

## How to Dimension 5G Network When Users Are Distributed on Roads Modeled by Poisson Line Process

Jalal Rachad, Ridha Nasri, Laurent Decreusefond

### ▶ To cite this version:

Jalal Rachad, Ridha Nasri, Laurent Decreusefond. How to Dimension 5G Network When Users Are Distributed on Roads Modeled by Poisson Line Process. VTC2019 (fall), IEEE, Sep 2019, Hawai, United States. hal-01793681

## HAL Id: hal-01793681 https://hal.science/hal-01793681v1

Submitted on 16 May 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# How to Dimension 5G Network When Users Are Distributed on Roads Modeled by Poisson Line Process

Jalal Rachad \*+, Ridha Nasri \*, Laurent Decreusefond + \*Orange Labs: Direction of Green transformation, Data knowledge, traffic and resources Modelling, 40-48 avenue de la Republique 92320 Chatillon, France +LTCI, Telecom ParisTech, Universite Paris-Saclay, 23 avenue d'Italie 75013, Paris, France Email:{\*jalal.rachad, \*ridha.nasri}@orange.com, +Laurent.Decreusefond@mines-telecom.fr

Abstract—The fifth generation (5G) New Radio (NR) interface inherits many concepts and techniques from 4G systems such as the Orthogonal Frequency Division Multiplex (OFDM) based waveform and multiple access. Dimensioning 5G NR interface will likely follow the same principles as in 4G networks. It aims at finding the number of radio resources required to carry a forecast data traffic at a target users Quality of Services (QoS). The present paper attempts to provide a new approach of dimensioning 5G NR radio resource (number of Physical Resource Blocks) considering its congestion probability, qualified as a relevant metric for QoS evaluation. Moreover, 5G users are assumed to be distributed in roads modeled by Poisson Line Process (PLP) instead of the widely-used 2D-Poisson Point Process. We derive the analytical expression of the congestion probability for analyzing its behavior as a function of network parameters. Then, we set its value, often targeted by the operator, in order to find the relation between the necessary resources and the forecast data traffic expressed in terms of cell throughput. Different numerical results are presented to justify this dimensioning approach.

*Index Terms*—5G network, Dimensioning, radio resources, Congestion probability, Poisson Line Process.

#### I. INTRODUCTION

5G New Radio (NR), compared to the previous generations, is expected to support new features in order to satisfy diverse use cases with different user requirements. NR physical design will be built upon the current 4G/4G+ key features. It aims to support a wide range of frequency bandwidths, high data rate applications (e.g., 3GPP enhanced Mobile Broad Band-eMBBservices) as well as vehicular-type communications with low latency constraints (e.g., 3GPP Ultra-Reliable Low-Latency Communication -URLLC- services). 5G-NR system comes with the scalable OFDM (Orthogonal Frequency Division Multiplex) technology having different subcarriers' spacing  $(\Delta f = 2^{\nu} 15 \text{ kHz}, \text{ where } \nu = 0 \text{ to } 4)$  and diverse spectrum bands [1]. A set of OFDM subcarriers constitutes the basic unit of radio resources that a cell can allocate to a mobile user. This set of subcarriers is called Physical Resource Block (PRB) in the 3GPP terminology. Furthermore, the allocation of PRBs to users is performed at each Time Transmit Interval (TTI) according to a predefined scheduling algorithm. The choose of this latter is mainly related to the fairness level

between users, i.e., the way that resources are allocated to users according to their channel qualities and their priorities, defined by the operator [2].

The available scientific literature of OFDMA based systems' dimensioning is quite rich but it is always considered as a hard task because of the presence of elastic data services; see for instance [3]–[6]. It was provided in [5] an analytical model for dimensioning OFDMA based networks with proportional fairness in resource allocation between users belonging to a specific class of services, i.e., required the same transmission rate. For a Poisson distribution of mobile users, [5] showed that the required number of resources in a typical cell follows a compound Poisson distribution. In addition, an upper bound of the outage probability was given. In [6], an adaptive resource allocation for multiuser OFDM system, with a set of proportional fairness constraints guaranteeing the required data rate, has been discussed. Despite the rich performance analysis made in [5] and [6], there is no explicit hint of how to dimension the radio resources given a predefined QoS.

Besides, the network geometry and the spatial distribution of users are important factors for system performance analysis and dimensioning exercise. Different models for network geometry and user distributions can be found in [7]–[10]. In particular, authors in [9] and [10] considered vehicular-type communication systems where the transmitting and receiving nodes are distributed in roads, modeled by Poisson Line Process (PLP). It seems that PLP is a relevant model for roads in urban environment and merits investigations when looking for performance analysis and dimensioning exercise of wireless cellular communications.

Always in the same context, we propose in this paper a dimensioning method for OFDM based 5G system with a proportional fair resources' allocation policy. Instead of considering a PPP spatial random distribution of users in the cell, we characterize at first the geometry of cities by a PLP model and then we distribute mobile users in the random roads. We derive the explicit formula of the congestion probability as a function of different system parameters. This metric is defined as the risk that requested resources exceed the available ones. It is often considered primordial for operators when it comes to resources dimensioning since it is related to the guaranteed quality of service. Then by setting the target congestion probability, we show how to dimension the number of PRBs given a forecast cell throughput.

The rest of this paper is organized as follows: In Section II, system models, including a short introduction of Poisson Line Process, are provided. Section III characterizes the proposed dimensioning model and provides an explicit expression of the congestion probability and an implicit relation between the number of required resources and the cell throughput. Numerical results are provided in Section IV. Section V concludes the paper.

#### II. SYSTEM MODEL AND NOTATIONS

When we want to model cellular networks, we often think about the network geometry, the shape of the cell, the association between cells and users and of course their spatial distribution. This latter is related to the geometry of the city where the studied cell area exists. The Geometry of the city, in turn, reveals linked to the spatial distribution of roads and buildings. Indoor users, which are distributed in buildings, are often modeled by a Poisson Point Process in  $\mathbb{R}^2$ , denoted here by 2D-PPP. However, outdoor users (e.g., pedestrians or vehicular) are always distributed in roads. To model the spatial distribution of roads, many models have been proposed in literature such as Manhattan model that uses a grid of horizontal and vertical streets, Poisson Voronoi and Poisson Line Process characterized by spatial random tessellations. In this work, roads are modeled according to a Poisson Line Process (PLP) for reasons of tractability.

#### A. Poisson Line Process

PLP is mathematically derived from a 2D-PPP. A PPP in  $\mathbb{R}^2$  with intensity  $\zeta$  is a point process that satisfies: *i*) the number of points inside every bounded closed set  $B \in \mathbb{R}^2$  follows a Poisson distribution with mean  $\zeta |B|$ , where |B| is the Lebesgue measure on  $\mathbb{R}^2$ ; *ii*) the number of points inside any disjoint sets of  $\mathbb{R}^2$  are independent [11].

Instead of points, the PLP is a random process of lines distributed in the plane  $\mathbb{R}^2$ . Each line in  $\mathbb{R}^2$  is parametrized in terms of polar coordinates  $(r,\theta)$  obtained from the orthogonal projection of the origin on that line, with  $r \in \mathbb{R}^+$  and  $\theta \in (-\pi,\pi]$ . Now we can consider an application T that maps each line to a unique couple  $(r,\theta)$ , generated by a PPP in the half-cylinder  $\mathbb{R}^+ \times (-\pi,\pi]$ . The distribution of lines in  $\mathbb{R}^2$  is the same as points' distribution in this half-cylinder; see [9] and [10] for more details.

In the sequel, we assume that roads are modeled by a PLP  $\phi$  with roads' intensity denoted by  $\lambda$ . The number of roads that lie inside a disk s of radius R is a Poisson random variable, denoted by Y. It corresponds to the number of points of the

equivalent 2D-PPP in the half-cylinder  $[0, R] \times (-\pi, \pi]$  having an area of  $2\pi R$ . Hence, the expected number of roads that lie inside s is  $\mathbb{E}(Y) = 2\pi\lambda R$ . Moreover, users are assumed to be distributed on each road according to independents 1D-PPP with the same intensity  $\delta$ . The mean number of users on a given road j is  $\delta L_j$ , with  $L_j$  is the length of road j. Besides, the number of roads that lie between two disks of radius  $R_1$ and  $R_2$  respectively, with  $R_1 \leq R_2$ , is  $2\pi\lambda(R_2 - R1)$ . Also, the number of distributed users in a road, parametrized by  $(r,\theta)$ and delimited by the two disks, is  $2\delta(\sqrt{R_2^2 - r^2} - \sqrt{R_1^2 - r^2})$ . Furthermore, the average number of users in the disk of radius R is calculated using the equivalent homogeneous 2D-PPP with intensity  $\lambda\delta$  in the disk area. Let u denotes the average number of users inside the disk s, it is written by

$$u = \lambda \delta \pi R^2. \tag{1}$$

For illustration, Fig. 1 shows a realization of the described Process  $\phi$ .



Fig. 1. A realization of the Poisson Line Process  $\phi$ .

#### B. Network model

We consider a circular cell s of radius R with a Base Station (BS), denoted also s and positioned at its center, transmitting with a power level P. Received power by a user located at distance x from s is  $Px^{-2b}/a$ , where 2b is the path loss exponent and a is the propagation constant. We assume that BS s allocates PRBs to its users at every TTI (e.g., 1 ms). Each PRB has a bandwidth denoted by W (e.g., W = 180kHz for scalable OFDM with subcarriers spacing of 15kHz).

Active users in the cell compete to have access to the available dimensioned PRBs. Their number is denoted by M. The BS allocates a given number n of PRBs to a given user depending on: i) the class of services he belongs to (i.e.,the transmission rate he requires) and ii) his position in the cell (i.e., the perceived radio conditions). Without loss of generality, we assume that there is just one class of service with a required transmission rate denoted by  $C^*$ .

A user located at distance x from s decodes the signal only if the metric "Signal to Interference plus Noise Ratio (SINR)"  $\Theta(x) = \frac{Px^{-2b}/a}{I+\sigma^2}$  is above a threshold  $\Theta^*$ , where I is the received co-channel interferences and  $\sigma^2$  is the thermal noise power. For performance analysis purpose, SINR  $\Theta(x)$  is often mapped to the user throughput by a link level curve. For simplicity of calculation, we use hereafter the upper bound of the well known Shannon's formula for MIMO system  $TX \times RX$ , with TX and RX are respectively the number of transmit and receive antennas. Thence, the throughput of a user located at distance x from s is

$$C(x) = \vartheta W log_2 \left(1 + \Theta(x)\right), \qquad (2)$$

with  $\vartheta = min(TX, RX)$ .

And then, the number of PRBs required by a user located at distance x from s is

$$n(x) = \left\lceil \frac{C^*}{C(x)} \right\rceil \le N,\tag{3}$$

where  $N = min(N_{max}, \lceil C^*/(\vartheta W \log_2(1 + \Theta^*)) \rceil)$ ,  $N_{max}$  is the maximum number of PRBs that a BS can allocate to a user (fixed by the operator) and  $\lceil . \rceil$  stands for the Ceiling function. It is obvious from (3) that users are fairly scheduled because a user with bad radio condition (with low value of C(x)) gets higher number of PRBs to achieve its transmission rate  $C^*$ . Let  $d_n$  be the distance from s that verifies, for all  $x \in$  $(d_{n-1}, d_n], n(x) = n$  with

$$n = \frac{C^*}{C(d_n)} \tag{4}$$

is an integer and

$$d_n = \begin{cases} 0 \text{ if } n = 0, \\\\ R, \text{ if } \exists k \in [1, N] \text{ such that } d_k = R \text{ and } n \ge k, \\\\ \left[\frac{a(I+\sigma^2)}{P} (2^{\frac{C}{n\vartheta W}} - 1)\right]^{\frac{-1}{2b}} \text{ otherwise,} \end{cases}$$

with  $\exists$  means that there exists at least one.

From (4), cell area s can be divided into rings with radius  $d_n$  such that for  $1 \leq n \leq N$ ,  $0 \leq d_{n-1} < d_n \leq R$ . The area between the ring of radius  $d_n$  and the ring of radius  $d_{n-1}$  characterizes the region of the cell where users require n PRBs to achieve the transmission rate  $C^*$ . We define the cell throughput by the sum over all transmission rates of users:

$$\tau = uC^*,\tag{5}$$

with u is recalled the average number of users inside s and derived in (1).

#### **III. PRESENTATION OF THE DIMENSIONING APPROACH**

Dimensioning exercise consists in evaluating the required radio resources that allow to carry a forecast data traffic given a target QoS. The QoS can be measured by the congestion probability metric or even by a target average user throughput. The present approach assesses the congestion probability as a function of many key parameters, in particular the number of PRBs M and the cell throughput  $\tau$ . To characterize this congestion probability, we need to evaluate the total requested PRBs by all users. In the remainder of this section, we will state some analytical results regarding the explicit expression of the congestion probability under the system model presented in the previous section.

#### A. Qualification of the number of total requested PRBs

Following [5], the number of users in each ring is modeled by a Poisson random variable denoted  $X_n$  with a parameter

 $\mu_n(\phi) = \delta \left( \alpha_n(\phi) - \alpha_{n-1}(\phi) \right),$ 

where

$$\alpha_n(\phi) = 2 \sum_{j=1}^{Y} \mathbb{1}_{(d_n > r_j)} \left( \sqrt{d_n^2 - r_j^2} \right).$$

Coefficient  $\alpha_n(\phi)$  stands for the length of the intersection between road j and ring n.  $r_j$  is the distance between the origin and its orthogonal projection on road j. Hence, we can define the total number of requested PRBs in the cell as the sum of demanded PRBs in each ring. It can be expressed as

$$\Gamma = \sum_{n=1}^{N} n X_n. \tag{7}$$

(6)

The random variable  $\Gamma$  is the sum of weighted Poisson variables. The evaluation of its distributions requires extensive numerical simulation. However, we fortunately derive here an analytical formula.

#### B. Congestion probability and dimensioning

The congestion probability, denoted by  $\Pi$ , is defined as the probability that the number of total requested PRBs in the cell is greater than the available PRBs fixed by the operator. In other words, it measures the probability of failing to achieve an output number of PRBs M required to guarantee a predefined quality of services:

$$\Pi(M,\tau) = \mathbb{P}(\Gamma \ge M). \tag{8}$$

The following theorem gives the explicit expression of the congestion probability conditionally to the PLP.

**Theorem 1.** Let  $\Gamma$  be defined as in (7) and  $\phi$  be a PLP defined as in subsection II.A, the probability that  $\Gamma$  exceeds a threshold M conditionally to  $\phi$  is

$$\mathbb{P}(\Gamma \ge M | \phi) = 1 - \frac{1}{\pi} e^{-\delta} \alpha_N \int_0^{\pi} e^{p_n(\phi)} \times \frac{\sin(\frac{M\theta}{2})}{\sin(\frac{\theta}{2})} \cos(\frac{M-1}{2} - q_n(\phi)) d\theta, \quad (9)$$

where

$$p_n(\phi) = \sum_{n=1}^N \mu_n(\phi) cos(n\theta)$$
 and  $q_n(\phi) = \sum_{n=1}^N \mu_n(\phi) sin(n\theta)$ .

Proof. See appendix A.

Theorem 1 is valid not only for PLP but also for every process of user distribution, in particular for 2D-PPP model for which the parameters  $\mu_n$  in (6) are adapted to  $\mu_n = u(d_n^2 - d_{n-1}^2)/R^2$ .

The final expression of the congestion probability can be derived by averaging over the PLP as follows:

$$\Pi(M,\tau) = \mathbb{E}_{\phi}[\mathbb{P}(\Gamma \ge M|\phi)].$$
(10)

By setting a target congestion probability  $\Pi^*$ , the required number of PRBs M is written as a function of  $\tau$  through the implicit equation  $\Pi(M, \tau) = \Pi^*$ . The output M of the implicit function constitutes the result of the dimensioning exercise.

#### IV. NUMERICAL RESULTS

For numerical purpose, we consider a cell of radius R = 0.3km with power level P = 43dBm and operating in a bandwidth of 20MHz. The downlink thermal noise power including the receiver noise figure is calculated for 20MHz to  $\sigma^2 = -93dBm$ . Since only users on roads are concerned, outdoor environment with propagation parameter a = 130dB and path loss exponent 2b = 3.5 is considered. We assume also that we have 8TX antennas in the BS and 2RX antennas in users' terminals. So, the number of possible transmission layers is at most 2. The SINR threshold is set to  $\Theta^* = -10dB$ .

Fig. 2 shows the explicit expression of the congestion probability compared with simulation results for three values of cell throughput  $\tau$ . We notice that the explicit expression of the congestion probability fits the simulated one. Moreover, it is obvious that an increase in cell throughput  $\tau$  generates an increase in the congestion probability because  $\tau$  is related to the number of users in the cell. When the intensity of users in roads  $\delta$  increases, the number of required PRBs on roads that lie inside the cell coverage area increases, thus the system experiences a high congestion probability. An other important factor that impacts system performances is the path loss exponent. The variations of this parameter has tremendous effect on the congestion probability: when 2b goes up, radio conditions become worse and consequently the number of demanded PRBs, to guarantee the required QoS, increases.

To see the impact of road geometry, we show in Fig. 3 the congestion probability with PLP model having different intensity compared with equivalent 2D-PPP model, i.e., PLP and 2D-PPP have the same user intensity. The first PLP model assumes high road intensity and low user intensity  $(\lambda = 100 km/km^2, \delta = 10 users/km)$  whereas the second PLP is configured with low road intensity and high user intensity  $(\lambda = 10 km/km^2, \delta = 100 users/km)$  so that  $\lambda\delta$ is the same for both PLP and 2D-PPP models. We observe that when road intensity becomes higher, users appear to be



Fig. 2. Congestion probability for different values of  $\tau$ 

distributed every where in the cell as in 2D-PPP model and hence the congestion probability for PLP gets closer to that of its equivalent 2D-PPP. However, congestion probability for PLP with low road intensity behaves differently from that of both PLP with higher road intensity and 2D-PPP. In other words, even if the mean number of users in the cell is the same, the geometry of the area covered by this cell has a significant impact on system performances.



Fig. 3. Simulated Congestion probability for PLP vs PPP with the same user intensity  $\lambda \delta = 1000 users/km^2$ .

During resource dimensioning exercise, the operator starts by defining a target congestion probability that can be tolerated for a given service. For different traffic forecasts, the number of PRBs is set to ensure that the congestion probability never exceeds its target. Fig. 4 shows the number of required PRBs that the operator should make it available when the expected cell throughput is known for two target values of congestion probability ( $\Pi^* = 1\%$  and  $\Pi^* = 5\%$ ) and for two road intensities ( $\lambda = 5km/km^2$  and  $\lambda = 15km/km^2$ ). Once again, we conclude that the number of dimensioned PRBs is very sensitive to the road intensity  $\lambda$  and this is in agreement with results of Fig. 3. Indeed, for a target congestion probability  $\Pi^* = 1\%$ , when  $\lambda$  increases from  $5km/km^2$  to  $15km/km^2$  (i.e., from 10 expected roads to 30) the number of dimensioned PRBs decreases by 23, for the same cell throughput value  $\tau = 25Mbps$ . For a given value of  $\tau$ , we can notice from (5) that the user intensity on roads  $\delta$  is inversely proportional to roads' intensity  $\lambda$ . Thus for fixed  $\tau$ , if  $\lambda$  increases  $\delta$  decreases and consequently the number of required PRBs decreases. Besides, we have shown previously that when  $\lambda$  is very high, the distribution of users becomes similar to a 2D-PPP. Thus, with a 2D-PPP model, one can have small values of dimensioned PRBs, which is considered optimistic compared to the real geometry of roads where more PRBs are required to guarantee the desired quality of services.



Fig. 4. Required PRB M as a function of cell Throughput  $\tau,$  for fixed transmission rate  $C^*=1Mbps.$ 

Dimensioning process is very important because it gives the operators a vision on how they should manage the available spectrum. If this dimensioned number exceeds the available one, the operator can for instance:

- aggregate fragmented spectrum resources into a single wider band in order to increase the available PRBs,
- activate capacity improvement features like carrier aggregation or dual connectivity between 5G and legacy 4G networks in order to delay investment on the acquisition of new spectrum bands,
- change the TDD (Time Division Duplexing), configuration to relieve the congested link,
- or even buy new spectrum bands.

#### V. CONCLUSIONS

In this paper, we have presented a resource dimensioning model for OFDM based 5G NR interface. Poisson Line Process have been used to model the geometry of roads covered by a typical cell. Outdoor users have been assumed to be distributed on that roads according to a 1D-Poisson Point Process. The explicit expression of the congestion probability has been derived and its behavior as a function of network parameters has been studied. Besides, we have established an implicit relationship between the required resources and the forecast traffic for a given target congestion probability. This relationship translates the dimensioning exercise that an operator can perform to look for the amount of required spectrum resources. Finally, we have shown how the spatial distribution of roads can hit the dimensioning process. Further extension to this approach could include the spatial distribution of indoor users according to a 2D-Poisson Point Process.

#### APPENDIX A

#### **PROOF OF THEOREM 1**

To prove theorem 1, we firstly calculate the moment generating function (i.e., Z-Transform) f(z) of the discrete random variable  $\Gamma$  of equation (7).

$$f(z) = \mathbb{E}(z^{\Gamma}) = \sum_{k=0}^{+\infty} z^k \mathbb{P}(\Gamma = k | \phi)$$
$$= \prod_{n=1}^{N} \sum_{k=0}^{+\infty} z^{nk} \mathbb{P}(X_n = k | \phi).$$
(11)

Since  $X_n$  is a Poisson random variable with parameter  $\mu_n$ , (11) is simplified to

$$f(z) = e^{-\delta \alpha_N} e^{\sum_{n=1}^N z^n \mu_n},$$
 (12)

with  $\alpha_N$  comes from the relation  $\sum_{n=1}^{N} \mu_n = \delta \alpha_N$ . It is obvious that f is analytic on  $\mathbb{C}$  and in particular inside the unit circle  $\omega$ . Cauchy's integral formula gives then the coefficients of the expansion of f in the neighborhood of z = 0:

$$\mathbb{P}(\Gamma = k | \phi) = \frac{1}{2\pi i} \int_{\omega} \frac{f(z)}{z^{k+1}} dz.$$
(13)

In (13), replacing f by its expression (12) and parameterizing z by  $e^{i\theta}$  lead to

$$\mathbb{P}(\Gamma = k | \phi) = \frac{1}{2\pi} e^{-\delta \alpha_N} \int_0^{2\pi} \frac{e^{\sum_{n=1}^N \mu_n e^{in\theta}}}{e^{ik\theta}} d\theta.$$
(14)

Since the congestion probability is defined by the CCDF (Complementary Cumulative Distribution Function) of  $\Gamma$ , then

$$\mathbb{P}(\Gamma \ge M | \phi) = 1 - \sum_{k=0}^{M-1} \mathbb{P}(\Gamma = k | \phi)$$
$$= 1 - \frac{1}{2\pi} e^{-\delta \alpha_N} \int_0^{2\pi} e^{\sum_{n=1}^N \mu_n e^{in\theta}} \sum_{k=0}^{M-1} e^{-ik\theta} d\theta.$$
(15)

The sum inside the right hand integral of (15) can be easy calculated to get the explicit expression of (9) after some simplifications.

#### REFERENCES

- 3GPP, "Technical specification group radio access network (nr); physical layer procedures for data," 3rd Generation Partnership Project (3GPP), TS 38.214, 12 2017, version 15.0.0.
- [2] R. D. Trivedi and M. Patel, "Comparison of different scheduling algorithm for Ite," *International Journal of Emerging Technology and Advanced Engineering*, vol. 4, no. 5, pp. 334–339, 2014.
- [3] R. Agarwal, V. R. Majjigi, Z. Han, R. Vannithamby, and J. M. Cioffi, "Low complexity resource allocation with opportunistic feedback over downlink ofdma networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, 2008.

- [4] A. K. Khattab and K. M. Elsayed, "Opportunistic scheduling of delay sensitive traffic in ofdma-basedwireless," in *proceedings of the 2006 International Symposium on on World of Wireless, Mobile and Multimedia Networks.* IEEE Computer Society, 2006, pp. 279–288.
- [5] L. Decreusefond, E. Ferraz, P. Martins, and T.-T. Vu, "Robust methods for Ite and wimax dimensioning," in *Performance Evaluation Methodologies and Tools (VALUETOOLS), 2012 6th International Conference on.* IEEE, 2012, pp. 74–82.
- [6] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser ofdm systems with proportional rate constraints," *IEEE transactions on wireless communications*, vol. 4, no. 6, pp. 2726–2737, 2005.
- [7] R. Nasri and A. Jaziri, "Analytical tractability of hexagonal network model with random user location," *IEEE Transactions on Wireless*

Communications, vol. 15, no. 5, pp. 3768-3780, 2016.

- [8] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on communications*, vol. 59, no. 11, pp. 3122–3134, 2011.
  [9] V. V. Chetlur and H. S. Dhillon, "Coverage analysis of a vehicular
- [9] V. V. Chetlur and H. S. Dhillon, "Coverage analysis of a vehicular network modeled as cox process driven by poisson line process," *arXiv* preprint arXiv:1709.08577, 2017.
- [10] C.-S. Choi and F. Baccelli, "An analytical framework for coverage in cellular networks leveraging vehicles," *arXiv preprint arXiv:1711.09453*, 2017.
- [11] R. Nasri and A. Jaziri, "Tractable approach for hexagonal cellular network model and its comparison to poisson point process," in *Global Communications Conference (GLOBECOM), 2015 IEEE.* IEEE, 2015, pp. 1–6.