Numerical simulation of RTM using boundary element method

Fabrice Schmidt, Pierre Lafleur, Florentin Berthet, Pierre Devos

NUMERICAL SIMULATION OF RTM USING BOUNDARY ELEMENT METHOD

F.M Schmidt
Enstimac, Route de Teillet, 81013 Albi CT Cedex 09 (FRANCE)

P. Lafleur
CRASP, Ecole Polytechnique, P.O. Box 6079, Stn Centre ville, Montreal, QC, H3C 3A7 (CANADA)

F. Berthet
Enstimac, Route de Teillet, 81013 Albi CT Cedex 09 (FRANCE)

P. Devos
Enstimac, Route de Teillet, 81013 Albi CT Cedex 09 (FRANCE)

ABSTRACT: Using mass conservation and Darcy's law to describe flow through isotropic porous media leads to a Laplace equation which may be solved numerically at each time step using the boundary integral method. For anisotropic porous media in which the permeability matrix is symmetric, the problem can be solved in the same way by rotating and stretching the coordinates. The numerical model has been compared with analytical solutions in the case of radial front flows. In addition, we have proceeded to a comparison between finite and boundary element methods in the case of a frontal injection.

1 INTRODUCTION

The Resin Transfer Moulding (RTM) process is widely used in the manufacturing of large components of fiber reinforced materials. In RTM, the resin is injected into a mould filled with reinforcing fibers. The reinforcement is composed of several layers of fibers mats or woven roving laid inside the mould cavity. The resin enters the mould through one or several injection gates and impregnate progressively the preform. Proper design of the vent and injection port location is crucial to prevent large air pocket from being trapped during flow. Therefore, the analysis of resin flow in the mould is a very important task in designing the RTM mould.

Several numerical methods have been developed to simulate the filling step based either on finite difference as in Li and Gauvin (1991) or control volume method as in James Lee et al. (1994) and on finite element method (Trochu et al. 1994). Following on Um and Lee (1991) and Hadavinia et al. (1995) in this study, the resin flow will be solved numerically by the boundary integral method (BEM). This method offers the great advantage of describing the flow by its boundary values only while reducing the problem dimensions by one. In addition, it permits to accurately capture the transient free surface.

2 GOVERNING EQUATIONS

The impregnation of a fibrous preform is usually modelled as a flow through anisotropic homogeneous porous media. It is governed by Darcy's law:

$$\vec{v} = -\frac{[K]}{\mu}\vec{\nabla}p \qquad (1)$$

where $\vec{v}$ is the velocity vector, $[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$ the permeability tensor, p the resin pressure and μ the viscosity (constant for a Newtonian resin). Combining (1) with the continuity equation gives :

$$\vec{\nabla}.\left(-\frac{[K]}{\mu}\vec{\nabla}p\right) = 0 \qquad (2)$$

Equation (2) can be reduced to Laplace's equation by rotating and stretching the coordinates. The new

coordinates (x,y) may be deduced from the previous coordinates (X,Y) using the following relationship :

$$\begin{cases} x = \dfrac{\alpha_1.X + \beta_1.Y}{\sqrt{\lambda_1}} \\ y = \dfrac{\alpha_2.X + \beta_2.Y}{\sqrt{\lambda_2}} \end{cases} \quad (3)$$

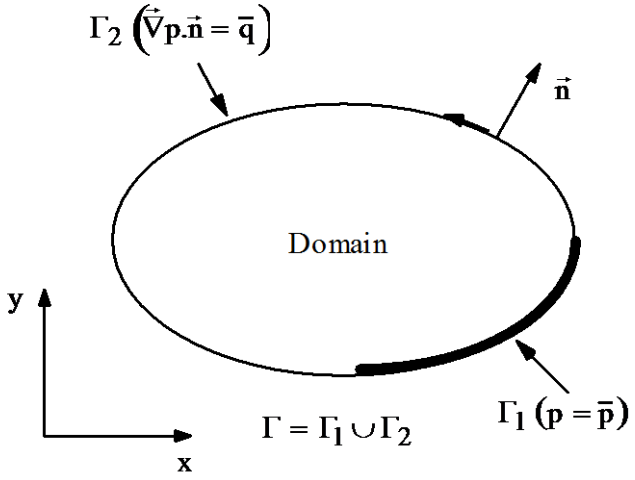
where $(\alpha_1,\beta_1,\alpha_2,\beta_2)$ and (λ_1,λ_2) are the components of the eigenvectors and the eigenvalues of the permeability matrix respectively. Hence, using the coordinates (x,y), equations (1) and (2) reduce simply to :

$$\vec{v} = -\frac{1}{\mu}\vec{\nabla}p \quad (4)$$

$$\Delta p = 0 \quad (5)$$

The boundary conditions are sketched in fig. 1. Γ_1 is the part of the boundary Γ where a known value of pressure $\bar{p}$ is applied; Γ_2 the part of the boundary Γ where a known value of pressure gradient $\bar{q}$ perpendicular to the solid wall is prescribed (for a non-penetration condition $\bar{q} = 0$).

Figure 1. Boundary conditions



3 NUMERICAL METHOD

3.1 *Boundary Integral Equation*

The transformed equation (5) is multiplied by weighting function p* and integrated over the calculation domain using Green's theorem to yield (Brebbia 1992):

$$\frac{p_i}{2} + \int_\Gamma p q^* d\Gamma = \int_\Gamma p^* q d\Gamma \quad (6)$$

where p_i is the value of the pressure at a point M_i located on the boundary (which is supposed to be smooth), q* pressure gradient associated with p*. The weighting function p* is the fundamental solution satisfying :

$$\Delta p^* + \delta_i = 0 \quad (7)$$

where δ_i is the Dirac delta function. For a bidimensional medium, p* and q* are given as (Ozisik 1980):

$$p^* = \frac{1}{2\pi}\ln\left(\frac{1}{r}\right) \quad \text{and} \quad q^* = \frac{1}{2\pi}\frac{-\vec{r}.\vec{n}}{r^2} \quad (8)$$

where r is the distance from the point M_i of application of the delta function to any point under consideration.

3.2 *Boundary Element Equation*

Dividing the boundary into N constant boundary elements and apply eq. (6), the following equation is obtained:

$$\frac{1}{2}p_i + \sum_{j=1}^{N}\int_{\Gamma_j} p q^* d\Gamma = \sum_{j=1}^{N}\int_{\Gamma_j} p^* q d\Gamma \quad (9)$$

where Γ_j represents each boundary element. For this type of element, the values of p and q are assumed to be constant over each element and equal to the value at the mid-element node. Equation (9) can be rewritten in matrix form as:

$$\sum_{j=1}^{N} H_{ij} p_j = \sum_{j=1}^{N} G_{ij} q_j \qquad (10a)$$

where

$$H_{ij} = \frac{1}{2}\delta_{ij} + \int_{\Gamma_j} q^* \, d\Gamma \qquad (10b)$$

$$G_{ij} = \int_{\Gamma_j} p^* \, d\Gamma \qquad (10c)$$

For the particular case of constant elements, the H_{ii} and G_{ii} integrals can be computed analytically (Brebbia 1992). Integrals like H_{ij} and G_{ij} $(i \neq j)$ are calculated using simple Gauss quadrature rules (4 points rule).

3.3 *Resin Front Updating*

At each time step Δt, the flow of resin inside the mould is regarded as quasi-steady. Unknown pressure p and pressure gradient q on the boundary are calculated by using eq. (10a). Then, the new location of the resin front is updated using a forward Euler integration scheme:

$$\vec{x}(t+\Delta t) = \vec{x}(t) + \Delta t(\vec{v}.\vec{n})\vec{n} \qquad (11)$$

Only the normal component of the resin velocity is needed on the resin front as there is no tangential component of the velocity. Using eq. (4), eq. (11) reduces to:

$$\vec{x}(t+\Delta t) = \vec{x}(t) - \Delta t\left(\frac{q}{\mu}\vec{n}\right) \qquad (12)$$

The updated nodes close to the mould which have moved out of the solid wall are relocated using an orthogonal condition.

3.4 *Automatic remeshing*

With an updated lagrangian formulation, the nodes of the mesh follow the kinematics evolution of the material points. This method may result in too distorted elements, when large deformations occur. An automatic remeshing procedure is used. The procedure consists in the following steps :

• addition of nodes on the current boundary (overdiscretisation);

• elimination of some of these nodes in order to generate an appropriate set of boundary nodes which must be compatible with the old mesh boundary and satisfy curvature and homogeneity conditions.

The curvature C_i at a given edge is calculated using the difference of the outwardly directed normal $\vec{n}_{i+1}$ et $\vec{n}_{i-1}$ of connected boundary elements:

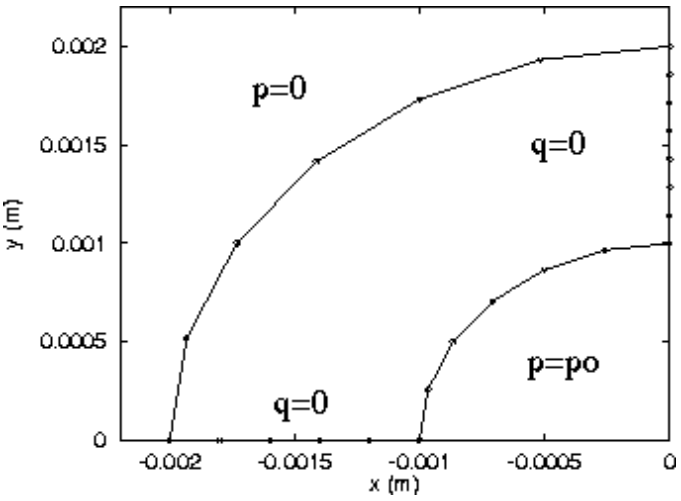
$$C_i = \left|\vec{n}_{i+1} - \vec{n}_{i-1}\right| \qquad (13)$$

4 APPLICATION

4.1 *Comparisons with Analytical Results*

Let us consider the radial impregnation of fibrous medium by an incompressible and Newtonian fluid (fig. 2).

Figure 2. Initial mesh and boundary conditions



If we assume that the porous medium is isotropic and homogeneous, the flow front should be of circular shape. Hammami et al. (1996) gave the following solution :

$$\left[\frac{R_f}{R_o}\right]^2\left[2\ln\left(\frac{R_f}{R_o}\right)-1\right]+1 = \frac{4Kp_o t}{\mu R_o^2} \qquad (14)$$

where R_f is the flow front radius at time t, R_o the inlet radius, p_o the constant injection pressure, t the

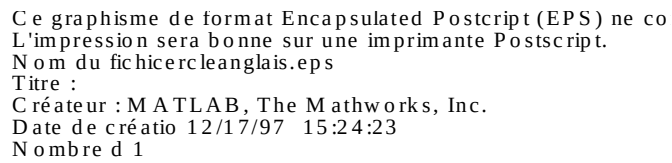
elapsed time, K the permeability. The values of the different parameters are given in tab. 1

Table 1. Processing and rheological parameters

$K\ (m^2)$	μ (Pa.s)	p_o (Pa)	R_o (m)
$4.\ 10^{-11}$	0.1	10^5	0.001

Fig. 3 shows the comparison between the flow front which has been computed using the boundary element method and the analytical solution (14) at t=200 s. The computation time takes less than 1 min. The number of nodes of the final mesh is 80. The time step used for the numerical simulation is Δt=0.1s. The agreement is fair considering the large stretching ratio $\frac{R_f(200)}{R_o} \cong 35$ and the boundary element model based on constant element.

Figure 3. Comparison between computed and analytical flow fronts at t=200s for the isotropic medium



If we consider now that the fibrous medium is anisotropic, the flow front should be of elliptic shape. The previous relationship has to be modified in order to compute the principal axis of the ellipse R_{fex} and R_{fey} to yield:

$$\left[\frac{R_{fex}}{R_{oe}}\right]^2 \cdot \left[2\ln\left(\frac{R_{fex}}{R_{oe}}\right) - 1\right] + 1 = \frac{4K_e p_o t}{\mu R_{oe}^2} \qquad (15)$$

and

$$R_{fey} = R_{fex}\left(\frac{K_{yy}}{K_{xx}}\right)^{\frac{1}{2}} \qquad (16)$$

where

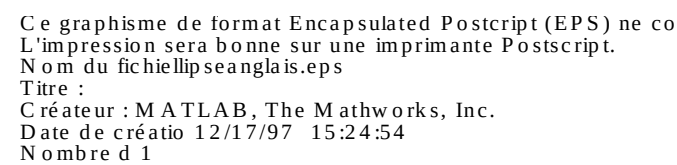
$$\begin{cases} K_e = \sqrt{K_{xx}.K_{yy}} \\ R_{oe} = R_o.\left(\frac{K_{xx}}{K_{yy}}\right)^{\frac{1}{4}} \end{cases} \qquad (17)$$

The values of the processing and rheological parameters are the same as in tab. 1 except for the permeabilities for which the values are:

$$K_{xx} = 6.5\,10^{-11} \text{ m and } K_{yy} = 1.5\,10^{-11} \text{ m}$$

Again the agreement between the flow front computed using the boundary element method and the analytical one is fair as shown in fig. 4 except near the solid wall (i.e. where the gradient pressure q is prescribed). This is due to the relocation of the nodes close to the mould which have moved out of the solid wall. Nevertheless, these comparisons with analytical solutions have permitted to test the capability of the boundary element model.

Figure 4. Comparison between computed and analytical flow fronts at t=200s for the anisotropic medium



4.2 *Comparison with Finite Element Simulation*

We proceed now to the simulation of a frontal injection of resin into a fibrous preform located between two parallel solid walls using finite element and boundary element methods. The resin is assumed to be an incompressible and Newtonian fluid which flows through an anisotropic one layer medium.

The finite element simulation has been carried out using a numerical model first developed by Hoareau et al. (1993) and derived from FORGE2® software. The numerical model is based upon an updated lagrangian finite element method (FEM).The domain is meshed using quadratic triangles (six nodes) as shown in fig. 5. The values of the processing and rheological parameters are given in tab. 2.

Table 2. Processing and rheological parameters

K_{xx} (m^2)	K_{yy}(m^2)	K_{xy} (m^2)	μ(Pa.s)	p_o(Pa)
$4\ 10^{-11}$	$4\ 10^{-11}$	$2.5\ 10^{-11}$	0.1	10^5

Figure 5. Finite element mesh at t=200s

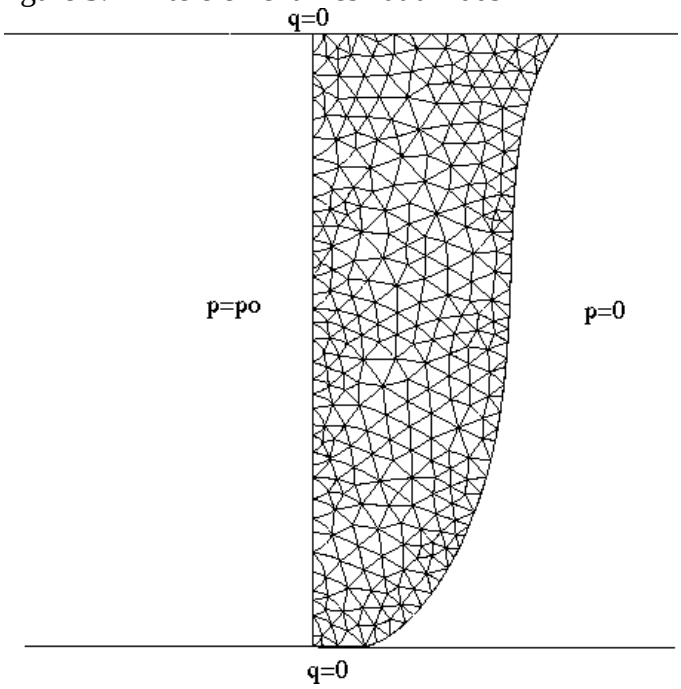


Fig. 6 presents the comparison between boundary element and finite element flow fronts at t=200s. In the centre of the flow front, the agreement is fair. Due to the different relocation schemes used in FEM and BEM simulations, the discrepancy between the two flow fronts increases for the nodes close to the mould. In addition, the remeshing procedure using the Delaunay's algorithm in FORGE2® may have a significant influence on the finite element solution.

Figure 6. Comparison between boundary element and finite element flow fronts at t=200s

Ce graphisme de format Encapsulated Postcript (EPS) ne co
L'impression sera bonne sur une imprimante Postscript.
Nom du fichicompBEEFang.eps
Titre :
Créateur : MATLAB, The Mathworks, Inc.
Date de créatio 12/17/97 14:20:18
Nombre d 1

5 CONCLUSION

The flow of a Newtonian resin through anisotropic and homogeneous media has been simulated using a boundary element method. The algorithm which is used to capture the transient front flow is rather simple and has permitted to accurately predict the front shape at a low cost.

The relocation scheme used for the nodes close to the mould which have moved out of the solid wall has to be improved in order to better take into account the contact between the resin and the mould.

In addition, the BEM simulations should be compared to experiments.

Further developments will include the simulation of the flow of resin through a typical RTM reinforcement in 3D. The boundary element procedure can be then applied to each subregion in turn (which is assumed to be piecewise homogenous) as if they are independent of each other.

REFERENCES

Brebbia C.A., Dominguez J. - Boundary Element : An introduction course. *Mc Graw Hill Book Company, Second edition, 1992*

Hadavinia H., Advani S. G. and Fenner R. T. - The evolution of radial fingering in a hele-Shaw cell using C^1 continuous Overhauser boundary element method. *Engineering Analysis Boundary Elements, Vol. 16,183-195, 1995*

Hoareau C., Trochu F., Gauvin R. and Vincent M. - *Proc. of ICCM-9, Madrid, Vol. 3, pp. 481-488, July 1993*

James Lee L., Young W. B. and Lin R. J. - Mold filling and cure modelling of RTM and SRIM processes. *Composites Structures Vol. 27, 109-120, 1994*

Hammami A., F. Trochu, R. Gauvin & Wirth S.- Directional Permeability Measurement of Deformed Reinforcement. *J. of Reinforced Plastics and Composites, Vol. 15, June 1996*

Li S. and Gauvin R. - Numerical Analysis of the Resin Flow in Resin Transfer Molding. *Journal of Reinforced Plastics and Composites, Vol. 10, 314-327, May 1991*

Ozisik N. - Heat Conduction. *Ed. John Wiley & Sons, 1980*

Trochu F., Gauvin R., Dong Ming Gao and Boudreault J.-F. - RTMFLOT, An Integrated Software Environment for the Computer Simulation of the Resin Transfer Molding Process. *Journal of Reinforced Plastics and Composites, Vol. 13, 262-270, March 1994*

Um M.-K. and Lee W. I. - A study on the Mold Filling Process In the Resin Transfer Molding. *Polymer Engineering and Science, Vol. 31, No. 11, 765-771, Mid-June 1991*