



9th INTERNATIONAL CONFERENCE
“INVERSE PROBLEMS: MODELING AND SIMULATION”
MAY 21-25, 2018, MALTA

Highly oscillatory integrals in optical imaging

Jean-Baptiste Bellet

Université de Lorraine, CNRS, IECL, F-57000 Metz, France

M23: Inverse Wave Propagation Problems without Phase Information
May 24, 2018

Outline

3D reflective tomography

Model problem

Geometrical modeling

Canonical models

Conclusion

Optical images

Visible

$[0.4\mu\text{m}, 0.7\mu\text{m}]$

Near infrared

$[0.7\mu\text{m}, 3\mu\text{m}]$



(a) Visible



(b) Near infrared

Passive

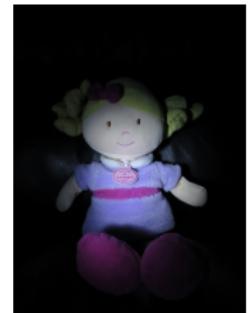
Ambient light

Active

Additional light source



(c) Passive



(d) Active

D. Schwen. Tree on Mt Victoria Devonport, New Zealand.
https://commons.wikimedia.org/wiki/File:Tree_example_VIS.jpg
https://commons.wikimedia.org/wiki/File:Tree_example_IR.jpg

Reflective tomography

1. Measure
 - ▶ Optical images
2. Reconstruct
 - ▶ Tomography solver
3. Visualize
 - ▶ Volume rendering
4. Identify



F. Knight, S. Kulkarni, R. Marino, and J. Parker (1989).

Tomographic Techniques Applied to Laser Radar Reflective Measurements.
Lincoln Laboratory Journal, 2(2).



G. Berginc and M. Jouffroy (2009).

Optronic system and method dedicated to identification for formulating three-dimensional images.
US patent 20110254924 A1, European patent 2333481 A1.



G. Berginc and M. Jouffroy (2009).

Simulation of 3D laser systems.

IEEE Geoscience and Remote Sensing Symposium, 2, 440-444.



G. Rigaud, J.-B. B., G. Berginc, I. Berechet, and S. Berechet (2016).

Reflective Imaging Solved by the Radon Transform.

IEEE Geoscience and Remote Sensing Letters, 13, 936-938.

Example: active optronic imaging

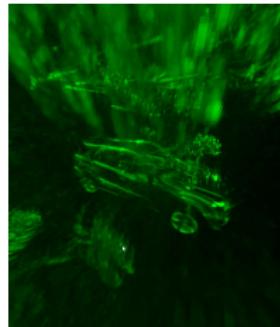
Input

- ▶ laser images (Thales)
- ▶ 181 images, 342x421



Output

- ▶ 3D volume $\simeq 400^3$ voxels
- ▶ Computation time: 4 seconds
- ▶ Display: real time, interactive



Computations on a GPU:
Nvidia Tesla C2075



G. Berginc, J.-B. B., I. Berechet, and S. Berechet, 2016
Optical 3D imaging and visualization of concealed objects.
Proc. SPIE, 9961, 99610Q.



Outline

3D reflective tomography

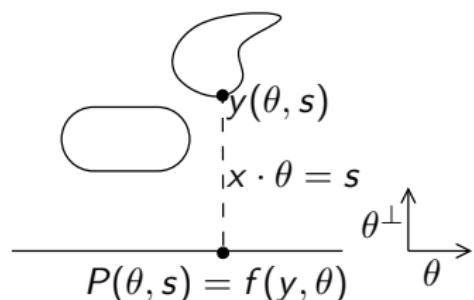
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Reflective orthographic projection



Scene

Opaque objects

Device

Screen, with angle θ

Light source (if active)

Projection

Ray: $x \cdot \theta = s$

Visible point: $y(\theta, s)$

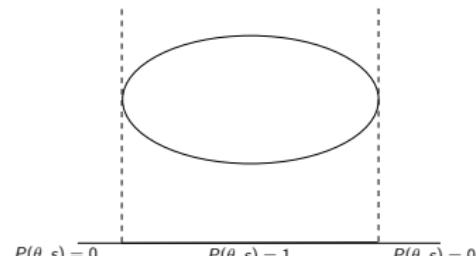
Records

$P(\theta, s) = f(y(\theta, s), \theta)$
 $s \in [-R, R], \theta \in \Theta$

Example: silhouette

Silhouette

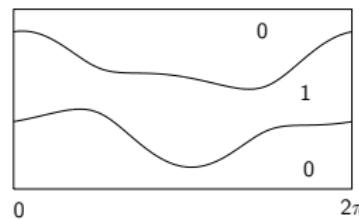
Binary image for a convex



(a) Silhouette

Records

Silhouettes: $(\theta, s) \mapsto P(\theta, s)$



(b) Records

Discontinuities

(θ, s) s.t. $x \cdot \theta = s$ is tangent

Filtered backprojection

Input: projection P

1. Filtering:

$$\tilde{P} = P \star \psi_\Omega = \int P(\theta, t) \psi_\Omega(s-t) dt, \quad \psi_\Omega(s) := \int_0^1 \sigma \cos(\Omega \sigma s) d\sigma$$

NB: $\frac{\Omega^2}{4\pi^2} \psi_\Omega$ is the usual Ram-Lak filter

2. Backprojection:

$$\mathcal{R}^*[\tilde{P}](x) = \int \tilde{P}(\theta, x \cdot \theta) d\theta, \quad x \in \mathbb{R}^2.$$

Output: filtered backprojection $\mathcal{R}^*[P \star \psi_\Omega]$

Tomography

Transmission tomography

- ▶ P is a Radon transform: $P = \mathcal{R}f = \int_{x \cdot \theta = s} f$
- ▶ Reconstruction formula: $\frac{\Omega^2}{4\pi^2} \mathcal{R}^*[P \star \psi_\Omega] \approx f$



F. Natterer and F. Wübbeling (2001).

Mathematical methods in image reconstruction.
SIAM

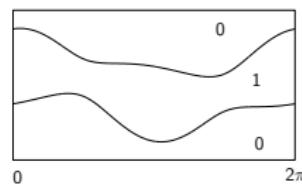
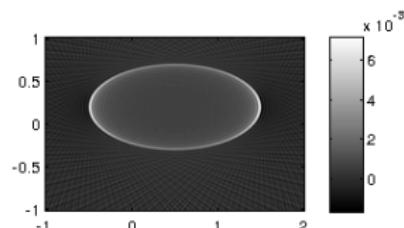
Reflective tomography

- ▶ P is a reflective projection
- ▶ Empirical solver: $\mathcal{R}^*[P \star \psi_\Omega]$

Numerical example



(a) Object

(b) Projection P (c) Filtered backprojection
 $\mathcal{R}^*(P \star \psi_\Omega)$

Observations

The highest values represent the initial object.

Theoretical question

Link the highest values with the initial geometry.

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High resolution modeling

Radial resolution Ω

In practice, the resolution $\Omega = \pi \times (\text{pixel density})$ is large

Highly oscillatory integral

$$\mathcal{R}^*[P * \psi_\Omega](x) = \int_0^1 \sigma \iint P(\theta, s) \cos(\Omega\sigma(x \cdot \theta - s)) ds d\theta d\sigma$$

Modeling

Geometrical modeling of the empirical reconstruction

Asymptotic expansion of $\mathcal{R}^*[P * \psi_\Omega](x)$, $\Omega \rightarrow \infty$

Resemblance (not purely fortuitous)

Geometrical optics is a high-frequency geometrical modeling!

Leading order terms

Theorem

Let P be a reflective projection. We essentially assume that P is piecewise smooth. The reconstruction $\Omega^2 \mathcal{R}^*[P * \psi_\Omega](x)$ is:

- ▶ $\mathcal{O}(\sqrt{\Omega})$
 - ▶ for some *convex portions of the original scene*,
 - ▶ for some *arcs of a large circle (artifacts)*,
- ▶ $\mathcal{O}(\log \Omega)$
 - ▶ for some *isolated points of the original scene*, such as corners,
 - ▶ for some *straight lines (in general, artifacts)*,
- ▶ $\mathcal{O}(1)$
 - ▶ *elsewhere (noise)*.



J.-B. B. (2017).

Analyse asymptotique et géométrique de la tomographie réfléctrice.
[<hal-01571707>](https://hal.archives-ouvertes.fr/hal-01571707)

Sketch of the proof

1. Describe geometrically the structure of P
2. Mono-frequency asymptotics

$$h(x, \lambda) = \int P(\theta, s) \cos(\lambda(x \cdot \theta - s)) d\theta ds, \quad \lambda \rightarrow \infty$$

- ▶ iterated divergence formula + stationary phase formula
- 3. Multi-frequency asymptotics

$$\mathcal{R}^*[P * \psi_\Omega](x) = \int_0^1 \sigma h(x, \Omega\sigma) d\sigma, \quad \Omega \rightarrow \infty$$

- ▶ Mellin transform method



N. Bleistein and R. A. Handelsman (1986).

Asymptotic Expansions of Integrals.
Dover.

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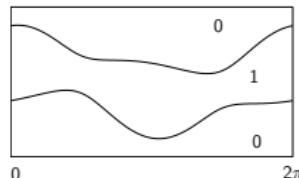
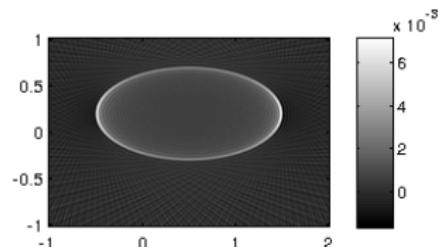
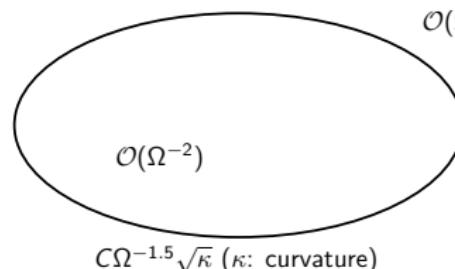
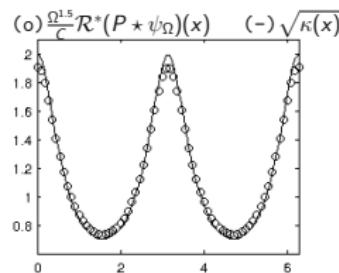
Canonical models

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Reconstruction from silhouettes

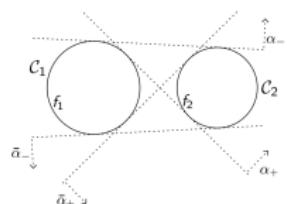


(a) Object

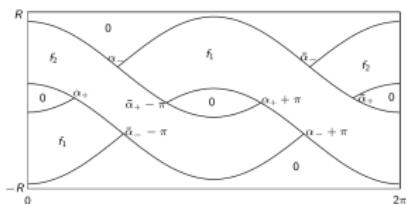
(b) Projection P (c) Reconstruction $\mathcal{R}^*(P * \psi_\Omega)$ (d) Asymptotics of $\mathcal{R}^*(P * \psi_\Omega)$ 

(e) Quality control

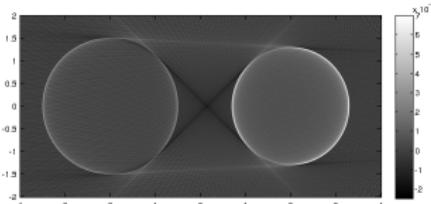
Reconstruction with occultations



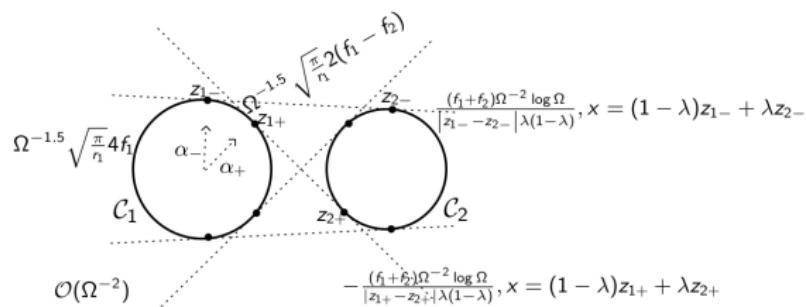
(a) Scene



(b) Reflective projection

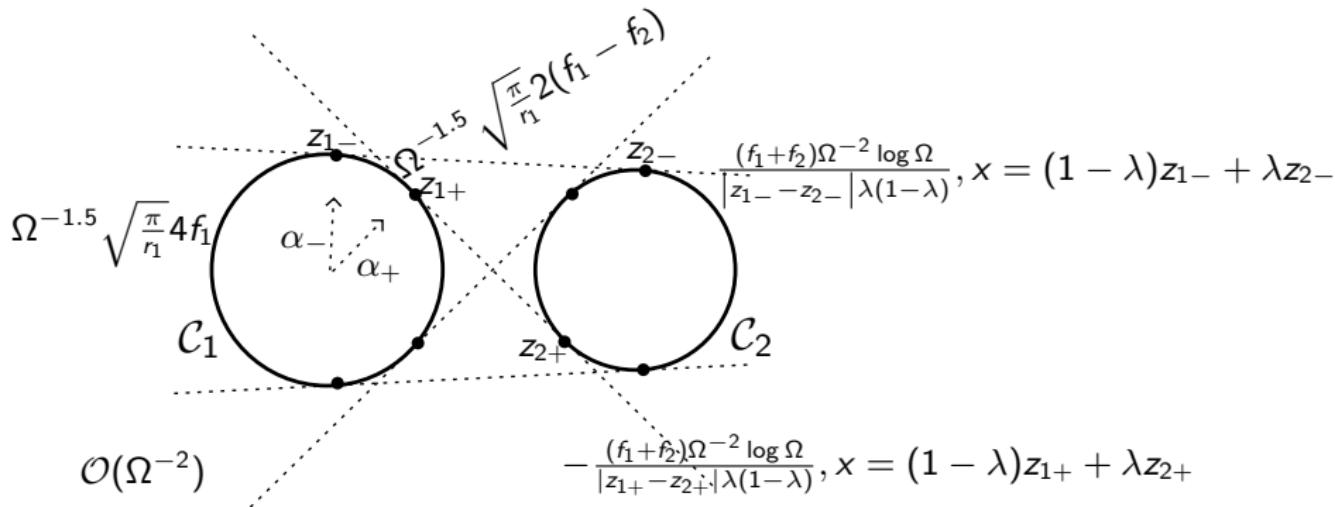


(c) Numerical reconstruction



(d) Asymptotics of the reconstruction

Reconstruction with occultations



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Conclusion

Synthesis

Reflective tomography

Phaseless optical imaging

Geometrical model

Quantifies

- ▶ well-reconstructed parts
- ▶ artifacts



J.-B. B. (2017).

Analyse asymptotique et géométrique de la tomographie réfléchissante. <[hal-01571707](#)>

Perspectives

Extensions

Other geometric problems



J.-B. B. and G. Berginc (2018).

Geometric problems from 3D reflective tomography. <[hal-01788342](#)>

Algorithmic improvements

Artifacts reduction

Noise removal

Acceleration of the solver



J.-B. B. (2018).

Multiresolution greedy algorithm dedicated to reflective tomography. <[hal-01678995](#)>

Thank you for your attention.