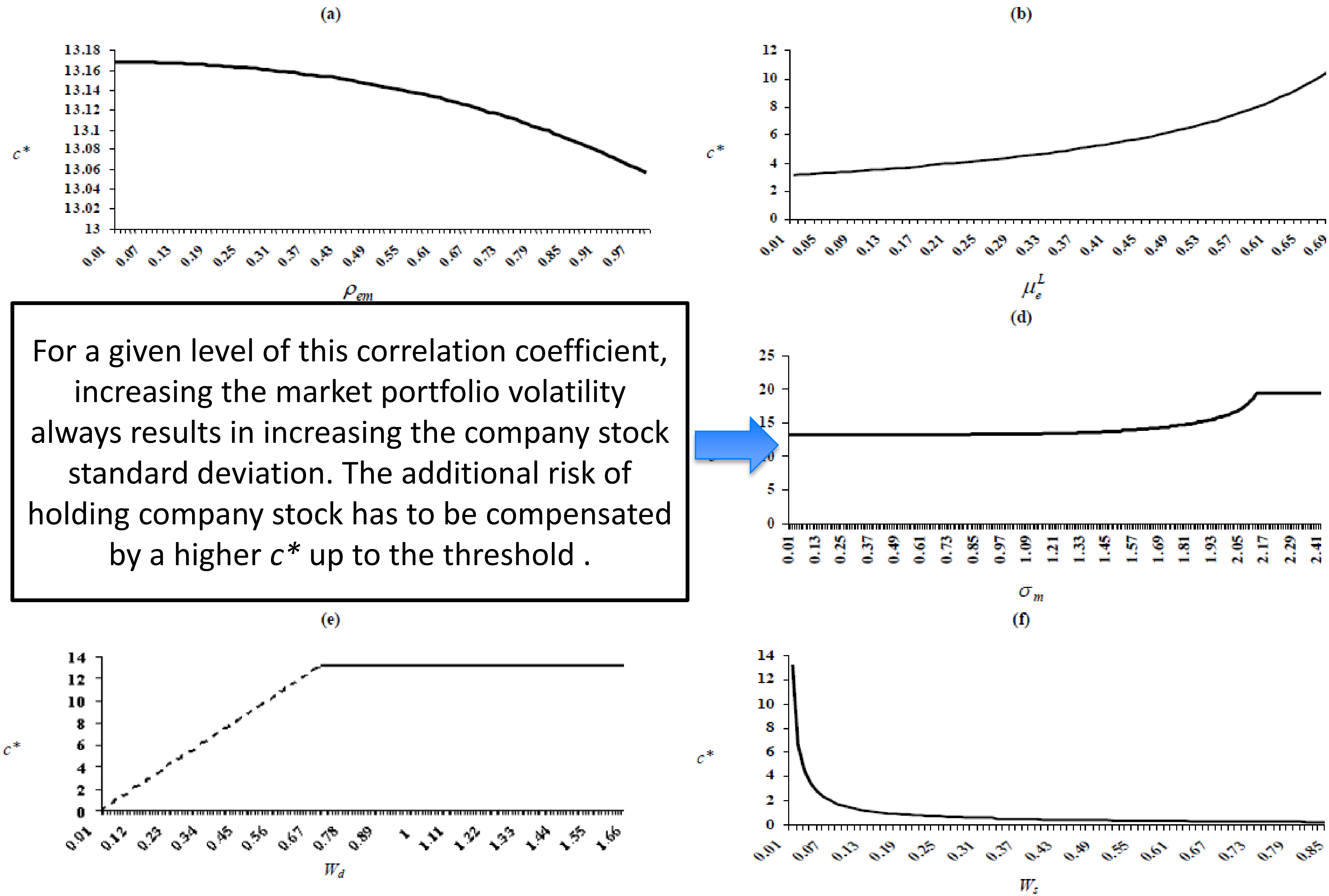
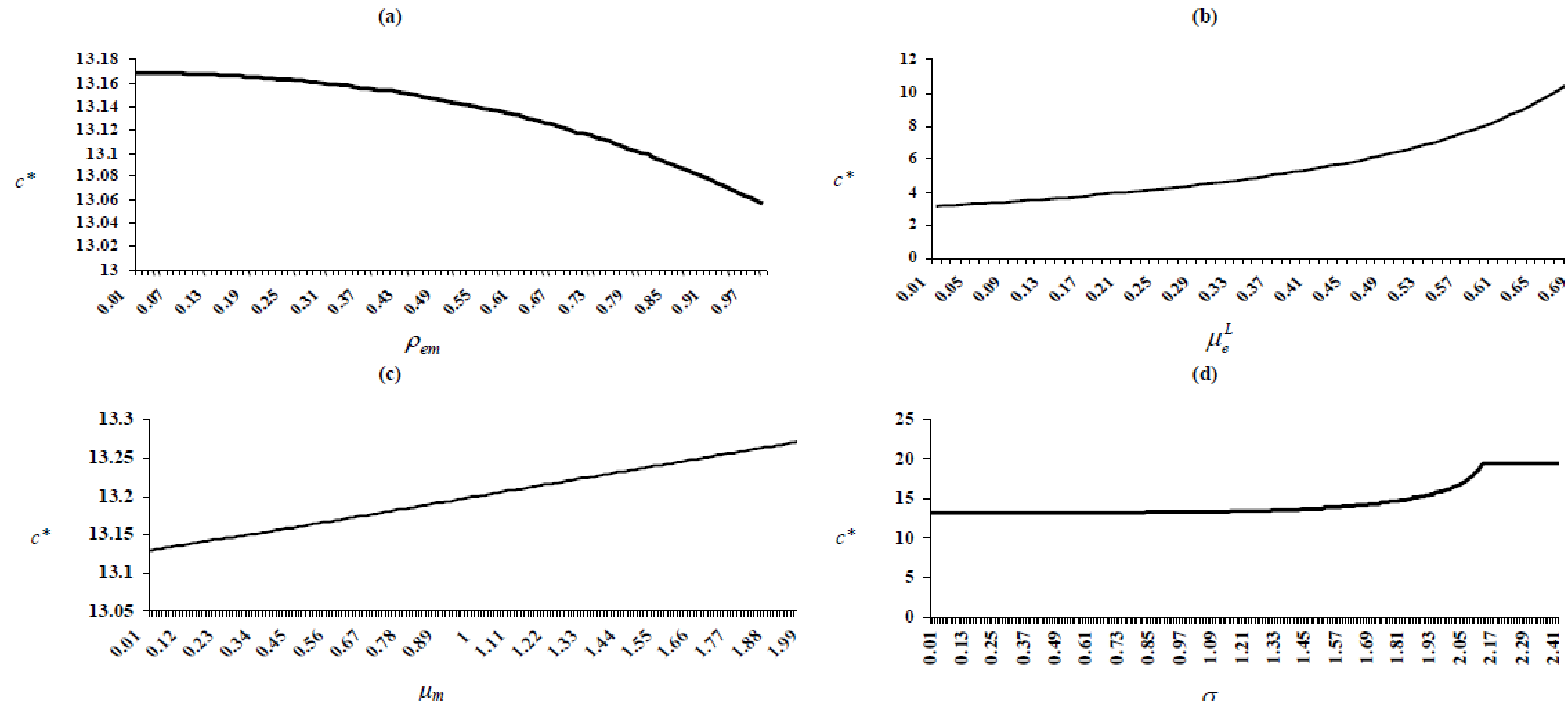
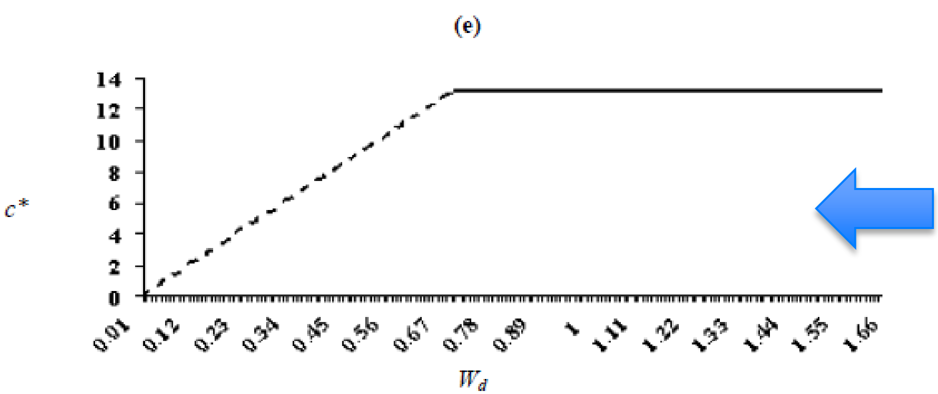


For a given level of this correlation coefficient, increasing the market portfolio volatility always results in increasing the company stock standard deviation. The additional risk of holding company stock has to be compensated by a higher c^* up to the threshold .

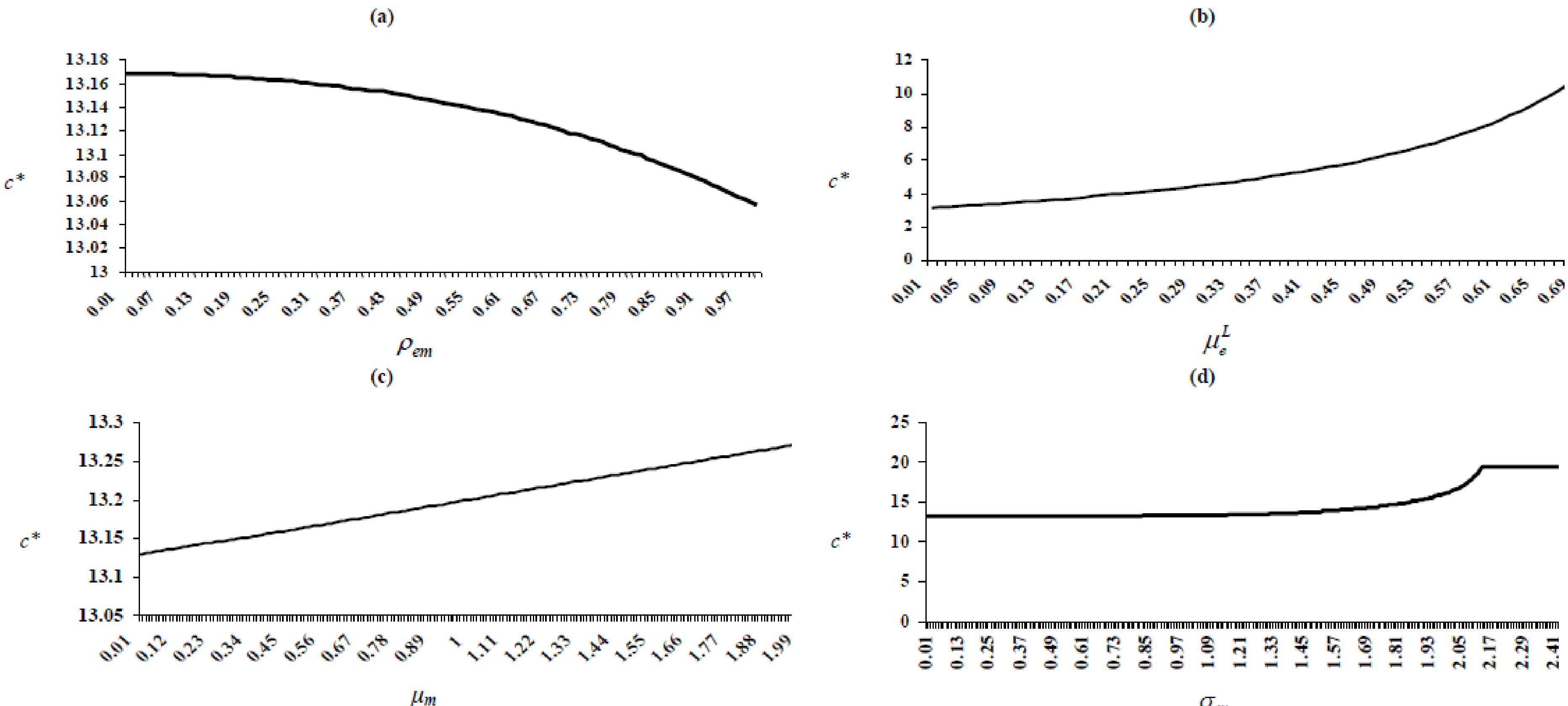
Figure 3: Relationships between c^* and the variables of the model



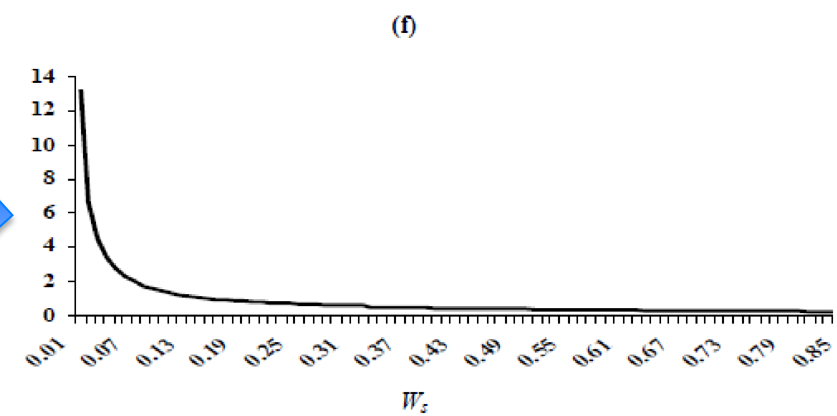


As entrepreneur’s wealth increases, expands to a certain level where it becomes high enough to compensate employee’s effort. Behind this level, the conditions of the perfect Nash equilibrum in sub-game are not satisfied.

Figure 3: Relationships between c^* and the variables of the model



The employee’s cost of exerting a high level of effort remains constant, so is the money value of company stock he is granted c^*W_s.



6. Concluding remarks

Identifying circumstances under which employee ownership is an optimal strategy.

- There is an optimal transfer of employee ownership that satisfies employee's risk preference and has an incentive effect.

Future work: Conditions under which company stock can be included in a long run investment strategy could be emphasized.

Employee ownership (EO):

A theoretical and empirical investigation of Management entrenchment vs reward management

Published in Economic Modelling in 2014

Nicolas AUBERT
Guillaume GARNOTEL
André LAPIED
Patrick ROUSSEAU

Outline

1. Literature: EO: a two-edged sword
2. The model
3. The employee ownership contracts
4. Empirical study (not presented)
5. Concluding remarks

1. Literature: EO a two edged sword

Employee Ownership has two outcomes:

- Bright side
  - Surveys: commitment/involvement, satisfaction/motivation
  - Secondary data: Turnover, absenteeism
- Dark side
  - "Employees and top managers are natural allies against takeover" (Pagano & Volpin, 2005, also Rauh, 2006)
  - The manager influences the level of EO (discount, matching contribution…) and generates "endorsement effect"

Our objectives:

- To investigate how these two outcomes are related
- To provide an empirical investigation

2. The model (1)

t=0

The nature chooses the management's type $k=\{G,B\}$: good (G) with a probability p_0 or bad (B) with a probability $1 - p_0$.

t=1

The manager defines the level of employee ownership c granted to the employees ($c \geq 0$).

t=2

The employee observes the level of c and decides the level of effort implemented $j=\{H,L\}$: high (H) or low (L).

FIGURE 1: TIME LINE

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FIGURE 1: TIME LINE

2. The model (2)

- Risk neutral manager's utility function
  - Without EO: $V^{0,j,k} = \int_{-\infty}^{+\infty} W_d(1 + r + \mu^{j,k}) f(r) dr \qquad (1)$

W_d: Manager's initial wealth
r: return on company stock with a probability distribution f(r)
μ: mean return on company stock with $\mu = f(j\text{:effort}, k\text{:manager's type})$

2. The model (2)

- Risk neutral manager's utility function
  - Without EO: $V^{0,j,k} = \int_{-\infty}^{+\infty} W_d(1 + r + \mu^{j,k}) f(r) dr$ (1)
  - With EO:

$$V^{c,j,k} = \int_{-\infty}^{+\infty} (W_d - cW_s)(1 + r + \mu^{j,k}) f(r) d(r)$$

W_d: Manager's initial wealth

r: return on company stock with a probability distribution f(r)

μ: mean return on company stock with $\mu = f(j$:effort, k:manager's type$)$

W_s: Employee's initial wealth

c: the manager's payment in company stock to the employee

2. The model (2)

W_s: Employee's initial wealth
Ψ: disutility of effort increasing with the level of effort
r_o: risk free asset return

- Risk averse worker's utility function
  - Without EO: $U^{0,j} = u[W_s(1+r_0)] - \psi(e^j)$ (3)

2. The model (2)

W_s: Employee's initial wealth
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- Risk averse worker's utility function
  - Without EO: $U^{0,j} = u[W_s(1+r_0)] - \psi(e^j)$ (3)
  - With EO:

$$U^{c,j} = \int_{-\infty}^{+\infty} [p_0 u[W_s(1+r_0) + cW_s(1+r+\mu^{j,G})] + (1-p_0)u[W_s(1+r_0) + cW_s(1+r+\mu^{j,B})]] f(r)dr - \psi(e^j) \quad (4)$$

2. The model (2)

W_s: Employee's initial wealth
Ψ: disutility of effort increasing with the level of effort
r_o: risk free asset return
p_o: proportion of good managers

- Risk averse worker's utility function
  - Without EO: $$U^{0,j} = u[W_s(1 + r_0)] - \psi(e^j) \tag{3}$$
  - With EO:

Good managers

$$U^{c,j} = \int_{-\infty}^{+\infty} [p_0 u[W_s(1 + r_0) + cW_s(1 + r + \mu^{j,G})] + (1 - p_0)u[W_s(1 + r_0) + cW_s(1 + r + \mu^{j,B})]] f(r)dr - \psi(e^j) \tag{4}$$

Bad managers

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  - Without EO: $U^{0,j} = u[W_s(1+r_0)] - \psi(e^j)$ (3)
  - With EO:

Good managers

$$U^{c,j} = \int_{-\infty}^{+\infty} [p_0 u[W_s(1+r_0) + cW_s(1+r+\mu^{j,G})] \qquad (4)$$
$$+ (1-p_0)u[W_s(1+r_0) + cW_s(1+r+\mu^{j,B})]]f(r)dr - \psi(e^j)$$

Bad managers

2. The model (3)

Assumption 1: The employee is risk averse and cautious[5].

$$\forall x : u'(x) > 0,\ u''(x) < 0,\ RRA(x) = -x\frac{u''(x)}{u'(x)} \leq 1 \tag{5}$$

Assumption 2: The employee's expected utility is strictly positive when company stock is not granted, and effort is low:

$$U^{0,L} = u[W_s(1 + r_0)] - \psi(e^L) > 0 \tag{6}$$

Assumption 3: The employee's expected utility is higher with employee ownership for a given level of effort:

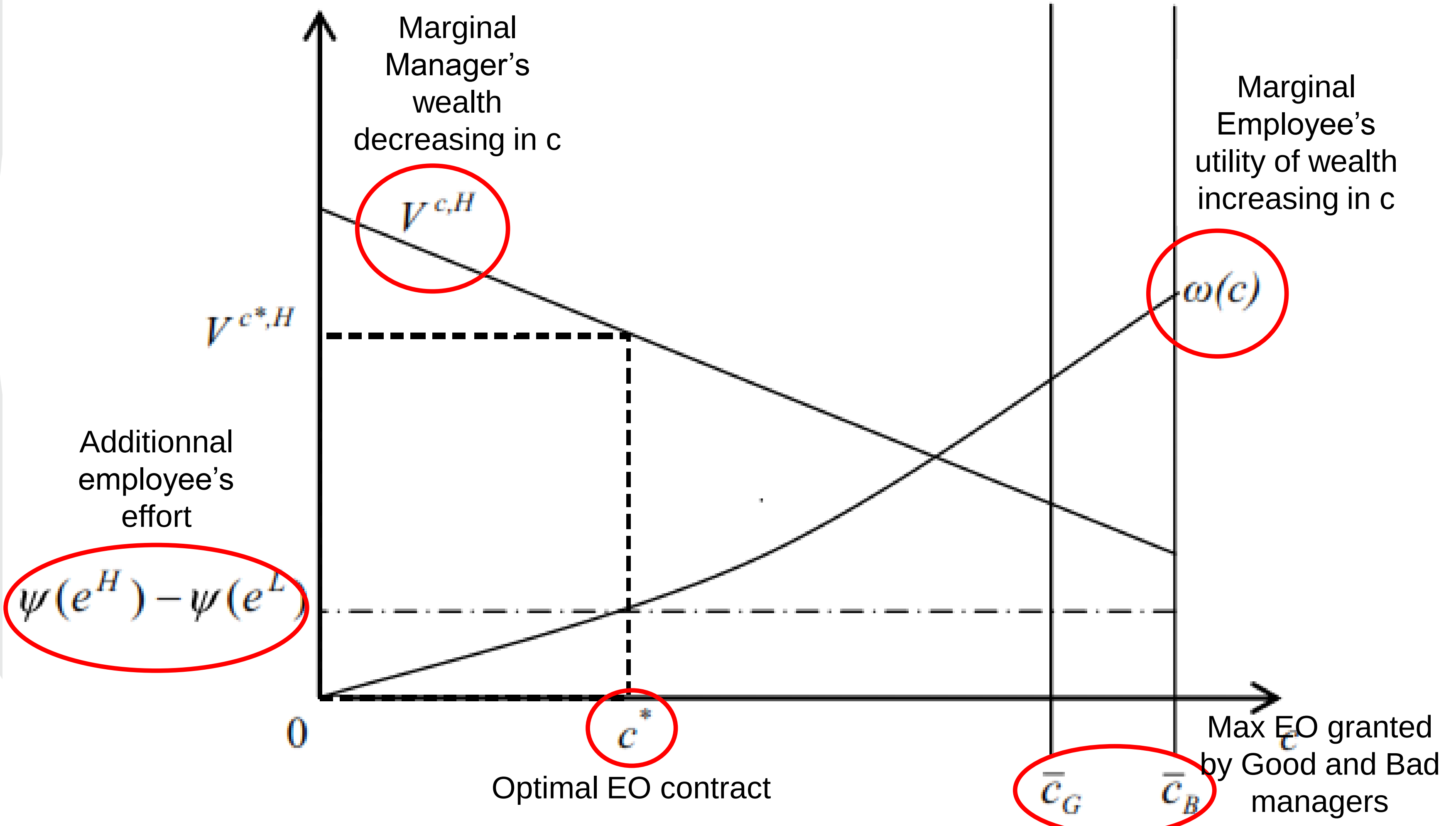
$$\begin{aligned} U^{c,j} = \int_{-\infty}^{+\infty} & [p_0 u[W_s(1+r_0) + cW_s(1 + r + \mu^{j,G})] \\ & + (1-p_0)u[W_s(1+r_0) + cW_s(1+r+\mu^{j,B})]]f(r)dr - \psi(e^j) \\ & \geq U^{0,j} = u[W_s(1+r_0)] - \psi(e^j),\ \forall c > 0,\ \forall j = \{H, L\} \end{aligned} \tag{7}$$

3. The EO contracts (proposition 1)

3.1. EO without threat of manager dismissal (Aubert et al, 2009)

- The employee exerts a high level of effort
- The manager grants c* below $\overline{c}_k$

◉ *c** is the optimal contribution $c^* \in (0, \overline{c}_k]$



3. The EO contracts (proposition 2)

3.2. EO with threat of manager dismissal (prop. 2)

- New variables whose relative positions on the figure is important (slides 29 & 30) :
  - Minimum performance condition for the manager: V_m
  - Minimum c guaranteeing manager cannot be dismissed: c_m

- Different strategies according to the level of V_m

i) No entrenchment: $V^{c^*,H,G} > V^{c^*,H,B} \geq \bar{V}_m$

ii) Everyone's entrenchment: $V_m > V^{c^*,H,G} > V^{c^*,H,B}$

iii) Bad manager's entrenchment: $V^{c^*,H,G} \geq \bar{V}_m > V^{c^*,H,B}$

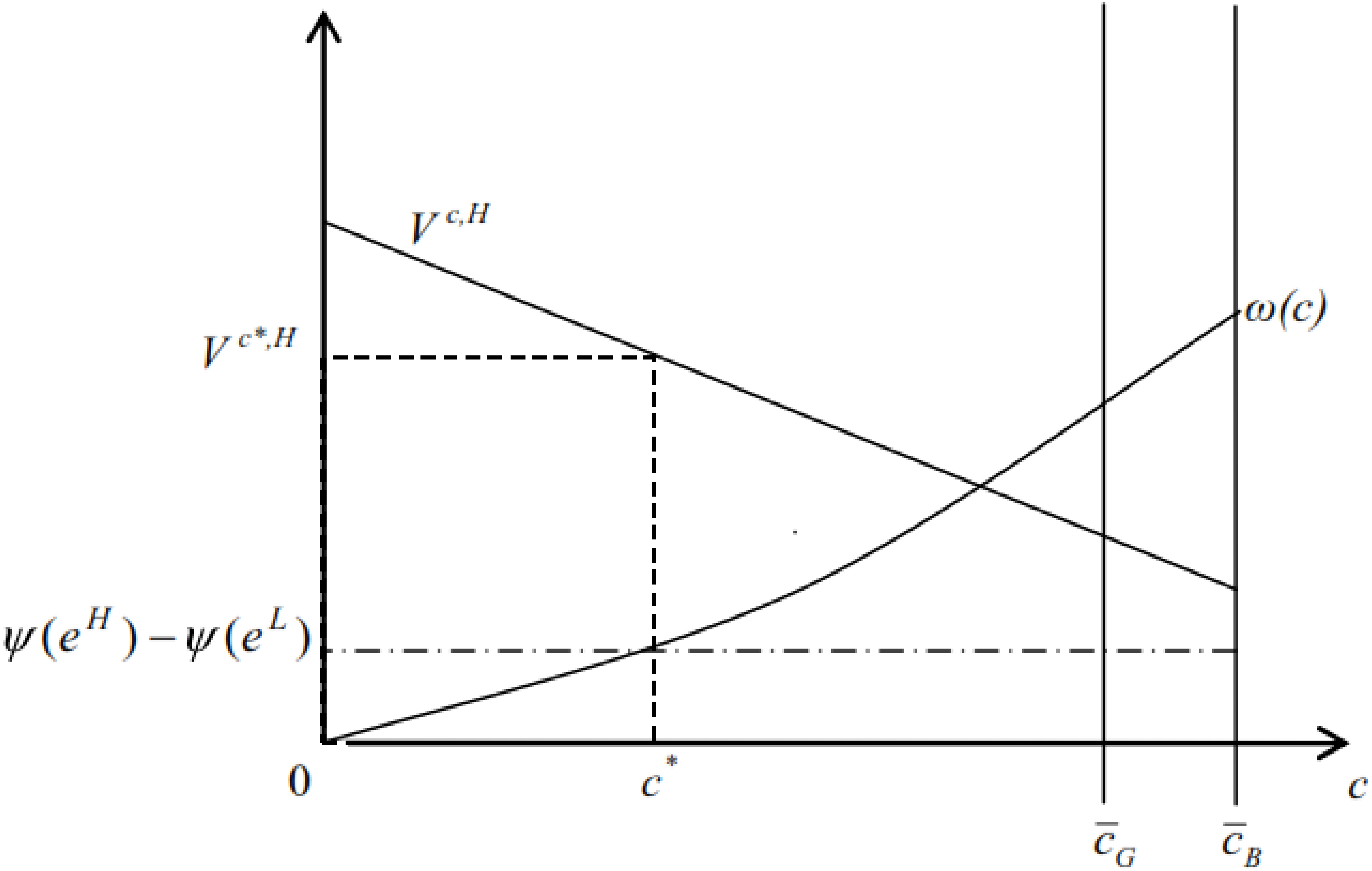
3. The EO contracts (prop. 2 continued)

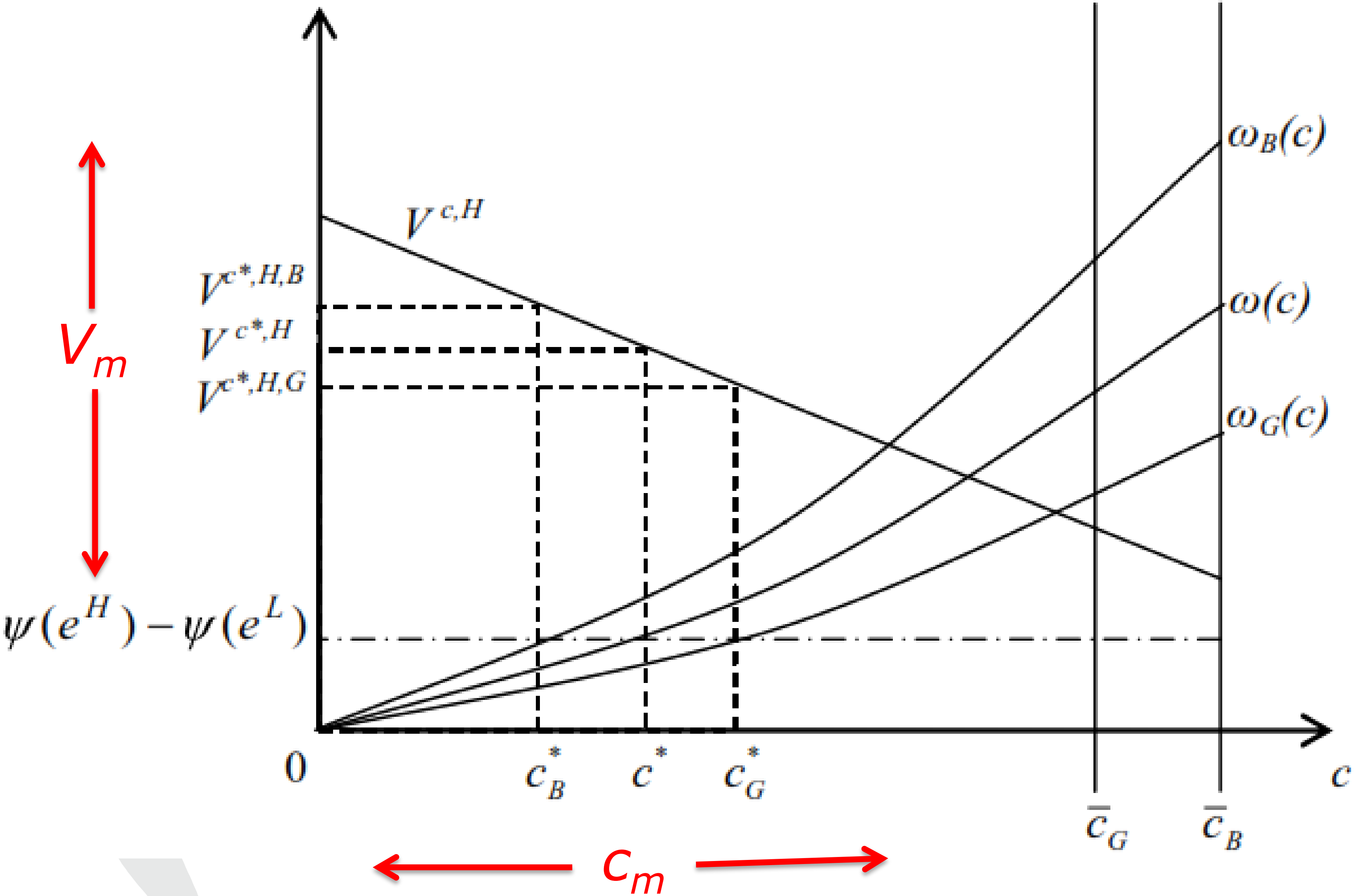
Let's focus on case iii (separating equilibrium) where the employee is able to identify the manager's type for sure

Interesting property: $c_B^* < c^* < c_G^*$

Equilibria :

- If $c_m < c^*$
  - Everyone plays c^*
- If $c_m > c^*$
  - Good manager plays c_G^* if they are not threatened
    $Max(c_G^*, c_m)$ if they are
  - Bad manager plays c_B^* if they are not threatened
    c_m if they are


$V^{c,H}$
$V^{c^*,H}$
$\omega(c)$
$\psi(e^H) - \psi(e^L)$
0
c^*
c
$\bar{c}_G$
$\bar{c}_B$


$V^{c,H}$
$V^{c^*,H,B}$
$V^{c^*,H}$
$V^{c^*,H,G}$
V_m
$\psi(e^H)-\psi(e^L)$
$\omega_B(c)$
$\omega(c)$
$\omega_G(c)$
0
c_B^*
c^*
c_G^*
c
$\bar{c}_G$
$\bar{c}_B$
C_m

3. The EO contracts (proposition 3)

3.3. Managerial behavior: cheating and commitment

- In the specific case where: $V^{c^*,H,G} \geq V_m > V^{c^*,H,B}$ and $c_G^* < c_m$

- Good and bad managers have an incentive to cheat
  - The bad plays c^*_G *(i.e. the good manager's contribution)*
  - The good plays c^*_B *(i.e. the bad manager's contribution)*

- Commitments help to solve this problem
  - EO should be defined before the nature selects the manager's type

4. Concluding remarks

A new perspective on employee ownership

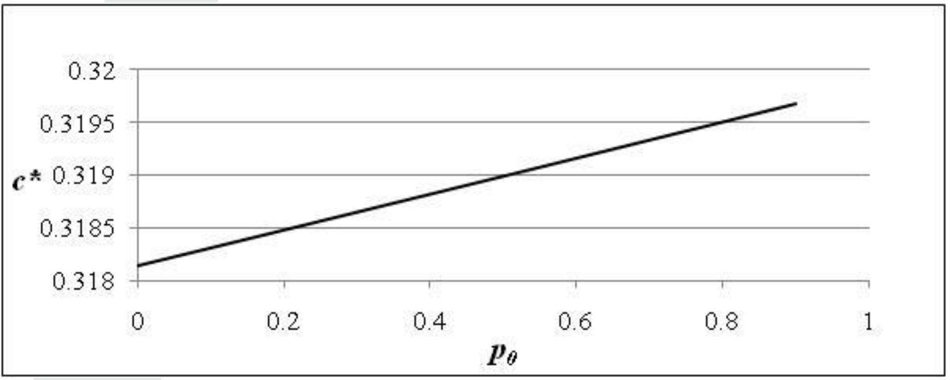
Theoretical predictions:

- Positive
  - EO is used as an entrenchment mechanism
  - Good managers signal themselves with EO
- Normative
  - Compensation policy involving EO should be defined before the manager is hired => compensation commitee

Empirical results validate prediction 1

Comparative statics: relationships with c* (1)

$\bar{c}_G$; c^*; c^*_B ; c^*_G



Relationship between:

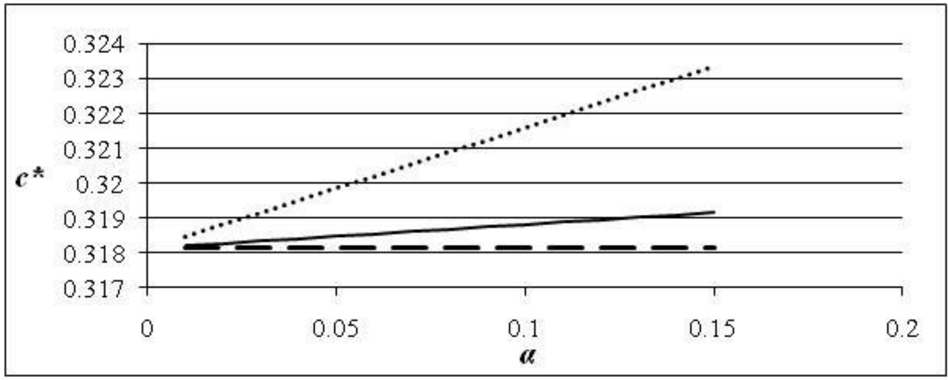
*The proportion of good managers (p0) and c**

Comparative statics: relationships with c* (1)

$\bar{c}_G$; c^* ; c^*_B ; c^*_G

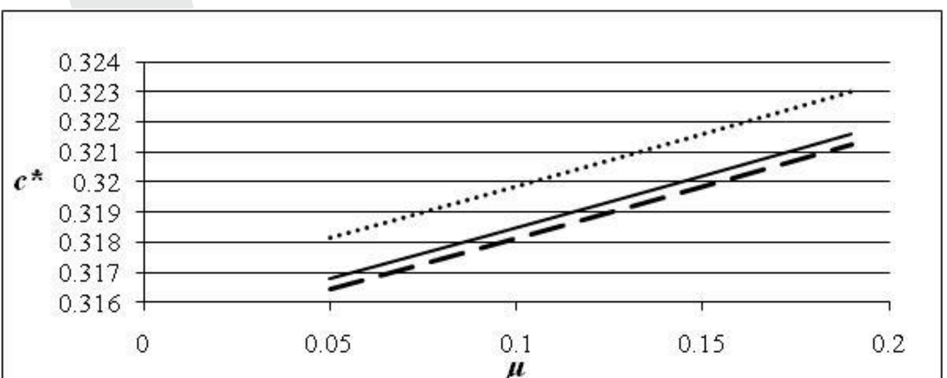
Relationship between:

*The mean return difference between good and bad managers (α) and c**



Comparative statics: relationships with c* (1)

$\bar{c}_G$; c^*; c^*_B; c^*_G



Relationship between:

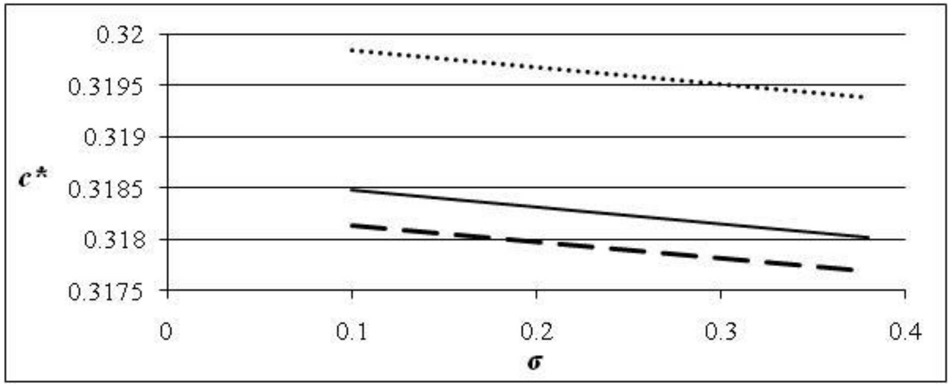
*The mean return on company stock (μ) and c**

Comparative statics: relationships with c* (1)

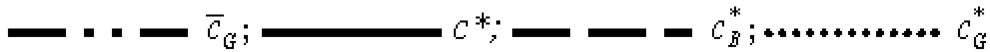
$\bar{c}_G$; c^*; c_B^*; c_G^*

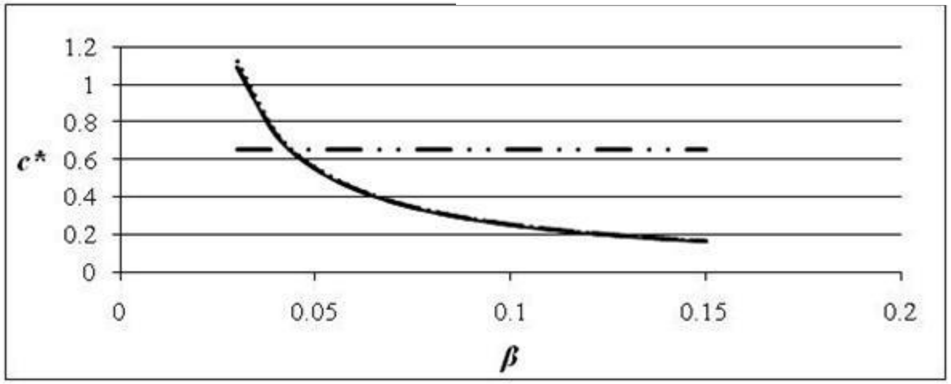
Relationship between:

*The company stock return risk (σ) and c**



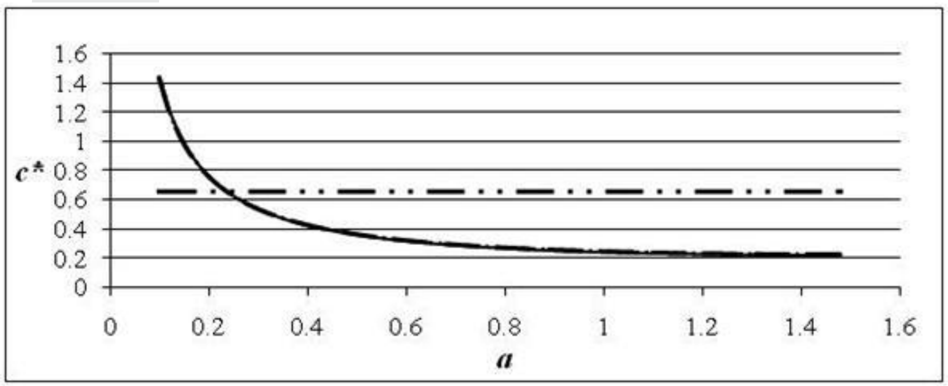
Comparative statics: relationships with c* (2)





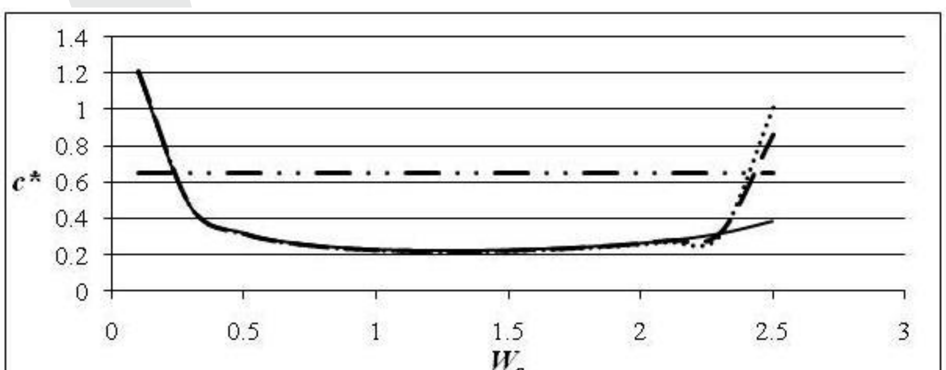
Relationship between:

*The mean return difference between high and low effort (β) and c**



Relationship between:

*The employee's absolute risk aversion (a) and c**



Relationship between:

*The employee's initial wealth (Ws) and c**

Optimal employee ownership contracts under ambiguity aversion

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Abstract

The aim of this paper is to introduce ambiguity aversion in the framework of employee ownership. We extend the result of Aubert et al (2014) in the framework of ambiguity using the multiple priors preferences of Gilboa and Schmeidler (1989). We provide a general result about optimal employee ownership under ambiguity. In this paper, we focus on the most common situation where employee ownership affects positively corporate performance but can be used as an entrenchment mechanism. The optimal contribution in company stock is determined in the situation described by a perfect subgame Nash equilibrium.

Contributions to the literature

The decision of implementing and developing employee ownership often lies with management. This paper is an attempt to investigate strategic interactions between employees and managers regarding employee ownership contracts assuming ambiguity aversion.
Managers have two motivations to offer company stock to their employees: to incentivize the employees or to keep their job. Another body of literature is behavioral finance. It started investigating why employees hold company stocks in their defined contribution pension plan back in the 2000s after Enron's bankrupt.

The major innovation of this paper is to introduce ambiguity aversion in a model's set up previously developed by Aubert et al (2014). They considered management monetary contributions to the development of employee ownership as revealing management's type. A good manager stimulates employee ownership because he wants to increase corporate performance whereas a bad manager develops it because it can help him to keep his position.

Our motivation is to extend Aubert et al (2014)'s model is to generalize it to situations where economic agents are affected by ambiguity aversion. We then extend their results.

Literature

Previous literature investigated cognitive biases motivating investment in company stock:
- excessive extrapolation of past returns and endorsement (Benartzi, 2001);
- loyalty and familiarity (Cohen, 2009; Huberman, 2001);
- framing effect (Benartzi and Thaler, 1999, 2002);
- risk myopia (Mitchell and Utkus, 2003);
- disposition effect (Choi et al, 2004);
- default heuristic (Madrian and Shea, 2001).

Ambiguity aversion has not been investigated so far either theoretically or empirically. However, ambiguity aversion affects less sophisticated individual investors like employees who are not always able to infer the probability distribution of stock returns.

The notion of ambiguity aversion provides a foundation for standard exercises of comparative statics in ambiguity for multiple priors preferences that are based on the size of the set of priors. This is the case of Hansen and Sargent (2011) multiplier preferences, which are easily seen to be probabilistically sophisticated. Remaining in the Von Neumann-Morgenstern formalization and particularly in Savage's model, Gilboa and Schmeidler (1989) propose accommodating ambiguity within economic decision making and assume that in the presence of ambiguity, individuals cannot identify a single probability distribution over states of nature. They thus consider multiple probability distributions and then evaluate their choices according to the worst probability distribution for that choice. This is the approach based on multiple priors.

Model

DEFINITION OF THE PARAMETERS

COMPENSATION SYSTEM

c	The amount of company stock granted to the employee. It takes the value of c or 0. c is a proportion of W_s, the employee's wealth
$i = \{0,c\}$	The level of the contribution paid by the manager to the employee in company stock

THE ECONOMY

$k = \{G,B\}$	Type of the manager, G for good and B for bad.
$j = \{H,L\}$	Level of the employee's effort: H for high and L for low
$P_0(G) = P_0$	Probability of having a good manager
$P_0(B) = 1 - P_0$	Probability of having a bad manager
$R + M^{j,k}$	Rate of return on company stock
R	R is a random variable with a zero mean, a volatility s and a density f(r).
$M^{j,k}$	The excess return of the company with mean $\mu^{j,k}$ and takes different values according to manager's type k and employee's effort j.
$\mu^{L,B} \leq Min\left(\mu^{L,G}, \mu^{H,B}\right)$ $\mu^{H,G} \geq Max\left(\mu^{L,G}, \mu^{H,B}\right)$	Rational conditions: Ceteris paribus, it follows that a better manager or higher employee effort increases the rate of return on the company stock.

EMPLOYER'S UTILITY FUNCTION

$V^{i,j,k}$	The risk neutral manager's utility expected function with $i = 0, c$
W_d	The positive level of employer's initial wealth totally invested in company stock.
cW_s	Amount of company stock granted to the employee

EMPLOYEE'S UTILITY FUNCTION

$U^{i,j}$	The Von Neuman Morgenstern risk averse employee's utility function
W_s	The positive level of employee's initial wealth
$\psi(e^j)$	The disutility of effort endured by the employee
r_0	Risk free asset rate of return

MODELLING AMBIGUITY (Maccheroni et al., 2006)

$\mathcal{P}_{R,\mu^{j,k,m}}$	Manager's compact set of multipriors
$\mathcal{P}_{R,\mu^{j,k,e}}$	Employee's compact set of multipriors
$\mathbb{P}_{R,\mu^{j,k,m}}$	Probabilities of the manager's set of multipriors
$\mathbb{P}_{R,\mu^{j,k,e}}$	Probabilities of the employee's set of multipriors
C_m	Manager's ambiguity index
C_e	Employee's ambiguity index

Utility functions

Employer's utility function

$$V^{0,j,k} = \left(W_d \ \mathbb{E}\left[1 + \mu^{0,j,k,m,*}\right] + C_m(\mathbb{P}_{R,\mu^{j,k,m,*}})\right),$$

for a given $\mathbb{P}^*_{m,0}\left(R, \mu^{j,k,m}\right)$ *in* Δ_m, *and* $V^{c,j,k}$ *corresponds to*

$$V^{c,j,k} = \left((W_d - cW_s)\mathbb{E}\left[\ \left(1 + \mu^{c,j,k,m,*}\right)\right] + C_m(\mathbb{P}^*_{m,c}\left(R, \mu^{j,k,m}\right)\right),$$

for another given $\mathbb{P}^*_{m,c}\left(R, \mu^{j,k,m}\right)$ *in* Δ_m.

Employee's utility function

$$\mathbb{E}\left(\begin{array}{c} p_0 u[W_s(1+r_0) + cW_s(1+R+M^{c,j,G,e*})] \\ +(1-p_0)u[W_s(1+r_0) + cW_s(1+R+M^{c,j,B,e*})] \end{array} + C_e(\mathbb{P}_{R,\mu^{c,j,e*}})\right)$$
$$-\psi(e^j). \quad (9)$$

for a given $\mathbb{P}_{R,\mu^{c,j,e*}}$ *in* Δ_e.

Propositions

Proposition 1

Suppose that both ambiguity indices C_m and C_e are null (as in the framework of Gilboa and Schmeidler, 1989). Assume also that we deal with the basic example (5). Assume that $M^{j,k}$ is characterized by its expectation $\mu^{j,k,\cdot}$ with $\mu^{j,k,\cdot} \in [\underline{\mu}^{j,k,\cdot}, \overline{\mu}^{j,k,\cdot}]$ Suppose also that ambiguity exists only on $\mu^{j,k,m} \in [\underline{\mu}^{j,k,m}, \overline{\mu}^{j,k,m}]$ and $\mu^{j,k,e} \in [\underline{\mu}^{j,k,e}, \overline{\mu}^{j,k,e}]$ with $\underline{\mu}^{j,k,m} = \underline{\mu}^{j,k,e}$. Then, under all previous assumptions, we recover exactly the model introduced by Aubert et al. (2013) with $\mu = \underline{\mu}^{L,B,m} = \underline{\mu}^{L,B,,e}; \mu + \alpha = \underline{\mu}^{L,G,m} = \underline{\mu}^{L,G,,e}; \mu + \beta = \underline{\mu}^{H,B,m} = \underline{\mu}^{H,B,e}$ and finally $\mu + \alpha + \beta = \underline{\mu}^{j,H,G} = \underline{\mu}^{j,H,G}$.

Proposition 2

Under assumption (9), the difference between the employee's utilities of wealth with high and low levels of effort defined by $\omega(c)$ has the following properties: $\omega(0) = 0$ and $\omega\prime(c) > 0$.

Proposition 3

If $C_m(\mathbb{P}^*_{m,c}\left(R, \mu^{j,k,m}\right)$ is decreasing with respect to c, the threshold $\widetilde{c_k}$ is the unique solution of Equation (14).

If the ambiguity index is null (i.e. $C_m = 0$), then:

$$\widetilde{c_k} = \frac{W_d}{W_s}\frac{\mu^{H,k,m,*} - \mu^{L,k,m,*}}{1 + \mu^{H,k,m,*}}. \quad (16)$$

Proposition 4

Under previous assumptions, for $\omega^{-1}[\psi(e^H) - \psi(e^L)] \leq \overline{c_G}$, there exists a unique $c^e = c^* \in (0, \overline{c_G}]$, which is the perfect subgame Nash equilibrium. With c^e, the employee expends a high level of effort ($j = H$) regardless of the manager's type.

The level c^* is given by the following relationship:

$$\omega(c^*) = \psi(e^H) - \psi(e^L). \quad (17)$$

Main references

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Reminder on ambiguity aversion

Famous example: Ellsberg's experiment

Risk averse: preference for the certain equivalent (ex: 10$) over a bet (ex: 50% chance to get 0 and 50% chance to get 20$ => expectation is 10$)

Ambiguity averse: preference for the 50/50 bet over a bet you do not know the distribution => distribution could be 0/100, 10/90, 50/50 etc. => multiple priors

Definition

Ambiguity aversion or uncertainty aversion refers to the preference for known risks (capacity to assign a probability to each outcome) over unknown risks (incapacity to do it).

More technically, the presence of ambiguity reflects the nonuniqueness of prior probabilities over sets of states. Rather than having in mind a unique probability to determine expected utility, the agent faces a whole set of possible probabilities, or so-called prior probabilities.

Modelling ambiguity: Gilboa & Schmeidler (1989)

Gilboa & Schmeidler (1989) develop a way to rationalize ambiguity

An ambiguity averse agent behaves as if he has multiple subjective probabilities

Maxmin expected utility: the agent has multiple probability distributions in mind and choose according to the worst probability distribution. They try to maximize the minimum expected utility of the act chosen.

Ex: in the Ellsberg's experiment, if they think they have a chance of winning comprised between 40% and 60%, they don't take the bet because 40% is lower than 50%