

TACS 4 BISE & SUETAN

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TACS 4 BISE & SUETAN
Birth of concepts

Olivier Maurice

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Prologue

BISE is for Bio Inspired System Engineering. TACS is for Tenfold Analysis of Complex Systems. The principle is to develop artificial systems being inspired by natural systems, with benefit of the TACS formalism. This book is a user's guide for BISE through TACS.

SUETAN is for Systems Under Extended Tensorial Analysis of Networks. It's a software that may be developed to help in system design.

This book is a set of proposals grouping various techniques in relation with system engineering under the tensorial analysis of networks formalism. With benefit of all previous works, the idea is to present tracks in order to develop a system engineering method able to integrate future technologies and in particular artificial intelligence and electronically increased materials. This little book must be seen as a unorganized set of ideas, the whole being tools helping for complex system modeling. System of systems, cyber-physical systems, etc., all are complex systems in general and belong to the wide thematic of systemic.

Chapitre 1

From classical to Bio Inspired System Engineering

Classical approach to make a system means to start from known parts (they were developed during a previous project) and assembling them in order to realize the customer needs. Natural systems think differently. In order to survive in given conditions and context, basic parts evolve to reach the best answer to this surviving objective.

The biggest difference comes from the capacity of natural systems to evolve but by remaining in adequacy with the rest of the world. With cyber-physical concept, this kind of capacity can be approached with artificial systems. That's the starting idea of BISE.

Coming form all my previous works, the tensorial analysis of networks has evolved to the tenfold analysis of complex systems. The objective and reason to live of the TACS formalism is to facilitate the estabishment of the relations between the real behaviours of systems and their mathematical expressions.

1.1 Natural processing

What is the most remarkable with natural systems is that they are all made based on the same elementary bricks. Humans have common elements with trees. The force of Nature is defined in the natural selection. Natural selection tests probabilistically new solutions and retains the best ones. In any cases, the basic collection of elements used to make the objects is relatively small. The number of functions can be listed :

— perception ;
— action ;
— reasoning and deciding.

Common objective is to survive. but depending on the environments, surviving means various things and various choices, various strategies.

Often in classical approaches, engineers think solution before to think innovation. It means that they are mainly inspired by previous experiences, of themselves or colleagues, starting from known solutions to reach an objective. Nature makes the same for part. She starts from a existing animal or plant to make them evolving and reaching a new version, perhaps better than the previous one, perhaps not. In human processes, we always try to reach immediately a better version. Doing that, we process by the same manner than "by fathers judgement" : we have a major chance to reproduce the previous system than to imagine a new one. There is without discussion some new domains where the approaches is radically new : IOT (internet of things) systems. In that case, people imagine new concepts that never existed before. But they principally concern software, not so hardware. If we want to follow the natural process, we may respect two steps :

1. Suppress older systems when new and better systems appear ;
2. test stochastic solutions without looking only to improve previous ones ;

You may say that this is not so far from what we do today ? After all, the human and economic world is similar to the natural one by the way that through competitiveness, it suppresses not optimized solutions. It's partly true if we consider the criterion of surviving as being purely the economical one. But a new criterion appears this last years, imagined from a long time but taking its place with force from a short time : the ecological criterion.

So today, the goal is not only to find the best economical solution but to find the best economical and ecological evolving. That's also the purpose of BISE.

1.2 An ADN for the BISE

ADN is first of all the key to manufacture any living been. It's a program that gives sense and objectives to an assembly of elementary parts. In an artificial system, elementary pieces may be some generic functions and the

software may allow to use them for a particular objective. Evolving is possible thanks to the internet network. The software can change in order to better use the hardware or for correcting some malfunctions. From this point of view, modern systems are similar to natural ones. The various molecules implied in the ADN can be associated with the various parts involved in the system. But the same physical subsystem can be activated differently thanks to the software. So, the ADN includes also the software component. And this software part gives the opportunity to the element to evolve.

Let's take the example of a sensor. We can dispose of a generic sensor, which needs a power supply and a bus connection. This sensor can measure various categories of pressure depending on a program embedded in the sensor microcomputer. But we understand that a first assumption is to dispose of a generic sensor. A sensor becomes an standard element of the collection of the possible elements callable by any ADN. We discern in these affirmations basic principles for BISE.

1.2.1 Global principles for BISE

A complex system can always be seen as a system of systems.

It's all the more true for natural systems as the smallest cell already embeds intelligence and complexity. We just have to look to the complexity of bacteria to undestand that from this point of view, natural systems are far from artificial systems in complexity and capacities. The more impressive capacity is perhaps the capacity of bacteria and some small living to sleep during years, waiting for adequate environment inviting them to come back to life.

Understanding the complexity of these small elements, we understand also that systems made of these elements are intrinsically system of systems.

Finally, beyond the elementary entities operator, each of these entities dissipates partly energy and consums partly energy. As a part of the dissipation can be used by other entities for sources, the whole dissipation or consumption at the system level is not the simple summation of the dissipations or consumptions at the entities level.

Now, taking the definition of Ricardo G. Sanfelice [16], a cyber-physical system is a system that combines physical and cyber components. The physical components come from nature or can be artificial made-man systems. The cyber side resends to softwares embedded in hardwares. The question of dissipation and sources for the cyber components is rarely said.

The classical representation of physical components comes from automaticians and is in a simple way written like :

$$\dot{x} = F(x, u) \tag{1.1}$$

x is often called a state variable, u an input and F a function giving the state variable evolving depending on the input u and the operator F.

The cyber component is written as a machine giving outputs $\mathcal{O}$ depending of some inputs ν, states η and logical machine k :

$$\mathcal{O} = k(\eta, \nu) \tag{1.2}$$

Coupling both physical and cyber systems means to find some bridge to exchange information between the state variables of the physical system x and the inputs and outputs of the cyber system. But using the classical automatics' writing, this bridge cannot be included in a common operator, while if we note the physical part as :

$$y = \zeta(u, x) \tag{1.3}$$

y being a dual state variable pointing out the state variable x value through some projection. We can note also the cyber system like : $\mathcal{O} - k(\eta, \nu) = 0$ or $k'(\mathcal{O}, \eta, \nu) = 0$. Then creating a coupling operator χ defined by morphisms linking x and $\nu : x \to \nu$, and another one linking $\mathcal{O}$ and x : $\mathcal{O} \to x$. Under these assumptions we obtain two equations associate in a single system :

$$\begin{cases} y = \zeta(u, x) + \chi(\mathcal{O}) \\ \\ 0 = \chi(x) + \mathcal{O} - k(\eta, \nu) \end{cases} \tag{1.4}$$

A bio-inspired system engineering works with entities able to have ports connectable through the operator χ. In other words, they are cyber-physical systems and more, the cyber side is finely diffused in all the system parts.

Now we remember that any system is composed of systems in a BISE approach. But as these systems also enclose cyber-physical systems, a BISE concept system is a cyber-physical system of systems (CPSoS).

1.3 Conclusion

We have presented in this chapter fundamental concepts proposed for CPSoS modelling. CPSoS seem to be the more complex kind of system that

can be imagined today, except living ones. Global organs are present in these systems as well as in the living systems : nervous (intelligence) networks, perception networks et actuators networks. The particularity of CPSoS is the capacity to exchange data through very long distance while living systems seem to be limited to some hundred of kilometres (for our current knowledge). But on another side their evolving remain very weak and they are not today capable of reproduction. Anyway, one of their principle interest is giving opportunity to create formalism able to model this systems, a first step to elaborate living systems models. This chapter submits a first presentation of this approach, based on the xTAN method[12,13,14] invented by the author. We understand easily that the notions discussed leads to the bio-inspired engineering technique.

Chapitre 2

Dynamic and perception

The purpose of this chapter is to introduce some cases or minding on system dynamic and perception. We want to present various problems, not really ordered, but to show how the tensorial analysis can be used in each situation before to see its implication in more complex problems.

> Everybody think see the world. But everybody feels only his own universe. It needs an incommensurable effort for imaging what the other feels - the author.

2.1 Classical dynamic

We can look at Newton's dynamic by a pragmatic way. Still recently, the equivalence between the inertia mass and the weighted mass was demonstrated with a remarkable accuracy. This result implies that masse is no more an intrinsic property of an object, but the cœfficient of a metric which gives the relation between force and impulse. Newton's equation can be written in classical mechanics like :

$$F_k = \frac{d}{dt}\left(m_{kq}v^q\right) \tag{2.1}$$

If the masse depends on time, this relation becomes :

$$F_k = \frac{dm_{kq}}{dt}v^q + m_{kq}\frac{dv^q}{dt} \tag{2.2}$$

A force is opposited to the displacement. We can make the assumption that this force can be given by $-K_{kq}v^q$ and that it is completed by a source of

movement f_k, we obtain :

$$f_k = \frac{dm_{kq}}{dt}v^q + m_{kq}\frac{dv^q}{dt} + K_{kq}v^q \tag{2.3}$$

Now if our body identified by the speeds v^q wears a little conductive loop. This loop in a magnetic field will respect the equation :

$$e_\nu = L_{\nu\mu}\frac{di^\mu}{dt} + R_{\nu\mu}i^\mu \tag{2.4}$$

The electromotive force (emf) e_ν is created by the loop movement in a constant magnetic field with speed v^q. It comes from the magnetic flux ϕ knowing :

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(xyB_z\right) \tag{2.5}$$

For a constant field and a movement in the only direction x, the emf is given by :

$$e = -yB_z v^x \tag{2.6}$$

By generalizing, our system is finally described by the equation :

$$f_\alpha = \zeta_{\alpha\beta}\omega^\beta \tag{2.7}$$

with :

$$f_\alpha = \begin{bmatrix} f_k \\ 0 \end{bmatrix} \tag{2.8}$$

$$\zeta_{\alpha\beta} = \begin{bmatrix} \frac{dm_{\alpha\beta}}{dt} + m_{\alpha\beta}\frac{d}{dt} + K_{\alpha\beta} & 0 \\ \epsilon_{\alpha\gamma\eta\beta}l^\gamma B^\eta & L_{\alpha\beta}\frac{d}{dt} + R_{\alpha\beta} \end{bmatrix} \tag{2.9}$$

and

$$\omega^\beta = \begin{bmatrix} v^\beta \\ i^\beta \end{bmatrix} \tag{2.10}$$

These terms can be changed depending on the studied cases, but the approach remains the same with this objective in final to dispose of a tensorial equation in a chosen configuration space. The choice must make appear the interesting variables, here for example the speed and the current.

Basic assumption of the classical dynamics is to set the time as invariant. Time remains constant and this, whatever the referential from which it is observed. That's why we can compute changing taking time as a reference. It behaves like a common rhythm to manage the values of various variables

during a physical process. How to do if this reference is no more a reference ? It means that time itself is a value that change depending on the referential from which it is seen. Time is no more an invariant. The unique solution is to find another invariant, another common reference.

2.2 Relativistic dynamic

If, in a given configuration space, time is locally invariant, the equations in this space are defined by classical dynamic. Rigorously, such a space perhaps doesn't exist. But if we are accurate on the domains where the assumption is valid, this space can exist in these limits. Sometimes, these assumptions can work at a scale level which is no more valid at a higher scale. Let's take the example of a simple transformer. At its macromodel level, the system is static and the mutual inductance is sufficient to describe with accuracy its behaviour. But if we look with details inside the transformer, we see currents and magnetic interactions coming from a relativistic interactions between these currents which are particles in movement. We understand that relativistic relations can be studied once the scale of the analysed system is well defined. Relativistic dynamic asks for defining the referential, the scale and its invariant.

2.2.1 Scale notion

A system of equation in one physics must be homogeneous in scale. It means that it must treat of variables that belong to the same scale. Two variables belong to two different scales if it is necessary to integrate to go from one variable to the other. For example going from quantum mechanics to classical mechanics needs an integration of the wave function. This doesn't mean that it is impossible to study simultaneously equations of various scales. It means only that these equations must be separated. The coupling functions between the various systems of equations associated with each scale will include integration operators.

We consider that if we can write for a variable A, a relation with a variable B like :

$$A = \int_{x^k} dx^k B \tag{2.11}$$

these two variables are of the same kind, but written in two different scales.

In this case, a system of equations that may grouped these two variables will not be homogeneous in scale.

2.2.2 Invariant

Having set the conditions for a good configuration space, we can try to understand the meaning of the invariant. When we derive a current, for example in the case of a inductance metric. We have :

$$e_2 = -M\frac{di^1}{dt} \tag{2.12}$$

This operation is actually complicated. The current i^1 belongs to another network, a first repository that we suppose to be far away. The derivation operation is performed in the receiving network, the second repository. M is a function that carries the current and its interaction from the first network to the second. It is clear that we are obliged to specify the nature of this function. As it stands, it links two different repositories without providing the slightest detail on this link. The simplest way to detail this function is to detail the interaction at the level of the electromagnetic field. We could have asked ourselves the question of how we could perceive the current since the repository 2 ? But this perception itself uses an exchange boson, a particle capable of transporting information from repository 1 to repository 2. In fact, the field is unavoidable.

The current i^1 accompanies an electromagnetic field $\mathbf{A}_1$ radiated in the surrounding immediate space. For the two reference systems to be different, they must be travelling at different speeds. If we have the expression that the repository 2 perceives a vector belonging to the repository 1, we can transform the field emitted in the repository 1 to know how it is perceived in the repository 2. To apprehend this problem we can take again the simple problem of the mirror and the vehicle. We consider a vehicle moving in the direction x to the speed v_x. This vehicle carries an axis perpendicular to its h height base. At the top of this axis is a mirror that reflects a light emitted at the base of the axis, parallel to the latter. The light moves in the direction y. Give us an invariant : the speed of light or speed. The distance travelled by the vehicle and measured from the repository 2 for a time t_2 is $x = v_x t_2$. The distance travelled by the light in the vehicle's reference frame is h.

The first thing is to write the expression of the invariant in both frames

of reference. In the reference 1 of the vehicle we have :

$$c = \frac{h}{t_1} \tag{2.13}$$

In repository 2, the distance travelled by the light is $\sqrt{h^2 + x^2}$. Fact :

$$c = \frac{\sqrt{h^2 + x^2}}{t_2} \tag{2.14}$$

From where :

$$t_2 = \left(\sqrt{1 + \frac{x^2}{h^2}} \right) t_1 \tag{2.15}$$

By replacing x with $v_x t_2$ we get quite easily :

$$t_2 = \left(\sqrt{1 - \frac{v_x^2}{c^2}} \right) t_1 \tag{2.16}$$

The time interval is shorter in the observer frame 2, static, than in the reference frame 1 in motion. Which means that time passes more quickly in the second repository. Conversely, the distances in the repository 1 in motion appear shorter views of the repository 2. We see that the process for establishing perceptions from one repository or another is as follows :

— defining an invariant ;
— writing this invariant in different referential ;
— then obtaining the transformation relations between referential.

2.2.3 An example for an electrical circuit

Consider a circuit made up of two branches connected and traversed by the same current of mesh J^1. Seen in each branch the space of the currents is of dimension 2 and the two currents of the two connected branches are i^1 and i^2. An invariant to both spaces of branches and meshes is power. The power of the first branch in branch space is $P_1 = R_{11}(i^1)^2$. This power is written $P_1 = R_{11}(J^1)^2$ in the space of the meshes. The developed power in the second branch is $P_2 = R_{22}(i^2)^2$ or $P_2 = R_{22}(J^1)^2$. We immediately deduce the connection :

$$\Omega = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2.17}$$

Once the connection - or *space change matrix* - is found, it becomes possible to rewrite the equations of the problem seen from the observer's reference frame (on this subject we could discuss the fact that the space of the meshes is for electrical circuits, the observation space).

The Lorentz transformation thus obtained transforms the coordinates t_1, x_1, y_1, z_1 of a moving origin space into the coordinates t_2, x_2, y_2, z_2 of the observation space :

$$\Lambda = \begin{bmatrix} \gamma & \frac{\gamma}{c^2}v^x & \frac{\gamma}{c^2}v^y & \frac{\gamma}{c^2}v^z \\ \gamma v^x & 1+\alpha(v^x)^2 & \alpha v^x v^y & \alpha v^x v^z \\ \gamma v^y & \alpha v^y v^x & 1+\alpha(v^y)^2 & \alpha v^y v^z \\ \gamma v^z & \alpha v^z v^x & \alpha v^y v^y & 1+\alpha(v^z)^2 \end{bmatrix} \tag{2.18}$$

with $\alpha = (\gamma - 1)/v^2$, v is the vectorial speed between the two spaces and $\gamma = (1 - v^2/c^2)^{-1/2}$.

Once the passing matrix is known, we can put an equation in the reference 1 and transform it into an equation in the reference 2. From this point of view, the transformation of Lorentz gives the passage for the coordinates, but not directly for the speeds. If we imagine two repositories in motion relating to the speed v along an axis x, the matrix of transformation of velocities between the two frames of reference is :

$$\Lambda = \begin{bmatrix} \gamma\beta(u_x - v) & 0 & 0 \\ 0 & \beta u_y & 0 \\ 0 & 0 & \beta u_z \end{bmatrix} \tag{2.19}$$

knowing that :

$$\beta = \left[\gamma\left(1 - \frac{u_x v}{c^2}\right)\right]^{-1}$$

The relation between the forces f_k and the velocities v^q printed on the mobiles is directly determined by the metric m_{ij}. This relation : $f_k = m_{kq}v^q$ is determined in a repository 1 by an observer who belongs to this repository. Likewise, a second law of the form $e_a = \zeta_{ab}i^b$ governs the electromagnetic interactions. For example, it is emissions radiated by equipment embedded in a vehicle. This radiation is similar to that measured in anechoic chamber as part of equipment qualification testing. Now this radiation, in practice, we want to observe it by ordinary receivers that are static, when the vehicle

that carries them is moving. We must perform the transformations :

$$\begin{cases} m_{\mu\nu} = \Lambda^k_\mu \Lambda^q_\nu m_{kq}, \quad f_\mu = \Lambda^k_\mu f_k \\ \\ \zeta_{\alpha\eta} = \Omega^a_\alpha \Omega^b_\eta \zeta_{ab}, \quad e_\alpha = \Omega^a_\alpha e_a \end{cases} \tag{2.20}$$

The transformations between the streams $\Omega^\eta_q i^q = i^\eta$ and $\Lambda^\mu_k v^k = v^\mu$ being the origin of the determination of their matrices, they are implicitly known.

In summary, the knowledge of an invariant and the endowment of a configuration space lead to the establishment of a transition matrix between the flow variables of two spaces attached to different reference frames.

Chapitre 3

Defining metrics

3.1 Origins

The concept of metrics is the central element of differential geometry. In a general way we establish a system of equations which one can represent by the expression :

$$e_k = \psi_k \left(m_{kq} \cdot f^q \right) \tag{3.1}$$

where ψ_k is a function dependent on m_{kq} operators acting on f^q streams. We then look for how the number e_k evolves as a function of the flow f^q. Either to calculate the terms :

$$\frac{\partial \psi_k}{\partial f^q} \tag{3.2}$$

These calculations generate a sequence of terms for each value taken by k for a value of q. This suite is used to define a vector, base of a vector space $\mathbf{b}_q$. Following the m_{kq} operator, the terms of $\mathbf{b}_q$ are simple or are themselves functions. If m_{kq} is a scalar, then the corresponding term is m_{kq} itself. For example if we consider a vector of functions of dimension 3 (k varies from 1 to 3) and three flows exist (q also varies from 1 to 3). We have :

$$\mathbf{b}_1 = \left[\begin{array}{ccc} \frac{\partial \psi_1}{\partial f^1} & \frac{\partial \psi_2}{\partial f^1} & \frac{\partial \psi_3}{\partial f^1} \end{array} \right] \tag{3.3}$$

If $\psi_1 = m_{11} f^1$, it is clear that $\mathbf{b}_1(1) = m_{11}$. Suppose a particular function ψ such that $\psi_k = m_{kq} \cdot f^q = m_{kk} f^k$. In that case :

$$\mathbf{b}_q = \delta_q^k m_{kk} \tag{3.4}$$

The G metric of the space defined by the basic vectors $\mathbf{b}_q$ is then given by :

$$G_{kq} = \langle \mathbf{b}_k | \, \mathbf{b}_q \rangle \tag{3.5}$$

In the particular space defined by the equation 3.4, the components of the G metric are all equal to $(m_{kk})^2$ and reduced to the components G_{kk}, the components G_{kq}, $k \neq q$ being equal to zero. If the elements of the metric m_{kk} are terms of dissipation, the term ω developed in the studied system and given by :

$$\omega = \sqrt{\sum_k (m_{kk} f^k)^2} = \sqrt{G_{kq} f^q} \tag{3.6}$$

the magnitude of a potential difference V, or a pulse p.

Take an electrical circuit, we get :

$$\omega = \sqrt{(R_1 i^1)^2 + (R_2 i^2)^2 + (R_3 i^3)^2} \tag{3.7}$$

which has the dimension of a difference of potentials. In mechanics :

$$\omega = \sqrt{(k_1 v^1)^2 + (k_2 v^2)^2 + (k_3 v^3)^2} \tag{3.8}$$

which has the dimension of a pressure pulse, etc. In any case, if we integrate ω with a generalized state variable that we will write x and derive from time, we get the power :

$$P = \frac{d}{dt} \int_x dx\omega \tag{3.9}$$

the power, which is an invariant. In the case of electromagnetism, the generalized variable is the load, and $P = i\omega \rightarrow iV$. In mechanics it is the integrated force on speed : $P = v\omega \rightarrow vF = v\dot{p}$.

The equation 3.9 shows clearly that given iso-flux, the power depends directly on ω which appears as a way to measure this power. It's ω that turns our variable into power and ω is defined as a quadratic value of the G product by the feed. Finally our metric element is directly given by G.

3.2 Inertia

If the m operator is a derivator, what happens to the components of the basic vectors ? We are getting :

$$\frac{\partial}{\partial f^a}\left(m_{kq}\frac{df^q}{dt}\right) = \frac{\partial}{\partial f^a} m_{kq}\left(\frac{df^q}{dt}\right) + m_{kq}\left(\frac{\partial}{\partial f^a}\frac{df^q}{dt}\right) \tag{3.10}$$

Using Poisson's bracket :

$$\left[\frac{\partial}{\partial f^a}\frac{d}{dt} - \frac{d}{dt}\frac{\partial}{\partial f^a}\right] = 0 \tag{3.11}$$

The second term of the equation 3.10 becomes null. Noting $\hat{\psi}$ the Jacobian matrix associated with the function ψ, this matrix has as a column the components of the basic vectors. Except for the existence of losses, or because the term $\partial_{f^a} m_{kq}$ is non-zero, these vectors themselves become null vectors. Finally the time derivation operators (or following a chosen invariant ω) constitute the kernel of the geometry of the function ψ. These are source terms just like the e_k pointers. The equation 3.1 can be rewritten :

$$e_k - m_{kq}\frac{d}{d\omega}f^q = \psi\left(m'_{kq}f^q\right) \tag{3.12}$$

The term $m_{kq}\partial_\omega f^q$ which opposes the source energy pulse of the system is called the inertia term. It still exists in dynamic systems [1]. For example, in electronics where it corresponds to inductive terms, an electrodynamic circuit cannot exist without inductors. The first term of the equation 3.10 uses the derivative with respect to the flow of the root of the component of the metric. We have :

$$\frac{\partial\omega}{\partial f^a} = \left[\frac{(m_{aa})^2 f^a}{\omega}\right]^{-1} \tag{3.13}$$

and so with a diagonal metric :

$$\frac{\partial}{\partial f^q}m_{ka}\left(\frac{df^a}{d\omega}\right) = \left[\frac{(m_{aa})^2 f^a}{\omega}\right]\frac{\partial}{\partial f^a}m_{kq} \tag{3.14}$$

The term between hook has the dimension of a component of $\hat{\psi}$. We will now name this dimension *impedance* or *impedance operator*. The dependence of the impedance (which can be therefore a mass, an electrical impedance, etc.) with the flux implies, if it must exist generally so as not to have a second term null in the absence of losses in 3.12 that the space can not be "flat". Let's see the consequences of this observation.

1. We will write $T_k = e_k - m_{kq}\frac{d}{d\omega}f^q$

3.3 Cartesian space and diagonality of G

For G to be purely diagonal and according to its definition, the basic vectors $\mathbf{b}_q$ must have no components in a common direction. In other words, in the scalar product $\langle \mathbf{b}_k |\, \mathbf{b}_q \rangle$ only the product exists :

$$G_{kq} = \delta_k^q \langle \mathbf{b}_k |\, \mathbf{b}_q \rangle \tag{3.15}$$

In this case the invariant is equal to :

$$\omega = \sqrt{(G_{aa} f^a)^2} \tag{3.16}$$

The preceding condition requires that none of the basic vectors have collinearity with another. They only have one component [2]. The corresponding space is called "**Cartesian**". And we have seen that if the studied system is devoid of losses and without dependence of the flux impedance, this space is the kernel of the morphism F such that :

$$F : \psi \xrightarrow{\partial_{fa}} \mathbf{b}_k \tag{3.17}$$

This kernel defines the source term T_k. But the physical systems complete this kernel by dissipation or curvature, which is a form of dissipation. The Cartesian space is therefore necessarily a reduced vision of reality, a vision that is only a local approximation of this reality. Consider electronics. The components - magnetic, resistive, capacitive - have models that always depend on the intensity of the current or the voltage at their terminals.

Another addiction that is interesting is that depending on the movement. Let's put ourselves in an observation frame of variables x', y', z', t'. We look at an electromotive force generated in this frame of reference :

$$e' = \frac{d}{dt'} \left(L i'^y \right) \tag{3.18}$$

After choosing geometric axes for the two referentials, parallel and relative speed u_y. A direct current i^y flows in a driver portion in the frame of reference observed along the direction y. This is an interesting problem because

2. for example $\mathbf{b}_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$

normally the electromotive force (fem) generated is zero because the current is constant. The electromotive force is expressed in the reference frame of the observer :

$$e' = \frac{d}{dt'}\left(\mu \frac{x'y'}{z'} i'^y\right) \tag{3.19}$$

Using the previous relations of passage we obtain by replacing the inductance by a canonical expression :

$$e' = \frac{d}{d\gamma t}\mu \left[\frac{\gamma\left(t + \frac{u^y}{c^2}y\right)}{\alpha(u^y)^2 y + z}\left(\frac{i^y - i_u^y}{1 - \frac{u^y v^y}{c^2}}\right)\right] \tag{3.20}$$

i_u^y est le courant qu^y. We obtain if u^y is constant :

$$e' = \mu \frac{1}{\gamma}\left[\frac{\gamma}{(\alpha(u^y)^2 y + z)}\left(\frac{i^y - i_u^y}{1 - \frac{u^y v^y}{c^2}}\right)\right] \tag{3.21}$$

For an observer in the same frame, at rest, there is no emf generated by the flow of direct current in the wire. On the other hand, for an external observer moving at the speed u^y with respect to the referential observed, a emf exists in this moving circuit. This emf is not created by the derivation of the observed current, which remains zero, but by the derivation of the expression of the inductance. If we study the e' functions and try to extract the metric, we get a constant value I_e comparable to losses that is expressed by :

$$I_e = \mu \frac{1}{(\alpha(u^y)^2 y + z)\left(1 - \frac{u^y v^y}{c^2}\right)} \tag{3.22}$$

It is clear that dispersions are above all eigenvectors of the axes of space. But the electromagnetic compatibility has a singularity from this point of view : the common impedance coupling, which is a partly resistive coupling term. If we consider three circuits including two coupled, one transmitter and the other receiver, the third being independent such as :

$$\begin{cases} e_1 - L_1 \frac{d}{dt} i^1 = R_1 i^1 + R_{12} i^2 \\ -L_2 \frac{d}{dt} i^2 = R_{12} i^1 + R_2 i^2 \\ e_3 - L_3 \frac{d}{dt} i^3 = R_3 i^3 \end{cases} \tag{3.23}$$

If we compute $\partial_{i^x} T_k$ we find the base :

$$\mathbf{b}_1 = \begin{bmatrix} R_1 \\ R_{12} \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} R_{12} \\ R_2 \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ R_3 \end{bmatrix} \tag{3.24}$$

and the metric :

$$G = \begin{bmatrix} (R_1)^2 + (R_{12})^2 & R_{12}(R_1 + R_2) & 0 \\ R_{12}(R_1 + R_2) & (R_{12})^2 + (R_2)^2 & 0 \\ 0 & 0 & (R_3)^2 \end{bmatrix} \tag{3.25}$$

The losses here generate a non-diagonal metric. We saw that the basic vectors were found in the Jacobian $\hat{\psi}$. We can write the system 3.23 in the form [3] :

$$e_k - L_{kx} \frac{d}{dt} i^x = b_{kx} i^x \tag{3.26}$$

Multiply each term by the transposed matrix of Jacobian $\left[\hat{\psi}^T\right]$:

$$\left[\hat{\psi}^T\right]_\mu^k \left(e_k - L_{kx} \frac{d}{dt} i^x \right) = \left[\hat{\psi}^T\right]_\mu^k b_{kx} i^x \tag{3.27}$$

but

$$\left[\hat{\psi}^T\right]_\mu^k b_{kx} = G_{\mu x} \tag{3.28}$$

and

$$T_\mu = \left[\hat{\psi}^T\right]_\mu^k \left(e_k - L_{kx} \frac{d}{dt} i^x \right) \tag{3.29}$$

Finally :

$$T_\mu = G_{\mu x} i^x \tag{3.30}$$

The space as described by G is not Cartesian. However, there is no moving repository here and all currents belong to the same and unique referential of the problem. We need to be able to find a proper repository that makes G diagonal. Let's note G in the form :

$$G = \begin{bmatrix} g_1 & g_2 & 0 \\ g_3 & g_4 & 0 \\ 0 & 0 & g_5 \end{bmatrix} \tag{3.31}$$

3. see O.Maurice, "*Elements of theory for electromagnetic compatibility and systems*" at Bookelis, 2017

Taking two matrices :

$$D = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad G' = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{3.32}$$

We want to check $DG = G'$, which allows us to find a new diagonal G' metric. Solving the previous equation we find :

$$\begin{cases} ag_1 + bg_3 = \alpha \\ ag_2 + bg_4 = 0 \\ cg_5 = 0 \\ dg_1 + eg_3 = 0 \\ dg_2 + eg_4 = \beta \\ fg_5 = 0 \\ gg_1 + hg_2 = 0 \\ gg_3 + hg_4 = 0 \\ ig_5 = \gamma \end{cases} \tag{3.33}$$

From where we deduce :

$$\begin{cases} c = 0 \\ f = 0 \\ g = h = 0 \\ a = \frac{g_4\alpha}{\Delta} \\ b = -\frac{g_2\alpha}{\Delta} \\ d = -\frac{g_3\beta}{\Delta} \\ e = \frac{g_1\beta}{\Delta} \\ i = \frac{\gamma}{g_5} \end{cases} \tag{3.34}$$

with $\Delta = g_1g_4 - g_2g_3$. We can impose that the metric be that of the eigenvalues to postpone the couplings in source terms. So :

$$G' = \begin{bmatrix} (R_1)^2 & 0 & 0 \\ 0 & (R_2)^2 & 0 \\ 0 & 0 & (R_3)^2 \end{bmatrix}$$
$$D = \frac{1}{\Delta} \begin{bmatrix} (R_1)^2\left((R_{12})^2 + (R_2)^2\right) & -(R_1)^2\left(R_{12}(R_1 + R_2)\right) & 0 \\ -(R_2)^2\left(R_{12}(R_1 + R_2)\right) & (R_2)^2\left((R_1)^2 + (R_{12})^2\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.35}$$

The equation 3.30 can be written :

$$T_\mu = G_{\mu x} i^x \Rightarrow D^\mu_\sigma T_\mu = D^\mu_\sigma G_{\mu x} i^x = G'_{\sigma x} i^x \tag{3.36}$$

that to say :

$$T'_\sigma = G'_{\sigma x} i^x \tag{3.37}$$

We can look at the particular case of solutions obtained without inertia, ie without derivative terms with respect to the invariant : without *acceleration*. These solutions constitute the **geodesic** of space. As :

$$\left[\hat{\psi}^T\right]^k_\mu = \begin{bmatrix} R_1 & R_{12} & 0 \\ R_{12} & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \tag{3.38}$$

we find that the source term of the equation 3.29 is reduced to :

$$\begin{gathered} T'_\sigma = D^\mu_\sigma \left[\hat{\psi}^T\right]^k_\mu e_k = \dots \\ \begin{bmatrix} \left[(R_1)^3 \left((R_{12})^2 + (R_2)^2\right) - (R_1)^2 (R_{12})^2 \left(R_1 + R_2\right)\right] e_1 \\ \left[(R_2)^2 R_{12} \left((R_1)^2 + (R_{12})^2\right) - R_1 (R_2)^2 R_{12} \left(R_1 + R_2\right)\right] e_1 \\ R_3 e_3 \end{bmatrix} \end{gathered} \tag{3.39}$$

The given metric equation 3.35 being purely diagonal and the solutions given by the equation 3.37 ; T' being defined by the previous relation, the basis of the space is :

$$\mathbf{b}_1 = \begin{bmatrix} R_1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ R_2 \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ R_3 \end{bmatrix} \tag{3.40}$$

the geodesics following the three elementary increments of measures R_1, R_2 and R_3 and the three curvilinear directions i^1, i^2 and i^3. The space is actually flat because wherever one is on the axes, in other words whatever the currents values i^1 to i^3, the components of the base keep the same values. In addition, the three directions are perpendicular two by two. Note that any vector $\vec{i} = i^k \mathbf{b}_k$ is a flux, like the distance ω.

3.4 The magnetic space

Curiously, it is the custom that engineers use the electric field more often than the magnetic field to model electronic systems in general. Yet when there are currents (so flows), there is a magnetic field. The electric field is primarily associated with the loads, and therefore with the state variables. But finally, is not there a trick because the field is not the most relevant ? If we take the vector potential in the Coulomb gauge, the question of the relevance of the electric or magnetic field is no longer relevant. The flux of the magnetic field that belongs to the cellular space of dimension 2 $\mathcal{T}^2$ can also be written :

$$\phi = S.\mathbf{B} = S.\nabla \times \mathbf{A} \tag{3.41}$$

This field has all its legitimacy since it is by its use that we demonstrate Neumann's formula of mutual inductance. It is also through it that one can obtain the correspondence between the photon and the free electromagnetic energy. But in what way can this field constitute the framework, the geodesic map of the electromagnetic space ? Geodesics are the abstract lines that follow a particle of mass null and not subjected to a force. The particle following the geodesics is in free fall. Maxwell's field is obtained by writing :

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \tag{3.42}$$

It is also legitimate to choose the potential covariant vector, because its expression as a function of the current involves an impedance, twice covariant. If we start from the 4-vector [4] (c, v) ; we obtain the vector potential by multiplying the velocity v by $\mu q \mathcal{G} \alpha^{-1}$; where $\mathcal{G}$ is a function of Green and α a function of the distance involved in $\mathcal{G}$. The product by the first term c gives the scalar potential divided by the celerity. The 4-potential vector is therefore $(\frac{\psi}{c}, \mathbf{A})$. But the 4-vector is a vector v^k. The 4-potential obtained is therefore of contravariant basis A^k which generates F^{ik}. The product by its dual must lead to the invariant that is the power. In the case of 4-vectors the mechanics are simple. If A_0 is the so-called "temporal" component of the 4) vector (its first term) and if A_k are the three spatial components, we have :

$$A_0 = A^0,\ A_1 = -A^1,\ A_2 = -A^2,\ A_3 = -A^3 \tag{3.43}$$

4. See, "*Theory of fields*" of L.Landau and E.Lifchitz, editions MIR

and so :

$$F_{ik} \leftarrow \partial_{[x^i,x^k]} \left(\frac{\psi}{c}, -\mathbf{A} \right) \tag{3.44}$$

By definition, $F_{ii} = 0$ and for example $F_{12} = \partial_{x^1} A_2 - \partial_{x^2} A_1$. The electromagnetic field is therefore of an inherently tensor nature. We can write :

$$F_{ik} = \begin{bmatrix} 0 & B_z & B_y & E_x/c \\ -B_z & 0 & B_x & E_y/c \\ B_y & -B_x & 0 & E_z/c \\ -E_x/c & -E_y/c & -E_z/c & 0 \end{bmatrix} \tag{3.45}$$

The lines of the F field define flow tubes to which we can associate an impedance operator : the reluctance. Between the currents or displacements of loads and the lines of field we have a relation partly dissipative :

$$\nabla_\alpha^{\beta\sigma} F_{\beta\sigma} = (\mu_{\alpha\beta} + \sigma_{\alpha\beta}) J^\beta \tag{3.46}$$

and a second equation which gives the emf :

$$\epsilon_\alpha^{\beta\sigma} F_{\beta\sigma} = 0 \tag{3.47}$$

The $\nabla_\alpha^{\beta\sigma}$ operator combines the partial derivations to give the Maxwell-Ampère equation while the $\epsilon_\alpha^{\beta\sigma}$ operator generates the rotational of the field for the first Maxwell-Faraday equation. This second equation includes the induction of charge displacement under Coulomb force. However, Poisson's equation is not explicitly given but is not useful in the dynamic expression of the field. More complete expressions are available.

The second equation interests us more particularly, the first giving the link between the particles and the field via the current density J.

This second equation is expressed classically by relations of the type :

$$\frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \tag{3.48}$$

But these relations are expressed much better - and in a more synthetic way - in an integral form :

$$\oint_{l(S)} d\mathbf{S} \cdot \mathbf{E}_l = -\frac{d}{dt} \phi_\eta \tag{3.49}$$

η is normal to the surface S. The integral of the electric field on a closed contour results in the product of an impedance operator on a mesh by a

current of mesh. Among these impedances is inevitably, as we have seen, an inductance L which reflects the inertia of the circuit with respect to external inductions. Let Z be the rest of the impedance of the circuit, we write :

$$Zi + \frac{d}{dt}(Li) = 0 \tag{3.50}$$

Since there can be no lossless circuits, Z includes a resistive portion R. Finally, the external field sources induce emf e in this circuit. First of all our fundamental equation 3.47 in the presence of these interactions with the environment becomes :

$$\epsilon_{\alpha}^{\beta\sigma} F_{\beta\sigma} = e_{\alpha} \tag{3.51}$$

then the equation 3.50 is now written in the presence of these sources and with only losses :

$$R_{\alpha\beta} i^{\beta} + \frac{d}{dt}\left(L_{\alpha\beta} i^{\beta}\right) = e_{\alpha} \tag{3.52}$$

In absence of "*acceleration*", ie for :

$$L_{\alpha\beta} \frac{d}{dt} i^{\beta} = 0 \tag{3.53}$$

(this is my definition), remains $e_{\alpha} = R_{\alpha\beta} i^{\beta}$. The basic vectors of the dissipative space are the geodesics of space here Cartesian. This is our previous discussion. Imagine a generalized variation (whatever the size of the space considered) of the inductance as a function of the current :

$$\frac{d}{dt} L_{\alpha\beta} i^{\beta} = \frac{\partial}{\partial i^{\sigma}} \frac{di^{\sigma}}{dt} L_{\alpha\beta} i^{\beta} \tag{3.54}$$

Writing :

$$\Omega_{\alpha\beta,\sigma} = \frac{\partial}{\partial i^{\sigma}} L_{\alpha\beta} \tag{3.55}$$

a term is added to the usual expression of the inductance in the function of the equation 3.1 which is :

$$\Omega_{\alpha\beta,\sigma} \frac{di^{\sigma}}{dt} i^{\beta} \tag{3.56}$$

When we derive this term in search of basic vectors, we obtain :

$$\frac{\partial}{\partial i^{\eta}} \left(\Omega_{\alpha\beta,\sigma} \frac{di^{\sigma}}{dt} i^{\beta} \right) = \frac{\partial}{\partial i^{\eta}} \Omega_{\alpha\beta,\sigma} \frac{di^{\sigma}}{dt} i^{\beta} + 0 + \Omega_{\alpha\beta,\sigma} \frac{di^{\sigma}}{dt} \delta_{\eta}^{\beta} \tag{3.57}$$

If the dependence of the induction as a function of the currents is of the first order, it remains as a component of the basic vectors :

$$\frac{\partial}{\partial i^{\beta}}\left(\Omega_{\alpha\beta,\sigma}\frac{di^{\sigma}}{dt}i^{\beta}\right)=\Omega_{\alpha\beta,\sigma}\frac{di^{\sigma}}{dt} \tag{3.58}$$

which has the dimension of an impedance, since it is twice covariant. In this case we have geodesics, even in the absence of resistive losses. Imagine a purely magnetic space (circuit) in a medium where only inductances subsist, with lossless electron plasmas. The equation 3.26 would become of the form :

$$e_{\alpha}-L_{\alpha\beta}\frac{d}{dt}i^{\beta}=\Omega_{\alpha\beta,\sigma}\frac{di^{\sigma}}{dt}i^{\beta} \tag{3.59}$$

We must specify the term L of this equation. If there are networks of inductors made of magnetic materials, there are necessarily mutual inductances between these elements, however weak they may be. If these interactions take place in a vacuum, the mutual inductances are expressed by :

$$L_{\alpha\beta}=\bar{\mu}_{0}\int_{x^{\alpha}}\int_{x^{\beta}}\frac{dx^{\alpha}dx^{\beta}}{d(\alpha,\beta)}\mathcal{G}_{\alpha\beta} \tag{3.60}$$

with $\bar{\mu}_{0}=\frac{\mu_{0}}{4\pi}$ et $\mathcal{G}$ a Green's function. On the other hand, the derivation of these mutuals as a function of the mesh currents is zero. In fact the metric is reduced to terms $\left(\Omega_{\alpha\alpha,\alpha}\right)^{2}$ since we suppose that the interactions are in a vacuum, so a priori without intensity dependence of the current (we will review this hypothesis). The space being cartesian, $\psi^{T}=\sqrt{G}$ and

$$T_{\sigma}=\left[\psi^{T}\right]_{\sigma}^{\alpha}\left(e_{\alpha}-L_{\alpha\sigma}i^{\sigma}\right)\equiv\Omega_{\sigma,\sigma}^{\alpha}\frac{di^{\sigma}}{dt}\left(e_{\alpha}-L_{\alpha\sigma}i^{\sigma}\right) \tag{3.61}$$

This Cartesian space is then completely defined by the equation :

$$T_{\sigma}=G_{\sigma\alpha}i^{\alpha} \tag{3.62}$$

But what are the geodesics ? According to the definition that I propose, we can not determine them by the equation 3.62 because the cancellation of all the terms in time derivative leads to the nullity of the two members. Yet just invert this equation and write :

$$i^{\alpha}=y^{\alpha\sigma}T_{\sigma}\leftrightarrow i^{\alpha}=\mathcal{R}^{\alpha\sigma}\phi_{\sigma} \tag{3.63}$$

with $y^{\alpha\sigma} = [G_{\sigma\alpha}]^{-1}$ et $\mathcal{R}$ the medium reluctance. Because the source term T necessarily includes the derivations of the magnetic flux ϕ. The equation 3.62 can be rewritten as a function of the magnetic flux. We can rewrite this equation in its covariant form. We only need to use the inverse of the reluctance named "*permeance*" U and :

$$\phi_\sigma = U_{\sigma\alpha} i^\alpha \tag{3.64}$$

The basic vectors are then defined by permeance and the metric is defined by a diagonal matrix containing the squares of permeances. But in this new representation, it is no longer the emf that are geometrical, but the magnetic fluxes. As the relation between the flux and the emf is a derivation operator, working on the flux allows to extract it and at the same time to define a non-zero metric, even in a space without losses. However, it is always particles that generate the flow. It is therefore under the condition that these particles do not cause losses in their displacements that this dynamic can be envisaged, which constitutes a very particular case.

3.5 Independence of mutual inductances in vacuum with respect to currents

A mutual inductance in the classical sense is given by Neumann's relation and is independent of currents :

$$M = \frac{\mu}{4\pi} \iint_{s_1\, s_2} \frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{r_{12}} \tag{3.65}$$

Looking at any current i on a length $\mathbf{x}$ as a load q at the speed $\mathbf{vb}$, we consider a current $q\mathbf{vb}$ in a circuit, itself in relative speed $\mathbf{u}$ with respect to a second observer circuit. The speed of the v_c charge perceived by the observer is given by :

$$\mathbf{v}_c = \frac{\mathbf{v} + \mathbf{u}}{1 + \mathbf{v} \cdot \mathbf{u} c^{-2}} \tag{3.66}$$

The particle radiates a field $\mu q \mathbf{v}_c G(r)$, $G(r)$ being the function of Green for a distance r between the source and the observer. But we can also directly calculate the field in the observer's frame of reference. If Λ is the repository

change matrix, with for example :

$$\Lambda = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{bmatrix} \tag{3.67}$$

($\beta = u/c$ et $\gamma = \sqrt{1-\beta^2}^{-1}$). By definition, we have the F' field in the given observation frame as a function of the F field in the observed repository given by :

$$F'_{\mu\nu} = \Lambda^{\sigma}_{\mu}\Lambda^{\eta}_{\nu}F_{\sigma\eta} \tag{3.68}$$

We can define A' with : $F'_{\sigma\eta} = A'_{\sigma v}x_{\eta}i^{v}$. If the emf e in reception are defined by :

$$e_{\alpha} = \frac{d}{ds}S^{\sigma\eta}_{\alpha}A'_{\sigma v}x_{\eta}i^{v} \tag{3.69}$$

The mutual inductances are defined by :

$$e_{\alpha} = M_{\alpha v}\frac{d}{ds}i^{v} \Rightarrow M_{\alpha v} = S^{\sigma\eta}_{\alpha}A'_{\sigma v}x_{\eta} \tag{3.70}$$

where the mutual inductance $M_{\alpha v}$ depends on the speed of the reference of the charges, but not on the speed of the charges themselves. As long as the u speed is constant, the problem is a special relativity problem and the mutual inductance retains its intrinsic properties. On the other hand, if the speed u is variable, the derivation of the A field is non-zero and we have :

$$e_{\alpha} = S^{\sigma\eta}_{\alpha}A'_{\sigma v}x_{\eta}\frac{d}{ds}i^{v} + S^{\sigma\eta}_{\alpha}x_{\eta}\frac{d}{dx^{i}}\frac{dx^{i}}{ds}A'_{\sigma v}i^{v} \tag{3.71}$$

but :

$$\frac{d}{dx^{i}}\frac{dx^{i}}{ds}A'_{\sigma v} = \frac{1}{s}\Gamma_{\sigma v,i}x^{i} \tag{3.72}$$

The coupling can then be defined more broadly by the operator :

$$\mathcal{M}_{\alpha v} = M_{\alpha v}\frac{d}{ds} + S^{\sigma\eta}_{\alpha}x_{\eta}\frac{1}{s}\Gamma_{\sigma v,i}x^{i} \tag{3.73}$$

the term added in the generalized relativistic framework depends on the variation of speed in the expression of the field. We could call "*Coriolis' Mutual*" [5] this second term. This term is used as the term of the metric component in the observation frame.

5. This term was used for the first time by Gabriel Kron.

Chapitre 4

Exploration of various cases

Any modern system of systems is made of electronics. The goal of this chapter is to give some illustration of how the tensorial analysis of networks can solve electronic circuits, even with quantum properties. We cannot cover all the possible interactions, but the examples studied should give all the materials and ideas to solve any other one in the same fields.

4.1 EMC & functional EMC

It is customary to call functional EMC (functional electromagnetic compatibility), the electromagnetic compatibility between the functions embedded in the same system. As the system is common, we can admit that in the vast majority of cases, the electronics belonging to this system are in the same frame of reference. The problem of the electromagnetic compatibility of electronics in different reference systems above all points to electronics in separate systems.

4.2 Fixed transmitter and mobile receiver

We imagine a fixed emitter, with a f_0 carrier. This emitter is positioned at the coordinates $(x_e, y_e, 0)$ in a referential R. A vehicle with a wired link is at the coordinate point $(x_v, y_v, 0)$ in a R' referential in uniform rectilinear motion at the speed v^x with respect to R. The field radiated by the antenna is perceived modified by the receiver. We must first calculate the field in the R' referential at the cabling level. Let's stay here in a classic setting. If we

can identify in the source referential a small current segment along the three directions of R, let's look at how the magnetic field from these segments is transformed, and seen from R'. That is, a speed between the two frames assumed in the only direction x. The transformation of the fields is given by [1] :

$$\begin{cases} B'_x = B_x \\ B'_y = \gamma \left(B_y + \beta \frac{E_z}{c} \right) \\ B'_z = \gamma \left(B_z + \beta \frac{E_y}{c} \right) \end{cases} \tag{4.1}$$

If our current segment in R is along the z axis, the magnetic field has two components following x and y, hence $B_z = 0$. But if we are in far-field interaction, $E_z = cB_y$ and

$$\begin{cases} B'_x = B_x \\ B'_y = \gamma \left(1 + \beta \right) B_y \\ B'_z = 0 \end{cases} \tag{4.2}$$

The coupling function for a loop located in R' with a normal in the y direction is expressed here by :

$$e_2 = -\chi_v z_v \frac{d}{dt} B'_y = -\chi_v z_v \frac{d}{dt} \gamma \left(1 + \beta \right) B_y \tag{4.3}$$

If the speed is constant, only the derivation of the field remains, with :

$$B_y = \mu \frac{i^1}{2\pi d(t)} \delta_{\frac{d(t)}{c}} \tag{4.4}$$

where $d(t)$ is the meitter-receptor distance depending on time : $d(t) = \sqrt{(v^x t)^2 + \Delta_y^2}$, $\Delta_y = y_v - y_e$. Finally the coupling function $e_2(i^1)$ is given by :

$$e_2 = -\chi_v z_v \frac{d}{dt} \left\{ \frac{\gamma \left(1 + \beta \right) \mu}{2\pi d(t)} exp \left(-j\omega_0 \frac{d(t)}{c} \right) i^1(t) \right\} \tag{4.5}$$

always at a constant speed and for a constant emitter current $i^1 exp\left(j\omega_0 t\right)$. We have a supposed constant factor :

$$\alpha = \chi_v z_v \frac{\gamma \left(1 + \beta \right) \mu}{2\pi} \tag{4.6}$$

1. $\beta = u/c$ and $\gamma = \sqrt{1 - \beta^2}^{-1}$

We find a first term from the derivative :

$$T = -j\omega_0 \frac{v^x t^2}{c d^2(t)} e^{-j\omega_0 \frac{d(t)}{c}} \tag{4.7}$$

a second one :

$$U = -\frac{v^x t^2}{d^3(t)} e^{-j\omega_0 \frac{d(t)}{c}} \tag{4.8}$$

and a third one :

$$V = \frac{1}{d(t)} e^{-j\omega_0 \frac{d(t)}{c}} \tag{4.9}$$

Finally :

$$e_2 = -\alpha \left\{ (T+U)\, i^1 + V \frac{d}{dt} i^1 \right\} \tag{4.10}$$

To compare with what this coupling function would have been in the same repository :

$$M_{21}^0 = -\chi_v z_y \frac{\mu}{2\pi d_0} exp\left(-j\omega_0 \frac{d_0}{c}\right) \frac{d}{dt} \tag{4.11}$$

If β is small enough, $\gamma \to 1$. Then :

$$M_{21}^0 = -\frac{\alpha}{d_0} exp\left(-j\omega_0 \frac{d_0}{c}\right) \frac{d}{dt} \tag{4.12}$$

if $v^x = 0$, $U = 0$ and $T = 0$; by choosing the special value of $d(t)$, d_0 we get $M_{21} = e_2/i^1 = M_{21}^0$. This equality in asymptotic limit tends to reinforce the relativistic result.

Now let's analyze the result. We want to compare :

$$\begin{cases} M_{21}^0 = -\alpha G(d_0) \frac{d}{dt}(\bullet) \\ \\ M_{21} = -\alpha\,(T+U)\,(\bullet) - \alpha G(d(t)) \frac{d}{dt}(\bullet) \end{cases} \tag{4.13}$$

The function vector related to the transmitter - receiver interaction that we consider is given by :

$$\begin{cases} e_1 = \left(r + L\partial_t + \frac{1}{C}\int_t dt\right) i^1 - \int_t dt \left\{ \begin{array}{c} M_{21}^0 \\ M_{21} \end{array} \right\} \partial_t i^2 \\ \\ 0 = -\int_t dt \left\{ \begin{array}{c} M_{21}^0 \\ M_{21} \end{array} \right\} \partial_t i^1 + (r' + L'\partial_t)\, i^2 \end{cases} \tag{4.14}$$

(r, L, C, r', L') are the components of the equivalent diagrams of transmit and receive antennas. From the system 4.14 we deduce the metric that depends on the dissipations and couplings by magnetic field in the only mobile case :

$$G = \begin{bmatrix} \left(r + \frac{t}{C}\right)^2 + \alpha^2 (T+U)^2 & -\alpha (T+U) \left[r + r' + \frac{t}{C}\right] \\ -\alpha (T+U) \left[r + r' + \frac{t}{C}\right] & (r')^2 + \alpha^2 (T+U)^2 \end{bmatrix} \tag{4.15}$$

The metric G^o in the static case being :

$$G^o = \begin{bmatrix} \left(r + \frac{t}{C}\right)^2 & 0 \\ 0 & (r')^2 \end{bmatrix} \tag{4.16}$$

The difference between the mobile case and the static case appear clearly. The moving metric has extra-diagonal terms while the static metric is purely diagonal. The moving space is curvilinear where the static space is flat. Inertia, on the other hand, is not fundamentally different. In the static case :

$$\mathcal{L}^o = [\psi]^T \begin{bmatrix} L & -\alpha G(d_0) \\ -\alpha G(d_0) & L' \end{bmatrix} \tag{4.17}$$

and in the dynamic case :

$$\mathcal{L} = [\psi]^T \begin{bmatrix} L & -\alpha G(d(t)) \\ -\alpha G(d(t)) & L' \end{bmatrix} \tag{4.18}$$

Evidently the evolution of the distance in the interaction changes the inertia, but its expression remains identical. This means that under the conditions we have chosen, the response to electromagnetic induction is intrinsic and does not depend on motion, which seems logical. How do we interpret the change in metrics and what are the consequences ?

4.2.1 Metric of a dynamic system

We can not calculate the metric of a system made of two bodies in relative motion. On the other hand, we can project the object in motion into the observation frame to translate its transformation and to be able to reason with a single frame of reference. The given metric equation 4.15 has extra-diagonal terms. If the metric is purely diagonal, the distance s between two points in the ψ plane is given in the static case by :

$$s = \sqrt{G^o_{11} i^1 i^1 + G^o_{22} i^2 i^2 + 2 G^o_{12} i^1 i^2} \tag{4.19}$$

As G^o_{12} is zero, remains only the distance "*pythagorician*" $\sqrt{G^o_{11}i^1i^1 + G^o_{22}i^2i^2}$. This distance has the dimension of a potential. This means that the increase in any one of the directions i^1 or i^2 makes it possible to traverse the plan in a continuous and regular way. By scanning all the possible values of i^1 for each value of i^2 we cover the ψ plan. Now consider the second metric. A distance in this new plan is given by :

$$s = \sqrt{G_{11}i^1i^1 + G_{22}i^2i^2 + 2G_{12}i^1i^2} \tag{4.20}$$

However, the expressions of the previous metrics do not predict a possible curvature. The figure 4.1 shows the surface described by the static metric G^o and the figure 4.2 that described by the dynamic metric G. On the diagonal,

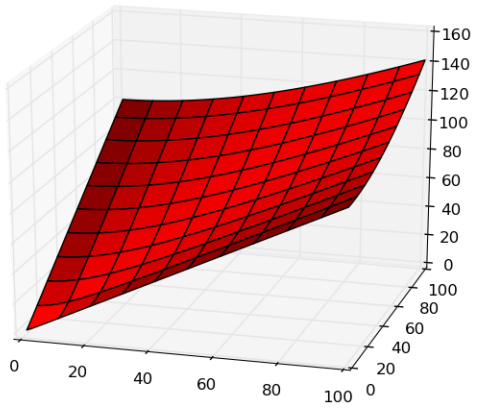


FIGURE 4.1 – Surface for a static metric

the distance covered s for $i^1 = x$ and $i^2 = y$ per G^o per unit is given by

$$s = \frac{\sqrt{x^2 + y^2}}{2} Cos\left(\frac{\pi}{4}\right) \tag{4.21}$$

Whereas in the case of the unitary second metric also, we get :

$$s = Cos\left(\frac{\pi}{4}\right)\sqrt{x^2 + y^2 + 2xy} \tag{4.22}$$

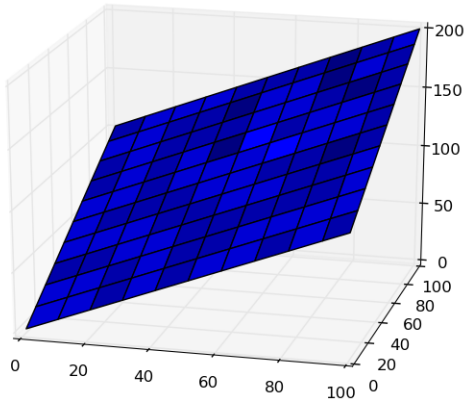


FIGURE 4.2 – Surface for a dynamic metric

In the first case we find $0.35\sqrt{2}$, so a smaller amplitude on the diagonal than on the edges for the same values of x and y. In the second case, for the same coordinates, the diagonal distance is identical and is equal to 0.7. This means that, in general, adding $2xy$ to the second metric raises the diagonal at the edges and the surface described is flat. In the first case it is concave.

But the curvature of the curvilinear surface is not expressed by its apparent geometric curvature. In the two previous cases, wherever we are on the ψ^o and ψ surfaces, the metric remains the same. The local rate of increase of the potential is constant on the whole surface. In the first case, if we share the available energy in both directions (meaning between the two uncoupled circuits), the resulting potential will be decreased. In the second case, it is maintained, the partition being redistributed by the coupling.

How to exploit this information ? The currents i^x can be generated by sources applied to the meshes that carry them. By studying the surface ψ we can detect whether excitations on certain axes generate significant increases in potentials. We indirectly see the co-space e_k. These conditions, in the case where these sources would be undesired sources, are obviously to be avoided and these maxima on the surface ψ are regions to avoid except that the system supports these maxima of constraints.

4.3 Exploring risk against an external field in a system

A recurring problem in EMC is to define the levels reported on the electronics by the presence of a field in the environment of the studied system. One solution is to provide us with a control line element, which we will place at different points of the system and with different loads, under the assumption that the line is not significantly different from a place to the other. Another problem that is not simple is the fact that the links can be differential or common mode. These are different strategies for hardening the system.

4.4 Analysis of a transistor oscillator

An oscillator is relaxation if a single alternation tends towards a stable state and the oscillation comes from the threshold control of a stage creating the dual alternation, itself then controlling the stage creating the first alternation, etc. Finally the oscillation results from opposite commands of non oscillating structures. Conversely, we can maintain a natural oscillator with losses like any real oscillator, the maintenance taking care to compensate these losses to maintain the oscillation to infinity. When an oscillator is serviced and started, the oscillation amplitude initially increases until a non-linear process is involved which will stabilize this amplitude. It is a minimum of losses, or the presence of a level non-linear material, etc.

We want to study here a particular structure inspired by another one called "Royer" named after its inventor. This is the basic schema shown in figure 4.3. We immediately understand from the view of this diagram that the oscillation is relaxation since each circuit connected to a switch (an NPN transistor) can not be intrinsically oscillating. Modify this structure by adding a capacitor to create a natural oscillator figure 4.4. It is this structure that we want to study. The first step for this goal is to develop the model for NPN transistors.

The principle is to activate the current in the coil T1 for example, the transistor T2 being blocked. T1 passing, the transplanting by the transformer induces on the bases of the transistors a ddp which blocks T1 and makes T2 passing. The current is reversed in the windings, etc. When we add capacitors, the purpose of the switches is to maintain the oscillation of the LC circuits formed by the transformer windings and the capacitors.

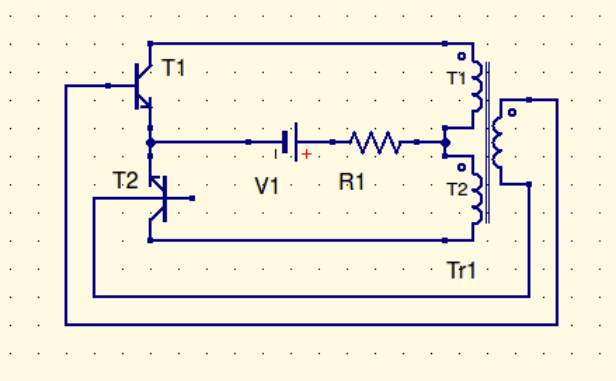


FIGURE 4.3 – Transistor oscillator schematic

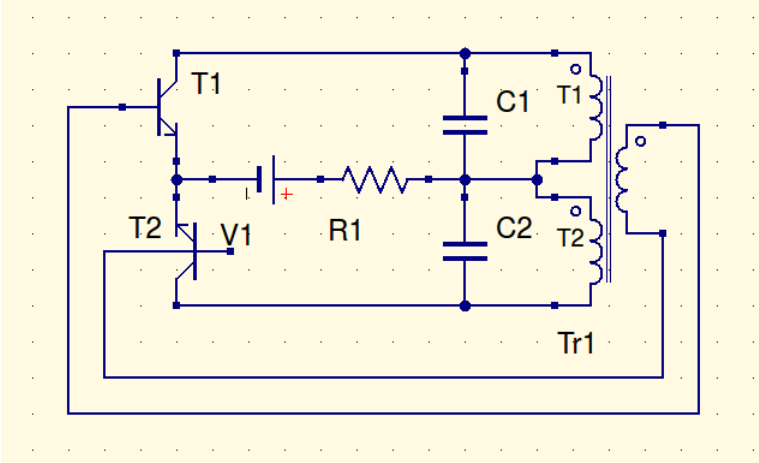


FIGURE 4.4 – Modified oscillator

4.4.1 First model of transistor

An NPN transistor is characterized by a collection of curves $I_c = f(V_{ce})$ which depend on the current flowing in the base-emitter junction of the transistor. The law giving the current in the collector as a function of the voltage across the transistor V_{ce} is of the form :

$$I_c = \left[-I_{cs} + \alpha V_{ce} + T_{cb}\left(1 - e^{-V_{ce}/V_O}\right)\right] \overset{V_{be}}{\mathcal{D}}_{>0} - I_{cs} \overset{V_{be}}{\mathcal{D}}_{<0} \tag{4.23}$$

The coefficient I_{cb} is a function of i_b basic current : $I_{cb} = \gamma i_b$; and α is a side that also depends on i_b : $\alpha = \sigma i_b$. The operators $\mathcal{D}$ are 0 or 1 depending on whether the parameter, here V_{be}, belongs to the index range or not. A typical pace of these characteristics as a function of i_b values is shown in figure 4.5.

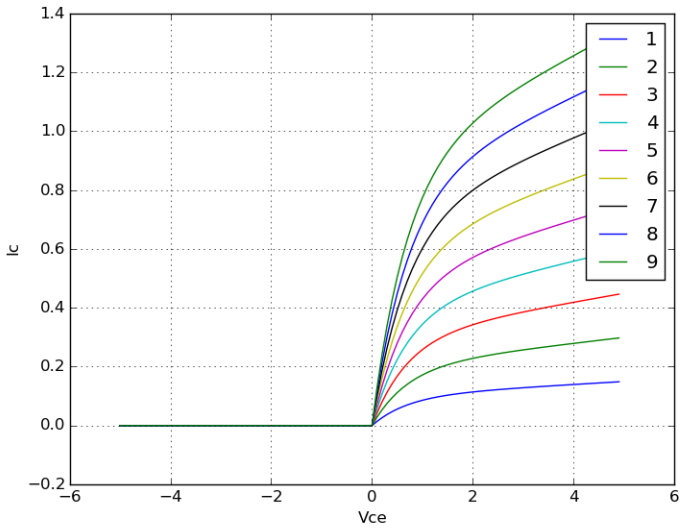


FIGURE 4.5 – NPN transistor characteristic

Intrinsically, the NPN transistor is a controlled current source and parameterized. To control the model of the transistor, we will first calculate a simple circuit, half the previous oscillator by taking a LC circuit powered by a generator and the NPN transistor can or not switch this circuit to the power supply. Figure 4.6 shows the scheme.

By associating the basic circuit i^b with two branches, and by naming i^c the collector-emitter current source we find the next impedance operator in

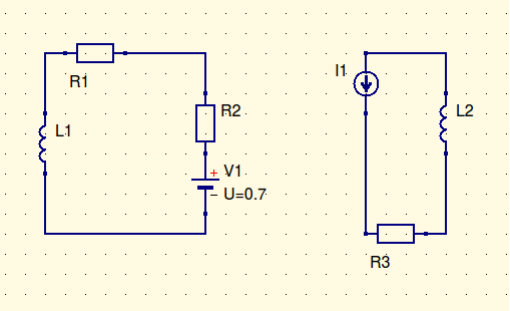


FIGURE 4.6 – NPN transistor schematic

the mesh-pair of nodes :

$$\zeta = \begin{bmatrix} L_1 p + R_1 + R_2 & 0 \\ 0 & L_2 p + R_3 \end{bmatrix} \tag{4.24}$$

By defining the condition $\alpha = 0$ (characteristic curve 4.7), the collector current can be written :

$$i^c = -I_{cs} + \gamma i^b \tag{4.25}$$

with $\gamma = \beta \left(1 - e^{-V_{ce}/V_0}\right) \overset{V_{be}}{\mathcal{D}}_{>0.7}$.

The system of equations to solve is given by :

$$\begin{cases} \left(L_1 p + R_1 + R_2\right) i^b = E_b \\ \\ \left(L_2 p + R_3\right) i^c = V_{ce} \end{cases} \tag{4.26}$$

In operation we can initiate a basic current to start. After which the collector current appears, depending on the base current and the collector - emitter voltage. This voltage is then updated according to the new collector current value, etc. The collector current is considered a source pair current source. This organization has the disadvantage of having to estimate the voltage V_{ce} in two steps to check if the junction is in saturation and to adjust the value of the collector current which depends on the voltage V_{ce}. We can look at the function of the NPN junction in another, less usual way. The figure 4.8 shows the returned curve of the previous $i^c(V_{ce})$ characteristic.

4.4.2 Another macromodel attempt

When the transistor is reverse biased, the collector - emitter junction is equivalent to a very strong resistor. The preceding curves have for equation :

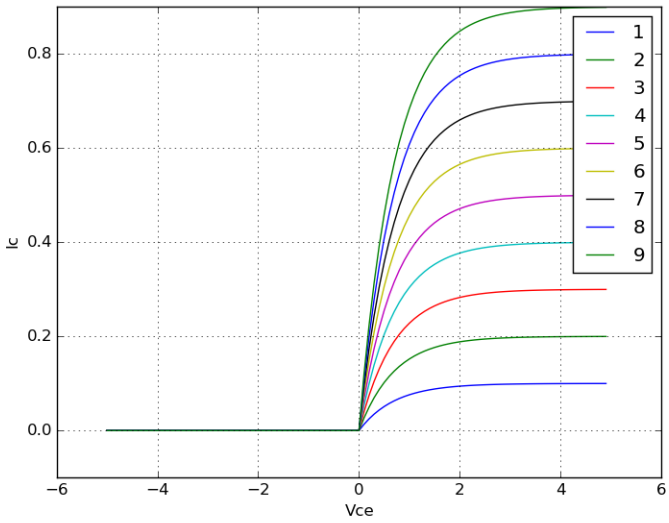


FIGURE 4.7 – Considered characteristics

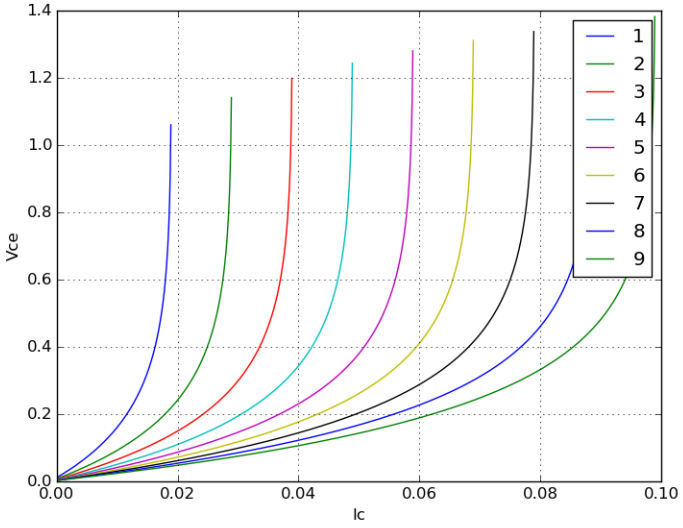


FIGURE 4.8 – Reverse characteristics

$$V_{ce}\left(i^b, i^c\right) = -V_0\, ln\left(1 - \frac{\left[i^c + I_{cs}\right]}{\beta i^b}\right) \tag{4.27}$$

The relationship between the voltage developed across the transistor and the current in its collector-emitter junction is an operator z_t parametrized by the base current : $V_{ce} = z_t\left(i^c\right)|_{i^b}$.

Let's calculate a circuit equipped with a transistor. Either a simple pair of meshes of equations :

$$\begin{cases} e_b - V_{be0}\overset{V_{be}}{\mathcal{D}}_{>0.7} = \left[\left(R_b + R_e\right)\overset{V_{be}}{\mathcal{D}}_{>0.7} + R_{bb}\overset{V_{be}}{\mathcal{D}}_{<0.7}\right] i^b \\ \\ E_{DC} = \left[z_t\left(i^c\right)|_{i^b} + R_c i^c\right]\overset{V_{be}}{\mathcal{D}}_{>0.7} + \left[R_{cb} + R_c\right]\overset{V_{be}}{\mathcal{D}}_{<0.7} i^c \end{cases} \tag{4.28}$$

E_{DC} is a continuous diet. How to solve such a system numerically ? It is rather painful because it is necessary to appeal to methods by approximations and convergences. In addition, the logarithm function does not drift well. It is better to find a macromodel that allows to conduct the calculation directly if it is possible. For that we are inspired by the graph presented figure 4.9.

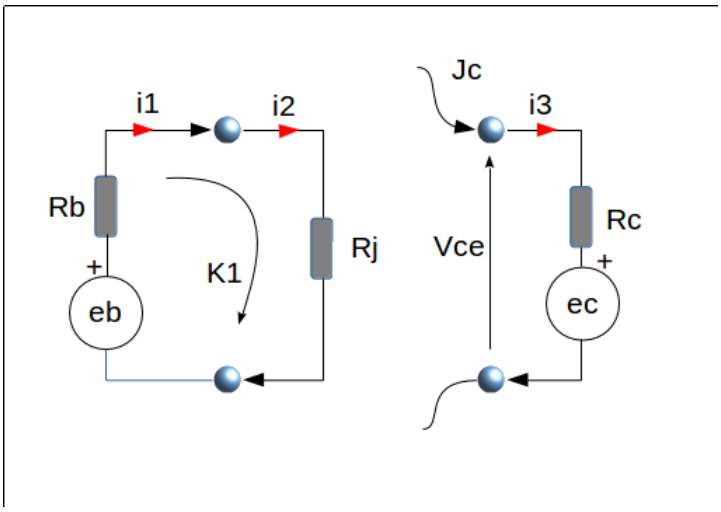


FIGURE 4.9 – Transistor macromodel

The collection of the three branches leads to the matrix impedance z

following :

$$z = \begin{bmatrix} R_b & 0 & 0 \\ 0 & R_j & 0 \\ 0 & 0 & R_c \end{bmatrix} \tag{4.29}$$

According to the scheme, the connection between branch currents and the K^1 mesh or imposed J^c is :

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.30}$$

By calculating the transformation $C^T z C$ we find the tensor ζ such that its matrix is :

$$\zeta = \begin{bmatrix} R_b + R_j & 0 \\ 0 & R_c \end{bmatrix} \tag{4.31}$$

and for the sources :

$$C^T e = \begin{bmatrix} e_b & e_c + V_{ce} \end{bmatrix} \tag{4.32}$$

The system of equations to be solved is then :

$$\begin{cases} e_b = (R_b + R_j)\, K^1 \\ V_{ce} = e_c + R_c J^c \\ J^c = \beta K^1 \end{cases} \tag{4.33}$$

that we can reorder this system to bring up the resolution algorithm and introduce the base junction threshold voltage V_{beo} as well as the collector conduction condition :

$$\begin{cases} K^1 = (e_b - V_{beo})\, (R_b + R_j)^{-1} \\ J^c = \beta K^1 \overset{K^1}{\mathcal{D}}_{>0} \\ V_{ce} = e_c + R_c J^c \end{cases} \tag{4.34}$$

We program the sequence of equations including the threshold voltage V_{beo} of the junction for a sample index t and a domain of computation of N samples for N/T periods :

— $e_b(t) = \left[1 + \frac{1}{2} Sin\left(2\pi \frac{t.dt}{T}\right)\right] e^{-\frac{t}{N}}$
— $K^1(t) = \frac{e_b(t) - V_{beo}}{R_b + R_j}$
— $J^c(t) = \beta K^1(t) \overset{K^1}{\mathcal{D}}_{>0}$
— $V_{ce}(t) = e_c + R_c J^c(t)$

The result obtained with this program is given in figure 4.10.

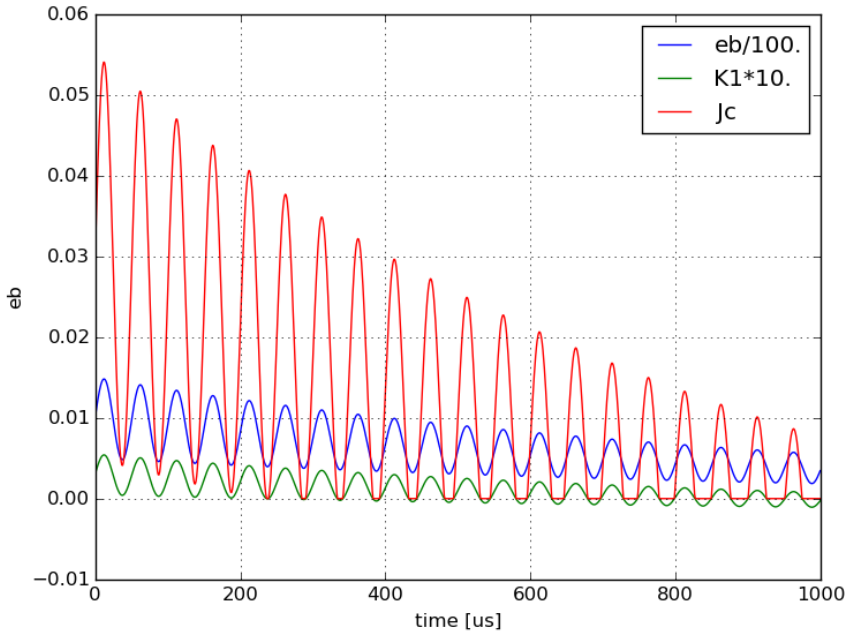


FIGURE 4.10 – Waveforms obtained

Let's talk about these results. At the beginning of the waveform for e_b the level is always higher than the diode voltage V_{beo}. The base current follows the applied control voltage with a peak value of approximately 2.5 mA. The collector current follows the basic control with a peak amplitude of 22.5 mA, which is a current amplification of about 10. When the command decreases and does not systematically exceed the basic threshold, the command is canceled and the output current is canceled. The resulting shape then becomes deformed, capping at 0 A minimum value.

The graph given in 4.9 thus represents the operation of an NPN transistor, except that it does not reproduce the saturation at the collector if the

collector current is greater than the available current. We must add a domain to translate this saturation. We write :

$$J^c(t) = \beta K^1(t) \overset{K^1}{\mathcal{D}}_{>0} \overset{K^1}{\mathcal{D}}_{<\frac{e_c}{\beta R_c}} + \frac{e_c}{R_c} \overset{K^1}{\mathcal{D}}_{>0} \overset{K^1}{\mathcal{D}}_{>\frac{e_c}{\beta R_c}} \tag{4.35}$$

This "empirical" model fails to account for the value of the basic resistance. We see that the development of a model from curves without further knowledge of the detail of the functioning of the component at the underlying scale is difficult. Let us take a microelectronic approach but try to adapt the formalism to the description constraint of the component.

4.4.3 Ebers-Moll's model

The Ebers-Moll model is based on the description of the transistor as a set of diodes. This approach describes the current as a function of the voltage developed across the diode, knowing its saturation current and this voltage, supplemented by the gain of the transistor. Remembering the important parameter :

$$\frac{kT}{q} = \alpha = 26mV \tag{4.36}$$

The emitter current for an NPN transistor is defined by :

$$i^E = I_{es}\left(e^{\frac{V_{be}}{\alpha}} - 1\right) \tag{4.37}$$

Knowing i^E we deduce i^C :

$$i^C = \frac{\beta}{\beta + 1} i^E \tag{4.38}$$

and the base current by :

$$i^b = \frac{i^C}{\beta} \tag{4.39}$$

Depending on the polarization of the transistor, we deduce if the transistor is saturated or linear. The coupling of the NPN transistor is expressed in admittance. We must therefore couple equations in admittance and impedance. This work has been studied in a general way to keep a coherence and a form

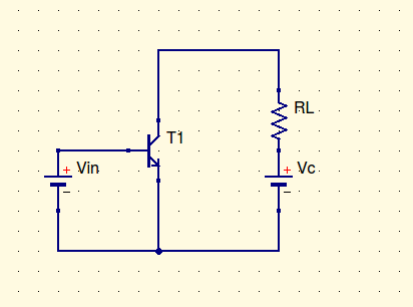


FIGURE 4.11 – Ebers-Moll

in adequacy with the description of networks in cellular topology [2]. Consider the diagram presented figure 4.11.

We have for this mock-up :

$$i^C = \frac{\beta}{1+\beta} I_{es} \left(e^{\frac{V_{in}}{\alpha}} + 1 \right) \tag{4.40}$$

This single equation may suffice to express the oscillator circuit with a current source determined by the base-emitter voltage. Typically $I_{es} = 10^{-14}$. It remains to express the saturation of the transistor if the emitter current called by the base voltage becomes greater than the available current of the circuit :

$$i^C_{max} = \frac{V_c - V_{cesat}}{R_L} \tag{4.41}$$

with $V_{cesat} \approx 0,2$. We complete this model :

$$i^C = 0,99\, I_{es} \left(e^{\frac{V_{in}}{\alpha}} + 1 \right) \left[1 - e^{-\frac{i^C}{i^C_{max}}} \right] \tag{4.42}$$

We can now resume the diagram of the oscillator on the previous basis whose limits and advantages we master.

4.4.4 Transistor oscillator

We consider the diagram of the oscillator given figure 4.12.

2. Maurice, O., & Reineix, A. (2010, April). Analyse tensorielle des réseaux hybridée comme outil théorique pour la CEM et l'électromagnétisme. In CEM2010 (pp. A-Reineix).

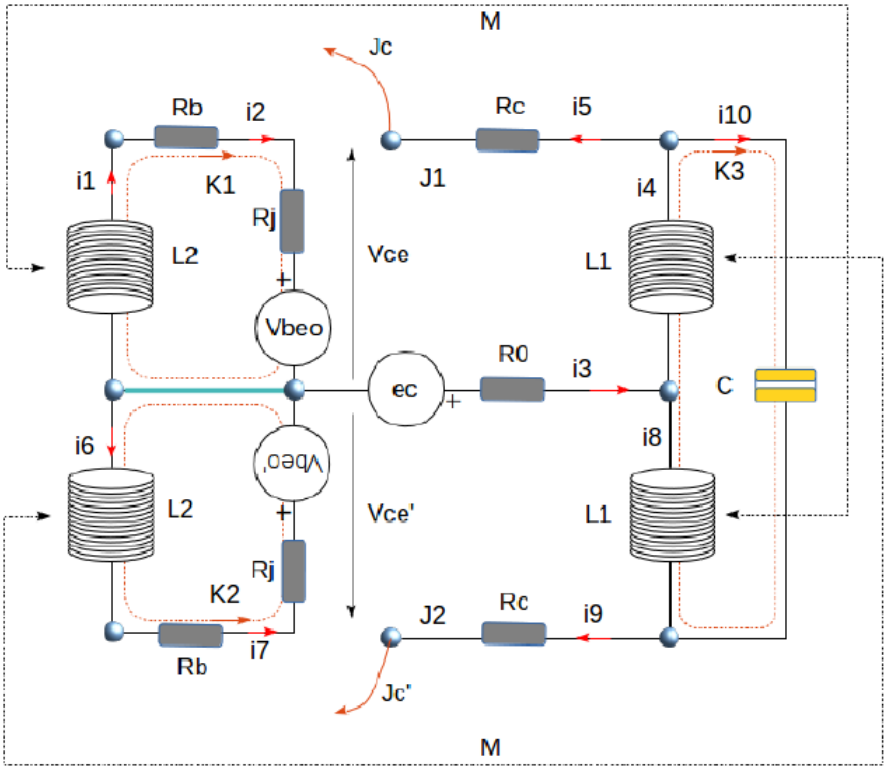


FIGURE 4.12 – Oscillator schematic

To write the matrices of the problem, we must define a connectivity between the currents on the branches and the mesh currents, including the source of current J. This kind of matrix can be constructed using a spreadsheet. Figure 4.13 shows the result for our circuit.

	A	B	C	D	E	F
1	branch current	K1	K2	K3	J1	J2
2	1	1	0	0	0	0
3	2	1	0	0	0	0
4	3	0	0	0	1	1
5	4	0	0	1	1	0
6	5	0	0	0	1	0
7	6	0	1	0	0	0
8	7	0	1	0	0	0
9	8	0	0	-1	0	1
10	9	0	0	0	0	1
11	10	0	0	1	0	0

FIGURE 4.13 – Spreadsheet for the connectivity matrix

Next step consists in changing of space, starting from the definition of the circuit in the branches space. First of all, we must give the definition of the impedance operator z (s is the Laplace operator) : 4.4.4.

$$z = \begin{bmatrix} L_2 s & 0 & 0 & 0 & 0 & 0 & 0 & -M_{18}s & 0 & 0 \\ 0 & R_b + R_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_1 s & 0 & -M_{46}s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -M_{64}s & 0 & L_2 s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_b + R_j & 0 & 0 & 0 \\ -M_{81}s & 0 & 0 & 0 & 0 & 0 & 0 & L_1 s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix} \tag{4.43}$$

There are two couplings through the transformers : one between the branches 1 and 8, and one between the branches 4 and 6.

To obtain the expression of ζ, the impedance operator in the mesh space, we compute the relation $C^T.z.C$ which gives the matrix shown equation 4.4.4.

$$\zeta = \begin{bmatrix} R_j + R_b + L_2 s & 0 & M_{18} s & 0 & -M_{18} s \\ 0 & R_j + R_b + L_2 s & -M_{46} s & -M_{46} s & 0 \\ M_{81} s & -M_{46} s & 2L_1 s + \frac{1}{sC} & L_1 s & -L_1 s \\ 0 & -M_{46} s & L_1 s & R_0 + R_e + L_1 s & R_0 \\ -M_{81} s & 0 & -L_1 s & R_0 & R_0 + R_c + L_1 s \end{bmatrix} \tag{4.44}$$

The tensorial equation $e_k = \zeta_{km} K^m$ is completed by the equations :

$$\begin{cases} J^1 = \beta K^1 \overset{K^1}{\mathcal{D}}_{K^1>0} \\ \\ J^2 = \beta K^2 \overset{K^2}{\mathcal{D}}_{K^2>0} \end{cases} \tag{4.45}$$

The source vector e_k is given by :

$$e_k = \begin{bmatrix} -V_{beo} \overset{K^1}{\mathcal{D}}_{K^1>0} & -V_{beo} \overset{K^2}{\mathcal{D}}_{K^2>0} & 0 & V_{ce}^1 & V_{ce}^2 \end{bmatrix} \tag{4.46}$$

and

$$R_j = R_{jp} \overset{K^1}{\mathcal{D}}_{K^1>0} + R_{jb} \overset{K^1}{\mathcal{D}}_{K^1<0} \tag{4.47}$$

(the same for R_j on K^2).

The problem is completely solved using the equation $e_q = \zeta_{qm} K^m$. This is not really a difficulty in time domain finite differences. What is more interesting is to study the influence of the load on the oscillator behaviour. It means to compute, knowing $e_b = \zeta_{ba} K^a$:

$$\frac{\partial \zeta_{ba}}{\partial R_c} \tag{4.48}$$

$$\frac{\partial \zeta_{ba}}{\partial R_c} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.49}$$

and so in that case :

$$\frac{\partial \zeta_{ba}}{\partial R_c} K^a = \frac{\partial}{\partial R_c} e_b \Rightarrow \frac{\partial}{\partial R_c} e_b = \delta_{bc} J^c \tag{4.50}$$

δ being the Kronecker symbol. It means here that the circuit depends on R_c only for the relation with the imposed current sources. R_c doesn't affect here other components of the problem.

Chapitre 5

Light, information & nervous transmission system

Any information network can be modelled using the technique of the *gamma matrix*[1]. Let's take an example to illustrate this technique. We imagine a channel made of a first frontier between a source and some propagation medium. This medium is closed at its end by a mirror. We work in the optical domain. On the left and on the right of the frontier we can name optical waves on the left (index g) and on the right (index d). We have propagation waves identified with a symbol p and backward waves identified with a σ. The incident wave on the frontier 1 is p_1^g on the left side. A wave can be reflected from this incident power σ_1^g, and another wave can be transmitted on the other side of the frontier p_1^d. This transmitted wave propagates and reach the left side of the extremity, p_2^g. A part can be reflected on this limit σ_2^g. Another part can be absorbed by the limit condition p_2. Finally the problem has six observables. We can define a wave vector v that has for components the various observables :

$$v = \left[\begin{array}{cccccc} p_1^g & \sigma_1^g & p_1^d & \sigma_1^d & p_2^g & \sigma_2^g \end{array} \right] \tag{5.1}$$

the first component is associated with the source of light. We must defined the cœfficients of reflection and transmission. We can construct a tab showing the correspondences between each waves. Figure 5.1 shows such a tab.

The rules to fill this kind of tab are :

1. relations of reflection or transmission can exist only at a frontier level ;

1. See reference on gamma matrices.

	p1g	s1g	p1d	s1d	p2g	s2g
p1g	0	0	0	0	0	0
s1g	r1gg	0	0	t1gd	0	0
p1d	t1dg	0	0	0	0	0
s1d	0	0	0	0	0	exp(-x)
p2g	0	0	exp(-x)	0	0	0
s2g	0	0	0	0	r2gg	0

FIGURE 5.1 – Spreadsheet for the gamma matrix

2. between waves that belong to two different frontiers, the relations are those of dispersion ;
3. in case of maintained source, a cœfficient 1 is put for the first propagation wave cell.

Applying the previous rules to construct the tab, we can define the γ matrix :

$$\gamma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ r^1_{gg} & 0 & 0 & t^1_{gd} & 0 & 0 \\ t^1_{dg} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-x} \\ 0 & 0 & e^{-x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r^2_{gg} & 0 \end{bmatrix} \tag{5.2}$$

We can now look at what appends if we apply γ to v : γv. At the beginning, the vector v has for only not zero component the source p^g_1. So :

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.3}$$

If we compute γv we obtain :

$$v = \begin{bmatrix} 0 & r^1_{gg} & t^1_{dg} & 0 & 0 & 0 \end{bmatrix} \tag{5.4}$$

As the source has a normalized amplitude, the wave amplitude that can be measured on port 2 (r^1_{gg}) is the reflected part of the incident wave. The rest of the power is transmitted through the frontier in the propagation medium. Now let's take a look to what appends if we apply two times γ :

$$\gamma\gamma v = \begin{bmatrix} 0 & 0 & 0 & 0 & t^1_{dg}e^{-x} & 0 \end{bmatrix} \tag{5.5}$$

here e^{-x} is a delay. The transmitted wave on the first frontier is propagated until the second frontier, with a delay x (x is something like $-x/cs$). We apply now three times γ :

$$\gamma\gamma\gamma v = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & r^2_{gg}t^1_{dg}e^{-x} \end{bmatrix} \tag{5.6}$$

the wave can go back depending on the reflection cœfficient r^2_{gg}, to propagate until the input of the channel making $\gamma\gamma\gamma\gamma v$:

$$\gamma\gamma\gamma\gamma v = \begin{bmatrix} 0 & 0 & 0 & r^2_{gg}t^1_{dg}e^{-2x} & 0 & 0 \end{bmatrix} \tag{5.7}$$

The previous wave is now located on the right of the first frontier. Let's multiply by γ one more time :

$$\gamma^5 v = \begin{bmatrix} 0 & t^1_{gd}r^2_{gg}t^1_{dg}e^{-2x} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.8}$$

The wave is transmitted to the measurement on port 2. But it lacks something : a part may be reflected to the medium. It lacks a cœfficient r^1_{dd} in our matrix. We have now :

$$\gamma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ r^1_{gg} & 0 & 0 & t^1_{gd} & 0 & 0 \\ t^1_{dg} & 0 & 0 & r^1_{dd} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-x} \\ 0 & 0 & e^{-x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r^2_{gg} & 0 \end{bmatrix} \tag{5.9}$$

and the wave vector becomes :

$$\gamma^5 v = \begin{bmatrix} 0 & t^1_{gd}r^2_{gg}t^1_{dg}e^{-2x} & r^1_{dd}r^2_{gg}t^1_{dg}e^{-2x} & 0 & 0 & 0 \end{bmatrix} \tag{5.10}$$

A part is measured on port 2, another part starts again to propagate in the medium, etc.

5.1 From classical to quantum dynamics

The wave vector gives intensities associated with the number of photons in the light. But if the source sends only one photon, the previous intensities and cœfficients becomes no more floating values but should becomes one or zero. It means that the cœfficients of the gamma matrix must become probabilities. the results of these probabilities transform the cœfficients in integer values 1 or zero. How it works ? If a component r has for amplitude 0.01, it means that the photon has 1 chance over 100 to go back from this frontier. We can compute a random function $P(100)$ that extracts one number between 100. The function $a = E\left[P(100)/100\right]$ has one chance over 100 to be equal to 1. r is so replaced by this probability a. The transmitted probability

is automatically given by the conservation law. As $r+t=1$, if t corresponds to the probability b, $b=1-a$. The same replacement is operated for all the components of the gamma matrix. Applying in this case the gamma matrix to the wave vector will propagate a photon over all the network studied. After each application of γ on a wave vector v, v must be projected from the probabilistic world of the quantum mechanics in the deterministic world by giving values to these probabilities. A toss is done at each time step to replace the components of v by the integers 1 or 0. After what the application of γ to the vector wave v can be read.

Let's take an example. If a component r has for amplitude 0.06 for example, it means that we have 6 chances over 100 to select a 1, and 94 chances over 100 to select a 0. To realize this kind of probability, we can construct a list made of six "1" and 94 "0". After what, we make a random selection in this list, random selection that gives the wanted amplitude.

We consider a source of light of normalized amplitude. This source illuminates a first equipment, a first frontier 1 that gives a reflection coefficient of $r_{gg}^1 = 0.02$. The reflected part is directed to a measurement unit that count the photons. Then behind the frontier, a medium of length x propagates the transmitted photons (with probability $p_1^d = \left(1 - r_{gg}^1\right) p_g^1$) until a second frontier with a delay of 2 ns. At this second frontier, the amplitude of probability to reflect the photon is $r_{gg}^2 = 0.85$. We want to model this optical channel. First we must define the γ-matrix.

The figure 5.2 shows the channel we consider.

The γ matrix is given by :

$$\gamma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_{gg}^1 & 0 & 0 & t_{gd}^1 & 0 & 0 & 0 \\ t_{dg}^1 & 0 & 0 & r_{gd}^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\theta x} & 0 \\ 0 & 0 & e^{-\theta x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{gg}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_{dg}^2 & 0 & 0 \end{bmatrix} \tag{5.11}$$

We start the process. The first application of γ at a given time t to v gives :

$$v(t) = \gamma v(0) = \begin{bmatrix} 0 & r_{gg}^1 & t_{dg}^1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.12}$$

One component of the wave vector can be written in general :

$$\psi^{(k)} = Ae^{-\theta s} \tag{5.13}$$

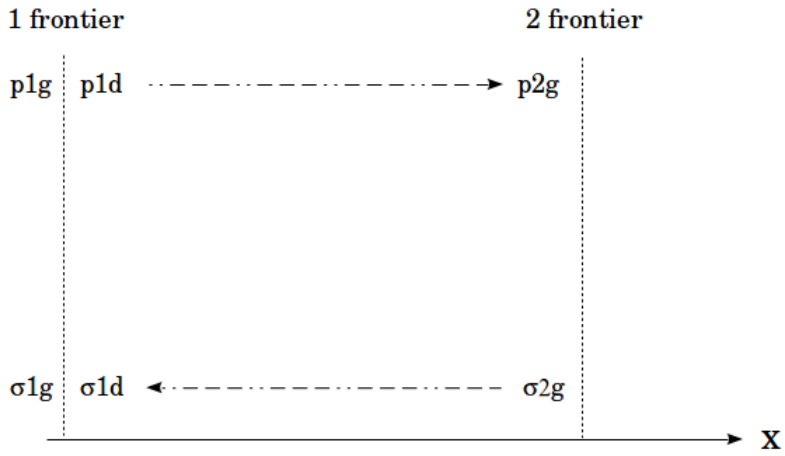


FIGURE 5.2 – Channel studied

s is Laplace's operator[2]. If we want to extract the phase property of this component, we can operate :

$$-\frac{1}{A}\left(\frac{\partial\psi^{(k)}}{\partial s}\right) = \theta \tag{5.14}$$

if n is the power of 10 which is defined by :

$$\forall(x,y),\ r^x_{yy} = R^x_{yy}.10^{-n} = \frac{R^x_{yy}}{10^n} \tag{5.15}$$

It means that we can make a distribution D^x_{yy} of numbers, only constituted with 0 and 1, having R "1" and $(10^n - R)$ "0". For example if $r^x_{yy} = 0.2$, $D^x_{yy} = [1,1,0,0,0,0,0,0,0,0]$, with $R = 2$ "1" and $10 - 2 = 8$ "0". A measurement applied to the vector v means to define a random vector variable $\tilde{u}$ which has for dimension the same dimension as D (equal to 10^n) and for which only one component is equal to 1 (randomly). The measurement m is so defined by :

$$m = \int_t dt\tilde{u}_x \cdot D^x_{yy}(t) \tag{5.16}$$

2. Remember that when the exponent is between parenthesis, it means that it points out a particular abstract component and not the whole vector

If we spot a particle on port r^1_{yy} by the couple (t, m), the values of the couple component are given by :

$$\left(-\frac{1}{A}\left(\frac{\partial\psi^{(k)}}{\partial s}\right),\ \int_t dt\tilde{u}_x \cdot D^x_{yy}(t)\right) \tag{5.17}$$

How to proceed in order to extract one measurement on one particular component (k) of v ? Starting from the state of v at a given time 5.18, this vector can be written :

$$v(t=1) = \gamma^1 v(0) = \left[\begin{array}{ccccccc} 0 & \psi^2 & \psi^3 & 0 & 0 & 0 & 0 \end{array}\right] \tag{5.18}$$

The component $\psi^{(x)}$ is a complex number, $\forall x$. We have for example $r^1_{gg} = 2.10^{-2}$ on port 2. So :

$$\psi^2 = 2.10^{-2} \Rightarrow R = 2,\ n = 2 \tag{5.19}$$

We apply the quantification : $D^2 \rightarrow \psi^2$ and $D^2 = [1]^2 + [0]^{100-2}$ where $[1]^2$ is a list of two "1" and $[0]^{98}$ is a list of 98 "0". As a consequence $t^2_{dg} = \left(1 - r^1_{gg}\right) = 98.10^{-2}$. Defining $\tilde{u} = [1]^x + [0]^{99}$, x being any location in $\tilde{u}$, we compute $m_2 = \tilde{u} \cdot D^2$. m_2 is the number of photon measured on port 2.

Until the order 5, nothing appears on port 2. After what :

$$\gamma^5 v = \left[\begin{array}{ccccccc} 0 & t^1_{gd}t^1_{dg}r^2_{gg}e^{-2\theta x} & r^1_{gd}r^2_{gg}t^1_{dg}e^{-2\theta x} & 0 & 0 & 0 & 0 \end{array}\right] \tag{5.20}$$

On port 2 we acquire $t^1_{gd}t^1_{dg}r^2_{gg}e^{-2\theta x}$. But these numbers are complex, and it is not easy to extract the delays from the amplitudes in this expression. We can define a first matrix γ where the delays are replaced by cœfficients 1 :

$$\gamma = \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r^1_{gg} & 0 & 0 & t^1_{gd} & 0 & 0 & 0 \\ t^1_{dg} & 0 & 0 & r^1_{gd} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r^2_{gg} & 0 & 0 \\ 0 & 0 & 0 & 0 & t^2_{dg} & 0 & 0 \end{array}\right] \tag{5.21}$$

and a second matrix $\bar{\gamma}$ where the amplitude cœfficients are replaced by 1 and delays appear :

$$\bar{\gamma} = \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\theta s} & 0 \\ 0 & 0 & e^{-\theta s} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{5.22}$$

with these new definitions :

$$v(t) = (\gamma + \bar{\gamma})^n \, v(0) \tag{5.23}$$

The measurement on port x becomes easier to define :

$$\left(-\frac{1}{s} \, log \left[(\bar{\gamma}^n v(0))^x \right], \int_t dt\tilde{u}(t) D^x(t) \right) \tag{5.24}$$

We can compute the evolving of the amplitude on port 2, by delineating this amplitude on a curve. That's what was done and shown figure 5.3.

The curve shows that the probability to capture a particle on port 2 is first around 1% while it becomes to be near of 98% in final. Note that this curve is obtained for a continuous excitation, i.e. when the γ_{11} component is equal to 1. The impulse response is given by the time derivative of this result.

Note that in fact all the time step are known. If we can define the length of the propagation structures, let say $x_1, x_2, x_3, \ldots$. Seen from a given port, the waves must make round trip each time they come back to this port. So the times to make these travels is twice the summation of the various travelling times made in the various media which can be crossed one, two, three times, etc. A particular travel time is given by :

$$\tau_1 = 2 \left(n\frac{x_1}{v_1} + m\frac{x_2}{v_2} + p\frac{x_3}{v_3} + \ldots \right) \tag{5.25}$$

The list $n, m, p, \ldots$ is a particular mode of propagation in the structure. We can compute all the possible modes with the equation 5.25 and by a simple sort ascending of the values memorized in a list T, each new component appearing in $\gamma^n v$ can be associate with the next component of T. This allows to trace the curve $v(t)$ easily for a defined propagation structure.

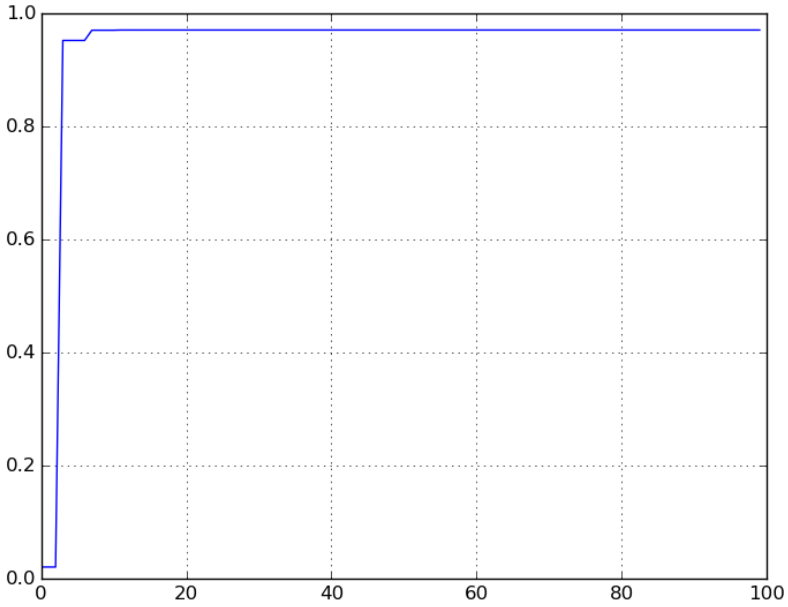


FIGURE 5.3 – Curve of the function A(t) on port 2

Chapitre 6

Mechanics

As much electronics could benefit of all the developments around schematics and graphs, as much it's not so evident for mechanics. In mechanics, initial positions appear as initial conditions. Then we want to compute the displacements either internal or external to the system depending on applied forces. Clearly the fluxes are speeds. The first property involved in mechanics is elasticity or plasticity. While the two fundamental Newton's laws says that both impulse and inertia moment are invariants. We write :

$$\mathbf{f} = \frac{d}{dt}\mathbf{p} \quad \tau = \frac{d}{dt}\mathbf{L} \tag{6.1}$$

Major property for structures is the elasticity. That's why we take a look first to this property.

6.1 Elasticity

Elasticity comes from Hook's idea that a body answers to an applied force, in some defined limits, proportionally to the force. i.e. :

$$\mathbf{f} = k\mathbf{u} \Rightarrow \mathbf{f} = \int_t dtk\mathbf{v} \tag{6.2}$$

k is called the elasticity constant, u is a displacement and v a speed.

The energy stored in the material coming from the elasticity is given by :

$$E = \frac{1}{2}ku^2 \tag{6.3}$$

The pressure if defined by $\sigma = F/S$, and Young's module by $E = \sigma/\epsilon$. ϵ is the deformation given by :

$$\epsilon = \frac{\Delta u}{u_0} \tag{6.4}$$

and finally :

$$\mathbf{f} = \sigma \mathbf{S} = \epsilon E \mathbf{S} = \frac{ES}{u_0} \Delta \mathbf{u} = k \Delta \mathbf{u} \tag{6.5}$$

leading to the relation between Hook's constant and Young's one :

$$k = \frac{ES}{u_0} \tag{6.6}$$

Similar definitions can be used for shear or compression constraints. Let's detail a little more these concepts.

6.1.1 Tensor of tensions

We have said that the change in length of the solid can be written :

$$\frac{\delta l}{l} = \frac{\sigma}{E} \tag{6.7}$$

E is Young's module and l the solid length. σ is the pressure. For a segment perpendicular to the force, the change in its length R is given by :

$$\frac{\delta R}{R} = -\frac{\nu}{E} \sigma \tag{6.8}$$

ν is called Poisson's cœfficient. This gives Poisson's law :

$$\frac{\delta R}{R} = -\nu \frac{\delta l}{l} \tag{6.9}$$

Hooke said that the variation of volume is proportional to the pressure, i.e. :

$$\frac{\delta V}{V} = -\chi \sigma \tag{6.10}$$

The three elastic constants (E, ν, χ) can be associated using a superposition principle.

We can make acting the forces in the three directions of the space writing :

$$\frac{\delta V}{V} = \frac{\Delta x^1}{x^1} + \frac{\Delta x^2}{x^2} + \frac{\Delta x^3}{x^3} = \frac{3\sigma}{E}(1 - \nu - \nu) \tag{6.11}$$

which gives :

$$\chi = \frac{3}{E}(1 - 2\nu) \tag{6.12}$$

The value $\nu = 1/2$ characterize a incompressible solid. For shearing, the observable is the angle of deformation, but the principle remains the same, we associate an angle to the tension for small deformations.

Deformation tensor

We can define the displacement $y(x + \delta x)$ of a point Q, neighbour of P under the form :

$$y_i\left(x^k + \delta x^k\right) = y_i\left(x^k\right) + D_{ik}\delta x^k \tag{6.13}$$

Leading to

$$D_{ik} = \frac{\partial}{\partial x^k} y_i\left(x^k\right) \tag{6.14}$$

D can be separated in its symmetric and asymmetric parts with :

$$D_{ik} = R_{ik} + \epsilon_{ik} \tag{6.15}$$

and :

$$\begin{cases} R_{ik} = \frac{1}{2}(D_{ik} - D_{ki}) = -R_{ki} \\ \\ \epsilon_{ik} = \frac{1}{2}(D_{ik} + D_{ki}) = +\epsilon_{ki} \end{cases} \tag{6.16}$$

R is the rotation tensor while ϵ is the deformation tensor.

6.2 General approach for the structure

A general equation for the structures may be :

$$m_{ik}\frac{dv^k}{dt} + F_{ik}\frac{dx^k}{dt} + (R_{ik} + \epsilon_{ik})\, x^k = f_i(t) \tag{6.17}$$

m is the inertia operator, F the dissipation operator. The equation can be rewritten :

$$m_{ik}\frac{dv^k}{dt} + F_{ik}v^k + (R_{ik} + \epsilon_{ik})\int_t dt v^k = f_i(t) \tag{6.18}$$

This equation answer to the first form of 6.1. A similar relation answers to the second form :

$$\frac{d}{dt}L_i + \Theta_{ik}v^k + \Omega_{ik}\int_t dtv^k = \tau_i(t) \tag{6.19}$$

Θ are dissipations in rotations, $\Omega_{ik}x^k$ are the torques apply on a structure. Let's make some example to see how to use these relations. First we study a simple structure made of timbers [1], shown figure 6.1.

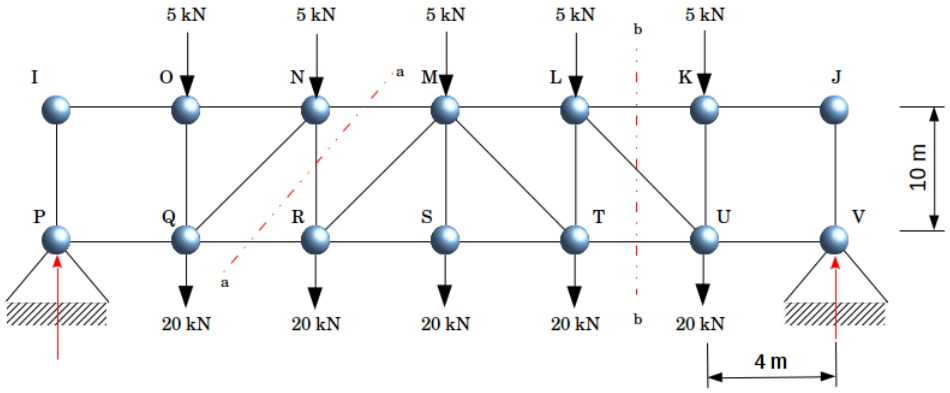


FIGURE 6.1 – A first structure

As we don't have any source or losses, our relation is reduced to :

$$\Omega_{ik}x^k = 0 \tag{6.20}$$

By replacement we obtain :

$$(4 + 8 + 12 + 16 + 20)\,\Omega_{00} - 24P = 0 \tag{6.21}$$

with $\Omega_{00} = 25.10^3$, this leads to $P = 62,5.10^3$ N.

If we want now to determine the force on the axis NR knowing the force P we can separate the domain in two using the aa axis which much cut

1. This example is extracted from Rea's problem solvers : Mechanics.

not more than three axis. Choosing as previously a y axis up, the simple summation of the forces $\sum_i f_i = 0$ gives :

$$-F_{NR} - 5.10^3 - 5.10^3 - 20.10^3 + 62,5.10^3 = 0 \Rightarrow F_{NR} = 32,5.10^3 \quad (6.22)$$

From the separate part on the right of the segment bb we can add two moments described figure 6.2.

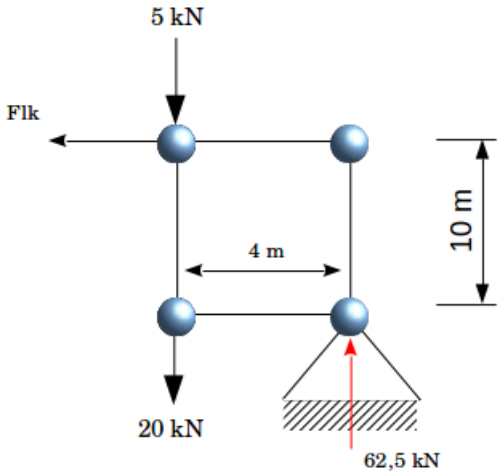


FIGURE 6.2 – Separate part in the structure

The summation leads to :

$$10F_{lk} + 4 \times 62,5.10^3 = 0 \Rightarrow F_{lk} = -25.10^3 \quad (6.23)$$

The negative sign says that the force has an opposite sense to the direction indicated on the figure.

6.3 Dynamic

Both equations 6.18 and 6.19 remain usable in dynamic. What we want to do now is to find a general approach for coupling separate pieces of mechanics in order to construct a system. We consider the system presented figure 6.3.

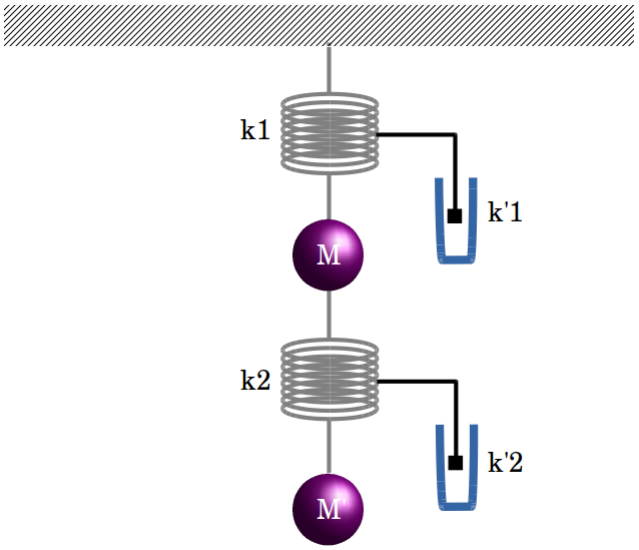


FIGURE 6.3 – Two coupled masses

Each mass M^x is a node where efforts are applied. Each spring has its own losses, represented by the friction part k'_x. Each spring has been tested before in order to characterize its properties k and k' and M. Any kind of mechanical piece can be characterized such a way. If s and s' are the abscissa associated with each part, we can write the system using 6.18 :

$$\begin{cases} M\frac{d^2}{dt^2}s + k'_1\frac{d}{dt}s + k_1 s = 0 \\ \\ M'\frac{d^2}{dt^2}s' + k'_2\frac{d}{dt}s' + k_2 s' = 0 \end{cases} \tag{6.24}$$

If we excite the first mass using a source of vibration $F_m Sin\,(\omega t)$, the previous system becomes :

$$\begin{cases} M\frac{d^2}{dt^2}s + k'_1\frac{d}{dt}s + k_1 s = F_m Sin\,(\omega t) \\ \\ M'\frac{d^2}{dt^2}s' + k'_2\frac{d}{dt}s' + k_2 s' = 0 \end{cases} \tag{6.25}$$

This can be written defining a mechanical impedance operator z attached with the system of masses :

$$z = \begin{bmatrix} M\frac{d^2}{dt^2}(\bullet) + k'_1\frac{d}{dt}(\bullet) + k_1 & 0 \\ 0 & M'\frac{d^2}{dt^2}(\bullet) + k'_2\frac{d}{dt}(\bullet) + k_2 \end{bmatrix} \tag{6.26}$$

Now we want to couple these two pieces for realizing the system shown figure 6.3. The first action is to separate in z the intrinsic properties attached with the branches from the properties coming from the environment. Noting $z = m + \sigma + \epsilon$ we define :

$$m = \begin{bmatrix} M\frac{d^2}{dt^2}(\bullet) & 0 \\ 0 & M'\frac{d^2}{dt^2}(\bullet) \end{bmatrix} \tag{6.27}$$

$$\sigma = \begin{bmatrix} k'_1\frac{d}{dt}(\bullet) & 0 \\ 0 & k'_2\frac{d}{dt}(\bullet) \end{bmatrix} \tag{6.28}$$

$$\epsilon = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{6.29}$$

The operators of mass m and friction σ belong to the branches space B while the constraints operator ϵ is attached with the environment.

We replace from now s and s' by v and v', k' by σ, and we use indices. Looking at the figure 6.4 we see the processes under a topological representation between the separate pieces characterization and the system construction (we make the assumption that the two masses are subject to the spring restoring forces alone).

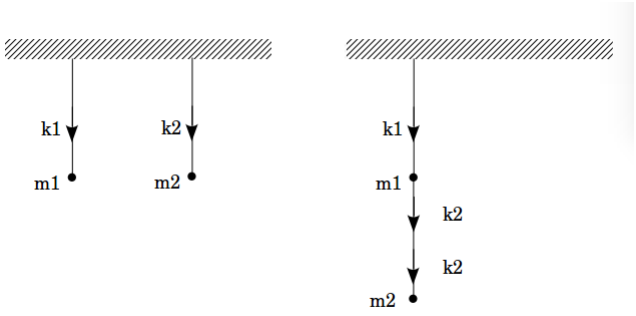


FIGURE 6.4 – Two coupled masses and the system construction

We must determine the relations between the external constraints and the masses in both cases. In the separate case, these relations lead to the constraints tensor given equation 6.29. When we connect the two systems, external constraints become involved following the relations :

	k_1	k_2
M	1	-1
M'	0	1

These relations lead to the connection matrix A with :

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \tag{6.30}$$

The constraints tensor in the branch space for the new system is defined by : $\epsilon = A.k.A^T$ and $f = A.f'$, which gives $(k \leftrightarrow \epsilon)$

$$\epsilon = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad f = \begin{bmatrix} F \\ 0 \end{bmatrix} \tag{6.31}$$

The system 6.25 can now be written :

$$\begin{cases} m_{11}\frac{d^2}{dt^2}s^1 + \sigma_{11}\frac{d}{dt}s^1 + k_{11}s^1 - k_{12}s^2 = F_m Sin\,(\omega t) \\ \\ -k_{21}s^1 + m_{22}\frac{d^2}{dt^2}s^2 + \sigma_{22}\frac{d}{dt}s^2 + k_{22}s^2 = 0 \end{cases} \tag{6.32}$$

Then we use :

$$s^1 = \int_t dtv^1 \quad s^2 = \int_t dtv^2 \tag{6.33}$$

we finally obtain :

$$\begin{cases} m_{11}\frac{d}{dt}v^1 + \sigma_{11}v^1 + k_{11}\int_t dtv^1 - k_{12}\int_t dtv^2 = F_m Sin\,(\omega t) \\ \\ -k_{21}\int_t dtv^1 + m_{22}\frac{d}{dt}v^2 + \sigma_{22}v^2 + k_{22}\int_t dtv^2 = 0 \end{cases} \tag{6.34}$$

Or, in general in a tensorial writing :

$$m_{ij}\frac{d}{dt}v^j + \sigma_{ij}v^j + \epsilon_{ij}\int_t dtv^j = f_i \tag{6.35}$$

which is our equation 6.18.

We have shown here how to link the Kron's spirit with mechanics. Each part must be characterized in a preliminary work to know its properties m', σ', k'. A system is construct reporting the external constraints on each part, which defines the connection matrix A. From the original equation :

$$m'_{ij}\frac{d}{dt}v^j + \sigma'_{ij}v^j + \epsilon'_{ij}\int_t dtv^j = f'_i \tag{6.36}$$

we obtain the equation for the system (U is the unity matrix) :

$$U^i_\alpha m_{ij} U^i_\beta \frac{d}{dt} v^\beta + U^i_\alpha \sigma_{ij} U^i_\beta v^\beta + A^i_\alpha \epsilon_{ij} A^j_\beta \int_t dt v^\beta = A^i_\alpha f_i \tag{6.37}$$

U is applied on the intrinsic properties and A on all external constraints. Same process is used for equation 6.19. Note that in a more general writing, we should employ :

$$\frac{d}{dt}\left(U^i_\alpha m_{ij} U^i_\beta\right) v^\beta + U^i_\alpha \sigma_{ij} U^i_\beta v^\beta + \int_t dt \left(A^i_\alpha \epsilon_{ij} A^j_\beta\right) v^\beta = A^i_\alpha f_i \tag{6.38}$$

Because the tensors m, ϵ and the connection A can change with time. As in electromagnetism, each impedance operator in mechanics is governed by domains setting their laws depending on temperature, pressure, etc.

6.3.1 Connection

The connection A is the major concept in mechanics when making a system. We start from a collection of pieces firstly separated. Then we have to identify the intrinsic efforts and the external ones. Working on the external efforts, we establish the relations between the constraints and each part, i.e. between the masses and the elasticities. This implies that any solid can be seen as a collection of masses.

Any solid can be identified mechanically looking at its mass center (also called gravity center). If we cut the solid in n parts, each part being of mass m_n and of location $\mathbf{P}_n$, the mass center is defined by :

$$\mathbf{G} = \frac{\sum_n m_n \mathbf{P}_n}{\sum_n m_n} \tag{6.39}$$

Both vectors can be projected in any referential. Knowing $\mathbf{G}$ we can compute the inertia moment L of equation 6.19 :

$$L = \sum_\alpha \mathbf{G}\mathbf{x}^\alpha \times m_{\alpha\alpha}(x^\alpha) \frac{d}{dt} x^\alpha \tag{6.40}$$

It means also that we can decompose any solid as a set of sub-solids. Starting from the mass center of one solid S, we can separate its volume in n solid angles, then in m sub-parts cutting the radius in m lengths. The mass of

each part can be easily computed : for a sub-volume V and a mass density ρ, the mass m_i of the sub-solid is

$$m_i = \iiint_V dV\rho \tag{6.41}$$

Depending on the use we want to do with a solid, we will separate it in various parts where some operations will be made to connect it with other parts. After what, if we create a hole in a solid in order to put a screw, this allows to construct a system but implies a weakened of the solid. For any system of linkage, the tensor ϵ says how the constraints are transmitted in the three directions and if some link breaking down occurs. The tensor ϵ must include domains to translate this kind of event. Let's take an example.

We consider a simple weaken, called to be submitted to a central weight. It means that in final, the weaken is attached by three points, two at the extremities giving a force that compensate the weaken weight, and one at the center applying an external force. The figure 6.5 shows the case imagined and the projection of the force vector on both axes intrinsic to the weaken. The forces can be projected following two directions : horizontal (x) one and vertical (y) one.

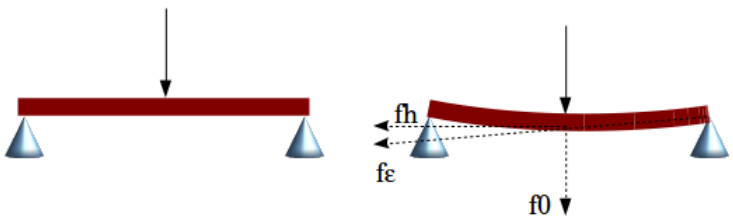


FIGURE 6.5 – A weaken submitted to one force

Applying the same method as previously without looking at losses by friction, we define first a mass tensor :

$$m = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{22} \end{bmatrix} \tag{6.42}$$

In this case $m_{11} = m_{22} = M/2$. For k (direction x), each original half weaken are in static state and :

$$k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{6.43}$$

The same can be described for the moment :

$$\omega = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix} \tag{6.44}$$

When we couple both half weaken we use the two connections (for k and ω) :

	k_1	k_2	ω_1	ω_2
m_1	1	-1	1	1
m_2	-1	1	1	1

which gives for the connection matrix on k, A and on ω, B :

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{6.45}$$

Knowing the whole weaken length χ we obtain the system equations after linking the two half weaken submitted to the force f_0 ($\tau_0 = f_0\chi$)

$$\begin{cases} \frac{d}{dt}U^i_\alpha m_{ij}U^j_\beta v^\beta + A^i_\beta \epsilon_{ij} A^j_\beta \int_t dt v^\beta = A^i_\alpha \left[f_0 Cos\left(\xi\right)\right]_i \\ \\ \frac{d}{dt}U^i_\alpha L_i + B^i_\beta \omega_{ij} B^j_\beta \int_t dt \dot{\theta}^\beta \frac{\chi}{2} = B^i_\alpha \left[\frac{1}{4} f_0 Sin\left(\xi\right)\right]_i \end{cases} \tag{6.46}$$

$\xi = arctg\left(y/x\right)$.

The behaviour of the material remains elastic until a given value of the deformation. Beyond this value, the material becomes plastic. How can we model this ? Intrinsic properties can be affect if degradation occurs on a material. If the material is broken in many parts, its mass will be modified for sure ! If the deformation becomes plastic, an offset in its location translates this modification. The law kx becomes $k'x + x_0$. It means that for example :

$$\epsilon_{ij} = \overset{y}{\mathcal{D}}_1 \epsilon_{ij}\left(\bullet\right) + \overset{y}{\mathcal{D}}_2 \left(\epsilon'_{ij}\left(\bullet\right) + x^j_0\right) \tag{6.47}$$

This capacity should be characterized during the characterization phase of the primitive mechanical element. An important aspect is the temperature dependence. It's clear that the elasticity of any material depends on temperature.

6.4 Vibrations dynamic

Considering a spring supporting a mass M, of rigidity k, for a curvilinear coordinate u we write :

$$-ku = M\frac{d^2u}{dt^2} \tag{6.48}$$

To the rigidity corresponds a recall couple $\Gamma = -C\alpha$ linked with the angle deviation α. For an inertia moment J we have :

$$-C\alpha = J\frac{d^2\alpha}{dt^2} \tag{6.49}$$

If p is Laplace's operator, we can write :

$$Mp^2u + ku = 0 \quad Jp^2\alpha + C\alpha = 0 \tag{6.50}$$

for sinusoidal solutions. The first equation leads to $u\,(Mp^2 + k) = 0$ which has for root

$$k - M\omega_0^2 = 0 \Rightarrow \omega_0 = \sqrt{\frac{k}{M}} \tag{6.51}$$

and for the deviation angle :

$$\omega_0 = \sqrt{\frac{C}{J}} \tag{6.52}$$

ω_0 is the eigenvalue of the oscillator in a free condition of oscillation. The cœfficients C and k can be determined in a static phase. The deformation of the solids submitted to a known static force F gives their recalls k or C.

6.4.1 Springs in parallel

If we consider a mass m_1 of value $M/2$ attached to a spring of rigidity k_1 and the same for a second mass m_2 of value $M/2$ and rigidity k_2 ; we have :

$$m = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{6.53}$$

and :

$$\epsilon = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{6.54}$$

Now if we attach both masses in order to make a single one, the connection becomes :

	k_1	k_2
m_1	1	1
m_2	1	1

We understand that if a recall k_x acts on a half mass, it acts also on the second half mass that participates to the whole mass M. If $g = d^2u/dt^2$, we obtain :

$$m_1 g = -(k_1 + k_2)u \quad m_2 g = -(k_1 + k_2)u \tag{6.55}$$

The result is when two springs are in parallel, the equivalent single spring is equal to the sum of the two separate springs.

Now if the two springs are in series, the total length x covered by the springs is $x = x_1 + x_2$. This length is in relation with a force F by $F = kx$. The force is applied on a mass attached at the extremity of the two springs in series, and is reported on each spring. So we have :

$$x = \frac{F}{k_1} + \frac{F}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2} \tag{6.56}$$

If we look at two free masses, as described figure 6.6.

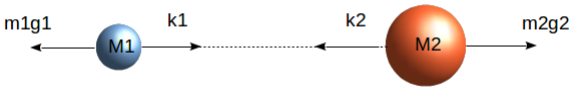


FIGURE 6.6 – Two free masses with a spring

Before to link the masses, they obey to the already seen relation :

$$-k_i x^i = m_i g_i \Leftrightarrow m_i g_i + k_i x^i = 0 \tag{6.57}$$

After linking them, each mass see the recall of the other. Following our method, it leads to both equations :

$$\begin{cases} m_1 g_1 + k_1 x^1 - k_2 x^2 = 0 \\ \\ -k_1 x^1 + m_2 g_2 + k_2 x^2 = 0 \end{cases} \tag{6.58}$$

Noting $x = x_1 - x_2$ and $g = g_1 - g_2$, we obtain of the previous system :

$$-kx\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g \tag{6.59}$$

Defining the reduced mass :

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{6.60}$$

This oscillator defined by $-kx = \mu g$ behaves like a single oscillator made with one mass and its eigenvalue is :

$$\omega_0 = \sqrt{\frac{k}{\mu}} \tag{6.61}$$

6.4.2 Moment expression

The relations in case of rotation can be always obtained from the translation relations. For example, starting from

$$\mathbf{F} = m\frac{d}{dt}\mathbf{v} \tag{6.62}$$

multiplying by the distance r we have :

$$\mathbf{F}r = mr\frac{d}{dt}\mathbf{v} \tag{6.63}$$

but $v = \theta r$ and so :

$$\mathbf{F}r = mr\frac{d}{dt}\theta r \Rightarrow \mathbf{\Gamma} = J\frac{d}{dt}\theta \tag{6.64}$$

which imply the inertia moment definition : $J = mr^2$.

6.5 Finite element approach

Figure 6.7 shows the problem we treat here [2]. On the figure, the edge are numbered with red letters and the nodes with black letters. The weight fixed on the last weaken implies a force F on the console. As the nodes 1 and 2 are attached to the wall, only the nodes 3,4 and 5 can move.

2. All the chapter follows the explanations given by *Pierre Thomas* in his book "Éléments finis pour l'ingénieur" at Lavoisier editor, 2006.

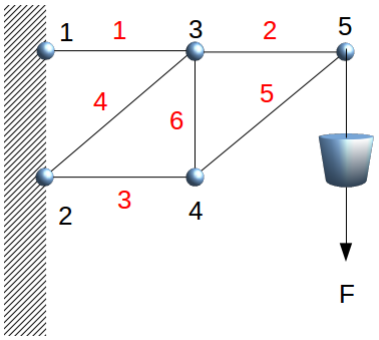


FIGURE 6.7 – The console considered

6.5.1 Assumptions on the weakens

Each weaken sees forces at each of its extremities, coming from F. We make the assumption that the weaken can turn on their extremities around screws. So the weaken are compressed or extended like springs, without twist or rotation. The weaken supports opposite efforts $\mathbf{T}$ that we associate with an axial vector $\mathbf{f}$ as shown figure 6.8.

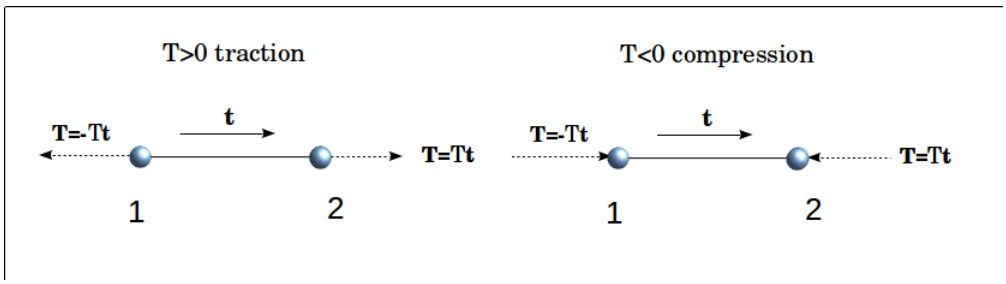


FIGURE 6.8 – Efforts on a weaken

Under the action of T, the weaken length changes of a value Δ. With an assumption of linearity, we have seen that this changing is given by :

$$\mathbf{T} = k\mathbf{\Delta} \tag{6.65}$$

We must be careful that this assumption doesn't imply that the absence of elongation means small displacements. For example a weaken can rotate without changing in length. In fact, the assumptions given when we declare

the problem with an initial geometry, we fix the mechanical behaviour of the problem.

Now to connect the weakens in order to make the console, we need to assure two properties :

1. the same displacements reported on the shared nodes : *continuity relation* ;
2. the opposition of the forces (action reaction Newton's law : *equilibrium relation*)

But as the console belongs to the same referential, the properties of each weaken defined in their referential must be projected in a common referential named "global referential". So we can write for each weaken involved in the console repaired by its extremities 1 and 2, the components of displacement u_1, v_1 and u_2, v_2 as the reported forces f_1, g_1 and f_2, g_2. Figure 6.9 shows this convention used for the whole weakens.

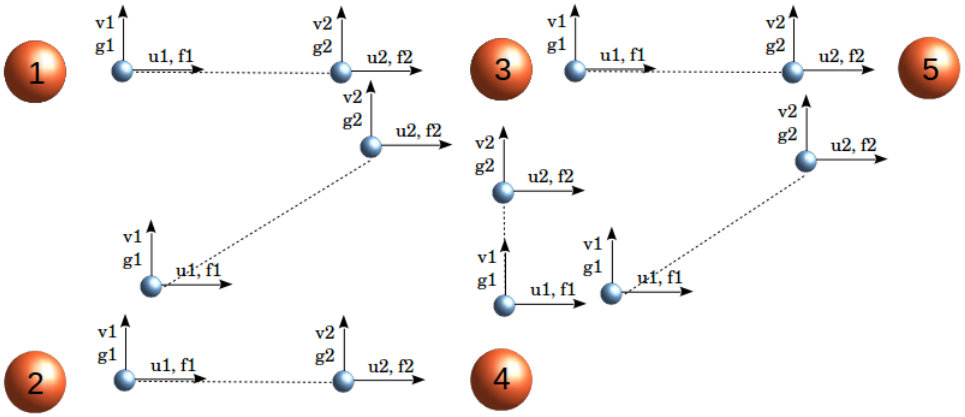


FIGURE 6.9 – Reported vectors

6.5.2 Mechanical behaviour

The elementary stiffness matrix is the key matrix to solve the relation $F_\alpha = k_{\alpha\beta}U^\beta$. F and U gives the forces and the displacements in each weaken

referential. Often we can use the small deformation assumption. Figure 6.10 shows the deformation of a weaken and the associated angle and lengths.

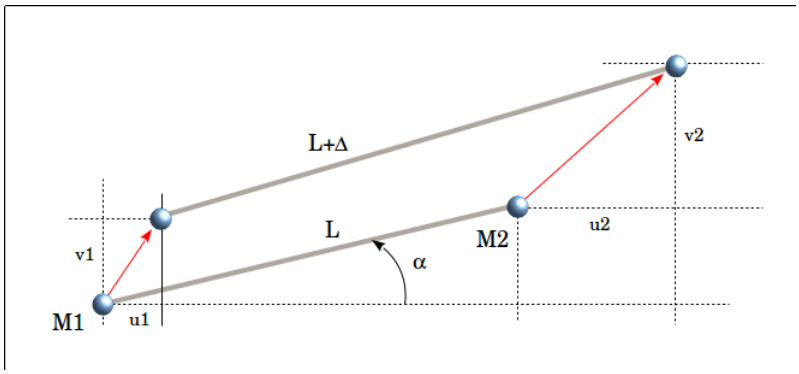


FIGURE 6.10 – Weaken deformation

The added length under deformation is given by (see *Pierre Thomas* reference) :

$$\Delta = \sqrt{(Lcos(\alpha) = u_2 - u_1)^2 + (Lsin(\alpha) + v_2 - v_1)^2} - L \tag{6.66}$$

This expression can be reduced using the small displacements assumption to reach :

$$\Delta \approx cos(\alpha)\,(u_2 - u_1) + sin(\alpha)\,(v_2 - v_1) \tag{6.67}$$

Each weaken stiffness matrix links the forces and the displacements following their axes direction. The equation is :

$$\mathbf{F} \cdot \mathbf{t} = k\mathbf{U} \cdot \mathbf{t} \tag{6.68}$$

Using the previous relations, we obtain for one element and its displacement at each node :

$$\begin{bmatrix} \mathbf{U}_1 \cdot \mathbf{t} \\ \mathbf{U}_2 \cdot \mathbf{t} \end{bmatrix} = \begin{bmatrix} cos(\alpha) & sin(\alpha) & 0 & 0 \\ 0 & 0 & cos(\alpha) & sin(\alpha) \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} \tag{6.69}$$

This relation can be written $s^a = C^a_b u^b$ where s are the coordinates attached with each weaken and $u^b \rightarrow (u, v)$ the coordinates attached with the common

referential (x, y). On another side we have :

$$\begin{bmatrix} f_1 \\ g_1 \\ f_2 \\ g_2 \end{bmatrix} = \begin{bmatrix} cos(\alpha) & 0 \\ sin(\alpha) & 0 \\ 0 & cos(\alpha) \\ 0 & sin(\alpha) \end{bmatrix} \begin{bmatrix} \mathbf{F}_1 \cdot \mathbf{t} \\ \mathbf{F}_2 \cdot \mathbf{t} \end{bmatrix} \tag{6.70}$$

or $f_k = C_k^q F_q$ where $f_k \rightarrow (f, g)$ are the forces applied at each node and for each direction in the common referential coming from the force source $\mathbf{f}$ projected on each axes of the local referential attached with the weaken axes. We can consider equation 6.68 to write it under the form :

$$\mathbf{F} \cdot \mathbf{t} = k\mathbf{U} \cdot \mathbf{t} \Leftrightarrow F_\alpha = k_{\alpha\beta} s^\beta \tag{6.71}$$

We can replace in this second equation s^β by $C_b^\beta u^b$, which gives :

$$F_\alpha = k_{\alpha\beta} C_b^\beta u^b \tag{6.72}$$

now we multiply on the left both members by C_k^α to obtain :

$$C_k^\alpha F_\alpha = C_k^\alpha k_{\alpha\beta} C_b^\beta u^b \tag{6.73}$$

Defining $\epsilon_{kb} = C_k^\alpha k_{\alpha\beta} C_b^\beta$, this leads to :

$$f_k = \epsilon_{kb} u^b \tag{6.74}$$

This relation gives the link between the forces applied on each weaken at their extremities and for all the directions of the common referential (or common space) and the displacements of the nodes of these extremities in the same common space. The solution with null forces $\epsilon_{kb} u^b = 0$ gives the kernel of the stiffness matrix. Once we have the expressions of the weaken properties in the common space, but for the separate problem, it remains to connect the weaken between each other to make the system. The figure 6.11 shows the structure with the numbers of the nodes, weakens, and weaken extremities.

If ω^σ are the displacements in the common space once the weaken are connected, we can find relations between ω^σ and u^σ writing :

$$u^\alpha = \mathcal{C}_\sigma^\alpha \omega^\sigma \tag{6.75}$$

For example, seeing figure 6.11 we can establish : $u_1(1) = \omega^1$ and $u_2(2) = \omega^3$, etc. It must be clear that ϵ is a diagonal matrix of sub-matrix k, each sub-matrix being of dimension 2x2 or 3x3, depending on the geometrical space

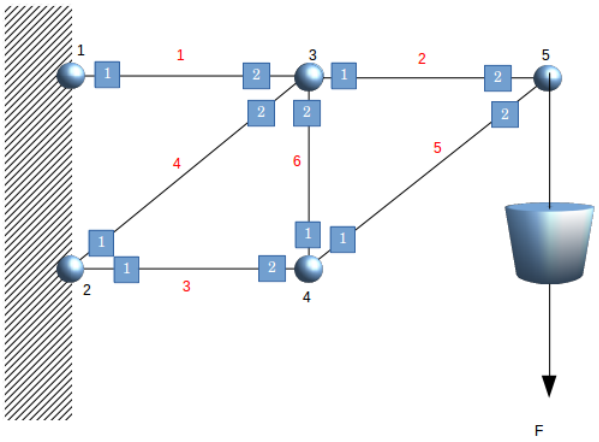


FIGURE 6.11 – Weaken connection

where the problem is studied (remember that in our case we work in two dimensions and local coordinates of the common space was u_1, v_1). We are now familiar with this kind of development. If h are the forces once the weaken connected, we can verify $h_\alpha = \mathcal{C}^\eta_\alpha f_\eta$. And starting from equation 6.74 we obtain :

$$f_k = \epsilon_{kb} u^b \Rightarrow f_k = \epsilon_{kb} \mathcal{C}^b_\sigma \omega^\sigma \Rightarrow \mathcal{C}^k_\alpha f_k = \mathcal{C}^k_\alpha \epsilon_{kb} \mathcal{C}^b_\sigma \omega^\sigma \tag{6.76}$$

Noting $\bar{\zeta}_{\alpha\sigma} = \mathcal{C}^k_\alpha \epsilon_{kb} \mathcal{C}^b_\sigma$, we finally obtain :

$$h_\alpha = \bar{\zeta}_{\alpha\sigma} \omega^\sigma \tag{6.77}$$

Which gives the solution of the constructed console. ζ is the stiffness tensor of the connected system in the common space.

6.6 Surfaces description

Any surface can be described by curvilinear coordinates. The transformation of these coordinates into a cartesian space is the key to define the system structure we want to use. As usual each part of structure is firstly defined in a local space attached with the part and giving a characterization of the part. Each part can be seen as a weaken as previously, the difference being that a part is a surface and not only a mono-directional object. The

curvilinear coordinates s^α of each part is locally transformed into a common space of coordinates u^β. Then after connection to make the system through $\mathcal{C}$, the complete structure is described. Finally a component of the structure is a tenfold made of two major objects :

1. the part geometrical description with u^η ;
2. the stiffness matrix of the part k ;
3. the electrical properties of the part σ, ϵ, μ

Two tensorial equations cover both mechanical and electronic properties of the system :

$$\begin{cases} h_\alpha = \bar{\zeta}_{\alpha\sigma}\omega^\sigma \\ \\ e_\alpha = \zeta_{\alpha\beta}k^\beta \end{cases} \tag{6.78}$$

But coupling must be added between the mechanical and electronic networks. The structure properties impacts the line propagation properties while the electronic adds weight on the structure part. The local environment (pressure, temperature) changes the structure and the electronics properties. The approach means to study and couple the structures and the electronics once their own global networks are defined, then to compute the local environment. As the impedance operators are defined on domains, the environment is automatically taken into account in the system working.

Dynamic behaviours follow exactly the same equations. The only difference comes from the fact that the forces and displacements changes with time.

Chapitre 7

Temperature

Temperature is perhaps one of the major concept of physics. Environments are described in term of electromagnetic fields, pressure, gravitation and temperature. We want to present here major ideas about temperature, in order to include this environment parameter in our studies.

7.1 Thermal dilatation cœfficient

Under some temperature ambiance, a piece of mater has its length changing. The relation between the temperature variation and the length variation is linear and we write :

$$\Delta L = \alpha L_0 \Delta T \tag{7.1}$$

L_0 is the nominal length of the piece. For nylon for example, the cœfficient is equal to 81.10^{-6}. The same idea can be applied to volume. If V_0 is the nominal volume of a piece of mater, the variation of volume ΔV is defined by :

$$\Delta V = \beta V_0 \Delta T \tag{7.2}$$

β is the cœfficient of volume dilation. In a given volume V, the pressure P on the volume walls is associated with the number of molecules N and their movement (giving a temperature T). This evidence is defined by the perfect gas relation :

$$PV = Nk_B T \tag{7.3}$$

k_B is Boltzmann's constant, one of the fundamental constant of nature.

7.2 Temperature topology

The heat Q cumulated when increasing the temperature of a body of mass m between two times t_1 and t_2 is given by :

$$Q = mc \int_{t_1}^{t_2} dT \tag{7.4}$$

The factor c is called the massive capacity of the material of the body. On another side, we take a look to the heat transfer. For a plate of surface S and length L, the heat transfer can be defined by :

$$\frac{dQ}{dt} = k_T S \frac{\Delta T}{L} \tag{7.5}$$

or in general :

$$\frac{dQ}{dt} = k_T S \frac{dT}{dx} \tag{7.6}$$

k_T is called the thermal conductivity, and translate the capacity of a body to transfer the heat through its surface and all along its length. As Q is energy, we can define the power P by :

$$P = \frac{dQ}{dt} \tag{7.7}$$

Noting $C_{th} = mc$ and $y_{th} = k_T S/L$ and :

$$\theta = \int_T dT \tag{7.8}$$

we obtain :

$$\int_t dtP = C_{th}\theta \Rightarrow \theta = \frac{1}{C_{th}} \int_t dtP \tag{7.9}$$

The second relation becomes :

$$P = y_{th}\theta \Rightarrow \theta = R_{th}P \tag{7.10}$$

with $R_{th} = 1/y_{th}$.

If we have a piece of matter heated by a electrical wire, a part P_s of the energy goes out through the admittance y_{th} : $P_s = y_{th}\theta$. Another part of the

energy P_b is used to heat the body of thermal capacity C_{th} : $P_b = C_{th} d\theta/dt$. Finally, P_e being the total energy transmitted to the wire :

$$P_e = C_{th}\frac{d\theta}{dt} + y_{th}\theta \tag{7.11}$$

Generalizing this relation, we create a natural space of nodes-pair sources or current of power P^k, the temperature changing θ_u giving :

$$P^k = C^{ku}\frac{d}{dt}\theta_u + y^{ku}\theta_u \tag{7.12}$$

The source of heat is a current driven in some electronic circuit. The temperature θ changes the resistances values or the solid length via the thermal dilatation cœfficient.

The three modes of heat transfer are :

— conduction ;

— convection ;

— radiation.

7.2.1 Conduction

The conduction is modelled by the thermal resistivity y_{th} with

$$y_{th} = k_T\frac{S}{l} \tag{7.13}$$

Some values of k_T are :

Matter	copper	Aluminium	Iron	Asbestos	Mica	Air
k_T (W/ $^\circ$C.m)	387	200	65	0,19	0,36	0,023

7.2.2 Convection

The convection conductivity y_c acts as the thermal conductivity. We define :

$$P_c = y_c\theta \tag{7.14}$$

y_c is linked with the surface of exchange S, and a given cœfficient of convection conductivity h_c.

$$y_c = h_c S \tag{7.15}$$

7.2.3 Radiation

The radiation modelling follows the same principle as for the convection modelling. We define a radiation conductivity by :

$$y_R = h_R S \tag{7.16}$$

which gives the relation between the power exchanged by radiation P_R and the temperature evolving :

$$P_R = y_R \theta \tag{7.17}$$

7.3 Thermal environment

Once identified the various sources of heat, a graph must be constructed to model the heat transfers between the system and the environment, and between the various organs inside the system. These exchanges must lead to a graph representing the heat fluxes with their various properties of conductivities or capacitances. The sources of heat create nodes-pair of current in this graph. The temperature variations θ_u becomes inputs as parameters for the domains of the impedance operators, or to set the length dilatation of solids. For many systems, we can conclude that :

- two graphs represents the system, one for electronics, one for thermal ;
- one graph represents the environment, one for electromagnetism, one for thermal.

Other graphs can be used, for pressure for example. These graphs are associated with tensorial equations including lagrangian operators, etc. These objects are grouped in tenfolds. It means that we have one tenfold for the system, and one tenfold for the environment.

Chapitre 8

General modelling of a cyber-physical system (CPS)

Let's recall what is a cyber-physical system : it's a system made with physical material and embedding cybernetic techniques : computers, numeric networks, cloud, etc. The material is modelled using the tensorial analysis of networks. The various networks are grouped in a whole system of systems under some equation like :

$$T_a = g_{ab}\psi^b \Leftrightarrow \begin{cases} h_\alpha = \bar{\zeta}_{\alpha\sigma}\omega^\sigma \\ \\ e_\alpha = \zeta_{\alpha\beta}k^\beta \end{cases} \tag{8.1}$$

Some of the emf e_a are made with known currents (current sources) multiplied by impedances, like $R_{ab}.J^b$. Through the mechanical process of a gamma matrix, this source associated with a first network is transported becoming the emf of another network.This process can translate the exchange of communication between the networks, while the electromagnetic interactions (like through antennas for example) are taken into account by a classical Green function. Both functions can represent somewhere the same physical base, but the gamma matrix becomes many more easier to use when numeric messages are involved (other cases can be interesting). It can be used for quantum process. The final equation for the CPS has the form :

$$\gamma_c^a e_a = \gamma_c^a \left(\zeta_{ab}k^b\right) \tag{8.2}$$

Any CPS is made of :

— a perception network - e_π ;
— an action network - k^a ;
— a nervous system and computer centres - $n_{\pi a}$;
— an energy network - e_E ;
— a waste management network - R_{uv} ;
— a communication network - γ_b^a ;
— a skeleton - s_{uv}.

To define completely the concept of CPS we now detail all the notions associated with the concept of system, usually declined in the science of system engineering [1].

8.1 Concept of system

There are many kind of definitions for system. Basically, a system is a set. Between others, it exists a definition that is more in accordance with our formalism : to say that a system is a box with inputs and outputs. Inside and outside the box we can discern fluxes (fluxes of energy, communication, etc.). A system can be seen as closed if no fluxes go from inside to outside, or open if fluxes can go from inside to outside. The fluxes k^a are the sources for cords under the xTAN formalism (see references) or for any kind of communication channel to other systems. As a matter of fact, all the system behaviour can be observed looking at these fluxes. The system organization is characterized by its fundamental operator ζ. This operator can change with time (dynamic system) or not (static system). A system cycle of life goes through six steps :

— Concept analysis
— feasibility
— development
— making
— start of service
— use
— out of service

A cell of a system can be seen as the box presented figure 8.1.

We will see that all the classical notions of system engineering can be declined using a mathematical representation of the system. This representation must include all the components of the system, the interactions between

1. See Luzeaux's reference.

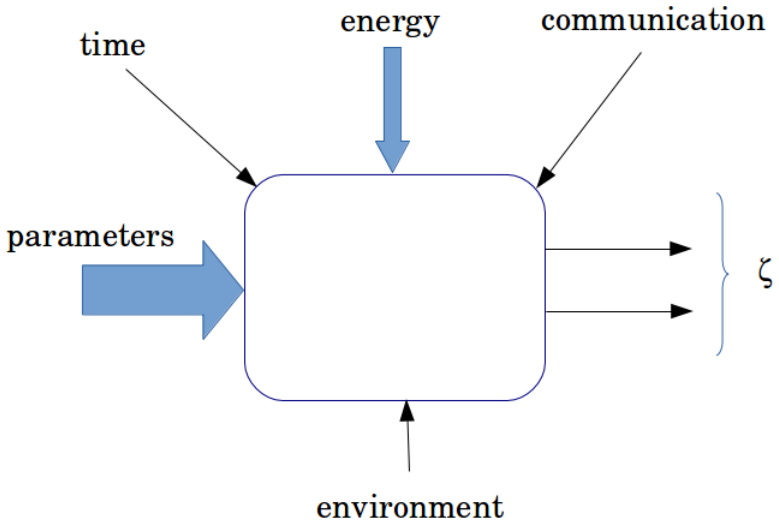


FIGURE 8.1 – Representation of a system cell

these components and the human factor. The difference between a cyber-physical system and a bio-inspired system holds in taking into account the human factor. Interactions involving models coming from the game theory will be used in the bio-inspired system engineering. That's the purpose of the next chapter. For the moment, we will explore the various thematics attached with the notion and development of systems. First of all, we recall the mathematical representation of a system under the xTAN method (extended tensorial analysis of networks, developed by the author), leading to the concept of manifold.

Usually a system is described by a set of state variables x. These variables evolve depending on commands u and external uncontrolled parameters θ. The equation of the system life can be written :

$$\frac{dx_i}{dt} = g_i\left(x, u, \theta\right) \tag{8.3}$$

This form focuses on the state variable changes without giving any particular meaning to each kind of variable. This, while their roles are very different.

The previous equation can be written :

$$u_i = \psi_i\left(x, \theta\right) \tag{8.4}$$

The function ψ_i includes the variable x and its time derivative. It remains that, except when studying the static state, the variable $\dot{x}$ is more interesting than x. $\dot{x}$ translates directly a flux which is our indicators for systems. There are no problems to work with $\dot{x}$ rather than with x. x is simply obtained through a time integral. We generalize the previous equation by :

$$u_i = \psi_i\left(x, \dot{x}, \ddot{x}, \theta\right) \tag{8.5}$$

ψ is any function applied to x and θ. But ψ changes with θ. The external parameters θ_j intervene through domains $\mathcal{D}$, changing the functions ψ_i depending on the environment influence. The reader will recognize in this description the technique previously used to model the oscillator. This technique allows to model any material system. Then, to go deeper in the system analysis, we make some operation with the objective of seeing the system as a complex geometrical object.

8.2 Tenfold analysis of complex systems (TACS) preliminaries

Now we must give us an algebra in order to identify any system and to model its evolution. Any system can be represented by a list of mathematical objects giving the time evolve of their fluxes. The nature of these fluxes point out automatically the physics concerned by the model. It's clear that an engineer using of these objects needs a draw corresponding to the real objects, to help understanding the graphs and equations associated in the model. So the system is identified with for one side a set of primitive objects used to make the system ($O\{\ldots\}$) and for second side a list of mathematical objects giving the system models (C, T, G). Each primitive object has a correspondence with our cell 8.1. T is the energy sources of a primitive part, G its metric and C a connectivity that constructs the primitive part. The simplest way to understand the approach is to apply it on a real system.

We take as example a pin-socket couple, under external vibrations and transmitting high voltage signal. The system is made of 5 objects at least :

— A pin and a socket (O_p, O_s) ;
— a resistor and a power supply (O_R, O_u) ;
— a vibration facility (O_v).

So for the moment : $O\{\ldots\} = \{O_p, O_s, O_R, O_u, O_V\}$. Is this set may be called a system ? No. In the Greek's meaning yes, but it limit the of the word to its basic mathematical meaning. Finally, a definition that pleases to me is given by Luzeaux : "a system can be seen as a black box where inputs and outputs materialize some number of fluxes". For the moment, our set being made of separate functions is not source of fluxes. Starting from these primitive objects O_i we have to connect them to hope making a system. Then, the fluxes analysis will give us the base to analyse the system. Connecting the primitive objects means to create connections between them. There are various kinds of connections :

— hard connections, mechanical mechanisms ;
— radiative interactions using antennas, etc. ;
— connections by wires or other guided waves ;
— near field interactions ;
— long distances connections through networks ;
— other kind of coupling connections.

The development of the system pass through steps from the analysis of

concept to the use then end of life of the system. First step concerns the concept analysis.

8.2.1 Concept analysis

For the majority of problems, it exists basic solutions. There is a group of fundamental solutions, but globally, there are often known. Innovation belongs more to the choice of technologies, to reach the same need. The story starts with the description of a need. In our case, the need is to connect a high voltage generator with a load using a pin-socket couple. The contact must be guaranteed for one thousand operations of connection, disconnection.

We can imagine that there are a lot of possibilities for this kind of contact. The offer is contained in a set of all available contacts $c\{\}$. Each kind of contact in the set $c\{\}$ can be modelled by a branch in a cellular topology. What does it mean ? If we power supplied the contact with a source of energy e, a current i (a flux) is created through the contact. This implies that a potential u is developed along the contact, associate with the work developed by the flux moving. We can write for this primitive element :

$$e = u + z.i \tag{8.6}$$

z is the law guiding the flux value depending on e. This law is influenced by the environment conditions : temperature, pressure, humidity, etc. The work u can be written somewhere as :

$$u = \int_a^b d\mathbf{x} \cdot \frac{\mathbf{J}}{\sigma} \tag{8.7}$$

that's Kirchhoff's law and fundamental relation. $\mathbf{J}$ is the flux, σ a contact property and $\mathbf{x}$ the curvilinear coordinate along the contact.

Since we use a spatial integration, we can associate an abstract branch which borders are the nodes a and b, defining the limits of the integral. This cellular object represents abstractly the contact, whatever it is, with its limits and properties. All the properties giving the relations between e and i must be contain in the operator z (here containing $1/\sigma$). Note that more than one branch can be necessary to reach this modelling quality. At least in general, one branch is needed for each physics : one for electronics, one for mechanics, etc. Finally the primitive object is a group of branch in multi-physic, see little networks, that will be parts in a wider network representing the system.

We understand that we can start from sets of contacts (pin-socket couple) $c\{\}$, high voltages $h\{\}$, wires $w\{\}$ and resistances $r\{\}$. The system conception consists in taking one element from each of these sets to construct the system. Basically we mus take one element to each of these sets.

A first approach in conception can be to test randomly the quadruplet obtained by taking one element to each set, then to verify if it is in accordance with the requirements. For the construction only, this strategy can be written, if A is a random function saying if the system is in accordance with the requirements :

$$P\left(A|\ c_i \in c\{\}\ ,\ h_i \in h\{\}\ ,\ w_i \in w\{\}\ ,\ r_i \in r\{\}\right) > 1 - \epsilon \tag{8.8}$$

It is clear that this probability is weak. If we look at the wires, between all the wires available in the market, a few are able to support high voltage. So, the probability to take a good wire between all the possible sample is already small.

The advantage of this strategy is that it can be rich for innovation. The fact to take randomly the element can lead to a quadruplet that nobody may have imagine, and perhaps the best efficient one. But as said before, the chance to select this combination is very thin. A long time may be spend before to find the good choice.

A more classical strategy may be to choice the best part coming from each set (higher efficiency, lower cost) and to make the system with these selections. For each part, this objective can be written, if f is an observation function based on defined criteria, returning the performance of the element :

$$\exists c_i \in c\{\}\ /\ f\left(c_i\right) > f\left(c_j\right),\ \forall c_j \in c\{\}\ with\, j \neq i \tag{8.9}$$

This approach, unfortunately the more used because the simplest one, doesn't lead necessarily to the best system. This comes form the fact that the system results emerge from couplings between its components, in addition to the components themselves. The lack of consideration for the system impact appears fastly. Even if a wire answer to the hight voltage and vibration by itself, its performance in vibrations can be inadequate when the contact will be connected to the wire. Here appears completely the gap between the separate component capacities and the system constraints. Another problem coming from the classical approach is that, as they tend to forget the system, they tend to forget the customer needs. This is because the customer needs doesn't appear in the decomposition of the target performances into each

system component performances. By the fact, the customer needs are memorized only in the original requirements and during the system construction. But it may be too late to reach the optimum solution, as seen in the previous example. Many round trip will be necessary to obtain the waited answer for the customer.

A new method recently developed tries to reinforce the customer oriented techniques : the LEAN. In this approach, the idea is to focus all the development on the customer needs. But it doesn't give proposals to improve the whole technique and leads to the same fundamental problem than the classical approach. It was imagined first for the making, then extended to R&D. I personally think that this technique is not the answer.

The approach considering the system problem is the systemic approach. For this concept, we accept the fact that the system properties cannot be obtained starting from the properties of its components. The systemic approach is somewhere a mix of the two previous approaches. The mathematical formulation of this strategy may be given at the component level by :

$$\exists c_i \in O_i \{c_i, h_i, w_i, r_i\} \ / \ f(O_i) > f(O_j), \forall O_j, with\, j \neq i \tag{8.10}$$

The difficulty is to find the component which answers adequately to the system needs and to its own performance objectives. This means to anticipate the impact of couplings. This seems to be impossible before the system exists. The key of success belongs to the capacity, once the global system architecture chosen, to decline a complete environment definition to all possible components implied in the system construction. This exist sometimes under the form of design rules. But the design rules have the defect of giving constraints, indeed beginning of solutions rather than objectives.

Someone could say that the fact to take into account the whole system constraints can be made through the classical approach also ? It's true but the big difference is that in the classical strategy, the system knowledge is made *a posteriori*, while in the systemic approach the system objectives is taken into account *a priori*. What we write for the classical approach :

$$P(A(t+\Delta t)|\ f(c_i(t)) > f(c_j), \forall c_j \in c\{\}\ with\, j \neq i) \tag{8.11}$$

and for the systemic approach :

$$P(f(O_i(t+\Delta t)) > f(O_j(t+\Delta t)), \forall O_j, with\, j \neq i|\ A(t)) \tag{8.12}$$

We see here that clearly, we converge to a mix of the previous concepts. If we resume both notations by $P(A|\ c_i)$ and $P(O_i|\ A)$, we have by one side :

$$P(A|\ c_i)\,P(c_i) = P(c_i|\ A)\,P(A) \tag{8.13}$$

while on the other side :

$$P(O_i|\ A)\,P'(A) = P(A|\ O_i)\,P(O_i) \tag{8.14}$$

If we accept $P'(A) > P(A)$ and $P(c_i) > P(O_i)$ then it implies for $P(O_i|\ A) \approx P(c_i|\ A)$:

$$P(A|\ O_i) > P(A|\ c_i) \tag{8.15}$$

which tends to give more weight to the systemic approach.

The mathematical transcription of systemic uses operators defined on domains.

We see that in the system engineering, two steps are preliminaries to the development :

— the concept analysis ;
— the feasibility.

It means that in this *top-down* approach, the time allotted to the "up" phase is significant. Unfortunately, this time is often neglected and reduced to "the part of the poor".

Concept analysis means to choice global system concept between various available. Some keyword for concepts are :

— *endoskeleton* or *exoskeleton* ;
— distributed system or local system ;
— system with central artificial intelligence or distributed artificial intelligence ;
— system with embedded actuators or remote actuators ;
— etc.

The question is : why answering one system kind rather than another ? Often the history shows that humans follow the nature in this choice. But not always. About space, it's clear that nature shows that exoskeletons seems to be the unique way to protect the living things embed in a spacecraft to protect them from the space environment. If we speak of transport, cars are exoskeleton systems while humans first use animals like horses whose are endoskeleton systems. An exception are the motorcycle : finally they are more inspired by natural mobilities than cars or planes. Does it mean that

it is impossible for a transport system embedding more than one or two persons to be a endoskeleton one ? For planes, the problem becomes similar to spacecraft. For this reason, we can think that exoskeleton structures are necessary. For cars, bus, etc., we see that more and more structures becomes opened structures, showing that endoskeleton are possible choices. But as they are nearest to natural systems, they call surely for higher capacities of conception becoming just now to be imaginable.

The advantage of exoskeleton are clear : this structure protects the organs versus the mechanical (and in some particular case the chemical) constraints of the external environment, including all the living being embed in the system. While in endoskeleton structures, the organs and embedded living being are exposed to these mechanical (and sometimes chemical) constraints coming from the external environment. The first disadvantage of the exoskeleton systems are their weight and some smaller performances in some domains. It seems that some of the first "large" animals was exoskeleton. It was a logical evolve for animals that were becoming more complex and must make face to the external constraint of water then atmosphere. It was an evolve from skin to exoskeleton. But increasing the dimensions, nature was obliged to evolve into endoskeletons. Figure 8.2 shows clearly the gain in mass of the endoskeleton structure. If we evaluate the weight of the skeleton in both case of exoskeleton - or shell and endoskeleton - or spine : for a given volume V of organs, the weight m_e of the shell made with a material of density ρ is given for an thickness δ and a volume surface S by :

$$m_e = \rho \delta S \tag{8.16}$$

The thickness δ is determined by the mechanical needs to support the whole weight and dynamic of the system. In case of endoskeleton, we have the skin weight $\rho_s \delta_s S$, and the spine weight determined by the whole weight to be able to support. But as the surface is reduced by a factor ten typically, for the same material, the skeleton weight is also reduced by ten. In another side, the skin weight can be drastically reduced by the skin material and its thickness. It means that this added organ is far from weighting the difference $\rho \delta S - \rho_s \delta_s S$. In global, the endoskeleton is lighter than the exoskeleton. Finally, what helps to decide is the external environment. We will now speak of environment and self-consistency for the constraints coming form the system itself.

When it is impossible to find a skin that protect the organs from the environment, a exoskeleton is necessary. If not, endoskeleton

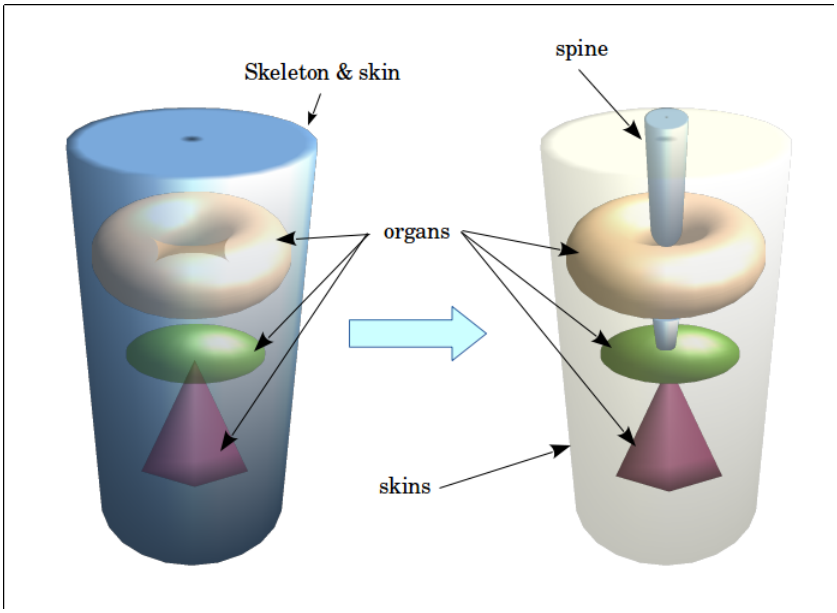


FIGURE 8.2 – Exo to endoskeleton evolve

should be preferable.

That's why insects are often exoskeleton systems. Because due to their small size, they may be often destroyed by their environment which includes bigger animals, falls, rain, etc.

For artificial human machines, extraterrestrial environment is generally the reason to choice exoskeletons. But the spatial environment in fact doesn't include planes. Probabley we may find planes in the future without exoskeleton. Some birds flight at the same altitude that standard planes (around 10 000 m).

In the deep ocean, fishes (and some mammal like the Sperm whale) can live. The same in some particular water zone where the water is very heat. It means that even in these hard environments, endoskeleton are possible solutions.

We may say that many classical systems are today thinking under exoskeleton strategy, because it was simplest to make them before, and it's hard to make something completely different today. But we will see that under a bio-inspired engineering, this kind of approach could be changed, with benefit of lighter systems and both electromechanical and intelligent skins. Spine

and other bones will be there to give a common rigid structure able to accept the various actuators and organs.

We can conclude the concept analysis saying that human being can benefit of all the research conducted by the Nature depending on all the mission allocated to the living being. For each kind of needs, there was various answers and kind of systems, very well adapted to their mission with time.

Our previous ideas can be practically enforced using classes. If we first speak of the system. It is made of a structure, of energy and information network (cables in general) and of organs (equipments). We can create classes for each of these elements. In one class, we put the properties - parameters - of the object and methods modelling the object behaviour. A class "architecture" groups one of each class of structure, etc. in order to make an architecture. Each component is chosen in a set of many of them. The result is a large proposal of architecture having various performances and costs. Based on this principle, many "test" can be try like do the Nature. We don't know in advance if such an architecture is interesting or not. It's only once the architecture is made that we can perform its properties. A similar definition of class can be made for the environment. The different classes can be instantiated in a program written to imagine the system through various assumptions of choices. In practice, a particular point merits to be cited. The methods must be declared as string, including parameters declared in the properties field.After what, when we have to realize the computations for evaluating the performances, these strings must becomes reachable equations. In python for example, a very interesting possibility exists : the function *eval* transforms a string in floating point equation. Each class of structure, cable, electronic has in its methods, one method that defines the impedance operator (in a general meaning). The architect class takes one class of each kind of component and tries various solutions of architecture. The process is explained figure 8.3.

Once an architecture is made, it remains to verify if it is in adequacy with the requirements. That's the purpose of the feasibility phase. The two phases of concept and feasibility can be explored in a loop before to find the best combination for the given objective. However the system construction can be guided by a preliminary study making a simulation of the system mission.

The system conception goes through the call of classes associated with the various system components : structure, cables, electronics, etc. How this mechanism can work ? The objective is to translate as soon as possible the system description using classes in models, to continue the development starting from these models rather than remaining in some textual descriptions.

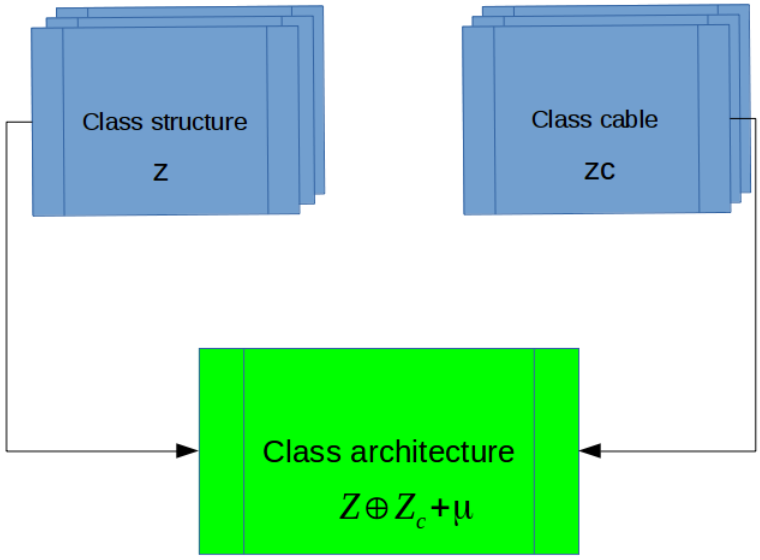


FIGURE 8.3 – Class architecture

If we consider a function y depending on t, but also on two other parameters g and v_0. Let's defined :

$$y(t) = v_0 t - gt^2 \rightarrow y(t; v_0, g) \tag{8.17}$$

Another function could be $g(t, A, a) = Aexp(-ax)$. We may defined these functions using classical functions in python, such as :

```
def y(t,vo):
    g=9.81
    return vo*t-0.5*g*t**2.

def g(x,a,A):
    return A*exp(-a*x)
```

The major problem of this programming is that it is strictly reserved for functions that has exactly the same number and type of parameters and variables. In classes, variables and parameters are visible by anybody, they behave as global variables. The previous function y may be defined as the next class :

```
class Y:
    def __init__(self,vo):
        self.vo=vo
        self.g=9.81
    def value(self,t):
        return self.vo*t-0.5*self.g*t**2.
```

We can then use the class by creating an instance of the class : $y = Y(3.)$. This instruction sets v_O to 3. To obtain the value $y(t = 0.1; v_0)$ we just have to complete the previous instruction by $v = y.value(0.1)$ which gives the result for $t = 0.1$. The keyword *self* return to the instance of the class. Making $y = Y(3.)$ we implicitly replace self by y. Calling $v = y.value(0.1)$ we obtain the result for $v_0 = 3$ and $t = 0.1$. The syntaxe

```
v=y.value(0.1)
```

is translated by python in :

```
v=Y.value(y,0.1)
```

Any sub-system is characterized by a tenfold including the tensor ζ, eventually sources E, and some other properties. For example we want to create a structure "ground plane". It's a generic classes describing a plate of thickness d, length l, width w, conductivity s. We may create :

```
class plate:
    def __init__(self,d,l,w,s):
        self.d=d
        self.l=l
        self.w=w
        self.s=s
        self.uo=4.*pi*1E-7
        self.Dx=[self.d,self.l,self.w]
    def Z(self,p):
        return 1./self.s*self.l/self.w*(1./self.d)+\
        sqrt((pi*p*self.uo*self.s)/(1J*2.*pi))
```

As this classe calls functions that belong to the package numpy, the use of this class must be preceded by a numpy calling :

```
from numpy import *
```

The dimensions of the plate are included in one of its properties, $self.D$. After an instance of the class plate, we can compare the requirements and the plate properties, comparing $\overset{x}{\mathcal{Q}}_i$ and $R.D$ if R is the instance of plate.

Now we can imagine two electronics. They belong to the same class *electronic*.

```
class electronic:
    def __init__(self,r,i):
        self.r=r
        self.iz=i
        self.Di=[-1E6,1E6]
        self.Dr=[1E6,1E6]
    def ZJ(self,p):
        return self.r+self.iz*p
```

This class allows to create the extremities of a line. The class line is given by :

```
class wire:
    def __init__(self,le,dia,cd):
        self.le=le
        self.dia=dia
        self.cd=cd
        self.uo=4.*pi*1E-7
        self.DI=[0.,1E3*self.dia]
    def ZW(self,p):
        return self.le/(self.cd*pi*self.dia)*\
        (4./self.dia+sqrt((pi*p*self.uo*self.cd)/(1J*2.*pi)))
```

With a plate, two loads and a wire we can construct a simple system made of the wire connecting two loads and installed over the plate.

Anybody may say : but how do we know that the chosen parts are in adequacy with the required system performances ? Here the frontier between system concept and feasibility is not so clear. A part of experience coming from the engineers intervenes in this process. But we may say that the major engine to make the first choices comes from evidences, themselves coming from nature observations. In the human being history, we see each time that humans take example on the natural systems to create their own artificial ones. Planes are a good example for that : in his first tentative, the human tries to copy birds. The planes were created step by step to conduct to the

artificial birds we known today.

8.2.2 Feasibility

Between all the objects possibly used and coming from the collection O_i, we must verify their compliance with the system requirements in its mission. This means to verify between others, the capacity to accept the environment conditions. The feasibility is this compliance between the component capabilities and the system needs. This information is included in the domain values and the associated laws in the definition of the operator z. The domain intersections lead to the system capability. The feasibility of a solution (a solution being a set O_i). By exploring the various combinations of components and adding the couplings between these components, we can highlight the lacks of the system prototyped. It is clear that to reach such a performance in the analyse, the models associated with each component, their domains definitions depending of the environment parameters must be accurate and pertinent.

Mathematically, we connect the components of O_i using a connectivity C which creates closed fluxes. The relation is simply given by :

$$b^k = C_a^k Q^a \tag{8.18}$$

Knowing this relation, the operator z is transformed in the closed fluxes space making $C^T z C = \zeta$. We then add the couplings to obtain the major operator representing the system and its domains, the whole making a manifold which is the system model. ζ contains all the mechanical and electrical information. Thermal exchanges are embedded in the domains settings. It means that the component of ζ seems like :

$$\zeta_{ij} = \overset{t}{\mathcal{D}}_{I1} L_1(\bullet) + \overset{t}{\mathcal{D}}_{I2} L_2(\bullet) + \dots \tag{8.19}$$

Once a kind of system is chosen (exoskeleton, endoskeleton) and the mission known the mathematical definition of feasibility can be detailed starting from the previous exposed concept.

The mission is declined by the actions made, the capacity of communication, the capacity of having sources of energy and the energy consumption, and the environment where the mission is realized. Once a kind of system chosen to answer to the customer needs ensure the possibility to cover the

actions specified, the mission and customer needs are principally declined in constraints and environment definition during the mission. In our example, it means that we have chosen to make a system with a plate ; mounting on this plate there is a connector to make the pin-socket contact, and some mechanical parts to realize the rest of the mock-up. The environment constraints are component or the source vector T (see equation 8.1). The target of robustness opposite the environment is described in domains. For temperature (t), pressure (p), intervals are defined and we know $\overset{t}{\mathcal{Q}}_i$, $\overset{p}{\mathcal{Q}}_j$ (other constraints may be defined as gravitation, etc.). For each elements chosen between others in the collections O_i, their tenfolds [2] should be defined on domains that cover the domains required by the customer needs. Noting $\overset{x}{\mathcal{D}}_i$ these domains we want to compare $\overset{x}{\mathcal{D}}_i$ and $\overset{x}{\mathcal{Q}}_i$ for all the system specifications. But it is clear that all the system organs won't see these levels of constraint. First exercise when making a new system is to propagate the constraints from the environment to each system part. The system must be represented as a set of sub-parts interconnected. Each sub-part is seen as a zone and receives the constraints and on another side transmits the constraints to enclosed zones. This study has various uncertainties, but at this phase of the project, we don't need to be very accurate. The figure 8.4 shows how the process works. In the feasibility phase, the major work is concentrated in this task of translating the environment constraints to the various parts of the system depending on its global architecture.

To determine this constraints propagation for the pressure we can use the same approach as for mechanics. A graph representing the various zones as nodes and their links as edges. Similar technique can be used for temperature as seen previously. That's what we will see later.

The computation of the adequacy between the virtual system and the future one that will be used is realized with benefit of the classes informations. In fact, the feasibility step is the transition step between the conception of the system and the beginning of its development. Let's take an example. We imagine a system made of one resistance.

2. A tenfold is a set of mathematical objects usable in the tensorial analysis of networks and leading to the manifold that represents a system or a part of the system. The tenfold concept was firstly developed in a thesis whose the author is the redactor : *Maurice, O. (2013). Introduction d'une théorie des jeux dans des topologies dynamiques (Doctoral dissertation, Limoges).*

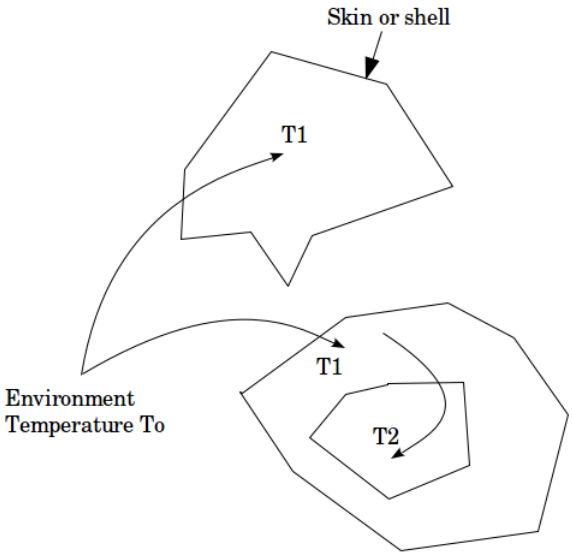


FIGURE 8.4 – Constraints propagation

Architecture side

The class architecture will call various sub-classes in order to make the system. Being very simple to illustrate this mechanism, we can create a structure "resist" defined for a first option by :

```
class resist1(object):
    def fixr(self):
        self.R=10.
        return self.R
```

Then the architecture is realized calling for each sub-class for structures, cables, electronics, etc. In our simple illustration it gives :

```
class architecture(electronique):
    R2=resist1()
    R3=R2.fixr()
```

If the temperature is the environment parameter, we first add the performance of the structure temperature as a class method :

```
class resist1(object):
```

```
    def D_T(self):
        return [1,2]

class archi(resist1):
    R=resist1()
```

When creating an instance of the class to define an architecture : *g=archi()*, we can after call for the architecture performances in temperature writing : *g.R.D_ T()* which return $[1, 2]$. The domain can be compared with the needs $\overset{x}{\mathcal{Q}}_i$. This second domain is inscribed in an environment class which may be :

```
class envir(object):
    def Q_T(self):
        return [1,3]
```

Here we would have got $Q > D$ and the structure *resist1* would not be compliant with the requirements. The structure class as the cable class or the electronic class have for method an impedance operator matrix (illustrated by R in the class resist1) constructed on functions declared in the property field. With a collection of instantiated classes, the architect class realizes the direct sum leading to the system definition operator ζ. This allows to evaluate the system response and its adequacy with the requirements. The intervals comparison indicates the system capability to work in the mission environment while the global operator ζ says if the system realizes the required mission. Starting from our previous example, we first take a look to the adequacy of the three components : structure, cable and electronics with the requirements. Defining

— the structure length tolerance $\overset{x}{\mathcal{Q}}_1 = [0.9, 1.1]$;
— the cable current strength requirements $\overset{I}{\mathcal{Q}}_1 = [0, 0.1]$;
— the electronics loads $\overset{r}{\mathcal{Q}}_1 = [100, 10^3]$.

First step of feasibility means to compare the required intervals $\overset{j}{\mathcal{Q}}_i$ and the informations of capacities include in the classes. It means to do there the comparisons :

— plate.Dx with $\overset{x}{\mathcal{Q}}_1$
— wire.DI with $\overset{I}{\mathcal{Q}}_1$
— electronic.Dr and $\overset{r}{\mathcal{Q}}_1$

In the case of the wire, we see that the model depends on a parameter that can be set for the cable : the diameter. The architect has to choice the adequate diameter to be compliant with this requirement. In final, this collection of organs can be a possible architecture for the needs. Now we have to construct the model associated with this system to verify that it can be a solution for the requirements, even for various mission profiles or environment constraints. In a general way, the previous exercise can necessitate to make a declination of constraints to evaluate the comparisons.

Declination of constraints

The problem is to calculate the evolving of the intervals $\overset{\alpha}{\mathcal{Q}}_\beta$ from outside to inside. The exercise is very difficult whatever the physics considered. The paradox is that the constraints can be declined only once the system is made, and we want to decline the constraints to determine the system ! Depending on the system architecture, the constraints are propagated more or less from the environment into the system body. This propagations determined by the coupling tensor μ. This tensor is the elastic tensor for mechanics, the electromagnetic coupling tensor for electronics, etc. There are no other ways than to make assumptions on these propagations. To make the system we use three major operations :

1. selection of the system components ;
2. connection of the components to make the system frame ;
3. adding the coupling mechanisms creating the system complete architecture and working.

It is clear that the first step of system concept allows only to reject options that are out of the requirements by an evident way. Feasibility needs to go further. To do this, the system is assembled and constraints are applied in order to make a first evaluation of the system response versus these constraints. That's the only solution taking into account that some coupling mechanisms in near fields cannot be evaluated without assembling the system. So, let's make the system !

We complete the previous classes with some added functionalities that was necessary to construct a system and determine its adequacy with the requirements.

#

```
class plate:
    def __init__(self,d,l,w,s):
        self.d=d
        self.l=l
        self.w=w
        self.s=s
        self.uo=4.*pi*1E-7
        self.Dx=[self.d,self.l,self.w]
    def Z(self,p):
        return 1./self.s*self.l/self.w*(1./self.d)+\
        sqrt((pi*p*self.uo*self.s)/(1J*2.*pi))

class electronic:
    def __init__(self,r,iL,iC):
        self.r=r
        self.izL=iL
        self.izC=iC
        self.Di=[-1E6,1E6]
        self.Dr=[1E6,1E6]
    def ZJ(self,p):
        if(self.izC==0.):
            return self.r+self.izL*p
        else:
            return self.r+self.izL*p+1./(self.izC*p)

class wire:
    def __init__(self,le,dia,cd):
        self.le=le
        self.dia=dia
        self.cd=cd
        self.uo=4.*pi*1E-7
        self.DI=[0.,1E3*self.dia]
        self.DP=[0.,200E-3]
    def ZW(self,p):
        return self.le/(self.cd*pi*self.dia)*\
        (4./self.dia+sqrt((pi*p*self.uo*self.cd)/(1J*2.*pi)))
```

```
#definig an architecture
struc=plate(1E-3,1.,0.2,1E-7)
cable=wire(0.5,1E-3,1E7)
load1=electronic(100.,1E-9,0.)
load2=electronic(1E3,1E-9,10E-9)
# component selection
p=1J*2.*pi*1E6 # test for 1 MHz frequency
Zb=[[struc.Z(p),0.,0.,0.],\
    [0.,cable.ZW(p),0.,0.],\
    [0.,0.,load1.ZJ(p),0.],\
    [0.,0.,0.,load2.ZJ(p)]]
#connections
C=[[1.],[1.],[1.],[1.]]
Zmi=dot(transpose(C),Zb)
Zm=dot(Zmi,C)
#adding coupling mechanism: here inertia tensor u
h=5E-2 # installation option for the cable over the groundplane
Zc=60.*log(4.*h/cable.dia)
c=3E8 # second installation option: no dielectric between
#the wire and the groundplane
L11=Zc/c*cable.le
u=L11*p
Zmc=Zm[0][0]+u
Bo=1E-3 # magnetic field of the environment
e=-cable.le*h*p*Bo # electrmotive force
K=e/Zmc # current induced in the cable
P=cable.ZW(p)*abs(sqrt(dot(K,conj(K))))
# is it in compliance with the wire capacity?
compare=max(cable.DP)/P
if (compare>1.):
    print "This cable can be used in the defined environment"
```

First, a collection of organs, including the cables, are available. In this collection we test an architecture assumption, made of one plate, one wire and two loads. The figure 8.5 shows the system we have construct.

After making the selection, we define the impedance operator tensor from the diagonal organization of the selected organs. This gives an object Z_b defined in the edge space.

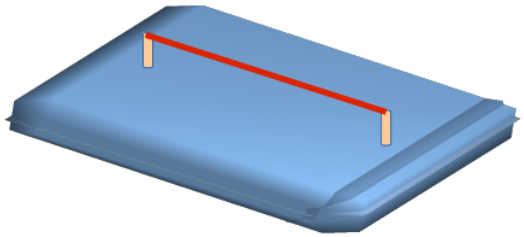


FIGURE 8.5 – Architecture realized

Then we must connect these organs between them. That's the role of the connectivity matrix C. Our system is made of one mesh (its genus is 1). Each element coming from the classes instances participates to the common mesh of the system, which is shown in the tab :

Organ	Instance	Common mesh
struc	plate	1
cable	wire	1
load1	electronic	1
load2	electronic	1

This tab leads to the connectivity matrix C which gives the impedance operator tensor in the mesh space $Z_m = C^T Z_b C$. But this operation gives the various meshes making the system global structure, without defining the couplings between the elements, including the inertia tensor[3]. Defining here for the unique mash involved $\mathcal{L} = L_{11}$, we compute $Z_m = Z_m + L_{11}p$. L_{11} is the inductance between the wire and the plate given by :

$$L_{11} = \frac{Z_c}{c} \quad Z_c = 60ln\left(4\frac{h}{\phi}\right) \tag{8.20}$$

ϕ is the wire diameter, h the height of the wire over the plate, c the propagation speed in the line made by the wire over the plate and Z_c the characteristic impedance of this line. Once the system constructed, we can calculate the power dissipated by the current induced in the line by an external magnetic

3. See *Elements of theory for electromagnetic compatibility and systems*, O.Maurice, Bookelis editor 2017.

field coming from the environment specification. Then we compare this value to the maximum power acceptable by the wire, which is the comparison between $\mathcal{Q}$ and $\mathcal{D}$. To do that, we have complete the class wire with the interval $wire.DP$ defined in power. We have realized the constraints declination through the global computation of the system. This operation can be made for all a set of possible architectures. But if it gives the compliance of the imagined system with the environment requirements, it remains to verify the compliance with the functional requirements. This is done by exciting the system through sources to provoke its actions in movement, perceptions, exchange of information, etc. These system response must be compared with the functional requirements. In fact, it has been done quasi implicitly during the system concept elaboration, but this gate must be verified once a detailed system description is available.

8.2.3 Development

At the start, the system is a simple idea, a concept. This concept can be seen as a construction made with elements coming from abstract sets. It's the same as previously, but the set $h\{\}$ contains no real component. It points out the kind of subsystem implicitly identified by the name of the set. This steps allows to define the large lines of the architecture before to detail the technologies involved in its realization. This realization goes through studies, theoretical analysis, simulations to end by some prototype not so far from the final system. This is the development phase.

The various objects coming from the selected set $O_i\{c_i, h_i, w_i, r_i\}$ are identified with their impedance operator ζ constructed from all the separate element in the object and from the connectivity C_a^k. The basic system is **electromechanical** and the **local environment is thermo-acoustical** while the **far environment is electromagnetic and gravitational**.

Often the first action made in classical systems concerns the structure. It is made of parts ζ_{mv} coupled each other. Then electrical parts ζ_{ev} are added and coupled each other and with the mechanical parts. First action means to directly summing both operators :

$$\zeta_{mv} \oplus \zeta_{ev} \tag{8.21}$$

then to add the couplings α_{me} and α_{em}. The same operations can be applied to the graph of each object. In fact the more usual technique starts from

the graph representation of the objects and guide the operations to be made on the corresponding cellular components. The couplings are embedded by extra-diagonal components because even in hard coupling case (when material is shared), a branch is shared between two meshes. This is translated by a coupling function which is the shared branch law. In other cases, for all far field interactions, cords wear the coupling functions.

The industrial management of the development is not so trivial. Dictionaries must be used to archive the knowledge and characteristics of the primitive objects selected. In up phases, each component is repaired by its major properties. These properties will be compared to the available ones in some list of parts $p\{\}$. The previous process is followed as described before.

If the coupling mechanism appears and is classically used in electromagnetic for Kron's formalism, The exercise had to be done for mechanics.

In the development phase, all details are explored and should be accurately defined at the end of this phase. Compared to the feasibility phase, added operations describe at a lower scale the different system parts in order to be nearer to the reality. Once the choices are definitively written, some simulations using meshed techniques (testing virtually the system) are done. They confirm the waited behaviours and can be completed by some experiments on prototypes. For example, when making the system principles, we developed some links for buses, power supply, ..., but not for the whole signals. The demonstration of feasibility do not need to consider all the electronics and their accurate definition. Only some representative signals are sufficient to demonstrate the pertinence of the architecture. But in the validation phase, the whole harnesses are taken into account in the (virtual or real) prototype and its behaviour in various environment is computed. A part of this detailed development is devoted to the suppliers. For that, the declination of constraints must be clearly written as a specification requirements given to the suppliers. For example, based on first assumption of impedances, the electromagnetic constraints are measured across the electronics input, using the input loads *electronic.ZJ*. Taking some margin, this value is transmitted to the equipment supplier as a requirement to be cover. The equipment supplier will develop his electronic equipment taking into account this target constraint. Even if the system is not described for all its signals, each kind of links is already described as functional link. And another very important technique intervene in order to make the system globally modelled : zoning. For various reason and first for security, it is important to separate the system in various zones sufficiently segregated between each other. With this pro-

perty, the system can be studied by parts. We can try to give this definitions under mathematical ones. The first step is to realize the direct summation of all the organs operators $z_i : z = \oplus_i z_i$. After what we add the couplings. Using zoning means that the coupling matrix presents weak coupling between some system parts. If z_a is a first sub-matrix of z and z_b a second sub-matrix of z, a weak coupling between z_a and z_b means that a single coupling function z_{ij}, $i \in [a]$, $j \in [b]$ exists and this coupling occurs without modifying the source characteristics. Under this assumption, the partial matrix attached to the couple z_a and z_b seems like :

$$\begin{bmatrix} z_a & z_{ij} \\ z_{ij} & z_b \end{bmatrix} \tag{8.22}$$

The second equation leads to $Z_{ij}k^j = Z_b k^b$ The two parts a and b can be separated reporting the coupling as an external constraint on the second part. (the same can be made with the first part). This principle allows to study the system by parts. This is a key technique to guarantee the system safety, its whole constitution being too much complex to be analysable. Finally two major assumption are made in order to conduct the system definition :

1. The system can be studied by part ;
2. typical impedances for the electronics are known.

This second point is a critical one. When making the first calculation on the system to see its feasibility, it's necessary to know the input impedances of the electronics. Under this assumption, declination of constraints, specifications, etc. can be made. Is this assumption reasonable ? More and more it is ! Modern buses, power supplies, numeric circuits present similar impedances. The diversion in their values can be covered by margins or parametric studies. Let's illustrate this approach with our previous example of a wire over a ground plane. The mesh impedance is simply $L_f p + R_f + Z_G + (Z_1 + Z_2)$. $L_f p$ and $R_f p$ are the impedance of the wire. Z_G the impedance of the ground plane. Z_1 and Z_2 are the impedances on the equipment inputs. Depending of the kind of electronic involved in the circuit, these two impedances take various complex values. The constraint reported on one equipment is defined by :

$$P = \frac{1}{4}\left\{ui^* + u^* i\right\} \tag{8.23}$$

u and i being the voltage across the load and the current flowing in the load.

In our case, this gives :

$$P = \frac{1}{4}\left\{Z_x k^0 (k^0)^* + Z_x^* (k^0)^* k^0\right\} = \frac{1}{2}\left|k^0\right|^2 \Re\left(Z_x\right) \tag{8.24}$$

The current is given by :

$$k^0 = \frac{e_0}{L_f p + R_f p + Z_G + (Z_1 + Z_2)} \tag{8.25}$$

or :

$$\left|k^0\right|^2 = \frac{e_0}{\left[\Re Z_G + R_f + \Re\left(Z_1 + Z_2\right)\right]^2 + \omega^2\left[L_f + \Im\left(Z_G + Z_1 + Z_2\right)\right]^2} \tag{8.26}$$

Often $\Re Z_G \approx R_f \to 0$. With $\Im Z_G << L_f$ we obtain :

$$\left|k^0\right|^2 = \frac{e_0}{\left[\Re\left(Z_1 + Z_2\right)\right]^2 + \omega^2\left[L_f + \Im\left(Z_1 + Z_2\right)\right]^2} \tag{8.27}$$

For $Z_x = Z_1 = Z_2$ this leads to :

$$P = \frac{1}{2}\frac{e_0 \Re Z_x}{\left[2\Re Z_x\right]^2 + \omega^2\left[L_f + 2\Im Z_x\right]^2} \tag{8.28}$$

For the majority of electronics (except electrical machines, etc.), $2\Im Z_x << L_f$ and :

$$P \approx \frac{1}{2}\frac{e_0 \Re Z_x}{\left[2\Re Z_x\right]^2 + \omega^2 L_f^2} \tag{8.29}$$

We see that under these assumptions, the result depends only of the real part of the loads. The problem is reduced by studying the order of value for this real part. And they are often known : for analogical, numerical signals it's typically 1 $M\Omega$. For hyper-frequencies, 50 Ω, etc.

But if we consider the voltage as criterion, the conclusion is different :

$$|u| = |Z_x|\left|k^0\right| = e_0^{1/2}\sqrt{\frac{\Re Z_x^2 + \Im Z_x^2}{\left[2\Re Z_x\right]^2 + \omega^2 L_f^2}} \tag{8.30}$$

In that case, the imaginary part influences the result. In that case we may consider the worst case given by the higher credible impedance, as much for the real part as for the imaginary part. The equipment input geometry gives many information for evaluating the inductance. The frequency band of the

circuit is another source of information to evaluate the capacitance of the impedance, and the resistance is sometimes given by the circuit consumption or the match condition in hyper-frequencies. Informations on the impedances can also be obtain reading data-sheets. The main problem comes from the impedance knowledge and circuit behaviour in out-band stress. The threshold also must be evaluated, as it is not the same for out-band or in-band.

For out-band, the threshold level is in general unknown. But many experiments made in the year 80' have shown the next results [4]

1. out-band criterion is defined in transmitted power ;
2. the transmitted continuous level (or in general "in-band" level) is a share of the total power.

The problem becomes to evaluate the cœfficient to be applied to the peak value of the transmitted power to calculate the effective in-band energy. This cœfficient is called cœfficient of detection efficiency. Without any more information, we can take 0,1 for value of this cœfficient. Knowing the level transmitted to the component through its impedance input schematic, the proportion of this level is compared with the functional in-band threshold in order to conclude on the disturbance risk.

8.2.4 On the risk of disturbances costing

The probability to provoke the disturbance of a component is associated with the distance between two functions [5]. If we test a component to a electrical stress, the number of components of exactly the same reference depending on the stress level seems like a Gaussian curve. By the fact, the probability that the component becomes disturbed for a stress superior to a given level u is a curve having the form shown figure 8.6.

This curve says that under a give level, no component are disturbed. Over a given level, no component exists being able to support the constraint, and an average of the tested components are disturbed for the level at mid rise of the curve. The constraint can be characterized by the same way. Beyond a given value, no constraint exists. Below a given value, it exists always a

4. see MAURICE, Olivier et PIGNERET, Jacques. Digital circuit susceptibility characterization to RF and microwave disturbances. In : Radiation and Its Effects on Components and Systems, 1997. RADECS 97. Fourth European Conference on. IEEE, 1997. p. 178-181.

5. see O.Maurice, *Elements of theory for electromagnetic compatibility and systems*, page 304. Bookelis editor 2017.

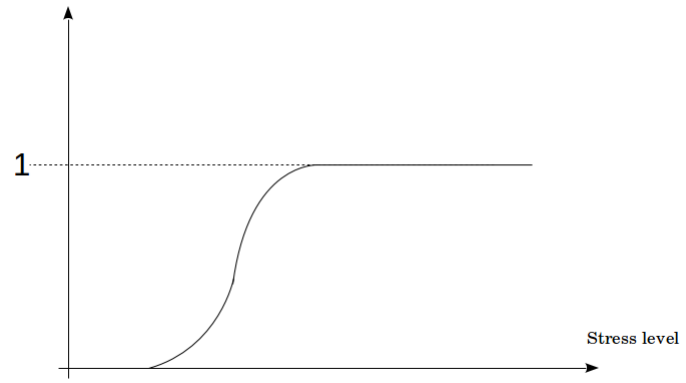


FIGURE 8.6 – Repartition function of disturbance

constraint presenting this level. The curve can be traced on the same graph than the one of the figure 8.6. The surface of intersection represents the probability to disturb the component (figure 8.7).

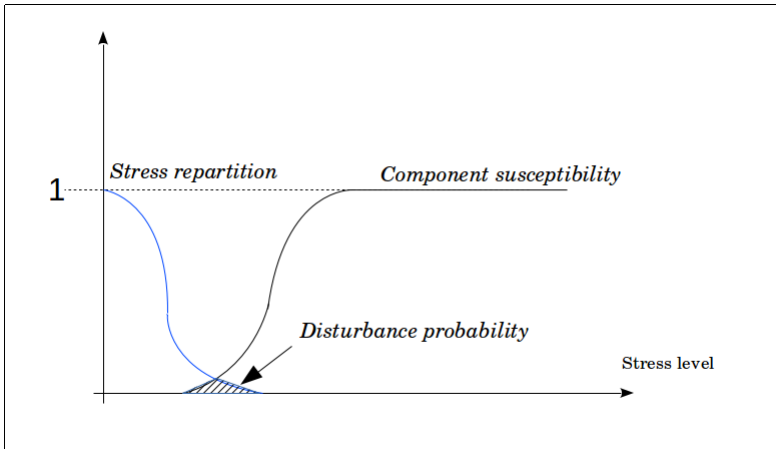


FIGURE 8.7 – Disturbance probability

A typical law that can be used for the repartition function is the sigmoid function. The susceptibility level s_l is something like :

$$P\left(s > s_l\right) = \left[1 + \alpha exp\left(-\frac{u}{s_l}\right)\right]^{-1} \tag{8.31}$$

Then the constraint function bounded by v_c has the form :

$$P\left(u > v_c\right) = 1 - \left[1 + \beta exp\left(-\frac{u}{v_c}\right)\right]^{-1} \tag{8.32}$$

and the probability P_{df} to disturb the component is given by :

$$P_{df} = \left(1 - \left[1 + \beta exp\left(-\frac{u}{v_c}\right)\right]^{-1}\right)\left(\left[1 + \alpha exp\left(-\frac{u}{s_l}\right)\right]^{-1}\right) \tag{8.33}$$

8.3 Application to system of systems (SoS)

System of systems are group of systems in interactions. These interactions can be communications, electromagnetic interactions, mechanical interactions (we can think in meteorite swarm), etc. Compare to systems, the difference is that there are no physical contacts between the systems. So the interactions are only through far field. it means that the SoS dimension is the operator ζ dimension. The system construction is realized by coupling various independent systems by waves. No connection by branches or near field interactions are used. The idea is to say that the system of systems doesn't change the intrinsic characteristics of each system. The best way to say that is the definition : *Definition : Giving a set of systems of operators* $\{\zeta_i\}$*, a system of systems is made by the direct summation of the systems* $\oplus_i \zeta_i$ *enriched by an interaction tensor only made of weak interactions cords.*

The approach can be perfectly similar to the one of systems. Each system can be seen as a class with its methods (capacities of actions) and properties. The architecture becomes the action of SoS conception. Adding the interactions like a communication network transforms the set of systems in a system of systems. A shared information is the basic and indispensable key to make a SoS. More than elsewhere, the SoS uses a particular interaction : the decision interaction. This kind of exchange calls for cords elaborated from the game theory.

There are three kinds of informations in SoS :

1. informations coming from outside - the environment ;
2. informations exchange between the systems in the SoS ;
3. informations exchange inside each system.

There is one level more than in systems, coming from the fact to group systems in order to make a system. It's true that this kind of separation can be made at lower scales, for example considering microprocessors in a system. With this point of view, any system is a SoS. The distinction comes from the definition of a system itself. If we consider a microprocessor, it's not a system in fact because it is not able to realize any mission by itself. If we accept in a system definition in the sense of artificial system that it leads to some mission in the society, our levels of interactions seems to be sufficient. Another way to discern SoS from systems is to say that in any case and whatever the criteria, the SoS has one more interaction layer than systems. The need of game theory to model all interactions for SoS is the first step to go from artificial systems to natural ones, or to be inspired by these natural systems. But before all, we must realize some software implementing this approach and then, having demonstrate the possible use of the formalism, it is possible to increase its complexity taking into account human factor.

Chapitre 9

SUETAN

SUETAN object is to implement previous techniques in a software for electromagnetic compatibility. The same approach can be used for other jobs.

9.1 Steps in the method

Equipments must be defined by a matrix z giving their structure (note that the approach we submit for systems can be beforehand used for equipments, considering them as systems !). In these matrix, some branches belongs to the equipment frontier and can be identified for having this property. Basically it's always possible to add a branch of high impedance (around 1 GΩ) without disturbing at all the nominal working of the equipment.

Then this impedance can be replaced by another impedance in order to translate connections with lines between two or more equipments. That's the second step when making a system. Last step consists in adding the interactions in relation with the structure (interactions between equipments or cables). Radiative ports can be easily created like probes coupled with the electronics inside the equipments. In these equipments there are also field ambiances imposed by the structures and leading to coupling mechanisms between the electronics and the structures openings.

In all cases the operators must cover the whole frequency band. It means that domains are used to point out different laws adapted to these frequency bands.

Another point is that the operator are written under Laplace's formalism. Basically, it covers both time and frequency domains. Depending on each cir-

cuit, various solutions will be used to compute non linear components. It can use canonical Laplace's function properties and non linear component response relatively to these properties. The non linear components can include generators creating the harmonics and setted by the source waveform across them.

These are here the basic principles for SUETAN.

9.2 Closing frontier edges

After realizing the instantiations of the equipments classes, we need to close their ports by operators in relation with their interactions kind with other equipments. In general this is limited to wire interactions.

In a class (which is the software expression of a tenfold) we find :

1. the equipment operator ;
2. the equipment sources ;
3. the ports definitions ;
4. the intrinsic signals natures and waveforms ;
5. the constraints (threshold levels probability, source level probability, parameter limits, etc.) ;
6. the equipment dimensions.

For a tenfold (or class) t_1, we have the i/O ports numbering 1. Remember that the operator in tenfold are described in the meshes space. If this tenfold is a simple sinus generator of source e_1, its operator may be given by $R_1 + PQ_1$ where PQ is the port default name and value. We can imagine a second equipment t_2 with an operator $R_2 + PQ_2$.

Now we decide to make a system with these two equipments. What is a line model to connect them ?

In low frequencies, a line is simply a telegraph cell. We have one constraint : our model must define operator for the frontiers then a cord to connect the i/O. So our line model must respect these rules. If we consider one telegraph cell, it is made of two inductance and one capacitor (eventually, it includes resistances and one conductance, but this is details). Connecting equipments t_1 and t_2 by ports PQ_1 and PQ_2 means to group the two equipments making

$t_1 \oplus t_2$ then to replace :

$$\begin{cases} PQ_1 \rightarrow \frac{Lx}{2}p + \frac{1}{Cxp} \\ \\ PQ_2 \rightarrow \frac{Lx}{2}p + \frac{1}{Cxp} \end{cases} \tag{9.1}$$

L and C being the line inductance per meter and capacitor per meter, x the line length and p Laplace's operator. The cord between the two meshes (each equipment is a mesh) is the shared branch, i.e. the capacitor. Then the whole operator for the connected two equipments is (we note $L/2 = L'$:

$$z - \begin{bmatrix} R_1 + L'xp + \frac{1}{Cxp} & -\frac{1}{Cxp} \\ \\ -\frac{1}{Cxp} & L'xp + \frac{1}{Cxp} + R_2 \end{bmatrix} \tag{9.2}$$

This model can be used in low frequencies, i.e. until the line length remains short compared to the wavelength. The limit frequency f_l can be defined by

$$f_l = \frac{c}{4x} \tag{9.3}$$

The previous model must be limited to this frequency domain. To indicate this condition, we multiply the corresponding laws by the domain restriction $\overset{\nu}{\mathcal{D}}_{BF}$ with $BF = [0, c/4x[$:

$$z = \begin{bmatrix} R_1 + \left(L'xp + \frac{1}{Cxp}\right)\overset{\nu}{\mathcal{D}}_{BF} & -\overset{\nu}{\mathcal{D}}_{BF}\frac{1}{Cxp} \\ \\ -\overset{\nu}{\mathcal{D}}_{BF}\frac{1}{Cxp} & \left(L'xp + \frac{1}{Cxp}\right)\overset{\nu}{\mathcal{D}}_{BF} + R_2 \end{bmatrix} \tag{9.4}$$

At higher frequencies, Branin's model must be used. In that case $PQ_1 = PQ_2 = z_c$ the characteristic impedance of the line and the two defined cords are :

$$\begin{cases} \alpha_{12} = (R_2 - z_c)\, e^{-\tau p} \\ \\ \alpha_{21} = (z_c - R_0)\, e^{-\tau p} \end{cases} \tag{9.5}$$

A second source $e_1 e^{-\tau p}$ being added to the t_2 sources. If $HF = [c/4x, f_{TEM}[$ (f_{TEM} being the limit frequency of the line in TEM mode, after what, modal propagation should be modelled) and if :

$$\begin{cases} \beta_{12} = -\frac{1}{Cxp} \\ \\ z_{bf} = \left(L'xp + \frac{1}{Cxp}\right) \end{cases} \tag{9.6}$$

The whole frequency band operator is :

$$z = \begin{bmatrix} R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} \\ \\ \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 \end{bmatrix} \tag{9.7}$$

For the first step that interests us, we finally realize the closing of the frontiers by :

$$\begin{cases} PQ_1 \rightarrow \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c \\ \\ PQ_2 \rightarrow \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c \end{cases} \tag{9.8}$$

Next step is to define the cords, thing that we have already computed for lines.

9.3 Adding cords for lines

First step of closing frontiers leads to the operator defined by :

$$z = \begin{bmatrix} R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & 0 \\ \\ 0 & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 \end{bmatrix} \tag{9.9}$$

after what we can add the cords coupling the i/O of the equipments in relation through the line. We obtain the operator described equation 9.7. The operation to create the lines, after the closing step is nothing more but nothing less than this. Note that there is a paradox in defining lines before to define the structure. If the line is a simple wire using the structure as second conductor, the theoretical expression of the characteristic impedance must be setted once the structure is defined. The technique is the same that the one used for the closing step. An abstract value is written in place of the definitive value depending on the structure.

We understand that for the theoretical analysis of our system, everything can be expressed in the order we submit. For two wires line, the characteristic impedance is fixed by the line itself. The structure can only disturb this value.

9.4 Grouping lines in harnesses

If we consider two lines l_1 and l_2 of operators z_1 and z_2 similar to the previous one. These two lines are grouped in a shared harness, which means to make $z' = z_1 \oplus z_2$. So :

$$z' = \begin{bmatrix} R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} & 0 & 0 \\ \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 & 0 & 0 \\ 0 & 0 & R_3 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} \\ 0 & 0 & \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_4 \end{bmatrix} \tag{9.10}$$

As previously, we have to add the couplings between the two lines. These couplings are separate like the lines in two domains : low and high frequency bands.

9.4.1 Grouping lines in low frequencies

Between each couple of lines we compute the mutual inductance, defined by Neumann's relation :

$$M = \frac{\mu_0}{4\pi} \oiint_{x,y} \frac{\mathbf{dx} \cdot \mathbf{dy}}{r_{xy}} \tag{9.11}$$

The mutual inductance is reported on each extremity of the opposite line. This add the coupling terms :

$$z' = \begin{bmatrix} R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} & -Mp & 0 \\ \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 & 0 & -Mp \\ -Mp & 0 & R_3 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} \\ 0 & -Mp & \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_4 \end{bmatrix} \tag{9.12}$$

The load accumulated on a first capacitor of a first line q generates an electric field E given by :

$$E = \frac{q}{4\pi\epsilon r^2} \tag{9.13}$$

The electromotive force induced on the second line is $e = hE$ if h is the line height. Noting :

$$C_{12} = \frac{h}{4\pi\epsilon r^2} \tag{9.14}$$

we find $e = q/C_{12}$. But $q = C_1 V$ with $V = 1/C_1 pi$. Finally :

$$\frac{e}{i} = \frac{1}{C_{12}} C_1 \frac{1}{C_1 p} = \frac{1}{C_{12} p} \tag{9.15}$$

and finally :

$$z' = \begin{bmatrix} R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} & -Mp - \frac{1}{C_{12} p} & 0 \\ \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 & 0 & -Mp - \frac{1}{C_{12} p} \\ -Mp - \frac{1}{C_{12} p} & 0 & R_3 + \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} \\ 0 & -Mp - \frac{1}{C_{12} p} & \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{bf} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_4 \end{bmatrix} \tag{9.16}$$

This result is due partly to the fact that there is no propagation in the model.

9.4.2 Grouping lines in high frequencies

Due to the propagation, the situation is more complicated but was studied by Jean-Paul Vabre (see reference). We define first two cœfficients α and β with :

$$\alpha = \frac{\gamma}{C+\gamma} \quad K = \frac{M(C+\gamma)}{L\gamma} \tag{9.17}$$

Under some assumptions, the near-end crosstalk is given by :

$$e_n = \alpha \frac{K+1}{2} z_c i_{opp} \tag{9.18}$$

where i_{opp} is the current of the opposite line and z_c its characteristic impedance. The far-end crosstalk is given by :

$$e_f = -\alpha(K-1)\tau p z_c i_{opp} \tag{9.19}$$

τ is the electrical length of the line. As the line is represented by a couple of meshes, the cords created by the two previous relations associates one mesh with two others. Figure 9.1 shows these cords. These two cords exist between all the couple of meshes of the two lines. The figure 9.2 shows the four cords that intervene in the coupling process. Finally for the near-end cross talk we have :

$$z'_{31} = -\overset{\nu}{\mathcal{D}}_{BF} \left(Mp + \frac{1}{C_{12} p} \right) + \overset{\nu}{\mathcal{D}}_{HF} \left(\alpha \frac{K+1}{2} z_c \right) \tag{9.20}$$

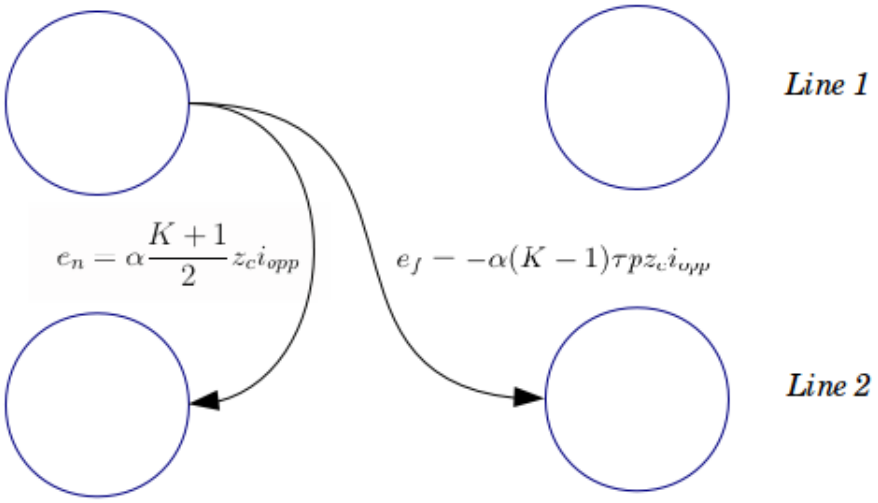


FIGURE 9.1 – Couplings between two lines

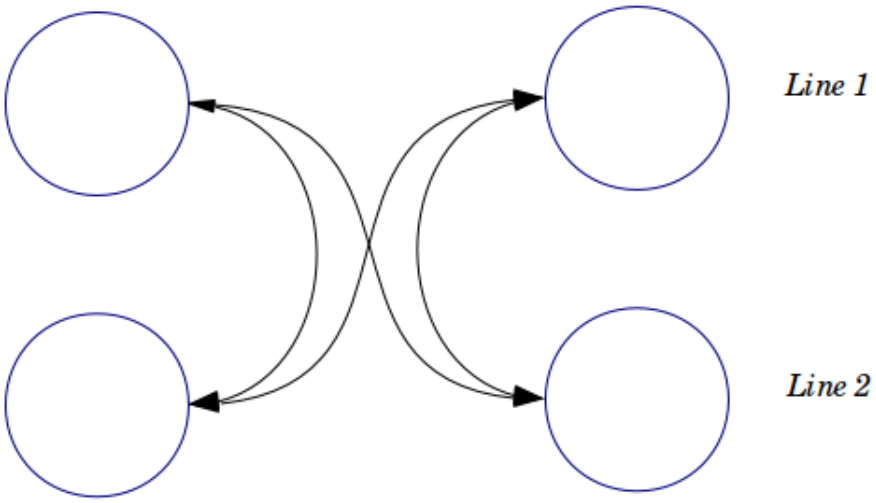


FIGURE 9.2 – Four couplings between two lines

and for the far-end crosstalk :

$$z'_{41} = -\overset{\nu}{\mathcal{D}}_{HF}\left(\alpha(K-1)\tau p z_c\right) \tag{9.21}$$

$z'_{42} = z'_{31}$, $z'_{32} = z'_{41}$, $z'_{13} = z'_{31}$, $z'_{24} = z'_{42}$, $z'_{14} = z'_{41}$ and $z'_{23} = z'_{32}$. The whole operator z' is now :

$$
\begin{array}{cccc}
R_1 + \overset{\nu}{\mathcal{D}}_{BF} z_{b_f} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} & -\overset{\nu}{\mathcal{D}}_{BF} \left(Mp + \frac{1}{C_{12^p}} \right) + \overset{\nu}{\mathcal{D}}_{HF} \left(\alpha \frac{K+1}{2} z_c \right) & -\overset{\nu}{\mathcal{D}}_{HF} \left(\alpha (K-1) \tau p z_c \right) \\
\overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{b_f} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_2 & -\overset{\nu}{\mathcal{D}}_{HF} \left(\alpha (K-1) \tau p z_c \right) & -\overset{\nu}{\mathcal{D}}_{BF} \left(Mp + \frac{1}{C_{12^p}} \right) + \overset{\nu}{\mathcal{D}}_{HF} \left(\alpha \frac{K+1}{2} z_c \right) \\
-\overset{\nu}{\mathcal{D}}_{BF} \left(Mp + \frac{1}{C_{12^p}} \right) + \overset{\nu}{\mathcal{D}}_{HF} \left(\alpha \frac{K+1}{2} z_c \right) & -\overset{\nu}{\mathcal{D}}_{HF} \left(\alpha (K-1) \tau p z_c \right) & R_3 + \overset{\nu}{\mathcal{D}}_{BF} z_{b_f} + \overset{\nu}{\mathcal{D}}_{HF} z_c & \overset{\nu}{\mathcal{D}}_{BF} \beta_{12} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{12} \\
-\overset{\nu}{\mathcal{D}}_{HF} \left(\alpha (K-1) \tau p z_c \right) & -\overset{\nu}{\mathcal{D}}_{BF} \left(Mp + \frac{1}{C_{12^p}} \right) + \overset{\nu}{\mathcal{D}}_{HF} \left(\alpha \frac{K+1}{2} z_c \right) & \overset{\nu}{\mathcal{D}}_{BF} \beta_{21} + \overset{\nu}{\mathcal{D}}_{HF} \alpha_{21} & \overset{\nu}{\mathcal{D}}_{BF} z_{b_f} + \overset{\nu}{\mathcal{D}}_{HF} z_c + R_4
\end{array}
\tag{9.22}
$$

With this operator the two lines are coupled in all the frequency band.

9.5 Immersing the harness in a structure, in low frequencies

The field behavior at low frequencies is completely different from the one at high frequencies. The influence of propagation and causality which doesn't exist in low frequencies changes the laws. As for lines, the low frequency field is well modeled using lumped elements inductances and capacitors. Another golden rule is to remember that the field cannot be stopped ! The field can be computed and exists everywhere. That's the combination of incident field and refracted one which leads sometimes to a null field in some space region.

At low frequencies, the equivalent schematic of a cavity is given figure 9.3.

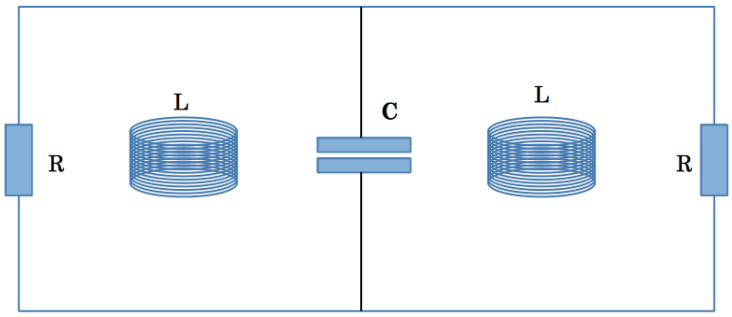


FIGURE 9.3 – Equivalent schematic of a cavity in low frequencies

The incident magnetic field creates an electromotive force given by $-pS.B$. This emf creates a current i that creates a magnetic field in opposition with the incident magnetic field. The global magnetic field at low frequencies is near to the incident one, but at higher frequencies, and more particularly beyond the skin effect, the reacted field is equal to the incident one and the total magnetic field becomes equal to zero. Mathematically, we have :

$$e_{LF} = -pS.B \Rightarrow \begin{bmatrix} e_{LF} \\ \\ e_{LF} \end{bmatrix} = \begin{bmatrix} R + \frac{1}{Cp} + Lp & -\frac{1}{Cp} - Mp \\ \\ -\frac{1}{Cp} - Mp & R + Lp + \frac{1}{Cp} \end{bmatrix} \begin{bmatrix} k^1 \\ \\ k^2 \end{bmatrix} \tag{9.23}$$

but $p \to 0$ so :

$$\Delta = \left(R + \frac{1}{Cp}\right)^2 - \frac{1}{C^2p^2} = R^2 + 2\frac{R}{Cp} \tag{9.24}$$

and

$$k^x = \frac{1}{\Delta} \begin{bmatrix} R + \frac{1}{Cp} & \frac{1}{Cp} \\ \\ \frac{1}{Cp} & R + \frac{1}{Cp} \end{bmatrix} \begin{bmatrix} e_{LF} \\ \\ e_{LF} \end{bmatrix} \tag{9.25}$$

and for example :

$$k^1 = \frac{1}{R}\left(\frac{R + 1/Cp}{R + 2/Cp}\right) e_{LF} \to \frac{e_{LF}}{2R} \tag{9.26}$$

which is the DC solution. The reaction field is so weak that it doesn't intervene in the current value. The figure 9.4 shows the mechanisms involved in low frequencies for the magnetic field.

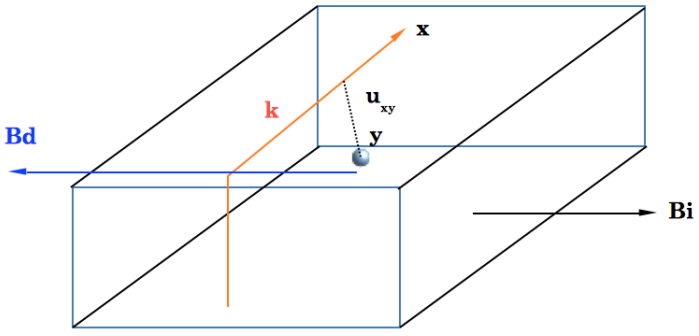


FIGURE 9.4 – Field process of a cavity in low frequencies

the reaction field radiated by the current in the cavity walls are given in any point y of the cavity volume by the Biot & Savart law :

$$B_d\left[y, i(x)\right] = \mu i \frac{d\mathbf{x} \times \mathbf{u}_{xy}}{4\pi (xy)^2} \tag{9.27}$$

If the source of B_i is far, the total field at point y is :

$$B_i + \int_x B_d\left[y, i(x)\right] \tag{9.28}$$

If we consider the particular example of a cylindrical cavity, $(xy) = r$ and in the center :

$$B_d = \int_\theta d\theta \mu i \frac{r.1}{4\pi r^2} = \mu \frac{i}{2r} \tag{9.29}$$

And the total field can be computed :

$$B = B_i + B_d = B_i \left(1 - \frac{\mu p \pi r}{4R}\right) \to B_i \tag{9.30}$$

We see that at very low frequencies ($p \to 0$), the incident magnetic field is not attenuated at all. But fortunately, the induced emf is also low. That this emf which is induced on the lines enclosed in the cavity. The equivalent schematic for the lines are also the low frequency one. Each meshes of the operator 9.16 will received an emf $-pS.B_i$ creating noise added to the functional sources.

The problem of low frequency fields inside a cavity leads to an impedance of the potential V inside the cavity which is very high. As soon as we increase a little the frequency, the major schematic becomes a RL one. Let's show that.

We can choice the meshes as desired. The branch currents are not affected by this choice. It means that whatever the choice, the connectivity leads to the same results in the branch space. This comes from the property of invariance associated with the tensorial analysis of the network. We can choice meshes leading for the same problem as before to the impedance operator :

$$z = \begin{bmatrix} R + Lp + R + Lp & R \\ R & R + Lp + \frac{1}{Cp} \end{bmatrix} \tag{9.31}$$

Under this assumption we find :

$$k^1 \approx \frac{e_{LF}}{2R + 2Lp} \qquad k^2 \approx \frac{e_{LF}}{R + 1/Cp} \to 0 \tag{9.32}$$

The current is determined by the losses and magnetic dimensions. the second cutoff frequency is given by :

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{9.33}$$

In low frequencies, the behavior is similar to a RL circuit. Then the capacitor intervenes near the resonant frequency. The induced current is driven by the resistance then the inductance. This inductance is defined, for a cavity of depth w by :

$$L = \mu\frac{S}{w} \tag{9.34}$$

S is the flux section, the same seen in the emf $-pS.B$. R is defined by $\rho P/(w.d)$. P is the section perimeter; d the wall depth and ρ the cavity conductivity. The current frequency evolving is :

$$k^x = \frac{-pS.B_i}{2R + 2Lp} \tag{9.35}$$

When the frequency (first cutoff R/L) is high enough this tends to :

$$k^x = -\frac{B_i}{\mu}w \tag{9.36}$$

or written in another way :

$$\oint_w dw.B_i = \mu k^x \tag{9.37}$$

which is Ampere's equation. This equation is the same for the reaction field $-B_d$. This implies that $B_d = -B_i$. So after this first cutoff frequency, the total magnetic field tends to zero. The field decreasing go faster and faster with frequency due to the skin effect. Figure 9.5 gives the curve of the induced current and of the total magnetic field inside the cavity.

We see that taking into account the skin effect in the resistance, the field amplitude decreases very fast to lead zero up to around 1 kHz. The total magnetic field amplitude is evaluated by :

$$|B_T| = \alpha\left|\frac{\mu}{w}B_i - k^x\right| \tag{9.38}$$

The constant $\beta = w/(2\pi\mu)$ can be called the magnetic structure constant. It gives the frequency beyond which the total field disappears.

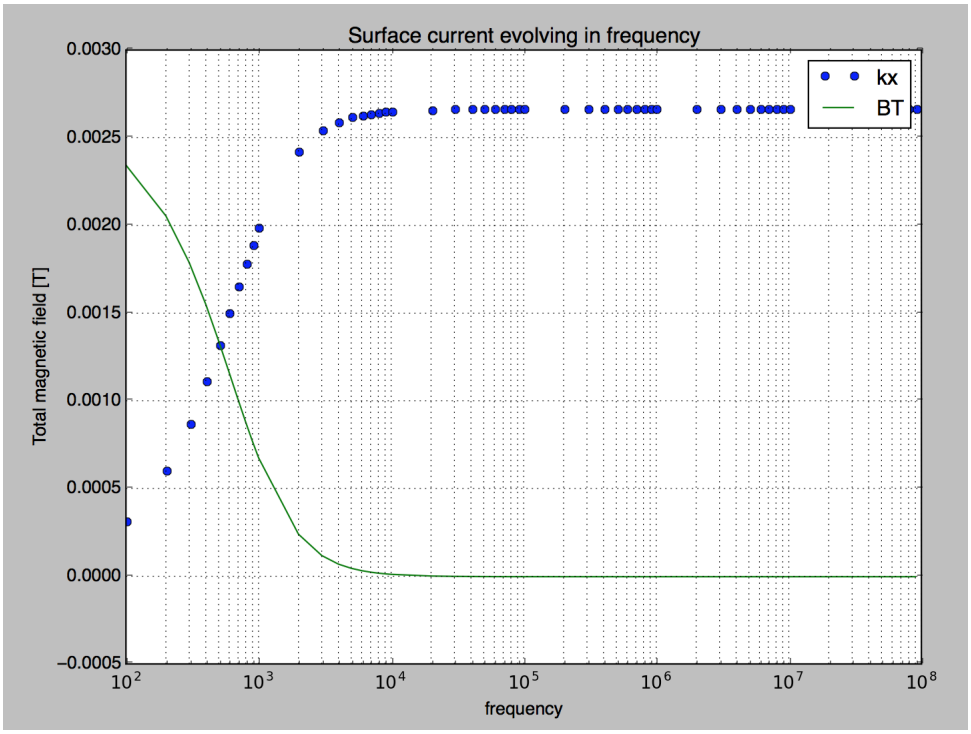


FIGURE 9.5 – Magnetic field attenuation inside a closed cavity

The incident electric field is cB_i while the electric field inside the cavity is obtained from the voltage across the capacitance :

$$E = \frac{1}{h}(Rk) \tag{9.39}$$

If there was not the skin effect, the electric field may exist at higher frequencies, respecting the relation

$$E = \frac{R}{h}k^1 + \frac{1}{hCp}\left(k^1 - k^2\right) \tag{9.40}$$

But due to the fact that the skin effect cutoff frequency arrives sooner than the RC cutoff frequency, the curve (given figure 9.6) is driven by the magnetic field and the single relation :

$$\oint_w dwB_T = \mu\sigma S.E_T \tag{9.41}$$

Without any openers in the cavity, the objects inside the cavity are completely isolated once the skin effect cutoff frequency is exceeded. Before this frequency, the emf induced in any object enclosed in the cavity can be computed, knowing the total field value. This gives the interaction between the cavity and the object. This interaction is symetric, i.e. it is the same for the interaction between the object and the cavity in the system impedance operator. Once the skin effect frequency exceeded, only field coming from openers can be transmitted inside the frequency, as the total field resulting from the interaction of the incident field and the induced currents is equal to zero. Note like in shielded cables, the common impedance coupling process injects voltage in objects connected to the cavity and this for frequencies below the skin effect cutoff frequency.

Each domain delimiting the low and high frequency domain can evolve differently, depending on the objects dimensions. That's why one constraint of this modeling is to keep the mesh numbering and association. But it is sometimes very difficult. Another solution, simplest when practiced consists in applying the frequency domains to the whole operators rather than on their components. It means that the whole system is seen for a known scale in time or frequency domain. Domains are kept at the component level when they are intrinsic to the device definitions. For example, for a diode, it can be the non linear voltage and current thresholds. The best method is probably to study the system starting from low frequencies and increasing after the frequencies, using operator adapted with each domain.

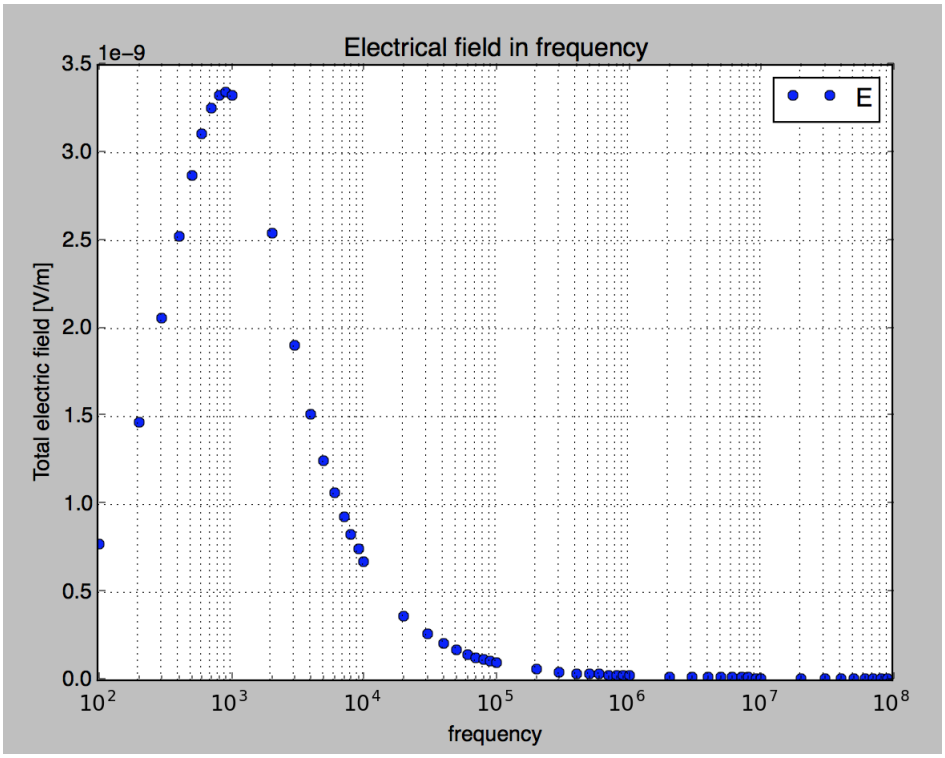


FIGURE 9.6 – Electric field attenuation inside a closed cavity

9.5.1 Making a system in low frequency assumption

Starting from 9.16 we can directly benefit of the expression z_{eq} of a couple of lines connected to two equipments used in low frequencies :

$$z_{eq} = \overset{\nu}{\mathcal{D}}_{BF} \begin{bmatrix} R_1 + z_{bf} & \beta_{12} & -Mp - \frac{1}{C_{12}p} & 0 \\ \beta_{21} & z_{bf} + R_2 & 0 & -Mp - \frac{1}{C_{12}p} \\ -Mp - \frac{1}{C_{12}p} & 0 & R_3 + z_{bf} & \beta_{12} \\ 0 & -Mp - \frac{1}{C_{12}p} & \beta_{21} & z_{bf} + R_4 \end{bmatrix} \tag{9.42}$$

On another side we have a closed cavity of operator :

$$z_{ca} = R + Lp \tag{9.43}$$

We make a system grouping z_{ca} and z_{eq} : $z_s = z_{ca} \oplus z_{eq}$:

$$z_s = \begin{bmatrix} R + Lp & 0 & 0 & 0 & 0 \\ 0 & R_1 + z_{bf} & \beta_{12} & -Mp - \frac{1}{C_{12}p} & 0 \\ 0 & \beta_{21} & z_{bf} + R_2 & 0 & -Mp - \frac{1}{C_{12}p} \\ 0 & -Mp - \frac{1}{C_{12}p} & 0 & R_3 + z_{bf} & \beta_{12} \\ 0 & 0 & -Mp - \frac{1}{C_{12}p} & \beta_{21} & z_{bf} + R_4 \end{bmatrix} \tag{9.44}$$

We write the interaction between the cavity and the couple of equipments :

$$\frac{e}{k} = p\frac{S_l}{2}\alpha = z_{eqca} \tag{9.45}$$

knowing that $B_T = \alpha\left(\bar{k} - k^x\right)$ and S_l the whole section between the lines and the cavity (a half value is associated with each half line under the model with two meshes). To this interaction is added a generator $e_B = -pS_l\alpha\bar{k}/2$. Another coupling mechanism comes from the shared part of the cavity used

by the lines enclosed in the cavity. If $R_l/2$ is this common impedance coupling value, we have :

$$z_s = \begin{bmatrix} R+Lp & \frac{R_l}{2}+z_{eqacav} & \frac{R_l}{2}+z_{eqacav} & \frac{R_l}{2}+z_{eqacav} & \frac{R_l}{2}+z_{eqacav} \\ \frac{R_l}{2}+z_{eqacav} & R_1+z_{bf} & \beta_{12} & -Mp-\frac{1}{C_{12}p} & 0 \\ \frac{R_l}{2}+z_{eqacav} & \beta_{21} & z_{bf}+R_2 & 0 & -Mp-\frac{1}{C_{12}p} \\ \frac{R_l}{2}+z_{eqacav} & -Mp-\frac{1}{C_{12}p} & 0 & R_3+z_{bf} & \beta_{12} \\ \frac{R_l}{2}+z_{eqacav} & 0 & -Mp-\frac{1}{C_{12}p} & \beta_{21} & z_{bf}+R_4 \end{bmatrix} \tag{9.46}$$

With the source vector $T = \begin{bmatrix} e_i & e_B & e_B & e_B & e_B \end{bmatrix}$, with $e_i = -pSB_i$, the tensorial equation $T_a = \zeta_{ab}k^b$ solve all the problem for the low frequency domain.

9.6 Immersing the harness in a structure, in high frequencies

There are two ways to model cavities. A first way uses resonators and a second way uses guidewaves structures. The first is the more physical one and the easier to use. But it requiers many dimensions when we want to cover a wide range of frequencies. At the contrary the second needs more effort to be understood but covers implicitly a large domain of frequencies.

9.6.1 Resonator approach

Any structure can be seen as a topological surface. Laces characterize these structures. There are two kinds of laces. Laces going around an empty volume and laces going in the direction of a volume axe. Figure 9.7 illustrates this mechanism. Across the laces surrounding the volume we can trace two lines perpendicular each other. The longitudinal line is kept as it is. Finally the three lines wear directions of field modes. Giving a name to each axe (for example a,b,c) we can construct after that an approximation of the field mode A_{abc}. Knowing the field distribution, we can compute the interaction between this field and various receivers insert inside the volume. The modal field is



FIGURE 9.7 – Volume topology

represented by a resonator. In this resonator, the energy of the magnetic field is included in the inductance, the electric field energy inside the capacitor and the losses are represented by the resistance. For a volume V, the magnetic energy W_B is given by :

$$W_B = \frac{1}{2\mu} \iiint_V dv B^2(a,b,c) \tag{9.47}$$

while for the electric field we have :

$$W_E = \frac{\epsilon}{2} \iiint_V dv E^2(a,b,c) \tag{9.48}$$

and we have : $1/2CV^2 = W_E$ and $1/2Li^2 = W_B$. Otherwise :

$$V = \int_a d\mathbf{a} \cdot \mathbf{E} \qquad i = \tfrac{1}{\mu} \oint_c d\mathbf{c} \cdot \mathbf{B} \tag{9.49}$$

The major difficulty in general comes from the estimation of the resistance value. The simplest way to determine its value comes from the quality cœfficient. The quality cœfficient Q is the ratio between the losses and the stored field energy. Losses can come from openers, losses in walls, absorbing materials inside the cavity volume, etc. It can be seen also as the ratio between the energy that flows outside the cavity (that's the case of losses) on the energy staying inside the cavity. At low frequencies we have seen that the losses are directly associated with the resistances of the wall including the

skin effect. What's happen at the first mode for example ? The currents in the wall follow the electric field limit condition through Ohm's relation $J = \sigma \cdot \mathbf{E}$. In fact rather than working with fields we may work with currents and loads and define current modes. But to compute the couplings between elements inside the cavity we need anywhere the fields. Feynman's had tried to compute interactions without fields. But in final it's not completely satisfying and fields must be kept !

For the first mode in a cubic cavity $E_{xyz} = E_{011}$, the electric field is given by :

$$E(x, y, z) = E_{0x} Sin\left(\pi \frac{y}{Y}\right) Sin\left(\pi \frac{z}{Z}\right) \tag{9.50}$$

The currents follow the wall. On the two planes (y, z) for $x = 0$ and $x = X$, the current converge to the load corresponding to the maximum amplitude of the electric field. So :

$$x = 0,\ x = X,\ (y, z) \in [0, Y],\ [0, Z] \Rightarrow \vec{k} = w \frac{B(x, y, z)}{\mu} \vec{u}_r \tag{9.51}$$

r being the radius of the circular referential associated with the plan. The power of losses P_R is given for these plans by :

$$P_R = \left\{ \int_0^{2\pi} \left[\int_0^R dr \rho \frac{1}{\delta r d\theta} \left(w \frac{B(x, y, z)}{\mu} \right)^2 \right]^{-1} \right\}^{-1} \tag{9.52}$$

The current amplitude is given by the magnetic field amplitude on the limit conditions. On the other walls, the distribution of the current is easily described. For example on the first wall :

$$P_R = \left\{ \int_0^Y \left[\frac{\rho}{\delta dy} \int_0^X dx \left(w \frac{B(x, y, z)}{\mu} \right)^2 \right]^{-1} \right\}^{-1} \tag{9.53}$$

in both cases δ is the skin depth. The resistance value can be obtained knowing the current trajectories. For the second case this is :

$$R^{-1} = \int_y dy \left[\int_x dx \frac{\rho}{\delta dy} \right]^{-1} \tag{9.54}$$

Making these computations, one resonator is associated with one mode. For each combination (x, y, z) of mode number we have a RLC circuit. Knowing

the field distribution associated with this mode through the assumption of laces axes, we can compute the emf induced by this modal field in various objects enclosed in the cavity. The field can be associated with the current in the RLC circuit. Let's take an example.

If in a rectangular cavity, the electric field is given by :

$$E_x = E_0 Sin\left(n\pi\frac{y}{Y}\right) Sin\left(m\pi\frac{z}{Z}\right) \rightarrow TE_{n0m} \tag{9.55}$$

Maxwell's equation $\nabla \times \mathbf{E}$ leads to :

$$\begin{cases} \frac{\partial \mathbf{E}}{\partial y} = -pB_z \\ \\ -\frac{\partial \mathbf{E}}{\partial z} = -pB_y \end{cases} \tag{9.56}$$

As on another side $\nabla \times \mathbf{B} = \mu\mathbf{J}$ we obtain :

$$\mu J_x = \frac{1}{p}\left(\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2}\right) \tag{9.57}$$

For a polarisation following axe x, the relation between the current and the field is given by the potential developed across the capacitor :

$$\frac{1}{Cp}k^x = V = \int_x d\mathbf{x} \cdot \mathbf{E}_x \tag{9.58}$$

If we place a little loop inside the cavity, perpendicular to the axes y at coordinates (x_l, y_l, z_l). The section of the loop is S_l and the emf e_l induced in the loop is :

$$e_l = -S_l \cdot pB_y(x_l, y_l, z_l) = -S_l\frac{\partial E_x(x_l, y_l, z_l)}{\partial z} \tag{9.59}$$

Replacing the electric field par its derivative and for the coordinates where the loop is located, this gives :

$$e_l = -S_l E_0 Sin\left(n\pi\frac{y_l}{Y}\right)\frac{m\pi}{Z}Cos\left(m\pi\frac{z_l}{Z}\right) \tag{9.60}$$

Knowing from the potential gradient that $\chi E_0 = 1/Cpk^x$, we obtain :

$$\zeta_{lx} = \frac{e_l}{k^x} = -\frac{S_l}{\chi Cp}Sin\left(n\pi\frac{y_l}{Y}\right)\frac{m\pi}{Z}Cos\left(m\pi\frac{z_l}{Z}\right) \tag{9.61}$$

That's this principle which is applied also in the case of a line enclosed in the cavity. The interaction operator ζ_{lx} is symmetric once more. It intervenes also between the loop and the resonator representing the modal field of the cavity.

What is finally the method ?

1. from the laces of the empty volume of the cavities, we determine for one polarisation the modes axes ;

2. for this field distribution and polarisation q, we obtain from Helmotz's equation :

$$\frac{\partial^2 E_q}{\partial x^2} + \frac{\partial^2 E_q}{\partial y^2} + \frac{\partial^2 E_q}{\partial z^2} + \left(\frac{\omega}{c}\right)^2 E_q = 0$$

. This leads to the dispersion K and the resonant frequencies $f_{nm,q}$;

3. knowing the resonances, we can compute for each of them the capacitor computing :

$$\frac{1}{2} C \chi^2 E_0^2 = \frac{1}{2} \epsilon \iiint_v dv \mathbf{E_q}(\mathbf{n}, \mathbf{m}) \cdot \mathbf{E_q}(\mathbf{n}, \mathbf{m})$$

;

4. The inductance is obtained from $\omega_0 = 1/\sqrt{LC}$;

5. the resistance is obtained computing the resistance of the current lines. This lines are perpendicular to the magnetic field near the walls and defined by the equation $\nabla \times \mathbf{B} = \mu \mathbf{J}$;

6. if an object is insert in the cavity, we compute the emf induced on this object and express it depending on the current in the resonator. The ratio of the emf on the current defines the interaction operator between the object and the cavity ζ_{oc}. The system is solved making the direct sum of the objects and the resonators and adding the coupling operators ζ_{oc}.

Let's apply the coupling process to a line inserted in the cavity.

The coupled harness is defined by the operator :

$$z = \overset{\nu}{\mathcal{D}}_{HF} \begin{bmatrix} R_1 + z_c & \alpha_{12} & \left(\alpha^{\frac{K+1}{2}} z_c\right) & -(\alpha(K-1)\tau p z_c) \\ \alpha_{21} & z_c + R_2 & -(\alpha(K-1)\tau p z_c) & \left(\alpha^{\frac{K+1}{2}} z_c\right) \\ \left(\alpha^{\frac{K+1}{2}} z_c\right) & -(\alpha(K-1)\tau p z_c) & R_3 + z_c & \alpha_{12} \\ -(\alpha(K-1)\tau p z_c) & \left(\alpha^{\frac{K+1}{2}} z_c\right) & \alpha_{21} & z_c + R_4 \end{bmatrix} \tag{9.62}$$

In first we make the direct summation of this operator with the operator of one mode : the resonator RLC. The we must compute the interaction operator between the cavity on this mode and the harness.

An emf e_t induced in the line at the extremity tx integrates the magnetic field perpendicular to the line section on its length. If we locate the line at the center of the cavity between abscissa z_a and z_b, we have if the line is of height x_l and using 9.56 :

$$e_t = -p \int_{z_a}^{z_b} dz x_l \frac{E_0}{p} Sin\left(n\pi \frac{y_0}{Y}\right) m\pi \frac{z}{Z} Cos\left(m\pi \frac{z}{Z}\right) \tag{9.63}$$

but $y_0 = Y/2$ and $E_0 = k^x/\chi Cp$ we obtain :

$$\zeta_{tx} = \frac{e_t}{k^x} = \int_{z_a}^{z_b} dz \frac{x_l}{\chi Cp} m\pi \frac{z}{Z} Cos\left(m\pi \frac{z}{Z}\right) \tag{9.64}$$

A similar emf is induced at the other extremity px of the line, with an opposite sign : $\zeta_{px} = -\zeta_{tx}$. To report the opencircuit emf, the expression computed previously is multiplied by two. Finally :

$$\zeta = \begin{bmatrix} R_{mn} + L_{mn}p + \frac{1}{C_{mn}p} & 2\zeta_{px} & 2\zeta_{tx} & 2\zeta_{px} & 2\zeta_{tx} \\ 2\zeta_{px} & R_1 + z_c & \alpha_{12} & \left(\alpha^{\frac{K+1}{2}} z_c\right) & -(\alpha(K-1)\tau p z_c) \\ 2\zeta_{tx} & \alpha_{21} & z_c + R_2 & -(\alpha(K-1)\tau p z_c) & \left(\alpha^{\frac{K+1}{2}} z_c\right) \\ 2\zeta_{px} & \left(\alpha^{\frac{K+1}{2}} z_c\right) & -(\alpha(K-1)\tau p z_c) & R_3 + z_c & \alpha_{12} \\ 2\zeta_{px} & -(\alpha(K-1)\tau p z_c) & \left(\alpha^{\frac{K+1}{2}} z_c\right) & \alpha_{21} & z_c + R_4 \end{bmatrix} \tag{9.65}$$

In this case the source vector T contains only the source for the cavity (source coming from an external field and some element inserted in the cavity to create the field). The equation $T_a = \zeta_{ab} k^b$ solve entirely the problem.

9.7 Conclusion for SUETAN

All SUETAN (it may be a software, but for the moment it's only a method while TACS4BISE is a formalism) is based on this principle. The object called to participate to the system construction are grouped on a table. A direct summation create a first operator without coupling between them (it means that the system remains virtual). Then connectivities and coupling applied on this tensor creates the final system. Steps of closings, connecting by wires, inserting in a structure etc. are parts of this operation. Once the system is created, the source can be moved from components to others in the source vector T to study assumptions of self energies or constraints coming from outside.

Often the question occurs to know if the numerical application, if there are some, may be realized in the time domain or in the frequency domain. It is clear that the frequency domain is easier for all the electromagnetic interactions. But even with non linear component, another solution exists, often forgotten. Small signal analysis considers that non linearities have a known state for a known polarity. This polarity comes from the zero component of the spectrum. Inside a time domain loop where the non linearities are fixed in term of impedance operator, a frequency loop can be computed using these fixed impedances for the non linearities. The spirit of this method means to cut the whole sources signal in subparts of constant values leading to constant local DC components and to constant polarities. During these steps, the spectrum is computed and the solution appears like a sequence of instantaneous spectrum. This approach is very efficient when the cutting is in relation with the system mission and a sort of partition of his states where each note is an observable system i/O (see O.Maurice "Elements of theory for electromagnetic compatibility and systems", 2017, Bookelis editor).

Chapitre 10

BISE approach for system conception

To be inspired by natural systems means two essential things :

— to think in evolution - capacity to evolve ;

— to see the system actions as results from system decisions.

These two properties in general do not belong to artificial systems. In particular, artificial systems are not able to reproduce themselves. But artificial systems can evolve thanks to electronics and informatics and with the intervention of artificial intelligence (AI). A game involve choices coming from various systems seen as gamers. Each gamer makes choices depending on the game results and on the other gamer choices. Starting from the actions of one system, enclosed in some variables k^x, and starting from the actions of a second system q^y, we construct a game through the construction of a pay-off matrix. This matrix says for each couple (k^x, q^y) what are the gains of each gamer. Then each gamer has its own AI analysing the gains and creating next command e_x or a_y whose determine next behaviour of the gamers.

The process is represented figure 10.1.

The major difference with classical cords is that this cord implies a group of system and not only two of them. Somewhere these cords concerns the whole system in one shot for each events step time. Another particularity, is that this cord calls for psychology and cognitive processes. So the computation is not so deterministic as classical cords calling for standard physical laws. By the fact, results can vary from one scientist to another. But simple reasoning allow to lead to similar results. Let's take an example. We want to model an engineer that must take a decision on some report. If he's very

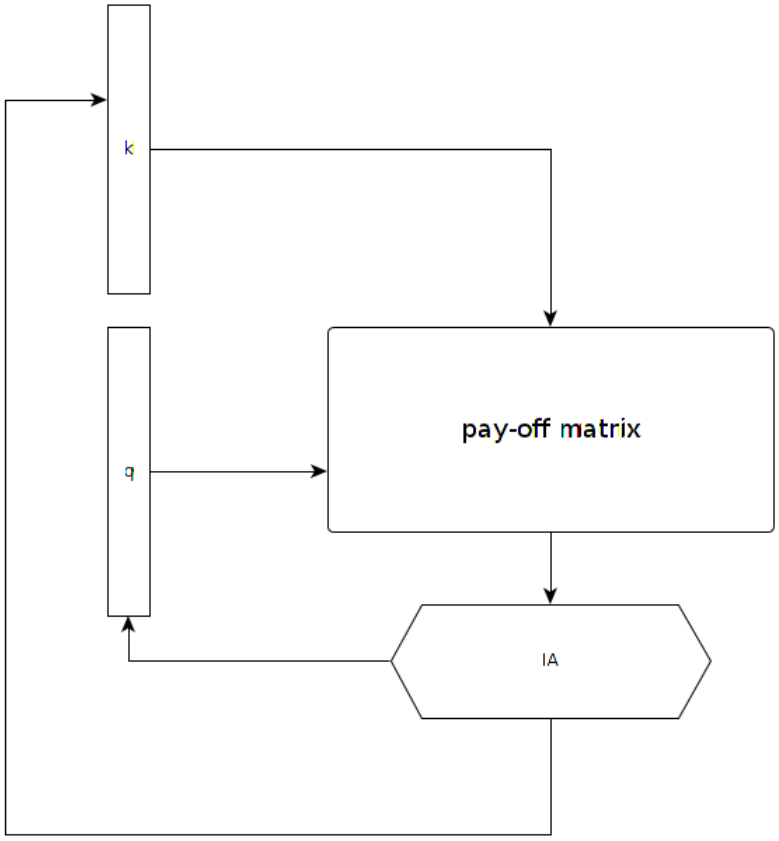


FIGURE 10.1 – IA process as cord

specialized in the topic presented in the report, his acceptance probability is directly in relation with the report quality. While if he doesn't know nothing on this topic, his acceptance probability becomes fifty percent.

10.1 System construction mechanism

To resume previous discussions, we can say that a classical system is made following the next steps :

1. defining a set of objects involved in the system, of operators z_{ii} ;
2. making the direct summation of these objects $\oplus_i z_{ii}$;
3. defining the construction of the system through its connectivity C : $\zeta_{\alpha\beta} = C^i_\alpha z_{ii} C^i_\beta$;
4. closing the equipment ports with the matched impedances of the lines $\zeta_{\alpha\beta} + Zc_{\alpha\beta}$;
5. adding Branin's lines between each equipment ports $\zeta_{\alpha\beta} + Zc_{\alpha\beta} + b_{\alpha\beta}$;
6. adding the cords between the equipments radiative ports, taking into account the environment (structure) field modes $g_{\alpha\beta} = \zeta_{\alpha\beta} + Zc_{\alpha\beta} + b_{\alpha\beta} + A_{\alpha\beta}$.

The source can evolve depending on time and defined by the gamma matrices $\gamma^j_\alpha e_j$. Step 1 to 6 are associated with the feasibility phase then the development phase. We want to study now how the human factor may be added to this process.

For game theory, no doubt that Eber's book is perhaps the best reference to begin discovering this thematic. Some of next examples are taking off from this book.

10.2 Game theory

We can explore some cases starting from choices already made by two gamers (it means that we don't care of how the systems arrive to these possibilities of choices). The game is presented through its pay-off matrix. This matrix is organized in a way that the couple (i, j) at the coordinates a, b gives the gains of the player i and player j for the choices a and b respectively. Let's consider the pay-off matrix :

	gamer 2 choice 1	gamer 2 choice 2
gamer 1 choice 1	7,3	7,4
gamer 1 choice 2	3,4	10,5

Noting Ξ the pay-off matrix, we have :

$$\Xi = \begin{bmatrix} 7 & 7 \\ 3 & 10 \end{bmatrix} \breve{g}_1 + \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \breve{g}_2 \tag{10.1}$$

where $\breve{g}_1$ and $\breve{g}_2$ are the two gamers. Now we can wonder if there is any Nash's equilibrium in this game ? Looking at the vector Ξ we can project its coordinates in the space of referential $(\breve{g}_1, \breve{g}_2)$. If we look at the vectors extracted from this representation, we obtain the figure 10.2.

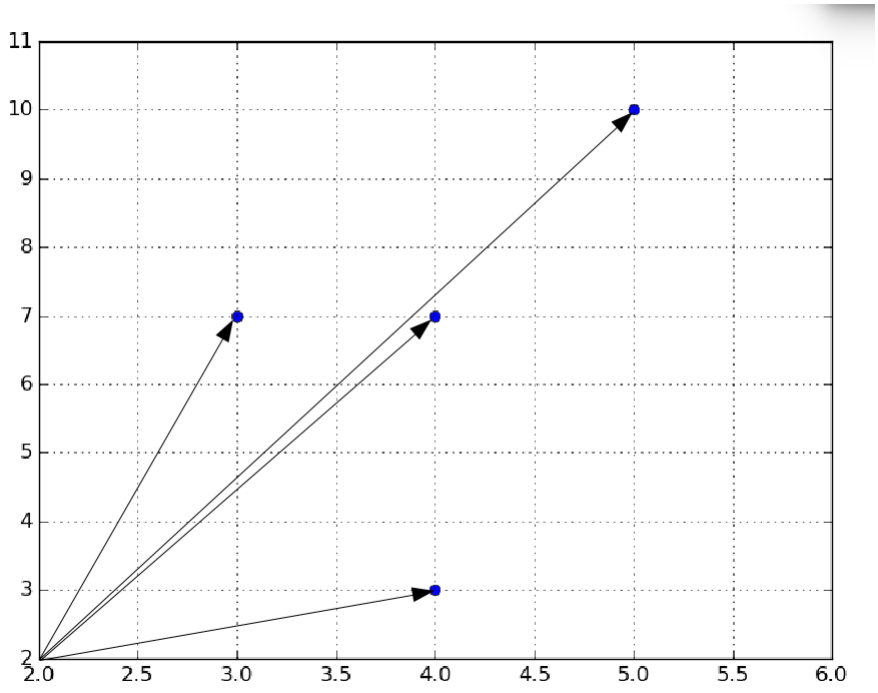


FIGURE 10.2 – Pay-off matrix as vectors

Seeing this figure, we understand that it exists a case for which both gamers wins the maximum gain. If any of the two gamers makes another choice, their gain would be lower. This vector points out the best choice for both

gamers and is called Nash's equilibrium. Nash's equilibrium is characterized here by the vector having the longer radius.

We consider now another case of pay-off matrix :

$$\Xi = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \breve{g}_1 + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \breve{g}_2 \tag{10.2}$$

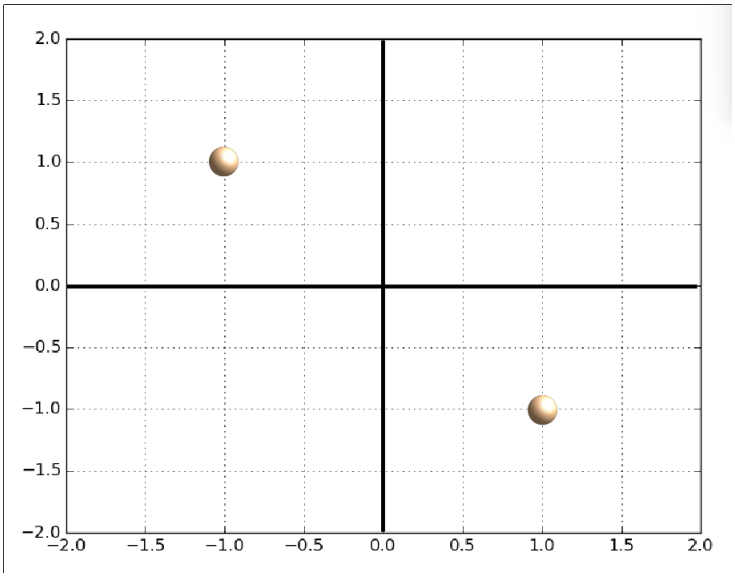


FIGURE 10.3 – Pay-off matrix as vectors : second case

Applying the previous rule, we don't see any Nash's equilibrium in this case. If any of the two gamers choices the value to win 1, the other loses 1, and the inverse. If A is the first choice of gain $+1$, the first player has the probability p_1 to play A and the second player has the probability p_2 to play A. Then, the probability for the first player to win playing A is p_1 multiplied by the gain equal to 1, plus $(1 - p_2)$ that the second player plays B (the second choice) multiplied by his gain in this case, to know -1. Finally the first player has the hope of earning playing A h_A given by :

$$h_A = 1 \cdot p_2 - 1 \cdot (1 - p_2) = 2p_2 - 1 \tag{10.3}$$

If the first player plays B, by the same way he wins :

$$h_B = -1 \cdot p_2 + 1 \cdot (1 - p_2) = 1 - 2p_2 \tag{10.4}$$

The optimized solution for the first player (if it exists) is obtained writing $h_A = h_B$. This leads to $p_2 = 0,5$. The game description is defined by :

	A	B
A	1,-1	-1,1
B	-1,1	1,-1

Knowing p_2, the second player has a chance p_1 to win given by : $p_1 = 0,5$. Under this new *mixed* form, a Nash's equilibrium appears giving both gamers the best strategy. The strategy here is defined for the couple of gamer : playing 50%. Nash's equilibrium is determined here looking at the hope of earnings. Here we have only one point, so Nash's equilibrium is evident. But how can we interpret this Nash's equilibrium based on mixed strategy (with hope of earning probability) ? There are two interpretations described by Nash himself. The first interpretation that I may call "the quantum interpretation" says that one gamer plays A or B with equal chance. So he don't have any preference between the two options. The second interpretation callable the "mass interpretation" says that if many persons play the game, there will be 50 % of them playing A and 50 % playing B.

In this new game, a player begins alone and can play A or C. If he plays A, he wins 1 piece and the second player wins 5 pieces. If he plays C, the second gamer plays. If the second gamer plays B, each gamer wins 0 piece, if the second gamer plays D, each gamer wins 2 pieces. The pay-off matrix can be defined by :

$$\Xi = \begin{bmatrix} 1 & 0 \\ x & 2 \end{bmatrix} \breve{g}_1 + \begin{bmatrix} 5 & 0 \\ x & 2 \end{bmatrix} \breve{g}_2 \tag{10.5}$$

The symbol x showing that this possibility doesn't exist. Time in this representation disappears, and we will see that this can modify the understanding of the situation. in fact the number x is known : if the first player choices A, he wins 1 and the second player wins noting, i.e. 0, even if its without playing ! So finally :

$$\Xi = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \breve{g}_1 + \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} \breve{g}_2 \tag{10.6}$$

Tracing the vectors on a graph we obtain the figure 10.4 where the arrow indicates the time passing.

This graph makes appear clearly two conclusions :

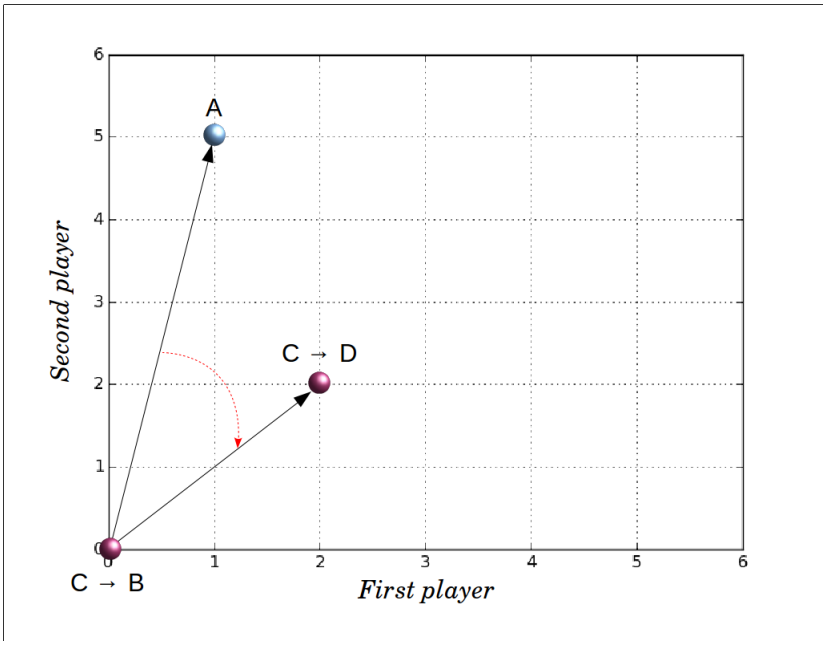


FIGURE 10.4 – Vector projection of the game

1. the best compromise is the combination $C \rightarrow D$;
2. because first player can win only 1 on the first choice, this best compromise is reachable.

Under this assumption, Nash's equilibrium is the combination $C \rightarrow D$. If second player would play first with the same pay-off matrix, the probability that he plays A would be high. We understand here that the game rules are fundamentals in the results.

Let's consider a new game. The process is shown figure 10.5.

A rational player, playing for his maximum interest should play C to share a new game with a second player, game where he can win 1000 rather than 900, under the assumption that gamer 2 plays N with him. We can trace a similar representation as previously (figure 10.6).

It's clear that the gamer 1 will play "C" which leads to the result $C \rightarrow N, N$ with the higher gain.

In this new game, the gamers play simultaneously. The gamer 2 can be of two types : A or B. Depending of his type, the gains are different and given by :

Joueur 1 A 900,900

C

Joueur 2

Joueur 1	N	R
N	1000,1000	0,800
R	800,0	800,800

FIGURE 10.5 – Pay-off matrix of a second game

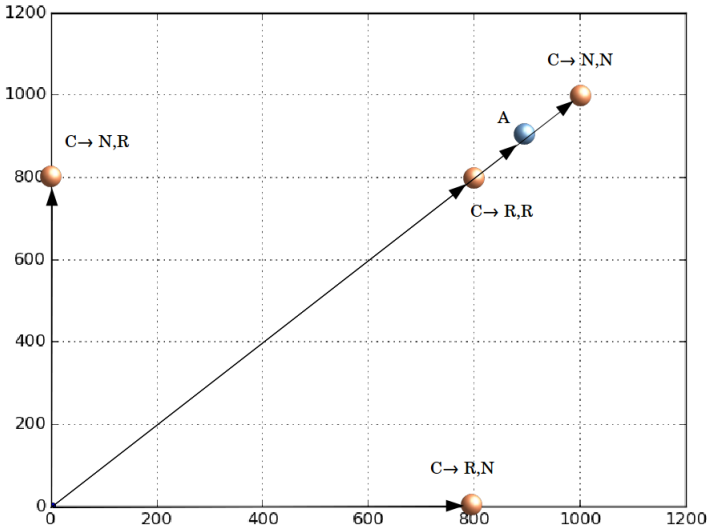


FIGURE 10.6 – Vector representation of the game

	player 2	type A		type B	
		N	R	N	R
player 1	N	3,1	2,0	3,0	2,1
	R	0,1	4,0	0,0	4,1

The second player can be of type A or B with the same probability. If player 1 plays "N". Player 2 will play N if he's of type A or R if he's of type B. The hope of earning h_e^1 of the first player is so :

$$h_e^1 = 3.P(A) + 2.P(B) = 2,5 \tag{10.7}$$

While if player 1 plays R his hope of earning h_e^1 is :

$$h_e^1 = 0.P(A) + 4.P(B) = 2 \tag{10.8}$$

Player 1 should play option "N". Is this result appears in a vectorial representation ? Figure 10.7 shows the couples obtained for each combination of earning and for the hope of earning in case of the two types of player 2.

As the player 1 doesn't know the type of the second player, he cannot choice between the two longer vectors for N, N or R, R. If he plays R and if player 2 is of type B (couple R, N), he looses the game. So this first deterministic representation doesn't give a solution. Now if we take a look to the hope of earnings, it becomes clear that the first player must choice N. In that case, both players keep a good hope of earning equal to $(2.5, 1)$ (type A) or $(2, 1)$ (type B). These probabilities are Bayesian ones. We speak of the probability for player 1 to choose N, knowing that player 2 is of type A, etc. : $P\left(N|\ A\right)$.

The vector representation can be seen under two ways. We can search for the best common gain that benefit at the maximum to both players. But we can also consider what is the best interest of the opponent, indicating what he's prepared to play. We can take a look to a very well known game : the prisoner's dilemma. Its pay-off matrix is :

	A	B
A	3,3	1,4
B	4,1	2,2

Nash's equilibrium of this case if B, B because if the first player plays A, the second player has interest to play B. But in A, B or B, A one of the two

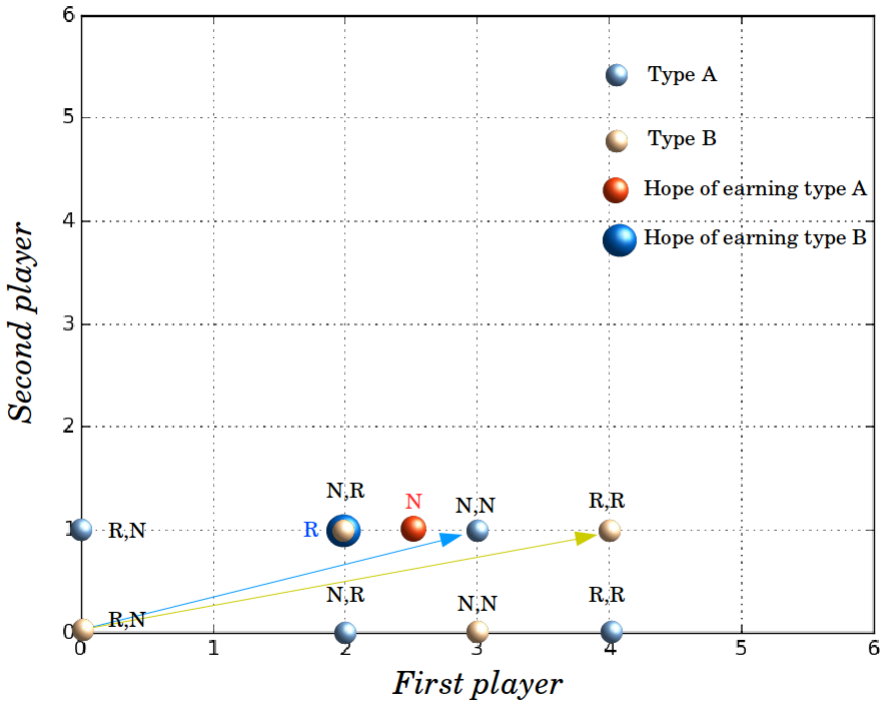


FIGURE 10.7 – Vector representation of the game

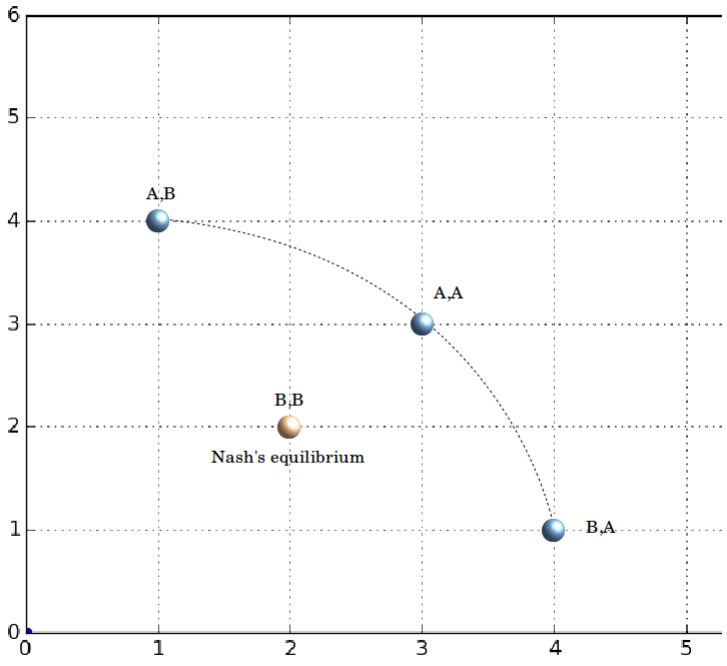


FIGURE 10.8 – Vector representation of the prisoner's dilemma

players has a low gain. If the first player plays B the second player should play B also. That's why B, B is Nash's equilibrium. Figure 10.8 shows the game through the vector representation.

In fact the maximum earning is reached for a combination that needs trust between the players. We understand that if the player agreed on a contract (implicit or explicit), they can increase the theoretical equilibrium. If the players plays individually, Nash's equilibrium is the best solution. Under the individual behaviour, the player look at the minimum risk whatever plays the first player knowing that they play simultaneously.

10.3 AI

The purpose here is not to expose classical AI but more to develop some ideas around this concept. What means AI ? What means intelligence ? Surely not something in relation with living being intelligence. It's more the notion of some electronics able to analyse perception with a capacity to learn, i.e. to increase its performance in analysis with the increasing number of cases studied. The inputs of AI are signals coming from various sensors. Each kind of signal is treated differently. What is called classically a neural network is for me a circuit. For a succession of signals k^α at a given time t_0, the circuits generate a vector of signature s_β identifying the information received.

If we imagine a vector a giving perceived actions and a vector r giving the corresponding answers, the behavior is defined by some function $r(a)$. Making the assumption that this function follows a known law L of unknown parameters, we can use the least squares method to determine these parameters. If we consider for example a simple law of the form $\alpha a_i + \beta$, for n samples we compute :

$$S = \sum_1^n (r_i - (\alpha a_i + \beta))^2 \tag{10.9}$$

in order to minimize S. This means that :

$$\frac{\partial S}{\partial \alpha} = 0 \quad \frac{\partial S}{\partial \beta} = 0 \tag{10.10}$$

It leads to the system of equations :

$$\begin{cases} \alpha \sum_1^n a_i^2 + \beta \sum_1^n a_i - \sum_1^n r_i a_i = 0 \\ \\ n\beta + \alpha \sum_1^n a_i - \sum_1^n r_i = 0 \end{cases} \tag{10.11}$$

Once the law determines, any solicitation a_i will result in an answer r_i : the system is modelled. For various outputs a_i of a system, the options that can be played r_i have to be defined. They are given by the function $y_i = \alpha a_i + \beta$. A learning phase must be followed to construct the law representing the behaviour. This technique (or similar ones) can be applied with more than only one function $r(a)$. Taking $a_i = t_i$, and a set of observables o_j, the cœfficients α_{ji} belongs to the functions $y_j = \alpha_{ji} t_i$. They are determined by minimizing $S = \sum_1^n (o_j - y_j)^2$. The set of polynomials of cœfficients α_{ji} and the associated laws (or polynomial orders) are the neuronal network (NN). It is clear that if the cœfficients were determined with an incomplete set of data a_i or with falsified responses r_i, the NN will have defects and might be unable to show a stable behaviour. The learning phase is a key phase for the NN pertinence and efficiency. Now, combining NN and game theory (GT) leads to an AI structure. The principle is to insert a NN between some outputs of a system and the possible set of choices presented in a GT. Then another NN is insert between the hope of earning vector of the GT and the set of commands (inputs) attached with the system. This triptych constituted the AI in the way I see it.

Many persons, and for first specialists may say that this is not AI. They will speak of *perceptrons*, etc. These concepts was developed thinking that the mind working could be similar to simple electrical devices. But with time, biologists learn us that neurons are of a complexity far way from any complex logical circuit.

The previous circuit can be complicated as desired adding couplings between each references and laws. Each polynomials can be seen as an operator applied to a given flux f^i. We obtain in a first step a diagonal matrix with all the r_i depending through laws of the f^i. Then we can add dependencies between decisions and fluxes.

Each neuron is constructed separately. The polynomial gives the law between the perception and the decision which leads after to some action. But the law can be changed depending on another decision taken by another neuron. Based on these principle we obtain a complex network which is the AI. As said before, the AI includes an action, a neuronal network giving possible choices to a game theory kernel. This kernel (pay-off matrix and hope of earnings) gives a decision and linked with this decision, a new action is operated.

A typical application concerns an autonomous vehicle. A pedestrian hesitates on crossing the road. He analyzes the vehicle movement using its

neuronal network. This NN takes into account the distance, the apparent vehicle speed, the road width, etc. Then this NN transmits two possible choices to a pay-off matrix. The same exercise is made by the vehicle. The pedestrian computes its maximum hope of earning and takes a decision to cross the road or not. This decision uses a translator to transmit the desired action to the corresponding machines. In this description, you note that we make no distinction between artificial and real networks or machines. The NN represents the minding of the human and the GT takes into account the capacity to decide. You may say "it's not game theory but decision helping theory". In fact not because, in many cases it is surely decision helping process, but in many other cases it's a game, a game to survive in the general sense of life. I mean that the decision is taken finally to guarantee surviving. It's typically the case for the autonomous vehicle problem.

Figure 10.9 shows the processes involved in this example.

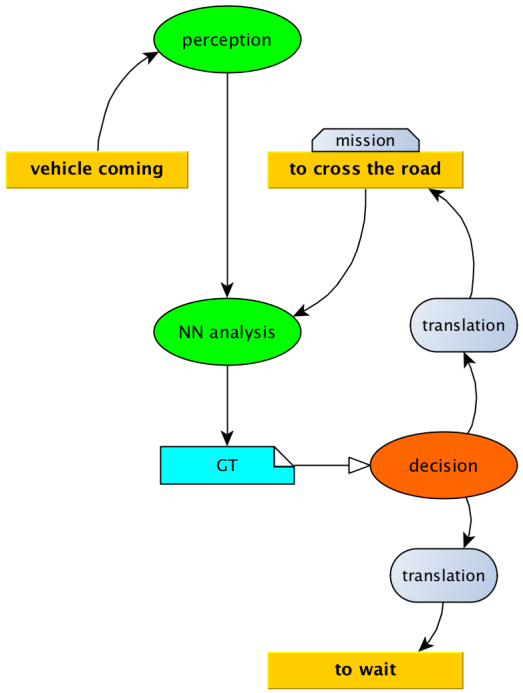


FIGURE 10.9 – Process involving an AI

Remaining modest on what we may understand on the human thinking,

we can say in our context that the first difference between AI and "*real I (RI)*" is that the complexity dimension of polynomials for RI are infinite compare to those of AI, and we are very very far from approaching this dimension.

The gamma matrices play an important role in this sequence. They transmit the source vector from each network to the other and gives the system rhythm.

To illustrate this mechanism we imagine a system made of two systems. Each of them are defined by an operator ζ given by :

$$\zeta_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10.12}$$

These operators describe the capacity of the systems to follow some location in a (x, y) plan (fluxes f^1 and f^2), and the fact that the systems transmit their coordinates to the other one (fluxes f^3 and f^4). At initial, the source vector of the system is $T_n = \left[\ e_x, e_y, 0, 0 \ \right]$. Receiving the coordinates of the second system, how the first system analyzes this and uses this for its next operations ?

The system receives the opposite coordinates b_x, b_y. Using an AI, the system translates this information in choices for a game. The possible choices are North, South, East and West (N,S,E,W). Rather than polynomials, the NN here is a suite of inequalities like :

$$\exists c \in [N, S, E, W] \ / \ if \Delta x = 0 \ and \ \Delta y > 0 \Rightarrow c = N \tag{10.13}$$

The second system moving, makes a choice c. The first system has the same possibilities c_0. A pay-off matrix explores the gains available for the 16 possible couples of choices coming from the two systems (N,N ; N,S ; etc.). The earnings ea_{ij} are defined by a law :

$$ea_{ij} = \left[\bar{d}_c \left(i, j, t \right) + d_\theta \left(i, j \right) \right]^{-1} \tag{10.14}$$

where $\bar{d}_c \left(i, j, t \right)$ is the average distance between a target that must be reached by the system of systems (SoS) and $d_\theta \left(i, j \right)$ the distance between the two

systems. The other system playing the choice c, the first system analyzes this looking at the different hopes of earnings associated with this choice c and its own possible choice c_0 in (N, S, E, W) (the probabilities can be basically fixed to $1/4$ for all choices). Its possible choice c_0 is guided by maximisation of the hope of earning ea_{cj} obtained when the second system plays c and the first one plays c_0. The choice $c_0 \in [N, S, E, W]$ must after be translated in a command for moving the first system. The same reasoning is applied seen from the second system. The translator has the form :

$$\begin{cases} if\ c_0 = N \Rightarrow e_y = A\left(1 - e^{-\tau p}\right) \\ if\ c_0 = S \Rightarrow e_y = -A\left(1 - e^{-\tau p}\right) \\ if\ c_0 = E \Rightarrow e_x = A\left(1 - e^{-\tau p}\right) \\ if\ c_0 = W \Rightarrow e_x = -A\left(1 - e^{-\tau p}\right) \end{cases} \tag{10.15}$$

τ is a pulse duration controlling a motor to create the system displacement on a Δx or Δy step. We can now draw the complete mechanism between the values of the second system displacement and the first system moving. This chain is represented figure 10.10.

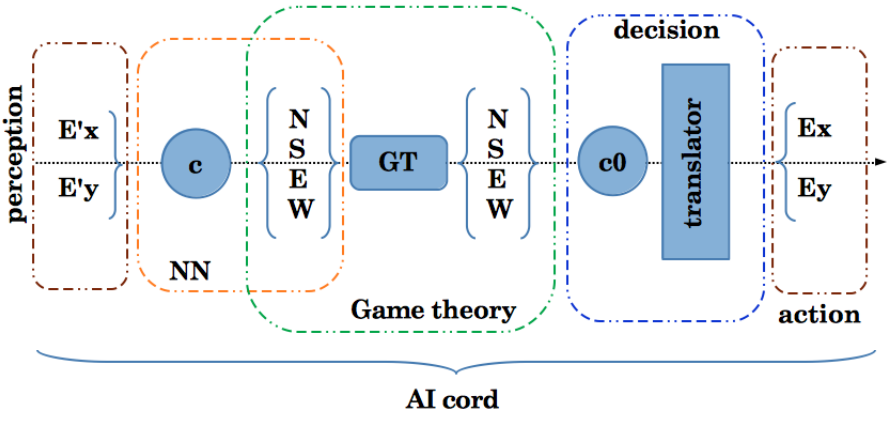


FIGURE 10.10 – Chain for an AI cord

The whole process represents an AI cord, taking the opposite system location e'_x, e'_y and giving the new displacement values e_x, e_y. If we note $\mathcal{A}_{xx}$

the part of cord using k'^x (associated with e'_x) and driving e_x, $\mathcal{A}_{xy}$ the cord using k'^x and driving e_y, etc. we model the SoS as usual by starting making the direct summation of the systems involved $\zeta_1 \oplus \zeta_2$ then adding cords, here AI ones. Finally we obtain the global operator :

$$\zeta = \begin{bmatrix} 1 & 0 & \mathcal{A}_{xx} & \mathcal{A}_{xy} & 0 & 0 & 0 & 0 \\ 0 & 1 & \mathcal{A}_{yx} & \mathcal{A}_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \mathcal{A}'_{xx} & \mathcal{A}'_{xy} \\ 0 & 0 & 0 & 0 & 0 & 1 & \mathcal{A}'_{yx} & \mathcal{A}'_{yy} \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10.16}$$

The operator is not symmetric and can be easily extends to N systems making the SoS.

We can remark that the AI which can be seen as a behavioral operator is included in a cord, i.e. an extra-diagonal component rather than in a diagonal component. When using behavioral operations, it is often easier to use cord to include this processes. Diagonal component model the limit condition, or frontier operators associated with these modelings. To have part of them as diagonal component may mean to detail their hardware. This is possible, and can be necessary sometimes in case of studies focused on their structure. But at the SoS level, we look for macromodels without keeping all the microdetails associated with these systems. We understand that in this case, cords are the simplest way to include AI in systems.

A system (an equipment, a vehicle) can be modeled using **eight fundamental kinds of components (8-fkc)** :

a cables or antennas, waveguides : techniques to transmit informations ;

b operators on the frontiers, interfaces between various environments (internal, external) and a system heart ;

c behavioral cords taking in charge complex system reasonings including AI ;

d sensors or elements for perception ;

e motors or elements as actuators ;

f a skeleton to assure the structure able to wear all these components ;

g some components used to evacuate the system heat and other waste ;

h a source of energy coming from outside.

Chapitre 11

Systemic concepts applied to BISE

11.1 Principles

Systems modeling was widely addressed by Gabriel Kron and affiliate authors using tensorial analysis of networks (TAN). Based on diakoptic principles, the method leads to the lagrangian of the problem, in a multi-physique writing. We expose here the major characteristics of this approach, focusing on multi-scale problems and how to take into account these inputs in a problem of system of systems.

11.2 Introduction

System of systems means many things like a group of animals or a group of robots exchanging datas through antennas, etc. We recall the history of TAN used for systems modeling and the systemic definition for systems. After, we recall the evolving of systems in order to detail what we consider as system of systems (SoS) in this paper, other considerations being possible. We are interest in cyber-physical systems seen as SoS : cyber-physical systems of systems (CPSoS). The cyber side refers to datas, numeric communications and intelligences while the physical side refers to the material (even living one) that support the intelligence and communications.

11.3 TAN history

Gabriel KRON had studied the systems starting from electrical machines which are complex systems. His personal approach for networks seeing them through the mesh space, has leaded to a particularly efficient formalism. Majority of hamiltonian approaches work onto branches and nodes spaces. Kron has extended applications in cellular topology to the mesh space, developing a method that gives the lagrangian of the problem. A first book written in 1973 gives a synthesis of his artwork on systems[1]. There was after the works of the French army direction : Angot, Papin, Kauffman write various mathematical books explaining Kron's technique in order to study complex electronic systems. But until there, interactions was limited to magnetic interactions and electronic applications[2][3]. Maurice & Co. submitted a generalized concept of cord, describing any interaction between two connex networks[4][5]. This new capacity leads to the opportunity of applying the TAN formalism to any kind of problem including biological, economical ones, etc.[6]. This is with these last developments that we study here cyber-physical systems of systems.

11.4 A definition for systems

By listening to the systemic community, a complex system is characterized by three properties :

— a lot of interactions ;

— a stochastic behavior even partially ;

— an emergence property.

This last property is often unknown. It means that the system has properties that cannot be observed at a lower scale. It clearly pose the concept of scales. It makes the assumption that edges exist between scales and that going from one scale level to another continuously is impossible. It doesn't mean that both scales cannot exist simultaneously. But the existence of one higher scale can be obtained from a lower one through an integration process which is not continuous[7].

Any system has an associate graph and an associate manifold. We pose as preliminary postulate that a system of systems (SoS) is a collection of connect graphs and a direct summation of manifolds. Each of these manifolds is a system of the SoS. The systems interact between them through cords

in the TAN formalism of the SoS. Each system is constructed from branches (T^1 elements of the cellular topology) connected through the connectivity C. Each system is characterized by an operator ζ giving all the particle speeds v^k inside the system for a given constraints vector e_m. The manifold associates with a system is defined by the equation $e_m = \zeta_{mk} v^k$. The interactions are defined by the extra-diagonal components of ζ : ζ_{mn}, $m \neq n$. The number of these interactions is an image of the system complexity. The stochastic property is intrinsic of the electronic components like microprocessors. What about the emergence property ? As non continuous change of scales we consider an integration from one observable that belongs at a lower scale. If we work at the same scale to define the system, it is possible that it doesn't present any emergence aspect. This property may appear at the SoS level.

11.5 SoS properties

A SoS S is defined by the direct summation of all the manifolds of a group of systems Q :

$$S = \oplus_k Q_k \tag{11.1}$$

The SoS S by definition includes the properties of the set of systems Q. May it have a new property associates with the union of the systems ? Typically, the new property that can appears when the systems Q are grouped and can communicate is associated with this capacity of communication that doesn't exist when the systems are separated. At least we can postulate that a new property of the SoS is the induced by this capacity. This capacity can guide a group behaviour of the systems. The SoS manifols S can be completed by a set of interactions G which are the Green's functions of radiation patterns for antennas. But more generally, the manifold S must be completed by two objects :

— the communication network between all systems Q_k ;

— the actions lade by all the systems and interpreted by all the systems.

The first network can be modelled by a special tensor G added to the manifold S or by a separate mechanism calling a technique named "gamma matrix"[8]. The second mechanism is more complex to express. In one system referential of a particular system, it starts from a set of flux K^x that are responsible for the actions of all the other systems. This flux is firstly **perceived** and interpreted through a complex matrix Λ. An operator of **attention** A decides if these actions are considered by the receiver. Then a Bayesian probability

P^c says what is the **answer** (**analysis** activity) of the receiver in front of these perceptions. This answer depends on a **pay-off matrix** ω which leads to the hope of earning $\omega_{\mu c}$. Then a last operator δ of decision translates these informations in a **decision** (a command) e_v taken by the receiver[9]. We translate these relations writing :

$$e_v = \delta_v^z \omega_{zc} P^c \left(e_c|\ A_y^z \Lambda_x^y K^x\right) \tag{11.2}$$

Note that this cognitive chain can be applied in fact to any relation between to systems, even for example electromagnetic ones. We can synthesize the whole process through a complex but unique operator h defined by :

$$h_{vx}\left(K^x\right) = \delta_v^z \omega_{zc} P^c \left(e_c|\ A_y^z \Lambda_x^y K^x\right) \tag{11.3}$$

The global reaction of a system inside the SoS becomes principally represented by the interaction h it has with all the other systems.

11.6 From SoS to cyber-physical SoS (CPSoS)

The communication using a cloud or anything else is included in the tensor G, added to ζ for S. This tensor has for sources some fluxes K. So the interaction h includes also the impact of the communications between the systems Q inside S[6]. Basically, the formalism contains all the elements for modelling the cyber-physical side. The difference comes from the fact that a cyber-physical SoS can contain systems that are very far between each other. But mathematically this property doesn't change the expression of the cognitive cord h, it only gives to the perception λ which includes the change of referential a specific writing. The difficult part for modelling such a CPSoS is not the communication network but the presence of multi physical layers and of various scales. The scale of description uses operators applied on fluxes which are the coordinates of the space. The flux vector embeds all the fluxes of all the physics concern by the SoS. There are currents, speeds, thermal powers, etc. If some relations need to use observable of lower scale, for example quantum observables, integrations are the key to go from this scale to a higher one : they are the change of dimension in a cellular topology. A collection of equations can accompany the system of equation of the CPSoS manifold to give the definitions of some local operator used in its own definition. The CPSoS manifold is completely defined by the tensorial

equation $e_q = \zeta_{qm} K^m$ plus the domain definitions of each parameter p, $\overset{p}{\mathcal{D}}$ that determines the laws L involved in ζ. We mean here that the operators ζ_{ij} are defined on various domains depending on parameters (temperature, etc.) :

$$\zeta_{ij} = \overset{p}{\mathcal{D}}_1 L^1_{ij} + \overset{p}{\mathcal{D}}_2 L^2_{ij} + \dots \tag{11.4}$$

The system of equations $e_q = \zeta_{qm} K^m$ can be seen as a vector of functions e_q. A base can be defined through the jacobian $\partial e_q / \partial K^m$. But various bases are defined for the various domains $\mathcal{D}$. We must ensure that the continuity between two local space attached with two local bases exists, i.e. :

$$det\left(\frac{\partial K^m\left[\mathcal{D}_i\right]}{\partial K^q\left[\mathcal{D}_j\right]}\right) \neq 0 \tag{11.5}$$

This constraint ensure that the manifold is continuous, i.e. that the CPSoS can be defined anywhere, for any values of the parameters and observables. Knowing the local bases, we can compute the metrics associated with these local spaces. It means that we can compute the distance between the center of a domain and any points in the same space. It means that we can compute the neighborhood of any point. We have defined a topology and the manifold defining the CPSoS[10].

11.6.1 Some more details on the numeric side

The cord can wear any kind of function. In particular it can wear numeric functions like sigmoid ones. Even if to model a whole system, it's better to use macro-models : it is always possible to synthesize a subsystem made of a wide number of components by a low dimension macro-model having the same transfer function. Rather than detailing a complete circuit made of digital gates we can create for example a function that outputs a "1" if the input word is the waited one. It is possible to study theoretically the communication network, whatever it is, by this kind of models. But the behaviour of the channel in front of piracy needs to be studied using the real and complete circuit. That's an example of a mechanism based on various scales.

11.6.2 Classical thematics considered for CPS

More than communication, the CPS' involve also the notions of control, mobility, architecture and security. They are composed of intelligence network, sensor network and actuators network[11]. We detail here each of these concepts.

CPSoS control

CPSoS control goes through the decisions δ, but also calling for the attention A and the kind of answers P of each system. The decision operator can be partly influenced by an exogen parameter. It can be an order given to the systems or a set of systems in answer to other systems behaviour.

Mobility

The mobility is translate by the values taken by some fluxes K^v in relation with the actuator in charge of the systems mobility. But the mobility influences also the perception of the systems between them through the operator Λ.

Architecture

The architecture is the description of the networks of intelligence, sensors and actuators. Three sub-graphs are associated to these networks.

Security

Security is today a major task for the CPS'. Electromagnetism and electromagnetic compatibility are the jobs addressed by this thematic as well as the TEMPEST and cryptography[12]. The perception of the systems actions can be faked due to the electromagnetic disturbances in the systems environment. It means that the operator Λ is polluted by a component of noise $\tilde{\Lambda}$. This pollution can lead to some erroneous decision $\tilde{\delta}$ and an incomprehensible behaviour of CPSoS. But we can even imagine that security passes through the attention operator A. It's clear that a strategy of CPSoS attack may be to focus the attention of one system in a direction opposite to the direction of attack. Once this system infected, the whole CPSoS can be infected step by step. Another strategy may be to deform the information of

decision. In that case, the operator Δ is corrupted and leads to abnormal decisions. There are many ways for a malicious system to disturb the whole CPSoS. There are general rules to avoid this kind of risk. First action must acts on Λ. Λ must be eventually corrected in order to transmit the good information. Then if an attack is identified, A must allow to bloc the access of this attack before it may infect the system further. If this protection fails, the only way to decrease the risk is to decrease the probability of answer to the faked information. This makes the assumption that the bayesian probability includes parameters influencing its values on the base of some trust indicators. The hope of earning can be modulated by the detection of a risk attached with some decision. Then the last barrier is linked with the decision operator which can also decide to exclude the hope of earning in relation with some source.

11.7 The pay-off matrix

The pay-off matrix says how many wins a system by making such a choice. It can be very factual in case of simple games with defined earnings, or very complex in case of CPSoS where the systems gains depends on many parameters at short and long time range. This matrix may be the key point of the system, and then of the SoS behaviour. Basically we can make the assumption that the systems behave depending on their hope of earnings. The bayesian probability P is first driven by the pay-off matrix ω. Then the decision is generally driven by the hope of earning, but can also be decided by the decision operator, defined itself by the psychological profile of the system.

In our previous illustration, we recognize the input (e'_x, e'_y) as the input data flux K^x, the transformation Λ realize the passage from (e'_x, e'_y) values to the corresponding choice c. The attention A makes the correspondences between the choice and the directions N, S, E, W. The Bayesian probability involving the input information $A\Lambda K$ and the choice made by the first system c, so $P^c\left(e_c|\ A^z_y \Lambda^y_x K^x\right)$ was taken here always equal to $1/4$.

The pay-off matrix ω_{zc} is here the tensor ea_{zc}. And the operator δ^z_v makes the correspondence with the commands e_x, e_y (it's the translator).

11.8 BISE : a conclusion for this first manual

BISE means to consider any system as made of a skeleton, a skin, muscles, nervous system. Made of an organ able to transform energy coming from the environment and organs to evacuate waste and heat. A system becomes so a SoS where all organs can be concepts with these notions in mind. The structure takes in charge part of the nervous network, the skin takes in charge part of the perception, etc.

Entities including microprocessors are able to conduct various operations of signal processing, memory management, computations, etc. More and more these kind of entities becomes generic (think in arduino, etc.) and a system becomes made of many of these entities with a central processor managing the whole and taking in charge the major memory. This central unit has another important role : it decides when some peripheral processors are out of use to report some actions to other organs. This is simplexity : ability to realize a mission even when some actors are no more available. This is a strong capacity of natural systems : the capacity to survive even in degraded mode. This capacity is reached seeing the system as a group of sub-systems having the 8-fkc. This architectures give also capacities to evolve. It remains two domains where nature is particular :

— reproduction and evolving through generations ;
— an intelligence in the way that beyond simple computations and complex algorithms, an abstract thinking exists in all animals. We must remember that Turing's criteria was never reached, though some say.

Nature is our inspiration but we are her children and we will not be able to remake her.

11.8.1 Aside : and what about quantum mechanics and safety ?

The change of scale between quantum and classical mechanics remain difficult, beyond all the relations available today. When the energy becomes very small, continuous behaviors becomes quantum behaviors. When we study a chain of transmission or reflexion of information, we can look at the various waves involved with high levels, in order to associate with this information amplitudes coming from reflexion and transmission cœfficients. That's what we discuss chapter 5. Quantum behaviors are similar to rare events. It gives models for particles of small energy or in other word for very small number

of particles. When there is a very high number of particles, they behaves classically, following thermodynamics.

Quantum mechanics is the manual for rare events. Safety is particularly interested in this kind of problem. Is it possible that a rare event occurs ? For example, if we consider a probability for an event a equal to 10^{-9}. If we make one jet. Is it possible that a arrives ? Theoretically yes, a can appear for one chance over 10^9. It's a poor chance, but it doesn't mean that this event cannot arrives on the first shoot. It's a paradox because reaching such a target in safety means that the risk a should never exist. In fact what we want to reach is a saturation phenomenon. Our first assumption is that we have an observer being able to observe one event at a time. If it's activated on one event, it cannot hear next events. It's typically the problems of critical events for safety on one mission.

Using this detector (observer), if a process results from a sequence of operations modeled using a gamma matrix and if a can appear during the first step for an input vector of information v, it means that γv gives a. Next steps can give high probabilities, reaching $1-a$ in final. So, $\gamma\gamma v \to 1-a$. In that case, the event a can exist on a first shoot as $\gamma v = a$ and the detector cannot see next events.

Now if the process implies first $1-a$ then a. First step being defined by $\gamma v = 1-a$, the chance to measure the event is that time near to be 1. So as the detector is one shoot measurement, it will always give the same response : $1-a$. The event a associated with $\gamma\gamma v$ will never been seen : that's the saturation phenomenon. For safety, this second process leads to the conclusion that the critical event a will never arrive. That's not the case in the previous sequence.

We understand that in a process resulting from a sequence of functions that can activate or not events, with a known probability, the fact to guarantee that a particular event cannot be realized depends on this sequence. Under some assumption and γ-matrix, rare events become event that never arrive. For other γ-matrix, they can appear on a first test with their own probability, but appearing on the first experiment. Their small probability meaning that the next experiments will not make appear these rare events.

If we imagine a bag of balls, if we take a ball at each random selection but without taking a new bag, the probability to select a rare event is $1/N$ on the first selection, then $1/(N-1)$, then $1/(N-2)$, etc. until 1. It means that the more selection we do, the more chance we have to select what was at the beginning a rare event. What may seems to be a disadvantage can be in fact

an advantage, because we finally know what is the rare event. That's always the difficulty in all jobs : if we make a system in order to avoid some event and if our conception is good, we will never seen this event, but somewhere we never demonstrate that it can exist and that it is what we think. The advantage with particles is that we can manipulate enormous numbers of particles while it is more difficult to have an enormous number of balls in a bag. When experiments are conducted on particles, two ways are available :

— sending one particle at each random selection ;
— sending a set of particles at each selection.

If we send one particle on each selection and if this particle is made each time exactly by the same process, we are in the case of playing with the same bag of balls without adding balls before the selection. Once a given number of selection made (say N), we are sure that law probabilities should appear (near $1/N$). Time going, we must see all the cases given by the theory. At the contrary if we make a set of particles each time (N particles sent each time) and if we create a process in order to make appear the rare events through γ^2 and not γ, the saturation process can be observed and the rare event may never appear.

This kind of reasoning, written differently, is made all the days by the safety workers. But it remains under this reasoning one strong assumption : we suppose that the rare event is known. And one specific property of rare events is sometimes to be unknown. How is that possible ? We should perfectly known our system and its environment ? It is also known (Gruyere principle) that the more critical rare events come from the coupling of systems, each of them being well controlled. Multi-physic and human factor intervene to create rare events that do not exist before to construct the whole system. Their gravity is all the more important that their construction is not predicted. So, all the process include to give some solution in case of crises are not efficient to face these events. These rare events are emergencies in the systemic thinking of systems. The proposed techniques in this book can allow reaching solutions in the system conception in order to find parries in front of these risks. γ^1 systems responses can be computed taking into account human factor cords and multi-physical conditions that lead to the system critical failure.

11.8.2 A simple circle for BISE

Finally, system engineering can be simply written. In general, the project begins by the customer needs. But in Nature, there are no customers ! Each system is its own customer. This difference makes that systems for humans are first tools before to be autonomous systems. Once this difference pointed out, we can consider that in both case, it exists a customer : in Nature, the customer is the system itself.

Accepting this assumption, the BISE sequence can be simply represented by the circle shown figure 11.1.

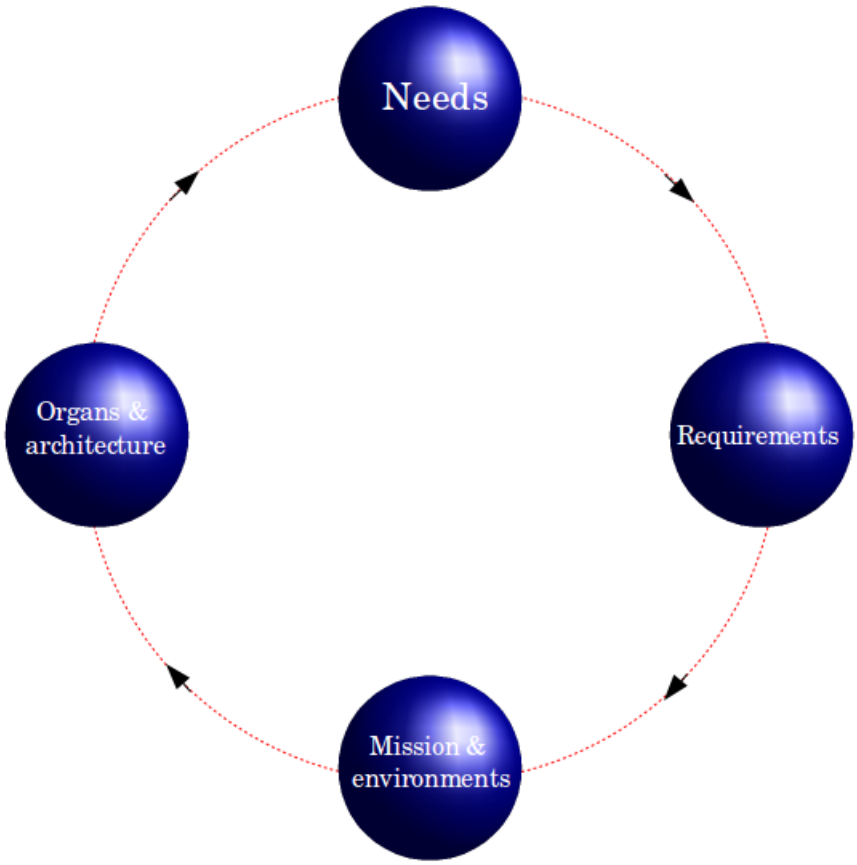


FIGURE 11.1 – BISE sequence

The needs step describes what is awaited from the system. What the system is supposed to do. In case of bio-system, this need is principally to survive in some ecological environment.

The requirements say what are the functions that must be garanteed anf with which safety target. The simplexity, which is a natural property, constructs strategies in order to reach these requirements by the most optimized architecture.

Mission and environment define the scenes that the system will encounter, the exchanges it will be called to assure, the locations where it will be able to find energy, system waste and dissipations, etc.

Organs and architecture define the whole system structure. It details the choices like an exo-skeleton for support or other options, etc.

This circle is explored until having the best solution for the problem considered,including post knowledge of the persons in charge of the system conception, material costs, etc.

Mathematically, needs are observables k^x defined in intervals I, $k^x \in I$. Requirements are probabilities $P(k^x)$ assigned to these observable objectives. Mission and environment are capacities in communications, speed, etc. They are parameters and standards giving criteria to judge of the system performance and of its adequation with the needs and requirements. Organs and architecture are firts classes in order to test hardware and software solutions to construct the system. This construction go ahead as and when that the circle is browsed.

This fourth step calls for the more persons. The architect on the base of one system, distributes the organs conception to various project leaders. Nervous and power supply networks are subcontracted by the same way. Common meetings allow to evaluate the system performance obtained with a first set of organs return back by the project leaders. adjustments are operated to take into account the coupling effects in multi-physics. Then a new circle is begun until converging to an optimal solution. The architect is in fact a team enclosing a system architect, a commercial, a financial, etc.

Various actions are :

Architect abstracts $\zeta_{\mu\nu}$ for the needs k^ν and environment T_μ. Construct $\oplus_\mu \zeta_{\mu\nu}$;

Project leaders look for ζ', considering mission & environment T_μ with $\exists k'^\nu \,/\, |y^{\nu\mu} T_\mu - k'^\nu| \to \epsilon \cap P\left(\epsilon > r |\, T_\mu \,, requirements\right) \to 0$;

Common meeting verifying is $T'_\mu = \left[\oplus_\mu \zeta'_{\mu\nu}\right] k^\nu \Rightarrow P\left(k^\nu\right) \in P\left(I\right)$, I

and $P(I)$ being defined by the requirements. Making adjustments for observables $P(k^{\nu}) \notin P(I)$, return back new objectives for ζ.

We see here that the fact to work as soon as possible with mathematical models allows to accurately define the technical actions that need to be leaded and avoid ambiguities and the absence of taking into account multiphysic effects, indeed human factor. This last point implies to control the AI cord, but this last interaction modeling that I propose needs yet more developments.

I hope that this compilation of tracks of solutions will inspire engineers. I think that the major axis of progress for system engineering comes from the fact to introduce more rigor and mathematics as soon as possible in its process. Often engineers exchange ideas rather than models. This leads to ambiguities, fake solutions and uncontroled hardware and softwares. This without forgetting that models allow to keep and transmit knowledge accurately, and that probabilities must be included in all the process flow.

Chapitre 12

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