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## Quantitative Convergence and Stability of Seismic Inverse Problems.

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# Quantitative Convergence and Stability of Seismic Inverse Problems.

Florian Faucher<sup>1</sup>,  
Hélène Barucq<sup>1</sup>, Henri Calandra<sup>2</sup> and Guy Chavent<sup>1</sup>.

**Reconstruction Methods for Inverse Problems**  
**Indam Workshop, Roma, Italy,**  
May 28<sup>th</sup> – June 1<sup>st</sup>, 2018.



Istituto Nazionale di Alta Matematica



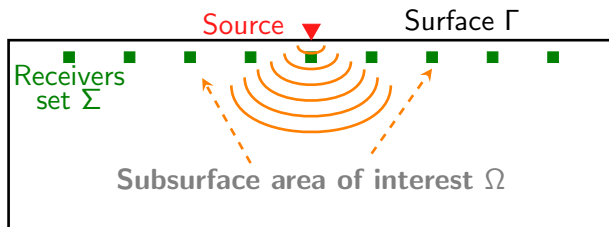
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<sup>1</sup>Inria Bordeaux Sud-Ouest, Project-Team Magique-3D.

<sup>2</sup>Total E&P

# Seismic inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



- ▶ Reflection (back-scattered) partial data,
- ▶ only from the surface of the (large) domain,

**nonlinear, ill-posed inverse problem.**

# Overview

- 1 Time-Harmonic Inverse Problem, FWI
- 2 Quantitative stability and convergence of FWI
  - Finite Curvature/Limited Deflection problem
  - Numerical convergence estimates
  - Numerical stability estimates
- 3 Numerical experiments
- 4 Conclusion

# Plan

## 1 Time-Harmonic Inverse Problem, FWI

# Time-harmonic wave equation

The forward problem wave equation depends on the medium:

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- ▶ elastic isotropic ( $\lambda, \mu, \rho$ )

$$-\rho\omega^2 \mathbf{u} - \nabla(\lambda \nabla \cdot \mathbf{u}) - \nabla \cdot (\mu [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]) = 0.$$



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$$-\rho\omega^2 \mathbf{u} - \nabla(\lambda \nabla \cdot \mathbf{u}) - \nabla \cdot (\mu [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]) = 0.$$

- ▶ anisotropy (stiffness tensor,  $\rho$ )

$$-\omega^2 \rho \mathbf{u} - \nabla \cdot (\underline{\underline{C}} \epsilon(\mathbf{u})) = 0.$$

Viscous behavior are considered with complex coefficients.

# Full Waveform Inversion (FWI)

FWI provides a quantitative reconstruction of the subsurface by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|F(m) - d\|^2.$$

- ▶  $d$  are the observed data,
- ▶  $F(m)$  represents the simulation using an initial model  $m$ :

$$F : m \rightarrow \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}})\}.$$



P. Lailly

The seismic inverse problem as a sequence of before stack migrations  
[Conference on Inverse Scattering: Theory and Application, SIAM, 1983](#)



A. Tarantola

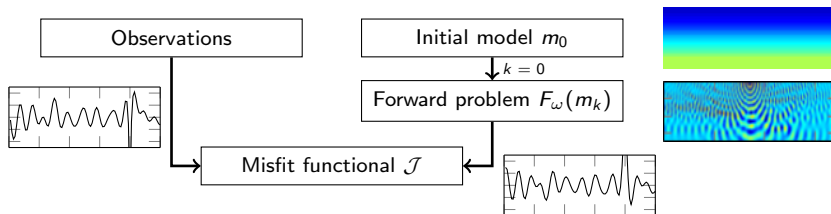
Inversion of seismic reflection data in the acoustic approximation  
[Geophysics, 1984](#)



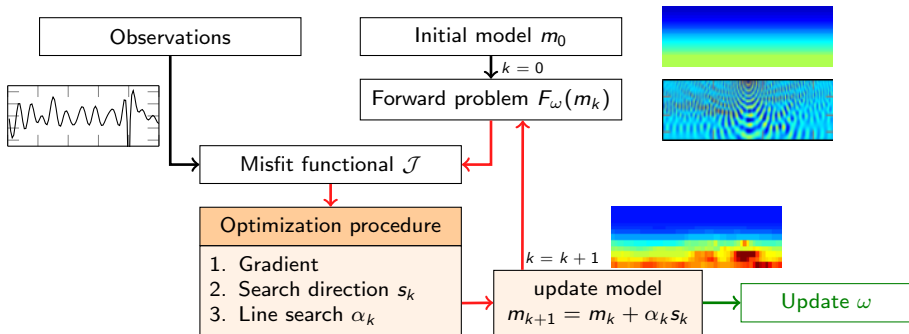
A. Tarantola

Inversion of travel times and seismic waveforms  
[Seismic tomography, 1987](#)

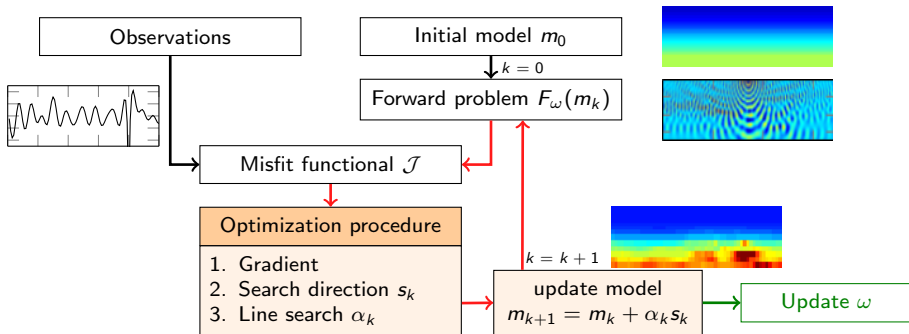
# FWI, iterative minimization



# FWI, iterative minimization



# FWI, iterative minimization



## Numerical methods

- ▶ Forward problem resolution with Discontinuous Galerkin methods,
  - ▶ parallel computation, HPC, large-scale optimization.
- 
- ▶ Multi-frequency algorithm, stability and convergence.

# Plan

- 2 Quantitative stability and convergence of FWI
  - Finite Curvature/Limited Deflection problem
  - Numerical convergence estimates
  - Numerical stability estimates

# Stability & Convergence

Helmholtz inverse problem from back-scattered **partial** data,  
**quantitative reconstruction with iterative optimization.**

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

## Convergence radius

- ▶ initial model needs to be within the radius of convergence.

## Stability

- ▶ ensures reconstruction accuracy.

# Stability & Convergence

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## Convergence radius

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## Stability

- ▶ ensures reconstruction accuracy.

**Numerical estimates** to guide the procedure

- ▶ frequency;
- ▶ parametrization;
- ▶ geometry;
- ▶ forward problem, ...



# Convergence (Finite Curvature/Limited Deflection)

## 1/ Convergence radius

- ▶ initial model needs to be within the radius of convergence.

Least squares minimization problem

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$



M. V. de Hoop, L. Qiu, O. Scherzer

An analysis of a multi-level projected steepest descent iteration for nonlinear inverse problems in Banach spaces subject to stability constraints

[Numerische Mathematik 2015](#)



G. Chavent

Nonlinear least squares for inverse problems: theoretical foundations and step-by-step guide for applications.

[Springer 2010](#)



G. Chavent and K. Kunisch

On weakly nonlinear inverse problems.

[SIAM Journal on Applied Mathematics 1996](#)

# Convergence (Finite Curvature/Limited Deflection)

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Least squares minimization problem

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

Finite Curvature/Limited Deflection guarantees **uniqueness** of the solution and **unimodality**: no local minimum.



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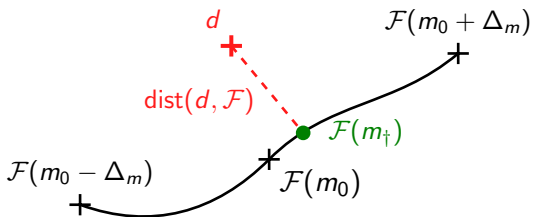


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# Condition 1/2: Finite Curvature problem



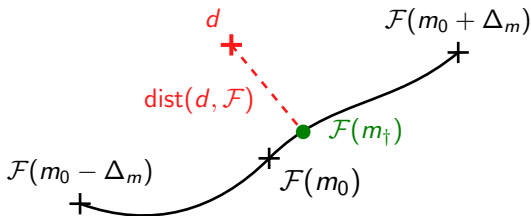
$$\forall m_0, \Delta_m \in \mathcal{M}, \quad P : t \in [0, 1] \rightarrow \mathcal{F}(m_0 + (2t - 1)\Delta_m).$$



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Nonlinear least squares for inverse problems: theoretical foundations and step-by-step guide for applications.  
Springer 2010

# Condition 1/2: Finite Curvature problem



$$\forall m_0, \Delta_m \in \mathcal{M}, \quad P : t \in [0, 1] \rightarrow \mathcal{F}(m_0 + (2t - 1)\Delta_m).$$

Finite Curvature controls the distance between data and attainable set,

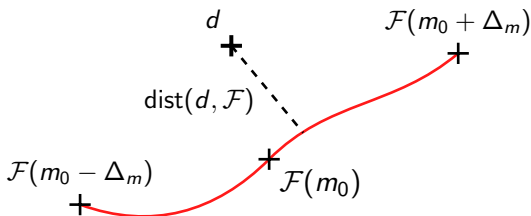
$$\text{dist}(d, \mathcal{F}) < \frac{\|P'(t)\|^2}{\|P''(t)\|}.$$



G. Chavent

Nonlinear least squares for inverse problems: theoretical foundations and step-by-step guide for applications.  
Springer 2010

# Condition 2/2: Limited Deflection problem



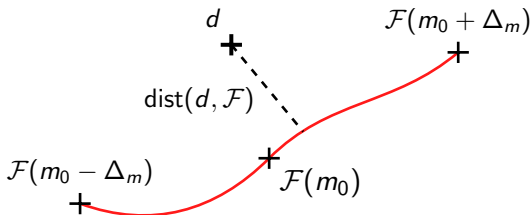
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# Condition 2/2: Limited Deflection problem



$$\forall m_0, \Delta_m \in \mathcal{M}, \quad P : t \in [0, 1] \rightarrow \mathcal{F}(m_0 + (2t - 1)\Delta_m).$$

Limited Deflection property controls the attainable set and model space,

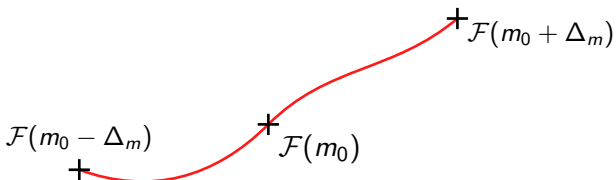
$$\Theta(P) \leq \int_0^1 \frac{\|P''(t)\|}{\|P'(t)\|} dt. \quad \text{FC/LD requires } \Theta(P) \leq \frac{\pi}{2}.$$



G. Chavent

Nonlinear least squares for inverse problems: theoretical foundations and step-by-step guide for applications.  
Springer 2010

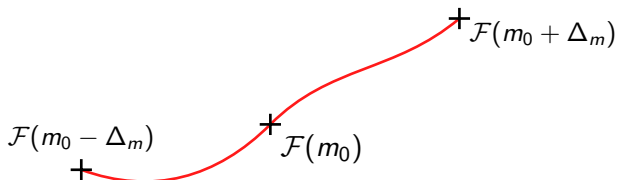
# Model space size estimation via Limited Deflection



For a given  $m_0$ , we estimate the maximal distance  $\Delta_m$  that still verifies the Limited Deflection property  $\Theta(P) \leq \frac{\pi}{2}$ .

**Larger size are required when missing a priori info.**

# Model space size estimation via Limited Deflection



For a given  $m_0$ , we estimate the maximal distance  $\Delta_m$  that still verifies the Limited Deflection property  $\Theta(P) \leq \frac{\pi}{2}$ .

- ▶  $\Delta_m$  is estimated in the direction  $\delta_k$ ,

$$\|\Delta_m^{\delta_k}\| \leq \frac{\pi}{4} \frac{\|\mathcal{F}'(m_0)(\delta_k)\|}{\|\mathcal{F}''(m_0)(\delta_k, \delta_k)\|}.$$

**Larger size are required when missing a priori info.**



# Estimation of the basin of attraction

Numerical estimate of the size  $\Delta_m^{\delta_k}$

- 1 with direction  $\delta_k$  (the geometry of the unknown),
- 2 with the frequency  $\omega$ ,
- 3 with the parametrization.

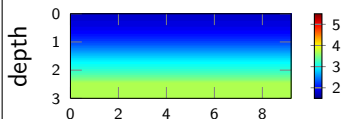
# Estimation of the basin of attraction

Numerical estimate of the size  $\Delta_{\delta_k}^m$

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- 3 with the parametrization.

**context:** Helmholtz equation with back-scattered data

$$(-\omega^2 m - \Delta)p = 0,$$



$m_0$  is a smooth velocity

- ▶ reflection data (top free surface),
- ▶ absorbing conditions on the sides,
- ▶ piecewise constant  $m$ .

# Estimation of the basin of attraction

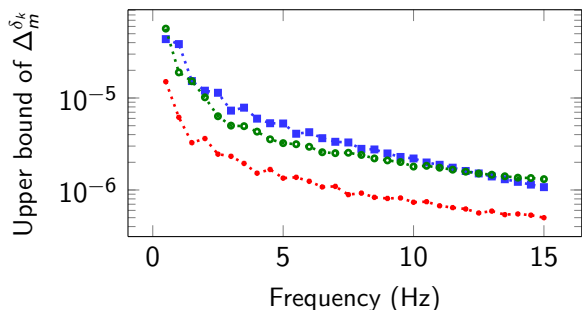
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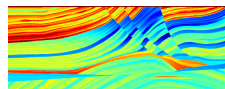
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(a) direction  $\delta_1$



(b) direction  $\delta_2$

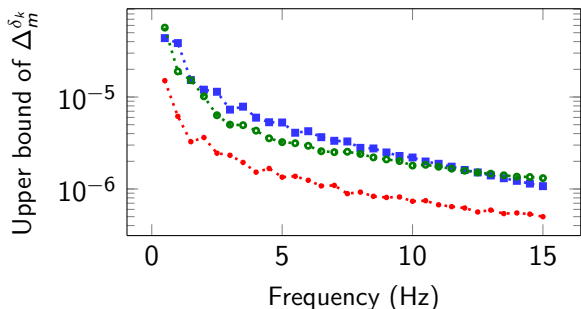


(c) direction  $\delta_3$

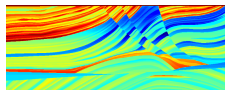
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Numerical estimate of the size  $\Delta_m^{\delta_k}$  with

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(a) direction  $\delta_1$



(b) direction  $\delta_2$



(c) direction  $\delta_3$

Low frequencies increase the size.

Reflecting objects complicate the procedure.

# Complex frequency

$$(-\omega^2 c^{-2} - \Delta)p = 0$$

- ▶  $-\omega^2 = (s + 2i\pi f)^2$
- ▶  $s = 0$ : Fourier domain  $\omega^2 = (2\pi f)^2$ ,
- ▶  $f = 0$ : Laplace domain  $-\omega^2 = s^2$ .



C. Shin and Y. H. Cha

Waveform inversion in the laplace domain ; Waveform inversion in the laplace fourier domain  
[Geophysical Journal International 2008–2009](#)



W. Ha, S. Pyun, J. Yoo and C. Shin

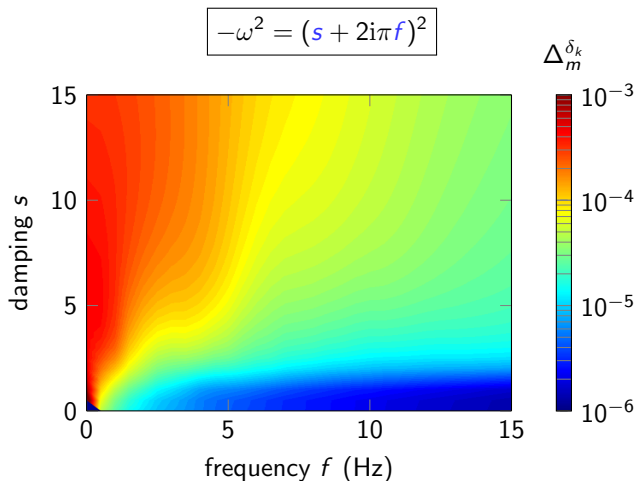
Acoustic full waveform inversion of synthetic land and marine data in the laplace domain ;  
[Geophysical Prospecting 2010](#)



P. V. Petrov and G. A. Newman

Three-dimensional inverse modelling of damped elastic wave propagation in the fourier domain.  
[Geophysical Journal International 2014](#)

# Complex frequency



**Selection of complex frequency.**

# Stability with frequency

## 2/ Stability

- ▶ ensures reconstruction accuracy.

The stability indicates how minimizing the data recovers the model

$$\|m_1 - m_2\| \leq C \|\mathcal{F}(m_1) - \mathcal{F}(m_2)\|.$$

The stability constant  $C$  depends on the frequency and the number of unknowns.



G. Alessandrini

Stable determination of conductivity by boundary measurement  
[Applicable Analysis 1988](#)



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates.  
[SIAM J. Math. Analysis 2016](#)



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
[arXiv:1702.04222, 2017](#)



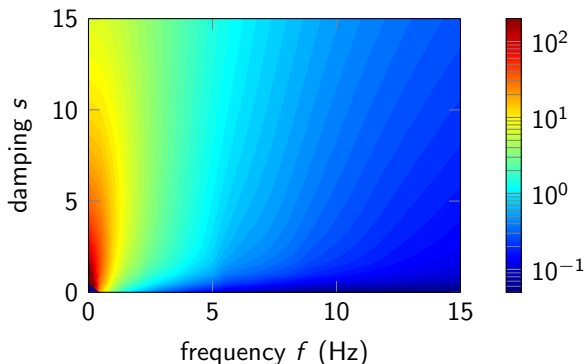
# Stability with frequency

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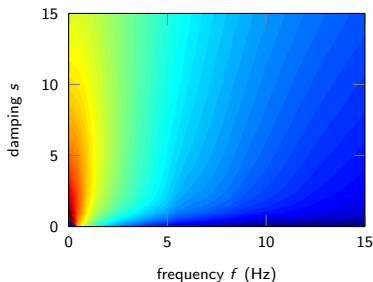
The stability indicates how minimizing the data recovers the model

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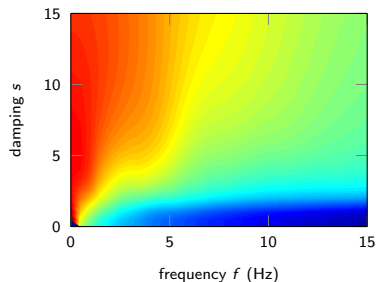


# Stability & Convergence estimates

From our estimates, we see different conditions for stability and convergence.



(a) stability



(b) convergence

Quantitative estimates provide an initial, comprehensive, relation to guide the iterative procedure.

# Plan

## 3 Numerical experiments

# Multi-level algorithm: stability and convergence

0./ No initial information on the subsurface

largest radius of convergence,

1./ Start with low frequency



high stability constant.

# Multi-level algorithm: stability and convergence

0./ No initial information on the subsurface

largest radius of convergence,

1./ Start with low frequency



high stability constant.

improve stability, *i.e.* resolution,

2./ Increase frequency

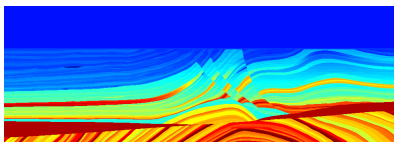


reduced radius of convergence.

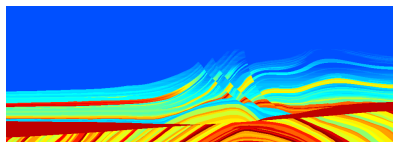
# Elastic Marmousi model

Multi-parameters inversion  $17 \times 3.5\text{km}$ , true models.

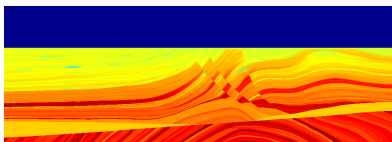
$$-\rho\omega^2\mathbf{u} - \nabla(\lambda\nabla\cdot\mathbf{u}) - \nabla\cdot(\mu[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]) = 0.$$



(a) P-wave speed  $\sqrt{\frac{\lambda+2\mu}{\rho}}$



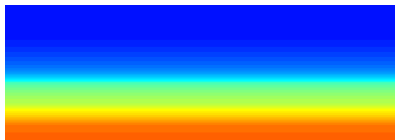
(b) S-wave speed  $\sqrt{\frac{\mu}{\rho}}$



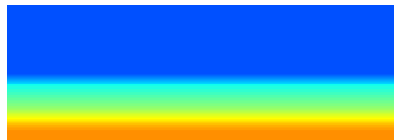
(c) Density

# Elastic Marmousi model

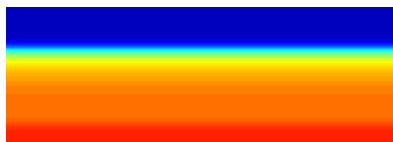
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(a) P-wave speed



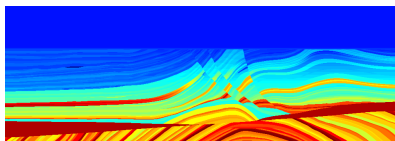
(b) S-wave speed



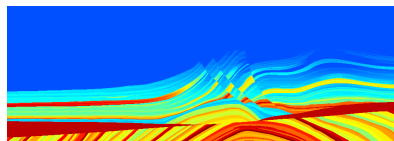
(c) Density

# Elastic Marmousi reconstruction $17 \times 3.5\text{km}$

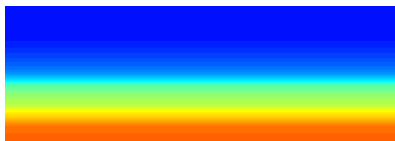
Frequency from 1 to 10Hz, unknown density, multi-level algorithm.



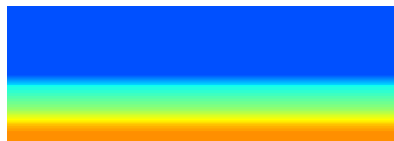
(a) True P-wave speed



(b) True S-wave speed



(c) Initial P-wave speed

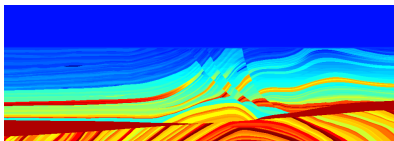


(d) Initial S-wave speed

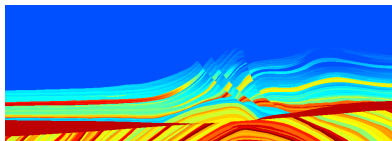


# Elastic Marmousi reconstruction $17 \times 3.5\text{km}$

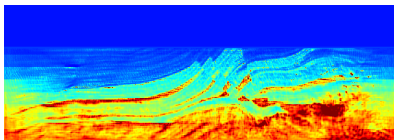
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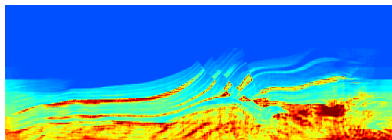
(a) True P-wave speed



(b) True S-wave speed



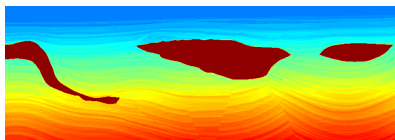
(c) 10Hz P-wave speed



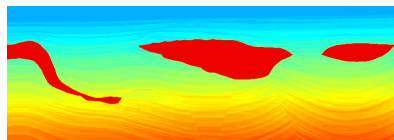
(d) 10Hz S-wave speed

# Elastic Pluto models

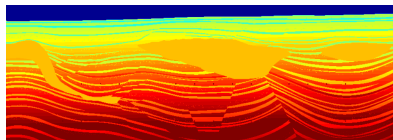
Multi-parameters inversion  $31 \times 7\text{km}$ , true models.



(a) P-wave speed



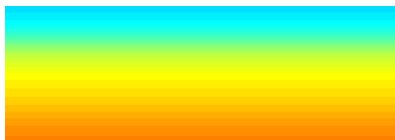
(b) S-wave speed



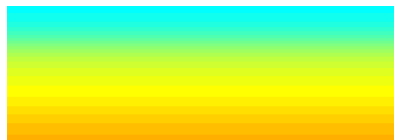
(c) Density

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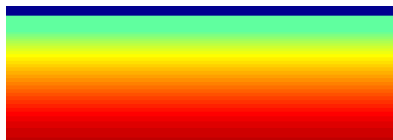
Multi-parameters inversion  $31 \times 7\text{km}$ , starting models.



(a) P-wave speed



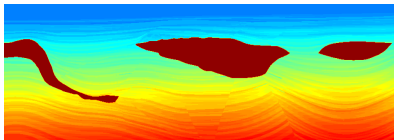
(b) S-wave speed



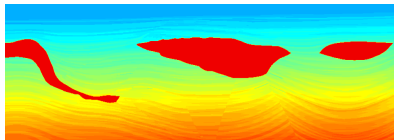
(c) Density

# Elastic Pluto reconstruction $31 \times 7$ km

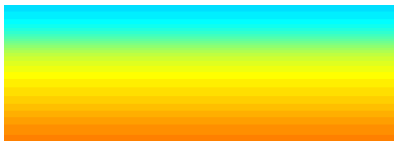
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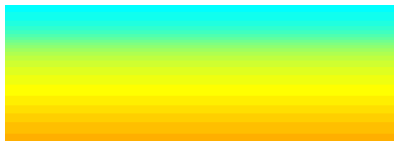
(a) True P-wave speed



(b) True S-wave speed



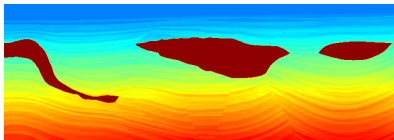
(c) Initial P-wave speed



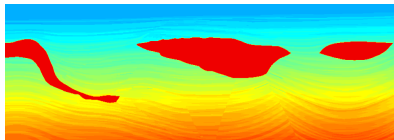
(d) Initial S-wave speed

# Elastic Pluto reconstruction $31 \times 7\text{km}$

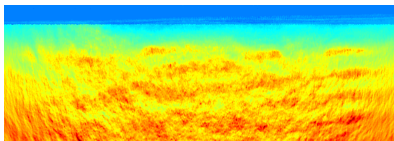
Frequency from 1 to 10Hz, unknown density, multi-level algorithm.



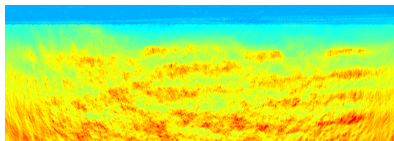
(a) True P-wave speed



(b) True S-wave speed



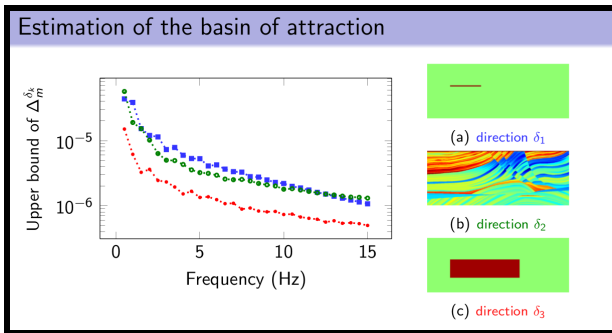
(c) 10Hz P-wave speed



(d) 10Hz S-wave speed

# Elastic Pluto reconstruction $31 \times 7\text{km}$

Reminder from the convergence analysis.



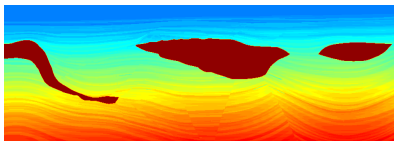
- ▶ Salt domes reduce the size of the radius of convergence.

## Solution

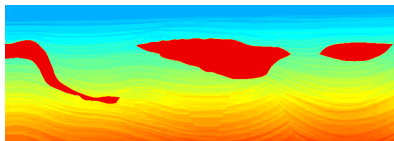
- ▶ Start with low or complex frequency.

# Elastic Pluto reconstruction $31 \times 7$ km

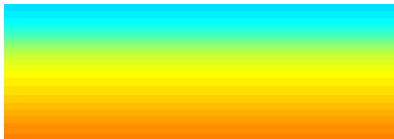
Using complex frequencies, unknown density, multi-level algorithm.



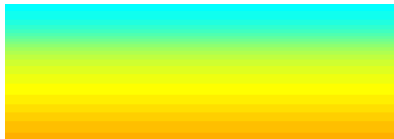
(a) True P-wave speed



(b) True S-wave speed



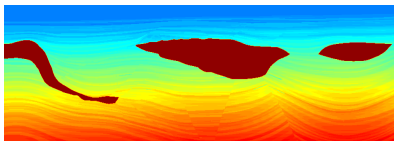
(c) Initial P-wave speed



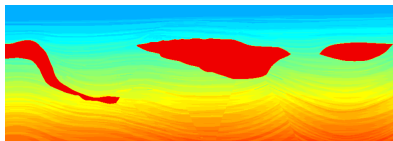
(d) Initial S-wave speed

# Elastic Pluto reconstruction $31 \times 7$ km

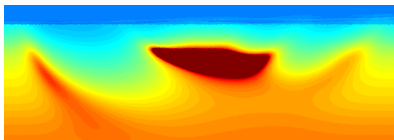
Using complex frequencies, unknown density, multi-level algorithm.



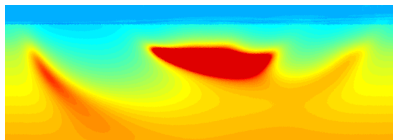
(a) True P-wave speed



(b) True S-wave speed



(c) (0Hz,10) P-wave speed

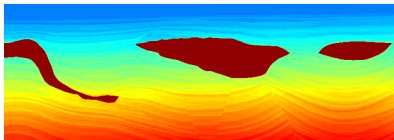


(d) (0Hz,10) S-wave speed

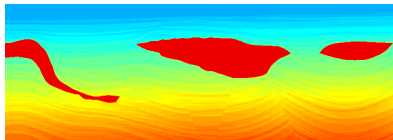


# Elastic Pluto reconstruction $31 \times 7\text{km}$

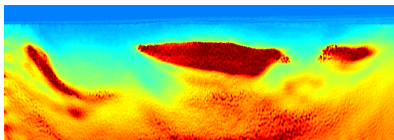
Using complex frequencies, unknown density, multi-level algorithm.



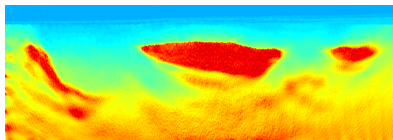
(a) True P-wave speed



(b) True S-wave speed



(c) 10Hz P-wave speed



(d) 10Hz S-wave speed

# Plan

## 4 Conclusion

# Conclusion

## Comprehensive FWI

- ▶ Quantitative stability and convergence analysis:
  - ▶ depending on the frequency and geometry,
  - ▶ **depending on the methods.**
- ▶ We can also quantify the noise robustness.

## Perspectives

- ▶ Analytical relation to develop between the two components,
- ▶ obtain conditional progression in frequency from stability and convergence estimates

# Quantitative reconstruction method for inverse problem

- ▶ Discontinuous Galerkin discretization in HPC framework,
- ▶ acoustic, elastic, anisotropy, viscosity; 2D, 3D, dual-sensors data.

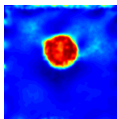
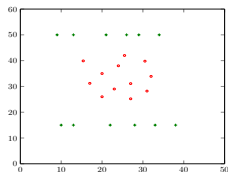
## **Applications and ongoing investigations:**

# Quantitative reconstruction method for inverse problem

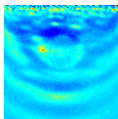
- ▶ Discontinuous Galerkin discretization in HPC framework,
- ▶ acoustic, elastic, anisotropy, viscosity; 2D, 3D, dual-sensors data.

## Applications and ongoing investigations:

- ▶ **Inverse scattering** using obstacles positions,
- ▶ **visco-elastic** reconstruction: five unknowns,



(a)  $\lambda$



(b)  $Q_\lambda$

- ▶  $(\lambda, \mu, \rho, Q_\lambda, Q_\mu)$ ,
- ▶ unknown  $Q$  does not prevent  $\lambda$  recovery,
- ▶ procedure to recover attenuation parameter?

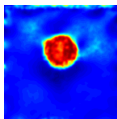
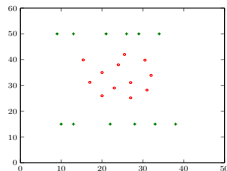
- ▶ **helioseismology**; **anisotropy**; model **parametrization**.

# Quantitative reconstruction method for inverse problem

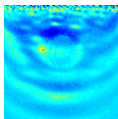
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- ▶ **helioseismology**; **anisotropy**; model **parametrization**.

THANK YOU

# APPENDIX

# Quantitative convergence for MBTT

Numerical estimate of the size  $\Delta_m^{\delta_k}$  the problem components

- 1 the frequency  $\omega$ ,
- 2 with direction  $\delta_k$  (the geometry of the unknown),
- 3 with the parametrization.

**The estimates allow a comparison of methods, in particular we want to compare the dependency of the optimization on the low frequencies.**



# Quantitative convergence for MBTT

Numerical estimate of the size  $\Delta_m^{\delta_k}$  the problem components

- 1 the frequency  $\omega$ ,
- 2 with direction  $\delta_k$  (the geometry of the unknown),
- 3 with the parametrization.

the MBTT method decomposes the model with a smooth part (propagator) and the reflectors:

$$c^{-2} = p + r = p + D\mathcal{F}^*(p)s.$$



F. Clément and G. Chavent

Waveform inversion through MBTT formulation – 1992



F. Clément, G. Chavent and S. Gómez

Migration-based traveltime waveform inversion of 2-d simple structures: A synthetic example  
Geophysics 2001



G. Chavent, K. Gadylshin and V. Tcheverda

Reflection fwi in mbtt formulation  
EAGE 2015

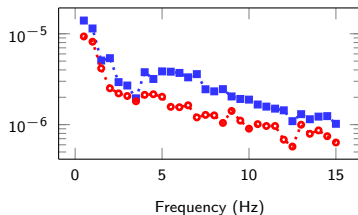
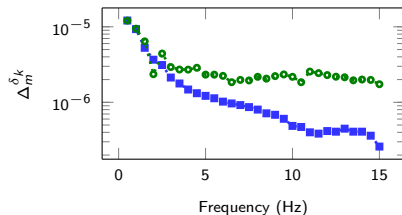
# Quantitative convergence for MBTT

Numerical estimate of the size  $\Delta_m^{\delta_k}$  the problem components

- 1 the frequency  $\omega$ ,
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- 3 with the parametrization.

Comparison of convergence with parametrization,

$$c^{-2} = p + D\mathcal{F}^*(p)s.$$



# Quantitative convergence estimates with frequency

Frequency progression for iterative algorithm,  $-\omega^2 = (s + 2i\pi f)^2$ .

## Other application

- ▶ Single frequency gives high radius than range of frequencies,
- ▶ can be apply for other minimization problem,
- ▶ ...

## Finite curvature indicates robustness to noise

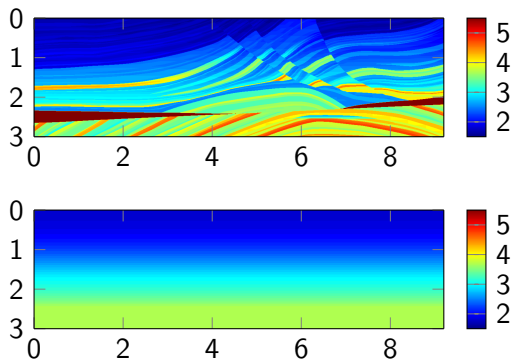
- ▶ Low and complex frequencies more affected.

## Stability

- ▶ Indicates the accuracy of the reconstruction.

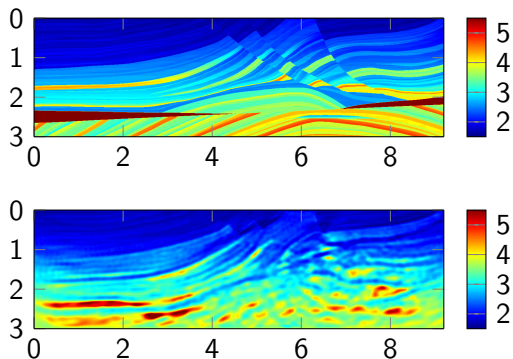
# Acoustic Marmousi reconstruction $9.2 \times 3\text{km}$

Starting available frequency is 4Hz.



# Acoustic Marmousi reconstruction $9.2 \times 3\text{km}$

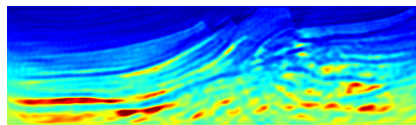
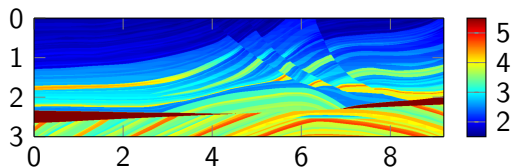
Starting available frequency is 4Hz.



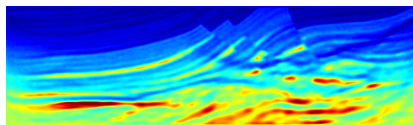
Using Fourier frequencies only: from 4 to 10Hz

# Acoustic Marmousi reconstruction $9.2 \times 3\text{km}$

Starting available frequency is 4Hz.



(a) Using Fourier frequencies only



(b) Using Complex frequencies from (4Hz, 7)