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# A Time-continuous Compartment Model for Building Evacuation 

Bertrand MAURY ${ }^{1,2}$, Sylvain FAURE ${ }^{1,2}$, Jacques ANGELÉ ${ }^{2}$ and Rémi BACHIMONT ${ }^{2}$

${ }^{1}$ Laboratoire de Mathématiques d’Orsay, Univ. Paris-Sud, CNRS, Université Paris-Saclay, 91405
Orsay, France
${ }^{2}$ Signactif, 260 avenue Aristide Briand - 92220 Bagneux - France.

Bertrand.Maury@math.u-psud.fr


#### Abstract

We propose here a general framework to estimate global evacuation times of complex buildings, and to dynamically investigate the dependence of this evacuation time upon various factors. This model relies on a network, which is in some way the skeleton of the building, the nodes of which are the bottlenecks or exit doors. Those nodes are connected by edges which correspond to portions of egress paths located within a given room. Such models have been proposed in a discrete setting. The model we propose takes the form of a continuous evolution equation of the differential type. It relies on a limited number of variables, namely the number of people gathered upstream each node, together with the number of people on their way from a node to the next one. The basic parameters of the model are the capacities of doors, and the time needed to walk from one node to the next one. In spite of its macroscopic character (the motions of pedestrians are not described individually), this approach allows to account for complex and nonlinear effects such as capacity drop at bottlenecks, congestion induced speed reduction, and possibly some dispersion in evacuees behaviors. We present here the basic version of the model, together with the numerical methodology which is used to solve the equations, and we illustrate the behavior of the algorithm by a comparison with experimental data.


## KEYWORDS :

modeling; human behavior; risk assessment; evacuation times; exit capacities;

## 1 Introduction

Providing accurate and robust estimates for evacuation times in complex buildings is a long-term challenge in public safety. A common dilemma resides in the opposition between microscopic and macroscopic approaches. Microscopic descriptions (see e.g. [5, 10, 11 allow for a precise description of evacuees interactions, possibly accounting for non uniformity of individual behavior (social tendencies, speed, ...), but they lead to higher, possibly prohibitive, computational times. Besides, they call for an accurate knowledge (at least statistically) of people characteristics, which is most of the time out of
reach. On the other hand, macroscopic models ( [6, 8) handle the crowd as a continuum, represented by a local density. The evolution of this density typically follows a conservative transport equation which expresses the "people conservation", and the core of the model lies in the manner the effective velocity is determined, based on individual tendencies and local density. This approach makes it possible to account for very large number of people at a reasonable computational cost. Yet, most models of this type are not able to reproduce some observable effects, like the Faster-is-Slower effect ([4), the Capacity Drop phenomenon (11), or the fluidizing role of an obstacle ([12]). Both approaches rely on a fine description of the behavior of individuals and their interactions with neighbors (finite number of finite-size individuals for the microscopic setting, infinitely many point particles for the macroscopic setting).

If one aims at predicting the evacuation time of complex building, it may be of interest to use a coarse grain description of the crowd, based on quantities that are directly observable and measurable. This approach, which we may call systemic, is based on a decomposition of the building into various compartments, corresponding to distincts areas (like rooms, halls, or corridors). Those compartments are connected by doors, and the balance of global headcounts in compartments is driven by fluxes between them. The model we propose here is based on a network defined as follows: the nodes of the network are the exits of the various compartments. Each compartment has a certain number of entrances, each of which is the exit of an compartment upstream. Each entrance of a given room is connected by an oriented edge to the exit. We shall follow the convention that the exit points to the entrances, in such a way that the arrows (oriented edges) express a dependence relation. More precisely, a node $i$ points to a node $j$ if $i$ is influenced by the situation of $j$ (possibly in the past). The model which is presented here is not new in its constitutive principles. In particular the Capacity Constrained Routing Approach presented in [7] is based on the same type of network. The same type of model is also proposed in [3] to represent car traffic networks. The main novelty of the approach lies in the nature of the model, which is continuous in time, whereas previous approaches were essentially discrete. This continuous character gives a sound theoretical character to the approach, which can be used in particular to design rigorous methodologies for parameter identification. It also makes it possible to tune up the time step, which is in our setting a discretization parameter, depending on the situation which is considered.

While allowing for faster-than-real-time computations, the large granularity of the description level a priori rule out the possibility to properly describe small scale interactions between individuals, and this model may not be used to investigate in any way the cause of the aforementionned phenomena. Yet, the most relevant parameters (in particular capacities and node-to-node travel time) can be made dynamic, and allow for an account of phenomena like the Capacity Drop, or the reduction of the walking speed in case of congestion.

## 2 Model description, mathematical formulation

Let us start with a toy problem: A certain quantity of people is accumulated upstream a single exit door. Since we aim at setting a continuous model, applicable to large numbers of entities, we represent this quantity by a real number $N \in \mathbb{R}_{+}$. The capacity of the exit, that is the maximal number of individuals which can go through it per unit time is denoted by $C$. We denote by $f=f(t) \geq 0$ the incoming flux, that is the upstream flux of pedestrians, and by $\Phi=\Phi(t) \geq 0$ the instantaneous flux through the door. The incoming flux is assumed to be known. The balance at the door writes $d N / d t=f-\Phi$. The core of the model relies in the expression of $\Phi$ as a function of the dynamic
variables $N, f$, and the static parameter $C$. By definition of the capacity it holds that $\Phi \in[0, C]$, and $\Phi=C$ whenever $N>0$. When $N=0, \Phi$ lies between 0 and $C$. Its value is $f$ when $f<C$, but it may happen that $\Phi$ saturates to $C$ if $f>C$. To sum up, the evolution problem can be written

$$
\left\{\begin{array}{rlrl}
\frac{d N}{d t} & =f-\Phi &  \tag{2.1}\\
\Phi & =\left\lvert\, \begin{array}{lll}
C & \text { if } & N>0 \\
\arg \min _{\tilde{\Phi} \in[0, C]}|\tilde{\Phi}-f| & \text { if } & N=0
\end{array}\right.
\end{array}\right.
$$

The extension to a many-room building is built in a similar manner, by accounting for the fact that people flowing through some passage node reach the next one after some transit time which is a parameter of the model. We denote by $R$ the number of room / compartments, $m_{i}$ the number of inlet accesses to room $i$. For $n=1, \ldots, m_{i}$, we denote by $\alpha_{i}^{n}$ the index of the room upstream the access $n$ to room $i$. The time spent by an individual to walk (in room $i$ ) from entrance $n$ to the exit is $T_{i}^{n}$. Finally, $\Phi_{i}$ is the instantaneous flow rate of people through the exit of $i$. The problem writes

$$
\begin{align*}
\frac{d N_{i}}{d t} & =-\Phi_{i}+\sum_{n=1}^{m_{i}} \Phi_{\alpha_{i}^{n}}\left(t-T_{i}^{n}\right), \quad i=1, \ldots, R  \tag{2.2}\\
\Phi_{i} & =\left|\begin{array}{ll}
C_{i} & \text { if } \quad N_{i}>0 \\
\arg \min _{\tilde{\Phi} \in\left[0, C_{i}\right]} \mid \tilde{\Phi}-\sum_{n=1}^{m_{i}} \Phi_{\alpha_{i}^{n}}\left(t-T_{i}^{n}\right)
\end{array}\right| \quad \text { for } i=1, \ldots, R .
\end{align*}
$$

## 3 Numerical solution

In spite of non-trivial mathematical issues (see [9), designing numerical algorithms to solve this sort of problems is straightforward. Let $\tau>0$ denote a time step. To simplify the presentation, we shall assume that the transfer times are whole multiples of this time step, and denote by $\tilde{T}_{i}=T_{i} / \tau \in \mathbb{N}$ the corresponding dimensionless times. We shall furthermore assume that people are initially gathered in the neighborhood of exits. The approximation of $N_{i}$ at time $t^{k}=k \tau$ is denoted by $N_{i}^{k}$.

The scheme is actually simpler than the continuous model, since it simply expresses that, at some time $t^{k}$, the number of individual walking through exit $i$ between $t^{k}$ and $t^{k+1}$ is $C_{i} \tau$ whenever there is enough people available to achieve this full capacity, and $N_{i}$ otherwise. In other words, the average flux in this time interval is either $C_{i}$ or $N_{i} / \tau$.

$$
\left\lvert\, \begin{align*}
& \Phi_{i}^{k+1}=\min \left(C_{i}, N_{i}^{k} / \tau\right), \quad i=1, \ldots, R, \\
& N_{i}^{k+1}=N_{i}^{k}-\tau \Phi_{i}^{k+1}+\sum_{n=1}^{m_{i}} \tau \Phi_{\alpha_{i}^{n}}^{k+1-\tilde{T}_{i}^{n}}, i=1, \ldots, R, \tag{3.1}
\end{align*}\right.
$$

where $\Phi_{i}^{k}$ is set to 0 for all $k<0$ (no evacuation before the initial time), and $R$ is the number of rooms. Assuming, as we did in the continuous setting, that all people are initially gathered upstream doors, we supplement this system with initial conditions $N_{1}^{0}, \ldots, N_{R}^{0}$. We may add an extra equation to keep an account of people who have evacuated the building at time $k$ :

$$
N_{0}^{k+1}=N_{0}^{k}+\tau \Phi_{1}^{k+1} .
$$



Figure 3.1: Topography: network, geodesic distance to the gate, and typical paths

It is also straightforward, for each room $i$, to keep an account of the number $\bar{N}_{i, n}$ of individuals who are on their way to the exit of room $i$, coming from the exit of room $\alpha_{i}^{n}$ :

$$
\bar{N}_{i, n}^{k+1}=\bar{N}_{i, n}^{k}+\tau \Phi_{\alpha_{i}^{n}}^{k+1}-\tau \Phi_{\alpha_{i}^{n}}^{k+1-\tilde{T}_{i}^{n}} .
$$

Global people balance is straightforwardly obtained by summing up all discrete equations:

$$
N_{0}^{k}+\sum_{i=1}^{R} N_{i}^{k}+\sum_{i=1}^{R} \sum_{n=1}^{m_{i}} \bar{N}_{i, n}^{k}=\text { Constant } .
$$

## 4 Illustration: comparison to experimental data

The model is tested in the configuration presented in [2], were a full set of experimental evacuations is presented. The topography is represented in Fig. 3.1 (left): it is made of 10 interconnected compartments. The underlying oriented network is represented on the left-hand side of the figure. Transit times are automatically computed as follows: the common target is defined as the gate upstream thestairs, denoted by 2 on the figure. The geodesic distance to the target, which corresponds to the length of the shortest path (accounting for walls and obstacles) to the target, is quasi-instantaneously computed by mean of a fast marching algorithm. More precisely, distances are computed at the center of cells of a cartesian grid which covers the whole domain, in a frontal way, starting from the target where is it set at 0 , and then propagating backward upstream the domain. We refer to 9 (Chapter 8) for methodological and implementational details concerning those computations. The travel times are computed from those distances by setting the speed of pedestrian at $1 \mathrm{~m} / \mathrm{s}$. Isolines of this geodesic distance is represented in Fig. 3.1 (middle). To illustrate the evacuation, we also represent some egress paths computed with a microscopic model (10), based on an initial random distribution of individuals (Fig. 3.1, right).

The time needed to get from one node to the other can then be computed as the difference between the geodesic distances divided by the speed of pedestrians. The initial number of individual is 86 ,


Figure 4.1: Computed number of people gathered upstream the exit gate vs. time, for scenarios 4 (top) and 5 (bottom)
distributed over the rooms 2 to 10 (circled black indexes in Fig. 3.1, left). The headcount in each room is indicated by the numbers in red. The effect of various gate widths have been experimentally studied, and we reproduce with the present model the two scenarios, referred to as scenario 4 and scenario 5 , respectively, like in $[2]$. In scenario 4 , the exit gate is wide open ( 1.24 m ), and the corresponding capacity is estimated at $2.78 \mathrm{Ps}^{-1}$. In scenario 5 , the door is reduce to half its width ( 0.62 m ), with a capacity of $1.72 \mathrm{Ps}^{-1}$. We run the numerical model is both situations, and we illustrate the results by representing the computed number of people gathered upstream the exit gate versus the time. Fig. 4.1 represents the plots in the two scenarios. The model makes it possible to recover the "state" of this sensitive point when the evacuation time goes on. As expected (see the figure at the top), this number first decreases. Then, around time 8 s , people coming from upstream rooms start to reach this node. The gate continues to work at full capacity, but the incoming fluxes are higher, which explains the increase. The little irregularities on the curve between times 8 and 17 corresponds to instants at which the first people coming from some given upstream room reach the gate node. Then, in a final phase (after time 25), all people still in the building are gathered at the gate, and the crowd flows out at full door capacity. The second plot corresponds to a smaller capacity: the initial decreasing phase is less efficient, then, in the second phase, the slope is higher (the incoming flux is the same as previously, but the outflow smaller). We recover, like in the experiments, a larger evacuation time, around 50 s , to compare to the 32 s of Scenario 4.

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