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First and second order rational solutions to the Johnson equation and rogue waves.

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Abstract

Rational solutions to the Johnson equation are constructed as a quotient of two polynomials in x , y and t depending on several real parameters. We obtain an infinite hierarchy of rational solutions written in terms of polynomials of degrees $2N(N+1)$ in x , and t , $4N(N+1)$ in y , depending on $2N-2$ real parameters for each positive integer N . We construct explicit expressions of the solutions in the cases $N=1$ and $N=2$ which are given in the following. We study the evolution of the solutions by constructing the patterns of their modulus in the (x, y) plane, and this for different values of parameters.

Key Words : Johnson equation, Fredholm determinants, wronskians, rational solutions, rogue waves.

PACS numbers :

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

We consider the Johnson equation (J) which can be written in the form

$$(u_t + 6uu_x + u_{xxx} + \frac{u}{2t})_x - 3\frac{u_{yy}}{t^2} = 0,$$

where subscripts x , y and t denote partial derivatives.

This equation first appears first in 1980, in a paper written by Johnson [1]. It gives the description of waves surfaces in shallow incompressible fluids [2]-[3]. This equation was widely accepted, and was later derived for internal waves in a stratified medium [4]. The physical model of this equation have the same degree

of universality as the Kadomtsev-Petviashvili equation [5].

The first solutions were constructed in 1980 by Johnson [1]. Various research were led and more general solutions of this equation were obtained, for example, some special solutions were found in [6]. In 2007, some important classes of solutions were obtained by using the Darboux transformation [7]. This equation is dissipative and there is no soliton-like solution with a linear front localized along straight lines in the (x, y) plane. The Johnson equation allows explaining the existence of the horseshoe like solitons and multisoliton solutions quite naturally. Other extensions have been considered as for example the elliptic case [8].

In this paper, we study rational solutions of the Johnson equation. We presents multi parametric families of rational solutions as a quotient of two polynomials in x, y and t depending on several real parameters. These solutions presented belong to an infinite hierarchy of rational solutions written in terms of polynomials of degrees $2N(N + 1)$ in x, t and $4N(N + 1)$ in y , depending on $2N - 2$ real parameters for each positive integer N . The study here is limited to the simplest cases where $N = 1$ and $N = 2$.

The patterns of the modulus of the solutions in the (x, y) plane for different values of time t and parameters are studied.

2 First order rational solutions

We consider the Johnson equation

$$(u_t + 6uu_x + u_{xxx} + \frac{u}{2t})_x - 3\frac{u_{yy}}{t^2} = 0, \quad (1)$$

We have the following result at order $N = 1$:

Theorem 2.1 *The function v defined by*

$$v(x, y, t) = -2 \frac{|n(x, y, t)|^2}{d(x, y, t)^2} \quad (2)$$

where

$$n(x, y, t) = 144x^2 + (6912t + 24y^2t)x + 108 + 576iyt + y^4t^2 + 82944t^2 \quad (3)$$

and

$$d(x, y, t) = 144x^2 + (6912t + 24y^2t)x - 36 + y^4t^2 + 82944t^2, \quad (4)$$

is a rational solution to the Johnson equation (1), quotient of two polynomials $|n(x, y, t)|^2$ and $d(x, y, t)^2$ of degree 4 in x, t and 8 in y .

Proof We have to replace the expression of the solution given by (2) and check that (1) is verified.

□

The modulus of the solution for different values of t are represented in the plane (x, y) . It can be seen that as t tends to 0, the front of the solution contracts and is concentrated in a small strip along the y axis.

When the time t grows, we get peaks on special curves.

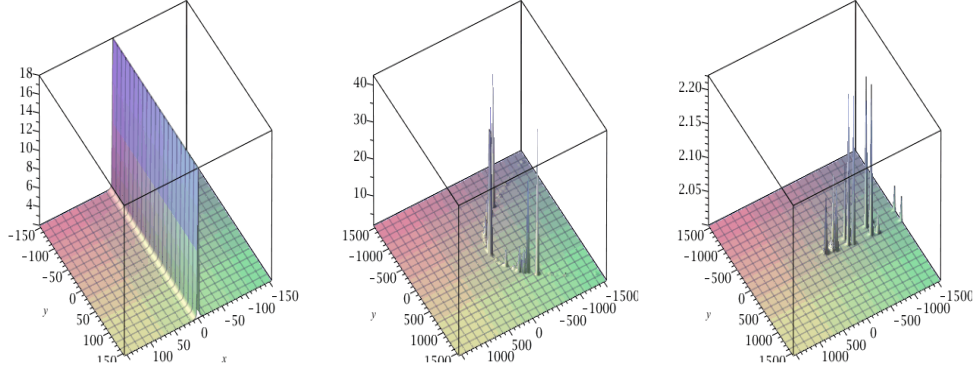


Figure 1. Solution of order 1 to (1), on the left for $t = 0$; in the center for $t = 0, 01$; on the right for $t = 1$.

3 Second order rational solutions depending on 2 parameters

The Johnson equation defined by (1) is always considered. For this second order, we get the rational solutions given by :

Theorem 3.1 *The function v defined by*

$$v(x, y, t) = -2 \frac{|n(x, y, t)|^2}{d(x, y, t)^2} \quad (5)$$

is a rational solution to the Johnson equation (1), quotient of two polynomials $|n(x, y, t)|^2$ and $d(x, y, t)^2$ of degree 12 in x, t and 24 in y depending on 2 real parameters a_1, b_1 .

Polynomials n and d are respectively given by

$n(x, y, t) = A(x, y, t) + iB(x, y, t)$, $d(x, y, t) = C(x, y, t) + iD(x, y, t)$ with

$$A(x, y, t) = \sum_{k=0}^6 a_k(y, t)x^k, \quad B(x, y, t) = \sum_{k=0}^6 b_k(y, t)x^k,$$

$$C(x, y, t) = \sum_{k=0}^6 c_k(y, t)x^k, \quad D(x, y, t) = \sum_{k=0}^6 d_k(y, t)x^k,$$

$$\mathbf{a}_6 = -2985984, \quad \mathbf{a}_5 = (-1492992y^2 - 429981696)t, \quad \mathbf{a}_4 = 17915904tyb_1 + (-311040y^4 - 143327232y^2 - 25798901760)t^2 + 11197440a_1^2 + 2239488b_1^2 - 6718464, \quad \mathbf{a}_3 = (-34560y^6 - 17915904y^4 - 5159780352y^2 - 825564856320)t^3 + (5971968y^3b_1 + 1719926784yb_1)t^2 - 17915904a_1b_1 + (3732480y^2a_1^2 + 746496y^2b_1^2 - 2239488y^2 - 71663616ya_1 + 1074954240a_1^2 + 214990848b_1^2 - 931627008)t, \quad \mathbf{a}_2 = (-2160y^8 - 995328y^6 - 358318080y^4 - 82556485632y^2 - 14860167413760)t^4 -$$

$$\begin{aligned}
& 2799360 a_1^4 - 3359232 a_1^2 b_1^2 - 559872 b_1^4 + (746496 y^5 b_1 + 286654464 y^3 b_1 + 61917364224 y b_1) t^3 + \\
& (466560 y^4 a_1^2 + 93312 y^4 b_1^2 - 279936 y^4 - 17915904 y^3 a_1 + 214990848 y^2 a_1^2 + 35831808 y^2 - 5159780352 y a_1 + \\
& 38698352640 a_1^2 + 7739670528 b_1^2 - 43858132992) t^2 + 8398080 a_1^2 + 8398080 b_1^2 + (-4478976 y^2 a_1 b_1 - \\
& 26873856 y a_1^2 b_1 - 8957952 y b_1^3 + 100776960 y b_1 - 1289945088 a_1 b_1) t + 8398080, \quad \mathbf{a}_1 = (-72 y^{10} - \\
& 20736 y^8 - 11943936 y^6 - 3439853568 y^4 - 495338913792 y^2 - 142657607172096) t^5 + (41472 y^7 b_1 + \\
& 11943936 y^5 b_1 + 3439853568 y^3 b_1 + 990677827584 y b_1) t^4 + 4478976 a_1^3 b_1 + 4478976 a_1 b_1^3 + (25920 y^5 a_1^2 + \\
& 5184 y^4 b_1^2 - 15552 y^6 - 1492992 y^5 a_1 + 13436928 y^4 a_1^2 - 4478976 y^4 b_1^2 + 25380864 y^4 - 573308928 y^3 a_1 + \\
& 3869835264 y^2 a_1^2 - 1289945088 y^2 b_1^2 + 2149908480 y^2 - 123834728448 y a_1 + 619173642240 a_1^2 + \\
& 123834728448 b_1^2 - 866843099136) t^3 + (-373248 y^4 a_1 b_1 - 4478976 y^3 a_1^2 b_1 - 1492992 y^3 b_1^3 + 16796160 y^3 b_1 - \\
& 1289945088 y^2 a_1^2 b_1 - 429981696 y b_1^3 + 3117367296 y b_1 - 30958682112 a_1 b_1) t^2 + (-466560 y^2 a_1^4 - \\
& 559872 y^2 a_1^2 b_1^2 - 93312 y^2 b_1^4 + 1399680 y^2 a_1^2 + 1399680 y^2 b_1^2 + 17915904 y a_1^3 + 53747712 y a_1 b_1^2 - \\
& 134369280 a_1^4 - 161243136 a_1^2 b_1^2 - 26873856 b_1^4 + 1399680 y^2 - 67184640 y a_1 + 618098688 a_1^2 + \\
& 188116992 b_1^2 + 1048080384) t, \quad \mathbf{a}_0 = (-y^{12} - 248832 y^8 - 20639121408 y^4 - 57063042868834) t^6 + \\
& 46656 a_1^6 + 139968 a_1^4 b_1^2 + 139968 a_1^2 b_1^4 + 46656 b_1^6 + (864 y^9 b_1 + 143327232 y^5 b_1 + 5944066965504 y b_1) t^5 + \\
& (540 y^9 a_1^2 + 108 y^8 b_1^2 - 324 y^8 - 41472 y^7 a_1 + 248832 y^6 a_1^2 - 248832 y^6 b_1^2 + 1327104 y^6 - 11943936 y^5 a_1 + \\
& 89579520 y^4 a_1^2 + 17915904 y^4 b_1^2 + 89579520 y^4 - 3439853568 y^3 a_1 + 20639121408 y^2 a_1^2 - 20639121408 y^2 b_1^2 - \\
& 990677827584 y a_1 + 3715041853440 a_1^2 + 743008370688 b_1^2 - 6191736422400) t^4 - 279936 a_1^4 - 2519424 a_1^2 b_1^2 - \\
& 2239488 b_1^4 + (-10368 y^6 a_1 b_1 - 186624 y^5 a_1^2 b_1 - 62208 y^5 b_1^3 + 699840 y^5 b_1 + 8957952 y^4 a_1 b_1 - \\
& 71663616 y^3 a_1^2 b_1 + 23887872 y^3 b_1^3 - 179159040 y^3 b_1 + 2579890176 y^2 a_1 b_1 - 15479341056 y a_1^2 b_1 - \\
& 5159780352 y b_1^3 + 16769286144 y b_1 - 247669456896 a_1 b_1) t^3 + (-19440 y^4 a_1^4 - 23328 y^4 a_1^2 b_1^2 - \\
& 3888 y^4 b_1^4 + 58320 y^4 a_1^2 + 58320 y^4 b_1^2 + 1492992 y^3 a_1^3 + 4478976 y^3 a_1 b_1^2 - 8957952 y^2 a_1^4 + \\
& 8957952 y^2 b_1^4 + 58320 y^4 - 5598720 y^3 a_1 - 22394880 y^2 a_1^2 - 165722112 y^2 b_1^2 + 429981696 y a_1^3 + \\
& 1289945088 y a_1 b_1^2 - 1612431360 a_1^4 - 1934917632 a_1^2 b_1^2 - 322486272 b_1^4 + 107495424 y a_1 + 9997074432 a_1^2 - \\
& 322486272 b_1^2 + 13436928000) t^2 - 734832 a_1^2 - 734832 b_1^2 + (373248 y^2 a_1^3 b_1 + 373248 y^2 a_1 b_1^3 + \\
& 1119744 y^2 a_1^4 b_1 + 2239488 y a_1^2 b_1^3 + 1119744 y b_1^5 - 28553472 y a_1^2 b_1 - 33032448 y b_1^3 + 107495424 a_1^3 b_1 + \\
& 107495424 a_1 b_1^3 - 35271936 y b_1 + 429981696 a_1 b_1) t + 2099520
\end{aligned}$$

$$\begin{aligned}
\mathbf{b}_6 &= 5971968, \quad \mathbf{b}_5 = (2985984 y^2 + 859963392) t, \quad \mathbf{b}_4 = (622080 y^4 + 286654464 y^2 + 51597803520) t^2 - \\
& 11197440 a_1^2 - 2239488 b_1^2 + (-17915904 y b_1 + 35831808 y) t + 13436928, \quad \mathbf{b}_3 = (69120 y^6 + 35831808 y^4 + \\
& 10319560704 y^2 + 1651129712640) t^3 + (-5971968 y^5 b_1 + 11943936 y^3 - 1719926784 y b_1 + 3439853568 y) t^2 + \\
& 17915904 y^4 a_1^2 + (-3732480 y^2 a_1^2 - 746496 y^2 b_1^2 + 4478976 y^2 + 71663616 y a_1 - 1074954240 a_1^2 - \\
& 214990848 b_1^2 + 1863254016) t, \quad \mathbf{b}_2 = (4320 y^8 + 1990656 y^6 + 716636160 y^4 + 165112971264 y^2 + \\
& 29720334827520) t^4 + 2799360 a_1^4 + 3359232 a_1^2 b_1^2 + 559872 b_1^4 + (-746496 y^5 b_1 + 1492992 y^4 a_1^2 + \\
& 286654464 y^3 b_1 + 573308928 y^3 - 61917364224 y b_1 + 123834728448 y) t^3 + (-466560 y^4 a_1^2 - 93312 y^4 b_1^2 + \\
& 559872 y^4 + 17915904 y^3 a_1 - 214990848 y^2 a_1^2 - 71663616 y^2 + 5159780352 y a_1 - 38698352640 a_1^2 - \\
& 7739670528 b_1^2 + 87716265984) t^2 - 8398080 a_1^2 - 8398080 b_1^2 + (4478976 y^2 a_1 b_1 + 26873856 y a_1^2 b_1 + \\
& 8957952 y b_1^3 - 100776960 y b_1 + 1289945088 a_1 b_1 + 53747712 y t - 16796160, \quad \mathbf{b}_1 = (144 y^{10} + \\
& 41472 y^8 + 23887872 y^6 + 6879707136 y^4 + 990677827584 y^2 + 285315214344192) t^5 + (-41472 y^7 b_1 + \\
& 82944 y^7 - 11943936 y^5 b_1 + 23887872 y^5 - 3439853568 y^3 b_1 + 6879707136 y^3 - 990677827584 y b_1 + \\
& 198135565168 y) t^4 - 4478976 a_1^3 b_1 - 4478976 a_1 b_1^3 + (-25920 y^6 a_1^2 - 5184 y^6 b_1^2 + 31104 y^6 + \\
& 1492992 y^5 a_1 - 13436928 y^4 a_1^2 + 4478976 y^4 b_1^2 - 50761728 y^4 + 573308928 y^3 a_1 - 3869835264 y^2 a_1^2 + \\
& 1289945088 y^2 b_1^2 - 4299816960 y^2 + 123834728448 y a_1 - 619173642240 a_1^2 - 123834728448 b_1^2 + \\
& 1733686198272) t^3 + (373248 y^4 a_1 b_1 + 4478976 y^3 a_1^2 b_1 + 1492992 y^3 b_1^3 - 16796160 y^3 b_1 + 1289945088 y a_1^2 b_1 + \\
& 429981696 y b_1^3 + 8957952 y^3 - 3117367296 y b_1 + 30958682112 a_1 b_1 - 859963392 y) t^2 + (466560 y^2 a_1^4 + \\
& 559872 y^2 a_1^2 b_1^2 + 93312 y^2 b_1^4 - 1399680 y^2 a_1^2 - 1399680 y^2 b_1^2 - 17915904 y a_1^3 - 53747712 y a_1 b_1^2 + \\
& 134369280 a_1^4 + 161243136 a_1^2 b_1^2 + 26873856 b_1^4 - 2799360 y^2 + 67184640 y a_1 - 618098688 a_1^2 - \\
& 188116992 b_1^2 - 2096160768) t, \quad \mathbf{b}_0 = (2 y^{12} + 497664 y^8 + 41278242816 y^4 + 1141260857376768) t^6 - \\
& 46656 a_1^6 - 139968 a_1^4 b_1^2 - 139968 a_1^2 b_1^4 - 46656 b_1^6 + (-864 y^9 b_1 + 1728 y^9 - 143327232 y^5 b_1 + \\
& 286654464 y^5 - 5944066965504 y b_1 + 11888133931008 y) t^5 + (-540 y^8 a_1^2 - 108 y^8 b_1^2 + 648 y^8 + \\
& 41472 y^7 a_1 - 248832 y^6 a_1^2 + 248832 y^6 b_1^2 - 2654208 y^6 + 11943936 y^5 a_1 - 89579520 y^4 a_1^2 - 17915904 y^4 b_1^2 - \\
& 179159040 y^4 + 3439853568 y^3 a_1 - 20639121408 y^2 a_1^2 + 20639121408 y^2 b_1^2 + 990677827584 y a_1 - \\
& 3715041853440 a_1^2 - 743008370688 b_1^2 + 12383472844800) t^4 + 279936 a_1^4 + 2519424 a_1^2 b_1^2 + 2239488 b_1^4 + \\
& (10368 y^6 a_1 b_1 + 186624 y^5 a_1^2 b_1 + 62208 y^5 b_1^3 - 699840 y^5 b_1 - 8957952 y^4 a_1 b_1 + 71663616 y^3 a_1^2 b_1 - \\
& 23887872 y^3 b_1^3 + 373248 y^5 + 179159040 y^3 b_1 - 2579890176 y^2 a_1 b_1 + 15479341056 y a_1^2 b_1 + 5159780352 y b_1^3 - \\
& 16769286144 y b_1 + 247669456896 a_1 b_1 - 51597803520 y) t^3 + (19440 y^4 a_1^4 + 23328 y^4 a_1^2 b_1^2 + 3888 y^4 b_1^4 - \\
& 58320 y^4 a_1^2 - 58320 y^4 b_1^2 - 1492992 y^3 a_1^3 - 4478976 y^3 a_1 b_1^2 + 8957952 y^2 a_1^4 - 8957952 y^2 b_1^4 - \\
& 116640 y^4 + 5598720 y^3 a_1 + 22394880 y^2 a_1^2 + 165722112 y^2 b_1^2 - 429981696 y a_1^3 - 1289945088 y a_1 b_1^2 + \\
& 1612431360 a_1^4 + 1934917632 a_1^2 b_1^2 + 322486272 b_1^4 - 107495424 y a_1 - 9997074432 a_1^2 + 322486272 b_1^2 - \\
& 26873856000) t^2 + 734832 a_1^2 + 734832 b_1^2 + (-373248 y^2 a_1^3 b_1 - 373248 y^2 a_1 b_1^3 - 1119744 y a_1^4 b_1 - \\
& 2239488 y a_1^2 b_1^3 - 1119744 y b_1^5 + 28553472 y a_1^2 b_1 + 33032448 y b_1^3 - 107495424 a_1^3 b_1 - 107495424 a_1 b_1^3 + \\
& 35271936 y b_1 - 429981696 a_1 b_1 - 33592320 y) t - 4199040
\end{aligned}$$

$$\begin{aligned}
\mathbf{c}_6 &= -2985984, \quad \mathbf{c}_5 = (-1492992 y^2 - 429981696) t, \quad \mathbf{c}_4 = 17915904 t y b_1 + (-311040 y^4 - 143327232 y^2 - \\
& 25798901760) t^2 + 11197440 a_1^2 + 2239488 b_1^2 + 2239488, \quad \mathbf{c}_3 = (-34560 y^6 - 17915904 y^4 - 5159780352 y^2 - \\
& 825564856320) t^3 + (5971968 y^3 b_1 + 1719926784 y b_1) t^2 + (3732480 y^2 a_1^2 + 746496 y^2 b_1^2 + 746496 y^2 + \\
& 1074954240 a_1^2 + 214990848 b_1^2 - 71663616) t, \quad \mathbf{c}_2 = (-2160 y^8 - 995328 y^6 - 358318080 y^4 - \\
& 82556485632 y^2 - 14860167413760) t^4 - 2799360 a_1^4 - 3359232 a_1^2 b_1^2 - 559872 b_1^4 + (746496 y^5 b_1 + \\
& 286654464 y^3 b_1 + 61917364224 y b_1) t^3 + (466560 y^4 a_1^2 + 93312 y^4 b_1^2 + 93312 y^4 + 214990848 y^2 a_1^2 +
\end{aligned}$$

$$\begin{aligned}
& 35831808 y^2 + 38698352640 a_1^2 + 7739670528 b_1^2 - 12899450880)t^2 - 5038848 a_1^2 - 5038848 b_1^2 + \\
& (-26873856 y a_1^2 b_1 - 8957952 y b_1^3 - 6718464 y b_1) t - 5038848, \quad \mathbf{c}_1 = (-72 y^{10} - 20736 y^8 - 11943936 y^6 - \\
& 3439853568 y^4 - 495338913792 y^2 - 142657607172096) t^5 + (41472 y^7 b_1 + 11943936 y^5 b_1 + 3439853568 y^3 b_1 + \\
& 990677827584 y b_1) t^4 + (25920 y^6 a_1^2 + 5184 y^6 b_1^2 + 5184 y^6 + 13436928 y^4 a_1^2 - 4478976 y^4 b_1^2 + \\
& 7464960 y^4 + 3869835264 y^2 a_1^2 - 1289945088 y^2 b_1^2 - 3009871872 y^2 + 619173642240 a_1^2 + 123834728448 b_1^2 - \\
& 371504185344) t^3 + (-4478976 y^3 a_1^2 b_1 - 1492992 y^3 b_1^3 - 1119744 y^3 b_1 - 1289945088 y a_1^2 b_1 - 429981696 y b_1^3 - \\
& 2042413056 y b_1) t^2 + (-466560 y^2 a_1^4 - 559872 y^2 a_1^2 b_1^2 - 93312 y^2 b_1^4 - 839808 y^2 a_1^2 - 839808 y^2 b_1^2 - \\
& 134369280 a_1^4 - 161243136 a_1^2 b_1^2 - 26873856 b_1^4 - 839808 y^2 - 26873856 a_1^2 - 456855552 b_1^2 - \\
& 456855552) t, \quad \mathbf{c}_0 = (-y^{12} - 248832 y^8 - 20639121408 y^4 - 570630428688384) t^6 + 46656 a_1^6 + \\
& 139968 a_1^4 b_1^2 + 139968 a_1^2 b_1^4 + 46656 b_1^6 + (864 y^9 b_1 + 143327232 y^5 b_1 + 5944066965504 y b_1) t^5 + \\
& (540 y^8 a_1^2 + 108 y^8 b_1^2 + 108 y^8 + 248832 y^6 a_1^2 - 248832 y^6 b_1^2 + 331776 y^6 + 89579520 y^4 a_1^2 + \\
& 17915904 y^4 b_1^2 + 161243136 y^4 + 20639121408 y^2 a_1^2 - 20639121408 y^2 b_1^2 - 82556485632 y^2 + 3715041853440 a_1^2 + \\
& 743008370688 b_1^2 - 3219702939648) t^4 + 279936 a_1^4 + 839808 a_1^2 b_1^2 + 559872 b_1^4 + (-186624 y^5 a_1^2 b_1 - \\
& 62208 y^5 b_1^3 - 46656 y^5 b_1 - 71663616 y^3 a_1^2 b_1 + 23887872 y^3 b_1^3 + 107495424 y^3 b_1 - 15479341056 y a_1^2 b_1 - \\
& 5159780352 y b_1^3 - 45148078080 y b_1) t^3 + (-19440 y^4 a_1^4 - 23328 y^4 a_1^2 b_1^2 - 3888 y^4 b_1^4 - 34992 y^4 a_1^2 - \\
& 34992 y^4 b_1^2 - 8957952 y^2 a_1^4 + 8957952 y^2 b_1^4 - 34992 y^4 - 22394880 y^2 a_1^2 + 49268736 y^2 b_1^2 - \\
& 1612431360 a_1^4 - 1934917632 a_1^2 b_1^2 - 322486272 b_1^4 + 35831808 y^2 + 2257403904 a_1^2 - 8062156800 b_1^2 - \\
& 14941863936) t^2 + 944784 a_1^2 + 944784 b_1^2 + (1119744 y a_1^4 b_1 + 2239488 y a_1^2 b_1^3 + 1119744 y b_1^5 - \\
& 1679616 y a_1^2 b_1 + 11757312 y b_1^3 + 11757312 y b_1) t + 419904
\end{aligned}$$

$$\begin{aligned}
& \mathbf{d}_6 = 5971968, \quad \mathbf{d}_5 = (2985984 y^2 + 859963392) t, \quad \mathbf{d}_4 = -17915904 t y b_1 + (622080 y^4 + 286654464 y^2 + \\
& 51597803520) t^2 - 11197440 a_1^2 - 2239488 b_1^2 - 4478976, \quad \mathbf{d}_3 = (69120 y^6 + 35831808 y^4 + 10319560704 y^2 + \\
& 1651129712640) t^3 + (-5971968 y^3 b_1 - 1719926784 y b_1) t^2 + (-3732480 y^2 a_1^2 - 746496 y^2 b_1^2 - 1492992 y^2 - \\
& 1074954240 a_1^2 - 214990848 b_1^2 + 143327232) t, \quad \mathbf{d}_2 = (4320 y^8 + 1990656 y^6 + 716636160 y^4 + \\
& 165112971264 y^2 + 29720334827520) t^4 + 2799360 a_1^4 + 3359232 a_1^2 b_1^2 + 559872 b_1^4 + (-746496 y^5 b_1 - \\
& 286654464 y^3 b_1 - 61917364224 y b_1) t^3 + (-466560 y^4 a_1^2 - 93312 y^4 b_1^2 - 186624 y^4 - 214990848 y^2 a_1^2 - \\
& 71663616 y^2 - 38698352640 a_1^2 - 7739670528 b_1^2 + 25798901760) t^2 + 5038848 a_1^2 + 5038848 b_1^2 + \\
& (26873856 y a_1^2 b_1 + 8957952 y b_1^3 + 6718464 y b_1) t + 10077696, \quad \mathbf{d}_1 = (144 y^{10} + 41472 y^8 + 23887872 y^6 + \\
& 6879707136 y^4 + 990677827584 y^2 + 285315214344192) t^5 + (-41472 y^7 b_1 - 11943936 y^5 b_1 - 3439853568 y^3 b_1 - \\
& 990677827584 y b_1) t^4 + (-25920 y^6 a_1^2 - 5184 y^6 b_1^2 - 10368 y^6 - 13436928 y^4 a_1^2 + 4478976 y^4 b_1^2 - \\
& 14929920 y^4 - 3869835264 y^2 a_1^2 - 1289945088 y^2 b_1^2 - 6019743744 y^2 - 619173642240 a_1^2 - 123834728448 b_1^2 + \\
& 743008370688) t^3 + (4478976 y^3 a_1^2 b_1 + 1492992 y^3 b_1^3 + 1119744 y^3 b_1 + 1289945088 y a_1^2 b_1 + 429981696 y b_1^3 + \\
& 2042413056 y b_1) t^2 + (466560 y^2 a_1^4 + 559872 y^2 a_1^2 b_1^2 + 93312 y^2 b_1^4 + 839808 y^2 a_1^2 + 839808 y^2 b_1^2 + \\
& 134369280 a_1^4 + 161243136 a_1^2 b_1^2 + 26873856 b_1^4 + 1679616 y^2 + 26873856 a_1^2 + 456855552 b_1^2 + \\
& 913711104) t, \quad \mathbf{d}_0 = 2 t^6 y^{12} - 864 t^5 y^9 b_1 + 497664 t^6 y^8 - 540 t^4 y^8 a_1^2 - 108 t^4 y^8 b_1^2 - 216 t^4 y^8 - \\
& 248832 t^4 y^6 a_1^2 + 248832 t^4 y^6 b_1^2 - 143327232 t^5 y^5 b_1 + 186624 t^3 y^5 a_1^2 b_1 + 62208 t^3 y^5 b_1^3 + 41278242816 t^6 y^4 - \\
& 663552 t^4 y^6 - 89579520 t^4 y^4 a_1^2 - 17915904 t^4 y^4 b_1^2 + 19440 t^2 y^4 a_1^4 + 23328 t^2 y^4 a_1^2 b_1^2 + 3888 t^2 y^4 b_1^4 + \\
& 46656 t^3 y^5 b_1 + 71663616 t^3 y^3 a_1^2 b_1 - 23887872 t^3 y^3 b_1^3 - 322486272 t^4 y^4 - 20639121408 t^4 y^2 a_1^2 + \\
& 20639121408 t^4 y^2 b_1^2 + 34992 t^2 y^4 a_1^2 + 34992 t^2 y^4 b_1^2 + 8957952 t^2 y^2 a_1^4 - 8957952 t^2 y^2 b_1^4 - 5944066965504 t^5 y b_1 - \\
& 107495424 t^3 y^3 b_1 + 15479341056 t^3 y a_1^2 b_1 + 5159780352 t^3 y b_1^3 - 1119744 t y a_1^4 b_1 - 2239488 t y a_1^2 b_1^3 + \\
& 1119744 t y b_1^5 + 1141260857376768 t^6 + 165112971264 t^4 y^2 - 3715041853440 t^4 a_1^2 - 743008370688 t^4 b_1^2 + \\
& 69984 t^2 y^2 + 22394880 t^2 y^2 a_1^2 - 49268736 t^2 y^2 b_1^2 + 1612431360 t^2 a_1^4 + 1934917632 t^2 a_1^2 b_1^2 + \\
& 322486272 t^2 b_1^4 - (2985984 y^2 + 859963392) t x^5 - 46656 a_1^6 - 139968 a_1^4 b_1^2 - 139968 a_1^2 b_1^4 - \\
& 46656 b_1^6 + 45148078080 t^3 y b_1 + 1679616 t y a_1^2 b_1 - 11757312 t y b_1^3 + (2985984 t y^2 + 859963392) t x^5 + \\
& 6439405879296 t^4 - 71663616 t^2 y^2 - 2257403904 t^2 a_1^2 + 8062156800 t^2 b_1^2 + (622080 t^2 y^4 + 286654464 t^2 y^2 - \\
& 17915904 t y b_1 + 51597803520) t^2 - 11197440 a_1^2 - 2239488 b_1^2 - 4478976) x^4 - (-17915904 t y b_1 + \\
& (622080 y^4 + 286654464 y^2 + 51597803520) t^2 - 11197440 a_1^2 - 2239488 b_1^2 - 4478976) x^4 - 279936 a_1^4 - \\
& 839808 a_1^2 b_1^2 - 559872 b_1^4 - 11757312 t y b_1 + (69120 t^3 y^6 + 35831808 t^3 y^4 - 5971968 t^2 y^3 b_1 + 10319560704 t^3 y^2 - \\
& 3732480 t y^2 a_1^2 - 746496 t y^2 b_1^2 - 1719926784 t^2 y b_1 + 1651129712640 t^3 - 1492992 t y^2 - 1074954240 t a_1^2 - \\
& 214990848 t b_1^2 + 143327232 t) x^3 - ((69120 y^6 + 35831808 y^4 + 10319560704 y^2 + 1651129712640) t^3 + \\
& (-5971968 y^3 b_1 - 1719926784 y b_1) t^2 + (-3732480 y^2 a_1^2 - 746496 y^2 b_1^2 - 1492992 y^2 - 1074954240 a_1^2 - \\
& 214990848 b_1^2 + 143327232) t x^3 + 29883727872 t^2 - ((4320 y^8 + 1990656 y^6 + 716636160 y^4 + 165112971264 y^2 + \\
& 29720334827520) t^4 + 2799360 a_1^4 + 3359232 a_1^2 b_1^2 + 559872 b_1^4 + (-746496 y^5 b_1 - 286654464 y^3 b_1 - \\
& 61917364224 y b_1) t^3 + (-466560 y^4 a_1^2 - 93312 y^4 b_1^2 - 186624 y^4 - 214990848 y^2 a_1^2 - 71663616 y^2 - \\
& 38698352640 a_1^2 - 7739670528 b_1^2 + 25798901760) t^2 + 5038848 a_1^2 + 5038848 b_1^2 + (26873856 y a_1^2 b_1 + \\
& 8957952 y b_1^3 + 6718464 y b_1) t + 10077696) x^2 + (4320 t^4 y^8 + 1990656 t^4 y^6 - 746496 t^3 y^5 b_1 + 716636160 t^4 y^4 - \\
& 466560 t^2 y^4 a_1^2 - 93312 t^2 y^4 b_1^2 - 286654464 t^3 y^3 b_1 + 165112971264 t^2 y^2 - 186624 t^2 y^4 - 214990848 t^2 y a_1^2 - \\
& 61917364224 t^3 y b_1 + 26873856 t y a_1^2 b_1 + 8957952 t y b_1^3 + 29720334827520 t^4 - 71663616 t^2 y^2 - 38698352640 t^2 a_1^2 - \\
& 7739670528 t^2 b_1^2 + 2799360 a_1^4 + 3359232 a_1^2 b_1^2 + 559872 b_1^4 + 6718464 t y b_1 + 25798901760 t^2 + \\
& 5038848 a_1^2 + 5038848 b_1^2 + 10077696) x^2 - 944784 a_1^2 - 944784 b_1^2 - ((144 y^{10} + 41472 y^8 + 23887872 y^6 + \\
& 6879707136 y^4 + 990677827584 y^2 + 285315214344192) t^5 + (-41472 y^7 b_1 - 11943936 y^5 b_1 - 3439853568 y^3 b_1 - \\
& 990677827584 y b_1) t^4 + (-25920 y^6 a_1^2 - 5184 y^6 b_1^2 - 10368 y^6 - 13436928 y^4 a_1^2 + 4478976 y^4 b_1^2 - \\
& 14929920 y^4 - 3869835264 y^2 a_1^2 - 1289945088 y^2 b_1^2 - 6019743744 y^2 - 619173642240 a_1^2 - 123834728448 b_1^2 + \\
& 743008370688) t^3 + (4478976 y^3 a_1^2 b_1 + 1492992 y^3 b_1^3 + 1119744 y^3 b_1 + 1289945088 y a_1^2 b_1 + 429981696 y b_1^3 + \\
& 2042413056 y b_1) t^2 + (466560 y^2 a_1^4 + 559872 y^2 a_1^2 b_1^2 + 93312 y^2 b_1^4 + 839808 y^2 a_1^2 + 839808 y^2 b_1^2 + \\
& 134369280 a_1^4 + 161243136 a_1^2 b_1^2 + 26873856 b_1^4 + 1679616 y^2 + 26873856 a_1^2 + 456855552 b_1^2 + \\
& 913711104) t x + (144 t^5 y^{10} + 41472 t^5 y^8 - 41472 t^4 y^7 b_1 + 23887872 t^3 y^6 - 25920 t^3 y^6 a_1^2 - 5184 t^3 y^6 b_1^2 - \\
& 11943936 t^4 y^5 b_1 + 6879707136 t^5 y^4 - 10368 t^3 y^6 - 13436928 t^3 y^4 a_1^2 + 4478976 t^3 y^4 b_1^2 - 3439853568 t^4 y^3 b_1 +
\end{aligned}$$

$$4478976 t^2 y^3 a_1^2 b_1 + 1492992 t^2 y^3 b_1^3 + 990677827584 t^5 y^2 - 14929920 t^3 y^4 - 3869835264 t^3 y^2 a_1^2 + 1289945088 t^3 y^2 b_1^2 + 466560 t y^2 a_1^4 + 559872 t y^2 a_1^2 b_1^2 + 93312 t y^2 b_1^4 - 990677827584 t^4 y b_1 + 1119744 t^2 y^3 b_1 + 1289945088 t^2 y a_1^2 b_1 + 429981696 t^2 y b_1^3 + 285315214344192 t^5 + 6019743744 t^3 y^2 - 619173642240 t^3 a_1^2 - 123834728448 t^3 b_1^2 + 839808 t y^2 a_1^2 + 839808 t y^2 b_1^2 + 134369280 t a_1^4 + 161243136 t a_1^2 b_1^2 + 26873856 t b_1^4 + 2042413056 t^2 y b_1 + 743008370688 t^3 + 1679616 t y^2 + 26873856 t a_1^2 + 456855552 t b_1^2 + 913711104 t(x) - 839808$$

Proof it is sufficient to replace the expression of the solution given by (5) in (1) and check that the relation is verified.

□

The modulus of the solution for different values of t is represented in the plane (x, y) . It can be seen that when the value of t increases, the maximum of the modulus of the solution decreases rapidly. We observe also the formation of horseshoes waves when time grows. The horseshoe multisoliton solutions correspond very well to real waves observed in thin films of shallow water. Waves with crossed parabolic profiles are also observed.

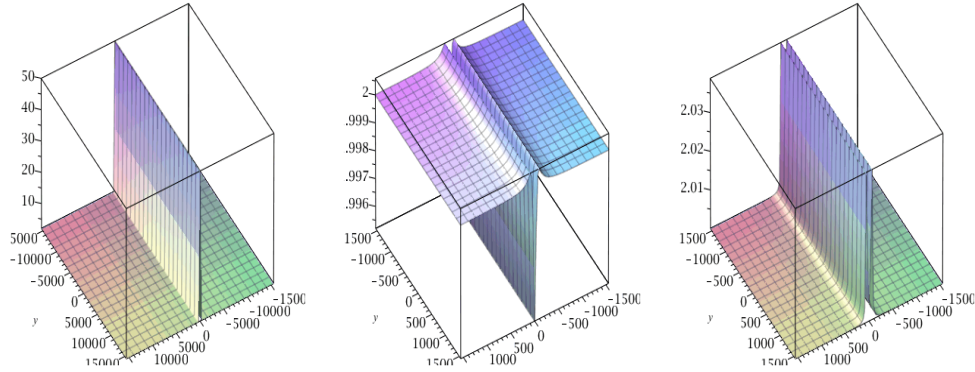


Figure 2. Solution of order 2 to (1) for $t = 0$, on the left $a_1 = 0, b_1 = 0$; ; in the center $a_1 = 100, b_1 = 0$; ; on the right $a_1 = 0, b_1 = 100$.

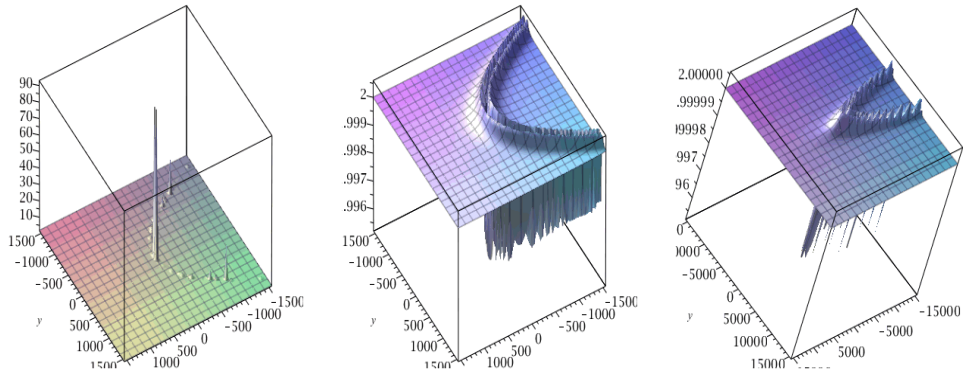


Figure 3. Solution of order 2 to (1) for $t = 0,01$, on the left $a_1 = 0, b_1 = 0$; ; in the center $a_1 = 10^2, b_1 = 0$; ; on the right $a_1 = 10^3, b_1 = 0$.

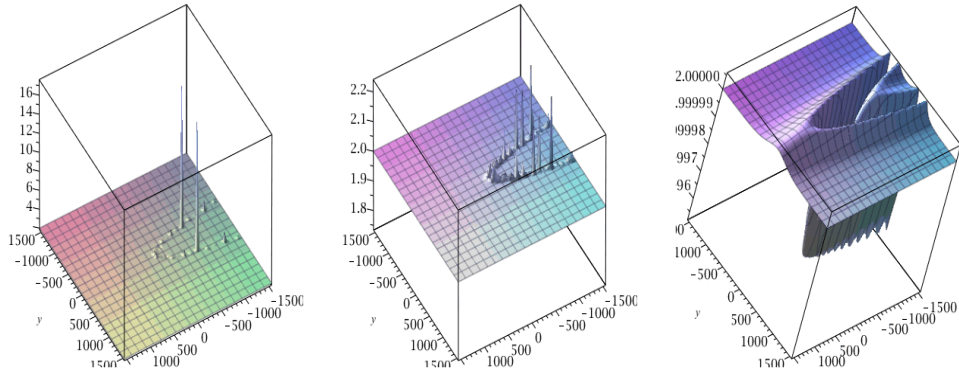


Figure 4. Solution of order 2 to (1) for $t = 0, 1$; on the left $a_1 = 0, b_1 = 0$; in the center $a_1 = 10, b_1 = 0$; on the right $a_1 = 10^3, b_1 = 0$.

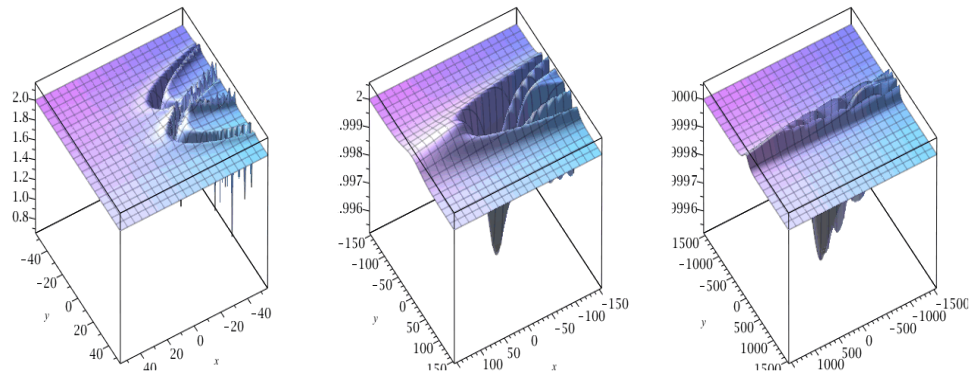


Figure 5. Solution of order 2 to (1) for $t = 1$; on the left $a_1 = 0, b_1 = 0$; in the center $a_1 = 10^2, b_1 = 0$; on the right $a_1 = 10^3, b_1 = 0$.

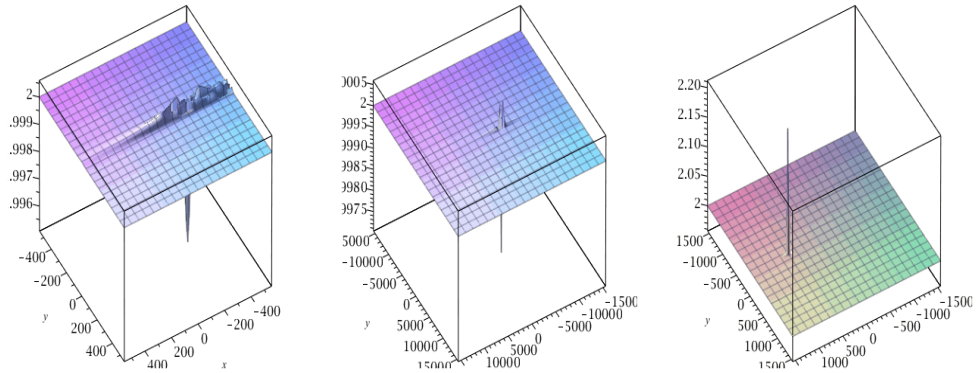


Figure 6. Solution of order 2 to (1); on the left for $t = 10, a_1 = 10^2, b_1 = 0$; in the center for $t = 100, a_1 = 10^3, b_1 = 0$; on the right for $t = 100, a_1 = 10^3, b_1 = 10^3$.

It is remarkable to note that coefficients of maximum degree in x , y , t are respectively the same ones, which explains the asymptotic behavior of solutions v : $\lim_{t \rightarrow \infty} v(x, y, t) = 2$, $\lim_{x \rightarrow \pm\infty} v(x, y, t) = 2$, $\lim_{y \rightarrow \pm\infty} v(x, y, t) = 2$.

4 Conclusion

Rational solutions to the Johnson equation of order 1 and 2 have been constructed here.

The following asymptotic behavior has been highlighted : $\lim_{t \rightarrow \infty} v(x, y, t) = 2$, $\lim_{x \rightarrow \pm\infty} v(x, y, t) = 2$, $\lim_{y \rightarrow \pm\infty} v(x, y, t) = 2$.

In a forthcoming publication we will to give the general method to construct rational solutions to the Johnson equation at order N .

We will show that these solutions can be written as a ratio of two polynomials of degrees $2N(N+1)$ in x , and t , $4N(N+1)$ in y , depending on $2N-2$ parameters. Moreover we will give a representation of these solutions in terms of Fredholm determinants, namely for every integer N these solutions can be written as the ratio of Fredholm determinants of order $2N$ depending on $2N-1$ parameters. We will construct also these solutions in terms of wronskians, i.e., they can be expressed as a quotient of wronskians of order $2N$ depending on $2N-1$ parameters.

We will present a proof of these present results and show the link with previous papers [14]-[16]-[32].

The method described in the present paper provides a powerful tool to get explicit solutions to the Johnson equation and to understand the behavior of rogue waves in the case of this equation.

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