

Supplemental material for Spontaneous lateral atomic recoil force close to a photonic topological material

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(Dated: May 14, 2018)

Optical force parameters

In Eq. (6) of the main text, $a_\theta \equiv |A_{\mathbf{k}_\parallel}|^2 \varepsilon_0 k_\parallel$ is a function only of θ , not of k_\parallel , where

$$|A_{\mathbf{k}_\parallel}|^2 = \frac{2}{\varepsilon_0} \left[k_\parallel + \frac{\Lambda(\omega_\theta, \omega_c, \omega_p)}{2\tilde{k}_\parallel} \right]^{-1} \quad (\text{S1})$$

with $\Lambda(\omega, \omega_c, \omega_p) = \partial_\omega (\varepsilon_t \omega) (\tilde{k}_\parallel^2 + k_x^2) + \partial_\omega (\varepsilon_a \omega) k_y^2 + \partial_\omega (\varepsilon_g \omega) 2k_x \tilde{k}_\parallel$. In addition,

$$\Gamma_{+, \theta} = \frac{1}{|\gamma|^2 k_\parallel^2} |(-i\mathbf{k}_\parallel - k_\parallel \hat{\mathbf{z}}) \cdot \boldsymbol{\gamma}|^2. \quad (\text{S2})$$

See [2] for the derivation of the quasi-static force.

Steady-state population for the pumped system

Without external compensation of atomic depopulation, the atom decays from an excited state to the ground state, $\rho_{ee}(t) = e^{-\Gamma t}$, where, for a z -directed dipole,

$$\Gamma = \frac{2|\gamma|^2 \omega_0^2}{\varepsilon_0 \hbar c^2} \text{Im}(\mathcal{G}_{zz}(\mathbf{r}_0, \mathbf{r}_0, \omega_0)), \quad (\text{S3})$$

and so the steady state recoil force is zero. Here, \mathcal{G}_{zz} is the zz -component of the standard electric Green dyadic defined as in [1]. In order to have a non-zero steady state recoil force, the atom can be pumped by an external laser. In the following we derive the steady state recoil force under continuous pumping.

In a general non-reciprocal, lossy, and inhomogeneous environment, for any arbitrary number of atoms, the master equation in the interaction picture is [1]

$$\frac{\partial \rho_s(t)}{\partial t} = -\frac{i}{\hbar} [V^{AF}, \rho_s(t)] + \mathcal{L} \rho_s(t) \quad (\text{S4})$$

where

$$\begin{aligned} \mathcal{L} \rho_s(t) = & \sum_i \frac{\Gamma_{ii}(\omega_0)}{2} \left(2\sigma_i \rho_s(t) \sigma_i^\dagger - \sigma_i^\dagger \sigma_i \rho_s(t) - \rho_s(t) \sigma_i^\dagger \sigma_i \right) \\ & + \sum_{i,j}^{i \neq j} \frac{\Gamma_{ij}(\omega_0)}{2} \left([\sigma_j \rho_s(t), \sigma_i^\dagger] + [\sigma_i, \rho_s(t) \sigma_j^\dagger] \right) \\ & + \sum_{i,j}^{i \neq j} g_{ij}(\omega_0) \left([\sigma_j \rho_s(t), -i\sigma_i^\dagger] + [i\sigma_i, \rho_s(t) \sigma_j^\dagger] \right) \end{aligned} \quad (\text{S5})$$

is the Lindblad super-operator and the dissipative and coherent coupling terms are, for linear polarization,

$$\begin{aligned} \Gamma_{ij}(\omega_0) &= \frac{2\omega_0^2}{\varepsilon_0 \hbar c^2} \sum_{\alpha, \beta=x,y,z} \gamma_{\alpha i} \text{Im}(\mathcal{G}_{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j, \omega_0)) \gamma_{\beta j}, \\ g_{ij}(\omega_0) &= \frac{\omega_0^2}{\varepsilon_0 \hbar c^2} \sum_{\alpha, \beta=x,y,z} \gamma_{\alpha i} \text{Re}(\mathcal{G}_{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j, \omega_0)) \gamma_{\beta j}. \end{aligned} \quad (\text{S6})$$

For a single atom, the above equation reduces to

$$\mathcal{L} \rho_s(t) = \frac{\Gamma(\omega_0)}{2} (2\sigma \rho_s(t) \sigma^\dagger - \sigma^\dagger \sigma \rho_s(t) - \rho_s(t) \sigma^\dagger \sigma). \quad (\text{S7})$$

The term describing the laser-atom interaction is

$$V_{AF} = -\hbar (\Omega e^{-i\Delta t} \sigma^\dagger + \Omega^* e^{i\Delta t} \sigma), \quad (\text{S8})$$

where $\Omega = \boldsymbol{\gamma} \cdot \mathbf{E}/\hbar$ is the Rabi frequency and $\Delta = \omega - \omega_0$ is the detuning parameter of the laser with respect to the atom transition frequency. Considering $\Delta = 0$, the dynamics of the density matrix elements in the basis $|e\rangle$,

$|g\rangle$, are

$$\begin{aligned}\frac{\partial \rho_{ee}(t)}{\partial t} &= -\Gamma \rho_{ee}(t) + i(\Omega \rho_{ge}(t) - \Omega^* \rho_{eg}(t)), \\ \frac{\partial \rho_{eg}(t)}{\partial t} &= -\frac{\Gamma}{2} \rho_{eg}(t) + i\Omega(\rho_{gg}(t) - \rho_{ee}(t)), \\ \frac{\partial \rho_{ge}(t)}{\partial t} &= -\frac{\Gamma}{2} \rho_{ge}(t) - i\Omega^*(\rho_{gg}(t) - \rho_{ee}(t)), \\ \frac{\partial \rho_{gg}(t)}{\partial t} &= \Gamma \rho_{ee}(t) + i(\Omega^* \rho_{eg}(t) - \Omega \rho_{ge}(t)).\end{aligned}\quad (\text{S9})$$

For a single atom, the density matrix ρ_s is a 2×2 Hermitian conjugate matrix with diagonal elements representing the excited and ground state probabilities satisfying $\rho_{ee}(t) + \rho_{gg}(t) = 1$ and the off-diagonal elements represent quantum interference satisfying $\rho_{eg}(t) = \rho_{ge}^*(t)$. Solving the above coupled differential equations simultaneously for $\rho_{ee}(t)$ gives

$$\begin{aligned}\rho_{ee}(t) &= \frac{2\Omega^2}{\lambda_1 \lambda_2} + \frac{\lambda_1(\lambda_1 + \Gamma/2) + 2\Omega^2}{\lambda_1(\lambda_1 - \lambda_2)} e^{\lambda_1 t} \\ &\quad + \frac{\lambda_2(\lambda_2 + \Gamma/2) + 2\Omega^2}{\lambda_2(\lambda_2 - \lambda_1)} e^{\lambda_2 t},\end{aligned}\quad (\text{S10})$$

where

$$\lambda_{1,2} = \frac{1}{2} \left(-3\Gamma/2 \pm \sqrt{\Gamma^2/4 - 16\Omega^2} \right), \quad (\text{S11})$$

It can be shown that under steady state conditions $t \rightarrow \infty$, $\rho_{ee,ss} = 4\Omega^2/(8\Omega^2 + \Gamma^2)$.

It should be mentioned that the laser not only provides an excitation to the atom, but it also supplies momentum. The time-averaged gradient force on the atom due

to the laser is

$$\mathcal{F}^{\text{laser}} = \frac{1}{2} \nabla \text{Re} \{ \gamma^* \cdot \mathbf{E} \}. \quad (\text{S12})$$

Considering a linearly-polarized atom, and supposing, for simplicity, that the substrate acts as a perfect mirror, $E_z \propto 2E_0 \sin(k_0 z)$, then the peak force reduces to $\mathcal{F}_x^{\text{laser}} = \text{Re} \{ \gamma_z^* k_0 E_0 \}$. If we suppose the intensity of the laser $\Omega = \gamma \cdot \mathbf{E}/\hbar$ is of GHz order, and that the electric dipole moment of the atom is of order 1D (Debye), then E_z is of the order 10^5 V/m. Considering a range of transition frequencies from far to near infrared, k_0 is of order $10^4 - 10^6$ m $^{-1}$, and the force applied to the atom by the laser beam is of order $10^{-17} - 10^{-19}$ N. The normalization constant $\mathcal{F}_0 = 3|\gamma|^2/16\pi d^4 \epsilon_0$ is of order 10^{-12} N for an atom a few nm above the interface. Therefore, it can be seen that the force due to the laser can be very weak comparing to the atomic recoil force, and can be ignored.

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