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SPOT WELD FATIGUE LIFE PREDICTION – NEW CRITERIA

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Abstract

This paper presents new criteria for fatigue life prediction of spot weld. It starts with the presentation of the database used for this study. Then, a sensitivity analysis is performed to determine the influential variable on the lifetime of spot weld. Two criteria for spot weld life prediction, based respectively on mechanical damage and fracture mechanics, are developed through a nonlinear multiple regression method of optimization. These criteria considerably improve the correlation between numerical and test results. Finally, an application of this study is led on a fatigue test of a full body-in-white with about 4000 spot welds.

1. Introduction

Spot-welding is the main technique of joining sheet metal in the automotive industry. In general, an automobile is assembled by about 4000 Spot Welds (SW). The fatigue failure of SW is critical for the safety of the vehicle structure and thus for the brand management of the car company. The number of both vehicle prototypes and design project time has decreased significantly the last decade to reduce total project costs and time. Therefore, the numerical life estimation of the SW must be accurate since designers require reliable information about the SW fatigue lifetime early in the design phase. SW fatigue is sensitive to numerous parameters [1]. In order to perform their influence, a sensitivity study is done [2]. An important database of SW fatigue tests is analyzed in order to try to cover these various parameters. This study reveals that constraints applied to the SW and the SW geometric parameters, such as specimen thickness and weld nugget diameter, have a high impact on the fatigue life. These factors must be taken into account in a SW fatigue life prediction criterion. First, SW fatigue life criteria based on mechanical damage [3] are analyzed and compared to the SW fatigue tests. The correlation between the fatigue test data and numerical results obtained with those criteria is not sufficient. A first new criterion depending on geometrical SW parameters is developed and the correlation with fatigue tests is significantly improved. Second, fracture mechanics approach for SW fatigue lifetime is investigated with four criteria (Lee [4], Pook [5], Wang [6] and Zhang [7]). The fatigue life estimation error with this approach is decreased compared to mechanical damage approach results. A second new criterion, based on stress intensity factor, is developed through a nonlinear multiple regression method of optimization [8]. This criterion considerably improves the correlation between numerical and test results. Finally, an application of this study is led on a fatigue test of a full body-in-white with about 4000 SW. A comparison of fatigue test data and numerical SW life prediction, obtained with all the criteria studied, is performed.

2. Database

A database of SW fatigue tests is used for this study. The welding process, to join the sheet metal of the specimen tests, is the same that used in Renault® vehicle production lines. The SW lifetime is determined experimentally, mostly between fifty thousand and one million cycles. Four steels, with low and high yield limit (from 178 to 680 MPa) and ultimate tensile strength (from 290 to 850MPa), used on body-in-white, make up the specimen tests for this study. For a given one, the material of the sheet metal assembled is identical and all the SW failure modes are nugget pullout (*Fig. 3-1*). The four different types of load conditions are illustrated (*Fig. 2-1*):

- i. Tensile loading condition
- ii. Shear loading condition
- iii. Mixture of tensile and shear loading conditions
- iv. Peel loading condition

Tensile, shear, mixture of tensile and shear conditions are tested with double U (2U) specimen tests (*Fig. 2-1*). One SW joins the two sheet metals. For the peel loading conditions, H-peel specimen test type is used and ten SW join the three sheet metals together.

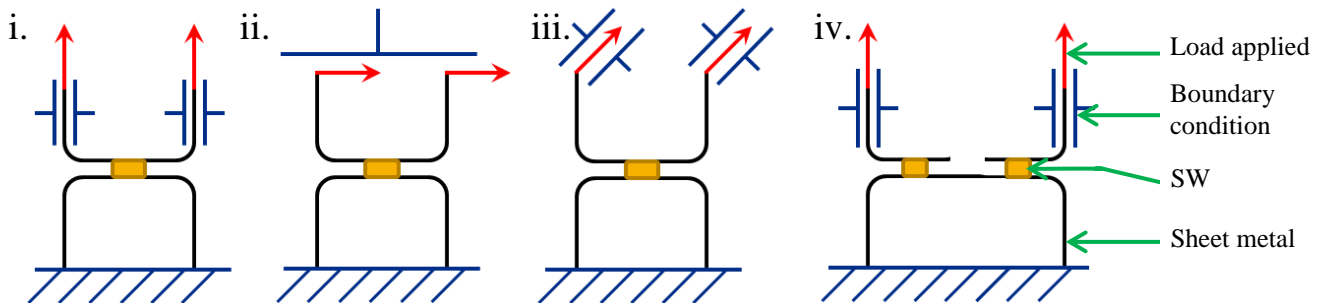


Fig. 2-1 : Diagrams of the four types of solicitations and their associated specimen tests boundary conditions

3. Study of influential variables on the spot weld lifetime

The objective of this study is to determine the influential variables of the database on SW lifetime. In order to consider the influence of SW parameters on tensile strength and failure simulation, Dancette et al [9] led a statistical study on a large tensile strength database. Dancette separated his analysis in two categories depending on the SW failure modes, the nugget pullout and interfacial fracture modes (*Fig. 3-1*). Depending on the load solicitation, Dancette found a relation between the diameter of the SW, the thickness of the specimen tests and the failure strength. Within the study involving several car manufacturers [1], R. Mohan Iyengar et al studied results in many endurance peel and shear tests with many grades of steel (low to high ultimate tensile strength). According to their research, the effects of the type of material and the mean stress is negligible compared to the geometric variables such as thickness and diameter of the SW (*Fig. 3-1*).

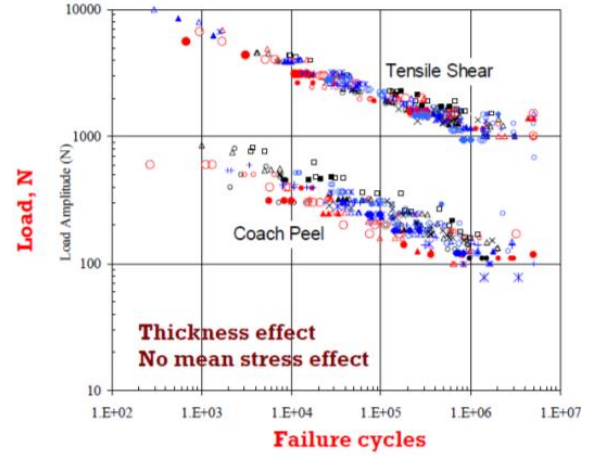
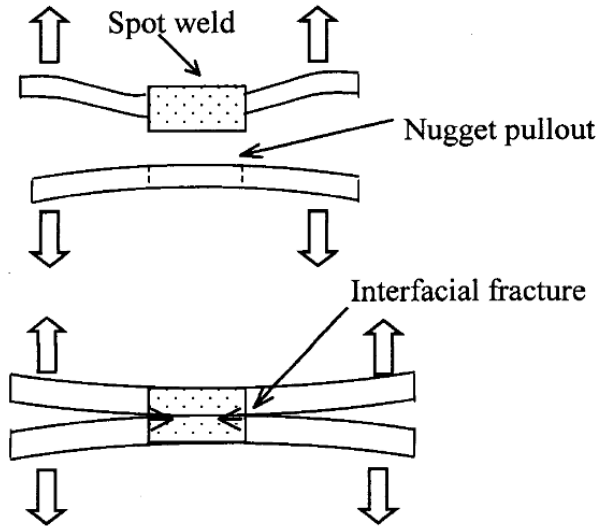


Fig. 3-1: Two failure modes of SW; nugget pullout and interfacial fracture [10](on the left), test results for coach peel and tensile shear loadings [1] (on the right)

In order to determine the variables that influence the SW fatigue life, a sensitivity analysis (SA) has been established. A sensitivity analysis is used to analyze, for mathematical or empirical model, the influence of the variability of the model input factors on the output variable, the response [2]. For this model, which combined a set of experimental input variables X_i , an experimental response Y from a function (f) is considered [2] :

$$\begin{aligned} f : \mathbb{R}^p &\rightarrow \mathbb{R} \\ X &\rightarrow Y = f(X) \end{aligned} \quad (3-1)$$

The set of input variables $X = \{X_1, \dots, X_p\}$ is made of experimental factors for the response Y . The objectives of a sensitivity analysis are to:

- determine the relationship between the factors and the response
- reduce the response variance if a model to predict the response is developed

In our analysis, the response Y is the number of cycles to failure and the available variables in our database are:

- a. the diameter of the SW and the sheet metal thickness ($\{dia, th\}$)
- b. the load amplitude applied to the specimen test (dF)
- c. the material of the sheet metals (mat)
- d. the loading ratio between the minimum and maximum applied load (Ra)

The loading ratio is always the same, equal to 0.1, for every test results. Thus, this variable is not taken into account in our analysis. Moreover, according to Mohan Iyengar et al [1], the load ratio is negligible compared to the load amplitude for endurance tests. So three factors, SW diameter and sheet metal thickness, load amplitude applied to the specimen tests, and the structure material, are considered in our analysis.

The Sobol method [11] is used to analyze the influence of the variable on the SW lifetime. This method determines the influence of the variable X_i on the variance of the response. So, the method analyses the variance of the response for a constant value (x_i) of the variable $V(Y | X_i = x_i)$. The expected value of all the possible combinations $V(Y | X_i = x_i)$ is taken into account. As a consequence, higher is the variance of $|X_i|$, higher is $E[V(Y | X_i)]$ also. The global variance is equal to:

$$V(Y) = E[V(Y | X_i)] + V(E[Y | X_i]) \quad (3-2)$$

The Sobol method uses $V(E[Y | X_i])$, via Eq. (3-3). This value is normalized in order to obtain the first order Sobol index S_i of the variable X_i :

$$S_i = \frac{V(E[Y | X_i])}{V(Y)} \quad (3-3)$$

The fatigue database of SW specimen tests is used to calculate the Sobol first order indexes. Index is calculated for the three variables ($\{dia, th\}, dF, mat$) and the three loading conditions; tensile, shear and peel loading conditions. Thus, three Sobol indexes per variable are calculated (Fig. 3-2).

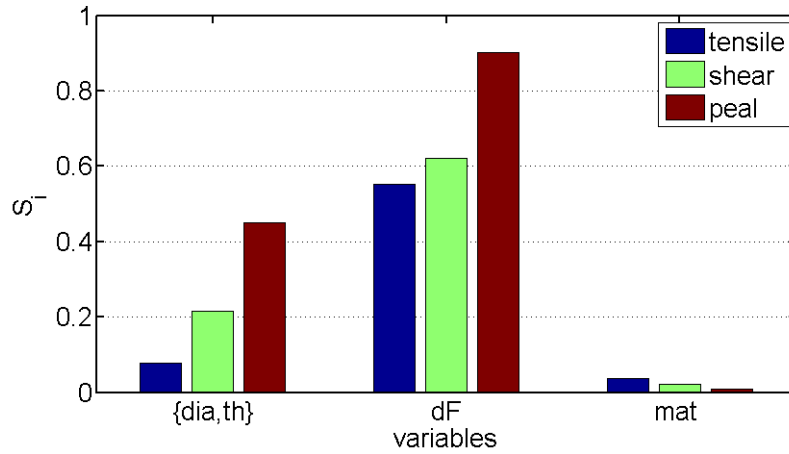


Fig. 3-2 : Sobol first order indexes (S_i) applied to the database for the three variables and loading conditions

Thanks to this sensitivity analysis, several observations can be deduced. First, the sheet metal thickness and SW diameter, $\{dia, th\}$ have a varying influence depending on the loading conditions. The Sobol index is low for tensile solicitations, but higher for the shear and peel loading conditions. Second, the load amplitude applied to the specimen test has a significant influence on the SW lifetime, whatever the load solicitation and with a Sobol first order index superior to 0.5 for every loading condition. Third, Sobol first order index of the material, whatever the loading conditions, is low, below 0.03. Therefore, for any SW fatigue life prediction criterion, it appears essential to take into account the diameter of the SW, sheet metal thickness, and the load amplitude applied to the specimen test. Compared to the two other variables, the sheet metal material can be neglected.

4. Spot weld fatigue life prediction

Two different approaches are studied for the SW fatigue life prediction; the mechanical damage and fracture mechanic approaches. For the two approaches, SW fatigue lifetime is calculated respectively from cyclic strain range and stress intensity factor.

4.1 Spot weld fatigue life prediction with cyclic strain range

First, the SW life prediction calculation method with cyclic strain range is presented. Then, this method is applied on the database presented in § 2. The calculated SW life with this method, N_{ca} , is compared to the experimental SW life, N_{ex} . A correlation error coefficient, R_N , is determined to compare the numerical and experimental results, with L_{ex} number of test results.

$$R_N = \sum_{j=1}^{L_{ex}} \frac{|N_{ca}^j - N_{ex}^j|}{\min(N_{ca}^j, N_{ex}^j)} \quad (4-1)$$

Because the correlation is not judged satisfying, a new method to predict the SW life, from cyclic strain range, is developed.

Spot weld fatigue life prediction calculation method

From the Finite Element Analysis (FEA), the strain range at the patch of the SW is extracted and strain history over time is calculated [3] (Fig. 4-1). The area enclosed by the nodes of the finite elements SW nugget is called patch [12]. The strain values are extracted from these patch elements. Normal strain is calculated for each finite elements of the patch every ten degrees normal plane of the element (critical plane method [13]). The normal cyclic strain ranges ($\Delta\epsilon$) obtained from these strain histories are calculated with a Rainflow counting method [14], for every patch elements' planes. One unique strain-life curve type (EN curve) relates the strain range to the SW lifetime (N_{ca}). Thus, a function g_1 , depending on the strain range, calculates the numerical lifetime.

$$N_{ca} = g_1(\Delta\epsilon) \quad (4-2)$$

The Miner's linear cumulative damage model [15] is used to calculate the SW life for every normal plane of the patch elements. The lowest lifetime calculated per SW is taken as the SW life prediction.

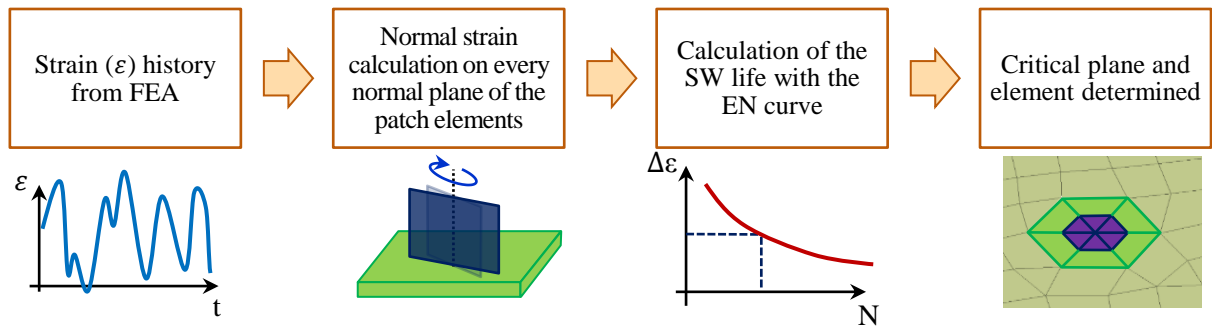


Fig. 4-1: SW fatigue life prediction with cyclic strain range method

Application of the SW fatigue life prediction calculation method on the specimen test database

For the four loading conditions of the database presented in § 2 (tensile, shear, mixt of tensile and shear, peel), the SW life prediction is calculated for all the specimen test results. Thus, the value of N_{ca} for each specimen test result is determined (Fig. 4-2). From them, R_N is calculated and is superior to 200%. The correlation between the experimental, N_{ex} , and the calculated SW lifetime, N_{ca} , obtained with the initial strain range criterion is not judged acceptable. In order to understand this poor correlation, the strain range ($\Delta\epsilon_{cri}$) of the most damaged patch element, in its critical plane, is extracted. For similar experimental SW lifetime values, the numerical critical strain ranges are distant, with a ratio between them higher than three for some cases. Therefore, determining a single strain-life, even for every loading condition, is not satisfactory.

New SW fatigue life prediction criterion

The function g_1 in Eq.(4-2) does not consider the diameter of the SW nugget and the sheet metal thickness. By examining the sensitivity analysis of the influential variables on the SW lifetime in § 3, these variables are influent on the SW experimental lifetime. As a consequence, a new function, g_2 , is introduced considering the SW diameter (dia) and the sheet metal thickness (th):

$$N_{ca} = g_2(\Delta\varepsilon, dia, th) \quad (4-3)$$

To obtain this new SW life prediction criterion, g_2 is developed through a nonlinear multiple regression method of optimization [8]. In this case, the correlation between experimental and test SW lifetime is improved. The correlation error coefficient, R_N , for every experimental test results is equal to 71%. In this case, the coefficient R_N is divided by 2.7 compared to the original strain range life prediction criterion (Fig. 4-2).

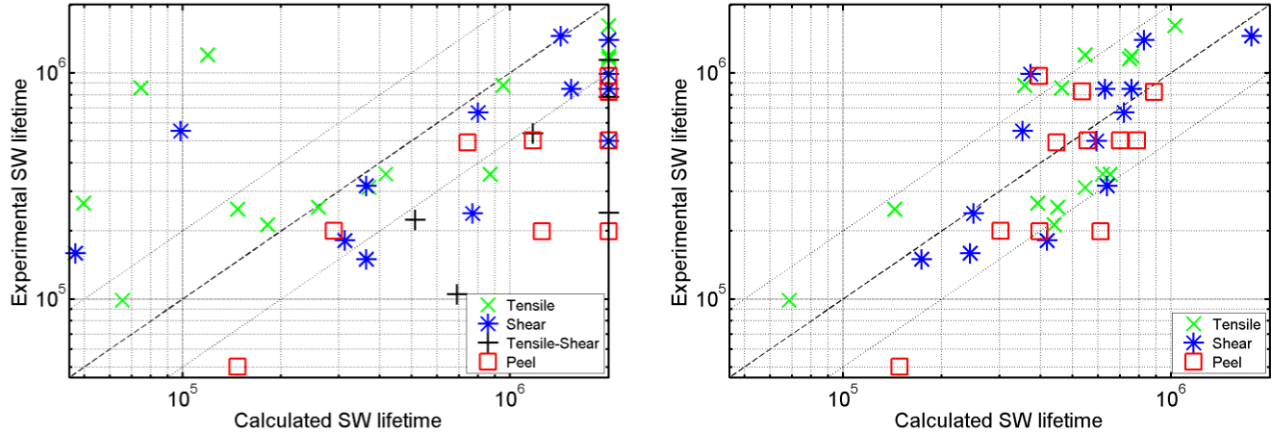


Fig. 4-2 : Experimental SW lifetime depending on the calculated SW lifetime with the initial cyclic strain range criterion (on the left) and the developed cyclic strain range criterion (on the right)

However, 24% of calculated lifetime via this new strain range criterion is more than twice higher or lower than the experimental lifetime, taken as reference. This percent is considered significant. It can be explained by the fact that the loading condition type is not considered in Eq. (4-3). Nevertheless, this new optimized criterion, taking into account the sheet metal thickness and SW diameter interaction, significantly improves the correlation between the experimental and calculated SW lifetime.

4.2 Spot weld fatigue life prediction with cyclic stress intensity factor range

First, the SW life prediction calculation method with cyclic stress intensity factor range is presented. Then, this method is applied on the database presented in § 2. The SW lifetime is calculated with four different methods; Lee [4], Pook [5], Wang [6] and Zhang [7]. Then, a new method is developed to predict the SW life, from cyclic stress intensity factor range.

Spot weld fatigue life prediction calculation method

From the finite element analysis, the strength tensor is obtained at the interface of the two sheet metal (Fig. 4-4). Thus, the force and moment histories are extracted for every SW. The stress intensity factors, in modes I and II, are calculated using the strength tensor. The stress intensity factors, for a crack at the SW interface, are represented in Fig. 4-3. k_I and k_{II} represent respectively the local mode I and II stress intensity factors, when the kinked crack length, d_{crack} , is greater than 0. When the kinked crack length is close to 0, the local stress intensity factors k_I and k_{II} can be as functions of the kink angle α and the global factors K_I and K_{II} for the main crack [16]. This assumption is considered for this study.

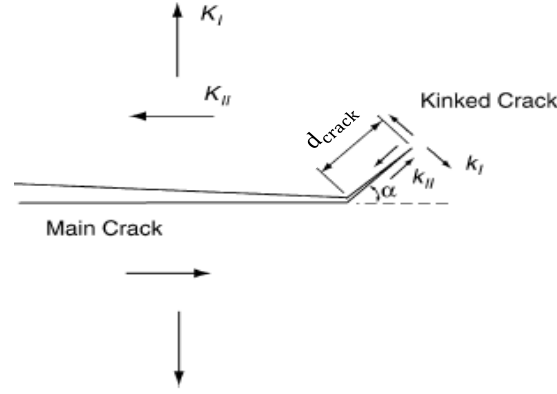


Fig. 4-3 : Main crack and a kinked crack with the kink length d_{crack} and the kink angle α [4]

An equivalent stress intensity factor, K_{eq} , resulting of a combination of the factors K_I and K_{II} , can be expressed [17] :

$$K_{eq} = \sqrt{K_I^2 + \beta K_{II}^2} \quad (4-4)$$

Equivalent stress intensity factor is calculated for each SW every ten degrees normal plane of the SW interface (critical plane method). The normal equivalent stress intensity ranges, ΔK_{eq} , are calculated from these K_{eq} histories with a Rainflow counting method. A stress intensity factor amplitude-life curve type ($\Delta K_{eq} - N$ curve), links ΔK_{eq} to the SW lifetime.

$$N_{ca} = g_3(\Delta K_{eq}) \quad (4-5)$$

Thus, the spot weld life (N_{ca}) is calculated.

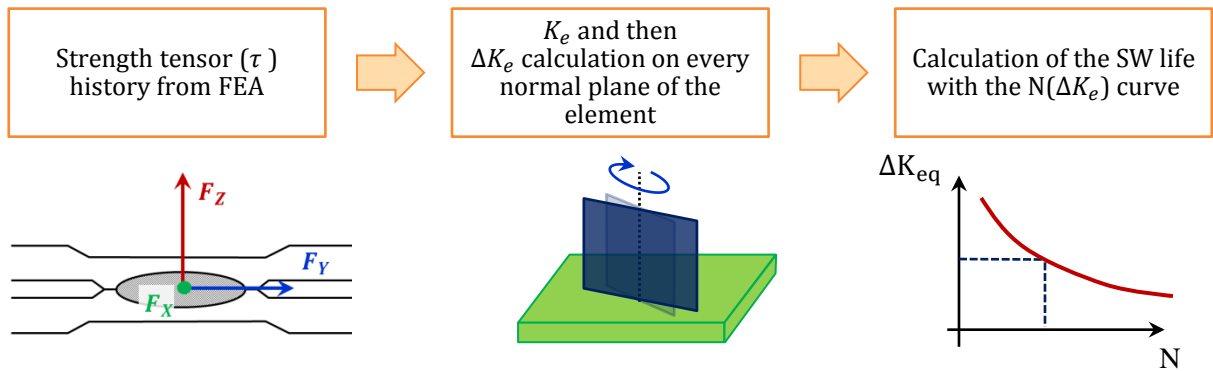


Fig. 4-4 : SW fatigue life prediction with cyclic stress intensity factor method

Selection of SW fatigue life prediction criteria

To determine the equations of the stress intensity factors $\{K_I, K_{II}\}$ (4-4) and of N_{ca} (4-5), different criteria are proposed in the literature. In this study, four stress intensity criteria to predict SW lifetime are selected and compared: Lee [4], Pook [5], Wang [6] and Zhang [7]. Lee attempts to determine his criterion to predict SW fatigue life, using different spot weld specimen tests, with a single parameter denoting the equivalent stress intensity factor. This K_{eq} depends of the strength tensor at the SW and

of the SW nugget diameter. The sheet metal thicknesses are not taken into account. Pook develops stress intensity factor criteria depending on the loading conditions and the specimen test geometries. He determines his criteria from circular plates, lap-shear and coach-peel specimen tests, and their fatigue test results corresponding. Pook criteria are complex to apply to body-in-white, since parameters depend on the specimen test geometry. Zhang develops his SW fatigue life criteria on analytical solutions for circular rigid inclusions in thin plates, with the J integral formulation.

Application of the SW fatigue life prediction calculation method on the specimen test database

The SW life, N_{ca} , is calculated with the four criteria described above. The database defined in § 2. is used to calculate the SW life for the three loading conditions: tensile, shear and peel. The SW life determined from the experimental and the calculated SW life prediction depends on the loading solicitation and the stress intensity factor. The R_N coefficient factor is higher than 200% for Wang criterion. For Lee criterion, the R_N coefficient factor is higher than 100%. This high correlation error coefficient for Lee criterion could be explained by the fact that the thickness of the sheet metal is not taken into account in the criterion formulae. In our database, for the same loading condition (specimen test type and SW diameter), some tests have different sheet metal thicknesses. In this case, the SW fatigue life results are different for the same load applied to the specimen test. As a consequence, the diameter is not the unique geometric variable to consider. Pook and Zhang have the best calculation-test SW lifetime correlation, with R_N error coefficients respectively equal to 77% and 66%. Considering R_N coefficient, Zhang criterion (Fig. 4-5), based on fracture mechanics, has a better calculation-test SW lifetime correlation than the optimized criterion, based on mechanical damage.

New SW fatigue life prediction criterion

Thirty percent of calculated lifetime via Zhang criterion is more than twice higher or lower than the experimental lifetime, taken as reference. This percent is considered too high. Therefore, a new criterion, based on stress intensity factor, is developed through a nonlinear multiple regression method of optimization. These criterion objectives are to be adapted to the database and so to decrease the correlation error between experimental and calculated SW lifetime. The new equivalent stress intensity factor (4-4) depends on the modes I and II stress intensity. K_I variables (4-6) are the sheet metal thickness, spot weld diameter, tension strength (F_{ten}), and the bending moments (M_{ben}). K_{II} variables (4-6) are shear strengths (F_{she}), the sheet metal thickness and the spot weld diameter.

$$K_I = K_{ten}(F_{ten}, dia, th) + K_{ben}(M_{ben}, dia, th) \quad (4-6)$$

$$K_{II} = K_{she}(F_{she}, dia, th)$$

With:

$$K_{ten} = F_{ten} \cdot f_{k1}(dia, th), K_{ben} = M_{ben} \cdot f_{k2}(dia, th) \\ K_{she} = F_{she} \cdot f_{k3}(dia, th) \quad (4-7)$$

$$N_{ca} = g_4(\Delta K_{eq}) = (A_2 \cdot \Delta K_{eq})^{1/A_1}$$

The functions (f_{k1}, f_{k2}, f_{k3}) are determined via nonlinear multiple regression method to be adapted to the database (4-7). Thus, a new equivalent stress intensity factor is developed, and the SW lifetime is predicted by the relation between the SW lifetime and the equivalent stress intensity factor range, depending on the constants $\{A_1, A_2\}$.

This new optimized criterion, based on fracture mechanics, considerably decreases the correlation error between numerical and test results; R_N coefficient is equal to 21%, divided at least by 3

compared to all the other criterion coefficients. Moreover, none calculated lifetime via this new criterion is more than twice higher or lower than the experimental lifetime, taken as reference (Fig. 4-5). The optimization method to obtain SW life prediction calculation equations, based on fracture mechanics adapted to the database, is effective and relevant.

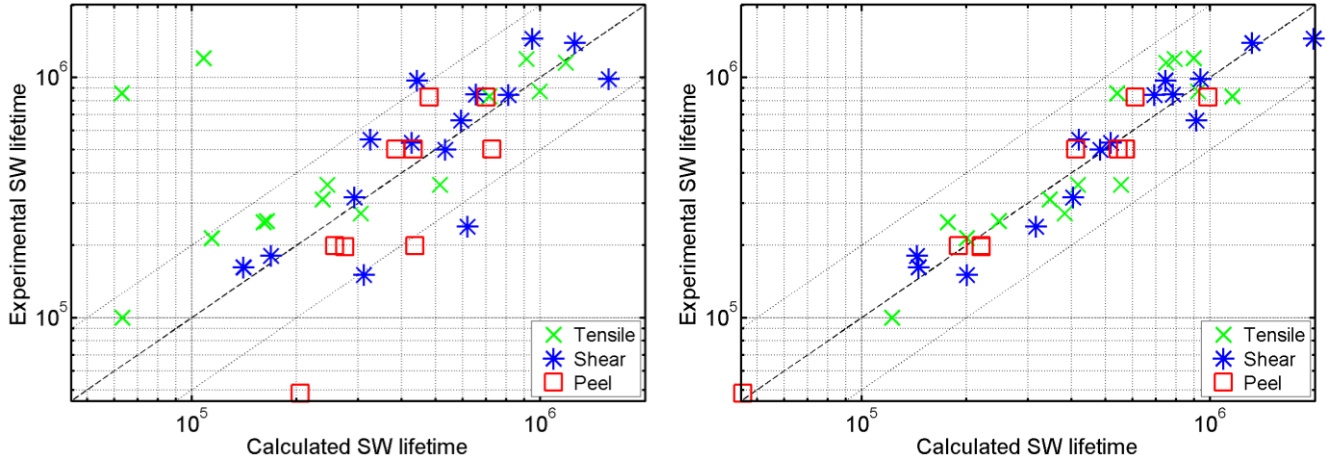


Fig. 4-5 : Some experimental SW lifetime depending on the calculated SW lifetime with the Zhang criterion (on the left) and the new stress intensity factor criterion (on the right)

5. Fatigue life calculation of a full body-in-white

An application of this study is led on a fatigue test of a full body-in-white. An automobile body is a complex structure consisting of about 450 structural parts that are mainly made of different steels. Spot-welding is the most common technique for connecting metal parts of an automotive body which is composed by about 4000 SW. At the fatigue test bench, the load applied to the body-in-white tends to represent a vehicle lifetime in extreme conditions for the fatigue of the vehicle. A specific body-in-white test bench results is studied, with numerous cracks appearing at SW during the fatigue test. Further to SW fatigue life prediction study, four life prediction criteria are tested on this full automotive vehicle: the first presented cyclic strain range criterion, developed cyclic strain range criterion, Zhang criterion and the developed stress intensity factor criterion. The calculation, a SW crack is considered if R_C coefficient is higher than 1:

$$R_C = C_S \cdot \frac{N_{bench}}{N_{ca}} \quad (5-1)$$

R_C coefficient compares predicted SW life, N_{ca} , to the test bench SW life, N_{bench} , with a security coefficient C_S superior to 1. Every cracks appeared are classified depending of their locations. Thus, SW crack locations are determined and R_C is calculated for every crack locations and every SW life prediction criteria. Then, a comparison of the number of cracks appearing at the durability bench and obtained by numerical calculation with these criteria is performed (Fig. 5-1).

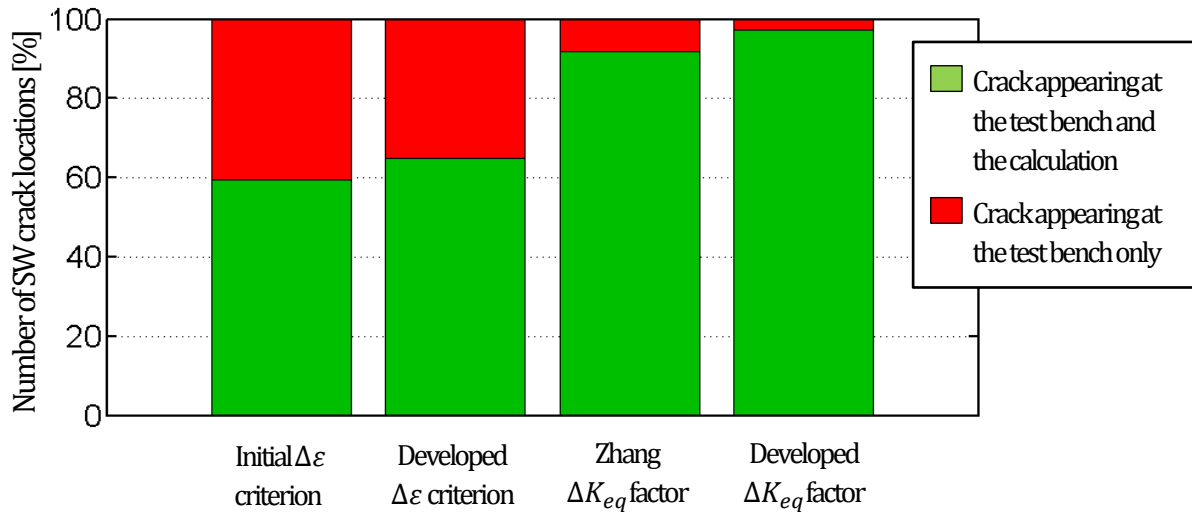


Fig. 5-1 : Number of SW crack locations appearing at the calculation and/or the test bench

Based on this fatigue test bench results, the initial strain range criterion has the worst calculation-test correlation. The percent of SW crack location common to the bench test and calculation is 59%, and 65% for the developed strain range criterion. The developed one has a better correlation, but the number of cracks revealed is considered not sufficient. The criteria based on fracture mechanics approach have a better test-calculation correlation. Calculation with Zhang allows to detect 92% of the SW cracks appearing at the fatigue test bench, and 97% with the developed stress intensity factor criterion. This proposed criterion, developed with the database presented in § 2, improves the correlation, with an increase of 38% for SW cracks appearing at the calculation and at the fatigue tests compared to the initial strain range criterion.

6. Conclusion

Spot-welding is the most frequently used technique for connecting metal parts of an automotive body. The fatigue failure of spot weld is critical for the safety, and needs to be predicted by means of numerical calculations. First, sensitive analysis is done to determine the influential variable on spot weld lifetime. Load amplitude applied to the spot weld and geometric parameters (sheet metal thicknesses, spot weld diameter) are significant factors on the variability of the spot weld lifetime, unlike sheet metal material. Second, two spot weld life prediction criteria, based on mechanical damage and fracture mechanics respectively, are developed through a nonlinear multiple regression method thanks to spot weld fatigue tests. These criteria considerably improves the correlation between numerical and test results compared to the initial criteria. Finally, an application of this study is led on a fatigue test of a full body-in-white with about 4000 spot welds. The spot weld life prediction criteria based on fracture mechanics predicts more cracks appearing at this test bench and the developed one has the best calculation-test correlation.

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