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## 1 Introduction and Problem Description

Large companies tend to manage their activities in a coordinated way. Accordingly, the integrated supply chain has proven to have a major impact on overall success in terms of cost, quality and service time. In this context, two logistics operations are often referred to in the literature as keys to achieving an efficient supply chain: inventory management and transportation. The coordination of these two operations is often known as the Inventory Routing Problem (IRP) (see [2]). The IRP aims to determine simultaneously the delivery quantities and the routes in order to minimize the inventory holding and transportation costs.

More complex systems have been studied to take into account current configurations such as urban distribution. In most cities, urban freight accounts for a large part of traffic, occupation of roads and pollution. Decentralized models of local distribution tend to replace centralized models in order to ensure a sustainable and efficient distribution in the last kilometers. According [3], city logistics appears to be one of the principal keys to managing the urban distribution, supporting economic, environmental and social aspects. Thus, consolidation facilities, called Distribution Centers (DCs), are introduced to coordinate freight flow without and within the urban areas. The DCs, located on the outskirts of the city, receive products through large vehicles and transfer them to smaller vehicles to deliver them to the final customers located in urban areas. Indeed, a two-echelon distribution system can provide an efficient way to reduce traffic, pollution and noises (see [4]). We present a Two-Echelon IRP (2E-IRP), which is a new extension of the IRP to the best of our knowledge. The customers must be served by a supplier strictly through DCs and routes must be defined in both echelons over a given time horizon. Each DC must meet the demands of a set of customers on the second echelon. Split and direct deliveries from the supplier to a customer are not allowed. For each DC, the set of customers that it must serve is given. Two different fleets of homogeneous vehicles are available, one at each echelon. Vehicles and inventory capacities at DCs and customers must be respected.

We address some variants of 2E-IRP in the context of replenishment policies and routing configuration. Three replenishment policies are modeled: Maximum Level (ML) and Order-Up-To Level (OU), often considered in IRPs; and a reorder point/reorder quantity policy ($R$,$Q$), often studied for multi-echelon systems, but apparently not yet considered in IRPs. For the ($R$,$Q$) policy, as no delays or lead-times are considered, we can assume $R$ equal to zero. Thus, this policy can be called Order Fixed Quantity (OFQ) and we only consider that the quantities delivered must be fixed by echelon. We also considered three possibles routing configurations: in the first, one vehicle is available for the supplier and for each DC, each one being able to perform at most one tour per time period; in the second, each echelon has a set of vehicles available; in the last, similarly to the first, but allowing multi-tours on the second echelon. We propose a Mixed Integer Linear Programming (MILP) formulation for a 2E-IRP. In addition, we analyze several valid inequalities available for IRPs and we introduced new ones inherent to the IRP within two echelons. An exact Branch-and-Cut (B&C) algorithm was developed to solve the problem. We ran experiments on a set of randomly generated instances.

## 2 Branch-and-Cut Algorithm

We developed a Branch-and-Cut algorithm based on the proposed MILP to solve the 2E-IRP. Initially, at each node of the Branch-and-Bound tree, a Linear Program (LP) is solved. The LP is a relaxation of the MILP without integrity and subtour elimination constraints, but with a subset of valid inequalities. Then, violated subtour elimination constraints are iteratively identified, by using a separation algorithm [5], and added to the model which is reoptimized until no violated constraints are found. If the current solution is non-integer, then branching occurs, otherwise a feasible solution is found. The process iterates until the upper bound is equal to the lower bound. The initial valid inequalities in the LP are chosen based on previous experiments with the proposed MILP.

We generated a new set of instances for the 2E-IRP, based on the parameters proposed by [1]. However, it was necessary to adapt some parameters of the instances generation to apply the OFQ inventory policy. Moreover, as in our problem, there are two echelons with dependent demands, we considered different parameters and data scales. We suppose that the first echelon is characterized by longer distances and DCs with higher capacity than customers. A set of five test instances was randomly generated for each combination of time horizon $T \in \{3, 6\}$ and number of DCs $M \in \{3, 4, 5\}$ yielding a total of 30 instances. For each instance, the number of customers per DC was randomly generated in the interval [5, 10]. We consider the number of vehicles equal to the number of DCs for the multi-vehicle case. When multi-tours are allowed, the vehicle capacity on the second echelon is set to one-third of the initially generated capacity.

All preliminary experiments were run on an Intel Core i5, 2.40 GHz and 8 GB RAM personal computer, with a maximum running time of one hour. The algorithm was implemented in C++ language using CPLEX 12.7.1 and one thread. We use the default setting in CPLEX for branching variable selection and for the node selection rule (best-bound-first strategy). The different inventory policies and routing configurations were compared on all test instances.

## 3 Conclusion

We presented an B&C algorithm to solve variants of 2E-IRP, a presumably new problem in the literature. We evaluated the impact on the total cost of the different inventory policies and routing configurations. The OU policy turned out to be very strictly, which implies high costs. Depending on the company needs, an intermediate option is the OFQ policy. Substantial savings can be achieved using the ML policy [1]. As regards the routing configurations, the multi-tours presented the highest costs on our test instances. However, it is important to consider this configuration because it can be the case of urban areas that only allow very small vehicles which can make multi-tours due to short distances. Preliminary tests show that the insertion of valid inequalities and the use of a B&C technique usually generates better results compared to the ones obtained by the complete formulation. In future work we recommend the addition of new features, such as transshipment between DCs or customers and uncertain data, to better represent the reality. We also suggest the use of Branch-and-Price algorithms.

## References


[1] C. Archetti, L. Bertazzi, G. Laporte and M.G. Speranza. *A Branch-and-Cut Algorithm for a Vendor-Managed Inventory-Routing Problem.* Transportation Sc., 41(3), 382–391, 2007.

[2] L.C. Coelho, J.-F. Cordeau and G. Laporte. *Thirty Years of Inventory Routing.* Transportation Sc., 48(1), 1–19, 2013.

[3] T.G. Crainic, N. Ricciardi and G. Storchi. *Models for Evaluating and Planning City Logistics Systems.* Transportation Sc., 43(4), 432–454, 2009.

[4] R. Cuda, G. Guastaroba and M.G. Speranza. *A Survey on Two-Echelon Routing Problems.* Computers & Operations Research, 55, 185–199, 2015.

[5] M. Padberg and G. Rinaldi. *A Branch-and-Cut Algorithm for the Resolution of Large-Scale Symmetric Traveling Salesman Problems.* SIAM Review, 33(1), 60–100, 1991.