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The Reduced Basis Method Applied to Aerothermal Simulations

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► **To cite this version:**

Jean-Baptiste Wahl, Christophe Prud'Homme, Yannis Hoarau. The Reduced Basis Method Applied to Aerothermal Simulations. CANUM, May 2018, Cap d'Agde, France. hal-01799506

HAL Id: hal-01799506

<https://hal.science/hal-01799506>

Submitted on 24 May 2018

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The Reduced Basis Method

Motivations

- Modeling : multi-physics non-linear models, complex geometries, genericity
- Uncertainty management / Risk analysis
- Optimization in early design, certification or operating phases

Objective 1: Fast

- Complex geometries
→ Large number of dofs
- Uncertainty quantification
→ Large number of runs

Objective 2: Reliable

- Field quality
→ Certified bounds
- Design optimization
→ Reach material limits

Main Idea

Weak formulation of the model : $a(u(\mu), v; \mu) = f(v; \mu)$

FEM Approximation:

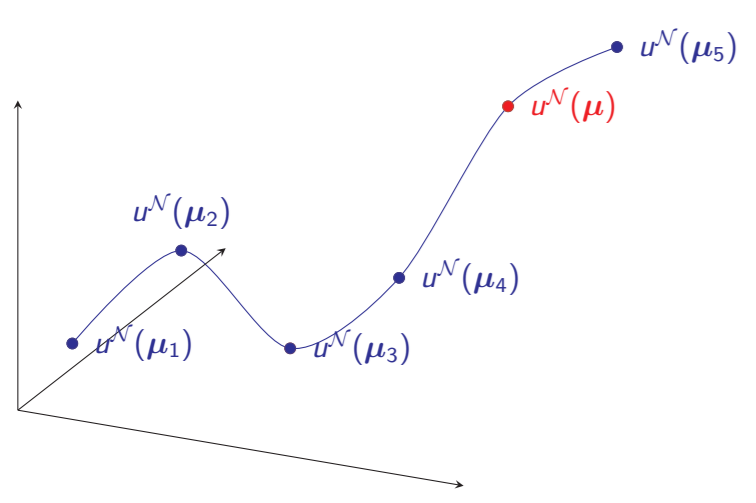
$$\underbrace{X^N = \text{span}\{\phi_1, \dots, \phi_N\}}_{\text{FEM approximation space}} \rightarrow u^N(\mu) = \sum_{i=1}^N u_i^N(\mu) \phi_i \rightarrow \underbrace{A^N(\mu) u^N(\mu) = F^N(\mu)}_{N \times N \text{ system expensive to solve}}$$

RB Approximation: $u^N(\mu) \approx u^N(\mu)$: linear combination of FEM solution

$$\underbrace{W^N = \text{span}\{u^N(\mu_1), \dots, u^N(\mu_N)\}}_{\text{RB approximation space}} \rightarrow u^N(\mu) = \sum_{i=1}^N u_i^N(\mu) \underline{u}^N(\mu_i) \rightarrow \underbrace{A^N(\mu) u^N(\mu) = F^N(\mu)}_{N \times N \text{ system cheaper to solve}}$$

Ingredients

- Set of parameters : \mathcal{D}^μ
- FEM "truth" approximation
 X^N : finite element approximation space of dimension $N \gg 1$
 $u^N(\mu) \in X^N$ is solution of $a(u^N(\mu), v; \mu) = f(v; \mu) \forall v \in X^N$
- RB approximation
Sample : $S_N = \{\mu_1 \in \mathcal{D}^\mu, \dots, \mu_N \in \mathcal{D}^\mu\}$
Approximation space : $W_N = \text{span}\{u^N(\mu_1), \dots, u^N(\mu_N)\}$ with $N \ll N$
Galerkin projection on W_N to determine RB coefficients



Efficient offline-online strategy

$$u^N(\mu) = \sum_{i=1}^N u_i^N(\mu) \underline{u}^N(\mu_i)$$

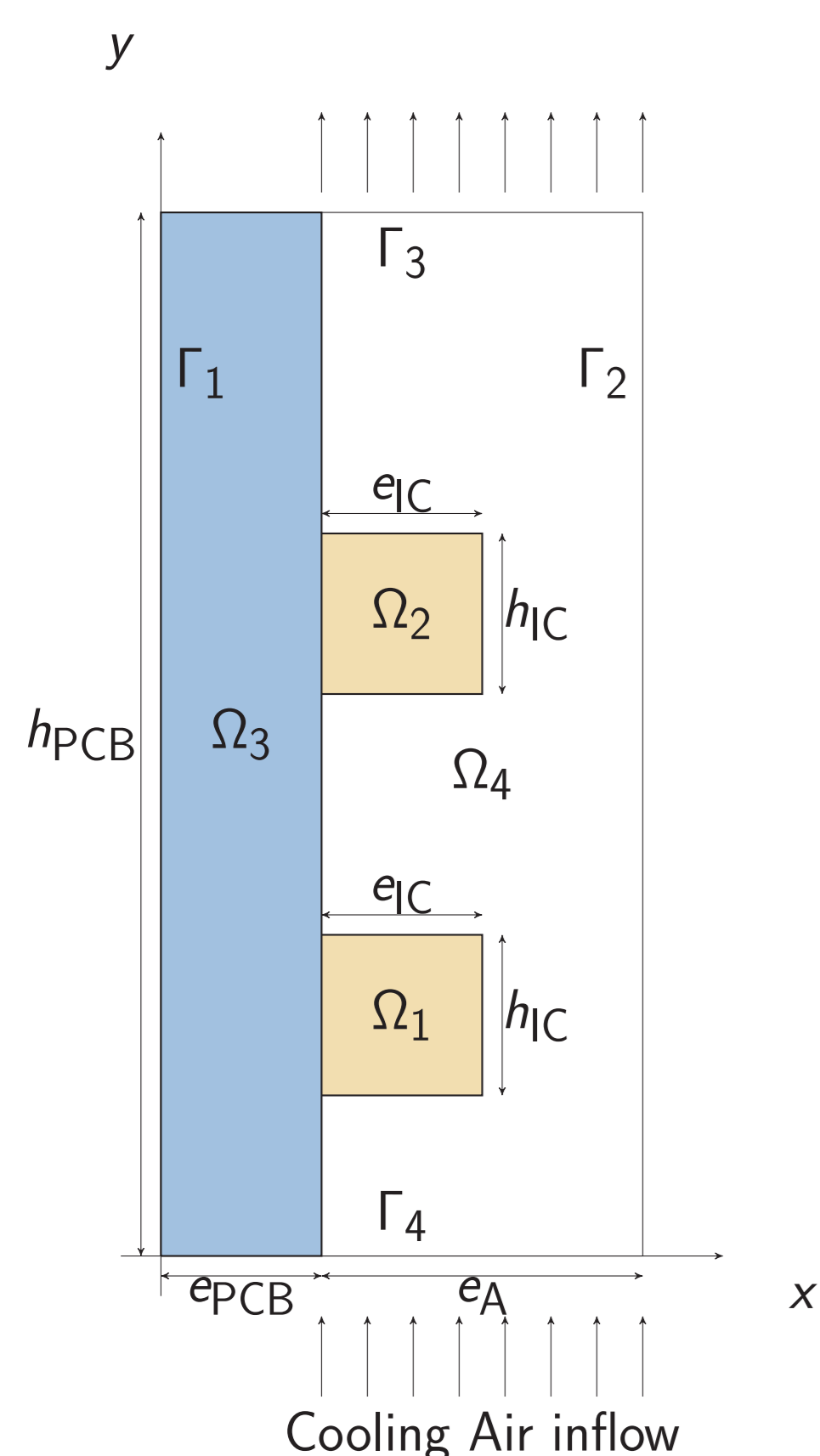
$N \times N$ system to solve : $\sum_{i=1}^N a(\underline{u}^N(\mu_i), v_k; \mu) u_i^N(\mu) = f(v_k; \mu)$, $1 \leq k \leq N$ If the parameter dependance can be

expressed as an affine decomposition :

$$a(u, v; \mu) = \sum_q \theta_q^a(\mu) a_q(u, v) \quad \text{and} \quad f(v; \mu) = \sum_q \theta_q^f(\mu) f_q(v)$$

$$\Rightarrow \sum_{i=1}^N \left[\sum_{q=1}^{Q_a} \theta_q^a(\mu) \underbrace{a_q(\underline{u}^N(\mu_i), \underline{u}^N(\mu_j))}_{\text{precomputed}} \right] u_i^N(\mu) = \sum_{q=1}^{Q_f} \theta_q^f(\mu) \underbrace{f_q(\underline{u}^N(\mu_j))}_{\text{precomputed}}$$

Numerical Results: Cooling of an Printed Circuit Board, Reduced Model



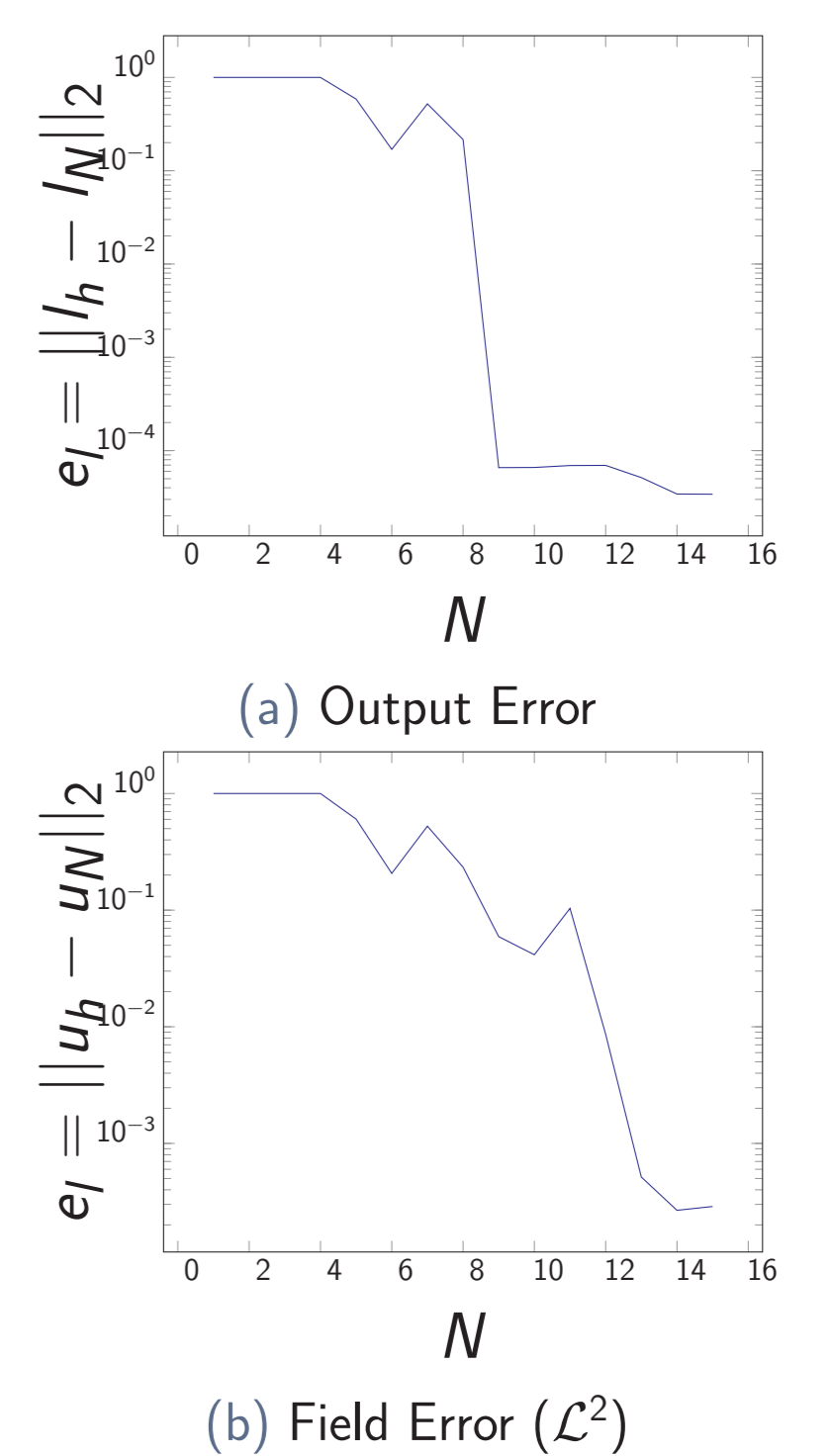
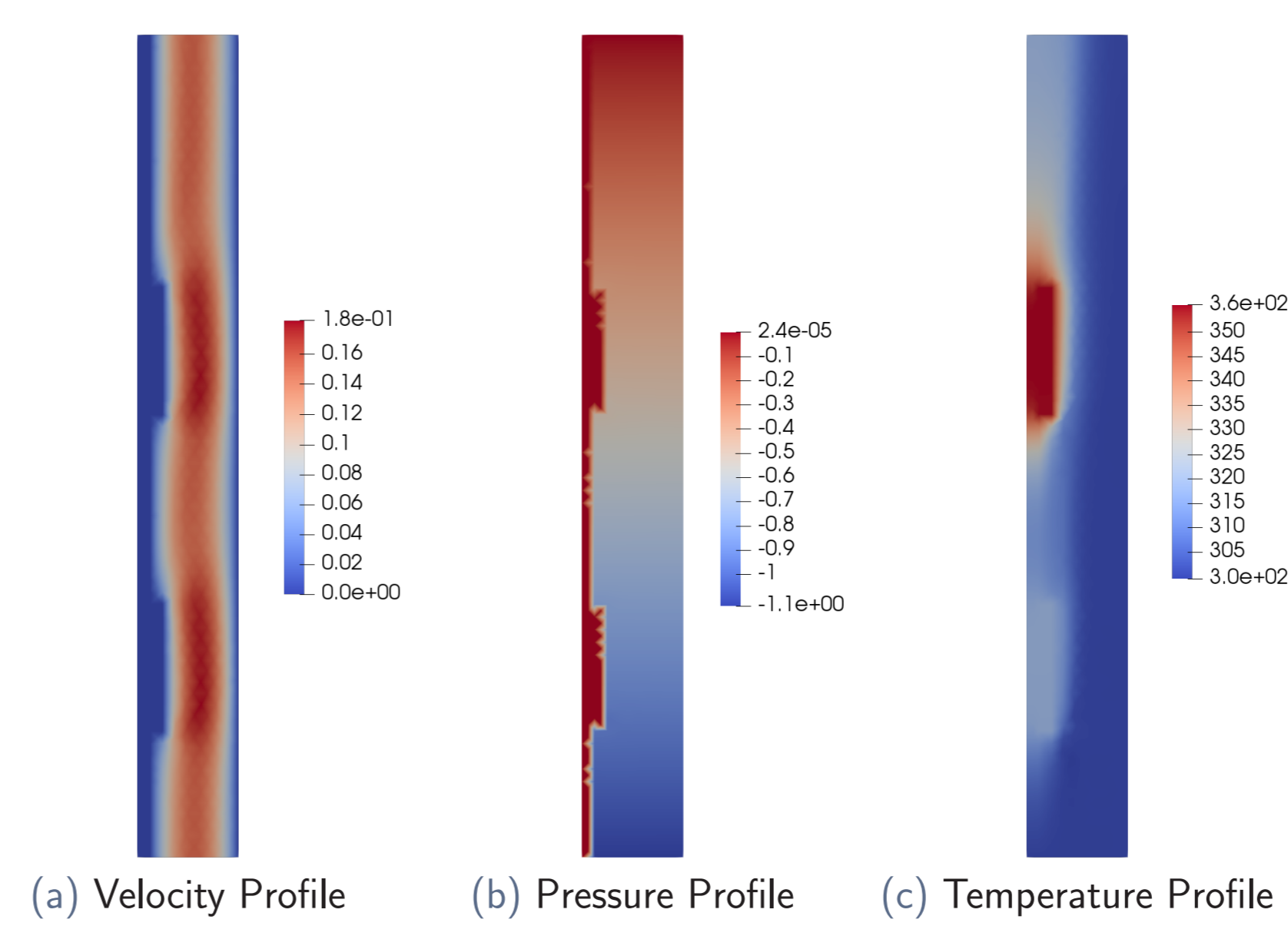
2D model representative of the neighboring of an electronic component submitted to a cooling air flow.

Physical Model

- Air thermal diffusivity: $\kappa_a = 2.7 \cdot 10^{-5}$
- Air kinematic viscosity: $\mu_a = 1.9 \cdot 10^{-5}$

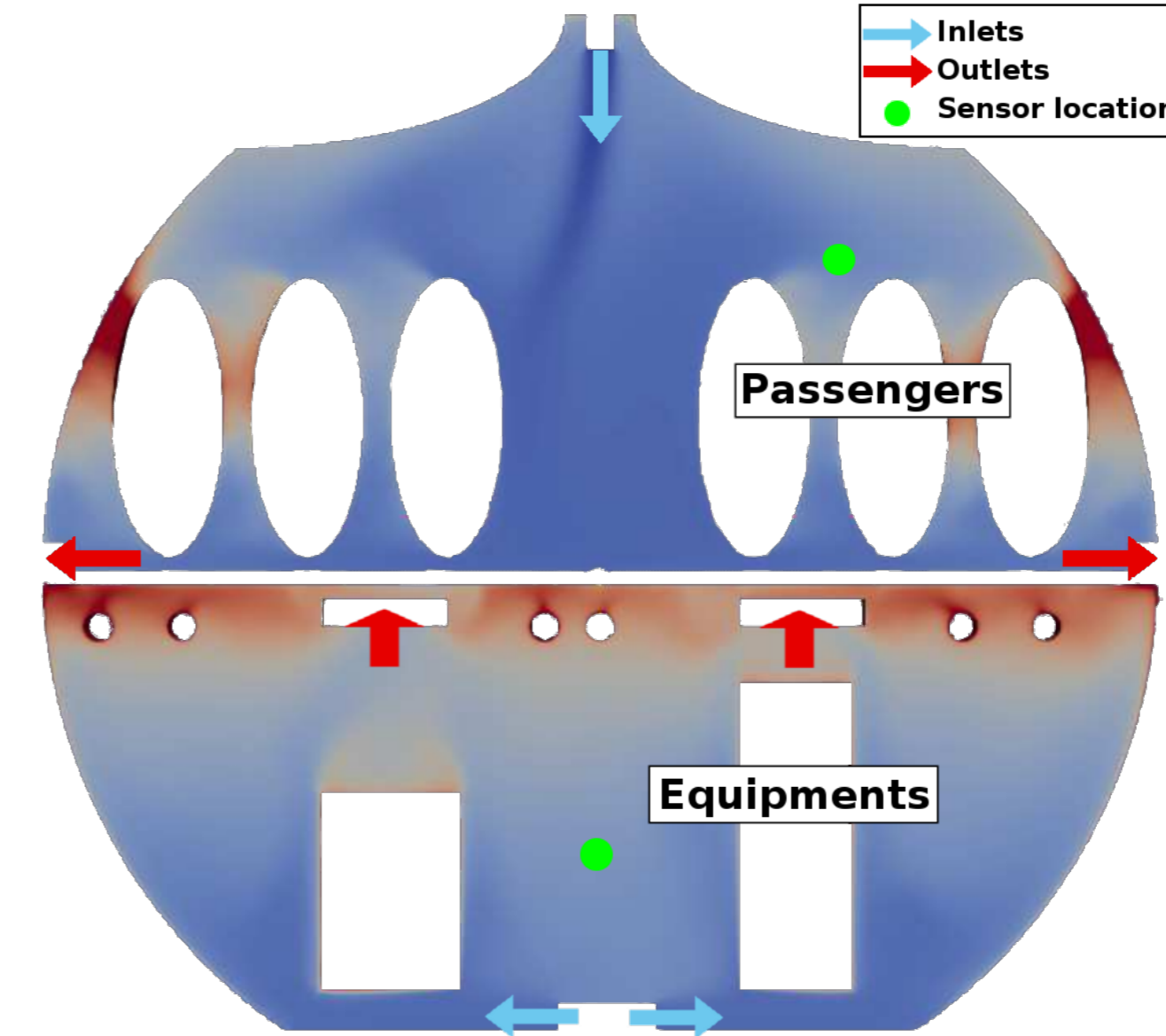
Parameters

- Q_1 and Q_2 : Heat sources from the two Integrated Circuits: $[0, 10^6]$
- κ_1 and κ_2 : Thermal conductivity of the two Integrated Circuits: $[0.2, 150]$
- D : The inflow rate: $[5 \cdot 10^{-4}, 10^{-2}]$



Convergence with respect to the size of the basis. Maximum of the error, evaluated on 100 RB approximations compared with the FEM approximations

Airbus Use-Case



Propose in the context of the ANR Project CHORUS
Objective : Apply the Certified Reduced Basis Methods on an aerothermal simulation in an avionic bay

Model :

- Steady Navier-Stokes/Heat transfer
- Incompressible Newtonian Fluid
- Boussinesq Approximation
- Turbulent Flow

Mathematical Model

Governing Equations

$$\begin{cases} \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mu \Delta \mathbf{u} = \rho \beta (T - T_0) \mathbf{g}, & \text{in } \Omega \times [0, T_f], \\ \nabla \cdot \mathbf{u} = 0, & \text{in } \Omega \times [0, T_f], \\ \mathbf{u} \cdot \nabla T - \kappa \Delta T = 0, & \text{in } \Omega \times [0, T_f], \end{cases} \quad (1)$$

+Boundary Conditions.

with

- ρ : fluid density ($\text{kg} \cdot \text{m}^{-3}$),
- $\kappa \approx 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$: thermal diffusivity ($\text{m}^2 \cdot \text{s}^{-1}$),
- $\mu \approx 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$: dynamic viscosity ($\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$),
- β : thermal expansion coefficient (K^{-1}),
- Length Scale $L \approx 1 \text{ m}$,
- $\Rightarrow Re \approx 10^5$, $Pr \approx 1$, $Pe \approx 10^5$.

Solving Strategy

- Finite Element Discretization
- Newton Algorithm with transient continuation
- Parallel implementation using Feel++ library: <http://www.feelpp.org/>

Challenge and Difficulties

- **Multi-physic coupled model**: simultaneous construction of the different reduced spaces
- **High Reynolds flow**: use of stabilization methods (SUPG/GLS) in the FEM and the RB model
- **Non-Linearity**: Newton algorithm with an affine decomposition of the Jacobian/Residual
- **Non-affine terms**: use of Empirical Interpolation Method (EIM) for discrete operators
- **Non-Linearity**: Use of Simultaneous EIM and RB (SER) algorithm to generate an affine approximation of the non-linear terms (stabilization terms)
- **Complex Formulation**: Due to geometric parameters, use of EIM to automatically recover the affine decomposition.

Perspectives

- Development of Efficient Error Estimators for the Reduced Model
- Reduction of the Coupled Turbulence Model

Sponsor



MSO4SC:
Mathematical Modeling, Simulation and Optimization
for Societal Challenges with Scientific Computing



The main objective of this project is to construct an e-infrastructure that provides, in a user-driven, integrative way, tailored access to the necessary services, resources and even tools for the fast prototyping, providing the service producers with the mathematical frameworks as well.

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