Proposition of a Method enabling Components Requisitions and Forecasts Analysis in Assemble-to-Order Systems

Mohammed Hichame Benbitour, Evren Sahin, Yves Dallery

Technical report

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Mohammed Hichame Benbitour, Evren Sahin, Yves Dallery
Laboratoire Génie Industriel, CentraleSupélec
3 Rue Joliot Curie, 91190 Gif-sur-Yvette, France
mohammed-hichame.benbitour@centralesupelec.fr
evren.sahin@centralesupelec.fr
yves.dallery@centralesupelec.fr

29 May 2018

Abstract

In this report, we aim at determining the probability distribution of components demand in Alpha plants. Alpha is one of the world's top ten auto parts makers (the company's name has been altered for confidentiality issues). Based on the case of Alpha, we develop a general method of components requisitions and forecasts analysis in Assemble-to-Order systems. This method allows to find the probability distribution of components demand and to estimate the parameters of this distribution. We apply the proposed method to the case of an Alpha plant situated in the west of France. We find that component demand follows a compound Poisson distribution.

Keywords: demand analysis; assemble-to-order; automotive industry;

1 Introduction

In most research papers on demand analysis, the term "demand" refers to the actual demand, and the term "forecast" refers to an estimation of demand. To respect the terminology used internally at Alpha, we use in this report the term "requisition" to refer to actual demand and the term "forecast" to refer to demand forecast. In this study, we consider the case of Alpha plants specialized on assembling car seats.

The analysis of requisitions and forecasts of goods -a finished good (FG), a sub-assembly or a component- in assembly systems is an important topic. Indeed, results obtained from this analysis can be used as input information for other processes such as: inventory management, production planning, capacity planning, etc. For example, inventory management models use a probability distribution to represent requisition (demand). In this report, we look for the probability distribution of components requisition in Alpha plants. Based on this case study, we develop a general method of components requisitions and forecasts analysis in Assemble-to-order (ATO) systems.

As defined by Song and Zipkin (2003), an ATO system includes components and finished goods; requisition occurs for finished goods, and the system keeps only inventory of components. The ATO system we consider in this report is the one that corresponds to Alpha plants. As described in Figure 1, Alpha is a first-tier supplier and works in a Just-in-Time setting with the OEM using synchronous delivery; the decoupling point between customer requisition and supply is at the level of components located at the first-tier supplier.

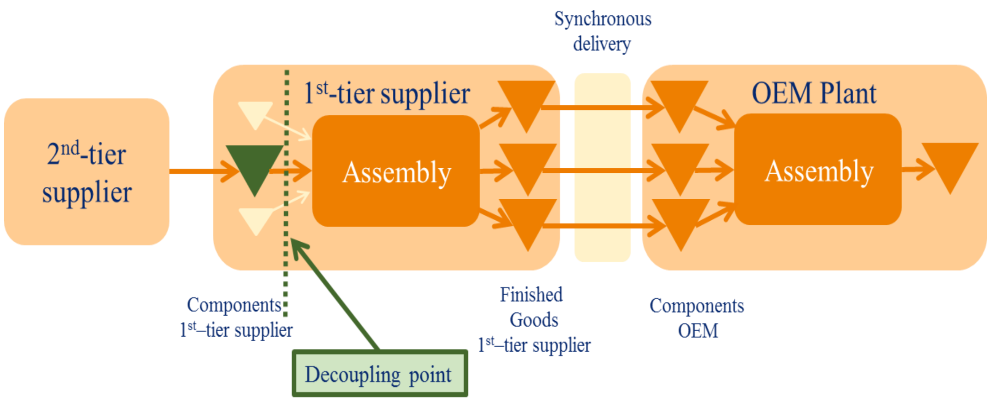


Figure 1. An example of a 1st-tier supplier ATO system

Before going further, we set the following definitions which will be used in the next sections of this report:

Alpha project: it is a contract between a customer (OEM) and Alpha where the latter supplies car parts (e.g., seats) for the same car model. Because of mass customization, one car model can have different references. Each project passes through four steps: birth (start of project), growth (increase of customer requisitions), maturity (steady requisitions), and decline (end of project). For example, Audi 4 is an Alpha project where the seats of Audi 4 are supplied to customer(s). A plant may work on more than one project at the same time.

Alpha FG: a FG represents the front and rear seats of a specific reference. For example, “Audi 4_electric seats_blue leather covers” represents one FG.

FG firm order: it is an order sent by customers for an Alpha FG. Customers can send during the same day many firm orders for the same FG. Each firm order must be totally satisfied four hours (in average) after its receiving, without backlogging. The four hours represent the demand lead time of each firm order.

FG requisition: conventionally, a FG requisition for day t is the sum of all firm orders of this FG (expressed in quantity) which have been satisfied during day t. For example, a FG j has a requisition of 47 units in day t.

FG forecast: it is an estimate of a future FG requisition sent by the customer to Alpha in the first day of each week.

Assembly coefficient: the number of units of a component required for the assembly of a FG using this component is called assembly coefficient. Assembly coefficients are given by in the Bill of Materials (BOM).

Component requisition: it is the sum of FGs requisitions which use the same component, multiplied by their respective assembly coefficients.

Component forecast: it is the sum of FGs forecasts which use the same component, multiplied by their respective assembly coefficients.

As it will be developed in this report, the proposed analysis method is applied to the case of an Alpha plant. We find that the probability distribution of each component requisition is a compound Poisson.

The work done is this document is related to two streams of literature: ATO systems and requisition analysis techniques. There are two papers (Atan, Ahmadi, Stegehuis, de Kok, & Adan, 2017; Song & Zipkin, 2003) that summarize literature in the ATO systems research field. Song and Zipkin (2003) classified papers into two categories depending on the problem addressed: 1) Performance evaluation and defining operating policies and 2) Design of ATO systems. They consider that there are two important issues to resolve in ATO systems: coordination (timing and quantity of supply orders) and allocation of components. The assumption relative to the distribution of components requisition in studied papers has a considerable impact on the analysis of ATO systems. Indeed, Song and Zipkin (2003) have pointed that data estimation (especially in large data sets) is not a trivial task. In this report, we propose a simple method to analyze historical components requisitions and forecasts in order to find the probability distribution of components requisition.

Atan et al (2017) classified ATO papers into periodic-review and continuous-review models; they also separated them depending on the number of FGs in the ATO system (one or multiple). In most studied systems, component requisition is assumed to arrive according to a Poisson and compound Poisson processes. Literature generally considers simple BOM configurations (small number of FGs and components). In this report, we propose a detailed analysis of the BOM where representative components are identified.

Many references have been developed in order to understand the nature of requisition and its decomposition into different patterns. The textbook (Hyndman & Athanasopoulos, 2014) was used to review the different characteristics of time series (historical component requisitions constitute a time series) and the relation between forecasts and requisitions. We also used (Feller, 1968) for understanding the usual probability distributions, their characteristics and the different processes behind them. Meyr (2004) and Zhang & Chen (2006) give examples of sharing requisitions and forecasts in the automotive industry. In our work, we propose a method of components requisitions analysis in ATO systems. We show interesting characteristics of components requisitions that can be useful in the determination of their probability distribution.

The remainder of this report is organized as follows. The proposed analysis method and the case study are given in sections 2 and 3 respectively. We provide conclusions in section 4.

2 Proposed Analysis method

We consider a plant that can be represented by an ATO system with multiple FGs and multiple components. Each FG requisition is random and independent from the other FGs requisitions. The probability distribution(s) of FGs and components requisition is (are) assumed to be unknown (FGs and components can have different probability distributions). Our objective is to find the probability distribution of each component requisition, and to estimate the parameters of this distribution such as mean, variance, etc.

If the random variable representing the requisition of a component x per time unit is denoted by R_x, then finding the probability distribution of R_x is to determine the probability $P(R_x=d)$ for each non-negative real d.

In the proposed analysis method, we assume that only requisitions and forecasts of FGs are known. In other terms, data on components requisitions and forecasts is not available. It has to be constructed. The different steps of this method are summarized in Figure 2. It starts by the

Preliminary analysis where: a) past FGs requisitions and forecasts are extracted, b) a suitable time unit to analyze the extracted data is defined and c) the Demand Summary Table (DST) of each FG is created. DST is a structured table that contains requisitions and forecasts. After the Preliminary analysis, the Detailed analysis aims at cleaning the DSTs of FGs and creating the DSTs of components by using the BOM. As we will see after, the DSTs of only a special group of components called "representative components" are created. Finally, the probability distribution of each component requisition is found, the forecast accuracy is evaluated and the parameters of the probability distribution are estimated.

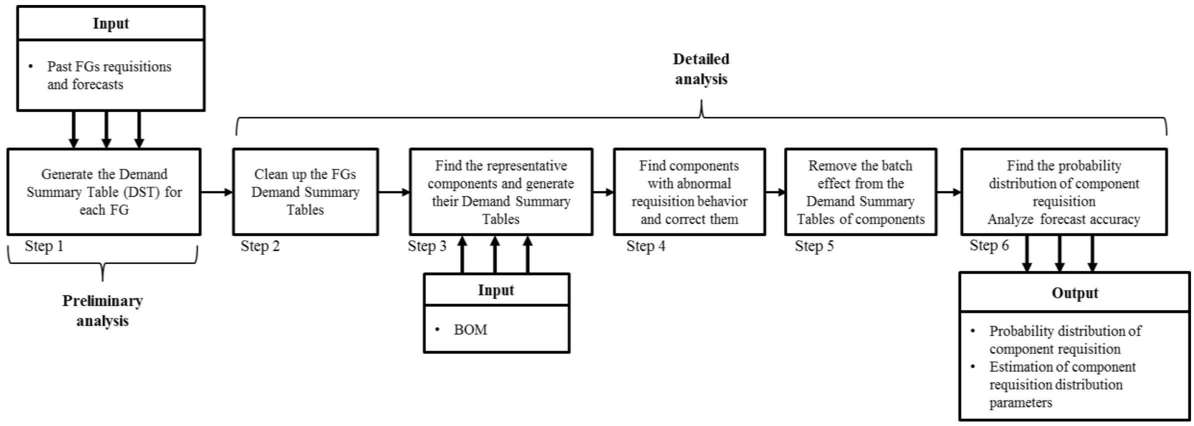


Figure 2. Proposed analysis method steps

Part 1: Preliminary analysis

Step 1: Generate the Demand Summary Table (DST) for each FG

- Inputs: extracted FGs requisitions and forecasts
- Output: FGs DST

First, we extract historical FGs requisitions and forecasts from the information system of the ATO system (e.g., ERP, customer invoices…) for a time horizon (interval) that is accepted to be representative of the behavior of requisition. There is no general rule to define the best extract time horizon. From the experience and know-how of plant managers, a suitable time horizon can be defined. For example, if requisition is seasonal, then the time horizon of extraction must contain at least one season.

For instance, an extraction can be made for a time horizon composed of *T* weeks (from week 1 to week *T*), where week 1 is the most recent week, called also the "last extraction week" and week *T* is the "first extraction week". We have for each week (From 1 to *T*) and for each FG:

- The requisitions that occurred during this week
- The forecasts made at the first day of this week for future delivery dates

Consequently, the extracted requisitions and forecasts in a week w ($1 \leq w \leq T$) and for a given FG y can be represented by Table 1. For each extracted FG requisition and forecast, we know:

- The quantity of this requisition/forecast
- The delivery date of this requisition/forecast

For FGs requisitions, all delivery dates are within week w. The delivery dates of FGs forecasts are during or after week w.

Table 1. Extracted data in a given week w ($1 \leq w \leq T$) for a given FG y

		Quantity	Delivery date
FG *y*	Requisition 1		Day 1 of week *w*
	Requisition 2		Day 2 of week *w*
	Requisition 3		Day 3 of week *w*
	Requisition …		Day ... of week *w*
	Last requisition in Week *w*		Last day of week *w*
	Forecast 1		Future date (the date can be in week *w* or later)
	Forecast 2		Future date (the date can be in week *w* or later)
	Forecast …		…
	Last forecast received in Week *w*		Future date (the date can be in week *w* or later)

Remark

There may be days in the extraction time horizon where a FG requisition (or forecast) does not exist. This case has two possible explanations:

- This FG requisition (or forecast) is null
- This FG requisition (or forecast) exists but it was not saved in the past data

After extracting the past FGs requisitions and forecasts, we find a suitable time unit to represent the extracted data. Requisitions and forecasts can be expressed per day, week, month or another customized time unit. Generally, the best time unit to choose is the smallest possible time unit because it gives the biggest sample size. We say possible time unit because if requisitions are announced per day without specifying the hour of delivery, then the time unit cannot be the hour. However, it is not always better to use the smallest possible time unit. In some cases, aggregation of requisitions (and forecasts) -from small to large time unit- can give interesting insights. The most suitable time unit would be the one that allows to depict clearly and completely the requisition behavior (trend, seasonal, randomness…). In the next steps, we assume that the time unit is the day without loss of generality.

From the extracted FGs requisitions and forecasts, we find the "first delivery date" and the "last delivery date". The first delivery date corresponds to the earliest announced delivery date in forecasts and requisitions of all FGs, and the last delivery date is the most recent one (similarly, in forecasts and requisitions). If there is a FG forecast with a delivery date after the last extraction week, then the last delivery date will be after the last extraction week also.

The extracted requisitions and forecasts of each FG can be summarized in a DST.

Definition of a DST (example): As shown in Table 2, the DST is a structured table that has as columns delivery dates and as rows requisitions and forecasts associated with each delivery date (from the first until the last). Forecasts can be made in the same week of delivery date (denoted by "Forecast W-0"), or in prior weeks. For instance, we denote by "Forecast W-1" the forecast made one week before the delivery date. A DST (represented in Table 2) is a matrix representation of extracted requisitions and forecasts (represented in Table 1). The transformation of Table 1 into Table 2 can be done using an application (e.g., in the case study we used an Excel VBA Macro to do the required transformation)

In the transformation application, the list of all days which exist between the first and last delivery dates is generated (columns of DST). For each delivery date, the corresponding requisition and forecasts are filled. If the requisition is unknown (it does not exist in the extracted data), we put "NR" in the DST. "NF" is put in the case of an unknown forecast. The number of delivery dates represents the sample size of the DST. If forecasts are given at most *Z* weeks prior to the delivery date, then the raw "Forecast W-*Z*" is the last raw in the DST.

Table 2. Example of Demand Summary Table (DST) of a FG

Year	First delivery date	…	2017	2017	2017	2017	2017	2017	2017	…	Last delivery date
Month			3	3	3	3	3	3	3		
Week			10	10	11	11	11	11	11		
Day			9	10	13	14	15	16	17		
Requisition	28		38	35	NR	29	31	27	NR		NR
Forecast W-0	29		38	33	NF	30	32	32	NF		NF
Forecast W-1	NF		37	NF	NF	NF	30	29	33		NF
Forecast W-2	NF		NF	NF	27	32	NF	30	31		NF
Forecast W-3	NF		30	37	32	29	32	NF	31		35
Forecast W-4	NF		NF	38	30	25	30	27	28		32
Forecast …	…		…	…	…	…	…	…	…		…
Forecast W-Z	NF		NF	42	29	35	40	34	NF		30

In the next step, the requisitions and forecasts of FGs are cleaned to remove incoherent information.

Part 2: Detailed analysis

Step 2: Clean up of FGs Demand Summary Tables

- Inputs: FGs DST
- Output: clean FGs DST

After generating the DSTs of FGs from the extracted requisitions and forecasts in step 1, it is possible that the DSTs contain incoherent information (either concerning forecasts or requisitions). The objective of this step is to clean the DSTs of FGs (garbage in, garbage out). If DSTs of FGs are already clean, this step is not necessary.

Incoherent information in a DST can have different forms. For example, some delivery dates may have requisitions which are outliers. Those delivery dates constitute exceptions that should be justified. Most importantly, those delivery dates do not represent "normal" requisition behavior and should be deleted from all the DSTs. In addition, the cases "NR" and "NF" have to be explained (see Remark in step 1).

In order to identify the delivery dates which cannot be exploited in the analysis, we analyze the total FGs requisitions (the sum of requisitions of all FGs). Delivery dates with "NR" are considered as zeros in the sum. In fact, the system managers know the total requisitions (the mix) better than the detailed requisitions (per FG), and they can identify the incoherent delivery dates by analyzing the mix more quickly than by analyzing the DST of each FG.

By representing graphically the variations of total requisitions, delivery dates with incoherent information can be easily detected. An upper and a lower limits of total requisitions can be fixed to decide what delivery dates to keep in the DST.

For example, we analyzed the total requisitions of a plant, depicted in Figure 3 (sample size of 100 delivery days), the upper and lower limits of total requisitions are fixed at ±25% of the average total requisitions (e.g., 196 units/day). The production capacity of this plant is assumed to be 250 units/day. It can be seen that total FGs requisitions in days 8 and 26 are null and less than the lower limit. In day 55, the total requisition is 322 units which is greater than production capacity (incoherent information). In addition, the total requisition in day 72 represents 7 units (less than the lower limit). Consequently, delivery dates 8, 26, 55 and 72 have to be removed from the DST because they do not represent a normal requisition behavior.

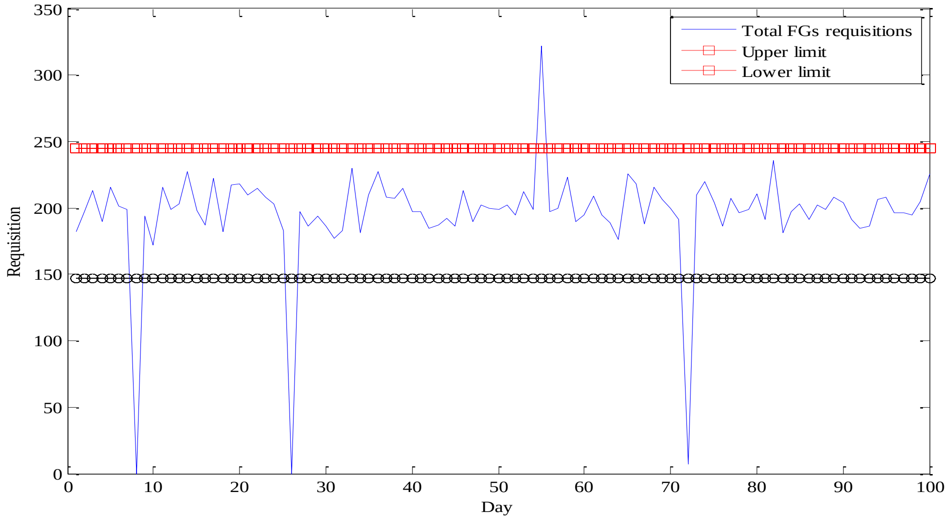


Figure 3. Example of total FGs requisitions analysis

After deciding the delivery dates to keep in the analysis, the DST of each FG must be cleaned. The information "NR" in the new tables is replaced by zero (null requisition).

We propose a conventional method to correct FG forecasts (replacing the information "NF" by a quantity): The information "NF" made in a week *w* is replaced by the most recent existing forecast made in weeks prior to week *w*. In the example of Table 2, Forecast W-2 to be delivered the 09/03/2017 takes the value 30. Forecasts W-1 and W-2 to be delivered the 10/03/2017 take the value 37. When the information "NF" cannot be corrected, it takes the value zero.

Step 3: Find the representative components and generate their Demand Summary Tables

- Inputs: FG DST, BOM
- Output: matrix of assembly coefficients, component DST

After cleaning the DSTs of FGs, the DST of each component can be generated using the BOM. The BOM is a list of components and sub-assemblies required to assemble the FGs. The BOM appears generally in a hierarchical format, with the highest level representing the FG and the bottom level representing the individual components required to assemble this FG (see the example in Figure 4). We assume that there is no stock of sub-assemblies and only individual components (supplied by external suppliers) are kept in stock.

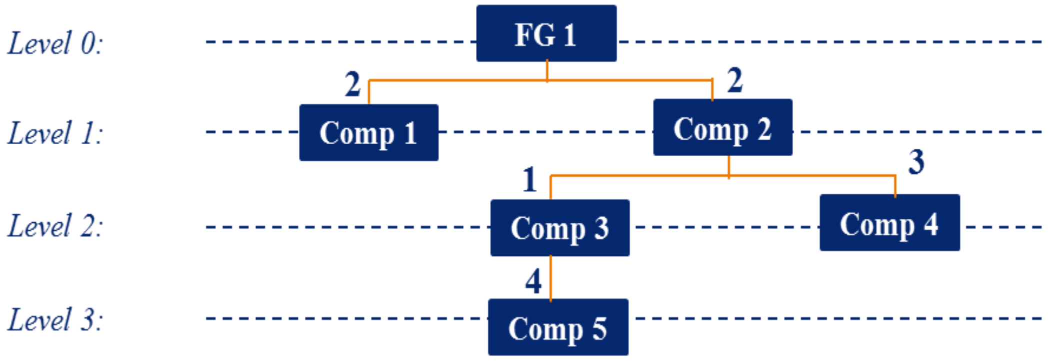


Figure 4. An example of a BOM

In Figure 4, the individual components are: Comp 1, Comp 4 and Comp 5 with assembly coefficients 2, 3 and 4 respectively. Comp 2 and Comp 3 represent sub-assemblies with assembly coefficients 2 and 1 respectively.

Another way to represent the information provided by the BOM is the use of the matrix of assembly coefficients. This matrix has as columns the FGs and as rows the individual components. The elements of this matrix are the assembly coefficients (e.g., Table 3). There are *m* components and *n* FGs. Each FG *j* uses "a_{ij}" units of component *i* where i=1:m and j=1:n. Assembly coefficients are assumed to be non-negative integers.

In ATO systems, the number of individual components is generally very high and it may be difficult to analyze the requisitions and forecasts of each component. Sometimes, there are similar components in the BOM which can be represented by one component called a "representative component". A more formal definition of similar components is given in the next lemma.

Lemma: Similar components

We say that two components *i* and *i'* are similar if:

There is a constant *p* that belongs to *Q**, where: $a_{ij} = p * a_{i'j}, \forall j = 1{:}n$.

This means that the assembly coefficients of component *i* are the same, multiple or devisor of assembly coefficients of component *i'*.

Example: Representative components

In the example provided in Table 3, it can be seen that components A, B, and C are similar, and one of these components can be picked as a representative component of them. Components D and F are also similar. For instance, we can say that A is a representative component of B and C, and D is a representative component of F. To save time, only the DSTs of representative components (A, D, E) are calculated instead of calculating the DSTs of the six components.

Table 3. Matrix of assembly coefficients with six components and five FGs

	FG1	FG2	FG3	FG4	FG5
Component A	1	1	0	1	1
Component B	2	2	0	2	2
Component C	7	7	0	7	7
Component D	2	1	1	1	2
Component E	2	0	0	0	2
Component F	4	2	2	2	4

To calculate a component DST, requisitions of this component are obtained by summing the requisitions of FGs which use this component multiplied by their respective assembly coefficients. Components forecasts are also calculated in a similar way.

Step 4: Find components with abnormal requisition behavior and correct them

- Inputs: component DST
- Output: clean component DST (If components DSTs are clean, this step is not necessary)

We represent in a graphical way the requisitions of representative components in order to detect components with abnormal requisition behavior. The abnormal requisition behavior can have many forms. For example, the plant may decide to replace a component by another new component. The two components (replaced and substitute) are used by the same FGs but each one has its own DST. The two components should be considered as the same component. Therefore, we create a new component with a DST equal to the sum of the DSTs of the two components. In the example below, it can be seen that component B replaces component A. Component C is the union between components A and B.

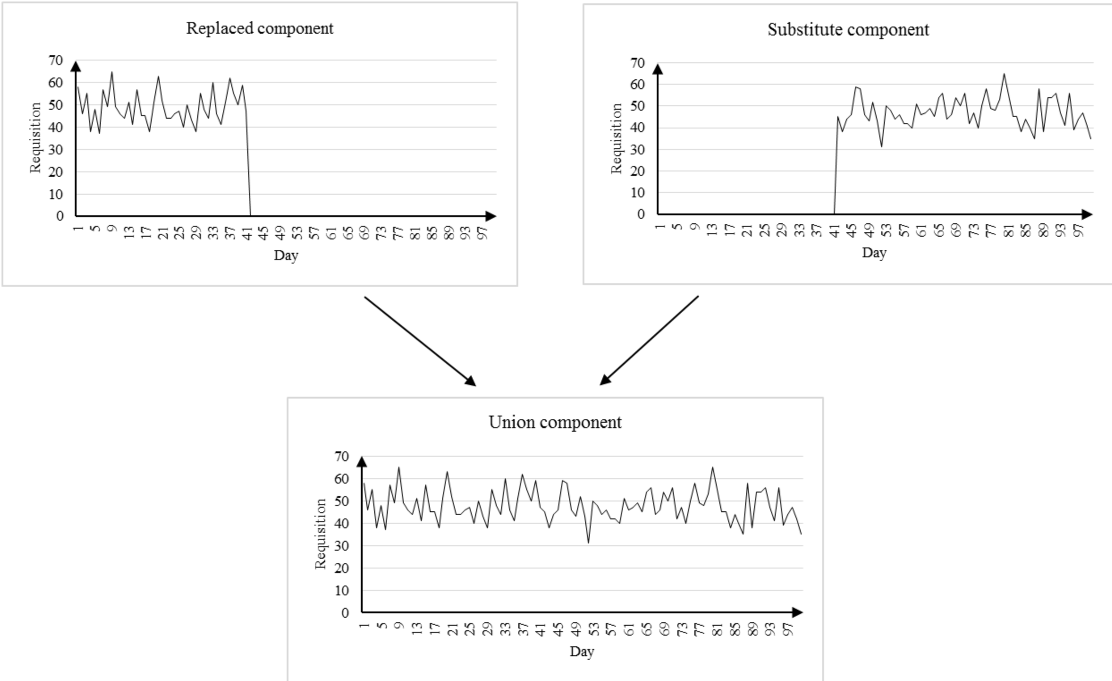


Figure 5. Example of replacement components

Step 5: Remove the batch effect from component DST

- Inputs: component DST, matrix of assembly coefficients
- Output: component DST where batch effect is removed

A component has an important characteristic: its requisition (and forecast) is a compound or batch variable. Indeed, a component requisition is based on two parameters:

1) The arrival process of FGs that use this component.
2) The number of component units required to assemble each arrived FG (this number is equal to the assembly coefficient of this FG).

Component requisition can be approximated by simpler parameters:

1) The arrival process of FGs that use this component.
2) Each FG arrival requires a number of component units that is equal to a weighted average assembly coefficient (weighted by the mean of FGs requisition) calculated for this component.

To analyze a component requisition, we first need to determine the arrival process of FGs that use it. To do so, we need to divide the requisitions of this component by its weighted average assembly coefficient (called also the batch size).

Let us explain this by giving two examples:

Example 1: a component used by 1 FG

Component x is used by FG y with an assembly coefficient a_{xy}. The arrival process of FG y characterized by the number of arrivals of FG y during day t which is equal to this FG y requisition in day t. Component x requisition can be viewed also as an arrival process.

If FG y has a determined arrival process, then the arrival process of component x is a compound version of the arrival process of FG y. The arrival of one unit of FG y demand induces the arrival of a_{xy} units of component x demand.

The probability distribution of component x requisition can be expressed by:

- $P(R_x = d) = P\left(\frac{R_x}{a_{xy}} = \frac{d}{a_{xy}}\right)$ for each non-negative real number d. $P\left(\frac{R_x}{a_{xy}} = \frac{d}{a_{xy}}\right)$ represents the probability that the requisition of component x (expressed in batch of size a_{xy}) is equal to $\frac{d}{a_{xy}}$.

Instead of analyzing the arrival process of component x, we first analyze the arrival process of this component in terms of batches (which is equal to the arrival process of FG y). In this example, the two analysis give the same result. In short, to find the probability distribution of component x requisition, we need to remove the batch effect from the requisitions and forecasts of this component. We divide all values stored in component x DST (all requisition and forecast values) by the assembly coefficient a_{xy}.

Example 2: a component used by 2 FGs

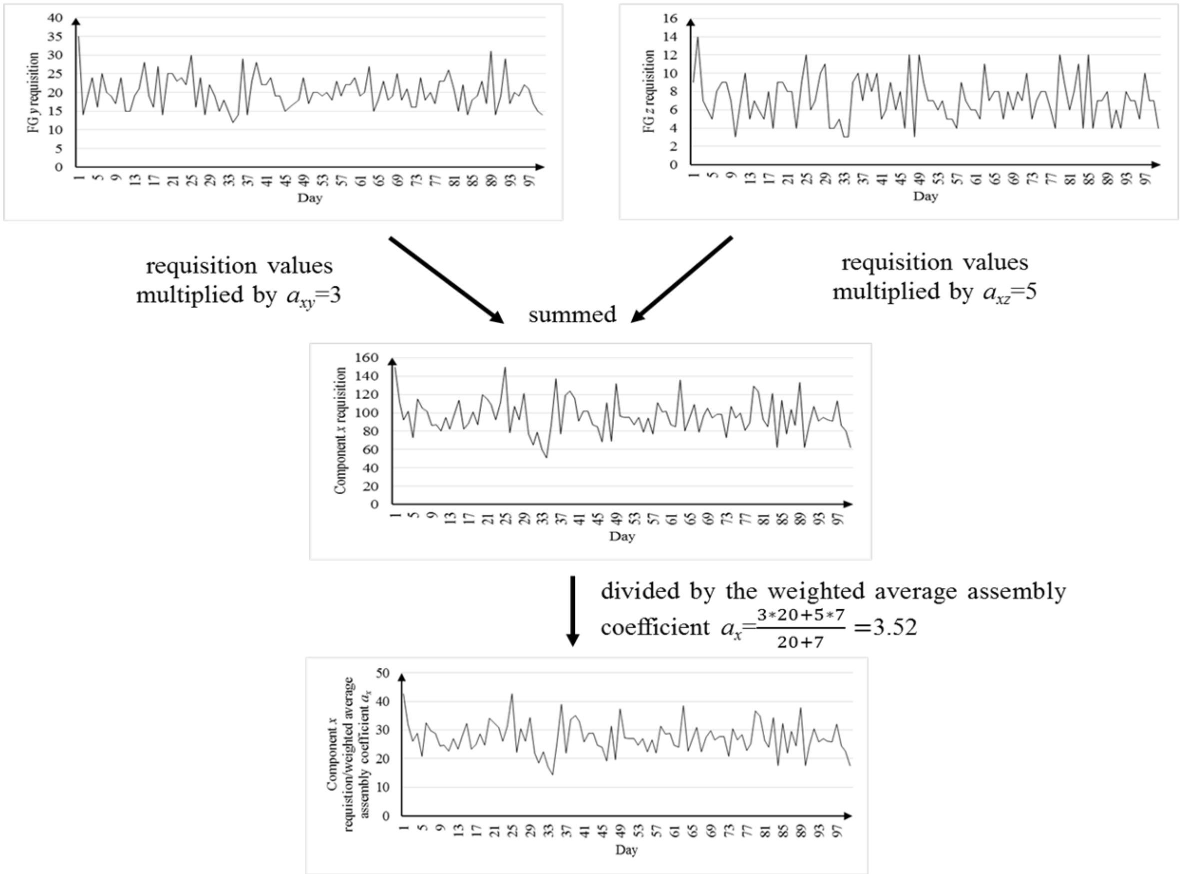
Component x is used by 2 FGs y and z with respective assembly coefficients a_{xy} and a_{xz}. The arrival process of component x can be approximated by a compound (batch) arrival process with a batch size equal to the average of assembly coefficients a_{xy} and a_{xz} weighted by the mean requisition of FGs y and z respectively. We denote the weighted average assembly coefficient of component x by a_x.

The probability distribution of component x requisition can be approximated by:

- $P(R_x = d) \cong P\left(\frac{R_x}{a_x} = \frac{d}{a_x}\right)$ for each non-negative real number d.

To find the probability distribution of component x requisition, we need to remove the batch effect from the requisitions and forecasts of this component obtained in step 3. We divide the values stored in component x DST (all requisition and forecast values) by the weighted average assembly coefficient a_x. By doing this, we obtain the DST of component x where batch effect is removed.

Figure 6 summarizes how the batch effect is removed from the requisitions of component x.



Delivery date	1	2	3	…	98	99	100
Requisition	150	112	92	…	86	80	62
Forecast W-0	112	112	104	…	95	90	101
Forecast W-1	89	85	94	…	91	82	89
Forecast W-2	91	100	107	…	89	108	81
Forecast W-3	88	102	98	…	97	86	99
Forecast W-4	86	100	85	…	94	104	109
Forecast …	…	…	…	…	…	…	…

DST of component *x*

a_x=3.52 →

Delivery date	1	2	3	…	98	99	100
Requisition/a_x	42.55	31.77	26.1	…	24.4	22.7	17.59
Forecast W-0 /a_x	31.77	31.77	29.5	…	26.95	25.53	28.65
Forecast W-1 /a_x	25.25	24.11	26.67	…	25.82	23.26	25.25
Forecast W-2 /a_x	25.82	28.37	30.36	…	25.25	30.64	22.98
Forecast W-3 /a_x	24.97	28.94	27.8	…	27.52	24.4	28.09
Forecast W-4 /a_x	24.4	28.37	24.11	…	26.67	29.5	30.92
Forecast …	…	…	…	…	…	…	…

DST of component *x* where batch effect is removed

Figure 6. Removing the batch effect from the requisitions of component *x*

Step 6: Find the best fitting probability distribution of component requisition and analyze forecast accuracy

- Inputs: component DST where batch effect is removed
- Output: probability distribution of component requisition, forecast accuracy, estimation of distribution parameters

In this step, we find the probability distribution of each component requisition. To do this, we use the DST of components without batch effect created in step 5. We describe in Figure 7 the elementary actions necessary to find the best fitting probability distribution:

- The requisitions (without batch effect) data constitute a time series. Hence, it can be decomposed into deterministic (trend, seasonal and cyclic) and non-deterministic (random) parts (see Hyndman and Athanasopoulos (2014)). In this study, we assume that only trends can be observed in requisitions (requisitions are not seasonal nor cyclic).
- We start by removing the trend from requisitions. In variability analysis, it is necessary to remove the trend (operation known as detrending) in order to have meaningful results (Wu, Huang, Long, & Peng, 2007). Matlab or R (softwares for statistical computing) can be used to remove trends (function detrend). Once trend is removed, we calculate the basic statistical indicators (to have an overall picture on requisitions) such as: Average, standard deviation Std (corrected), Coefficient of variation CV, etc.
- In order to find the best fitting probability distribution for requisition (without batch effect); usual distributions such as Binomial, Poisson, Normal or Geometric can be tested using goodness-of-fit tests (e.g., Pearson's chi-squared test and Kolmogorov–Smirnov test). The probability distribution of component requisition can be with one parameter such as the Poisson distribution (mean) and with two parameters such as the Normal distribution (mean and standard deviation). Probability distributions with more than two parameters are not considered in our study.
- When the sample size is small, the use of goodness-of-fit tests may be controversial. The results of these tests may be inaccurate (McDonald, 2009) and sometimes more than one probability distribution can be accepted by these tests. In this case, alternative tests can be used. For example, the ratio $\frac{Std}{\sqrt{Average}}$ is calculated to test the Poisson distribution (in a Poisson process: $\frac{Std}{\sqrt{Average}} = 1$). Skewness and kurtosis estimators can be calculated to test the normal distribution.
- If no usual probability distribution is a good fit for requisition (without batch effect), an empirical probability distribution can be developed using descriptive statistics.

Once the distribution of a component requisition is determined, one needs to find the values of its parameters (such as the mean, variance, etc.). For this, we will use the forecast values of components (without batch effect).

Figure 8 describes the elementary actions to evaluate the forecasts accuracy and to estimate the parameters of the found probability distribution. The commonly used accuracy measures are calculated for each forecast raw of the DST obtained in step 5. (Forecast W-0 /a_x, Forecast W-1 /a_x, etc.):

- Average Error (where Error = Forecast-Requisition for each delivery date)
- Average absolute Error (where absolute Error = |Error| for each delivery date)
- Forecast quotient = $\frac{\text{Average Forecast}}{\text{Average requisition}}$

We can say that the forecast accuracy is good if the "Average absolute Error" is near to 0 and the Forecast quotient is near to 1.

For the estimation of parameters of probability distribution, the standard deviation Std can be estimated using the corrected standard deviation estimator:

$$\text{Std} = \sqrt{\frac{1}{\text{Number of delivery dates} - 1} * \sum_{t=1}^{\text{Number of delivery dates}} \left(\text{Requisition}_t - \text{Average requisition}\right)} \quad (1)$$

The mean requisition can be estimated by:

$$\text{Mean requisition} = \frac{\text{Average Forecast}}{\text{Forecast quotient}} \quad (2)$$

Illustrative example of calculation of mean requisition

We are at the end of day t-1, we have the past requisitions and forecasts of a component for the past 10 days. We have also the forecasts for the next 10 days (sent by the customer). For simplification, we assume that the batch size is equal to 1, i.e., there is no batch effect (in the opposite case, the batch effect must be removed from requisitions and forecasts before any analysis). Let us assume that we know the demand probability distribution of requisition, and we want to estimate the mean requisition for the next 5 days. To calculate this parameter, we first calculate the Forecast quotient over the past ten days:

Delivery date	t-10	t-9	t-8	t-7	t-6	t-5	t-4	t-3	t-2	t-1
Requisition	25	20	19	17	24	15	15	19	21	28
Forecast W-1	21	24	20	19	22	20	16	19	23	27
Error	-4	4	1	2	-2	5	1	0	2	-1

$$\text{Forecast quotient} = \frac{\text{Average Forecast W-1}}{\text{Average requisition}} = \frac{21.10}{20.30} = 1.04$$

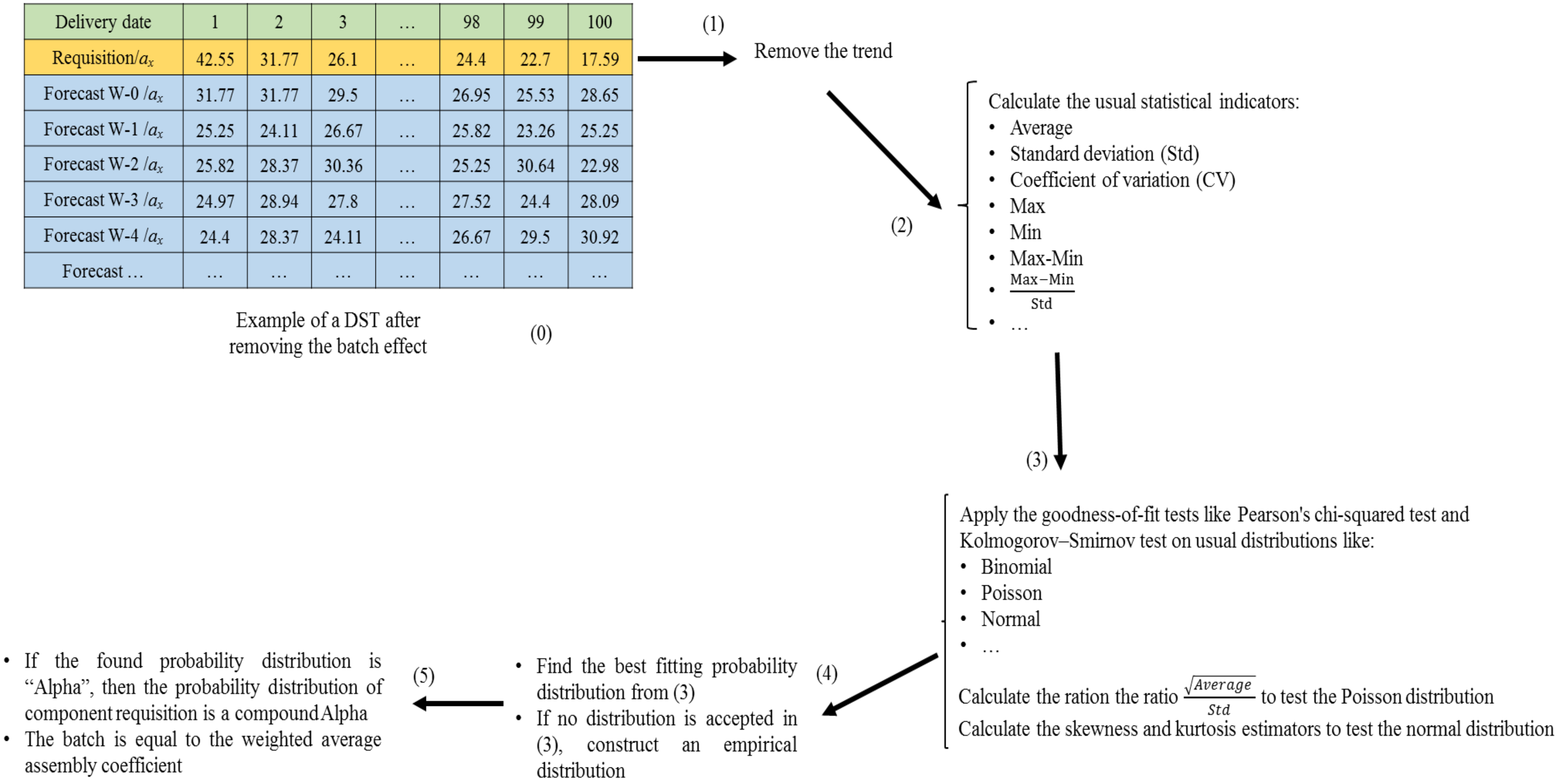
Then we calculate average forecast over the next 5 days:

Delivery date	t	t+1	t+2	t+3	t+4
Forecast W-1	18	21	25	27	31

$\text{Average forecast} = 24.40$

As a result, the mean requisition for the next 5 days can be estimated by:

$$\text{Mean requisition} = \frac{\text{Average Forecast}}{\text{Forecast quotient}} = \frac{24.40}{1.04} = 23.47$$



Delivery date	1	2	3	…	98	99	100
Requisition/a_x	42.55	31.77	26.1	…	24.4	22.7	17.59
Forecast W-0 /a_x	31.77	31.77	29.5	…	26.95	25.53	28.65
Forecast W-1 /a_x	25.25	24.11	26.67	…	25.82	23.26	25.25
Forecast W-2 /a_x	25.82	28.37	30.36	…	25.25	30.64	22.98
Forecast W-3 /a_x	24.97	28.94	27.8	…	27.52	24.4	28.09
Forecast W-4 /a_x	24.4	28.37	24.11	…	26.67	29.5	30.92
Forecast …	…	…	…	…	…	…	…

Figure 7. Finding the best fitting probability distribution of a component requisition

Delivery date	1	2	3	…	98	99	100
Requisition/a_x	42.55	31.77	26.1	…	24.4	22.7	17.59
Forecast W-0 /a_x	31.77	31.77	29.5	…	26.95	25.53	28.65
Forecast W-1 /a_x	25.25	24.11	26.67	…	25.82	23.26	25.25
Forecast W-2 /a_x	25.82	28.37	30.36	…	25.25	30.64	22.98
Forecast W-3 /a_x	24.97	28.94	27.8	…	27.52	24.4	28.09
Forecast W-4 /a_x	24.4	28.37	24.11	…	26.67	29.5	30.92
Forecast …	…	…	…	…	…	…	…

Example of a DST after removing the batch effect (0)

(1)

For each raw that represents a forecast, calculate:

- Average Error (where Error=Forecast-Requisition)
- Average absolute Error (where absolute Error=|Error|)
- Forecast quotient=Average forecast/Average requisition
- Average Ratio (where Ratio=Forecast/Requisition)
- Average quadratic Error (where quadratic Error=Error^2)
- …

(2)

Mean requisition=Average forecast/Forecast quotient

(3)

The standard deviation of requisition probability distribution can be estimated by calculating the standard deviation of past requisitions (without batch effect)

Figure 8. Measuring forecast accuracy and estimating the parameters of the distribution of a component requisition

3 Case study

As seen in section 2, the proposed analysis method concentrates on analyzing the probability distribution of a component requisition instead of analyzing the probability distribution of FGs requisition. We justify this choice by the fact that:

- If a component *x* is used by *n* FGs and the *n* FGs requisitions have different probability distributions, then it would not be possible to deduce the probability distribution of component *x* requisition used by the *n* FGs.

We applied the proposed analysis method to the case of an Alpha plant situated in the west of France. This plant manufactures customized seats for two projects (five-door car models). There are 264 different FGs. The plant works with a relatively constant production rate of 332 FG per day. Subsection 3.1 describes the generation of FGs DSTs.

3.1 Preliminary analysis

We started by demanding an extract of FGs requisitions and forecasts from plant managers. The available data were within this horizon: the first extraction week is week 42 of year 2014 and the last extraction week is week 13 of year 2015. In order to have the largest sample size possible, we decided to use the "day" as a time unit for the analysis. The first delivery date is the 13/10/2014 and the last delivery date is the 28/08/2015. The initial sample size is 320 days. We filled the DSTs of all FGs using an Excel VBA application (we transformed the extracted requisitions and forecasts into FGs DSTs). Figure 9 gives an example of a FG DST.

	A	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS	CT	CU	CV
1	Year	2015	2015	2015	2015	2015	2015	2015	2015	2015	2015	2015	2015	2015	2015
2	Month	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	Day	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4	Week	2	2	2	2	2	3	3	3	3	3	3	3	4	4
5	Requisition	1	17	12	19	31	0	13	19	18	16	33	0	0	23
6	Forecast W-0	0	21	20	17	24	0	19	22	14	29	30	0	0	20
7	Forecast W-1	0	0	0	0	0	0	18	19	14	20	32	0	0	12
8	Forecast W-2	0	0	0	0	0	0	0	0	0	0	0	0	0	28
9	Forecast W-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	Forecast W-4	0	22	19	17	13	0	0	0	0	0	0	0	0	0
11	Forecast W-5	0	20	19	19	0	0	11	17	16	6	15	0	0	0
12	Forecast W-6	0	20	19	19	0	0	12	12	11	11	11	0	0	14
13	Forecast W-7	0	20	19	19	0	0	12	12	11	11	11	0	0	15
14	Forecast W-8	0	20	19	19	0	0	12	12	11	11	11	0	0	15
15	Forecast W-9	0	0	0	0	0	0	12	12	11	11	11	0	0	15
16	Forecast W-10	13	13	12	12	0	0	0	0	0	0	0	0	0	15
17	Forecast W-11	0	0	0	0	0	0	13	13	13	12	12	0	0	0
18	Forecast W-12	13	13	12	12	0	0	0	0	0	0	0	0	0	13
19	Forecast W-13	11	10	10	10	0	0	13	13	13	12	12	0	0	0
20	Forecast W-14	NF	NF	NF	NF	NF	0	11	10	10	10	10	0	0	13
21	Forecast W-15	NF	NF	NF	NF	NF	NF	NF	NF	NF	NF	NF	NF	0	11

Figure 9. DST of an Alpha FG

3.2 Detailed analysis

We studied the total FGs requisitions depicted in Figure 10. We did not use an upper and a lower limit (as proposed in step 2 of the analysis method) because the total FGs requisition per day was relatively steady for all days except the case where requisitions were null. It can be seen that requisitions in the right half of the graph are null. This is due to the fact that all requisitions after the last extraction week are unknown (the delivery dates after the last extraction week are announced by forecasts). During the end of the year, the plant was closed because of holidays and for that reason there was no requisition. In the left part of the graph, the requisitions are unknown because they were not recorded. The other days where requisitions are null represent no-working days.

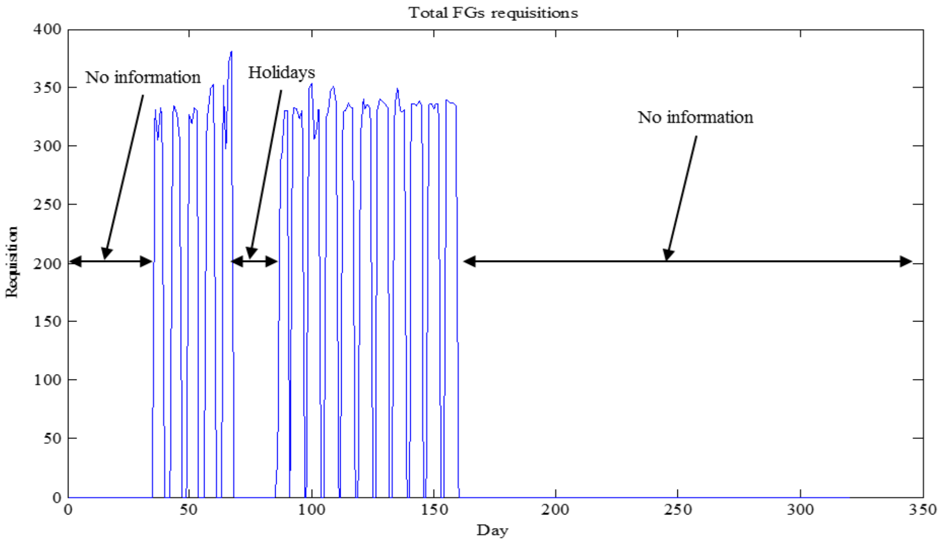


Figure 10. Total FGs requisitions-Case study

After removing all delivery dates with null total requisition, we find the graph below. The new sample size is 74 days and the average requisition is 332 FG/day (Figure 11). The DSTs of all FGs are updated (only the 74 days are kept). We correct forecasts using the conventional method given at the end of step 2 in the analysis method.

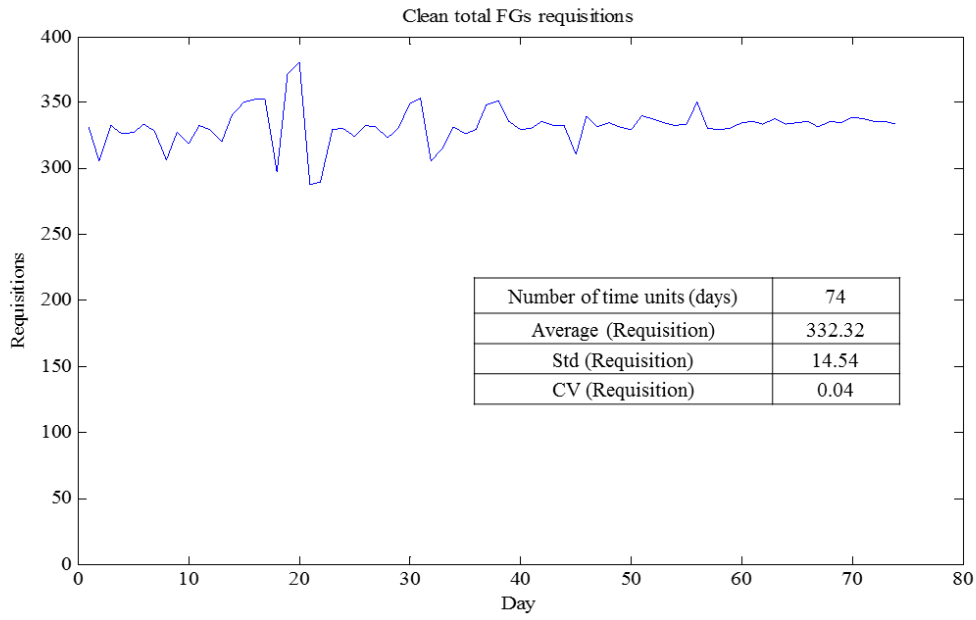


Figure 11. Clean total FGs requisition-Case study

Once the DSTs of FGs are cleaned, we analyzed the BOM. As it is shown in Table 4, there are 689 individual components, 236 of them are representative components. The DSTs of representative components are generated using an Excel VBA Macro. We represent graphically some components requisition patterns in Figure 12. It can be seen that there are different requisition behaviors: decreasing trend, increasing trend, low runner (slow mover), substitution…etc. We found that there are five components which have been replaced by others. We created five new components as described in the example of step 4. The number of studied components becomes now 231.

Table 4. Summary of the BOM

Number of FGs	264
Number of individual components (lowest level)	689
Number of representative components	236
Number of all components (including sub-assemblies)	1017

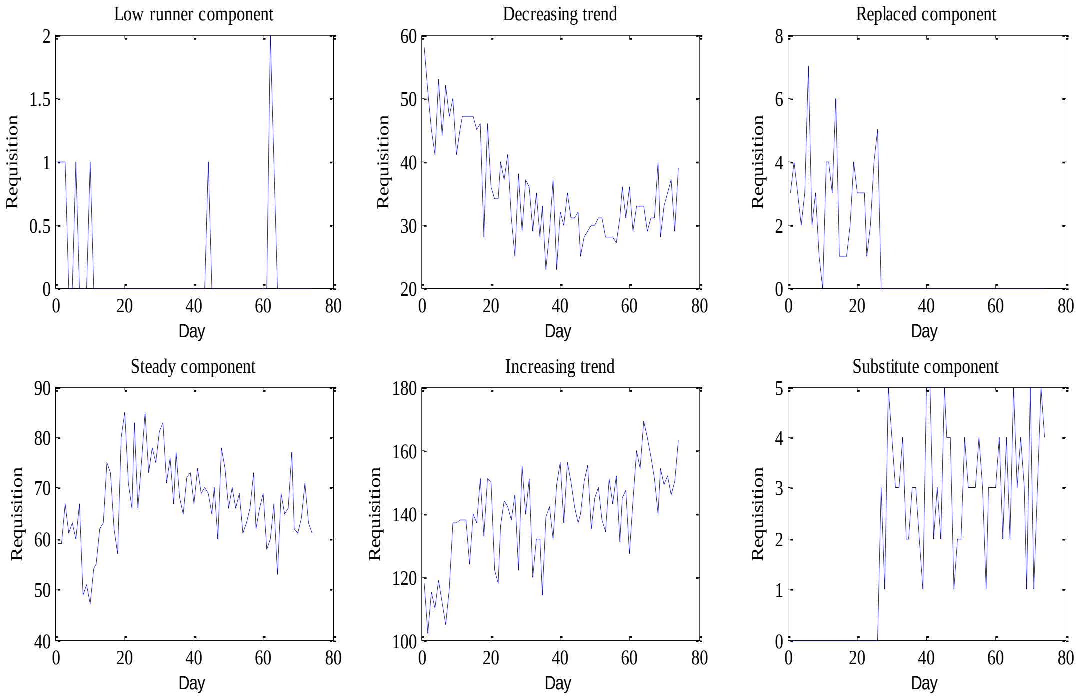


Figure 12. Some components requisition patterns

We removed the batch effect from the DST of each component (requisitions and forecasts are divided by the weighted average assembly coefficient associated with the component). After, we removed the trend from requisitions using the function "detrend" in Matlab as shown in the example below.

Example: Removing trend from requisition

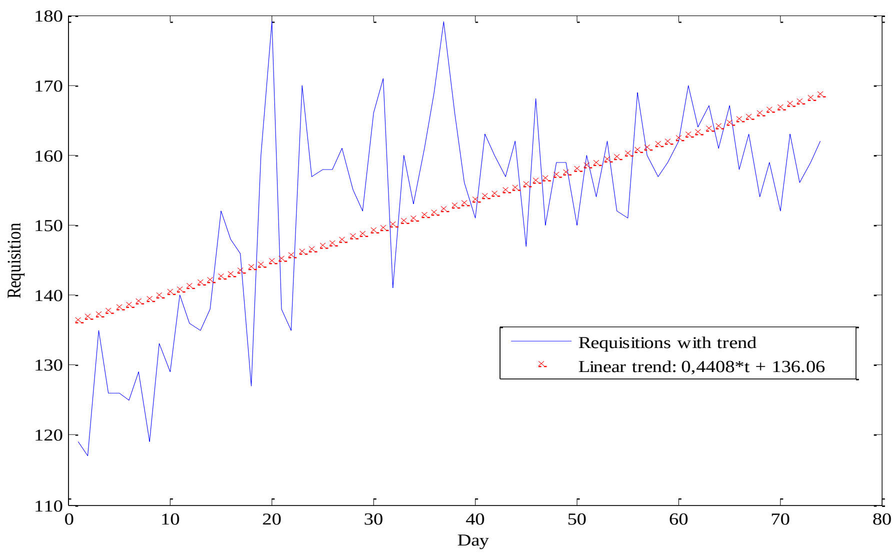


Figure 13. Component requisition (without batch effect) with a linear trend

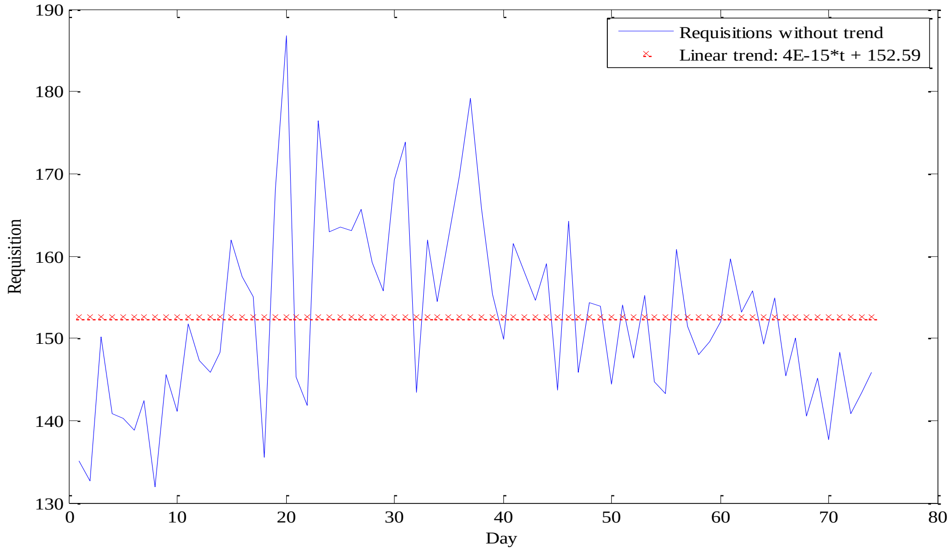


Figure 14. Component requisition (without batch effect) after removing the trend

We calculated the basic statistical indicators of requisition values (after removing the batch effect) over 74 periods. The average, Std and CV=$\frac{\text{Average}}{\text{Std}}$ are depicted in Figure 15 (components are ordered in decreasing order of average requisition). We draw attention to the fact that if a component is used by all FGs, then its average requisition (without batch effect) is equal to the average total FGs requisition.

There are 123 components which have less than 5% of the average total requisition (332 units/day), 48 components with a requisition between 5% and 15%, and 60 components with more than 15%.

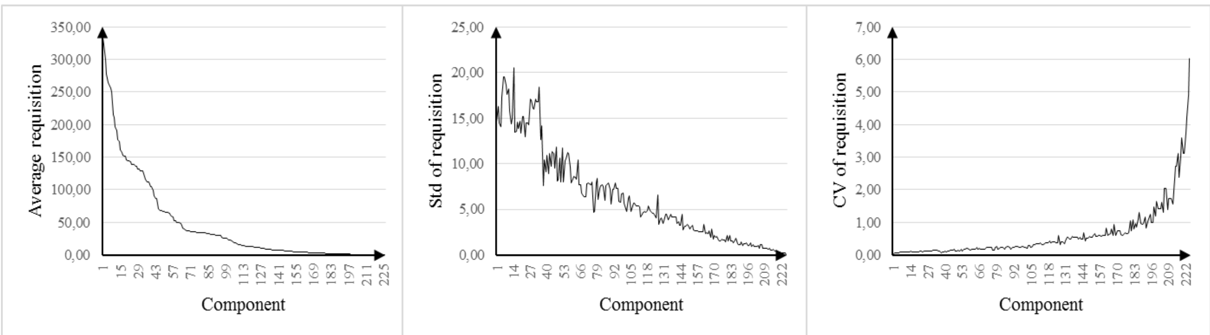


Figure 15. Variations of Average requisition, Std and CV (without batch effect)

Because the sample size is 74 (relatively small), we found that more than one probability distribution (Binomial, Poisson and Normal) can be accepted by the goodness-of-fit tests. Another test is used: we calculate the ratio $\frac{\text{Std of requisition}}{\sqrt{\text{Average requisition}}}$ to compare it to 1. We remind that in a Poisson process $\frac{\text{Std of requisition}}{\sqrt{\text{Mean requisition}}}$=1. Among the 74 values, the minimum and maximum of ratio $\frac{\text{Std of requisition}}{\sqrt{\text{Average requisition}}}$ are 0.69 and 1.9 respectively. There are 31 components with a ratio greater than 1.30. The average ratio for all components is 1.11 with a coefficient of variation equal to 0.17. We can conclude that component requisition (without batch effect) follows a Poisson probability distribution. Component requisition distribution is therefore a compound Poisson with a batch size equal to the weighted average assembly coefficient.

Hence, the probability distribution of a component *x* requisition can be expressed by:

- $P(R_x = d) \cong P\left(\frac{R_x}{a_x} = \frac{d}{a_x}\right) = \frac{e^{-\beta_x} * \beta_x^d}{d!}$ for each non-negative integer d.

where β_x is the mean requisition (expressed in terms of batches of size a_x where a_x is the weighted average assembly coefficient of component *x*). β_x can be estimated using forecasts.

3.3 Forecast accuracy analysis and estimation of the value of mean requisition $\boldsymbol{\beta_x}$

Figures 16 and 17 depict the forecast accuracy measures: Forecast quotient and Average absolute Error for forecasts made one week before delivery date and forecasts made three weeks before delivery date. It can be seen that forecast accuracy is good except for components with low requisition (these components are called “Low Runners” in the studied plant). The forecast accuracy decreases from week 1 to week 3 (before delivery date). This result is intuitive because the more forecasts are far from the delivery date, the less accurate they are. Because the forecast accuracy is good, forecasts can be used to estimate the mean requisition: $\beta_x = \frac{\text{Average forecast}}{\text{Forecast quotient}} \cong$ Average forecast.

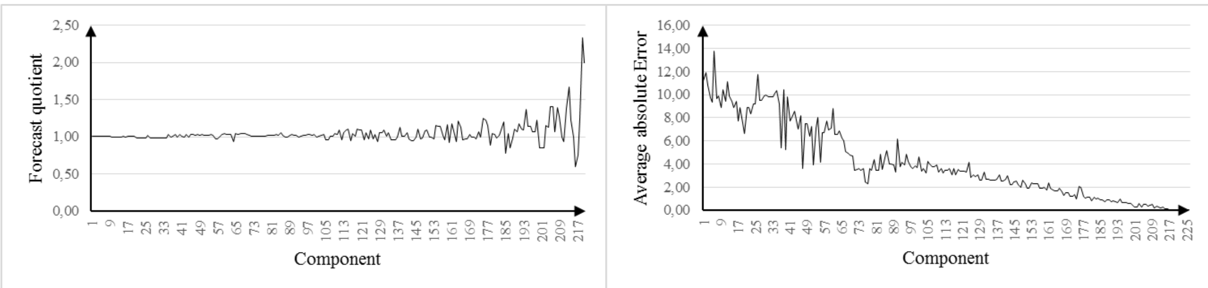
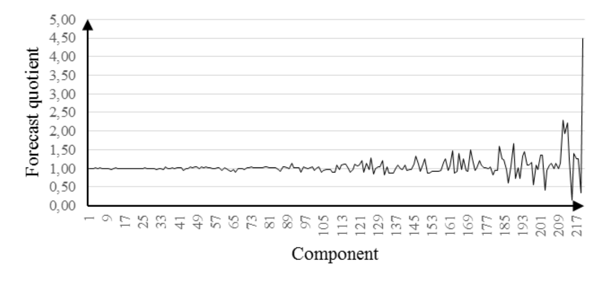


Figure 16. Forecasts (without batch effect) made one week before delivery date (W-1)



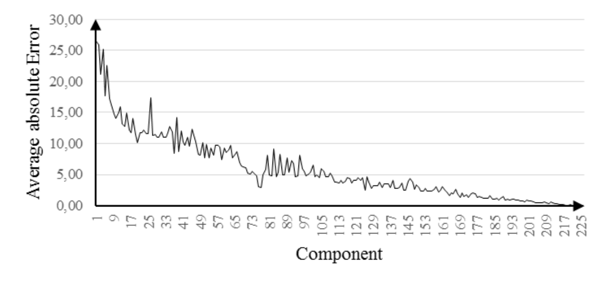


Figure 17. Forecasts (without batch effect) made three weeks before delivery date (W-3)

3.4 Interpretation of results

The probability distribution of component requisition this Alpha plant can be interpreted in this way:

The sum of FGs requisitions per day is relatively constant (within this interval [288,381], average=332 and CV=0.04). If we say that each FG, has a "percentage of use" equal to the average requisition of this FG divided by the average sum of FGs requisitions, then the requisition of each FG can be viewed as a binomial process (see (Feller, 1968)). This analogy is explained in the Table below:

Table 0.1. Analogy between a binomial process and a FG requisition

	Binomial process	FG requisition
Experiments	Number of Bernoulli trials	Total requisitions (332 units/day)
Probability of success	Constant probability of success	FG use percentage $\left(= \frac{\text{Average requisition of FG}}{\text{Average sum of FGs requisitions}}\right)$
Result (random variable)	Number of successes	FG requisition per day

Because FGs have a small use percentage (interval=[0%,13%], average=0.6% and CV=2.52), the binomial process can be approximated by a Poisson process. Hence, we deduce that a FG requisition can be viewed as a Poisson arrival process. This result can be verified statistically: the average ratio $\frac{Std}{\sqrt{Average}}$ for all FGs is 1.19 with a CV equal to 0.20.

On the other hand, each component *x* is used by a set of FGs. The arrival of each unit of FG that uses component *x* induces the arrival of a_x units of this component on average. The sum of arrivals of FGs that use component *x* follows a Poisson process (the sum of Poisson processes is a Poisson process). As a result, the arrival process of component *x* can be viewed as a compound Poisson with a batch size equal to a_x (a_x is the weighted average assembly coefficient).

We think that the results of this case study stay true for each Alpha plant that has a relatively steady total FGs requisition.

4 Conclusion

In this report, we proposed a method to analyze components requisitions and forecasts in an ATO system. The proposed method has as inputs the past FGs requisitions and forecasts, and the BOM. It enables to determine the probability distribution of component requisition and the parameters of this distribution.

The proposed method has shown its efficiency for many reasons: it allows to clean quickly the input data (cleaning the total FGs requisitions instead of cleaning each FG requisition), it is easily implementable in computer calculation software (for example, Matlab and Excel VBA were used in the case study), and it facilitates visualization and manipulation of data (Matrix format of DST).

The proposed method was applied to the case of an Alpha plant with 264 FGs and 689 components. We found that for each component the probability distribution of the requisition can be approximated by a compound Poisson distribution. The forecast accuracy is shown to be very good, and hence, it can be used to estimate the parameter of this distribution (mean requisition). We think that these results probably hold true for all Alpha plants that have a relatively steady total FGs requisition.

Our work has some limits: the sample size used in the case study was small (74 days) and larger sample sizes would provide more reliable results. Some would also say that the test of the ratio $\frac{Std}{\sqrt{Average}}$ is not sufficient to prove that Poisson distribution is a good fit. The choice of this test can be justified by two facts:

- If Poisson distribution is a good fit then at least the ratio $\frac{Std}{\sqrt{Average}} = 1$.
- Intuitively, the arrival process of FGs is a Poisson process as explained in subsection 3.4.

An important point concerning the use of this method and analyzing requisition is related to the extraction of useful input data. As a perspective, we propose to integrate the DST of FGs in the ERP system in order to be filled in a continuous way. The continuously filled DSTs can be also improved by adding comments to each filled day (column), hence data analyzers can interpret more quickly incoherent information.

It would also be of interest to generate forecasts of requisition using a forecasting method (e.g., exponential smoothing) in order to improve the estimation of probability distribution parameters especially for components with low requisition.

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