

# OPTIMAL STOPPING FOR MEASURE-VALUED PIECEWISE DETERMINISTIC MARKOV PROCESSES, APPLICATION TO POPULATION DYNAMICS.

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**Abstract:** In our work, we study an optimal stopping problem for specific Markov processes: Piecewise Deterministic Markov Processes (PDMP). Those time-continuous processes were formalized by Davis in [2]. The only source of randomness in those Markov processes comes from the jumps. The jump times are drawn in a Poisson-like fashion. Between jumps, the process follows a deterministic trajectory, given by the flow of some differential equation. In Davis' work, the state space of PDMP is in  $\mathbb{R}^d$  for some integers  $d$ . We extend this vision to measure-valued processes.

We introduce such measure-valued processes in order to model population dynamics problems, when the population size is small. The population characteristics can be described as follows: let  $n$  be the size of the population at some time  $t$ . Let  $x_i$  be a real number, representing some biological trait of the  $i$ -th individual. It can be the size, the weight, the concentration of some protein... So, at this time  $t$ , information about the population can be summarized with a locally finite measure  $\mu := \sum_{i=1}^n \delta_{x_i}$ . The process, at time  $t$ , takes this value  $\mu$ . It is related to a growth fragmentation model [1].

We investigate an optimal stopping problem for measure-valued PDMPs. The purpose is to select a stopping time  $\tau$  in order to maximize some mean reward  $g$  of the PDMP  $(X_t)_t$ :

$$\sup_{\tau} \mathbb{E}[g(X_{\tau})].$$

To solve our optimal stopping problem, we imitate the technique from the paper [4]. This paper considers the specific case of  $\mathbb{R}^d$ -valued processes. The optimal performance is called the value function of the optimal stopping problem. We prove that this value function can be recursively constructed by iterating a dynamic programming operator. We illustrate this work with a toy model of cell division. In particular, we prove that controlling the whole population is not equivalent to controlling a tagged cell, unlike other classical problems [3].

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