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**Broadband transmission losses and time dispersion maps from
full-wave time-domain numerical simulations in ocean acoustics**

Alexis Bottero,^{1, a)} Paul Cristini,¹ Dimitri Komatitsch,¹ and Quentin Brissaud²

¹⁾*Aix Marseille Univ., CNRS, Centrale Marseille, LMA, Marseille,
France.*

²⁾*Division of Geological and Planetary Sciences, Mail Code 252-21,
California Institute of Technology, Pasadena, CA 91125, USA.*

bottero@lma.cnrs-mrs.fr

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1 **Abstract:** In this letter, we present a procedure for the calculation of
2 transmission loss maps from time-domain full-wave numerical simula-
3 tions that can be generalized to arbitrary time sequences and to elastic
4 media. It provides an insight into the understanding of how energy
5 spreads into a complex configuration. In addition, we propose to gen-
6 erate time dispersion maps that give complementary information on
7 how the energy is distributed over time. These maps are generated at
8 a negligible additional numerical cost. To illustrate the kind of results
9 that can be obtained with this procedure, we consider the classical two-
10 dimensional upslope wedge with a fluid bottom and compare our results
11 with those previously obtained based on a parabolic equation. Then,
12 for the same configuration, we present maps for an elastic bottom and
13 for non-monochromatic signals.

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^{a)} Author to whom correspondence should be addressed.

14 **1. Introduction**

15 In underwater acoustics, several numerical methods are available in the literature for the
16 solution of acoustic wave propagation problems in complex marine environments. Most of
17 them solve the Helmholtz equation, thus providing a frequency-domain solution. This is the
18 traditional way that wave propagation problems in the ocean are handled, and results from
19 these simulations are presented as transmission loss (TL) curves or transmission loss maps.
20 The reason for this choice is the high variability of the ocean, which generates strong signal
21 fluctuations and makes the analysis of time signals difficult. When time-domain solutions are
22 sought, they are often obtained after Fourier transform of the results of several frequency-
23 domain simulations but, in some situations, it may be more convenient to generate the results
24 directly in the time domain. Recently, a time-domain spectral-element method ([Komatitsch
25 and Tromp, 1999](#)) has been shown to efficiently solve full-wave propagation problems in
26 ocean acoustics ([Bottero *et al.*, 2016](#); [Cristini and Komatitsch, 2012](#)). Beyond its capability
27 of handling complex geometries and rheologies accurately, as any finite-element technique,
28 the time-domain spectral-element method runs efficiently on very large computers, thus
29 providing a drastic reduction of the duration of numerical simulations, which is one of its
30 attractive properties. Nevertheless, this property is partly lost if, instead of the time-domain
31 wave equation, it is the Helmholtz equation that is solved in the framework of a spectral-
32 element method. Consequently, the choice of solving the time-domain wave equation is
33 mandatory to take advantage of the possibility of running such numerical simulations on large
34 supercomputers. The reason for this situation is the behavior of linear system solvers used

35 for the inversion of the mass matrix when solving in the frequency domain. They typically
36 exhibit decreasing performance when increasing the number of cores and have difficulties
37 scaling when more than a thousand processor cores or so are used. This is a significant
38 limitation because such a number of cores is not that high in terms of current standards, and
39 even less so in the future. Consequently, when solving wave propagation problems requiring
40 a high number of processor cores, the question of the generation of frequency-domain results
41 from time-domain numerical simulations arises, since the underwater acoustics community is
42 used to analyzing results in the frequency domain. A simple way of addressing this question
43 consists in storing all time signals for all receiver positions and then performing a Fourier
44 transform to convert them to the frequency domain. However, this solution is realistic from
45 a technical point of view only if the number of receivers is small to moderate because of
46 the amount of storage (in memory or to disk) that this operation requires when the number
47 of recording points and/or the number of time steps computed are large to very large. In
48 practice, this limits the generation of frequency-domain results from time-domain simulations
49 to the creation of a small number of transmission loss curves, which implies that producing
50 2D transmission loss maps will require an amount of storage too large to be handled. The
51 objective of this letter is thus to present an efficient way of creating transmission loss maps
52 from time-domain full-wave numerical simulations that avoids the storage of individual time
53 signals. Since the source time signal can be arbitrary chosen, the transmission loss maps can
54 be evaluated for quasi-monochromatic signals as well as for signals with a wider bandwidth,
55 leading to the possibility of analyzing the influence of bandwidth on the distribution of

56 acoustic energy inside the domain. In addition, time dispersion maps can also be calculated
 57 on the fly during the simulation, providing an insight into the structure of the received time
 58 signals. All these quantities are obtained at a negligible additional numerical cost.

59 The letter is organized as follows: Sec. 2 is devoted to the definition of the different
 60 physical quantities that we want to study, and to how one can compute them efficiently in a
 61 time-domain numerical simulation. Then, in Sec.3, we provide and discuss some examples of
 62 the evaluation of these quantities within the framework of a time-domain spectral-element
 63 method. Wave propagation over a fluid and then over an elastic upslope wedge is considered
 64 for several source bandwidths. We finally draw some conclusions in Sec.4.

65 **2. Generalization of the calculation of transmission losses and evaluation of** 66 **signal time spreading**

67 In this section, we define the different physical quantities that we want to study and show how
 68 they can be calculated on-the-fly in a time-domain full-wave numerical simulation. These
 69 quantities will allow for the evaluation of the transmission losses and of the time structure of
 70 signals at all the discrete points of the spatial domain under study. Let us note $u_x(\mathbf{x}, t)$ and
 71 $u_z(\mathbf{x}, t)$ the horizontal and vertical displacement field, respectively, and $P(\mathbf{x}, t)$ the pressure
 72 field at time t and position $\mathbf{x} = (x, z)$. $\dot{u}(\mathbf{x}, t) = \sqrt{\dot{u}_x(\mathbf{x}, t)^2 + \dot{u}_z(\mathbf{x}, t)^2}$ is the norm of the
 73 particle velocity field. The instantaneous energy per unit volume field in the fluid is given
 74 by ((Jensen *et al.*, 2011) pp.11-12):

$$\mathcal{E}(\mathbf{x}, t) = \frac{1}{2}\rho\dot{u}^2(\mathbf{x}, t) + \frac{1}{2}\frac{P^2(\mathbf{x}, t)}{\rho(\mathbf{x})c^2(\mathbf{x})}, \quad (1)$$

75 where $\rho = 1000 \text{ kg.m}^{-3}$ is the density of water and $c(\mathbf{x})$ is the distribution of sound velocity.
 76 Likewise, in a linear isotropic solid medium the instantaneous energy field reads ([Achenbach,](#)
 77 [1973](#)):

$$\mathcal{E}(\mathbf{x}, t) = \frac{1}{2}\rho(\mathbf{x})\dot{u}^2(\mathbf{x}, t) + \frac{1}{2}\sum_{i,j}\epsilon_{ij}(\mathbf{x}, t)\sigma_{ij}(\mathbf{x}, t). \quad (2)$$

78 where $\epsilon(\mathbf{x}, t)$ and $\sigma(\mathbf{x}, t)$ are the second-order strain and stress tensor fields, respec-
 79 tively. Let T_f refer to the duration that is considered, then the integrated energy field reads:

80

$$E(\mathbf{x}) = \int_0^{T_f} \mathcal{E}(\mathbf{x}, t) dt. \quad (3)$$

81 This physical parameter represents the amount of energy received at a given position inside
 82 the model at time T_f . It is similar to the radiated seismic energy introduced by ([Boatwright](#)
 83 [and Choy, 1986](#)) and evaluated from body waves measurements, or to the T-Phase Energy
 84 Flux (TPEF) proposed by ([Okal, 2003](#)) to characterize the energy generated by an earthquake
 85 source in the form of a T-wave. Then, knowing the energy E_0 of the emitting source, it is
 86 possible to evaluate the transmission losses for time T_f as:

$$TL(\mathbf{x}) = -10 \log \frac{E(\mathbf{x})}{E_0} \quad (4)$$

87 Energy is determined by an integral in time, but with the value that we get we have no
 88 knowledge on how this energy is distributed within the time interval that we are considering.
 89 For a given energy value, time spreading can be very different depending on the propagation
 90 path followed by the signal. It is therefore very useful to get such a piece of information. In

91 order to calculate it, we first define the maximum energy field by:

$$M(\mathbf{x}) = \max_{t < T_f} \mathcal{E}(\mathbf{x}, t). \quad (5)$$

92 This field gives the maximum of the instantaneous energy for each point and provides a way
 93 of defining an "effective" time dispersion for a signal as:

$$\mathcal{T}(\mathbf{x}) = 2 \frac{E(\mathbf{x})}{M(\mathbf{x})}. \quad (6)$$

94 This quantity is homogeneous to a duration. It represents the duration of the triangle-shaped
 95 signal that has the same energy and maximum amplitude as those that we have calculated. It
 96 is therefore a measure of the time spreading of the signal. Time-domain full-wave numerical
 97 simulations provide access to these physical parameters at each time step δt and thus allow
 98 for the computation of transmission losses and time dispersion maps on-the-fly during the
 99 run at a negligible additional computational cost. In practice, at iteration $i + 1$ and position
 100 \mathbf{x} , one can evaluate:

$$E_{i+1}(\mathbf{x}) = E_i(\mathbf{x}) + \mathcal{E}(\mathbf{x}, t_{i+1})\delta t \quad (7)$$

101

$$M_{i+1}(\mathbf{x}) = \max[M_i(\mathbf{x}), \mathcal{E}(\mathbf{x}, t_{i+1})] \quad (8)$$

102

$$\mathcal{T}_{i+1}(\mathbf{x}) = 2 \frac{E_{i+1}(\mathbf{x})}{M_{i+1}(\mathbf{x})} \quad (9)$$

103 Time domain full-wave numerical methods can thus also provide an at-a-glance view of how
 104 the acoustic energy emitted by a source is distributed inside a complex heterogeneous model.

105 In the following sections, we will show examples of the maps that can be obtained based on
106 the calculation of these physical quantities.

107 **3. Application to wave propagation in a 2D upslope wedge**

108 In order to validate our approach and to present new results, we choose to investigate the clas-
109 sical two-dimensional (2D) wedge benchmark problem, whose characteristics can be found in
110 Section 6.9.2 of (Jensen *et al.*, 2011). This configuration illustrates several wave propagation
111 phenomena such as mode conversion and mode cutoff due to a varying water column depth.
112 In the context of an attenuating fluid bottom, transmission loss maps for a monochromatic
113 source have already been published and can thus be used for comparison. In addition, with
114 our approach, the transmission loss concept can be extended to broadband source signals
115 and to elastic media. This is an interesting feature that enables us to determine the energy
116 spreading in the whole computational domain. As a consequence these new maps may help
117 to understand the physical mechanisms that lead to the distribution of acoustic energy inside
118 a given configuration. For the sake of illustration, let us first generate a transmission loss
119 map from a full-wave time-domain numerical simulation for exactly the same configuration
120 as in (Jensen *et al.*, 2011). These previous results were obtained with a split-step imple-
121 mentation of the Thomson-Chapman parabolic equation using a Greene wide-angle source
122 to initialize the solution. It is worth noting that the results that will be generated using
123 our approach will be obtained based on a time-domain full-wave numerical simulation that
124 makes no approximation. As a consequence, comparing our results with those obtained with
125 a parabolic equation will also lead to a comparison of the two numerical methods. Then, in

126 a second step, we will consider the influence of the bandwidth of the emitted signal for both
 127 a fluid and an elastic bottom. For this purpose, we define a source time function signal by:

$$s(t) = \begin{cases} \frac{A}{2} (1 - \cos(\Delta\pi t) \sin(2\pi f_0 t)) & \text{if } 0 < t < \frac{2}{\Delta} \\ 0 & \text{else} \end{cases} \quad (10)$$

128 where A is the maximum amplitude of the source, f_0 is the dominant frequency of the source
 129 signal, and Δ is its bandwidth. This type of time sequence will allow us to control the band-
 130 width of the emitted signal by changing the value of parameter Δ . Quasi-monochromatic
 131 signals that will provide results close to frequency-domain calculations as well as narrowband
 132 or broadband signals can be generated. We will thus be able to study the influence of the
 133 bandwidth on the spreading of acoustic energy inside the computational domain. In addition
 134 to a fluid viscoacoustic bottom, we will also consider a solid viscoelastic bottom by adding
 135 a shear velocity of $c_s = 600 \text{ m.s}^{-1}$ and a shear attenuation coefficient $\alpha_S = 0.5 \text{ dB}.\lambda_S^{-1}$ to
 136 the sediment characteristics. In the spectral-element method, viscoacoustic or viscoelastic
 137 effects are represented based upon three generalized Zener standard linear solids placed in
 138 parallel, with different relaxation times for each, to mimic a constant Q quality factor over
 139 the frequency band under study in the simulation (Komatitsch and Tromp, 1999). In order
 140 to avoid spurious reflections from the sides of the computational domain, for all configura-
 141 tions the domain is extended up to a range of 20 km and down to a depth of 1 km and
 142 equipped with perfectly matched absorbing layers (PMLs, (Xie *et al.*, 2016)).

143 The first results are shown in Fig. 1, which provides the comparison between
 144 a monochromatic TL map (Fig. 1(a)) taken from (Jensen *et al.*, 2011) and a quasi-

145 monochromatic TL map (Fig. 1(b)) generated from a time-domain full-wave numerical
 146 simulation.

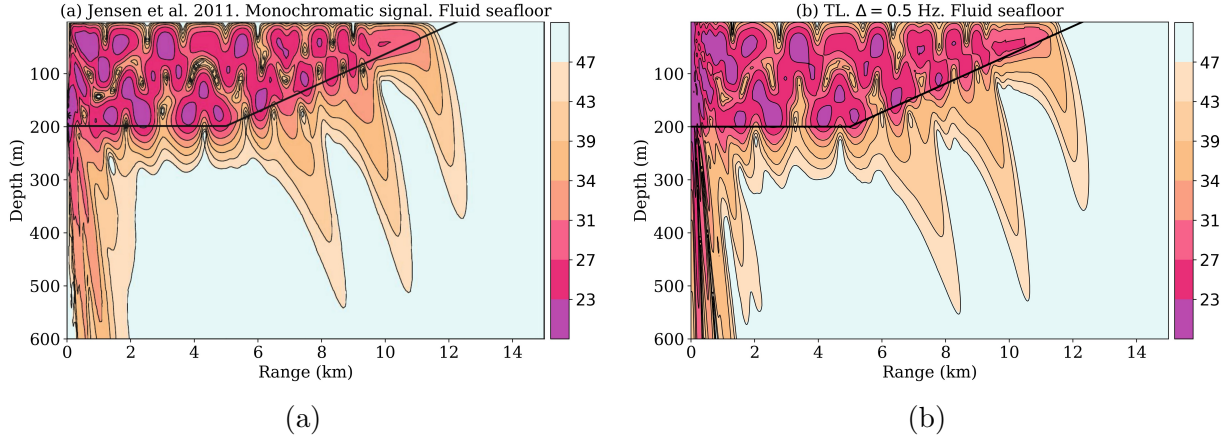


Fig. 1. (Color online) (a) Original TL map, in dB, from reference (Jensen *et al.*, 2011) page 514. (b) TL map from a full-wave simulation using an quasi-monochromatic source ($f_0 = 25$ Hz, $\Delta = 0.5$ Hz). Figure (a) is reproduced (slightly modified) from Figure 6.11 of (Jensen *et al.*, 2011) with permission from publisher Springer. The black line indicates the seabed interface.

147 The quasi-monochromatic signal was generated using a bandwidth $\Delta = 0.5$ Hz. We
 148 performed several simulations with signals having smaller bandwidth but did not notice
 149 any changes in the TL map. We thus consider this value of the bandwidth as a good
 150 approximation of a monochromatic signal for this configuration. Note that this is a fluid
 151 only configuration. The two TL maps are very similar. Mode cutoff of the three modes,
 152 which exist in the flat part of the model, are recovered almost identically in both cases.
 153 The main differences are observed for short ranges. This is not surprising since the parabolic
 154 equation, which is used to generate the results of Fig 1(a), has angular limitations. Moreover,

155 a source with a limited aperture was used, contrary to our simulation, which implements a
156 point source. Therefore the discrepancies between the two TL maps are attributed to the
157 known inaccuracy of PE codes for steep angles. Nevertheless, interference structures in the
158 water column are very close, except at the end of the wedge where again the grazing angles
159 are steepened because of the varying depth and thus cannot be handled correctly through
160 numerical modeling based on the parabolic equation. This comparison may be seen as a first
161 answer to the question raised by the author of (Buckingham, 1992) on the accuracy of TL
162 maps obtained using the parabolic equation for an upslope fluid wedge. Based on the results
163 that we obtain, it can be considered that the accuracy is very good.

164 Since our numerical method works in the time domain, we can also consider signals
165 with different bandwidths in order to evaluate how this parameter may influence the spread-
166 ing of energy inside the computational domain. Figure 2 (b) represents the TL map for a
167 signal with a bandwidth $\Delta = 8.0$ Hz.

168 A smoothing of the interference structure is observed in the water column mainly
169 from the beginning of the wedge to its end. The reason for this situation lies in the fact
170 that for each frequency there is a different modal structure, with different grazing angles
171 and therefore different cutoff depths varying continuously with frequency. This smoothing
172 is also observed in the sediment. The complex structure that was observed below the source
173 also disappears.

174 Adding a shear velocity to the sea floor leads to a very different structure of the
175 leaking of acoustic energy in the bottom (Figure 2 (c)). The leaking of energy associated to

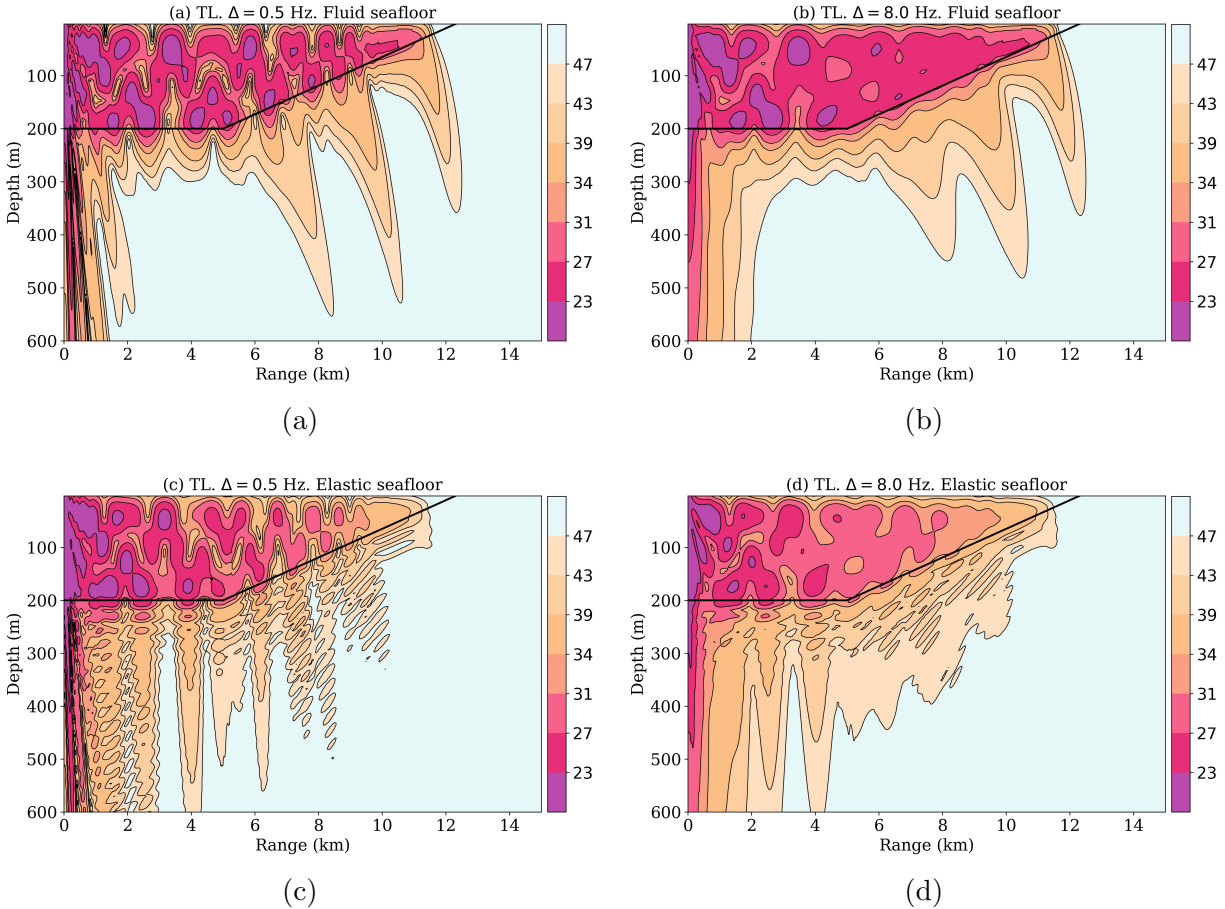


Fig. 2. (Color online) Effect of source bandwidth and bottom elasticity on transmission losses for an upslope wedge. (a) TL map, in dB, for a fluid bottom using a quasi-monochromatic source ($f_0 = 25$ Hz, $\Delta = 0.5$ Hz). (b) TL map, in dB, for a fluid bottom using a broadband source ($f_0 = 25$ Hz, $\Delta = 8.0$ Hz). (c) TL map, in dB, for an elastic bottom using a quasi-monochromatic source ($f_0 = 25$ Hz, $\Delta = 0.5$ Hz). (d) TL map, in dB, for an elastic bottom using a broadband source ($f_0 = 25$ Hz, $\Delta = 8.0$ Hz).

176 the presence of shear waves is strong and dominant. The associated narrow beams are almost
 177 vertical because of the low velocity of shear waves in this configuration. Below the sloping
 178 interface, it can be seen that the beams exhibit an interference structure due to the leaking,
 179 in this case, of both the shear waves and the propagating modes. This structure is also seen

180 in the near field but, in this case, it is generated by the leaking of evanescent modes. Similar
181 results were presented in (Abawi and Porter, 2007) (top figure of their Figure 1). It can
182 also be noted that the leaking of the first mode is strongly affected by the presence of shear
183 waves, as it does not penetrate deep into the sediment. There is much less energy in the
184 water column at the end of the wedge than in the fluid-only configuration. A large amount
185 of energy is captured by the shear waves of the bottom. The structure of the sound field in
186 the sediment suggests that if another interface is considered, i.e. if we consider an elastic
187 layer over a semi-infinite half-space, the presence of shear waves is critical and may generate
188 complex effects because of the potential interaction between these beams and this interface.
189 As in the pure fluid case, increasing the bandwidth (Figure 2 (d)) leads to a smoothing of
190 the energy levels mainly in the area of the slope.

191 As mentioned in the previous section, another type of information can be extracted
192 from time-domain simulations. This piece of information is related to the spreading of energy
193 with time at a receiver location. Indeed, for a given energy level, the time structure of the
194 received signal can be very different and provide additional information on the propagation
195 process that led to this received signal. This is particularly useful e.g. for T-waves analysis
196 because the time structure of a received signal is different depending on the source mechanism
197 that led to the generation of this signal. In order to illustrate the kind of information that is
198 provided by this type of map, we performed a full-wave time domain numerical simulation
199 for a fluid bottom and a broadband signal ($f_0 = 25$ Hz, $\Delta = 25.0$ Hz).

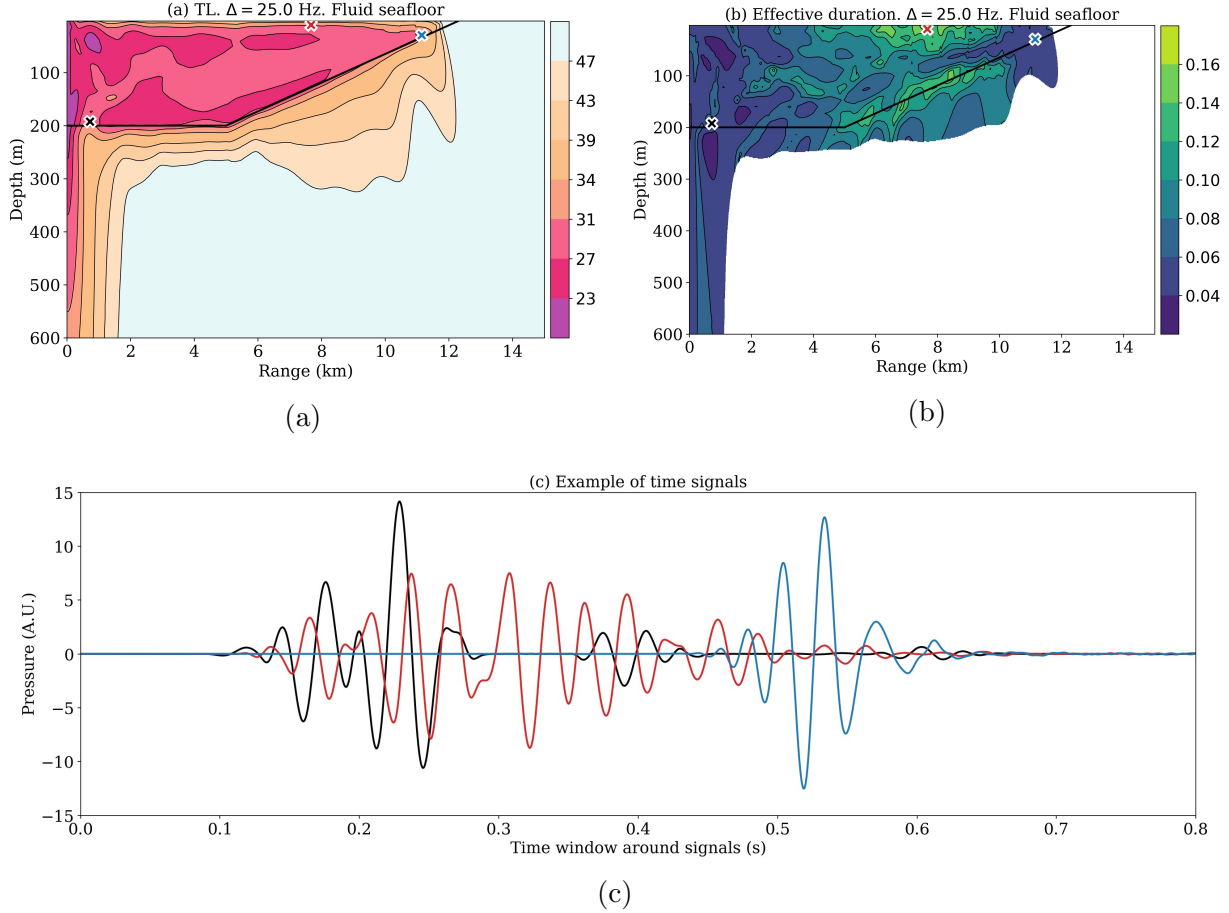


Fig. 3. (Color online) Figure illustrating the use of time dispersion maps. The geometry is the same as in Fig 1. (a) TL map, in dB, using a broadband source ($f_0 = 25$ Hz, $\Delta = 25.0$ Hz). (b) Associated time dispersion map $\mathcal{T}(\mathbf{x})$ (in seconds). This field is shown only for transmission losses that are below 43 dB. (c) Arbitrary time-shifted pressure signals recorded at the positions indicated by the crosses in (a) and (b). The color of each curve corresponds to the color of the respective cross. The wave energy is about the same at the three positions, but the time dispersion is different.

200 Figure 3 (a) represents the TL map, and Figure 3 (b) represents the time dispersion
 201 map for TL values lower than 43 dB in order to avoid showing very weak signals.
 202 Finally, Figure 3 (c) provides time sequences associated to receivers located at the position

203 of the crosses indicated in Figures 3 (a) and (b). These positions were chosen so that
204 the energy level is similar for all positions. The time sequences are arbitrary shifted for
205 visualization purposes. The time dispersion map exhibits complicated structures reflecting
206 the various time structures that can be generated in this configuration. Signals tend to be
207 more dispersed with range, especially right beneath the sea surface or along the interface
208 with the sea bottom. Nevertheless, at the end of the wedge, at the cutoff depth of mode 1,
209 signals tend to be narrower. This effect is clearly visible in Figure 3 (c).

210 4. Conclusions

211 We have presented an efficient procedure to compute transmission losses and time disper-
212 sion maps from time-domain full-wave numerical simulations. This procedure allowed us to
213 extend the notion of transmission losses to non-monochromatic signals and to elastic me-
214 dia. Some results using this procedure were obtained for a 2D wedge configuration in ocean
215 acoustics. In the case of a simulation in the frequency domain and for a fluid bottom, these
216 results were compared with results previously obtained using a parabolic equation, showing
217 that both methods give similar results and that the main structures of the sound field are
218 correctly captured by the parabolic equation. Discrepancies between the two approaches
219 were found in regions where the parabolic approximation is known to fail. As a result, our
220 approach can provide reference solutions for more complex configurations because the nu-
221 merical simulations that are performed are realized without any such approximation and are
222 able to handle complex geometries accurately. In future work we expect to present results

223 on T-wave propagation or on pile driving, in which this approach may bring new insights
224 into the mechanisms of wave propagation.

225 Our SPECFEM open source spectral-element software package used in this study
226 is freely available at geodynamics.org; it contains all the tools needed to reproduce the
227 simulations presented in this letter.

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