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Non destructive testing using non linear vibroacoustic

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ABSTRACT. Several recent experimental studies show that it is possible to detect defects in a structure by considering its vibro-acoustic response to an external actuation. On this topic there is a vast literature in applied physics. We recall some papers related to the use of the frequency response for non destructive testing; in particular generation of higher harmonics, cross-modulation of a high frequency by a low frequency: We intend to present simple spring mass models, simple bar and beam models with damage and use **asymptotic expansions and numerical methods** to try to get results which show some similarity with the experiments of [P.D 03]. Asymptotic expansions have been used for at least a century and for example has been used recently for numerical approximation of bifurcation of structures in PotierFerry-Cochelin and coworkers (1993) [E.H 93]. **The key idea is to look at the solution in the frequency domain for the experiments and consequently for the numerics.**

RÉSUMÉ.

KEYWORDS: Non destructive testing, non linear vibroacoustic

MOTS-CLÉS : Essais non destructifs, vibroacoustique non linéaire

1. Some previous papers

- In Ekimov-Didenkulov-Kasakov (1999), [A.E 99],
- In Zaitsev-Sas (1999), [V.Z 99],
- Other results may be found in Sedunov-Tsionsky-Donskoy(2002) [D.D 02],Sutin-Donskoy (1998), [A.S 98], Moussatov-Castagnede-Gusev(2002), [GUS 02] ...
- GDR 2501 (Etude de la propagation ultrasonore en milieux inhomogènes en vue du controle non destructif)
- In Vanderborck-Lagier-Groby (2003) [P.D 03], "a vibro-acoustic method, based on frequency modulation, is developed in order to detect defects on aluminum and concrete beams.

Very recent experiments

- have been performed on a real bridge by G. Vanderborck with four prestressed cables: two undamaged cables, a damaged cable and a safe one but damaged at the anchor;
- See Vanderborck-Lagier(2004) [VAN] for a presentation of the results of the experiment with a new post processing graphic presentation of experimental results.

Difficulties of the experiments:

- non linearities of the shakers (including piezoelectric actuators)
- Natural non linearities: supports, links of complex multi structures as air planes, bridges etc

2. Simplest mechanical example

in which we can exhibit intermodulations. We consider a 1 d.o.f example of a spring mass system with a non linear spring.

$$\ddot{y} = -\omega_0^2 y - \frac{k_3}{m} y^3 + \epsilon \frac{F}{m} \sin(\alpha t) \quad \text{with } \omega_0^2 = \frac{k}{m} \quad (2.1)$$

$$\text{with initial conditions } y(0) = \epsilon \eta_1, \quad \dot{y} = \epsilon v_1 \quad (2.2)$$

We are going to solve this equation **symbolically** with an asymptotic expansion with respect to ϵ : $y = \epsilon y_1 + \epsilon^2 y_2 + \epsilon^3 y_3 + \dots$; then **numerically**...

The linear case

The first term is solution of:

$$\ddot{y}_1 = -\omega_0^2 y_1 + \frac{F}{m} \sin(\alpha t) \quad \text{with } y_1(0) = \eta_1, \quad \dot{y}_1 = v_1 \quad \text{which gives:} \quad (2.3)$$

$$\text{with } \phi = \frac{F}{m(\alpha^2 - \omega_0^2)}, \quad y_1 = \eta_1 \cos(\omega_0 t) + \left(\frac{v_1}{\omega_0} + \frac{\alpha}{\omega_0} \phi \right) \sin(\omega_0 t) - \phi \sin(\alpha t) \quad (2.4)$$

Remarque 1. If we set $\eta_1 = 0$, $v_1 = 0$, then the term of pulsation ω_0 has magnitude $\frac{\alpha}{\omega_0}$ times the magnitude of the term of pulsation α ; this is not a good choice for the non linear case in which $\frac{\alpha}{\omega_0}$ is of order 100; *it seems good choice* $\eta_1 = 0$, $v_1 = \omega_0 \left(-\frac{\alpha}{\omega_0} + 1 \right) \frac{F}{m(\alpha^2 - \omega_0^2)} = (-\alpha + \omega_0) \phi$

Other terms

The term y_2 is zero but the **third term** satisfies:

$$\ddot{y}_3 = -\omega_0^2 y_3 + \frac{k_1}{m} y_1^3 \quad (2.5)$$

to simplify, we assume $\eta_1 = 0$ and set $\phi = \frac{F}{m(\alpha^2 - \omega_0^2)}$, $\psi = \frac{v_1}{\omega_0} + \frac{\alpha\phi}{\omega_0}$ so that:

$$y_1^3 = (\psi \sin(\omega_0 t) - \phi \sin(\alpha t))^3 = \quad (2.6)$$

$$-1/4\psi^3 \sin(3\omega_0 t) + 3/2\psi(1/4\psi^2 + \phi^2) \sin(\omega_0 t) \quad (2.7)$$

$$-3/4\psi\phi^2 \sin((\omega_0 + 2\alpha)t) - 3/4\psi\phi^2 \sin((\omega_0 - 2\alpha)t) \quad (2.8)$$

$$+3/4\psi^2\phi \sin((\alpha + 2\omega_0)t) + 3/4\psi^2\phi \sin((\alpha - 2\omega_0)t) + \quad (2.9)$$

$$-3/2(\psi^2\phi + 1/2\phi^3) \sin(\alpha t) + 1/4\phi^3 \sin(3\alpha t) \quad (2.10)$$

If we go on in the expansion, we get terms of pulsation $\alpha + 4\omega_0$, $\alpha + 6\omega_0$ etc

General tendency:

– The pick of $3\omega_0$ is much larger than the pick in $\alpha \pm 2\omega_0$ which are the most natural picks in the experiments;

– it is delicate to find datas such that the secondary picks at $\alpha \pm 2\omega_0$

– actually appear when the differential equation is solved numerically.

– Question: algorithm and software for detecting the secondary picks?

– then find (by optimization) datas such that the secondary picks are important: criteria for damage.

3. Transverse vibrations: vibrating masses on stretched cables in large displacement

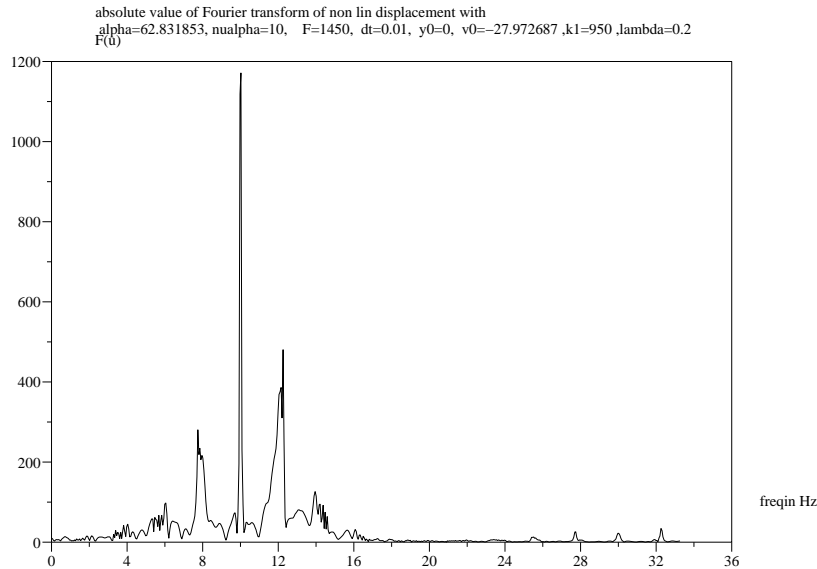
Work performed with Theissen (doctoral student of U. Muenster); Erasmus students N. Goris and I. Altrogge worked on this topic during their stay in UNSA (2004-2005). We consider n masses attached to horizontal springs (or cables) which are in tension T_0 , at rest ; the tension is positive when the cable is in traction which is assumed; at rest the mass m_i is submitted to the force T the masses are moving (vertically) transversely to the springs; we denote by upper case letters quantities in the rest position and lower case in the current configuration.

3.1. Masses in vertical displacement

Here we assume that the masses can move only vertically.

3.1.1. A possible damage of a cable

is breakage of several fibers, this will cause decrease of rigidity k_1 say for cable 1.



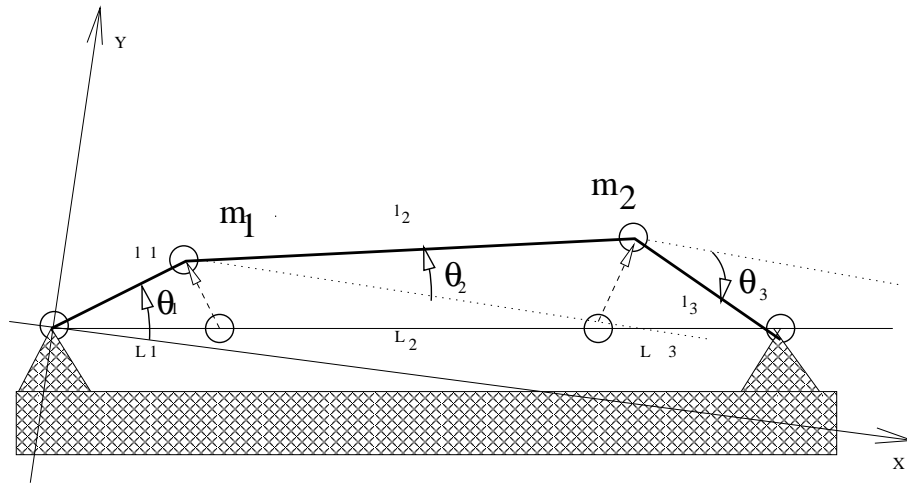
- Let us start with undamaged cables of same rigidity k . If we note L_0 , the common length of the unstressed cables, and L their common stressed length, their tension is $T_0 = k(L - L_0)$;
- now, after damage, $k_1 < k = k_2$, cable 1 becomes longer and cable 2 shorter, $L_1 > L_2$, the tension goes down to $T_{00} = k_1(L_1 - L_0) = k_2(L_2 - L_0)$;
- note the limit case of cable 1 broken is $k_1 = 0$ so that the cable 2 gets length L_0 but the system is no longer working properly!
- Before such a breakdown, if the change of tension is substantial, this causes a substantial change of the fundamental frequency; indeed, this is the routine monitoring of cable bridges!
- The nonlinear vibroacoustic testing aims at monitoring the cables before such a substantial change.

4. A non linear string model

A model of non linear string has been introduced first by Kirchoff in 1877 and rederived by Carrier in 1945.

$$y_{tt} - T \left(\int_0^l y_x^2 \right) y_{xx} = f \tag{4.1}$$

Several mathematical studies of this type of equations have been performed recently (Medeiros(1994), Clark- Lima (1997)).



Two masses on stretched cables (cable 2 damaged) moving freely

5. Masses free to move in a plane

Equation of the dynamics

$$m_i x_i'' = -T_i \cos(\theta_i) + T_{i+1} \cos(\theta_{i+1}) + f_i \quad i = 1 \dots n \quad (5.1)$$

$$m_i y_i'' = -T_i \sin(\theta_i) + T_{i+1} \sin(\theta_{i+1}) + g_i \quad i = 1 \dots n \quad (5.2)$$

6. Bar models with defects

Bar models with longitudinal waves (dynamical traction and compression) are considered.

$$\rho \frac{\partial^2 u}{\partial t^2} - \frac{\partial n}{\partial x} = f(x, t) \quad (6.1)$$

With a non linear stress-strain law:

$$n = E \left(A \frac{\partial u}{\partial x} + \epsilon \chi_{[a, b]} \left(\frac{\partial u}{\partial x} \right)^3 \right) \quad (6.2)$$

We could as well assume that the applied load is of order epsilon without any assumption on the nonlinearity. Assuming ϵ to be small an approximate solution is searched for with the following "ansatz": $u = u_0 + \epsilon u_1 + \dots$

Theoretical justification of the expansions: the situation is complex in full generality: non linear hyperbolic equations exhibit a singularity after a finite time! But: the experiments are performed during a short time interval and the Fourier transforms are computed on these time

intervals! Following a suggestion of Guy Metivier we are addressing the problem during a small initial time interval in which the solution is smooth: plan to use an approximation of the equation with a fixed point method proposed in Majda. In any case we should smooth the characteristic function (the material is changing smoothly)!

7. Conclusion

- Some simple models governed by ODE or PDE show intermodulations;
- But what is the relative level of secondary peaks for a given set of data deserves investigations: indeed it is also the difficulty of the real experiments
- Need to include other behaviors: shocks, friction
- Need of more precise models: non linear beams including tractional, flexural, torsional effects
- Mixture of local models for the defect and global models for the undamaged structure to obtain precise results at low computational cost.

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