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# Newtonian Fluid Computations in Lagrangian Variables with a Remeshing Technique. Application to the Filling Stage of the Diecasting Process

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## ABSTRACT

The Navier Stokes incompressible model is used to describe 2-dimensional metal casting flow. In a die-filling simulation the free moving boundary must be known precisely. For this reason the Lagrangian formulation is chosen instead of the Eulerian one. The computation domain and the velocity field are approached by means of the finite element method. A non-structured mesh is built by using quadratic six-noded triangular elements. In time the mesh degenerates. A remeshing technique, based on the Delaunay algorithm, provides a new computation mesh. As for the application, we present the computation of a die-filling case with low Reynolds number.

## 1. Introduction.

From the practitioner's point of view, the numerical simulation of the filling stage in the casting process is of major interest for many reasons :

- the prediction of defects, such as the incomplete filling of some regions or the merging of metal free surfaces at too low a temperature,
- the possible optimisation of filling conditions, especially the gate systems,
- the determination of the temperature field at the end of the filling, in order to define consistent initial conditions for a thermomechanical model of the solidification stage,
- and more generally, to be able to study the feasibility of new operating conditions or new processes.

Such industrial ambitious objectives have induced the development of various numerical models using different approaches. The finite difference and finite volume method have been first worked out and remain the more frequently used, with many variations : Marker And Cell (MAC) technique [1], Sola-Vof method... [2-4]. Their main drawbacks are :

- a spatial description which is mandatory structured and regular,
- the approximations by finite differences of the derivatives included in the Navier Stokes equations,
- a coarse description of the external surfaces of the computation domains : mould/fluid interface, free surface of the fluid.

The finite element method presents the major advantage of the more precise integral formulation, associated to a more general spatial discretisation. The method can be adapted to either Eulerian (the mesh is fixed in space) [5-6] or Lagrangian formulation (the mesh is then convected with the material flow). The latter is original and has been developed by very few authors for the simulation of fluid flow [7-9]. The better representation of free surfaces ("front tracking" procedures can be avoided), and also the easy introduction of surface tensions make it especially attractive.

However, in the case of very large deformations, such as those encountered in die filling, this method can give satisfying results only when it is associated to a remeshing technique of the fluid domain, in order to regenerate periodically elements having a correct shape ratio. Ramaswamy gives simple examples of a remeshing technique with application to wave propagation [9] with moderate distortions. In this paper, we are mainly concerned with a general method for automatic remeshing to cope with the fluid flow simulation in the casting process. The flow is considered isothermal. In the first part, the Lagrangian formulation of Navier Stokes equations is briefly outlined. Then, the capability of the remeshing technique is demonstrated with an example.

## 2. Theoretical formulation.

### 2.1 Navier Stokes equation in Lagrangian variables.

The motion of a liquid melt is well governed by a Newtonian incompressible fluid flow model. To respect precisely the motion of the fluid free surfaces the Lagrangian variables are used. This consists of taking the time  $t$  and the particle coordinates  $x_0$  at an initial time, as variables. Let  $x(x_0, t)$  be the coordinates at time  $t$  of the particle initially at  $x_0$ . The velocity and the acceleration of the particle may be expressed with the following partial derivatives

$$V = \frac{\partial x}{\partial t} \quad (1)$$

$$\dot{V} = \frac{\partial V}{\partial t} \quad (2)$$

The Navier Stokes equations postulate the conservation of linear momentum and mass in any point of the body at any time. In Lagrangian variables they take the following form

$$\rho \frac{\partial V}{\partial t} = \nabla \cdot \sigma + \rho g \quad (3)$$

$$\nabla \cdot V = 0 \quad (4)$$

where the stress tensor  $\sigma$  satisfy the linear constitutive law

$$\sigma = -pI + 2\mu D \quad (5)$$

in which  $I$  is the unit tensor and  $D$  is the tensor of rates of deformation

$$D(V) = \frac{1}{2} (\nabla V + \nabla V^t) \quad (6)$$

the superscript  $t$  denoting the transpose of a tensor. In the above equations  $\rho$  is the density,  $\mu$  the dynamic viscosity,  $g$  is the gravity and  $p$  is the hydrostatic pressure.

In the momentum equation (3) we can note the simplification of the inertial term characteristic of the Lagrangian formulation. While in the Eulerian one a non linear convective term must be added.

The mechanical problem needs boundary conditions. They take the following form [10]

$$\sigma \cdot n = \tau \frac{dt}{ds} \quad \text{on } \Gamma_f \quad (7)$$

$$V = 0 \quad \text{on } \Gamma_b \quad (8)$$

where  $\Gamma_f$  is the free surface,  $\Gamma_b$  the fluid-mould interface,  $n$  the unit normal at the free surface,  $s(x_0, t)$  the curvilinear coordinates on the free boundary at time  $t$ ,  $t$  is the unit tangent at the free surface, and  $\tau$  is the surface tension stress.

The momentum equation is a first order partial derivative equation which needs an initial value for the velocity. Commonly we will take

$$V = 0 \quad \text{in } \Omega_0 \quad (9)$$

where  $\Omega_0$  is the open which represent the initial position of the fluid.

### 2.2 Weak formulation.

We adopt a perturbation (penalization) method, proposed by Temam[11], to treat the continuum equation (4)

$$\varepsilon p + \nabla \cdot V = 0 \quad \varepsilon \rightarrow 0^+ \quad (10)$$

It is then possible to eliminate the pressure  $p$  from the momentum equation (3). The elimination of  $p$  is made to simplify the formulation and save in the sequel computational cost.

A weak formulation of the momentum equation may be obtain by multiplying it by any virtual velocity field  $V^*$ , which satisfy the boundary condition (8), and by summing it over  $\Omega_0$ . By using (10) we obtain



$$\forall V^*(+B.C.) \quad \rho \int_{\Omega_0} \frac{\partial V}{\partial t} \cdot V^* d\Omega_0 = \frac{1}{\varepsilon} \int_{\Omega_0} \nabla(\nabla \cdot V) \cdot V^* d\Omega_0 \\ + 2\mu \int_{\Omega_0} (\nabla \cdot D(V)) \cdot V^* d\Omega_0 + \rho \int_{\Omega_0} g \cdot V^* d\Omega_0 \quad (11)$$

By means of integrations by part and by using equations (10) (5) and (7) we obtain while  $V^*$  is null on  $\Gamma_b$

$$\forall V^*(+B.C.) \quad \rho \int_{\Omega_0} \frac{\partial V}{\partial t} \cdot V^* d\Omega_0 = -\frac{1}{\varepsilon} \int_{\Omega_0} \nabla \cdot V \nabla \cdot V^* d\Omega_0 \\ - 2\mu \int_{\Omega_0} \text{tr}(D(V)D(V^*)) d\Omega_0 + \rho \int_{\Omega_0} g \cdot V^* d\Omega_0 - \tau \int_{\Gamma_f} \dot{t} \cdot \frac{dV^*}{ds} d\Gamma_f \quad (12)$$

where  $\text{tr}$  is the trace operator.

Neglecting inertial and surface tension effects we will obtain an elasticity problem to be solved, where the constants  $1/\varepsilon$  and  $2\mu$  are the Lamé coefficients.

### 2.3 Time integration with a Lagrangian updating.

Now we look at the integration scheme. The time length is divided in  $N$  time steps  $\Delta t = T/N$ . Let us denote  $t_i = i\Delta t$  an intermediate time.

The first step consists to find at time  $t_1$  the location  $x(x_0, t_1)$ , the

velocity  $V(x_0, t_1)$  and the acceleration  $\frac{\partial V}{\partial t}(x_0, t_1)$  of any fluid particle initially

located in  $x_0$  with the velocity  $V(x_0, 0)$  and the acceleration  $\frac{\partial V}{\partial t}(x_0, 0)$ . We

have three unknowns. We search for them as the solution of equation (12) which satisfy also the two following integration rules:

$$V(x_0, t_1) = V(x_0, 0) + \Delta t \left\{ (1-\theta) \frac{\partial V}{\partial t}(x_0, 0) + \theta \frac{\partial V}{\partial t}(x_0, t_1) \right\} \quad (13)$$

$$x(x_0, t_1) = x(x_0, 0) + \Delta t V(x_0, t_1) + \frac{\Delta t^2}{2} \left\{ (1-\theta) \frac{\partial V}{\partial t}(x_0, 0) + \theta \frac{\partial V}{\partial t}(x_0, t_1) \right\} \quad (14)$$

where  $\theta$  is the explicit/implicit parameter, which belongs to  $[0, 1]$ . This scheme is Newmark [12] type. Note that the momentum equation (12) is not set on  $\Omega_1$  that means an explicit treatment of the momentum conservation. Equation (13) is implicit because it links the unknown velocity with the unknown acceleration. The updating of the particle coordinates by means of (14) will be made after the simultaneous solution of equations (12)-(13).

When we have found  $x(x_0, t_1)$ , the velocity  $V(x_0, t_1)$  and the acceleration

$\frac{\partial V}{\partial t}(x_0, t_1)$ , we start to compute the second time step by a Lagrangian updating.

At time  $t_1$  a new Lagrangian coordinates  $\tilde{x}_0$  is defined. Location, velocity and acceleration of particles are reinitialized :

$$\tilde{x}_0 = x(x_0, t_1) \quad (15)$$

$$V(\tilde{x}_0, 0) = V(x_0, t_1) \quad (16)$$

$$\frac{\partial V}{\partial t}(\tilde{x}_0, 0) = \frac{\partial V}{\partial t}(x_0, t_1) \quad (17)$$

and the new unknowns  $x(x_0, t_2)$ ,  $V(x_0, t_2)$ ,  $\frac{\partial V}{\partial t}(x_0, t_2)$  are noted:

$$x(x_0, t_2) = x(\tilde{x}_0, t_1) \quad (18)$$

$$V(x_0, t_2) = V(\tilde{x}_0, t_1) \quad (19)$$

$$\frac{\partial V}{\partial t}(x_0, t_2) = \frac{\partial V}{\partial t}(\tilde{x}_0, t_1) \quad (20)$$

They are the solution of relations (13)-(14) in which  $x_0$  is replaced by  $\tilde{x}_0$  and of relation (12) set on  $\Omega_{\Delta t}$ .

We repeat the algorithm while  $i\Delta t < T$ . This updated Lagrangian formulation is a sequence of computations on successive current configurations.

### 2.4 Space discretization by finite elements.

Now we look at space and velocity field approximations. The open  $\Omega_0$  is approached by a set of  $n_e$  elementary cells  $\Omega_e$ . Each point  $x_0$  of  $\Omega_e$  is located by means of the nodes location  $\{x_{0,e}\}$  and the matrix  $[N]$  of the shape functions.

$$x_0 = [N] \{x_{0,e}\} \quad (21)$$



Isoparametric element will be used in the sequel. Let us denote by  $\{v_{1,e}\}$  the unknown velocities of the cell nodes, by  $\{v_{0,e}\}$  and  $\{\dot{v}_{0,e}\}$  respectively their velocity and acceleration at the previous time step. In each cell starting from these vectors we can obtain the associated field by means of the matrix  $[N]$ .

Let  $\langle \nabla \cdot N \rangle$  be the vector which links the divergence of a vector field in any point of  $\Omega_e$  in term of nodal values

$$\nabla \cdot V = \langle \nabla \cdot N \rangle \{v_{1,e}\} \quad (22)$$

Let  $[DN]$  be the matrix which links in vectorial form the strain rate tensor in term of nodal velocities

$$\{D(V)\} = [DN] \{v_{1,e}\} \quad (23)$$

The free boundary  $\Gamma_f$  of  $\Omega_0$  is approached by  $n_b$  element sides  $\Gamma_b$ . A boundary particle is located by means of the matrix  $[\tilde{N}]$

$$x_0(s) = [\tilde{N}(s)] \{x_{0,b}\} \quad (24)$$

where  $s$  is the curvilinear coordinate defined on  $\Gamma_b$  and  $\{x_{0,b}\}$  the nodal points of  $\Gamma_b$ . The unit tangent at  $\Gamma_b$  in  $s$  is denoted by  $\vec{t}_b(s)$ .

### 2.5 Momentum equation in terms of a linear system.

We use for the virtual velocity  $V^*$  the same approximation as for the velocity field (Galerkin method). Using the above approximations (21)-(24), and the numerical integration scheme (13), the momentum equation (12) becomes the linear system

$$C V = F \quad (25)$$

where the vector  $V$  is the assemblage of the free nodal velocity components

$$V = A_{e=1}^{n_e} \{v_{1,e}\} \quad (26)$$

$C$  is the matrix defined by

$$C = A_{e=1}^{n_e} (C_\rho^e + C_\varepsilon^e + C_\mu^e) \quad (27)$$

$F$  is the vector of the applied forces

$$F = A_{e=1}^{n_e} (F_\rho^e + F_g^e) + A_{b=1}^{n_b} F_\tau^b \quad (28)$$

The elementary mass matrix  $C_\rho^e$  is defined by

$$C_\rho^e = \frac{\rho}{\theta \Delta t} \int_{\Omega_e} [N]^t [N] d\Omega_e \quad (29)$$

$C_\varepsilon^e$  the elementary matrix which comes from the incompressibility is set by

$$C_\varepsilon^e = \frac{1}{\varepsilon} \int_{\Omega_e} \langle \nabla \cdot N \rangle^t \langle \nabla \cdot N \rangle d\Omega_e \quad (30)$$

The viscous elementary matrix takes the following form

$$C_\mu^e = 2\mu \int_{\Omega_e} [DN]^t [DN] d\Omega_e \quad (31)$$

The inertial forces are given by

$$\langle F_\rho^e \rangle = \frac{\rho}{\theta} \left\{ \frac{1}{\Delta t} \langle v_{0,e} \rangle + (1-\theta) \langle \dot{v}_{0,e} \rangle \right\} \int_{\Omega_e} [N]^t [N] d\Omega_e \quad (32)$$

The gravity forces is done by

$$\langle F_g^e \rangle = \rho \langle g \rangle \int_{\Omega_e} [N] d\Omega_e \quad (33)$$

The forces given by surface tension are

$$\langle F_\tau^b \rangle = \tau \int_{\Gamma_b} \langle \vec{t}_b(s) \rangle \left[ \frac{d\tilde{N}}{ds}(s) \right] ds \quad (34)$$

The momentum equation is linear while an implicit treatment of the inertial term in term of velocity. Hughes et al. [13] use a lumped mass matrix (diagonal) while here we use the full mass matrix (Galerkin one).

In the sequel we use the six node triangular quadratic element. We use the four Gauss points integration rule to evaluate the different elementary matrix and vectors defined by equations (29)-(34). The matrix of incompressibility is exactly integrated. If a reduced integration is used, with the one node formula, some numerical problems occur, like zig-zag pattern velocity field.



### 3. Automatic remeshing.

#### 3.1 Metal forming calculation and remeshing.

A lot of work has been presented on the mesh generation problem and a number of algorithms of varying degrees of automation have been proposed in two dimensions for a variety of applications. For metal forming calculation, the remeshing step was concerned primarily with remedying to mesh distortions and consequently was only understood as reshaping an existing finite element mesh. A radical alternative to the structured mesh updating is the use of triangles. This element gives unstructured meshes more flexible in treating complex geometries while retaining an appropriate mesh point distribution. Moreover several triangle mesh generation schemes are now available and can be used for general purpose. More particularly we are concerned with large deformation occurring in metal forming or unsteady flow and our problem is to create several meshes for only one calculation.

The mesh generator presented here, was developed to solve the remeshing stage, in Lagrangian finite element analysis (updated Lagrangian description or incremental scheme) and more particularly for the numerical simulation of the forging process [14]. The drawback of classical reshaping method is finally the lack of flexibility of the structured grid because of the constraint of the connectivity. Another difficulty well solved with the help of unstructured grid is the dynamic mesh adaptation to the contact problem.

Obviously, the calculation presented here needs a fully automatic remeshing procedure and shows the effectiveness of our mesh generation scheme which seems very general and well adapted for any formulation using a convected mesh (Lagrangian element) and a time integration scheme.

#### 3.2 The remeshing scheme.

The remeshing main procedure verifies at each step of calculation the distortion of elements and the accuracy of the mesh boundary (penetration of the lines into the die or other obstacles, free surfaces curvature ...) and decides itself to remesh or not according to prescribed tolerances.

In case of remeshing, a new boundary is deduced from the old mesh fitting perfectly the external geometric constraint (contact adaptivity). A generalized algorithm of the Delaunay triangulation ([15],[16],[17]) is used to create a mesh fitting exactly the prescribed boundary without internal node. At this stage, points can be created into elements according to the method proposed in [16] and [17]. The mesh is refined by adding internal nodes in order to obtain elements having the best possible shape. It is easy to show that this condition is sufficient to finish the nodes generation algorithm.

Two procedures are used to perfect the mesh. The first one is a topological operation : exchanging the diagonal between two adjacent triangles [18], and the second one is a mesh regularisation using a Laplace algorithm (internal nodes are moved to the barycenter of neighbouring nodes). A combination of these two procedures in a global and iterative algorithm gives a mesh very close to the optimal one. At last, the time depending parameters are interpolated from the old grid to the new one and the calculation may continue.

Each stage of that remeshing scheme is fully automatic and does not need user intervention (although it is possible to change the tolerances and the mesh size parameters during computation). Moreover the history of remeshing is automatically stored and usable by the post processing system as a time depending solution.

### 4. Die filling example.

We look at a filling case by injection. Figure 1 presents the dimension of the mould. The fluid is initially located in the channel of injection with zero initial velocity and acceleration.

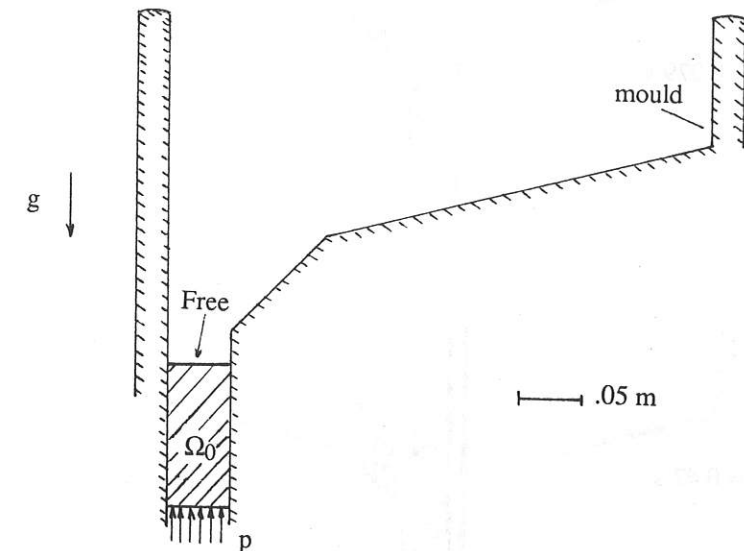


Figure 1. Initial configuration and mould dimensions.

The upper bound of the fluid is a free surface. We suppose a perfect adhesion between the fluid and the die. The lower bound is submitted to a constant pressure  $p=85000 \text{ N.m}^{-2}$ . To simulate material injection the location of the nodes on the pressurized bound are not updated. The fluid is a steel in semisolid state. Its viscosity is  $100 \text{ N.m}^{-2}\text{s}$ , the density is  $7000 \text{ Kg.m}^{-3}$ , the gravity is  $g=10 \text{ m.s}^{-2}$ , the surface tension is equal to  $0.9 \text{ N.m}^{-2}$ .

The computation is stopped after 1860 time steps of  $2 \cdot 10^{-3} \text{ s}$ . That requires 132 remeshings of the fluid. The CPU time is around 2 hours on DN10000 Apollo computer.

Figures 2 - 3 show the time sequence of liquid filling the mould and meshes. After 2.5 s. the fluid reaches the top of the mould. Figures 4 - 5 show velocity fields. The velocity is maximum in the injection channel. Its main value is  $1.5 \text{ m.s}^{-1}$ , giving a Reynolds number of 5.25 in the channel. We can note on Figures 2 - 3 that the meshes degenerate essentially where injection is made. An arbitrary Lagrangian-Eulerian formulation (ALE) may be interesting in the channel to reduce the mesh distortions. We can note the smoothness of the free flow surface which is a consequence of surface tension effects.



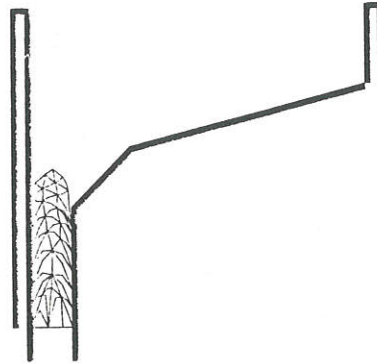
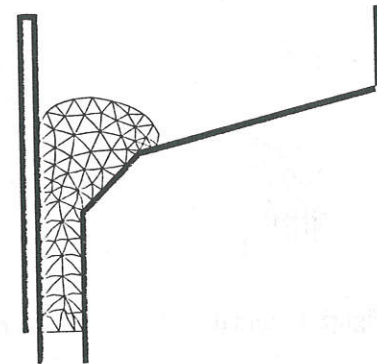
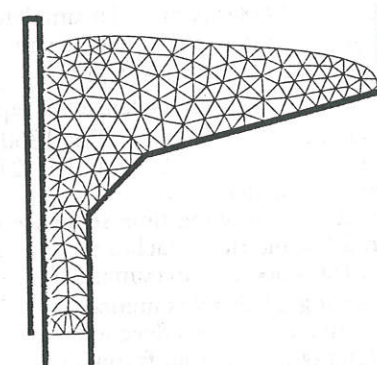
$t = 0.079 \text{ s}$  $t = 0.47 \text{ s}$  $t = 1.76 \text{ s}$ 

Figure 2. Computed mesh (the plot are time .079 .47 1.76 s.)

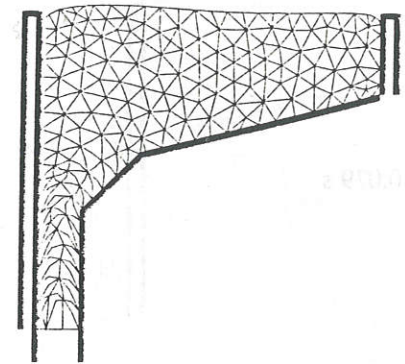
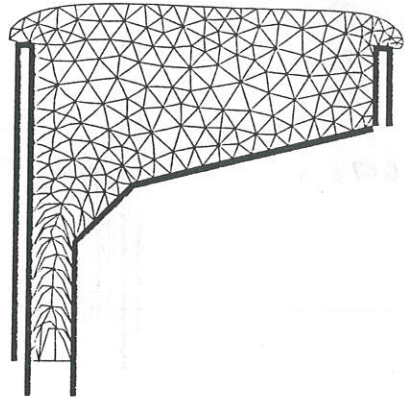
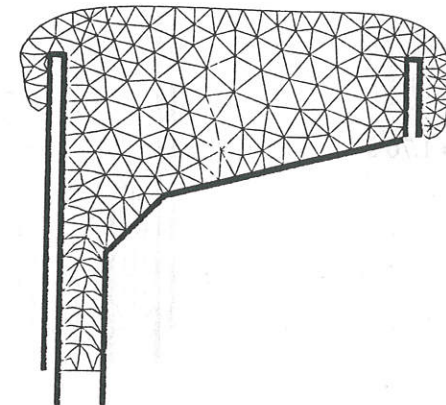
 $t = 2.52 \text{ s}$  $t = 3.32 \text{ s}$  $t = 3.87 \text{ s}$ 

Figure 3. Computed mesh (the plot are time 2.52 3.32 3.87 s.)

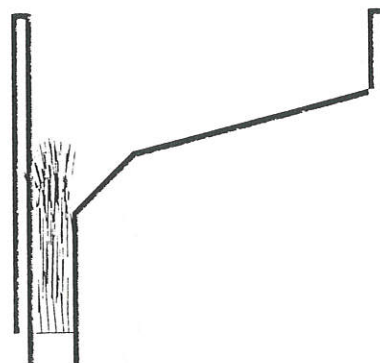
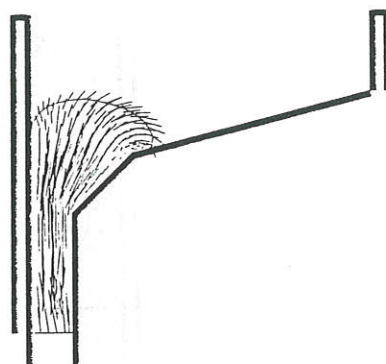
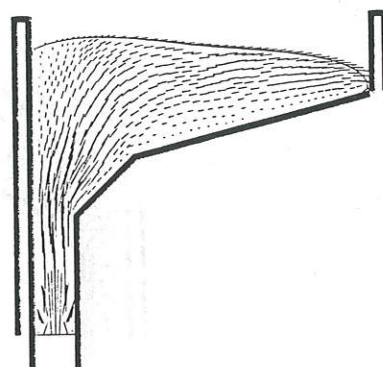
$t = 0.079 \text{ s}$  $t = 0.47 \text{ s}$  $t = 1.76 \text{ s}$ 

Figure 4. Computed velocity vectors (the plot are time .079 .47 1.76 s.)

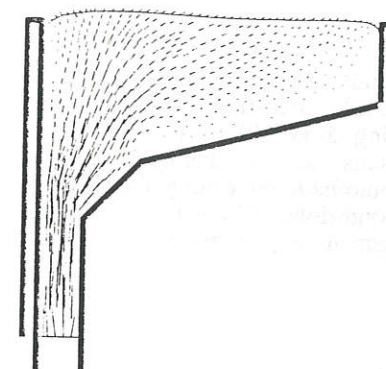
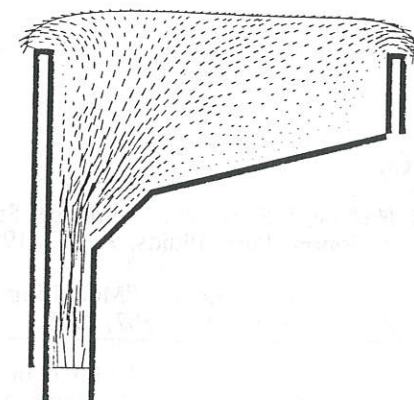
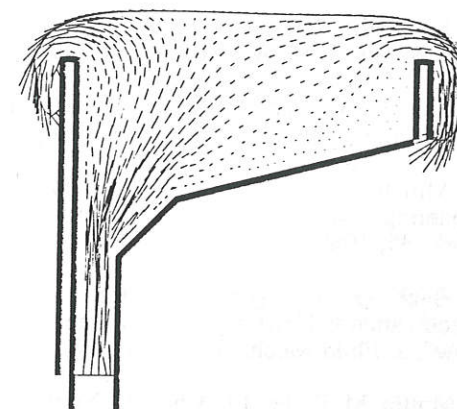
 $t = 2.52 \text{ s}$  $t = 3.32 \text{ s}$  $t = 3.87 \text{ s}$ 

Figure 5. Computed velocity vectors (the plot are time 2.52 3.32 3.87 s.)



## 5. Conclusion.

Our updated Lagrangian formulation of the Navier-Stokes equations is attractive. It allows to treat practical cases with low computation cost, the viscosity being around 100 Pa.s. Numerical comparisons of our solutions with analytical results and with other numerical schemes are in current development.

The automatic remeshing algorithm permit us to treat mould geometries and filling conditions of industrial complexity. With the automatic remeshing procedure there are no theoretical limitations on the flow geometry and its time-length.

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