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A Hybrid FDTLM-IE Method for Efficient-Scattering Problems Modeling

M. Attia* M. Ney† T. Aguili ‡

Abstract — **A rigorous modelling of open space problems or matched guided structures including radiation phenomena needs an enormous computational effort. This effort is more important when applying a volumic method that requires the meshing of the entire computational domain and the use of absorbing boundary conditions. In this work we present a hybrid formulation of the frequency-domain TLM method (FDTLM) with an integral field equation. This hybrid method combining the advantages of both numerical methods is applied to model a diffraction problem where an infinte metallic square cylinder is exposed to an incident plane wave. As compared to the standard FDTLM, the hybrid method avoids the use of absorbing boundary condition and, thus, reduces the total computational TLM domain.**

1 INTRODUCTION

The frequency-domain formulation of the TLM method (FD-TLM) has been developed to be suitable for narrow band applications or dispersive problems. Despite the success of the TLM method, it still has an important limitation. As for any numerical method with volumic formulation, when analyzing structures, the entire computational domain has to be meshed. This may present a real handicap, especially when modeling structures open to free space such radiation and scattering problems. Even if many solutions have been developed for improving the performance of some boundary conditions, the problem of meshing persists. The perfectly matched layers, for example, have been developed for the FDTLM. However, they should be placed far enough from any discontinuity and surrounding layers have to be meshed as fields must also be computed within the PML [1]. In this context, combining, the integral field equations with the frequency domain TLM presents an efficient way to overcome this problem by avoiding the use of any absorbing boundary conditions and reducing the volume to be meshed. Actually, by applying the equivalence principle [2], it is possible to partition the 3D space into sub-regions. Thus, complex objects are embedded in a discretised subdomain where the FD-TLM method is applied. Certainly, this presents the main strength of the TLM method very suitable in modeling inhomogeneity and complex structures in a more straightforward way. Once the problem is resolved inside the subdomain and the fields are computed along the Huygen's surface, the field at any point in the space is, then, easily obtained by applying the Green function. Different publications dealt with this hybrid method [3] but all of them developed time domain formulation of TLM. One research work, treated a frequency formulation of this hybridization. But, it consists on a 2D formulation applied to specific metamaterials structures [4]. Since then, no further works have been developed to investigate the FDTLM-IE hybridization. This paper is organized as follow: Section 2 presents a brief look back at the frequency domain formulation used in this work. Then, Section 3 deals with the development of the hybrid FDTLM-IE method. Section 4 presents some numerical results of the proposed method applied to compute the RCS of an infinite square metallic cylinder. Finally we end with a conclusion in Section 5.

2 A BRIEF BACK UP TO THE FDTLM FORMULATION

The Frequency domain TLM formulation used in this work is derived from the TD formulation by applying a Fourier transform. More computational details are presented in a paper published by Chen and Ney [5]. We present in this section the main steps of the formulation. Let's Consider the incident and scattering voltages impulses vectors $[V^i]$ and $[V^r]$ of all the cells in the computational domain. Thus, $[V^i]$ and $[V^r]$ are one-column vectors of dimension $N_b \times N_n$ where N_b is the number of branches of the TLM node and N_n, is the total number of nodes in the structure. The TD-TLM formulation is based on 2 principal equations relating incident and reflected voltages:

$$[V^r]_k = [S][V^i]_k \tag{1}$$

$$[V^i]_{(k+1)} = [C][V^r]_k + [V^i_{source}]_{(k+1)} \tag{2}$$

*IMT Atlantique Bretagne, pays de la Loire Ecole Mines-Telecom (France), National Engineering School of Tunis (Tunisia), e-mail: attia.meriam@gmail.com

†IMT Atlantique Bretagne, pays de la Loire Ecole Mines-Telecom (France), e-mail: michel.ney@telecom-bretagne.eu

‡National Engineering School of Tunis Syscom Laboratory (Tunisia), e-mail: taoufik.aguili@gmail.com

where the subscript k represents the time step, $[C]$ the connecting matrix which describes the transmission between adjacent nodes and $[S]$ the scattering matrix that incorporates Maxwell's equations in their discrete form. When applying a DFT to (1) and (2) for a given angular frequency ω, subscript k vanishes and one obtains:

$$([I] - \gamma[C][S])[V^i(\omega)] = [V^i_{source}(\omega)] \quad (3)$$

where $\gamma = exp(j\omega\Delta t)$, Δt is a fixed time step and $[I]$ is the identity matrix. By solving the linear system (3), we obtain the incident voltages and then all the field components at the center of each node.

3 FORMULATION OF THE SCATTERING PROBLEM

Let's consider an infinite metallic square cylinder exposed to an electromagnetic field E^{inc} and H^{inc}. The structure is considered as two-dimensional electromagnetic problem. We aim to find the scattered field at a given point of the space.

3.1 Using a classic FDTLM method

The total field can be decomposed into a scattered and incident fields. Thus one can write:

$$E^{tot} = E^{inc} + E^{scat} \quad (4)$$

$$H^{tot} = H^{inc} + H^{scat} \quad (5)$$

where E^{inc} and H^{inc} are defined as the electric and magnetic incident fields. Based on the equivalence principle, this problem is equivalent to that of fig. 1 by assuming the presence of magnetic current source M_s and electric current source J_s on the connecting surface defined as follow:

$$M_s = E^{inc} \times n \quad (6)$$

$$J_s = n \times H^{inc} \quad (7)$$

where n is the inward unit vector of the connecting surface. Thus one obtains two components of the magnetic surface currents denoted M_x and M_y and one electric current component J_z. By using the equivalences voltages/electric filed and current/magnetic field in the TLM node centers [6] one can note:

*On the interface parallel to the x axis :

$$\begin{array}{l} M_y = ({}^2V_6^r + {}^2V_6^i) - ({}^1V_{10}^r + {}^1V_{10}^i) \\ J_z = \frac{1}{Z_0}({}^2V_6^i - {}^2V_6^r) + ({}^1V_{10}^i + {}^1V_{10}^r) \end{array} \quad (8)$$

*On the interface parallel to the y axis :

$$\begin{array}{l} M_x = ({}^2V_7^r + {}^2V_7^i) - ({}^1V_5^r + {}^1V_5^i) \\ J_z = \frac{1}{Z_0}({}^2V_7^i - {}^2V_7^r) + ({}^1V_5^i + {}^1V_5^r) \end{array} \quad (9)$$

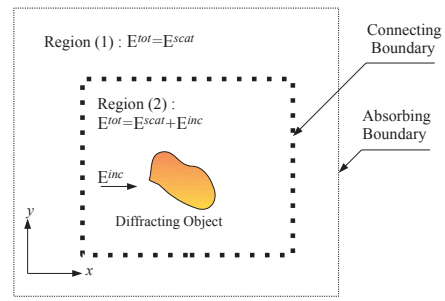


Figure 1: Equivalent problem of an obstacle illuminating by an electromagnetic wave using pure TLM method.

which leads to the following connecting boundary conditions:

$$\begin{array}{l} {}^2V_6^i = \frac{1}{2}(Z_0 J_z + M_y) + {}^1V_{10}^r \\ {}^2V_7^i = \frac{1}{2}(Z_0 J_z + M_x) + {}^1V_5^r \\ {}^1V_{10}^i = \frac{1}{2}(Z_0 J_z + M_y) + {}^2V_6^r \\ {}^1V_5^i = \frac{1}{2}(Z_0 J_z - M_x) + {}^2V_7^r \end{array} \quad (10)$$

where ${}^1V^i$, ${}^1V^r$, ${}^2V^i$ and ${}^2V^r$ are the incident and reflected impulses from region (1) and (2) respectively.

3.2 Using a hybrid FDTLM-IE method

By applying a hybrid formulation between integral equations and FDTLM method, one can avoids the use of absorbing boundary conditions by inserting a Huygen's interface denoted ($\mathcal{C}$) and applying the free-space Green's function. Thus, the new formulation of the problem is depicted in fig.2. Equation

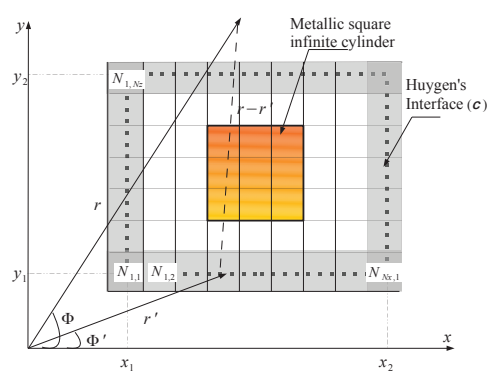


Figure 2: Coordinate system for the equivalent problem. External nodes (on the Huygen's interface) are in Grey color.

(4) is then written as function of the free space Green function as follow:

$$E_z^{scat}(r) = -\xi \oint_c J_z(r') H_0^{(2)}(k \|r - r'\|) dl' \quad (11)$$

Then, the total tangential field on the Huygen's surface ($\mathcal{C}$), according to (4,5) and (11) is:

$$E_z^{(\mathcal{C})}(r) = E^{(inc)} - \xi \oint_c J_z(r') H_0^{(2)}(k \|r - r'\|) dl' \tag{12}$$

where $\xi = \frac{\omega\mu}{4}$, $r' \in \mathcal{C}$ and $H_0^{(2)}$ is the Hankel function of second type and order 0 and J_z is the surface current density. On the other hand, according to (3), the electric field at the surface can be computed with the FD-TLM from the incident voltage as function of connection matrix $[C]$, scattering matix $[S]$ and the excitation voltages. If we class the voltages in TLM domain in two classes $V^{(\mathcal{C})}$ (external voltages on the surface ($\mathcal{C}$) separating domain 1 and 2) and V^{int}(interior voltage inside TLM domain), then equation (3) becomes:

$$\left({}^{[I]}-\gamma \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \right) \begin{bmatrix} V^{(int)} \\ V^{(\mathcal{C})} \end{bmatrix}^i = \begin{bmatrix} V^{(int)} \\ V^{(\mathcal{C})} \end{bmatrix}^{inc} \tag{13}$$

One should note that the TLM/IE interface must pass through the centers of TLM nodes (figure 2). Because equations (4) and (5) deal with total field component which are computed at the center of nodes in TLM method. The resolution of the linear system (13) needs the insertion of the appropriate boundary conditions at the surface ($\mathcal{C}$) which are unknown. At this stage, we use equation (11) to find the relationship between $V^{(\mathcal{C})}$ and V^{int} . To do this, the electric field as well as the surface current have to be expressed in terms of incident voltages. The surface current density at the node center can be expressed as function of the voltages as follow:

$$J_z = \frac{1}{2Z_0\Delta l} \left(V_{11}^i - V_{12}^r + V_1^i - V_3^i \right) \tag{14}$$

The electric field E_z is also a linear combination of the voltages at the node center:

$$E_z = \frac{1}{2\Delta l} \left(V_7^i + V_{10}^i + V_5^i + V_6^i \right) \tag{15}$$

where V^r are reflected voltages from the node center, Z_0 is the free-space impedance and Δl is the node size. Next consider a node located at (x_l, y_m) on (S), with $l = 1 \cdots N_x$ and $m = 1 \cdots N_y$ (N_x and N_y are the number of nodes on the surface ($\mathcal{C}$) along x and y axis respectively). As the structure is rectangular, we propose the use of Cartesian coordinates to compute electric and magnetic field on the rectangular Huygen's interface. So, according to figure 2 we note: $r = xcos(\Phi) + ysin(\Phi)$ and $r' = x'cos(\Phi') + y'sin(\Phi')$. Inserting (15) and (14) into (12), yields E_z at each node center (16).

$$\begin{aligned} E_z^{(x_l,y_m)} &= \tfrac{1}{2}\left(V_7^i + V_{10}^i + V_5^i + V_6^i\right)^{(x_l,y_m)} = V^{TE_{10}} - \xi\, [\\ &\sum_{k=1}^{N_x} (V_{center})^{(N_{k,1})} \int_{a_x}^{b_x} H_0^{(2)}(x_l, y_m, x', y_1) dx' \\ &- \sum_{k=1}^{N_x} (V_{center})^{(N_{k,N_y})} \int_{a_x}^{b_x} H_0^{(2)}(x_l, y_m, x', y_{N_y}) dx' \\ &+ \sum_{k=1}^{N_y} (V_{center})^{(N_{N_x,k})} \int_{a_y}^{b_y} H_0^{(2)}(x_l, y_m, x_{N_x}, y') dy' \\ &- \sum_{k=1}^{N_y} (V_{center})^{(N_{1,k})} \int_{a_y}^{b_y} H_0^{(2)}(x_l, y_m, x_1, y') dy' \,] \end{aligned} \tag{16}$$

where $V_{center} = V_{11}^i - V_{12}^i + V_1^i - V_3^i$, $\xi = \frac{\omega\mu}{4}$, a_x, b_x, a_y and b_y are defined as follows:

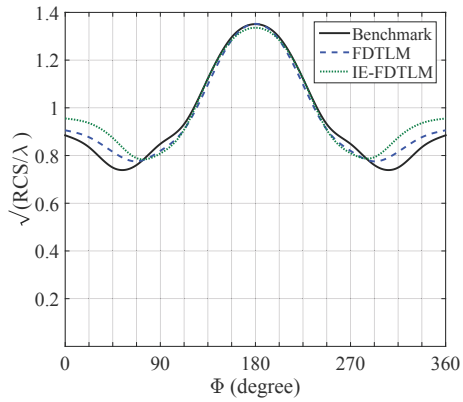
$$\begin{array}{ll} a_x = x_k - \Delta l/2, \text{for } (k = 2 \cdots N_x); & a_1 = x_1 \\ b_x = x_k + \Delta l/2, \text{for } (k = 1 \cdots N_{x-1}); & b_{N_x} = N_x \\ a_y = x_k - \Delta l/2, \text{for } (k = 2 \cdots N_y); & a_1 = y_1 \\ b_y = x_k + \Delta l/2, \text{for } (k = 1 \cdots N_{y-1}); & b_{N_y} = N_y \end{array}$$

4 SIMULATION RESULTS

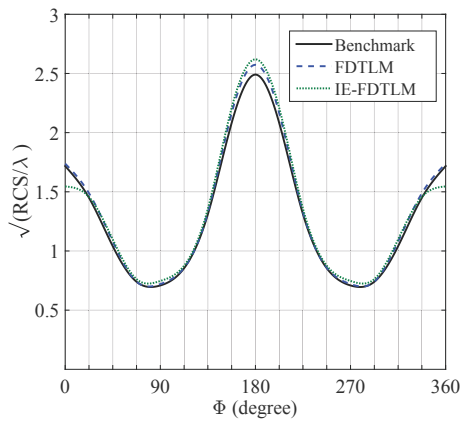
As we have already mentioned we aim to compute the diffracted fields of a perfectly conducting square cylinder exposed to an incident plane wave. The Huygen's interface is placed at $6\Delta l$ from the cylinder boundaries. The scattered far-field was computed at several frequency range where simulations were performed forward ($\phi = 180°$) and back ($\phi = 0°$). The (RSC/λ) in (dB) is computed and compared to the benchmark values [7] at frequency points for $ka = 0.8$, $ka = 2$ and $ka = 6$ where a is the radius of the cylinder ($a = 15.5\Delta l$) and k is the free space phase constant. Results of figure 3 show a good accuracy when computing the Radar Cross section RCS of the proposed structure. The hybrid FDTLM-IE results are in excellent agreement with the analytical results. In terms of computing time, we compared in Table 1 the results obtained with a classic FDTLM (using the perfectly matched layers PML as absorbing boundary conditions) and the new hybrid formulation FDTLM-IE. Obviously, we can observe that for nearly the same accuracy, the hybrid method needs only almost 1/5 of the computing time required for FDTLM method with a small deterioration of the precision (the relative error increases from 0.95% to 1.6%).

Method	# of nodes	CPU	Relative error
FDTLM	22500	35.6 min	1.72%
FDTLM-IE	4900	7.3 min	2.1%

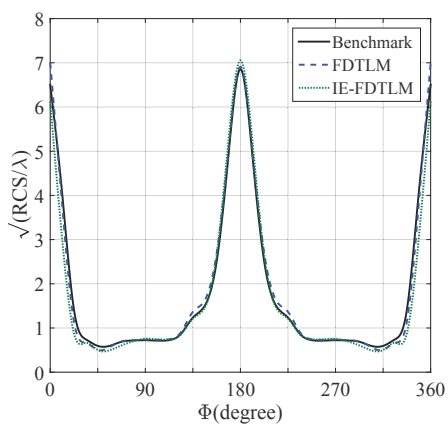
Table 1: Comparison between FDTLM and FDTLM-IE for $ka = 0.8$.



(a)



(b)



(c)

Figure 3: Scattered far field pattern for the infinite cylinder: (a):$ka = 0.8$, (b):$ka = 2$, (c): $ka = 8$

5 CONCLUSION

In this work, a hybrid FDTLM-IE method is used to analyze a two-dimensional electromagnetic scattering problem. Both of the classic and hybrid method was detailed and applied to compute the RCS of an infinite metallic square cylinder exposed to a plane wave source. Results showed a good agreement between the classic FDTLM (using ABC), the hybrid FDTLM-IE and a benchmark test with an important gain in CPU time and memory ressources when using the hybrid method.

References

[1] M. Attia, M. Ney and T. Aguili, "Absorbing boundary conditions for the FD-TLM method: The matched layer and the one-way equation technique". *Int. Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 27, pp. 6278, 2014.

[2] K.M. Chen, "A Mathematical Formulation of the Equivalence Principle". *IEEE Trans. on microwave theory and techniques*, vol. 37, 1989.

[3] L. Pierantoni, S. Lindenmeier and P. Russer. "Efficient analysis and modelling of the radiation of microstrip lines and patch antennas by the TLM-integral equation (TLM-IE) method". *Int. Journal of Num. Modelling*, Vol. 12, pp 329-340, 1999.

[4] M. Zedler, G.V. Eleftheriades, "Hybridisation of 2D frequency domain TLM with the MoM-discretised 2D-EFIE", in *Proc.* European Microwave Conference, 2010 pp. 1429-1432.

[5] Z. Chen and M. Ney, "On the Relationship Between the Time-Domain and Frequency-Domain TLM Methods". *IEEE Antennas and Wireless Prop. Letters*, vol. 7, pp. 46-49, 2008.

[6] Z. Chen, "The transmission Line Matrix (TLM) method and its boundary treatments", *PhD dissertation*, university of Ottawa, Canada 1992.

[7] N. R. S. Simons and E. Bridges, "Application of the TLM method to two-dimensional scattering problems". *Int. Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 5, pp. 93110, 1192.