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Piecewise Polynomial Model of the Aerodynamic Coefficients of the Generic Transport Model and its Equations of Motion*

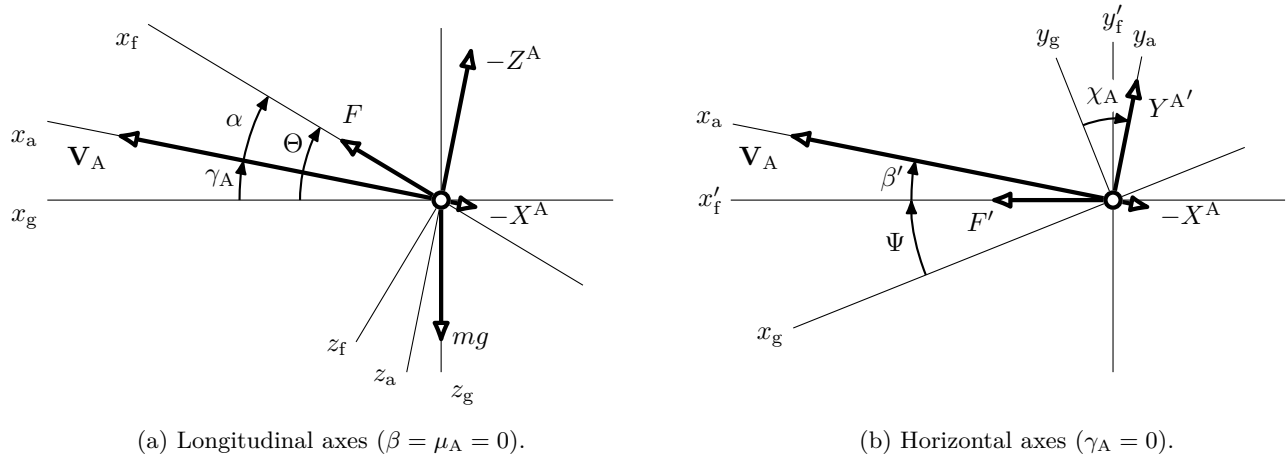
Torbjørn Cunis¹, Laurent Burlion², and Jean-Philippe Condomines³

Abstract

The purpose of this document is to illustrate the piecewise polynomial model which has been derived from wind-tunnel measurement data of the Generic Transport Model (GTM) using the `pwpfit` toolbox. For implementation details and use in MATLAB, please refer to the source code at <https://github.com/pwpfit/GTMpw>.

PRELIMINARIES

If not stated otherwise, all variables are in SI units. We will refer to the following axis systems of ISO 1151-1: the *body axis system* (x_f, y_f, z_f) aligned with the aircraft's fuselage; the *air-path axis system* (x_a, y_a, z_a) defined by the velocity vector \mathbf{V}_A ; and the *normal earth-fixed axis system* (x_g, y_g, z_g) . The orientation of the body axes with respect to the normal earth-fixed system is given by the attitude angles Φ, Θ, Ψ and to the air-path system by angle of attack α and side-slip β ; the orientation of the air-path axes to the normal earth-fixed system is given by azimuth χ_A , inclination γ_A , and bank-angle μ_A . (Fig. 1.)



(a) Longitudinal axes ($\beta = \mu_A = 0$).

(b) Horizontal axes ($\gamma_A = 0$).

Fig. 1: Axis systems with angles and vectors. Projections into the plane are marked by '.

I. AERODYNAMIC COEFFICIENTS

The piecewise polynomial models of the aerodynamic coefficients are given

$$C_{\odot}(\alpha, \beta, \dots) = \begin{cases} C_{\odot}^{pre}(\alpha, \beta, \dots) & \text{if } \alpha \leq \alpha_0, \\ C_{\odot}^{post}(\alpha, \beta, \dots) & \text{else,} \end{cases} \quad (1)$$

where $C_{\odot} \in \{C_X, C_Y, C_Z, C_l, C_m, C_n\}$ are polynomials in angle of attack, side-slip, surface deflections, and normalized body rates; and the boundary is found at

$$\alpha_0 = 16.111^\circ. \quad (2)$$

The polynomials in low and high angle of attack, C_{\odot}^{pre} , C_{\odot}^{post} , are sums

$$C_{\odot}^{pre} = C_{\odot\alpha}^{pre}(\alpha) + C_{\odot\beta}^{pre}(\alpha, \beta) + C_{\odot\xi}^{pre}(\alpha, \beta, \xi) + C_{\odot\eta}^{pre}(\alpha, \beta, \eta) + C_{\odot\zeta}^{pre}(\alpha, \beta, \zeta) + C_{\odot p}^{pre}(\alpha, \hat{p}) + C_{\odot q}^{pre}(\alpha, \hat{q}) + C_{\odot r}^{pre}(\alpha, \hat{r}); \quad (3)$$

$$C_{\odot}^{post} = C_{\odot\alpha}^{post}(\alpha) + C_{\odot\beta}^{post}(\alpha, \beta) + C_{\odot\xi}^{post}(\alpha, \beta, \xi) + C_{\odot\eta}^{post}(\alpha, \beta, \eta) + C_{\odot\zeta}^{post}(\alpha, \beta, \zeta) + C_{\odot p}^{post}(\alpha, \hat{p}) + C_{\odot q}^{post}(\alpha, \hat{q}) + C_{\odot r}^{post}(\alpha, \hat{r}). \quad (4)$$

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In the following subsections, we present the polynomial terms obtained using the `pwfit` toolbox. Coefficients of absolute value lower than 10^{-2} have been omitted for readability.

A. Domain of low angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{pre} = -0.039 + 0.244\alpha + 4.453\alpha^2 - 17.398\alpha^3; \quad (5)$$

$$C_{Z\alpha}^{pre} = -0.017 - 5.241\alpha - 1.865\alpha^2 + 28.463\alpha^3; \quad (6)$$

$$C_{m\alpha}^{pre} = 0.119 - 1.465\alpha + 8.130\alpha^2 - 31.986\alpha^3; \quad (7)$$

$C_{Y\alpha}^{pre}$, $C_{l\alpha}^{pre}$, $C_{n\alpha}^{pre}$ are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{X\beta}^{pre} = 0.012 - 0.048\alpha - 1.433\alpha^2 + 0.035\alpha\beta + 0.033\beta^2 + 5.481\alpha^3 - 0.135\alpha^2\beta; \quad (8)$$

$$C_{Y\beta}^{pre} = -1.086\beta - 0.185\alpha\beta + 0.734\alpha^2\beta + 0.213\beta^3; \quad (9)$$

$$C_{Z\beta}^{pre} = -0.033 + 0.204\alpha + 1.730\alpha^2 + 0.065\beta^2 - 9.930\alpha^3 + 0.073\alpha^2\beta + 2.857\alpha\beta^2; \quad (10)$$

$$C_{l\beta}^{pre} = -0.066\beta - 0.298\alpha\beta + 1.191\alpha^2\beta; \quad (11)$$

$$C_{m\beta}^{pre} = -0.221\alpha - 3.304\alpha^2 - 0.025\alpha\beta - 0.725\beta^2 + 13.192\alpha^3 + 0.199\alpha^2\beta + 0.462\alpha\beta^2 - 0.016\beta^3; \quad (12)$$

$$C_{n\beta}^{pre} = 0.116\beta - 0.022\alpha\beta - 0.363\alpha^2\beta + 0.056\beta^3. \quad (13)$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{X\xi}^{pre} = 0.013\beta\xi - 0.012\xi^2 - 0.071\alpha\beta^2 - 0.148\alpha\beta\xi + 0.063\alpha\xi^2; \quad (14)$$

$$C_{Y\xi}^{pre} = 0.323\beta - 0.020\xi - 0.410\alpha\beta + 0.015\alpha\xi + 3.141\alpha^2\beta + 0.022\alpha^2\xi - 0.532\beta^3 - 0.703\beta\xi^2; \quad (15)$$

$$C_{Z\xi}^{pre} = -0.088\beta\xi - 0.010\xi^2 - 0.323\alpha\beta^2 + 0.446\alpha\beta\xi - 0.095\alpha\xi^2; \quad (16)$$

$$C_{l\xi}^{pre} = 0.013\beta - 0.070\xi + 0.036\alpha\xi + 0.286\alpha^2\beta + 0.230\alpha^2\xi - 0.047\beta^3 + 0.027\beta^2\xi - 0.043\beta\xi^2 + 0.039\xi^3; \quad (17)$$

$$C_{m\xi}^{pre} = 0.321\beta^2 + 0.031\beta\xi - 0.096\xi^2 - 0.931\alpha\beta^2 - 0.471\alpha\xi^2; \quad (18)$$

$$C_{n\xi}^{pre} = 0.067\beta - 0.058\alpha\beta + 0.344\alpha^2\beta + 0.043\alpha^2\xi - 0.110\beta^3 - 0.094\beta\xi^2 + 0.016\xi^3. \quad (19)$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{X\eta}^{pre} = -0.011\beta - 0.011\eta + 0.012\alpha\beta + 0.262\alpha\eta - 0.161\eta^2 - 0.778\alpha^2\eta + 0.097\alpha\beta^2 - 0.022\alpha\beta\eta - 0.012\alpha\eta^2 + 0.017\beta^3 - 0.070\eta^3; \quad (20)$$

$$C_{Y\eta}^{pre} = -0.323\beta + 0.410\alpha\beta - 3.141\alpha^2\beta + 0.532\beta^3; \quad (21)$$

$$C_{Z\eta}^{pre} = -0.033\beta - 1.087\eta + 0.148\alpha\beta + 0.029\beta^2 + 0.019\beta\eta + 0.020\eta^2 - 0.120\alpha^2\beta + 1.726\alpha^2\eta + 0.036\alpha\beta^2 - 0.055\alpha\beta\eta + 0.194\alpha\eta^2 - 0.018\beta^3 + 0.616\beta^2\eta + 0.024\beta\eta^2 + 1.518\eta^3; \quad (22)$$

$$C_{l\eta}^{pre} = -0.013\beta - 0.286\alpha^2\beta + 0.047\beta^3; \quad (23)$$

$$C_{m\eta}^{pre} = -0.107\beta - 3.706\eta + 0.412\alpha\beta - 0.144\alpha\eta + 0.061\beta^2 + 0.053\beta\eta - 0.481\eta^2 - 0.136\alpha^2\beta + 10.402\alpha^2\eta + 0.434\alpha\beta^2 - 0.045\alpha\beta\eta + 2.857\alpha\eta^2 + 0.013\beta^3 + 1.376\beta^2\eta + 0.221\beta\eta^2 + 2.544\eta^3; \quad (24)$$

$$C_{n\eta}^{pre} = -0.067\beta + 0.058\alpha\beta - 0.344\alpha^2\beta + 0.110\beta^3. \quad (25)$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{X\zeta}^{pre} = 0.011\beta - 0.012\alpha\beta + 0.014\beta^2 + 0.039\beta\zeta - 0.054\zeta^2 - 0.197\alpha\beta^2 - 0.141\alpha\beta\zeta + 0.064\alpha\zeta^2 - 0.017\beta^3; \quad (26)$$

$$C_{Y\zeta}^{pre} = -0.013\beta + 0.041\alpha\beta + 0.080\alpha\zeta - 0.284\alpha^2\beta - 0.595\alpha^2\zeta + 0.089\zeta^3; \quad (27)$$

$$C_{Z\zeta}^{pre} = 0.033\beta - 0.148\alpha\beta - 0.024\beta^2 - 0.024\beta\zeta + 0.022\zeta^2 + 0.120\alpha^2\beta + 0.017\alpha\beta^2 + 0.085\alpha\beta\zeta - 0.055\alpha\zeta^2 + 0.018\beta^3; \quad (28)$$

$$C_{l\zeta}^{pre} = -0.016\beta - 0.081\zeta + 0.191\alpha\beta + 0.069\alpha\zeta - 1.012\alpha^2\beta - 0.121\alpha^2\zeta + 0.061\beta^3 + 0.042\beta^2\zeta + 0.124\zeta^3; \quad (29)$$

$$C_{m\zeta}^{pre} = 0.107\beta - 0.412\alpha\beta - 0.265\beta^2 - 0.191\beta\zeta + 0.156\zeta^2 + 0.136\alpha^2\beta + 0.337\alpha\beta^2 + 0.794\alpha\beta\zeta - 0.260\alpha\zeta^2 - 0.013\beta^3; \quad (30)$$

$$C_{n\zeta}^{pre} = -0.122\zeta + 0.094\alpha\beta + 0.160\alpha\zeta + 0.393\alpha^2\beta - 0.493\alpha^2\zeta + 0.108\beta^3 - 0.029\beta^2\zeta + 0.022\beta\zeta^2. \quad (31)$$

Polynomials in angle of attack and normalized body p -rate:

$$C_{Y\hat{p}}^{pre} = 2.281\hat{p} - 7.166\alpha\hat{p} + 39.769\hat{p}^2 - 54.568\alpha^2\hat{p} + 689.390\alpha\hat{p}^2 + 8193.600\hat{p}^3; \quad (32)$$

$$C_{l\hat{p}}^{pre} = -14.046\hat{p} - 13.426\alpha\hat{p} + 25.305\hat{p}^2 + 175.810\alpha^2\hat{p} + 1247.500\alpha\hat{p}^2 - 988.450\hat{p}^3; \quad (33)$$

$$C_{n\hat{p}}^{pre} = -2.106\hat{p} - 1.812\alpha\hat{p} - 135.930\hat{p}^2 + 64.789\alpha^2\hat{p} - 296.100\alpha\hat{p}^2 + 4450.500\hat{p}^3; \quad (34)$$

$C_{X\hat{p}}^{pre}$, $C_{Z\hat{p}}^{pre}$, $C_{m\hat{p}}^{pre}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body q -rate:

$$C_{X\hat{q}}^{pre} = 63.167\hat{q} + 0.076\alpha^2 + 677.000\alpha\hat{q} + 440.640\hat{q}^2 + 0.126\alpha^3 + 1193.100\alpha^2\hat{q} + 2.281.300\alpha\hat{q}^2 - 1.043.400\hat{q}^3; \quad (35)$$

$$C_{Z\hat{q}}^{pre} = 0.091\alpha - 1875.100\hat{q} - 0.203\alpha^2 - 1872.400\alpha\hat{q} + 4.243.400\hat{q}^2 - 0.283\alpha^3 - 4745.800\alpha^2\hat{q} - 15.335.000\alpha\hat{q}^2 - 166.980\hat{q}^3; \quad (36)$$

$$C_{m\hat{q}}^{pre} = -0.021 - 0.033\alpha - 2383.500\hat{q} + 0.496\alpha^2 - 199.420\alpha\hat{q} + 1.792.400\hat{q}^2 + 0.908\alpha^3 + 1574.400\alpha^2\hat{q} + 7.267.100\alpha\hat{q}^2 - 2.980.200.000\hat{q}^3; \quad (37)$$

$C_{Y\hat{q}}^{pre}$, $C_{l\hat{q}}^{pre}$, $C_{n\hat{q}}^{pre}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body r -rate:

$$C_{Y\hat{r}}^{pre} = 44.179\hat{r} + 49.371\alpha\hat{r} + 2891.700\hat{r}^2 + 92.222\alpha^2\hat{r} - 5041.500\alpha\hat{r}^2 + 10.357\hat{r}^3; \quad (38)$$

$$C_{l\hat{r}}^{pre} = 9.679\hat{r} + 43.079\alpha\hat{r} - 15.899\hat{r}^2 + 48.546\alpha^2\hat{r} + 243.490\alpha\hat{r}^2 - 1.593.100\hat{r}^3; \quad (39)$$

$$C_{n\hat{r}}^{pre} = -9.646\hat{r} - 14.456\alpha\hat{r} - 1548.100\hat{r}^2 - 53.131\alpha^2\hat{r} + 3188.800\alpha\hat{r}^2 + 4313.400\hat{r}^3; \quad (40)$$

$C_{X\hat{r}}^{pre}$, $C_{Z\hat{r}}^{pre}$, $C_{m\hat{r}}^{pre}$ are zero due to lack of GTM measurement data.

B. Domain of high angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{post} = 0.019 - 0.130\alpha + 0.169\alpha^2 - 0.022\alpha^3; \quad (41)$$

$$C_{Z\alpha}^{post} = -0.365 - 2.711\alpha + 1.647\alpha^2 - 0.369\alpha^3; \quad (42)$$

$$C_{m\alpha}^{post} = 0.247 - 2.847\alpha + 2.748\alpha^2 - 1.105\alpha^3; \quad (43)$$

$C_{Y\alpha}^{post}$, $C_{l\alpha}^{post}$, $C_{n\alpha}^{post}$ are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{X\beta}^{post} = 0.016\alpha - 0.023\alpha^2 + 0.053\beta^2 - 0.061\alpha\beta^2; \quad (44)$$

$$C_{Y\beta}^{post} = -0.887\beta - 0.867\alpha\beta + 0.637\alpha^2\beta + 0.213\beta^3; \quad (45)$$

$$C_{Z\beta}^{post} = -0.219 + 0.690\alpha + 0.018\beta - 0.452\alpha^2 - 0.068\alpha\beta + 0.939\beta^2 + 0.029\alpha^3 + 0.055\alpha^2\beta - 0.251\alpha\beta^2; \quad (46)$$

$$C_{l\beta}^{post} = 0.014\beta - 0.277\alpha\beta + 0.103\alpha^2\beta; \quad (47)$$

$$C_{m\beta}^{post} = -0.247 + 1.291\alpha + 0.036\beta - 1.970\alpha^2 - 0.087\alpha\beta - 1.023\beta^2 + 0.838\alpha^3 + 0.028\alpha^2\beta + 1.520\alpha\beta^2 - 0.016\beta^3; \quad (48)$$

$$C_{n\beta}^{post} = 0.150\beta - 0.279\alpha\beta + 0.129\alpha^2\beta + 0.056\beta^3. \quad (49)$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{X\xi}^{post} = -0.068\beta^2 - 0.036\beta\xi + 0.032\xi^2 + 0.140\alpha\beta^2 + 0.024\alpha\beta\xi - 0.095\alpha\xi^2; \quad (50)$$

$$C_{Y\xi}^{post} = 1.026\beta - 0.020\xi - 2.635\alpha\beta + 0.026\alpha\xi + 2.152\alpha^2\beta - 0.016\alpha^2\xi - 0.532\beta^3 - 0.703\beta\xi^2; \quad (51)$$

$$C_{Z\xi}^{post} = -0.112\beta^2 + 0.061\beta\xi - 0.124\xi^2 + 0.093\alpha\beta^2 - 0.086\alpha\beta\xi + 0.309\alpha\xi^2; \quad (52)$$

$$C_{l\xi}^{post} = 0.078\beta - 0.046\xi - 0.195\alpha\beta + 0.013\alpha\xi + 0.152\alpha^2\beta - 0.047\beta^3 + 0.027\beta^2\xi - 0.043\beta\xi^2 + 0.039\xi^3; \quad (53)$$

$$C_{m\xi}^{post} = 0.201\beta^2 + 0.041\beta\xi - 0.205\xi^2 - 0.505\alpha\beta^2 - 0.032\alpha\beta\xi - 0.082\alpha\xi^2; \quad (54)$$

$$C_{n\xi}^{post} = 0.130\beta - 0.237\alpha\beta + 0.024\alpha\xi + 0.187\alpha^2\beta - 0.017\alpha^2\xi - 0.110\beta^3 - 0.094\beta\xi^2 + 0.016\xi^3. \quad (55)$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{X\eta}^{post} = -0.023\eta - 0.016\alpha\beta + 0.123\alpha\eta + 0.075\beta^2 - 0.154\eta^2 + 0.013\alpha^2\beta - 0.126\alpha^2\eta - 0.164\alpha\beta^2 - 0.037\alpha\eta^2 + 0.017\beta^3 - 0.070\eta^3; \quad (56)$$

$$C_{Y\eta}^{post} = -1.026\beta + 2.635\alpha\beta - 2.152\alpha^2\beta + 0.532\beta^3; \quad (57)$$

$$C_{Z\eta}^{post} = -0.013\beta - 1.307\eta + 0.058\alpha\beta + 1.429\alpha\eta + 0.032\beta^2 - 0.260\eta^2 - 0.048\alpha^2\beta - 0.583\alpha^2\eta + 0.026\alpha\beta^2 + 1.188\alpha\eta^2 - 0.018\beta^3 + 0.616\beta^2\eta + 0.024\beta\eta^2 + 1.518\eta^3; \quad (58)$$

$$C_{l\eta}^{post} = -0.078\beta + 0.195\alpha\beta - 0.152\alpha^2\beta + 0.047\beta^3; \quad (59)$$

$$C_{m\eta}^{post} = -3.589\eta + 2.488\alpha\eta + 0.268\beta^2 + 0.038\beta\eta + 0.148\eta^2 - 0.441\alpha^2\eta - 0.303\alpha\beta^2 + 0.622\alpha\eta^2 + 0.013\beta^3 + 1.376\beta^2\eta + 0.221\beta\eta^2 + 2.544\eta^3; \quad (60)$$

$$C_{n\eta}^{post} = -0.130\beta + 0.237\alpha\beta - 0.187\alpha^2\beta + 0.110\beta^3. \quad (61)$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{X\zeta}^{post} = 0.016\alpha\beta - 0.097\beta^2 - 0.020\beta\zeta - 0.040\zeta^2 - 0.013\alpha^2\beta + 0.198\alpha\beta^2 + 0.068\alpha\beta\zeta + 0.015\alpha\zeta^2 - 0.017\beta^3; \quad (62)$$

$$C_{Y\zeta}^{post} = -0.099\beta - 0.076\zeta + 0.333\alpha\beta + 0.221\alpha\zeta - 0.237\alpha^2\beta - 0.177\alpha^2\zeta + 0.089\zeta^3; \quad (63)$$

$$C_{Z\zeta}^{post} = 0.013\beta - 0.058\alpha\beta - 0.047\beta^2 + 0.048\alpha^2\beta + 0.101\alpha\beta^2 + 0.031\alpha\beta\zeta + 0.045\alpha\zeta^2 + 0.018\beta^3; \quad (64)$$

$$C_{l\zeta}^{post} = -0.069\beta - 0.104\zeta + 0.118\alpha\beta + 0.165\alpha\zeta - 0.079\alpha^2\beta - 0.175\alpha^2\zeta + 0.061\beta^3 + 0.042\beta^2\zeta + 0.124\zeta^3; \quad (65)$$

$$C_{m\zeta}^{post} = -0.316\beta^2 + 0.190\beta\zeta + 0.096\zeta^2 + 0.518\alpha\beta^2 - 0.559\alpha\beta\zeta - 0.048\alpha\zeta^2 - 0.013\beta^3; \quad (66)$$

$$C_{n\zeta}^{post} = 0.451\beta - 0.304\zeta - 1.755\alpha\beta + 0.791\alpha\zeta + 1.304\alpha^2\beta - 0.434\alpha^2\zeta + 0.108\beta^3 - 0.029\beta^2\zeta + 0.022\beta\zeta^2. \quad (67)$$

Polynomials in angle of attack and normalized body p -rate:

$$C_{Y\hat{p}}^{post} = 2.714\hat{p} - 33.368\alpha\hat{p} + 3086.500\hat{p}^2 + 33.135\alpha^2\hat{p} - 10146\alpha\hat{p}^2 + 8193.600\hat{p}^3; \quad (68)$$

$$C_{l\hat{p}}^{post} = 3.365\hat{p} - 33.324\alpha\hat{p} + 346.110\hat{p}^2 + 26.371\alpha^2\hat{p} + 106.590\alpha\hat{p}^2 - 988450\hat{p}^3; \quad (69)$$

$$C_{n\hat{p}}^{post} = 8.850\hat{p} - 28.449\alpha\hat{p} - 879.480\hat{p}^2 + 20.942\alpha^2\hat{p} + 2348.200\alpha\hat{p}^2 + 4450.500\hat{p}^3; \quad (70)$$

$C_{X\hat{p}}^{post}$, $C_{Z\hat{p}}^{post}$, $C_{m\hat{p}}^{post}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body q -rate:

$$C_{X\hat{q}}^{post} = 0.038 - 0.176\alpha + 1260.000\hat{q} + 0.245\alpha^2 - 4151.300\alpha\hat{q} + 1750800\hat{q}^2 - 0.097\alpha^3 + 3226.500\alpha^2\hat{q} - 2378300\alpha\hat{q}^2 - 1043400\hat{q}^3; \quad (71)$$

$$C_{Z\hat{q}}^{post} = 0.050 - 0.411\alpha - 5274.900\hat{q} + 0.977\alpha^2 + 11599\alpha\hat{q} - 3051200\hat{q}^2 - 0.677\alpha^3 - 9654.600\alpha^2\hat{q} + 10607000\alpha\hat{q}^2 - 166980\hat{q}^3; \quad (72)$$

$$C_{m\hat{q}}^{post} = 0.115 - 0.412\alpha - 662.240\hat{q} + 0.431\alpha^2 - 8186.500\alpha\hat{q} + 6062800\hat{q}^2 - 0.172\alpha^3 + 8209.000\alpha^2\hat{q} - 7919900\alpha\hat{q}^2 - 298020000\hat{q}^3; \quad (73)$$

$C_{Y\hat{q}}^{post}$, $C_{l\hat{q}}^{post}$, $C_{n\hat{q}}^{post}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body r -rate:

$$C_{Y\hat{r}}^{post} = 222.670\hat{r} - 676.410\alpha\hat{r} + 5458.500\hat{r}^2 + 415.860\alpha^2\hat{r} - 14170\alpha\hat{r}^2 + 10357\hat{r}^3; \quad (74)$$

$$C_{l\hat{r}}^{post} = -7.843\hat{r} + 181.470\alpha\hat{r} + 14.963\hat{r}^2 - 222.030\alpha^2\hat{r} + 133.730\alpha\hat{r}^2 - 1593100\hat{r}^3; \quad (75)$$

$$C_{n\hat{r}}^{post} = -27.260\hat{r} + 41.592\alpha\hat{r} - 837.820\hat{r}^2 - 29.673\alpha^2\hat{r} + 662.790\alpha\hat{r}^2 + 4313.400\hat{r}^3; \quad (76)$$

$C_{X\hat{r}}^{post}$, $C_{Z\hat{r}}^{post}$, $C_{m\hat{r}}^{post}$ are zero due to lack of GTM measurement data.

II. EQUATIONS OF MOTION

We have the aerodynamic forces and moments in body axis

$$\begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f = \frac{1}{2} \rho S V_A^2 \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}; \quad \begin{bmatrix} L^A \\ M^A \\ N^A \end{bmatrix}_f = \frac{1}{2} \rho S V_A^2 \begin{bmatrix} b C_l \\ c_A C_m \\ b C_n \end{bmatrix} + \begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f \times (\mathbf{x}_{cg} - \mathbf{x}_{cg}^{\text{ref}}); \quad (77)$$

and the resulting forces by rotation into the air-path axis system

$$X_a = (X_f^A \cos \alpha + Z_f^A \sin \alpha + X_f^F \cos \alpha) \cos \beta + Y_f^A \sin \beta; \quad (78)$$

$$Y_a = -(X_f^A \cos \alpha + Z_f^A \sin \alpha + X_f^F \cos \alpha) \sin \beta + Y_f^A \cos \beta; \quad (79)$$

$$Z_a = -X_f^A \sin \alpha + Z_f^A \cos \alpha - X_f^F \sin \alpha; \quad (80)$$

with the thrust $X_f^F = F$ (engines aligned with the x_f -axis). The resulting forces lead to changes in air speed, side-slip, air-path inclination, and azimuth by

$$\dot{V}_A = \frac{1}{m} (X_a - mg \sin \gamma_A); \quad (81)$$

$$\dot{\beta} = \frac{1}{m V_A} Y_a; \quad (82)$$

$$\dot{\gamma}_A = -\frac{1}{m V_A} (Z_a \cos \mu_A + mg \cos \gamma_A); \quad (83)$$

$$\dot{\chi}_A = -\frac{1}{m V_A} Z_a \sin \mu_A. \quad (84)$$

For a symmetric plane ($I_{xy} = I_{uz} = 0$), the resulting moments in body axis are given as

$$L_f = L_f^A - q r (I_z - I_y) + p q I_{zx}; \quad (85)$$

$$M_f = M_f^A + M_f^F - p r (I_x - I_z) - (p^2 - r^2) I_{zx}; \quad (86)$$

$$N_f = N_f^A - p q (I_y - I_x) - q r I_{zx}; \quad (87)$$

with $M_f^F = l_t F$ (engines symmetric to the x_f - y_f -plane and shifted vertically from the origin by l_t) and the inertias I_x, I_y, I_z, I_{zx} . The changes of angular body rates are then given as

$$\dot{p} = \frac{1}{I_x I_z - I_{zx}^2} (I_z L_f + I_{zx} N_f); \quad (88)$$

$$\dot{q} = \frac{1}{I_y} M_f; \quad (89)$$

$$\dot{r} = \frac{1}{I_x I_z - I_{zx}^2} (I_{zx} L_f + I_x N_f). \quad (90)$$

Here, the normalized body rates have been used with

$$\begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} = \frac{1}{2V_A} \begin{bmatrix} b p \\ c_A q \\ b r \end{bmatrix} \iff \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2V_A \begin{bmatrix} b^{-1} \hat{p} \\ c_A^{-1} \hat{q} \\ b^{-1} \hat{r} \end{bmatrix} \quad (91)$$

and

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{q}} \\ \dot{\hat{r}} \end{bmatrix} = \frac{1}{V_A} \left(\frac{1}{2} \begin{bmatrix} b \dot{p} \\ c_A \dot{q} \\ b \dot{r} \end{bmatrix} - \dot{V}_A \begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} \right). \quad (92)$$

The change of attitude is finally obtained by rotation into normal earth-fixed axis system:

$$\dot{\Phi} = p + q \sin \Phi \tan \Theta + r \cos \Phi \tan \Theta; \quad (93)$$

$$\dot{\Theta} = q \cos \Phi - r \sin \Phi; \quad (94)$$

$$\dot{\Psi} = q \sin \Phi \cos^{-1} \Theta + r \cos \Phi \cos^{-1} \Theta. \quad (95)$$

III. LONGITUDINAL MODEL

The longitudinal model is restricted to the x_a - z_a -plane, assuming $\beta = \mu_A = \chi_A = 0$.

A. Aerodynamic Coefficients

The longitudinal aerodynamic coefficients, C_L, C_D, C_m , are obtained as

$$C_L(\alpha, \eta) = C_{L\alpha}(\alpha) + C_{L\eta}(\alpha, \eta) \quad \text{with}$$

$$C_{L\alpha}(\alpha) = \begin{cases} 0.017 + 5.234\alpha + 1.985\alpha^2 - 30.060\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.279 + 3.251\alpha - 3.235\alpha^2 + 0.708\alpha^3 & \text{else} \end{cases}, \quad (96)$$

$$C_{L\eta}(\alpha, \eta) = -0.000 + 0.003\alpha + 0.521\eta - 0.072\alpha^2 - 0.416\alpha\eta + 0.089\eta^2 + 0.051\alpha^3 + 0.039\alpha^2\eta - 0.293\alpha\eta^2 - 0.479\eta^3; \quad (97)$$

$$C_D(\alpha, \eta) = C_{D\alpha}(\alpha) + C_{D\eta}(\alpha, \eta) \quad \text{with}$$

$$C_{D\alpha}(\alpha) = \begin{cases} 0.029 - 0.110\alpha + 2.364\alpha^2 + 3.948\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ -0.170 + 1.427\alpha + 0.719\alpha^2 - 0.486\alpha^3 & \text{else} \end{cases}, \quad (98)$$

$$C_{D\eta}(\alpha, \eta) = +0.008 - 0.012\alpha + 0.112\eta + 0.040\alpha^2 + 0.183\alpha\eta - 0.069\eta^2 - 0.053\alpha^3 - 0.043\alpha^2\eta - 0.070\alpha\eta^2 - 0.628\eta^3; \quad (99)$$

$$C_m(\alpha, \eta) = C_{m\alpha}(\alpha) + C_{m\eta}(\alpha, \eta) \quad \text{with}$$

$$C_{m\alpha}(\alpha) = \begin{cases} 0.117 - 1.475\alpha + 8.475\alpha^2 - 32.729\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.144 - 2.456\alpha + 2.304\alpha^2 - 0.950\alpha^3 & \text{else} \end{cases}, \quad (100)$$

$$C_{m\eta}(\alpha, \eta) = +0.014 + 0.165\alpha - 1.968\eta - 0.410\alpha^2 + 1.365\alpha\eta - 0.415\eta^2 + 0.186\alpha^3 - 0.144\alpha^2\eta + 0.948\alpha\eta^2 + 1.356\eta^3. \quad (101)$$

B. Equations of motion

The longitudinal equations of motion are given as

$$\dot{V}_A = \frac{1}{m} \left(T \cos \alpha - \frac{1}{2} \rho S V_A^2 C_D(\alpha, \eta, q) - mg \sin \gamma_A \right), \quad (102)$$

$$\dot{\gamma}_A = \frac{1}{m V_A} \left(T \sin \alpha + \frac{1}{2} \rho S V_A^2 C_L(\alpha, \eta, q) - mg \cos \gamma_A \right), \quad (103)$$

$$\dot{q} = \frac{1}{I_y} \left(l_t T + \frac{1}{2} \rho S c_A V_A^2 C_m(\alpha, \eta, q) - \frac{1}{2} \rho S V_A^2 C_Z(\alpha, \eta, q) (x_{cg}^{\text{ref}} - x_{cg}) + \frac{1}{2} \rho S V_A^2 C_X(\alpha, \eta, q) (z_{cg}^{\text{ref}} - z_{cg}) \right); \quad (104)$$

with

$$\Theta = \alpha + \gamma. \quad (105)$$

MATLAB SOURCE CODE

The source code for the aerodynamic coefficients and the equations of motion can be found at:

<https://github.com/pwpfit/GTMpw>

NOMENCLATURE

α	= Angle of attack (rad);
α_0	= Low-angle of attack boundary ($^\circ$);
β	= Side-slip angle (rad);
γ_A	= Air-path inclination angle (rad);
ζ	= Rudder deflection (rad), negative if leading to positive yaw moment;
η	= Elevator deflection (rad), negative if leading to positive pitch moment;
μ_A	= Air-path bank angle (rad);
ξ	= Aileron deflection (rad), negative if leading to positive roll moment;
ϱ	= Air density ($\varrho = 1.200 \text{ kg m}^{-3}$);
χ_A	= Air-path azimuth angle (rad);
Θ	= Pitch angle (rad);
Φ	= Bank angle (rad);
Ψ	= Azimuth angle (rad);
b	= Reference aerodynamic span ($b = 2.088 \text{ m}$);
c_A	= Aerodynamic mean chord ($c_A = 0.280 \text{ m}$);
g	= Standard gravitational acceleration ($g \approx 9.810 \text{ m s}^{-2}$);
l_t	= Engine vertical displacement, positive along z_f -axis ($l_t = 0.100 \text{ m}$);
m	= Aircraft mass ($m = 26.190 \text{ kg}$);
p	= Roll rate (rad s^{-1});
q	= Pitch rate (rad s^{-1});
r	= Yaw rate (rad s^{-1});
x_{cg}, z_{cg}	= Longitudinal position center of gravity ($x_{cg} = -1.450 \text{ m}$, $z_{cg} = -0.300 \text{ m}$);
$x_{cg}^{\text{ref}}, z_{cg}^{\text{ref}}$	= Longitudinal position reference center of gravity ($x_{cg}^{\text{ref}} = -1.460 \text{ m}$, $z_{cg}^{\text{ref}} = -0.290 \text{ m}$);
C_l	= Aerodynamic coefficient moment body x_f -axis (\cdot);
C_m	= Aerodynamic coefficient moment body y_f -axis (\cdot);
C_n	= Aerodynamic coefficient moment body z_f -axis (\cdot);
C_D	= Aerodynamic drag coefficient, force negative air-path x_a -axis (\cdot);
C_L	= Aerodynamic lift coefficient, force negative air-path z_a -axis (\cdot);
C_X	= Aerodynamic coefficient force body x_f -axis (\cdot);
C_Y	= Aerodynamic coefficient force body y_f -axis (\cdot);
C_Z	= Aerodynamic coefficient force body z_f -axis (\cdot);
D	= Drag force, positive along negative air-path X_a -axis ($L = -X_a^A$, N);
F	= Thrust force (N), positive along body x_f -axis;
L	= Lift force, positive along negative air-path Z_a -axis ($L = -Z_a^A$, N);
L_f	= Roll moment (N m), mathematically positive around x_f -axis;
M_f	= Pitch moment (N m), mathematically positive around y_f -axis;
N_f	= Yaw moment (N m), mathematically positive around z_f -axis;
S	= Wing area ($S = 0.550 \text{ m}^2$);
V_A, \mathbf{V}_A	= Aircraft speed and velocity <i>relative to air</i> ($V_A = \ \mathbf{V}_A\ _2$, m s^{-1});
X^A	= Resulting force along air-path x_a -axis (N);
X_f^A	= Aerodynamic force along body x_f -axis (N);
X_f^F	= Thrust force along body x_f -axis (N);
Y^A	= Resulting force along air-path y_a -axis (N);
Y_f^A	= Aerodynamic force along body y_f -axis (N);
Z^A	= Resulting force along air-path z_a -axis (N);
Z_f^A	= Aerodynamic force along body z_f -axis (N);
$(\cdot)^{\text{post}}$	= Domain of high angle of attack;
$(\cdot)^{\text{pre}}$	= Domain of low angle of attack;
x_a, y_a, z_a	= Air-path axis system;
x_f, y_f, z_f	= Body axis system;
x_g, y_g, z_g	= Normal earth-fixed axis system;