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# Piecewise Polynomial Model of the Aerodynamic Coefficients of the Generic Transport Model and its Equations of Motion

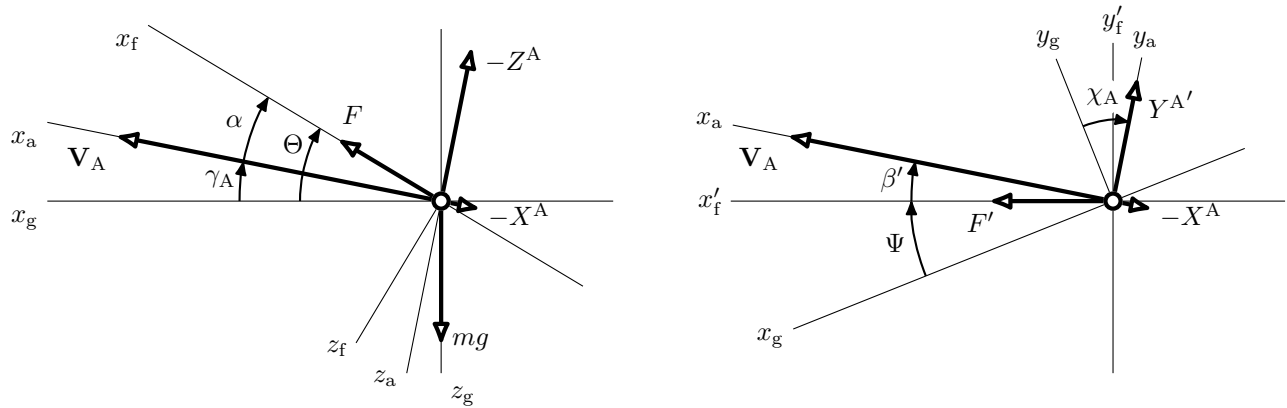
Torbjørn Cunis<sup>1</sup>, Laurent Burlion<sup>2</sup>, and Jean-Philippe Condomines<sup>3</sup>

## Abstract

The purpose of this document is to illustrate the piecewise polynomial model which has been derived from wind-tunnel measurement data of the Generic Transport Model (GTM) using the `pwppfit` toolbox. For implementation details and use in MATLAB, please refer to the source code at <https://github.com/pwppfit/GTMpr>.

## PRELIMINARIES

If not stated otherwise, all variables are in SI units. We will refer to the following axis systems of ISO 1151-1: the *body axis system*  $(x_f, y_f, z_f)$  aligned with the aircraft's fuselage; the *air-path axis system*  $(x_a, y_a, z_a)$  defined by the velocity vector  $\mathbf{V}_A$ ; and the *normal earth-fixed axis system*  $(x_g, y_g, z_g)$ . The orientation of the body axes with respect to the normal earth-fixed system is given by the attitude angles  $\Phi, \Theta, \Psi$  and to the air-path system by angle of attack  $\alpha$  and side-slip  $\beta$ ; the orientation of the air-path axes to the normal earth-fixed system is given by azimuth  $\chi_A$ , inclination  $\gamma_A$ , and bank-angle  $\mu_A$ . (Fig. 1.)



(a) Longitudinal axes ( $\beta = \mu_A = 0$ ).

(b) Horizontal axes ( $\gamma_A = 0$ ).

Fig. 1: Axis systems with angles and vectors. Projections into the plane are marked by '.

## I. AERODYNAMIC COEFFICIENTS

The piecewise polynomial models of the aerodynamic coefficients are given

$$C_{\odot}(\alpha, \beta, \dots) = \begin{cases} C_{\odot}^{pre}(\alpha, \beta, \dots) & \text{if } \alpha \leq \alpha_0, \\ C_{\odot}^{post}(\alpha, \beta, \dots) & \text{else,} \end{cases} \quad (1)$$

where  $C_{\odot} \in \{C_X, C_Y, C_Z, C_l, C_m, C_n\}$  are polynomials in angle of attack, side-slip, surface deflections, and normalized body rates; and the boundary is found at

$$\alpha_0 = 16.111^\circ. \quad (2)$$

The polynomials in low and high angle of attack,  $C_{\odot}^{pre}$ ,  $C_{\odot}^{post}$ , are sums

$$C_{\odot}^{pre} = C_{\odot\alpha}^{pre}(\alpha) + C_{\odot\beta}^{pre}(\alpha, \beta) + C_{\odot\xi}^{pre}(\alpha, \beta, \xi) + C_{\odot\eta}^{pre}(\alpha, \beta, \eta) + C_{\odot\zeta}^{pre}(\alpha, \beta, \zeta) + C_{\odot p}^{pre}(\alpha, \hat{p}) + C_{\odot q}^{pre}(\alpha, \hat{q}) + C_{\odot r}^{pre}(\alpha, \hat{r}); \quad (3)$$

$$C_{\odot}^{post} = C_{\odot\alpha}^{post}(\alpha) + C_{\odot\beta}^{post}(\alpha, \beta) + C_{\odot\xi}^{post}(\alpha, \beta, \xi) + C_{\odot\eta}^{post}(\alpha, \beta, \eta) + C_{\odot\zeta}^{post}(\alpha, \beta, \zeta) + C_{\odot p}^{post}(\alpha, \hat{p}) + C_{\odot q}^{post}(\alpha, \hat{q}) + C_{\odot r}^{post}(\alpha, \hat{r}). \quad (4)$$

<sup>1</sup>ONERA – The French Aerospace Lab, Department of Information Processing and Systems, Centre Midi-Pyrénées, Toulouse, 31055, France and ENAC, Université de Toulouse, Drones Research Group; now with the University of Michigan, e-mail: [tcunis@umich.edu](mailto:tcunis@umich.edu).

<sup>2</sup>Rutgers, The State University of New Jersey, Department of Mechanical & Aerospace Engineering, Piscataway, NJ 08854, USA, e-mail: [laurent.burlion@rutgers.edu](mailto:laurent.burlion@rutgers.edu).

<sup>3</sup>ENAC, Université de Toulouse, Drones Research Group, Toulouse, 31055, France, e-mail: [jean-philippe.condomines@enac.fr](mailto:jean-philippe.condomines@enac.fr).

In the following subsections, we present the polynomial terms obtained using the `pwfit` toolbox. Coefficients of absolute value lower than  $10^{-2}$  have been omitted for readability.

### A. Domain of low angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{pre} = -0.039 + 0.244\alpha + 4.452\alpha^2 - 17.394\alpha^3; \quad (5)$$

$$C_{Z\alpha}^{pre} = -0.017 - 5.241\alpha - 1.866\alpha^2 + 28.466\alpha^3; \quad (6)$$

$$C_{m\alpha}^{pre} = 0.119 - 1.465\alpha + 8.129\alpha^2 - 31.983\alpha^3; \quad (7)$$

$C_{Y\alpha}^{pre}$ ,  $C_{l\alpha}^{pre}$ ,  $C_{n\alpha}^{pre}$  are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{X\beta}^{pre} = 0.012 + 0.013\alpha - 2.049\alpha^2 + 0.027\alpha\beta + 0.066\beta^2 + 9.808\alpha^3 + 0.249\alpha^2\beta - 0.572\alpha\beta^2 - 10.651\alpha^4 - 1.178\alpha^3\beta + 1.936\alpha^2\beta^2 - 0.043\beta^4; \quad (8)$$

$$C_{Y\beta}^{pre} = -1.024\beta + 0.219\alpha\beta - 0.307\alpha^2\beta + 0.070\beta^3 + 4.667\alpha^3\beta - 1.062\alpha\beta^3; \quad (9)$$

$$C_{Z\beta}^{pre} = -0.035 - 0.018\alpha + 2.911\alpha^2 + 0.024\beta^2 - 6.328\alpha^3 - 0.098\alpha^2\beta + 4.018\alpha\beta^2 - 8.805\alpha^4 + 0.328\alpha^3\beta - 8.609\alpha^2\beta^2 + 0.165\beta^4; \quad (10)$$

$$C_{l\beta}^{pre} = -0.144\beta + 0.177\alpha\beta + 0.052\alpha^2\beta + 0.203\beta^3 + 3.495\alpha^3\beta - 1.113\alpha\beta^3; \quad (11)$$

$$C_{m\beta}^{pre} = 0.068 - 0.205\alpha - 9.113\alpha^2 + 0.021\alpha\beta - 1.396\beta^2 + 54.917\alpha^3 + 0.170\alpha^2\beta - 1.633\alpha\beta^2 - 83.622\alpha^4 - 0.990\alpha^3\beta + 11.760\alpha^2\beta^2 - 0.027\alpha\beta^3 + 1.164\beta^4; \quad (12)$$

$$C_{n\beta}^{pre} = 0.228\beta - 0.228\alpha\beta - 0.230\beta^3 - 1.697\alpha^3\beta + 0.539\alpha\beta^3. \quad (13)$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{X\xi}^{pre} = 0.033\alpha - 0.262\alpha^2 + 0.042\beta^2 + 0.014\beta\xi + 0.135\xi^2 + 1.195\alpha^3 + 0.094\alpha^2\beta - 0.147\alpha\beta^2 + 0.016\alpha\beta\xi - 0.129\alpha\xi^2 - 2.732\alpha^4 - 0.308\alpha^3\beta + 0.993\alpha^2\beta^2 - 0.575\alpha^2\beta\xi + 0.161\alpha^2\xi^2 - 0.111\beta^4 - 0.048\beta^2\xi^2 - 0.012\beta\xi^3 - 0.499\xi^4; \quad (14)$$

$$C_{Y\xi}^{pre} = 0.080\beta - 0.022\xi - 1.925\alpha\beta - 0.036\alpha\xi + 0.501\alpha^2\beta + 0.229\alpha^2\xi - 0.021\beta^3 - 0.244\beta\xi^2 - 1.469\alpha^3\beta - 0.751\alpha^3\xi + 3.189\alpha\beta^3 + 5.135\alpha\beta\xi^2 + 0.227\alpha\xi^3; \quad (15)$$

$$C_{Z\xi}^{pre} = -0.030\alpha - 0.322\alpha^2 - 0.033\beta^2 - 0.140\beta\xi + 0.029\xi^2 + 3.097\alpha^3 + 0.094\alpha^2\beta - 0.328\alpha\beta^2 + 0.385\alpha\beta\xi - 0.247\alpha\xi^2 - 5.289\alpha^4 - 0.308\alpha^3\beta + 0.142\alpha^2\beta^2 + 0.169\alpha^2\beta\xi - 0.795\alpha^2\xi^2 + 0.109\beta^4 + 0.027\beta^3\xi - 0.058\beta^2\xi^2 + 0.197\beta\xi^3 + 0.118\xi^4; \quad (16)$$

$$C_{l\xi}^{pre} = 0.021\beta - 0.079\xi - 0.158\alpha\beta + 0.046\alpha\xi + 0.149\alpha^2\beta + 0.698\alpha^2\xi - 0.066\beta^3 + 0.027\beta^2\xi - 0.036\beta\xi^2 + 0.069\xi^3 - 0.508\alpha^3\beta - 1.316\alpha^3\xi + 0.338\alpha\beta^3 + 0.016\alpha\beta^2\xi + 0.453\alpha\beta\xi^2 - 0.139\alpha\xi^3; \quad (17)$$

$$C_{m\xi}^{pre} = -0.039 + 0.059\alpha + 0.163\alpha^2 + 0.401\beta^2 - 0.013\beta\xi - 0.564\xi^2 + 2.568\alpha^3 + 0.094\alpha^2\beta - 1.548\alpha\beta^2 + 0.068\alpha\beta\xi - 0.439\alpha\xi^2 - 4.594\alpha^4 - 0.308\alpha^3\beta - 1.295\alpha^2\beta^2 - 0.048\alpha^2\beta\xi + 0.356\alpha^2\xi^2 + 0.085\beta^4 + 0.082\beta^3\xi + 0.084\beta^2\xi^2 + 0.038\beta\xi^3 + 2.201\xi^4; \quad (18)$$

$$C_{n\xi}^{pre} = -0.249\alpha\beta + 0.014\alpha\xi + 0.041\alpha^2\beta + 0.053\alpha^2\xi + 0.029\beta^3 - 0.017\beta\xi^2 + 0.016\xi^3 + 0.337\alpha^3\beta + 0.048\alpha^3\xi + 0.337\alpha\beta^3 + 0.049\alpha\beta^2\xi + 0.690\alpha\beta\xi^2 - 0.122\alpha\xi^3. \quad (19)$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{X\eta}^{pre} = -0.036\alpha - 0.292\alpha^2 + 0.148\alpha\eta - 0.102\eta^2 + 1.173\alpha^3 - 0.411\alpha^2\eta + 0.033\alpha\beta^2 - 0.011\alpha\beta\eta + 0.105\alpha\eta^2 - 0.036\eta^3; \quad (20)$$

$$C_{Y\eta}^{pre} = -0.267\beta + 1.036\alpha\beta - 3.141\alpha^2\beta + 0.532\beta^3; \quad (21)$$

$$C_{Z\eta}^{pre} = 0.041\alpha - 0.016\beta - 0.526\eta + 0.205\alpha^2 + 0.074\alpha\beta + 0.050\alpha\eta - 0.012\beta^2 - 1.770\alpha^3 - 0.060\alpha^2\beta + 0.815\alpha^2\eta + 0.306\alpha\beta^2 - 0.028\alpha\beta\eta + 0.181\alpha\eta^2 + 0.297\beta^2\eta + 0.012\beta\eta^2 + 0.645\eta^3; \quad (22)$$

$$C_{l\eta}^{pre} = -0.012\beta + 0.057\alpha\beta - 0.286\alpha^2\beta + 0.047\beta^3; \quad (23)$$

$$C_{m\eta}^{pre} = 0.028 + 0.032\alpha - 0.054\beta - 1.851\eta - 0.172\alpha^2 + 0.206\alpha\beta - 0.172\alpha\eta - 0.199\beta^2 + 0.026\beta\eta - 0.173\eta^2 - 0.300\alpha^3 - 0.068\alpha^2\beta + 5.136\alpha^2\eta + 0.816\alpha\beta^2 - 0.023\alpha\beta\eta + 0.897\alpha\eta^2 + 0.693\beta^2\eta + 0.111\beta\eta^2 + 1.324\eta^3; \quad (24)$$

$$C_{n\eta}^{pre} = -0.057\beta + 0.142\alpha\beta - 0.344\alpha^2\beta + 0.110\beta^3. \quad (25)$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{X\zeta}^{pre} = -0.011 + 0.026\alpha + 0.351\alpha^2 + 0.022\beta^2 + 0.091\beta\zeta - 2.731\alpha^3 - 0.042\alpha\beta^2 - 0.040\alpha\beta\zeta + 0.013\alpha\zeta^2 + 6.059\alpha^4 + 0.053\alpha^2\beta^2 + 0.245\alpha^2\beta\zeta - 0.211\alpha^2\zeta^2 - 0.011\beta^4 - 0.042\beta^2\zeta^2 - 0.130\beta\zeta^3 - 0.056\zeta^4; \quad (26)$$

$$C_{Y\zeta}^{pre} = 0.187\beta + 0.308\zeta + 0.820\alpha\beta - 0.501\alpha\zeta + 0.160\alpha^2\beta + 1.251\alpha^2\zeta - 0.455\beta^3 - 0.224\beta^2\zeta + 0.168\beta\zeta^2 - 0.235\zeta^3 + 14.007\alpha^3\beta - 6.802\alpha^3\zeta - 2.757\alpha\beta^3 + 0.307\alpha\beta^2\zeta - 0.678\alpha\beta\zeta^2 + 1.093\alpha\zeta^3; \quad (27)$$

$$C_{Z\zeta}^{pre} = -0.106\alpha + 0.016\beta + 0.254\alpha^2 - 0.074\alpha\beta + 0.055\beta^2 + 0.054\beta\zeta + 0.117\zeta^2 + 3.459\alpha^3 + 0.060\alpha^2\beta + 0.089\alpha\beta^2 - 0.367\alpha\beta\zeta + 0.063\alpha\zeta^2 - 6.397\alpha^4 - 0.984\alpha^2\beta^2 + 1.934\alpha^2\beta\zeta - 1.049\alpha^2\zeta^2 - 0.099\beta^4 - 0.140\beta^3\zeta - 0.047\beta^2\zeta^2 - 0.043\beta\zeta^3 - 0.122\zeta^4; \quad (28)$$

$$C_{l\zeta}^{pre} = 0.024\zeta + 0.027\alpha\beta - 0.027\alpha\zeta + 0.150\alpha^2\beta + 0.068\alpha^2\zeta - 0.013\beta^3 - 0.026\beta^2\zeta - 0.017\zeta^3 + 0.600\alpha^3\beta - 0.251\alpha^3\zeta - 0.172\alpha\beta^3 + 0.046\alpha\beta^2\zeta + 0.020\alpha\beta\zeta^2 + 0.040\alpha\zeta^3; \quad (29)$$

$$C_{m\zeta}^{pre} = -0.016 - 0.039\alpha + 0.054\beta + 0.335\alpha^2 - 0.206\alpha\beta + 0.032\beta^2 - 0.340\beta\zeta + 0.298\zeta^2 - 7.653\alpha^3 + 0.068\alpha^2\beta + 0.723\alpha\beta^2 - 0.354\alpha\beta\zeta + 0.021\alpha\zeta^2 + 23.281\alpha^4 - 1.037\alpha^2\beta^2 + 0.588\alpha^2\beta\zeta - 1.013\alpha^2\zeta^2 - 0.311\beta^4 - 0.304\beta^3\zeta + 0.213\beta^2\zeta^2 + 0.680\beta\zeta^3 - 0.247\zeta^4; \quad (30)$$

$$C_{n\zeta}^{pre} = 0.040\beta - 0.145\zeta + 0.109\alpha\beta + 0.033\alpha\zeta + 0.606\alpha^2\beta - 0.264\alpha^2\zeta - 0.108\beta^3 + 0.114\beta^2\zeta - 0.052\beta\zeta^2 + 0.088\zeta^3 - 1.391\alpha^3\beta + 1.698\alpha^3\zeta - 0.387\alpha\beta^3 - 0.098\alpha\beta^2\zeta + 0.077\alpha\beta\zeta^2 - 0.090\alpha\zeta^3. \quad (31)$$

Polynomials in angle of attack and normalized body  $p$ -rate:

$$C_{Y\hat{p}}^{pre} = -0.071\hat{p} - 0.138\alpha\hat{p} - 0.013\hat{p}^2 - 0.862\alpha^2\hat{p} + 0.383\alpha\hat{p}^2 + 14.142\hat{p}^3; \quad (32)$$

$$C_{l\hat{p}}^{pre} = -0.266\hat{p} - 0.247\alpha\hat{p} - 0.015\hat{p}^2 + 3.159\alpha^2\hat{p} + 0.540\alpha\hat{p}^2 - 2.843\hat{p}^3; \quad (33)$$

$$C_{n\hat{p}}^{pre} = -0.083\hat{p} - 0.044\alpha\hat{p} - 0.065\hat{p}^2 + 1.221\alpha^2\hat{p} + 0.073\alpha\hat{p}^2 + 5.811\hat{p}^3; \quad (34)$$

$C_{X\hat{p}}^{pre}$ ,  $C_{Z\hat{p}}^{pre}$ ,  $C_{m\hat{p}}^{pre}$  are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body  $q$ -rate:

$$C_{X\hat{q}}^{pre} = -0.030\alpha + 0.851\hat{q} + 0.076\alpha^2 + 12.434\alpha\hat{q} + 571.330\hat{q}^2 + 0.127\alpha^3 + 23.482\alpha^2\hat{q} + 2196.700\alpha\hat{q}^2 + 2529.200\hat{q}^3; \quad (35)$$

$$C_{Z\hat{q}}^{pre} = -32.946\hat{q} - 0.203\alpha^2 - 32.151\alpha\hat{q} + 1401.800\hat{q}^2 - 0.282\alpha^3 - 80.578\alpha^2\hat{q} + 1239.200\alpha\hat{q}^2 + 2345.900\hat{q}^3; \quad (36)$$

$$C_{m\hat{q}}^{pre} = -0.024 - 0.033\alpha - 42.378\hat{q} + 0.496\alpha^2 - 2.991\alpha\hat{q} + 765.020\hat{q}^2 + 0.909\alpha^3 + 29.585\alpha^2\hat{q} + 2214.800\alpha\hat{q}^2 + 2370.400\hat{q}^3; \quad (37)$$

$C_{Y\hat{q}}^{pre}$ ,  $C_{l\hat{q}}^{pre}$ ,  $C_{n\hat{q}}^{pre}$  are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body  $r$ -rate:

$$C_{Y\hat{r}}^{pre} = 0.748\hat{r} + 0.865\alpha\hat{r} + 0.881\hat{r}^2 + 1.625\alpha^2\hat{r} - 1.485\alpha\hat{r}^2 + 2.662\hat{r}^3; \quad (38)$$

$$C_{l\hat{r}}^{pre} = 0.147\hat{r} + 0.755\alpha\hat{r} + 0.862\alpha^2\hat{r} + 0.125\alpha\hat{r}^2 - 5.986\hat{r}^3; \quad (39)$$

$$C_{n\hat{r}}^{pre} = -0.337\hat{r} - 0.249\alpha\hat{r} - 0.471\hat{r}^2 - 0.912\alpha^2\hat{r} + 1.028\alpha\hat{r}^2 + 19.902\hat{r}^3; \quad (40)$$

$C_{X\hat{r}}^{pre}$ ,  $C_{Z\hat{r}}^{pre}$ ,  $C_{m\hat{r}}^{pre}$  are zero due to lack of GTM measurement data.

### B. Domain of high angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{post} = 0.019 - 0.130\alpha + 0.169\alpha^2 - 0.022\alpha^3; \quad (41)$$

$$C_{Z\alpha}^{post} = -0.365 - 2.712\alpha + 1.647\alpha^2 - 0.369\alpha^3; \quad (42)$$

$$C_{m\alpha}^{post} = 0.247 - 2.847\alpha + 2.748\alpha^2 - 1.105\alpha^3; \quad (43)$$

$C_{Y\alpha}^{post}$ ,  $C_{l\alpha}^{post}$ ,  $C_{n\alpha}^{post}$  are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{X\beta}^{post} = 0.101\alpha - 0.239\alpha^2 + 0.025\beta^2 + 0.199\alpha^3 + 0.160\alpha\beta^2 - 0.011\beta^3 - 0.055\alpha^4 - 0.148\alpha^2\beta^2 + 0.010\alpha\beta^3 - 0.043\beta^4; \quad (44)$$

$$C_{Y\beta}^{post} = -0.393\beta - 2.181\alpha\beta + 1.680\alpha^2\beta - 0.591\beta^3 - 0.402\alpha^3\beta + 1.287\alpha\beta^3; \quad (45)$$

$$C_{Z\beta}^{post} = -0.040 + 0.183\alpha - 0.268\alpha^2 + 0.039\alpha\beta - 0.293\beta^2 + 0.168\alpha^3 - 0.088\alpha^2\beta + 3.329\alpha\beta^2 - 0.039\alpha^4 + 0.056\alpha^3\beta - 2.148\alpha^2\beta^2 + 0.165\beta^4; \quad (46)$$

$$C_{l\beta}^{post} = 0.074\beta - 0.339\alpha\beta + 0.111\alpha^2\beta - 0.142\beta^3 + 0.113\alpha\beta^3; \quad (47)$$

$$C_{m\beta}^{post} = 0.357 - 2.954\alpha - 0.096\beta + 7.802\alpha^2 + 0.510\alpha\beta - 0.589\beta^2 - 7.667\alpha^3 - 0.736\alpha^2\beta - 1.740\alpha\beta^2 + 2.469\alpha^4 + 0.296\alpha^3\beta + 1.931\alpha^2\beta^2 - 0.040\alpha\beta^3 + 1.164\beta^4; \quad (48)$$

$$C_{n\beta}^{post} = 0.197\beta - 0.186\alpha\beta - 0.286\alpha^2\beta - 0.238\beta^3 + 0.160\alpha^3\beta + 0.567\alpha\beta^3. \quad (49)$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{X\xi}^{post} = 0.079 - 0.611\alpha + 1.476\alpha^2 + 0.135\beta^2 - 0.025\beta\xi - 0.083\xi^2 - 1.489\alpha^3 - 0.253\alpha\beta^2 - 0.013\alpha\beta\xi + 0.897\alpha\xi^2 + 0.531\alpha^4 + 0.194\alpha^2\beta^2 + 0.026\alpha^2\beta\xi - 0.717\alpha^2\xi^2 - 0.111\beta^4 - 0.048\beta^2\xi^2 - 0.012\beta\xi^3 - 0.499\xi^4; \quad (50)$$

$$C_{Y\xi}^{post} = -1.486\beta - 0.059\xi + 4.744\alpha\beta + 0.127\alpha\xi - 4.548\alpha^2\beta - 0.097\alpha^2\xi + 2.041\beta^3 + 2.727\beta\xi^2 + 0.121\xi^3 + 2.605\alpha^3\beta + 0.031\alpha^3\xi - 4.143\alpha\beta^3 + 0.013\alpha\beta^2\xi - 5.429\alpha\beta\xi^2 - 0.185\alpha\xi^3; \quad (51)$$

$$C_{Z\xi}^{post} = 0.038 - 0.332\alpha + 0.762\alpha^2 - 0.043\beta^2 - 0.277\xi^2 - 0.480\alpha^3 - 0.326\alpha\beta^2 - 0.034\alpha\beta\xi + 0.676\alpha\xi^2 + 0.054\alpha^4 + 0.262\alpha^2\beta^2 - 0.033\alpha^2\beta\xi - 0.206\alpha^2\xi^2 + 0.109\beta^4 + 0.027\beta^3\xi - 0.058\beta^2\xi^2 + 0.197\beta\xi^3 + 0.118\xi^4; \quad (52)$$

$$C_{l\xi}^{post} = -0.107\beta - 0.035\xi + 0.419\alpha\beta - 0.042\alpha\xi - 0.500\alpha^2\beta + 0.084\alpha^2\xi + 0.082\beta^3 + 0.035\beta^2\xi + 0.194\beta\xi^2 + 0.030\xi^3 + 0.253\alpha^3\beta - 0.032\alpha^3\xi - 0.188\alpha\beta^3 - 0.013\alpha\beta^2\xi - 0.365\alpha\beta\xi^2; \quad (53)$$

$$C_{m\xi}^{post} = -0.062 + 0.636\alpha - 1.619\alpha^2 - 0.520\beta^2 + 0.051\beta\xi - 0.061\xi^2 + 1.436\alpha^3 + 1.727\alpha\beta^2 - 0.202\alpha\beta\xi - 2.612\alpha\xi^2 - 0.434\alpha^4 - 1.297\alpha^2\beta^2 + 0.105\alpha^2\beta\xi + 1.727\alpha^2\xi^2 + 0.085\beta^4 + 0.082\beta^3\xi + 0.084\beta^2\xi^2 + 0.038\beta\xi^3 + 2.201\xi^4; \quad (54)$$

$$C_{n\xi}^{post} = -0.170\beta + 0.013\xi + 0.458\alpha\beta - 0.034\alpha\xi - 0.227\alpha^2\beta + 0.037\alpha^2\xi + 0.328\beta^3 + 0.017\beta^2\xi + 0.397\beta\xi^2 - 0.045\xi^3 + 0.161\alpha^3\beta - 0.021\alpha^3\xi - 0.726\alpha\beta^3 - 0.017\alpha\beta^2\xi - 0.781\alpha\beta\xi^2 + 0.095\alpha\xi^3. \quad (55)$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{X\eta}^{post} = 0.042 - 0.276\alpha - 0.011\eta + 0.546\alpha^2 + 0.061\alpha\eta + 0.039\beta^2 - 0.063\eta^2 - 0.293\alpha^3 - 0.063\alpha^2\eta - 0.088\alpha\beta^2 - 0.032\alpha\eta^2 - 0.036\eta^3; \quad (56)$$

$$C_{Y\eta}^{post} = -0.608\beta + 1.973\alpha\beta - 2.152\alpha^2\beta + 0.532\beta^3; \quad (57)$$

$$C_{Z\eta}^{post} = -0.100 + 0.559\alpha - 0.594\eta - 0.926\alpha^2 + 0.029\alpha\beta + 0.585\alpha\eta + 0.110\beta^2 - 0.035\eta^2 + 0.452\alpha^3 - 0.024\alpha^2\beta - 0.230\alpha^2\eta - 0.127\alpha\beta^2 + 0.319\alpha\eta^2 + 0.297\beta^2\eta + 0.012\beta\eta^2 + 0.645\eta^3; \quad (58)$$

$$C_{l\eta}^{post} = -0.049\beta + 0.150\alpha\beta - 0.152\alpha^2\beta + 0.047\beta^3; \quad (59)$$

$$C_{m\eta}^{post} = 0.090 - 0.372\alpha - 1.880\eta + 0.436\alpha^2 + 1.470\alpha\eta + 0.019\beta\eta - 0.087\eta^2 - 0.139\alpha^3 - 0.341\alpha^2\eta + 0.076\alpha\beta^2 + 0.589\alpha\eta^2 + 0.693\beta^2\eta + 0.111\beta\eta^2 + 1.324\eta^3; \quad (60)$$

$$C_{n\eta}^{post} = -0.070\beta + 0.142\alpha\beta - 0.187\alpha^2\beta + 0.110\beta^3. \quad (61)$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{X\zeta}^{post} = 0.016 - 0.140\alpha + 0.478\alpha^2 + 0.082\beta^2 + 0.203\beta\zeta - 0.122\zeta^2 - 0.669\alpha^3 - 0.316\alpha\beta^2 - 0.463\alpha\beta\zeta + 0.456\alpha\zeta^2 + 0.298\alpha^4 + 0.274\alpha^2\beta^2 + 0.325\alpha^2\beta\zeta - 0.307\alpha^2\zeta^2 - 0.011\beta^4 - 0.042\beta^2\zeta^2 - 0.130\beta\zeta^3 - 0.056\zeta^4; \quad (62)$$

$$C_{Y\zeta}^{post} = 3.051\beta - 0.205\zeta - 11.790\alpha\beta + 1.702\alpha\zeta + 14.450\alpha^2\beta - 2.320\alpha^2\zeta - 2.302\beta^3 - 0.260\beta^2\zeta - 0.182\beta\zeta^2 + 0.353\zeta^3 - 6.130\alpha^3\beta + 1.104\alpha^3\zeta + 3.811\alpha\beta^3 + 0.433\alpha\beta^2\zeta + 0.566\alpha\beta\zeta^2 - 0.996\alpha\zeta^3; \quad (63)$$

$$C_{Z\zeta}^{post} = 0.220 - 1.248\alpha + 2.437\alpha^2 - 0.029\alpha\beta - 0.083\beta^2 + 0.142\beta\zeta - 0.081\zeta^2 - 2.074\alpha^3 + 0.024\alpha^2\beta + 0.345\alpha\beta^2 - 0.163\alpha\beta\zeta + 0.591\alpha\zeta^2 + 0.654\alpha^4 - 0.149\alpha^2\beta^2 + 0.107\alpha^2\beta\zeta - 0.422\alpha^2\zeta^2 - 0.099\beta^4 - 0.140\beta^3\zeta - 0.047\beta^2\zeta^2 - 0.043\beta\zeta^3 - 0.122\zeta^4; \quad (64)$$

$$C_{l\zeta}^{post} = 0.129\beta - 0.471\alpha\beta + 0.047\alpha\zeta + 0.546\alpha^2\beta - 0.070\alpha^2\zeta - 0.091\beta^3 - 0.019\beta^2\zeta + 0.012\beta\zeta^2 - 0.203\alpha^3\beta + 0.031\alpha^3\zeta + 0.106\alpha\beta^3 + 0.021\alpha\beta^2\zeta - 0.010\alpha\beta\zeta^2 - 0.033\alpha\zeta^3; \quad (65)$$

$$C_{m\zeta}^{post} = 0.429 - 3.325\alpha + 8.084\alpha^2 + 0.245\beta^2 - 0.974\beta\zeta + 0.199\zeta^2 - 7.822\alpha^3 - 0.363\alpha\beta^2 + 2.617\alpha\beta\zeta + 0.150\alpha\zeta^2 + 2.615\alpha^4 + 0.142\alpha^2\beta^2 - 1.957\alpha^2\beta\zeta - 0.220\alpha^2\zeta^2 - 0.311\beta^4 - 0.304\beta^3\zeta + 0.213\beta^2\zeta^2 + 0.680\beta\zeta^3 - 0.247\zeta^4; \quad (66)$$

$$C_{n\zeta}^{post} = 0.149\beta - 0.146\zeta - 0.223\alpha\beta + 0.055\alpha\zeta + 0.015\alpha^2\beta + 0.184\alpha^2\zeta - 0.377\beta^3 + 0.176\beta^2\zeta - 0.036\beta\zeta^2 + 0.067\zeta^3 - 0.013\alpha^3\beta - 0.106\alpha^3\zeta + 0.568\alpha\beta^3 - 0.318\alpha\beta^2\zeta + 0.020\alpha\beta\zeta^2 - 0.015\alpha\zeta^3; \quad (67)$$

Polynomials in angle of attack and normalized body  $p$ -rate:

$$C_{Y\hat{p}}^{post} = -0.058\hat{p} - 0.590\alpha\hat{p} + 0.970\hat{p}^2 + 0.582\alpha^2\hat{p} - 3.112\alpha\hat{p}^2 + 14.142\hat{p}^3; \quad (68)$$

$$C_{l\hat{p}}^{post} = 0.044\hat{p} - 0.589\alpha\hat{p} + 0.133\hat{p}^2 + 0.464\alpha^2\hat{p} + 0.013\alpha\hat{p}^2 - 2.843\hat{p}^3; \quad (69)$$

$$C_{n\hat{p}}^{post} = 0.113\hat{p} - 0.504\alpha\hat{p} - 0.240\hat{p}^2 + 0.369\alpha^2\hat{p} + 0.695\alpha\hat{p}^2 + 5.811\hat{p}^3; \quad (70)$$

$C_{X\hat{p}}^{post}$ ,  $C_{Z\hat{p}}^{post}$ ,  $C_{m\hat{p}}^{post}$  are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body  $q$ -rate:

$$C_{X\hat{q}}^{post} = 0.034 - 0.209\alpha + 22.961\hat{q} + 0.246\alpha^2 - 76.364\alpha\hat{q} + 821.140\hat{q}^2 - 0.097\alpha^3 + 59.647\alpha^2\hat{q} + 1308.300\alpha\hat{q}^2 + 2529.200\hat{q}^3; \quad (71)$$

$$C_{Z\hat{q}}^{post} = 0.015 - 0.385\alpha - 91.233\hat{q} + 0.978\alpha^2 + 199.050\alpha\hat{q} + 1312.200\hat{q}^2 - 0.677\alpha^3 - 165.610\alpha^2\hat{q} + 1557.900\alpha\hat{q}^2 + 2345.900\hat{q}^3; \quad (72)$$

$$C_{m\hat{q}}^{post} = 0.128 - 0.470\alpha - 11.370\hat{q} + 0.432\alpha^2 - 145.970\alpha\hat{q} + 1029.300\hat{q}^2 - 0.173\alpha^3 + 145.910\alpha^2\hat{q} + 1275.000\alpha\hat{q}^2 + 2370.400\hat{q}^3; \quad (73)$$

$C_{Y\hat{q}}^{post}$ ,  $C_{l\hat{q}}^{post}$ ,  $C_{n\hat{q}}^{post}$  are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body  $r$ -rate:

$$C_{Y\hat{r}}^{post} = 3.868\hat{r} - 11.818\alpha\hat{r} + 1.677\hat{r}^2 + 7.267\alpha^2\hat{r} - 4.315\alpha\hat{r}^2 + 2.662\hat{r}^3; \quad (74)$$

$$C_{l\hat{r}}^{post} = -0.154\hat{r} + 3.156\alpha\hat{r} + 0.019\hat{r}^2 - 3.867\alpha^2\hat{r} + 0.042\alpha\hat{r}^2 - 5.986\hat{r}^3; \quad (75)$$

$$C_{n\hat{r}}^{post} = -0.639\hat{r} + 0.714\alpha\hat{r} - 0.239\hat{r}^2 - 0.510\alpha^2\hat{r} + 0.203\alpha\hat{r}^2 + 19.902\hat{r}^3; \quad (76)$$

$C_{X\hat{r}}^{post}$ ,  $C_{Z\hat{r}}^{post}$ ,  $C_{m\hat{r}}^{post}$  are zero due to lack of GTM measurement data.

## II. EQUATIONS OF MOTION

We have the aerodynamic forces and moments in body axis system

$$\begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f = \frac{1}{2}\rho S V_A^2 \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}; \quad \begin{bmatrix} L^A \\ M^A \\ N^A \end{bmatrix}_f = \frac{1}{2}\rho S V_A^2 \begin{bmatrix} b C_l \\ c_a C_m \\ b C_n \end{bmatrix} + \begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f \times (\mathbf{x}_{cg} - \mathbf{x}_{cg}^{ref}); \quad (77)$$

and the weight force by rotation into the body axis system

$$X_g^G = -g \sin \Theta; \quad (78)$$

$$Y_g^G = -g \sin \Phi \cos \Theta; \quad (79)$$

$$Z_g^G = -g \cos \Phi \cos \Theta; \quad (80)$$

The resulting forces lead to changes in the velocity vector, in body axis system, by

$$\dot{\mathbf{V}}_{Af} = \frac{1}{m} \begin{bmatrix} X^A + X^G + X^F \\ Y^A + Y^G \\ Z^A + Z^G \end{bmatrix}_f + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{V}_{Af}. \quad (81)$$

with the thrust  $X_f^F = F$  (engines aligned with the  $x_f$ -axis). For a symmetric plane ( $I_{xy} = I_{yz} = 0$ ), the resulting moments in body axis are given as

$$L_f = L_f^A - q r (I_z - I_y) + p q I_{zx}; \quad (82)$$

$$M_f = M_f^A + M_f^F - p r (I_x - I_z) - (p^2 - r^2) I_{zx}; \quad (83)$$

$$N_f = N_f^A - p q (I_y - I_x) - q r I_{zx}; \quad (84)$$

with  $M_f^F = l_t F$  (engines symmetric to the  $x_f$ - $y_f$ -plane and shifted vertically from the origin by  $l_t$ ) and the inertias  $I_x, I_y, I_z, I_{zx}$ . The changes of angular body rates are then given as

$$\dot{p} = \frac{1}{I_x I_z - I_{zx}^2} (I_z L_f + I_{zx} N_f); \quad (85)$$

$$\dot{q} = \frac{1}{I_y} M_f; \quad (86)$$

$$\dot{r} = \frac{1}{I_x I_z - I_{zx}^2} (I_{zx} L_f + I_x N_f). \quad (87)$$

Here, the normalized body rates have been used with

$$\begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} = \frac{1}{2V_A} \begin{bmatrix} b p \\ c_a q \\ b r \end{bmatrix} \iff \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2V_A \begin{bmatrix} b^{-1} \hat{p} \\ c_a^{-1} \hat{q} \\ b^{-1} \hat{r} \end{bmatrix} \quad (88)$$

and

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{q}} \\ \dot{\hat{r}} \end{bmatrix} = \frac{1}{V_A} \left( \frac{1}{2} \begin{bmatrix} b\dot{p} \\ c_a\dot{q} \\ b\dot{r} \end{bmatrix} - \dot{V}_A \begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} \right). \quad (89)$$

The change of attitude is finally obtained by rotation into normal earth-fixed axis system:

$$\dot{\Phi} = p + q \sin \Phi \tan \Theta + r \cos \Phi \tan \Theta; \quad (90)$$

$$\dot{\Theta} = q \cos \Phi - r \sin \Phi; \quad (91)$$

$$\dot{\Psi} = q \sin \Phi \cos^{-1} \Theta + r \cos \Phi \cos^{-1} \Theta. \quad (92)$$

### III. LONGITUDINAL MODEL

The longitudinal model is restricted to the  $x_a$ - $z_a$ -plane, assuming  $\beta = \mu_A = \chi_A = 0$ .

#### A. Aerodynamic Coefficients

The longitudinal aerodynamic coefficients,  $C_L, C_D, C_m$ , are obtained as

$$C_L(\alpha, \eta) = C_{L\alpha}(\alpha) + C_{L\eta}(\alpha, \eta) \quad \text{with} \quad (93)$$

$$C_{L\alpha}(\alpha) = \begin{cases} 0.017 + 5.234\alpha + 1.985\alpha^2 - 30.060\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.279 + 3.251\alpha - 3.235\alpha^2 + 0.708\alpha^3 & \text{else} \end{cases},$$

$$C_{L\eta}(\alpha, \eta) = -0.000 + 0.003\alpha + 0.521\eta - 0.072\alpha^2 - 0.416\alpha\eta + 0.089\eta^2 + 0.051\alpha^3 + 0.039\alpha^2\eta - 0.293\alpha\eta^2 - 0.479\eta^3; \quad (94)$$

$$C_D(\alpha, \eta) = C_{D\alpha}(\alpha) + C_{D\eta}(\alpha, \eta) \quad \text{with} \quad (95)$$

$$C_{D\alpha}(\alpha) = \begin{cases} 0.029 - 0.110\alpha + 2.364\alpha^2 + 3.948\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ -0.170 + 1.427\alpha + 0.719\alpha^2 - 0.486\alpha^3 & \text{else} \end{cases},$$

$$C_{D\eta}(\alpha, \eta) = +0.008 - 0.012\alpha + 0.112\eta + 0.040\alpha^2 + 0.183\alpha\eta - 0.069\eta^2 - 0.053\alpha^3 - 0.043\alpha^2\eta - 0.070\alpha\eta^2 - 0.628\eta^3; \quad (96)$$

$$C_m(\alpha, \eta) = C_{m\alpha}(\alpha) + C_{m\eta}(\alpha, \eta) \quad \text{with} \quad (97)$$

$$C_{m\alpha}(\alpha) = \begin{cases} 0.117 - 1.475\alpha + 8.475\alpha^2 - 32.729\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.144 - 2.456\alpha + 2.304\alpha^2 - 0.950\alpha^3 & \text{else} \end{cases},$$

$$C_{m\eta}(\alpha, \eta) = +0.014 + 0.165\alpha - 1.968\eta - 0.410\alpha^2 + 1.365\alpha\eta - 0.415\eta^2 + 0.186\alpha^3 - 0.144\alpha^2\eta + 0.948\alpha\eta^2 + 1.356\eta^3. \quad (98)$$

#### B. Equations of motion

The longitudinal equations of motion are given as

$$\dot{V}_A = \frac{1}{m} \left( F \cos \alpha - \frac{1}{2} \rho S V_A^2 C_D(\alpha, \eta, q) - mg \sin \gamma_A \right), \quad (99)$$

$$\dot{\gamma}_A = \frac{1}{m V_A} \left( F \sin \alpha + \frac{1}{2} \rho S V_A^2 C_L(\alpha, \eta, q) - mg \cos \gamma_A \right), \quad (100)$$

$$\dot{q} = \frac{1}{I_y} \left( l_t F + \frac{1}{2} \rho S c_a V_A^2 C_m(\alpha, \eta, q) - \frac{1}{2} \rho S V_A^2 C_Z(\alpha, \eta, q) (x_{cg}^{\text{ref}} - x_{cg}) + \frac{1}{2} \rho S V_A^2 C_X(\alpha, \eta, q) (z_{cg}^{\text{ref}} - z_{cg}) \right); \quad (101)$$

with

$$\Theta = \alpha + \gamma. \quad (102)$$

### MATLAB SOURCE CODE

The source code for the aerodynamic coefficients and the equations of motion can be found at:

<https://github.com/pwpfit/GTMpw>



APPENDIX

A. Spline-based Longitudinal Coefficients

The purpose of this model is to provide an application example of a spline-based longitudinal aircraft model. It has neither been designed nor evaluated for engineering purposes. For the longitudinal equations of motion, refer to Section III-B and note

$$C_D(\alpha, \dots) = -C_X(\alpha, \dots) \cos \alpha - C_Z(\alpha, \dots) \sin \alpha \quad (103)$$

$$C_L(\alpha, \dots) = C_X(\alpha, \dots) \sin \alpha - C_Z(\alpha, \dots) \cos \alpha \quad (104)$$

The longitudinal aerodynamic coefficients are obtained as

$$C_{\odot}(\alpha, \eta, \hat{q}) = C_{\odot\alpha}(\alpha) + C_{\odot\eta}(\alpha, \eta) + C_{\odot\hat{q}}(\alpha, \hat{q}), \quad (105)$$

where  $C_{\odot} \in \{C_X, C_Z, C_m\}$ ,  $\hat{q} = c_a q / (2V_A)$ , and

$$C_{\odot\alpha}(\alpha, \eta) = \begin{cases} C_{\odot\alpha}^{(1)} & \text{if } \alpha \in (-\infty; \alpha_1), \\ C_{\odot\alpha}^{(2)} & \text{if } \alpha \in [\alpha_1; \alpha_2), \\ C_{\odot\alpha}^{(3)} & \text{if } \alpha \in [\alpha_2; \alpha_3), \\ C_{\odot\alpha}^{(4)} & \text{if } \alpha \in [\alpha_3; \alpha_4), \\ C_{\odot\alpha}^{(5)} & \text{if } \alpha \in [\alpha_4; \infty); \end{cases} \quad (106)$$

$$C_{\odot\eta}(\alpha, \eta) = \begin{cases} C_{\odot\eta}^{(1)} & \text{if } \alpha \in (-\infty; \alpha_1), \\ C_{\odot\eta}^{(2)} & \text{if } \alpha \in [\alpha_1; \alpha_2), \\ C_{\odot\eta}^{(3)} & \text{if } \alpha \in [\alpha_2; \alpha_3), \\ C_{\odot\eta}^{(4)} & \text{if } \alpha \in [\alpha_3; \alpha_4), \\ C_{\odot\eta}^{(5)} & \text{if } \alpha \in [\alpha_4; \infty); \end{cases} \quad (107)$$

$$C_{\odot\hat{q}}(\alpha, \hat{q}) = \begin{cases} C_{\odot\hat{q}}^{(1)} & \text{if } \hat{q} \in (-\infty; \hat{q}_1), \\ C_{\odot\hat{q}}^{(2)} & \text{if } \hat{q} \in [\hat{q}_1; \hat{q}_2), \\ C_{\odot\hat{q}}^{(3)} & \text{if } \hat{q} \in [\hat{q}_2; \hat{q}_3), \\ C_{\odot\hat{q}}^{(4)} & \text{if } \hat{q} \in [\hat{q}_3; \hat{q}_4), \\ C_{\odot\hat{q}}^{(5)} & \text{if } \hat{q} \in [\hat{q}_4; \infty); \end{cases} \quad (108)$$

with

$$\alpha_1 = 5^\circ, \quad \alpha_2 = 15^\circ, \quad \alpha_3 = 25^\circ, \quad \alpha_4 = 45^\circ; \quad (109)$$

$$\hat{q}_1 = -0.200^\circ, \quad \hat{q}_2 = -0.075^\circ, \quad \hat{q}_3 = 0.075^\circ, \quad \hat{q}_4 = 0.200^\circ; \quad (110)$$

and

$$C_{X\alpha}^{(1)}(\alpha) = -0.025 - 0.002\alpha + 0.827\alpha^2; \quad (111)$$

$$C_{X\alpha}^{(2)}(\alpha) = -0.204 + 2.778\alpha - 7.493\alpha^2; \quad (112)$$

$$C_{X\alpha}^{(3)}(\alpha) = 0.217 - 1.218\alpha + 1.628\alpha^2; \quad (113)$$

$$C_{X\alpha}^{(4)}(\alpha) = 0.022 - 0.110\alpha + 0.114\alpha^2; \quad (114)$$

$$C_{X\alpha}^{(5)}(\alpha) = -0.052 + 0.024\alpha + 0.063\alpha^2; \quad (115)$$

$$C_{Z\alpha}^{(1)}(\alpha) = -0.028 - 4.949\alpha + 0.837\alpha^2; \quad (116)$$

$$C_{Z\alpha}^{(2)}(\alpha) = 0.155 - 8.239\alpha + 14.537\alpha^2; \quad (117)$$

$$C_{Z\alpha}^{(3)}(\alpha) = -0.815 - 0.298\alpha - 1.648\alpha^2; \quad (118)$$

$$C_{Z\alpha}^{(4)}(\alpha) = -0.467 - 2.158\alpha + 0.786\alpha^2; \quad (119)$$

$$C_{Z\alpha}^{(5)}(\alpha) = -1.052 - 0.996\alpha + 0.254\alpha^2; \quad (120)$$

$$C_{m\alpha}^{(1)}(\alpha) = 0.157 - 1.724\alpha + 1.806\alpha^2 + 11.969\alpha^3; \quad (121)$$

$$C_{m\alpha}^{(2)}(\alpha) = 0.565 - 10.419\alpha + 59.277\alpha^2 - 118.320\alpha^3; \quad (122)$$

$$C_{m\alpha}^{(3)}(\alpha) = 7.543 - 64.304\alpha + 174.300\alpha^2 - 160.370\alpha^3; \quad (123)$$

$$C_{m\alpha}^{(4)}(\alpha) = -0.866 + 1.176\alpha - 1.875\alpha^2 + 0.681\alpha^3; \quad (124)$$

$$C_{m\alpha}^{(5)}(\alpha) = -1.053 + 1.752\alpha - 2.210\alpha^2 + 0.561\alpha^3; \quad (125)$$

$$C_{X\eta}^{(1)}(\alpha, \eta) = -0.010\eta + 0.195\alpha\eta - 0.082\eta^2 + 0.628\alpha^2\eta + 0.213\alpha\eta^2 - 0.036\eta^3; \quad (126)$$

$$C_{X\eta}^{(2)}(\alpha, \eta) = 0.011\eta - 0.009\alpha\eta - 0.060\eta^2 + 0.165\alpha^2\eta - 0.043\alpha\eta^2 - 0.036\eta^3; \quad (127)$$

$$C_{X\eta}^{(3)}(\alpha, \eta) = 0.060\eta - 0.175\alpha\eta - 0.043\eta^2 + 0.087\alpha^2\eta - 0.106\alpha\eta^2 - 0.036\eta^3; \quad (128)$$

$$C_{X\eta}^{(4)}(\alpha, \eta) = 0.027\eta - 0.079\alpha\eta - 0.056\eta^2 + 0.042\alpha^2\eta - 0.076\alpha\eta^2 - 0.036\eta^3; \quad (129)$$

$$C_{X\eta}^{(5)}(\alpha, \eta) = -0.281\eta + 0.586\alpha\eta - 0.133\eta^2 - 0.305\alpha^2\eta + 0.021\alpha\eta^2 - 0.036\eta^3; \quad (130)$$

$$C_{Z\eta}^{(1)}(\alpha, \eta) = -0.595\eta + 0.058\alpha\eta + 0.039\eta^2 + 0.045\alpha^2\eta + 0.023\alpha\eta^2 + 1.142\eta^3; \quad (131)$$

$$C_{Z\eta}^{(2)}(\alpha, \eta) = -0.626\eta + 0.461\alpha\eta + 0.012\eta^2 - 0.452\alpha^2\eta + 0.332\alpha\eta^2 + 1.142\eta^3; \quad (132)$$

$$C_{Z\eta}^{(3)}(\alpha, \eta) = -0.631\eta + 0.299\alpha\eta + 0.051\eta^2 + 0.224\alpha^2\eta + 0.183\alpha\eta^2 + 1.142\eta^3; \quad (133)$$

$$C_{Z\eta}^{(4)}(\alpha, \eta) = -0.666\eta + 0.516\alpha\eta - 0.337\eta^2 - 0.084\alpha^2\eta + 1.074\alpha\eta^2 + 1.142\eta^3; \quad (134)$$

$$C_{Z\eta}^{(5)}(\alpha, \eta) = -1.116\eta + 1.625\alpha\eta + 0.312\eta^2 - 0.768\alpha^2\eta + 0.247\alpha\eta^2 + 1.142\eta^3; \quad (135)$$

$$C_{m\eta}^{(1)}(\alpha, \eta) = -1.879\eta - 0.047\alpha\eta - 0.227\eta^2 - 0.845\alpha^2\eta + 0.378\alpha\eta^2 + 1.409\eta^3; \quad (136)$$

$$C_{m\eta}^{(2)}(\alpha, \eta) = -2.006\eta + 1.410\alpha\eta - 0.260\eta^2 - 0.807\alpha^2\eta + 0.756\alpha\eta^2 + 1.409\eta^3; \quad (137)$$

$$C_{m\eta}^{(3)}(\alpha, \eta) = -1.916\eta - 0.071\alpha\eta - 0.704\eta^2 + 3.530\alpha^2\eta + 2.449\alpha\eta^2 + 1.409\eta^3; \quad (138)$$

$$C_{m\eta}^{(4)}(\alpha, \eta) = -1.612\eta + 0.765\alpha\eta + 0.802\eta^2 + 0.017\alpha^2\eta - 1.002\alpha\eta^2 + 1.409\eta^3; \quad (139)$$

$$C_{m\eta}^{(5)}(\alpha, \eta) = -1.555\eta + 0.535\alpha\eta - 0.777\eta^2 + 0.219\alpha^2\eta + 1.009\alpha\eta^2 + 1.409\eta^3; \quad (140)$$

as well as

$$C_{X\hat{q}}^{(1)}(\alpha, \hat{q}) = -0.003 - 1.776\hat{q} - 0.010\alpha - 276.480\hat{q}^{(2)} - 2.319\hat{q}\alpha - 0.005\alpha^2; \quad (141)$$

$$C_{X\hat{q}}^{(2)}(\alpha, \hat{q}) = 0.001 + 0.689\hat{q} + 0.004\alpha + 120.080\hat{q}^{(2)} + 1.492\hat{q}\alpha - 0.005\alpha^2; \quad (142)$$

$$C_{X\hat{q}}^{(3)}(\alpha, \hat{q}) = 0.001 + 1.338\hat{q} + 0.003\alpha + 232.350\hat{q}^{(2)} + 0.558\hat{q}\alpha - 0.005\alpha^2; \quad (143)$$

$$C_{X\hat{q}}^{(4)}(\alpha, \hat{q}) = 0.001 + 1.897\hat{q} + 0.004\alpha + 144.900\hat{q}^{(2)} - 0.699\hat{q}\alpha - 0.005\alpha^2; \quad (144)$$

$$C_{X\hat{q}}^{(5)}(\alpha, \hat{q}) = 0.001 + 2.271\hat{q} - 0.009\alpha - 7.040\hat{q}^{(2)} + 2.981\hat{q}\alpha - 0.005\alpha^2; \quad (145)$$

$$C_{Z\hat{q}}^{(1)}(\alpha, \hat{q}) = -0.008 - 25.814\hat{q} + 0.138\alpha + 2806.400\hat{q}^{(2)} + 22.164\hat{q}\alpha + 0.004\alpha^2; \quad (146)$$

$$C_{Z\hat{q}}^{(2)}(\alpha, \hat{q}) = -0.010 - 32.148\hat{q} + 0.019\alpha + 1120.800\hat{q}^{(2)} - 11.921\hat{q}\alpha + 0.004\alpha^2; \quad (147)$$

$$C_{Z\hat{q}}^{(3)}(\alpha, \hat{q}) = -0.014 - 36.128\hat{q} + 0.029\alpha + 507.060\hat{q}^{(2)} - 3.943\hat{q}\alpha + 0.004\alpha^2; \quad (148)$$

$$C_{Z\hat{q}}^{(4)}(\alpha, \hat{q}) = -0.010 - 40.109\hat{q} + 0.019\alpha + 1120.800\hat{q}^{(2)} + 4.036\hat{q}\alpha + 0.004\alpha^2; \quad (149)$$

$$C_{Z\hat{q}}^{(5)}(\alpha, \hat{q}) = -0.008 - 46.443\hat{q} + 0.138\alpha + 2806.400\hat{q}^{(2)} - 30.049\hat{q}\alpha + 0.004\alpha^2; \quad (150)$$

$$C_{m\hat{q}}^{(1)}(\alpha, \hat{q}) = 0.008 - 41.631\hat{q} - 0.013\alpha + 300.350\hat{q}^{(2)} - 10.716\hat{q}\alpha - 0.081\alpha^2; \quad (151)$$

$$C_{m\hat{q}}^{(2)}(\alpha, \hat{q}) = 0.009 - 40.268\hat{q} + 0.009\alpha + 632.510\hat{q}^{(2)} - 4.284\hat{q}\alpha - 0.081\alpha^2; \quad (152)$$

$$C_{m\hat{q}}^{(3)}(\alpha, \hat{q}) = 0.009 - 40.445\hat{q} + 0.012\alpha + 674.850\hat{q}^{(2)} - 2.169\hat{q}\alpha - 0.081\alpha^2; \quad (153)$$

$$C_{m\hat{q}}^{(4)}(\alpha, \hat{q}) = 0.009 - 39.916\hat{q} + 0.009\alpha + 381.400\hat{q}^{(2)} - 0.193\hat{q}\alpha - 0.081\alpha^2; \quad (154)$$

$$C_{m\hat{q}}^{(5)}(\alpha, \hat{q}) = -0.001 - 35.258\hat{q} + 0.008\alpha - 208.640\hat{q}^{(2)} + 0.192\hat{q}\alpha - 0.081\alpha^2; \quad (155)$$

## NOMENCLATURE

$\alpha$	= Angle of attack (rad);
$\alpha_0$	= Low-angle of attack boundary ( $^\circ$ );
$\beta$	= Side-slip angle (rad);
$\gamma_A$	= Air-path inclination angle (rad);
$\zeta$	= Rudder deflection (rad), negative if leading to positive yaw moment;
$\eta$	= Elevator deflection (rad), negative if leading to positive pitch moment;
$\mu_A$	= Air-path bank angle (rad);
$\xi$	= Aileron deflection (rad), negative if leading to positive roll moment;
$\varrho$	= Air density ( $\varrho = 1.200 \text{ kg m}^{-3}$ );
$\chi_A$	= Air-path azimuth angle (rad);
$\Theta$	= Pitch angle (rad);
$\Phi$	= Bank angle (rad);
$\Psi$	= Azimuth angle (rad);
$b$	= Reference aerodynamic span ( $b = 2.088 \text{ m}$ );
$c_A$	= Aerodynamic mean chord ( $c_A = 0.280 \text{ m}$ );
$g$	= Standard gravitational acceleration ( $g \approx 9.810 \text{ m s}^{-2}$ );
$l_t$	= Engine vertical displacement, positive along $z_f$ -axis ( $l_t = 0.100 \text{ m}$ );
$m$	= Aircraft mass ( $m = 26.190 \text{ kg}$ );
$p$	= Roll rate ( $\text{rad s}^{-1}$ );
$q$	= Pitch rate ( $\text{rad s}^{-1}$ );
$r$	= Yaw rate ( $\text{rad s}^{-1}$ );
$x_{cg}, z_{cg}$	= Longitudinal position center of gravity ( $x_{cg} = -1.450 \text{ m}$ , $z_{cg} = -0.300 \text{ m}$ );
$x_{cg}^{\text{ref}}, z_{cg}^{\text{ref}}$	= Longitudinal position reference center of gravity ( $x_{cg}^{\text{ref}} = -1.460 \text{ m}$ , $z_{cg}^{\text{ref}} = -0.290 \text{ m}$ );
$C_l$	= Aerodynamic coefficient moment body $x_f$ -axis ( $\cdot$ );
$C_m$	= Aerodynamic coefficient moment body $y_f$ -axis ( $\cdot$ );
$C_n$	= Aerodynamic coefficient moment body $z_f$ -axis ( $\cdot$ );
$C_D$	= Aerodynamic drag coefficient, force negative air-path $x_a$ -axis ( $\cdot$ );
$C_L$	= Aerodynamic lift coefficient, force negative air-path $z_a$ -axis ( $\cdot$ );
$C_X$	= Aerodynamic coefficient force body $x_f$ -axis ( $\cdot$ );
$C_Y$	= Aerodynamic coefficient force body $y_f$ -axis ( $\cdot$ );
$C_Z$	= Aerodynamic coefficient force body $z_f$ -axis ( $\cdot$ );
$D$	= Drag force, positive along negative air-path $X_a$ -axis ( $L = -X_a^A$ , N);
$F$	= Thrust force (N), positive along body $x_f$ -axis;
$L$	= Lift force, positive along negative air-path $Z_a$ -axis ( $L = -Z_a^A$ , N);
$L_f$	= Roll moment (N m), mathematically positive around $x_f$ -axis;
$M_f$	= Pitch moment (N m), mathematically positive around $y_f$ -axis;
$N_f$	= Yaw moment (N m), mathematically positive around $z_f$ -axis;
$S$	= Wing area ( $S = 0.550 \text{ m}^2$ );
$V_A, \mathbf{V}_A$	= Aircraft speed and velocity <i>relative to air</i> ( $V_A = \ \mathbf{V}_A\ _2$ , $\text{m s}^{-1}$ );
$X^A$	= Resulting force along air-path $x_a$ -axis (N);
$X_f^A$	= Aerodynamic force along body $x_f$ -axis (N);
$X_f^F$	= Thrust force along body $x_f$ -axis (N);
$Y^A$	= Resulting force along air-path $y_a$ -axis (N);
$Y_f^A$	= Aerodynamic force along body $y_f$ -axis (N);
$Z^A$	= Resulting force along air-path $z_a$ -axis (N);
$Z_f^A$	= Aerodynamic force along body $z_f$ -axis (N);
$(\cdot)^{\text{post}}$	= Domain of high angle of attack;
$(\cdot)^{\text{pre}}$	= Domain of low angle of attack;
$x_a, y_a, z_a$	= Air-path axis system;
$x_f, y_f, z_f$	= Body axis system;
$x_g, y_g, z_g$	= Normal earth-fixed axis system;