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Towards real time computation of 3D magnetic field in parametrized Polyhelix magnets using a reduced basis Biot-Savart model

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Laboratoire National des Champs Magnétiques Intenses (LNCMI)

Magnetic Field (T: Tesla)

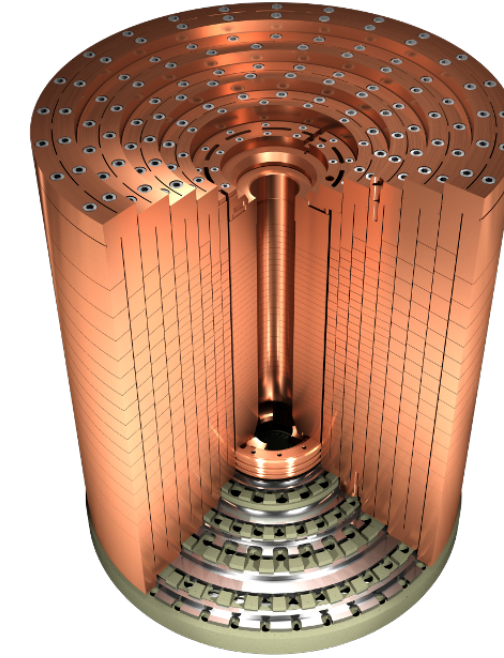
Human brain: 10^{-12} T
Earth: $5 \cdot 10^{-5}$ T
MRI: 1T
Superconductor: 24T

Static Magnetic Field

Grenoble: 37T, soon 43T

Pulsed Magnetic Field

Toulouse: 99T



Objectives

- Design a Reduced Basis method for Biot & Savart law, allowing to change experimental (conductivity, heat transfer coefficient,...) or geometric (helices shape, radius,...) parameters.
- Compare Finite Element and Reduced Basis (RB) results, and illustrate the use of the method with different applications.

Non-linear Electro-Thermo-Magnetic

Electric

$$\begin{cases} -\nabla \cdot \sigma(T) \nabla V = 0 & \text{in } \Omega_C \\ -\sigma(T) \nabla V \cdot \mathbf{n} = 0 & \text{on } \Gamma_N \\ V = V_D & \text{on } \Gamma_I \\ V = 0 & \text{on } \Gamma_O \end{cases}$$

Thermic

$$\begin{cases} -\nabla \cdot \kappa(T) \nabla T = \sigma(T) \nabla V \cdot \nabla V & \text{in } \Omega_T \\ -\kappa(T) \nabla T \cdot \mathbf{n} = h(T - T_w) & \text{on } \Gamma_C \\ -\kappa(T) \nabla T \cdot \mathbf{n} = 0 & \text{on } \Gamma_E \end{cases}$$

Magnetostatic

Biot & Savart law:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_{\Omega_C} \frac{-\sigma(T) \nabla V(\mathbf{r}) \times (\mathbf{x} - \mathbf{r})}{|\mathbf{x} - \mathbf{r}|^3} d\mathbf{r} \quad \forall \mathbf{x} \in \Omega_M$$

V electric potential

T temperature

$\sigma(T)/\kappa(T)$ electric/thermal conductivity

h exchange coefficient

T_w water's temperature

Reduced Basis Method

Reduced Basis Method (RBM)

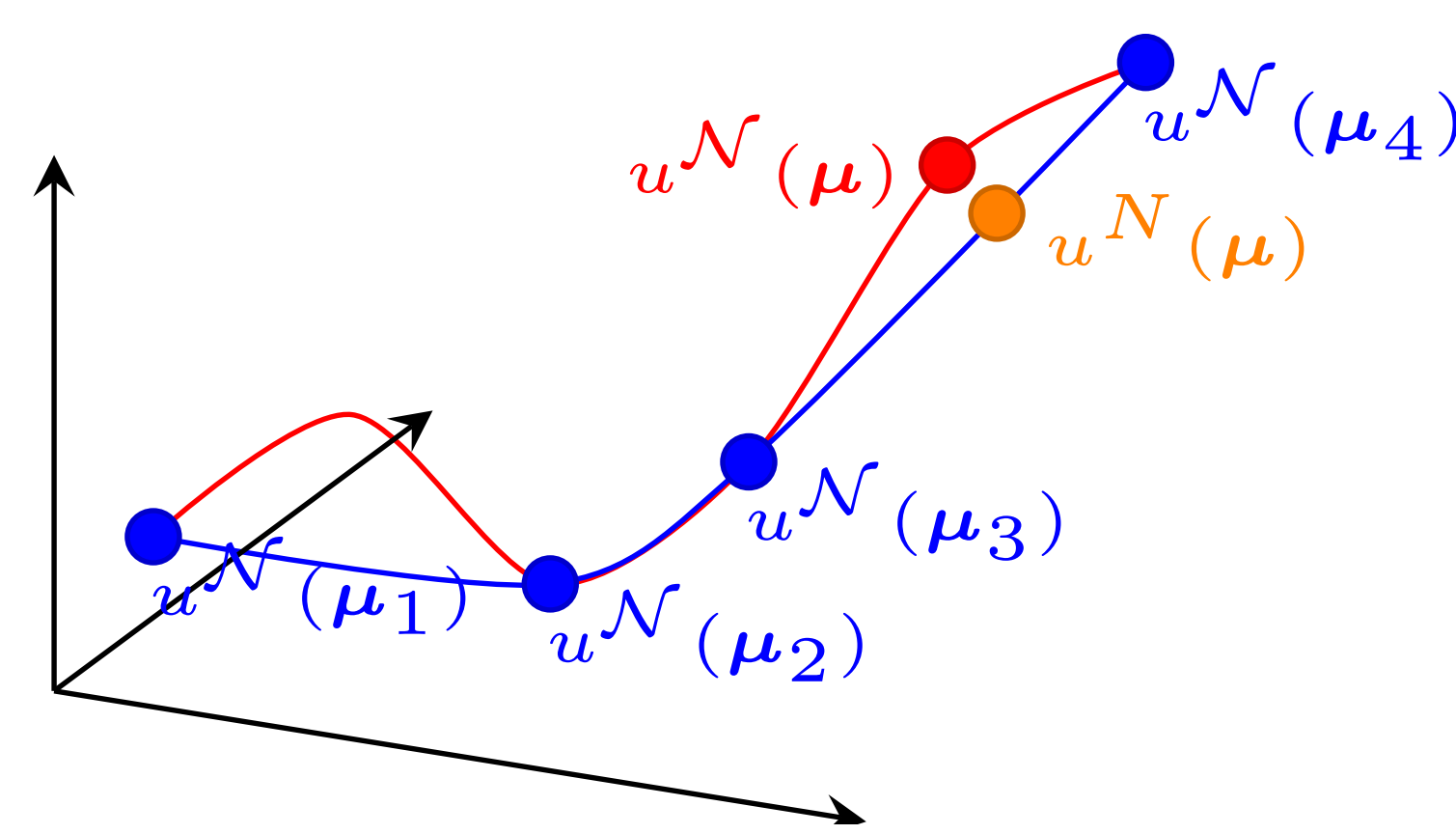
- Use a more appropriate basis, made of solutions
- Use **affine decomposition** to have an efficient computation

$$a(u, v, \boldsymbol{\mu}) = \sum_{q=0}^{Q^a} \Theta_q^a(\boldsymbol{\mu}) a_q(u, v)$$

• Offline phase:

- Using POD or greedy algorithm, find the best parameters to approach the problem
- Build the basis, and pre-compute matrices and vectors

- **Online phase:** solve a reduced problem for a parameter, real-time



Empirical Interpolation Method (EIM)

In case of non-affine or non-linear problem, need to solve a large number of problem

Simultaneous EIM RBM (SER)

Reduce the number of solution to compute during the offline phase

Biot & Savart Reduced Basis (BSRB)

Reduced Basis for V and EIM for $\sigma(T)$:

$$V(\mathbf{r}; \boldsymbol{\mu}) = \sum_{n=1}^N V_n(\boldsymbol{\mu}) \xi_n^V(\mathbf{r}) \quad \sigma(\mathbf{r}; \boldsymbol{\mu}, T) = \sum_{m=1}^{M_\sigma} \beta_m^\sigma(\boldsymbol{\mu}, T) q_m^\sigma(\mathbf{r})$$

Reduced Basis for \mathbf{B} :

$$\mathbf{B}(\mathbf{x}; \boldsymbol{\mu}) = \sum_{n=1}^N \sum_{m=1}^{M_\sigma} V_n(\boldsymbol{\mu}) \beta_m^\sigma(\boldsymbol{\mu}, T) \frac{\mu_0}{4\pi} \int_{\Omega_C} \frac{-q_m^\sigma(\mathbf{r}) \nabla \xi_n^V(\mathbf{r}) \times (\mathbf{x} - \mathbf{r})}{|\mathbf{x} - \mathbf{r}|^3} d\mathbf{r}$$

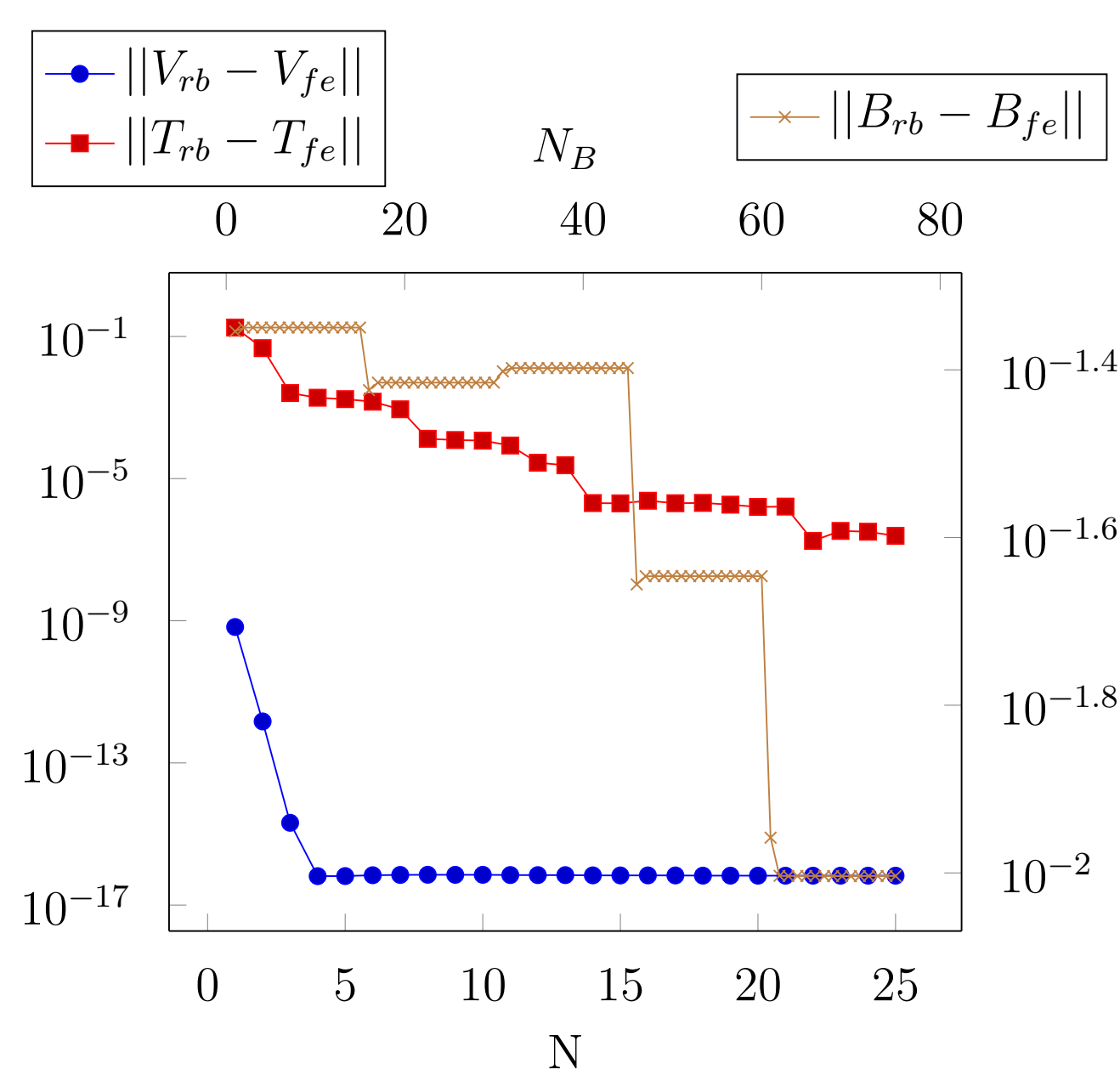
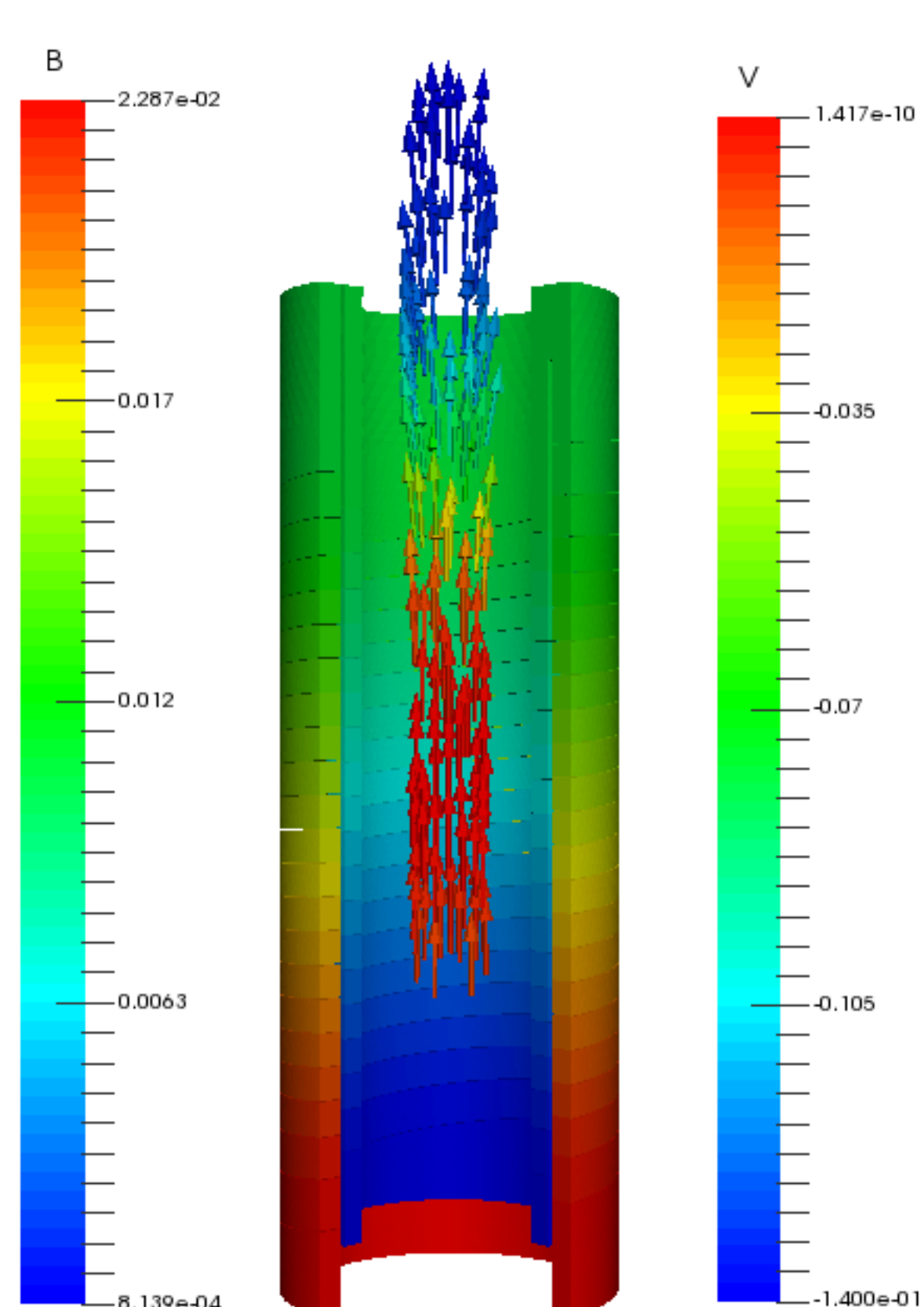
Geometric parameters:

Ω_C and the distance $|\mathbf{x} - \mathbf{r}|$ depends on the parameters.

⇒ Need to use the discrete version of EIM to interpolate the magnetic field:

$$\mathbf{B}(\mathbf{x}; \boldsymbol{\mu}) \approx \sum_{m=1}^{M_B} \beta_m^B(\boldsymbol{\mu}) \mathbf{q}_m^B(\mathbf{x})$$

Comparison between Finite Element and Reduced Basis

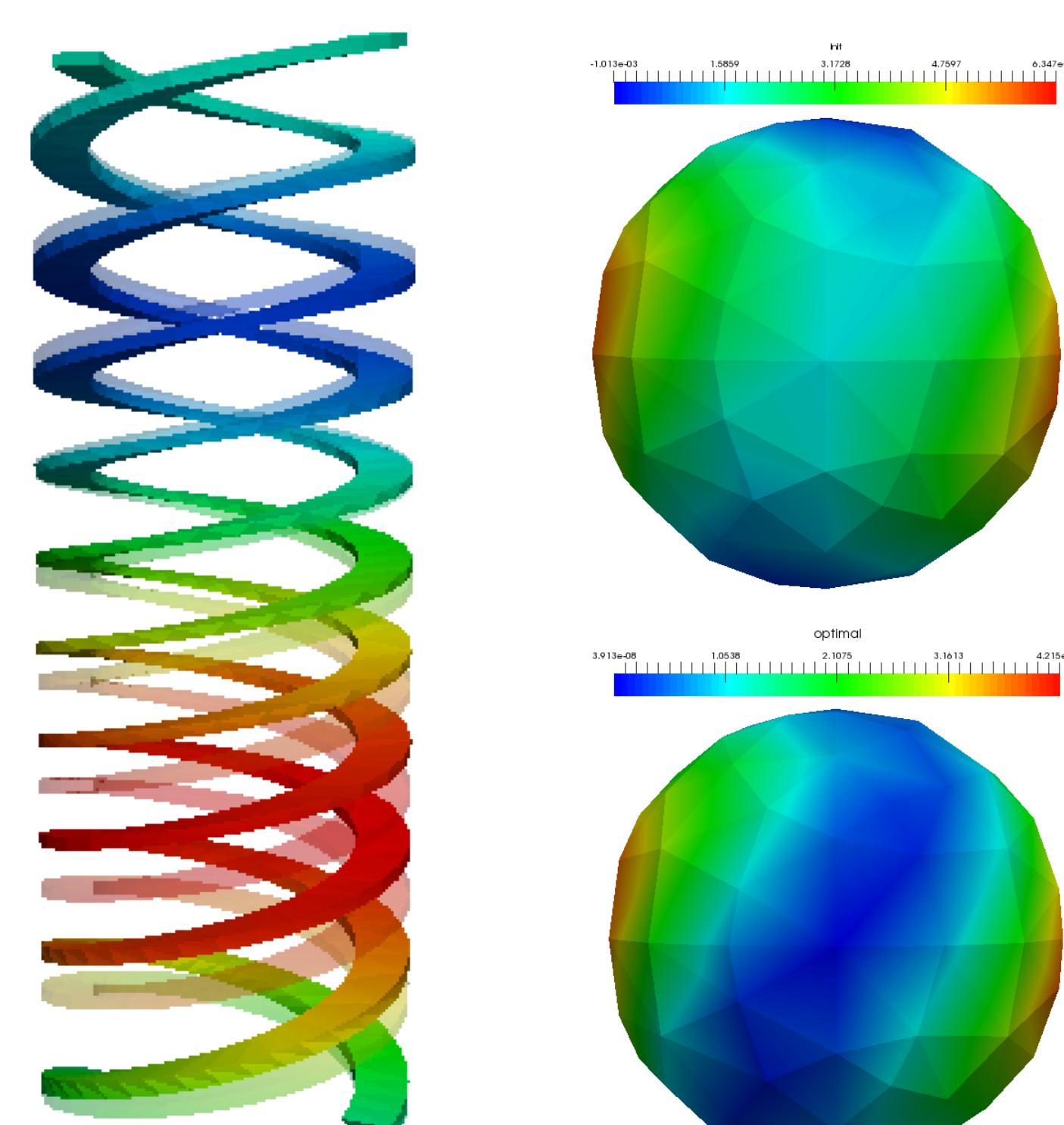


Parameters: $\sigma_0, k_0, L, \alpha, h, T_w, V_D$

Computing time:

	dofs	FE	RB
V	1 350 000	45s	0.05s
B	3000	110s	0.0005s

Optimization of the homogeneity by controlling the shape of the helices



Shape of the cuts of the helices

before/after

initial and optimized homogeneity in a ball at the center of the helices

Ingredients:

- ϕ : geometric transformation on the helices (Ω_C)

$$\phi = \begin{pmatrix} x \cos(\alpha) + y \sin(\alpha) \\ -x \sin(\alpha) + y \cos(\alpha) \\ z \end{pmatrix}$$

- α : Bezier curve which control points p_k are the parameters of the optimization.

$$\alpha(t) = \sum_{k=0}^n C_n^k t^k (1-t)^{n-k} p_k \quad (1)$$

	init	optimal	gain
$\max(B_z)$	0.41337	0.4051	-2%
$\min(B_z)$	0.41076	0.4034	-1.79%
$\frac{\max(B_z)}{\min(B_z)} - 1$	5381.9	3544.9	34%

Conclusions

- A Finite Element, full 3D, parallel, multi-physics model, coupling thermoelectric, magneto static (Maxwell and Biot & Savart) and mechanic.
- A Reduced Basis method for Biot & Savart allowing a real time computation for the magnetic field in the region of interest.
- Use of the Discrete Empirical Interpolation Method (DEIM) to deal with geometric parameters, giving the opportunity to do geometrical optimization.
- Use of the Simultaneous EIM and RB (SER) to deal with non linearities and to accelerate the DEIM offline phase.
- In the future, accelerate the Biot & Savart reduced basis construction.
- Integrate the RB models into a tool for magnet designers without numerical simulation knowledge to visualize the 3D magnetic field maps in real time while changing parameters.