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## Damage detection based on a stochastic version of the Volterra series

Luis G G Villani<sup>1</sup>, Samuel da Silva<sup>1</sup> and Americo Cunha Jr<sup>2</sup>

1 UNESP – Universidade Estadual Paulista, Faculdade de Engenharia de Ilha Solteira, Brazil, [luisgustavovillani@gmail.com](mailto:luisgustavovillani@gmail.com), [samuel.silva13@unesp.br](mailto:samuel.silva13@unesp.br).

2 UERJ – Universidade do Estado do Rio de Janeiro, Instituto de Matemática e Estatística, Brazil, [americo@ime.uerj.br](mailto:americo@ime.uerj.br).

### Abstract

This paper aims the damage detection in a clamped-free beam, presenting nonlinear behavior for large displacements, considering the data variation related with the presence of uncertainties. For this, a stochastic version of the Volterra series, expanded through a random version of the Kautz functions, is implemented, using Monte Carlo simulations to estimate the underlying statistics. The measurements were performed in different days and conditions, to consider the data variation and the presence of nonlinear behavior in the analysis. Two different damage indexes are proposed, based on the advantage of the use of Volterra series, related with the capability to separate linear and nonlinear contributions in the total system response. The results have shown that the nonlinear approach is more sensible to the presence of inserted damage (loss of mass), presenting better results than the classical linear approach.

### 1. Introduction

Many structures and systems can operate in nonlinear regime of motion, depending on the working conditions that they are subjected. So, they exhibit complex responses with harmonic distortion, jumps, modal interactions, bifurcation, and possible chaos [1]. In Structural Health Monitoring (SHM) problems, this nonlinear behavior can be confused with the presence of inserted damage, mainly when the classical approaches, based on linear metrics, are implemented [2]. In these situations, more sophisticated methods have to be used, making the approach able to make difference between the intrinsic nonlinear behavior of the system and the presence of damage.

On the other hand, any measurement is uncertain in nature, because of the variability related, for example, with the presence of noise, environment conditions, sensors sensibility, and others [3]. These uncertainties can reflect in data variation and to influence the model's parameters estimation. Then, this variability can also difficult the application of damage detection procedures, considering real structures and systems. Therefore, it is essential a statistical analysis to ensure the confidence of the obtained results and to reduce the number of false positives in the SHM approaches applied [4].

In this sense, this paper proposes the use of a stochastic version of Volterra series as an effective tool to take into account measurement uncertainties in the process of damage detection in a clamped-free beam underlying large displacements, using for this the Monte Carlo method to construct estimations of the statistical quantities of interest. The Volterra series are expanded using random Kautz functions to reduce the number of terms to be identified and the processing time. The damage inserted, associated to change of mass in connections (bolt loosed), is detected based on comparison between

linear and nonlinear contributions of the Volterra series in the total response, estimated in reference and damaged conditions. The results have shown that to consider the nonlinear terms in the analysis improves the capability of the approach to detect damage with a lower level of false alarms.

## 2. Detecting Damage Using Stochastic Volterra series

In this paper, a stochastic version of the classical Volterra series is proposed as a tool to detect damage in a structure operating in nonlinear regime of motion and that it is subject to data variation. In general, the systems uncertainties can be classified in two main groups, the *data uncertainties* and the *model uncertainties* [3]. The *model uncertainties* are related with the lack of knowledge about the physics of the problem, so, they can be reduced or even eliminated with the increase of knowledge about the system behavior [5]. This type of uncertainties is not taken into account in this work. Thus, it is assumed that the Volterra series are able to describe the dynamics behavior of the structure. On the other hand, the *data uncertainties* are related with the model parameters variation, caused by data variation, environment conditions, noise, and others [6]. In this work, the *data uncertainties* are considered through the use of a parametric probabilistic approach. In this sense, the model parameters subjected to uncertainties are assumed as random variables or random process, defined on the probabilistic space  $(\Theta, \Sigma, \mathbb{P})$ , where  $\Theta$  is a sample space,  $\Sigma$  is a  $\sigma$ -algebra over  $\Theta$ , and  $\mathbb{P}$  is a probability measure. It is assumed that any random variable  $\theta \in \Theta \mapsto \mathbb{Y}(\theta) \in \mathbb{R}$  in this probabilistic setting, with probability distribution  $P_{\mathbb{Y}}(dy)$  on  $\mathbb{R}$ , admits a probability density function (PDF)  $y \mapsto p_{\mathbb{Y}}(dy)$  with respect to  $dy$ .

### 2.1 Stochastic version of the Volterra series

The discrete-time Volterra series can describe the nonlinear relationship between input and output signals through the generalization of the convolution concept [7]. For better understanding about the Volterra series formulation and more details considering the deterministic analysis, some other works are suggested [8-9]. Thus, consider a nonlinear system with a single output, that it is a random process  $(\theta, k) \in \Theta \times \mathbb{Z}_+ \mapsto \mathbf{y}(\theta, k)$ , as a consequence of a single input, that it is deterministic  $(k) \in \mathbb{Z}_+ \mapsto u(k)$ . The stochastic version of the Volterra series can describe the random output of the nonlinear system as

$$\mathbf{y}(\theta, k) = \sum_{\eta=1}^{\infty} \sum_{n_1=0}^{N_1-1} \dots \sum_{n_{\eta}=0}^{N_{\eta}-1} \mathbb{H}_{\eta}(\theta, n_1, \dots, n_{\eta}) \prod_{i=1}^{\eta} u(k - n_i), \quad (1)$$

where the random process  $(\theta, n_1, \dots, n_{\eta}) \in \Theta \times \mathbb{Z}_+^{\eta} \mapsto \mathbb{H}_{\eta}(\theta, n_1, \dots, n_{\eta})$  represents the random version of the  $\eta$ -order Volterra kernel and  $N_1, \dots, N_{\eta}$  the number of terms used. In this work the Volterra series will be truncated in the third order term ( $\eta = 3$ ), because of the characteristic of the nonlinear system in study, that can be well approximated by a Duffing oscillator [10]. The main advantage of the use of Volterra series in the description of nonlinear systems is its capability in separate linear and nonlinear contributions in the total response through the kernels convolution

$$\mathbf{y}(\theta, k) = \underbrace{\mathbf{y}_1(\theta, k)}_{\text{linear}} + \underbrace{\mathbf{y}_2(\theta, k)}_{\text{quadratic}} + \underbrace{\mathbf{y}_3(\theta, k)}_{\text{cubic}} + \dots + \mathbf{y}_\eta(\theta, k), \quad (2)$$

nonlinear

where

$$\begin{aligned} \mathbf{y}_1(\theta, k) &= \sum_{n_1=0}^{N_1-1} \mathbb{H}_1(\theta, n_1) u(k - n_1), \\ \mathbf{y}_2(\theta, k) &= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \mathbb{H}_2(\theta, n_1, n_2) u(k - n_1) u(k - n_2), \\ \mathbf{y}_3(\theta, k) &= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \sum_{n_3=0}^{N_3-1} \mathbb{H}_3(\theta, n_1, n_2, n_3) u(k - n_1) u(k - n_2) u(k - n_3). \end{aligned} \quad (3)$$

There are some drawbacks in the use of this formulation, like the high number of terms, the difficulty in the series convergence and the model stability. So, in order to reduce the high number of terms the kernels can be expanded in a orthonormal bases, such as the Kautz functions, that will be considered in this work because of its capability in describe oscillatory systems [11-12]. Then, it can be considered

$$\mathbb{H}_\eta(\theta, n_1, \dots, n_\eta) \approx \sum_{i_1=1}^{J_1} \dots \sum_{i_\eta=1}^{J_\eta} \mathbb{B}_\eta(\theta, i_1, \dots, i_\eta) \prod_{j=1}^{\eta} \Psi_{i_j}(\theta, n_j), \quad (4)$$

where  $(\theta, i_1, \dots, i_\eta) \in \Theta \times \mathbb{Z}_+^\eta \mapsto \mathbb{B}_\eta(\theta, i_1, \dots, i_\eta)$  represents the  $\eta$ -order random Volterra kernel, expanded in the Kautz basis,  $J_1, \dots, J_\eta$  are the number of Kautz functions used in each orthonormal projection and the random process  $(\theta, n_j) \in \Theta \times \mathbb{Z}_+ \mapsto \Psi_{i_j}(\theta, n_j)$  represents the random version of the  $i_j$ -th Kautz filter. It is important to observe that the Kautz functions are assumed as random, because their definition depends of the system dynamics and, in this work, the system output, which is related with its dynamic behavior, is considered as random.

Now the random system output can be estimated through multiple convolutions between the random Volterra kernels, represented in the orthonormal basis, and the deterministic input signal filtered by the random Kautz functions

$$\mathbf{y}(\theta, k) = \sum_{\eta=1}^{\infty} \sum_{i_1=1}^{J_1} \dots \sum_{i_\eta=1}^{J_\eta} \mathbb{B}_\eta(\theta, i_1, \dots, i_\eta) \prod_{j=1}^{\eta} \mathbb{I}_{i_j}(\theta, k), \quad (5)$$

where the random process  $(\theta, k) \in \Theta \times \mathbb{Z}_+ \mapsto \mathbb{I}_{i_j}(\theta, k)$  represents the deterministic input signal, filtered by the random Kautz functions, i. e.,

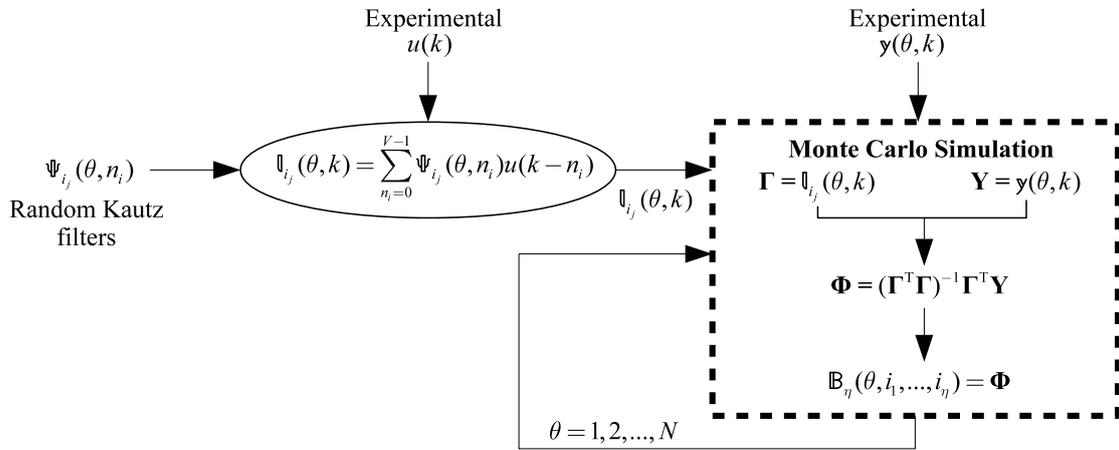
$$\mathbb{I}_{i_j}(\theta, k) = \sum_{n_i=0}^{V-1} \Psi_{i_j}(\theta, n_i) u(k - n_i), \quad (6)$$

where,  $V = \max\{J_1, \dots, J_\eta\}$ .

Then, the estimation of the coefficients can be done considering the least squares method and Monte Carlo simulations. In this regard, considering each realization  $\theta$ , the input signal filtered  $\mathbb{I}_j(\theta, k)$  can be used to complete the matrix  $\Gamma$ , and the random experimental output  $\mathbf{y}(\theta, k)$  used to complete the vector  $\mathbf{Y}$ . Finally, the coefficients estimation is made considering

$$\Phi = (\Gamma^T \Gamma)^{-1} \Gamma^T \mathbf{Y}, \quad (7)$$

where,  $\Phi$  is composed by the terms of the orthonormal kernels  $\mathbb{B}_\eta$  for each realization  $\theta$ . The procedure is repeated until the Monte Carlo convergence is achieved. Figure 1 shows a flowchart describing the kernels estimation procedure used in this work, considering the Monte Carlo simulations.



**Figure 1. Flowchart of the random kernels coefficients estimation based on Monte Carlo simulations.**

## 2.2 Random version of the Kautz functions

The deterministic version of the Kautz functions can be found in [13]. The functions definition is related with the system dynamics, expressed as natural frequencies and damping ratios. As, in this work, the system response is assumed as random, the modal parameters are also considered as random. So, the Kautz functions are modeled as random process, of generalize can be rewritten as

$$\Psi_{\eta, 2j-1}(\theta, z) = \frac{z \sqrt{[1 - \mathbf{b}_\eta(\theta)^2][1 - \mathbf{c}_\eta(\theta)^2]}}{z^2 + \mathbf{b}_\eta(\theta)[\mathbf{c}_\eta(\theta) - 1]z - \mathbf{c}_\eta(\theta)} \left\{ \frac{-\mathbf{c}_\eta(\theta)z^2 + \mathbf{b}_\eta(\theta)[\mathbf{c}_\eta(\theta) - 1] + 1}{z^2 + \mathbf{b}_\eta(\theta)[\mathbf{c}_\eta(\theta) - 1]z - \mathbf{c}_\eta(\theta)} \right\}, \quad (8)$$

and

$$\Psi_{\eta, 2j}(\theta, z) = \Psi_{\eta, 2j-1}(\theta, z) \frac{z - \mathbf{b}_\eta(\theta)}{\sqrt{1 - \mathbf{b}_\eta(\theta)^2}}, \quad (9)$$

being the random parameters  $\theta \in \Theta \mapsto \mathbf{b}_\eta(\theta)$  and  $\theta \in \Theta \mapsto \mathbf{c}_\eta(\theta)$  defined as

$$\mathbb{b}_\eta(\theta) = \frac{[\mathbb{W}_\eta(\theta) + \overline{\mathbb{W}_\eta(\theta)}]}{1 + \mathbb{W}_\eta(\theta)\overline{\mathbb{W}_\eta(\theta)}}, \quad (10)$$

where  $\theta \in \Theta \mapsto \mathbb{W}_\eta(\theta)$  and  $\theta \in \Theta \mapsto \overline{\mathbb{W}_\eta(\theta)}$  represent, respectively, the random Kautz poles in discrete domain, related with each Volterra kernel, and its complex conjugate. The discrete poles can be related with the continues poles

$$\mathbb{W}_\eta(\theta) = \exp\left(\frac{\mathbb{S}_\eta(\theta)}{F_s}\right), \quad (11)$$

where  $F_s$  is the sampling frequency. Finally, the Kautz poles, for each kernel, are defined as

$$\mathbb{S}_\eta(\theta) = -\xi_\eta(\theta)\omega_\eta(\theta) - j\omega_\eta(\theta)\sqrt{1-\xi_\eta(\theta)^2}, \quad (12)$$

where  $\theta \in \Theta \mapsto \xi_\eta(\theta)$  and  $\theta \in \Theta \mapsto \omega_\eta(\theta)$  are the parameters of the Kautz poles and  $\eta$  represents the number of the Volterra kernel considered in the analysis. When it is considered the linear analysis  $\xi_1(\theta)$  and  $\omega_1(\theta)$  can be approximated, respectively, by the damping ratio  $\theta \in \Theta \mapsto \zeta_n(\theta)$  and natural frequency  $\theta \in \Theta \mapsto \omega_n(\theta)$ , that are considered random variables as a consequence of the data variation. The definition of the parameters related with the high order Volterra kernels it is more complicated, because they are not exactly the natural frequencies and damping ratios, but combinations of these values. In this work, it is proposed a linear approximation of these parameters, considering the kernels of second and third order

$$\begin{aligned} \xi_2(\theta) &= p_1 \zeta_n(\theta), \\ \omega_2(\theta) &= p_2 \omega_n(\theta), \\ \xi_3(\theta) &= p_3 \zeta_n(\theta), \\ \omega_3(\theta) &= p_4 \omega_n(\theta). \end{aligned} \quad (13)$$

where the relationships,  $p_1$ ,  $p_2$ ,  $p_3$  and  $p_4$  can be determined through the minimization of the error function

$$\min_{\Delta} J = \sum_{k=1}^N [\hat{y}(k) - y(k)], \quad (14)$$

where  $\hat{y}(k)$  is the system response obtained through the Volterra model,  $y(k)$  is the experimental system response, and  $\Delta$  the vector with the coefficients  $p_1$ ,  $p_2$ ,  $p_3$  and  $p_4$ . The optimization is realized only for a representative data, and not in all Monte Carlo simulations, this methodology reduces the computational costs. So, in this work, the system's mean response is used in the optimization procedure. Then, the Kautz functions poles are obtained through the modal parameters estimated in each Monte Carlo simulation and the coefficients estimated.

### 2.3 Damage detection approach based on novelty detection

The damage detection is done based on the variation of the Volterra kernels contributions in the total system response and novelty detection. As mentioned before, the main advantage of the Volterra series is the capability to separate linear and nonlinear contributions through the kernels convolution, and this advantage allows the comparison between the contributions before and after the damage occurrence. So, this characteristic allows the analysis of which term of the response is more sensible to the presence of damage. In this sense, two damage indexes can be calculated, in reference condition, considering the contributions obtained through the Eq. (3)

$$\mathbf{y}_{lin}^{ref}(\theta, k) = \mathbf{y}_1^{ref}(\theta, k), \quad (15)$$

$$\mathbf{y}_{nlin}^{ref}(\theta, k) = \mathbf{y}_2^{ref}(\theta, k) + \mathbf{y}_3^{ref}(\theta, k),$$

where  $\mathbf{y}_{lin}^{ref}(\theta, k)$  is a matrix  $N_s \times k$  representing the linear contribution, composed by only the first kernel contribution, and  $\mathbf{y}_{nlin}^{ref}(\theta, k)$  is also a matrix  $N_s \times k$  representing the nonlinear contribution composed by the sum of the second and third kernels contributions. Additionally, the index  $m$  is used to represent the two different indicators calculated. So, the general notation  $\mathbf{y}_m^{ref}(\theta, k)$  is used from here, with  $m = lin$  or  $nlin$ , depending on the number of kernels considered in the analysis.  $N_s$  is the number of Monte Carlo simulations considered and  $k$  represents the discrete-time. It is observed that the contributions of the total response obtained, in reference condition, are random processes, estimated with statistic confidence and not simple curves. Therefore, to detect system variations related with damage, is needed to determine if the new contributions obtained through models identified in unknown system condition are contained in these random processes. To create a classification damaged/healthy state a distance-based method to outliers' detection is used [14]. In this sense, two different damage indexes, in reference condition, based on the standardized Euclidian distance between vectors, can be calculated as

$$\lambda_m^{ref}(\theta) = \sum_{n=1}^{N_s} \sqrt{[\mathbf{y}_m^{ref}(\theta, k) - \mathbf{y}_m^{ref}(n, k)]V^{-1}[\mathbf{y}_m^{ref}(\theta, k) - \mathbf{y}_m^{ref}(n, k)]^T}, \quad (16)$$

where  $\lambda_{lin}^{ref}(\theta)$  and  $\lambda_{nlin}^{ref}(\theta)$  are, respectively, the linear and nonlinear indexes calculated in reference condition, that are random variables, and  $V$  is a diagonal matrix  $N_s \times k$  in which the diagonal elements are the standard deviations of the columns of  $\mathbf{y}_{lin}^{ref}(\theta, k)$  or  $\mathbf{y}_{nlin}^{ref}(\theta, k)$ . The indexes are calculated in a stochastic sense, because the model is stochastic and the model response is assumed as a random process. It is expected that the statistic confidence of the model does the approach able to differ data variation and damage. With the indexes identified, their probability density function (PDF) can be estimated considering Kernel Density Estimation approach [15-16]

$$\hat{p}_{\lambda_m^{ref}}(\lambda_m^{ref}) = \frac{1}{N_s h} \sum_{i=1}^{N_s} K\left(\frac{\lambda_m^{ref} - \lambda_{m,i}^{ref}}{h}\right), \quad (17)$$

where  $\hat{p}_{\lambda_m^{ref}}(\lambda_m^{ref})$  represents an approximation of the true PDF of  $\lambda_m^{ref}(\theta)$ ,  $\lambda_{m,i}^{ref}$  is the  $i$ th realization of the random variable  $\lambda_m^{ref}(\theta)$ ,  $K$  is the kernel used in the transformation, a

Gaussian kernel in this work, and  $h$  is a smoothing parameter. The main difficulty in Kernel Density Estimation is the choice of optimal value to the smoothing parameter, then the cross-validation was used to set the better value to  $h$  [17]. The Kernel Density Estimation is used because the index distribution is assumed as unknown a priori and the large number of Monte Carlo simulations provides sufficient samples of the index to estimate its density with this approach.

Then, a threshold of the reference condition can be defined based on the empirical distribution estimated [18]

$$\Lambda_m = \{\lambda_m^{ref} \text{ such that } \int_{\lambda_m^{ref}}^{+\infty} \hat{p}_{\Lambda_m^{ref}}(\lambda_m^{ref}) d\lambda_m^{ref} = \beta\}, \quad (18)$$

where  $\beta$  is the probability of false alarms considered and  $\Lambda_m$  is the threshold estimated. Now, with the model constructed in the reference condition and the threshold calculated, new Volterra models can be calculated in unknown conditions (healthy or damaged) and compared with the stochastic reference system response

$$\lambda_m^{unk} = \sum_{n=1}^{N_s} \sqrt{[y_m^{unk}(k) - y_m^{ref}(n, k)]V^{-1}[y_m^{unk}(k) - y_m^{ref}(n, k)]^T}, \quad (19)$$

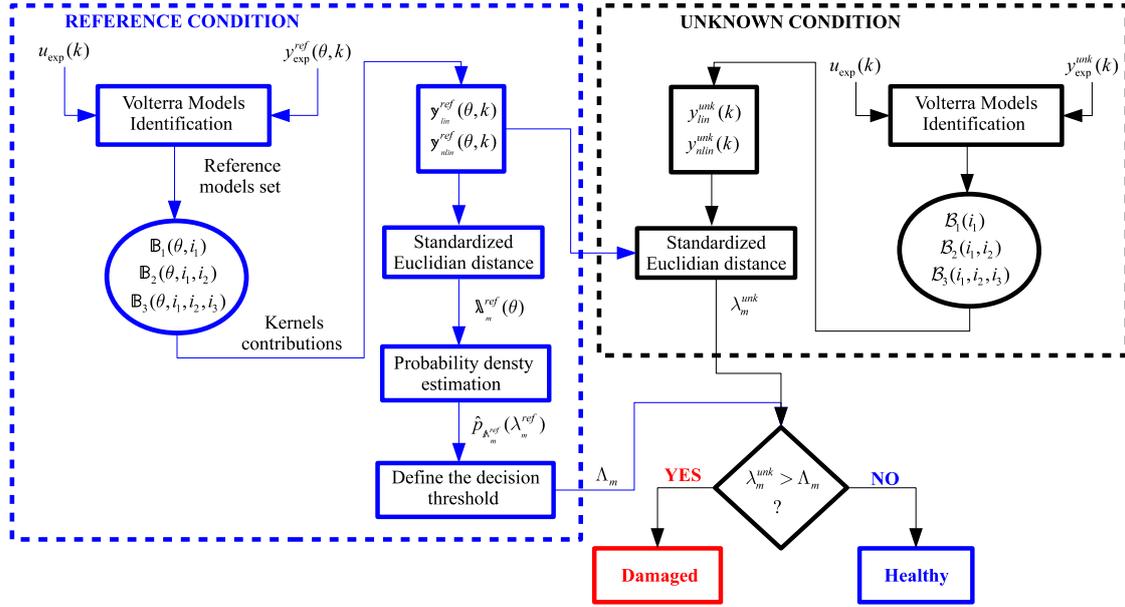
where  $y_m^{unk}(k)$  represents the kernels contributions of the Volterra series identified in the unknown condition. So, the hypothesis test can be applied to determine if the system is in healthy or damaged condition

$$\begin{cases} H_0 : \lambda_m^{unk} \leq \Lambda_m \\ H_1 : \lambda_m^{unk} > \Lambda_m \end{cases}, \quad (20)$$

where the null hypothesis  $H_0$  represents the healthy condition and  $H_1$  the damaged. Figure 2 shows a flowchart of the damage detection approach proposed in this work, based on a stochastic version of the Volterra series.

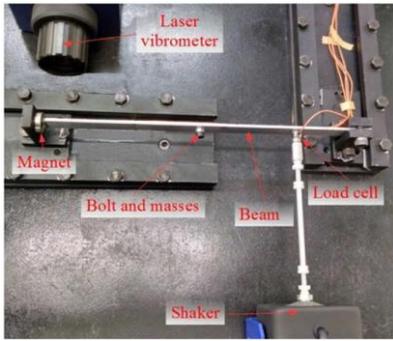
### 3. Experimental damage detection in a nonlinear beam

The experimental setup considered was the same used in [9] and [19], composed by a clamped-free beam with dimensions of  $300 \times 18 \times 3$  [mm<sup>3</sup>] and a magnet positioned near to the free end, to generate a nonlinear interaction with the beam (Figure 1). To emulate the presence of damage a bolted connection is positioned 150 [mm] from the free end with four nuts with 1 [g] each one, so the reference condition considers the 4 nuts and the damage is simulated through the loss of mass. Table 1 shows the structural conditions considered in the analysis. The signals acquisition was done considering sampling frequency of 1024 Hz and 4096 samples and using a *m+p VibPilot* data acquisition system. The structure was excited using a *MODAL SHOP* shaker (Model Number: K2004E01) attached 50 mm from the clamped, considering three different levels of voltage amplitude 0.01 V (low level), 0.10 V (medium level) and 0.15 V (high level). Finally, a vibrometer laser *Polytec* (Model: OFV-525/ -5000-S) and a *Dytran* load cell (Model: 1022V) are used to measure the velocity at the free end of the beam and the force excitation, respectively.

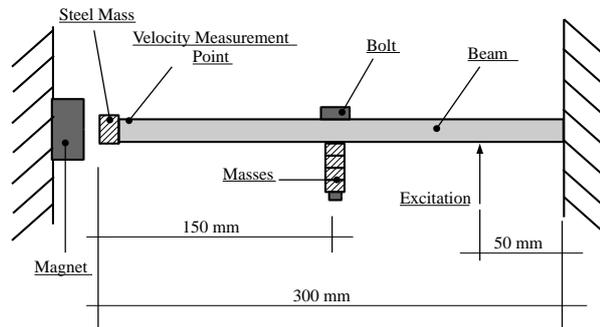


**Figure 2. Flowchart of the damage detection approach based on Volterra kernels contributions and novelty detection.**

The stepped sine test was performed to show the nonlinear behavior of the system, close to the first vibration mode of the structure, when the high level of input is applied. Figure 3 shows the results obtained, it is observed the jump phenomena, characteristic of nonlinear systems, when 0.15 V of voltage amplitude is applied, so the structure presents nonlinear behavior for large displacements. Additionally, to consider the data variation in the procedure, the measurements were repeated 160 times in different days during 2 weeks, and then, a white noise with 5% of the signals' RMS value was randomly added to the data, generating 2048 synthetic experimental realizations to be used in the Monte Carlo simulations. To exemplify the difficulty of detect damage in this situation, figure 4 shows the envelope of the FRFs curves, with 99% of confidence, in reference condition, and different levels of damage. It is observed that it is not possible to differentiate the damaged and healthy conditions, mainly when the level of the damage is low (3 nuts). Thus, to use only the inspection of the output in time or frequency domain is not enough to correctly detect the structural state of the system.



(a) Photo.

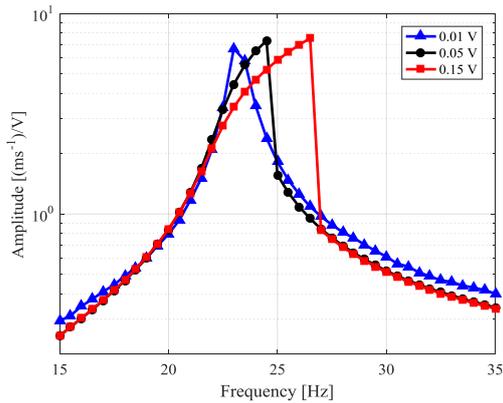


(b) Scheme.

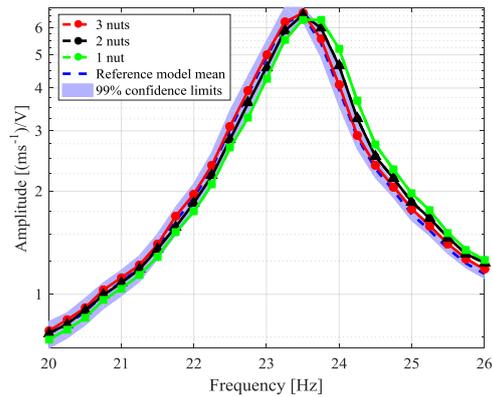
**Figure 3. Experimental setup [9].**

**Table 1. Structural conditions.**

State	Ref	Dam I	Dam II	Dam III	Rep
<b>Structural condition</b>	4 nuts (reference)	3 nuts	2 nuts	1 nut	4 nuts (repair)



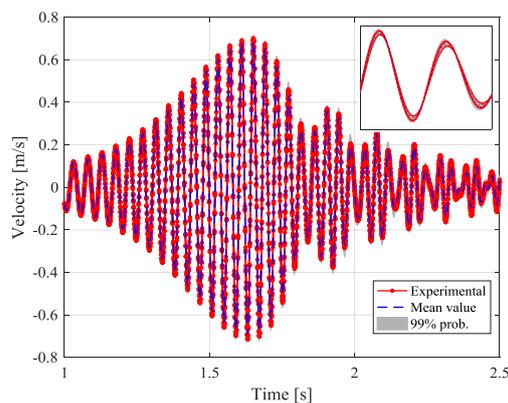
**Figure 4. Stepped sine test considering different levels of input amplitude.**



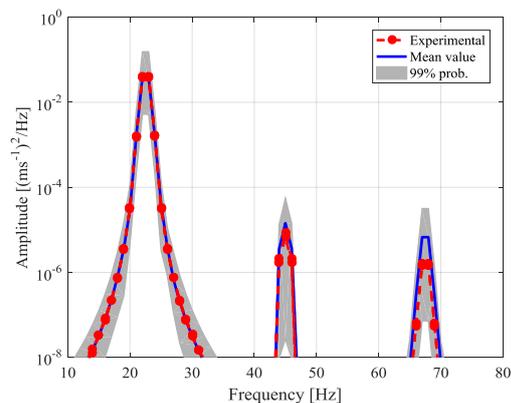
**Figure 5. FRF representing the data and damage variations.**

### 2.1 Volterra models estimation

The estimation of the Volterra kernels was done in two steps, first a low level (0.01 V) chirp, varying the frequency from 10 to 50 Hz with rate of 10 Hz/s, was applied to estimate the first kernel (linear), and then, a high level (0.15 V) chirp, varying the frequency from 10 to 50 Hz with rate of 10 Hz/s, was applied to estimate the second and third kernels (nonlinear). The procedure is repeated 2048 times, to achieve the Monte Carlo convergence and to construct the stochastic reference model. The reference model was validated in time and frequency domains, considering as input the same chirp signal used in the identification process and a single tone sine with frequency close to the first vibration mode of the structure (~23 Hz) and high level of amplitude (0.15 V). Figures 6 and 7 show the results obtained in the validation process. It is observed that the probability bands of the model, with 99% of confidence, can describe the nonlinear system behaviour, in all frequencies components, and the data variation.



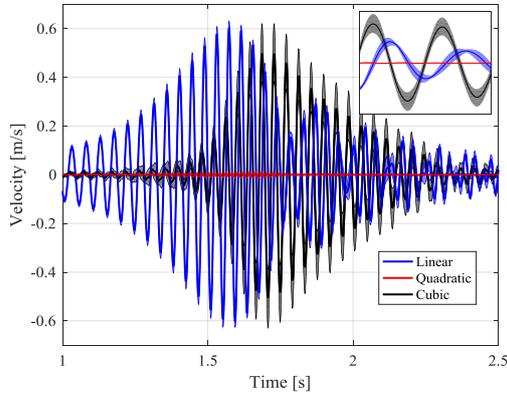
**Figure 6. System output, considering a chirp input with high level of amplitude (0.15 V).**



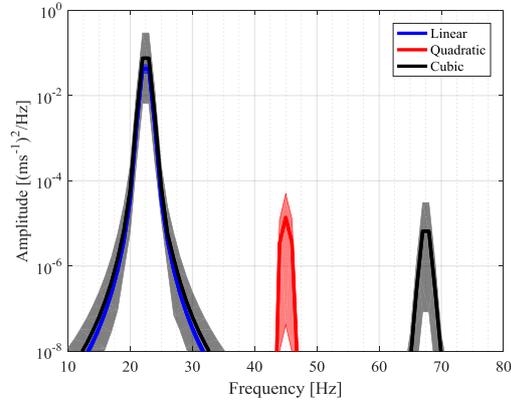
**Figure 7. PSD of system output, considering a sine input with high level of amplitude (0.15 V).**

The main advantage of Volterra series approach is the capability to separate linear and nonlinear contributions through the kernels convolution. This characteristic can be used to detect damage in the system. So, to exemplify, figures 8 and 9 shows the kernels contributions in the total response, considering the responses showed in figures 6 and 7. The first and third kernels have high contribution in the total response in time domain, the second kernel has low contribution because the output is approximated symmetric.

In frequency domain, the linear kernel contributes only to the first frequency component, the second kernel contributes only to the second harmonic and the third kernel contributes to the first and third harmonics. This occurs because of the mathematical definition of the third kernel that it has dependence of the first kernel.



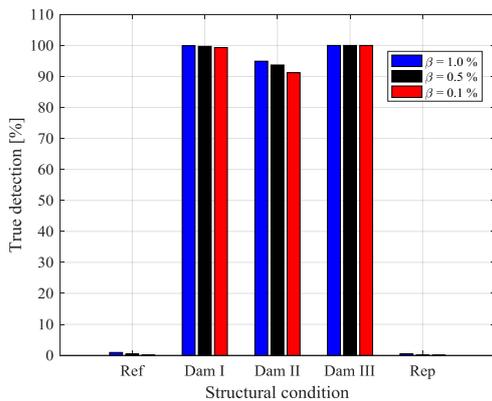
**Figure 8. Kernels contributions in the total output, considering a chirp input with high level of amplitude (0.15 V).**



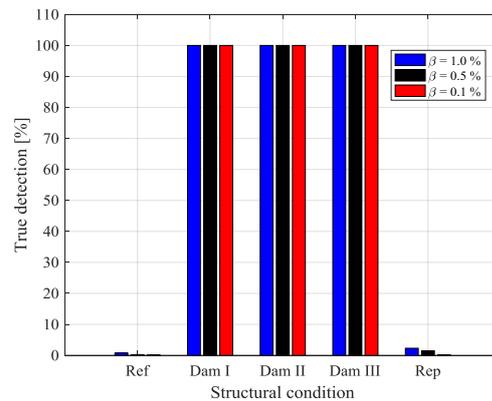
**Figure 9. PSD of kernels contributions, considering a sine input with high level of amplitude (0.15 V).**

## 2.2 Damage detection

After the identification of the set of reference models, the stochastic model obtained is used in the damage detection process. Then, new Volterra models are identified in each structural condition, and again, to consider the data variation, 2048 realizations were considered. So, the methodology explained in section 2.3 is applied, considering the two different indexes proposed (linear and nonlinear). The results were obtained considering a chirp input signal, with high level of voltage amplitude and varying the frequency from 10 to 50 Hz with rate of 10 Hz/s. Figure 10 shows the probability to detect damage using the proposed indexes, considering the 5 different structural conditions and 3 values of  $\beta$ . The nonlinear index has better performance, because it is more sensible to the presence of the damage. The linear index cannot classify all damage conditions as damaged.



**(a) Linear index.**



**(a) Nonlinear index.**

**Figure 10. Probability of damage detection for different structural conditions.**

Finally, the receiver operating characteristics (ROC) curve was computed (Farrar 2012). This curve shows the relation between false alarms and true detections obtained

considering different values of  $\beta$  and structural conditions. Figure 11 shows the results obtained to the two indexes proposed. The nonlinear index has better performance than the linear index, with lower number of false alarms and higher number of true detections. Therefore, although the damage emulated (loss of mass) it has linear characteristic, the nonlinear metric proposed present better results, because the nonlinear terms of the response are more sensible to the presence of damage, without high influence of the data variation.

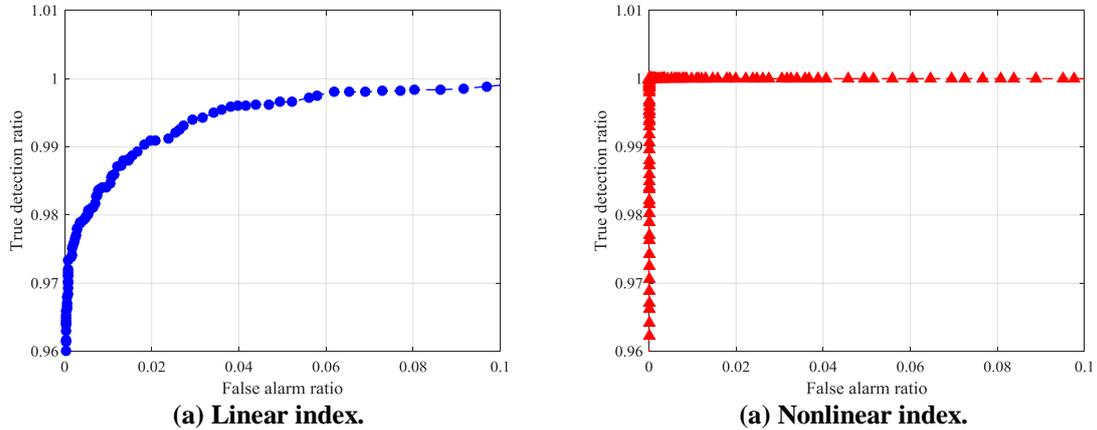


Figure 11. Receiver operating characteristics (ROC) curve.

### 3. Conclusions

This paper has proposed an approach based on a stochastic version of the Volterra series, estimated based on Monte Carlo simulations, to detect damage in a nonlinear system considering data variation related with uncertainties. The results have shown that the nonlinear index proposed has better performance than the classical linear approach, despite the linear characteristic of the damage emulated (loss of mass). This occurs, probably, because the third kernel's mathematical definition is related with the linear kernel, how can be observed on the cubic contribution in the first harmonic of the system's output PSD (Fig. 9), doing the cubic contribution more sensible to the presence of damage. These results prove the advantage in using nonlinear metrics when the monitored structure presents nonlinear behavior. Besides that, the problem is more difficult to solve when uncertainty in the date is assumed and it can change or fluctuate by chance the metrics. In order to separate these effects, the proposed method in this paper has shown adequate to allow identifying the source of nonlinearities or presence of damage in an uncertainty scenario.

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