Logarithmic Mathematical Morphology: a new framework robust to illumination changes

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LOGARITHMIC MATHEMATICAL MORPHOLOGY: A NEW FRAMEWORK ROBUST TO ILLUMINATION CHANGES

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ABSTRACT

A new set of mathematical morphology (MM) operators robust to illumination changes in images is defined thanks to the Logarithmic Image Processing (LIP) model which is based on physical laws of acquisition and which is consistent with human vision. The fundamental operators of logarithmic-MM, the logarithmic-dilation and the logarithmic-erosion, are defined with the LIP-addition of a structuring function. The combination of these two adjunct operators gives morphological filters, namely the logarithmic-opening and closing, useful for pattern recognition. The mathematical relation existing between "classical" dilation and erosion - with an additive structuring function - and their logarithmic-versions is established facilitating their implementation. Results on simulated and real images show that logarithmic-MM is more efficient on low-contrasted information than "classical" MM.

Index Terms— Mathematical Morphology, contrast variations, illumination changes, Logarithmic Image Processing, pattern recognition

1. INTRODUCTION

Images are functions whose values are bounded between 0 and M (e.g. 256 for 8 bits images). These values depends of the illumination conditions. During the acquisition, some parts of the image may be underexposed to light with dark values close to 0, whereas other parts may be overexposed to light with bright values close to M. Therefore, the processing should not be the same in the bright and in the dark parts. When processing grey-level images by Mathematical Morphology (MM) [1, 2, 3], dark parts and bright parts are processed in the same way and in some cases the transformed image may have values that exceed the upper limit M.

The aim of this paper is to overcome this issue by defining morphological operators few sensitive to lighting variations thanks to an appropriate model, the Logarithmic Image Processing (LIP) one [4, 5, 6, 7] which allows to brighten or darken images in a way compatible with the human visual system [8]. Previously, a morphological transform, the LIP-top-hat, was defined with the LIP model in [6]. Another model, the Symmetric Logarithmic Image Processing (SLIP) model [9] was combined with wavelets in [10]. The SLIP modifies the LIP model by making it symmetric for grey-levels. However, this modification, interesting from a mathematical point of view, is not related to the physics of acquisition.

This paper constitutes the first attempt to define morphological operators robust to lighting variations. Such a property makes the definition of Logarithmic-Mathematical Morphology of the utmost importance for many applications where the acquisition depends on the illumination (e.g. industry, outdoor scenes, video, forensics, medical images, large image databases, etc.) [11, 12, 13, 14, 15].

The paper is organised as follows: i) after a reminder about MM and the LIP model, ii) MM will be defined in the Logarithmic-additive framework. The morphological properties of the operators will be verified and iii) the Logarithmic-MM will be illustrated and compared to the classical MM.

2. PREREQUISITES

2.1. Mathematical Morphology

MM introduced by Matheron and Serra [1, 2, 16] is defined on complete lattices [17, 18].

Definition 1. Complete lattice. Given a set $\mathscr{L}$ and a partial order $\leq$ on $\mathscr{L}$, $\mathscr{L}$ is a complete lattice if every subset $\mathscr{X}$ of $\mathscr{L}$ has an infimum (a greatest lower bound), $\wedge\mathscr{X}$, and a supremum (a least upper bound), $\vee\mathscr{X}$.

The least element O and the greatest element I are two important elements of the lattice $\mathscr{L}$.

A grey-level image is a function f: $D \subset \mathbb{R}^n \rightarrow [O, M[$, with $M \in \mathbb{R}$. The space of images is denoted $\mathcal{I} = [O, M[^D$. The (bounded) set of images $\overline{\mathcal{I}} = [0, M]^D$ and the set of functions $\overline{\mathbb{R}}^D$, $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ are complete lattices with the order $\leq$. For $\overline{\mathcal{I}}$, the least and greatest elements are the constant functions equal to zero, f_0, and M, f_M. The supremum and infimum are respectively, for any $\mathscr{X} \subset \overline{\mathcal{I}}$:

$$\left(\wedge_{\overline{\mathcal{I}}}\mathscr{X}\right)(x) = \wedge_{[0,M]}\{f(x) : f \in \mathscr{X},\ x \in D\} \quad (1)$$

$$\left(\vee_{\overline{\mathcal{I}}}\mathscr{X}\right)(x) = \vee_{[0,M]}\{f(x) : f \in \mathscr{X},\ x \in D\}. \quad (2)$$

Definition 2. Erosion, dilation [18]**.** Given $\mathscr{L}_1$ and $\mathscr{L}_2$ two complete lattices, a mapping $\psi \in \mathscr{L}_2^{\mathscr{L}_1}$ is

- an erosion iff $\forall \mathscr{X} \subset \mathscr{L}_1, \psi(\wedge \mathscr{X}) = \wedge \psi(\mathscr{X})$, then $\varepsilon = \psi$;
- a dilation iff $\forall \mathscr{X} \subset \mathscr{L}_1, \psi(\vee \mathscr{X}) = \vee \psi(\mathscr{X})$, then $\delta = \psi$.

As the definitions of these mappings apply even to the empty subset of $\mathscr{L}_1$, we have: $\varepsilon(I) = I$ and $\delta(O) = O$.

Definition 3. Adjunction [3]**.** Let $\varepsilon \in \mathscr{L}_2^{\mathscr{L}_1}$ and $\delta \in \mathscr{L}_1^{\mathscr{L}_2}$ be operators between complete lattices $\mathscr{L}_1$ and $\mathscr{L}_2$; the pair (ε, δ) is called an adjunction between $\mathscr{L}_1$ and $\mathscr{L}_2$ if for all $X \in \mathscr{L}_1, Y \in \mathscr{L}_2$ there is

$$\delta(Y) \leq X \Leftrightarrow Y \leq \varepsilon(X). \tag{3}$$

Proposition 1. If (ε, δ) is an adjunction, then ε is an erosion and δ is a dilation [3].

When using an additive structuring function $b \in [O, M]^{D_b}$, $D_b \subset D$, invariant by translation (in D), the previously defined dilation δ and erosion ε, in the same lattice $(\overline{\mathbb{R}}^D, \leq)$, or $(\overline{\mathcal{I}}, \leq)$, can be simplified [19, 17]:

$$\delta_b(f)(x) = \vee \{f(x-h) + b(h), h \in D_b\} = (f \oplus b)(x) \tag{4}$$

$$\varepsilon_b(f)(x) = \wedge \{f(x+h) - b(h), h \in D_b\} = (f \ominus b)(x). \tag{5}$$

The symbols $\oplus$ and $\ominus$ represent the extension to functions [2] of Minkowski operations between sets [20, 21].

Definition 4. Opening, closing [2, 3]**.** An operator $\psi \in \mathscr{L}^{\mathscr{L}}$ on the complete lattice $\mathscr{L}$ is called:

- an opening if ψ is increasing ($\forall X, Y \in \mathscr{L}$, if $X \leq Y$ then $\psi(X) \leq \psi(Y)$), anti-extensive ($\forall X \in \mathscr{L}$, $\psi(X) \leq X$) and idempotent ($\psi \circ \psi = \psi$),
- a closing if ψ is increasing, extensive ($\forall X \in \mathscr{L}$, $X \leq \varphi(X)$) and idempotent.

Proposition 2. Let $(\varepsilon, \delta) \in \mathscr{L}_2^{\mathscr{L}_1} \times \mathscr{L}_1^{\mathscr{L}_2}$ be an adjunction between $\mathscr{L}_1$ and $\mathscr{L}_2$, then $\delta\varepsilon$ is an opening on $\mathscr{L}_1$ and $\varepsilon\delta$ is a closing on $\mathscr{L}_2$ [22, 3].

Openings (resp. closing) are morphological filters which are commonly used to remove the bright peaks (resp. the dark "holes") [19, 23].

2.2. Logarithmic Image Processing

The LIP model, introduced by Jourlin et al. [4, 5, 6, 7], is a mathematical framework for image processing based on the physical law of transmittances. Its consistency with the human visual model [8] makes it suitable not only for images acquired with transmitted light but also for images acquired with reflected light. Due to the relation, $T_f = 1 - f/M$, between the transmittance $T_f(x)$ at point x and the grey level $f(x)$, the grey scale is inverted in the LIP-model: 0 corresponds to the white extremity, when no obstacle is placed between the light source and the sensor, whereas M corresponds to the black value, when the source cannot be transmitted through the obstacle. The addition of two images corresponds to the superposition of two obstacles generating the images f and g:

$$f \mathbin{\triangleq} g = f + g - f.g/M. \tag{6}$$

The multiplication of an image f by a real number λ is deduced from equation 6, $\lambda \mathbin{\triangleq} f = M - M\,(1 - f/M)^{\lambda}$, and corresponds to a variation of thickness (or absorbance) of the object. The opposite function $\triangle f$ of f and the difference between two grey level functions f and g are expressed by:

$$\triangle f = (-f)/(1 - f/M) \tag{7}$$

$$f \mathbin{\triangle} g = (f - g)(1 - g/M). \tag{8}$$

Let us note that $\triangle f$ is not an image (as it takes negative values) and $f \mathbin{\triangle} g$ is an image if and only if $f \geq g$.

Property 1 (*A physical property*). The negative values $\triangle f$, with $f \geq 0$, corresponds to light intensifiers and can be used to compensate the light attenuation due to an object [7].

Property 2 (*Mathematical properties*). Let $\mathcal{F} =]-\infty, M[^D$ be the space of functions with values in $]-\infty, M[$. The space $(\mathcal{F}, \mathbin{\triangleq}, \mathbin{\triangle})$ is a real *vector space* and $(\mathcal{I}, \mathbin{\triangleq}, \mathbin{\triangle})$ represents the positive cone of this vector space [6, 7].

3. LOGARITHMIC-MATHEMATICAL MORPHOLOGY

MM is defined on the lattice of functions with real values $\overline{\mathbb{R}}^D$. When performing a dilation by a (non-flat) structuring function, the dilated function may have values which exceed the range $[0, M[$. In order to solve this issue and to perform operations few sensitive to light variations, let us extend MM with Logarithmic-MM, on the lattice of functions $\overline{\mathcal{F}} = [-\infty, M]^D$ with values in $[-\infty, M]$.

Proposition 3. Let $f \in \overline{\mathcal{F}}$ be a function and $b \in]-\infty, M[^{D_b}$ a structuring function, the mappings in $\overline{\mathcal{F}}$ defined by

$$\delta_b^{\mathbin{\triangle}}(f)(x) = \vee \{f(x-h) \mathbin{\triangle} b(h), h \in D_b\} \tag{9}$$

$$\varepsilon_b^{\mathbin{\triangle}}(f)(x) = \wedge \{f(x+h) \mathbin{\triangle} b(h), h \in D_b\} \tag{10}$$

form an adjunction $(\varepsilon_b^{\mathbin{\triangle}}, \delta_b^{\mathbin{\triangle}})$, with $\varepsilon_b^{\mathbin{\triangle}}$ an erosion and $\delta_b^{\mathbin{\triangle}}$ a dilation.

Definition 5. $\varepsilon_b^{\mathbin{\triangle}}$ a logarithmic-erosion and $\delta_b^{\mathbin{\triangle}}$ is called a logarithmic-dilation.

Proof: of proposition 3.

- $\delta_b^{\mathbin{\triangle}}$ is a dilation (def. 2), because $\forall f, g \in \overline{\mathcal{F}}, \forall x \in D$,
$\delta_b^{\mathbin{\triangle}}(f \vee g)(x) = \vee_{h \in D_b} \{((f \vee g)(x-h)) \mathbin{\triangle} b(h)\}$
$= \vee_{h \in D_b} \{(f(x-h) \mathbin{\triangle} b(h)) \vee (g(x-h) \mathbin{\triangle} b(h))\}$,
$\mathbin{\triangle} b(h)$ preserves the order $\leq$ (increasing operator)
$= [\vee_{h \in D_b} \{f(x-h) \mathbin{\triangle} b(h)\}] \vee [\vee_{h \in D_b} \{g(x-h) \mathbin{\triangle} b(h)\}]$
$= \delta_b^{\mathbin{\triangle}}(f)(x) \vee \delta_b^{\mathbin{\triangle}}(g)(x)$.
In addition, with $b(h) \in]-\infty, M[$, we have:
$\delta_b^{\mathbin{\triangle}}(O)(x) = \delta_b^{\mathbin{\triangle}}(f_{-\infty})(x) = \vee_{h \in D_b}\{(-\infty(x-h) \mathbin{\triangle} b(h))\}$
$= \vee_{h \in D_b}\{-\infty(1 - b(h)/M) + b(h)\} = -\infty(x)$
$= O(x)$.
- Similarly, $\forall f, g \in \overline{\mathcal{F}}$, $\varepsilon_b^{\mathbin{\triangle}}(f \wedge g) = \varepsilon_b^{\mathbin{\triangle}}(f) \wedge \varepsilon_b^{\mathbin{\triangle}}(g)$ and $\varepsilon_b^{\mathbin{\triangle}}(I) = \varepsilon_b^{\mathbin{\triangle}}(f_M) = M = I$. Therefore, $\varepsilon_b^{\mathbin{\triangle}}$ is an erosion.
- $(\varepsilon_b^{\mathbin{\triangle}}, \delta_b^{\mathbin{\triangle}})$ is an adjunction because:

$$\begin{aligned}
\delta_b^{\mathbin{\triangle}}(f) \leq g &\Leftrightarrow \forall x \in D,\ \vee_{h \in D_b}\{f(x-h) \mathbin{\triangle} b(h)\} \leq g(x) \\
&\Leftrightarrow \forall x \in D, \forall h,\ f(x-h) \mathbin{\triangle} b(h) \leq g(x) \\
&\Leftrightarrow \forall y \in D, \forall h,\ f(y) \leq g(y+h) \mathbin{\triangle} b(h) \\
&\Leftrightarrow \forall y \in D,\ f(y) \leq \wedge_{h \in D_b}\{g(y+h) \mathbin{\triangle} b(h)\} \\
&\Leftrightarrow f \leq \varepsilon_b^{\mathbin{\triangle}}(g)
\end{aligned} \tag{11}$$

Proposition 4. The *negative function* [3] is $f^*(x) = \triangle f(x)$.

Indeed, we have $(f^*)^* = f$.

Proposition 5. Let $\overline{b}(x) = b(-x)$ be the reflection of the structuring function b, the logarithmic-erosion $\varepsilon_b^{\triangle}$ and dilation $\delta_b^{\triangle}$ are dual by their *negative function*:

$$(\delta_b^{\triangle}(f^*))^* = \varepsilon_{\overline{b}}^{\triangle}(f) \quad \text{and} \quad (\varepsilon_b^{\triangle}(f^*))^* = \delta_{\overline{b}}^{\triangle}(f). \tag{12}$$

Proof: Demonstration of the first relation:

$$\begin{aligned}(\delta_b^{\triangle}(f^*))^*(x) &= \triangle(\vee_{h\in D_b}\{\triangle f(x-h) \mathbin{\triangle} b(h)\}) \\ &= \wedge_{h\in D_b}\{f(x-h) \triangle b(h)\} \\ &= \wedge_{h\in D_{\overline{b}}}\{f(x+h) \triangle \overline{b}(h)\} = \varepsilon_{\overline{b}}^{\triangle}(f)(x).\end{aligned}$$

Similarly, we have $(\varepsilon_b^{\triangle}(f^*))^* = \delta_{\overline{b}}^{\triangle}(f)$.

As $(\varepsilon_b^{\triangle}, \delta_b^{\triangle})$ is an adjunction, an opening and a closing can be defined as demonstrated in [3].

Proposition 6. Given an adjunction $(\varepsilon_b^{\triangle}, \delta_b^{\triangle})$, the operator $\gamma_b^{\triangle} = \delta_b^{\triangle}\varepsilon_b^{\triangle}$ is an opening and $\varphi_b^{\triangle} = \varepsilon_b^{\triangle}\delta_b^{\triangle}$ is a closing (by adjunction). $\gamma_b^{\triangle}$ is called a logarithmic-opening and $\varphi_b^{\triangle}$ a logarithmic-closing.

Proposition 7. Let $f \in \overline{\mathcal{F}}$ be a function and $b \in]-\infty, M[^{D_b}$ a structuring function, the logarithmic-dilation $\delta_b^{\triangle} f$ and the logarithmic-erosion $\varepsilon_b^{\triangle} f$ are related to the dilation δ and erosion ε by:

$$\delta_b^{\triangle} f = M(1 - \exp(-\delta_{\acute{b}}(\acute{f}))) \tag{13}$$

$$\varepsilon_b^{\triangle} f = M(1 - \exp(-\varepsilon_{\acute{b}}(\acute{f}))). \tag{14}$$

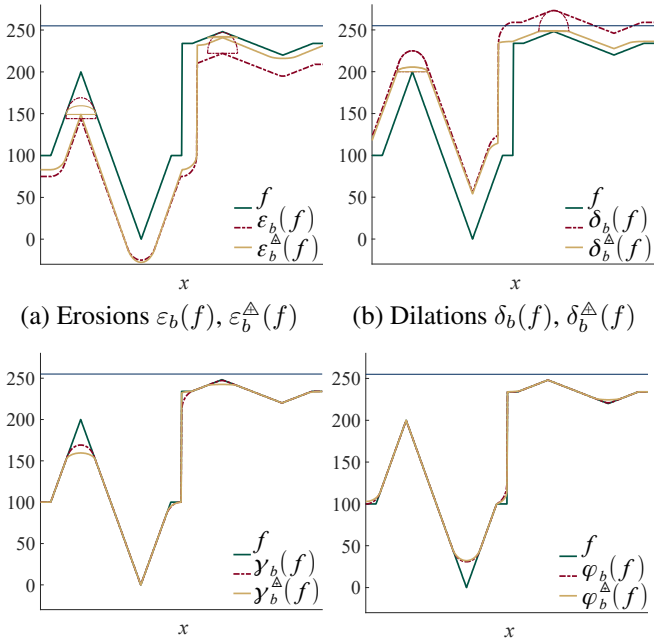
with $\acute{f} = -\ln(1 - f/M)$, $\acute{f} \in \overline{\mathbb{R}}$.

Proof: The dilation δ_b and the erosion ε_b are mappings of the lattice $\overline{\mathbb{R}}^D$, whereas the logarithmic-dilation $\delta_b^{\triangle}$ and erosion $\varepsilon_b^{\triangle}$ are mappings of the lattice $[-\infty, M]^D$. In order to link these operations, a bijective mapping (i.e. an isomorphism) is needed between these two lattices. Such an isomorphism $\xi : \overline{\mathbb{R}}^D \to [-\infty, M]^D$ and its inverse ξ^{-1} are both defined in [5] by $[\xi(f)](x) = M(1 - \exp(-f/M))$ and $\left[\xi^{-1}(f)\right](x) = -M\ln(1 - f/M)$. As increasing bijections, ξ ans ξ^{-1} distribute over infima and suprema. Therefore, the dilation $\delta_b^{\triangle}$ can be expressed by:

$$\begin{aligned}\delta_b^{\triangle} f(x) &= \xi \circ \xi^{-1}(\vee_{h\in D_b}\{f(x-h) \mathbin{\triangle} b(h)\}) \\ &= \vee_{h\in D_b}\{M(1 - e^{\ln\left(1-\frac{f(x-h)\triangle b(h)}{M}\right)})\} \\ &= M(1 - e^{\wedge_{h\in D_b}\ln\left((1-\frac{f(x-h)}{M})(1-\frac{b(h)}{M})\right)}) \\ &= M(1 - e^{-\vee_{h\in D_b}\left\{-\ln\left(1-\frac{f(x-h)}{M}\right)-\ln\left(1-\frac{b(h)}{M}\right)\right\}}) \\ &= M(1 - e^{-\delta_{\acute{b}}(\acute{f})})\end{aligned} \tag{15}$$

Similarly, we have $\varepsilon_b^{\triangle} = M(1 - e^{-\varepsilon_{\acute{b}}(\acute{f})})$.

Remark: Using proposition 7, the implementation of the logarithmic dilation and erosion becomes easier as they are related to the "classical " dilation and erosion which exist in many image analysis toolboxes.



(a) Erosions $\varepsilon_b(f)$, $\varepsilon_b^{\triangle}(f)$ (b) Dilations $\delta_b(f)$, $\delta_b^{\triangle}(f)$

(c) Openings $\gamma_b(f)$, $\gamma_b^{\triangle}(f)$ (d) Closings $\varphi_b(f)$, $\varphi_b^{\triangle}(f)$

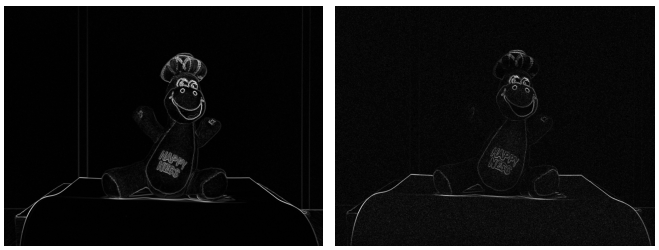
Fig. 1. Comparison between "classical" operations of MM ((a) $\varepsilon_b(f)$, (b) $\delta_b(f)$, (c) $\gamma_b(f)$, (d) $\varphi_b(f)$) and operations of Logarithmic-MM ((a) $\varepsilon_b^{\triangle}(f)$, (b) $\delta_b^{\triangle}(f)$, (c) $\gamma_b^{\triangle}(f)$, (d) $\varphi_b^{\triangle}(f)$). (a) and (b) the structuring function is represented for both peaks and for all operations $\varepsilon_b(f)$, $\varepsilon_b^{\triangle}(f)$, $\delta_b(f)$ and $\delta_b^{\triangle}(f)$.

4. RESULTS

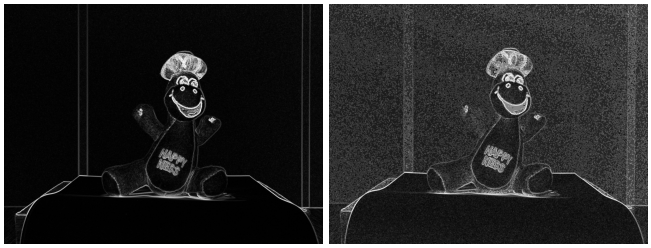
In figure 1, the "classical" operations of MM and those of Logarithmic-MM are compared on a simulated signal. For each operation of Logarithmic-MM, the amplitude of the structuring function (sf) changes according to the grey-level values of the image because the LIP-laws, $\triangle$ or $\triangle$, are used in equations 9 and 10, whereas for all the operations of "classical" MM the amplitude of the sf remains the same. In figure 1 (b), the values of the logarithmic-dilation $\delta_b^{\triangle}(f)$ always remain below the upper limit $M = 256$, whereas for the "classical" dilation $\delta_b(f)$, some grey-levels may exceed the limit M. This property is due to the LIP addition law $\triangle$. In figure 1 (a), the lowest values of both erosions $\varepsilon_b^{\triangle}(f)$ and $\varepsilon_b(f)$ are negative by definition of both operations with the laws $\triangle$ (eq. 10) and $-$ (eq. 5). For real functions $\overline{\mathbb{R}}^D$, the negative values have no physical justification, whereas, in the LIP model they can be interpreted as light intensifiers [7]. In figure 1 (c) (resp. (d)), the disparity between the openings $\gamma_b(f)$ and $\gamma_b^{\triangle}(f)$ (resp. closings $\varphi_b(f)$ and $\varphi_b^{\triangle}(f)$) is greater for the grey-levels close to $M = 256$ than for those close to zero. Indeed, the LIP model decreases the grey-level differences close to M.

In figure 2, two images of the same scene are acquired at two different exposure time (i.e. shutter speed): a bright image f and a dark image f^d. Both images f and f^d are complemented ($f^c = M - 1 - f$) before computing a morphological gradient $\varrho_b(f) = \delta_b(f) - \varepsilon_b(f)$ [23] and its logarithmic version $\varrho_b^{\triangleq}(f) = \delta_b^{\triangleq}(f) \triangleq \varepsilon_b^{\triangleq}(f)$. For comparison purpose, the amplitudes of each gradient are scaled between 0 and 255 (Fig. 2 (c,d,e,f)). The logarithmic-gradient of the dark image $\varrho_b^{\triangleq}([f^d]^c)$ (Fig. 2 f) finds much more contours than the "classical" one $\varrho_b([f^d]^c)$ (Fig. 2 d). Even on the bright image f, the logarithmic-gradient $\varrho_b^{\triangleq}(f^c)$ (Fig. 2 e) finds more contours than the "classical" one $\varrho_b^{\triangleq}(f)$ (Fig. 2 c) especially on the darkest parts.

(a) Bright image f (b) Dark image f^d



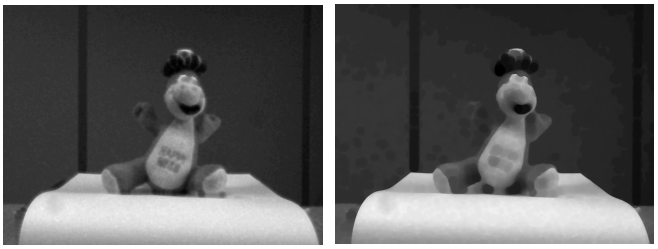
(c) Gradient $\varrho_b(f^c)$ (d) Gradient $\varrho_b([f^d]^c)$



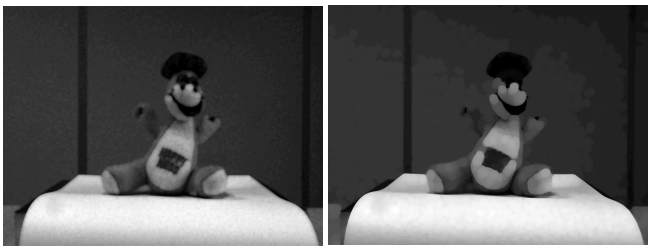
(e) L-gradient $\varrho_b^{\triangleq}(f^c)$ (f) L-gradient $\varrho_b^{\triangleq}([f^d]^c)$

Fig. 2. Comparison between (c, d) the morphological gradients ϱ_b and (e, f) the Logarithmic-morphological gradients $\varrho_b^{\triangleq}$ on (a) a bright image f (acquired with an exposure time of 1/40 s) and (b) on a dark image of the same scene (exposure time of 1/800 s). The structuring function b is an hemisphere of radius 2 pixels.

In figure 3, an opening $[\gamma_b([f^d]^c)]^c$, a closing $[\varphi_b([f^d]^c)]^c$, a logarithmic-opening $[\gamma_b^{\triangleq}([f^d]^c)]^c$ and a logarithmic-closing $[\varphi_b^{\triangleq}([f^d]^c)]^c$ are compared on the complement of the dark image f^d (Fig. 2 b), using an hemisphere of radius 15 pixels as structuring function. For comparison purpose, the amplitudes of each filtered image are scaled between 0 and 255. The "classical" opening and closing (Fig. 3 a, c) have a limited effect in terms of transformation whereas the logarithmic-opening and closing have a more important effect (Fig. 3 b, d). E.g. on the body of the soft toy monster, the words are removed with the logarithmic-opening and closing and not with the "classical" opening and closing.



(a) Opening $[\gamma_b([f^d]^c)]^c$ (b) L-opening $[\gamma_b^{\triangleq}([f^d]^c)]^c$



(c) Closing $[\varphi_b([f^d]^c)]^c$ (d) L-closing $[\varphi_b^{\triangleq}([f^d]^c)]^c$

Fig. 3. (a) Opening $[\gamma_b([f^d]^c)]^c$ and (b) logarithmic-opening $[\gamma_b^{\triangleq}([f^d]^c)]^c$ on the dark image f^d. (c) Closing $[\varphi_b([f^d]^c)]^c$ and (d) logarithmic-closing $[\varphi_b^{\triangleq}([f^d]^c)]^c$. The structuring function b is an hemisphere of radius 15 pixels. The grey-levels are scaled between 0 and 255.

Those examples illustrate the robustness of the logarithmic morphological operators on images with important contrast variations caused by different illumination conditions.

5. CONCLUSION AND PERSPECTIVES

Logarithmic-mathematical morphology is introduced in this paper. The fundamental operators of logarithmic-dilation $\delta_b^{\triangleq}$ and erosion $\varepsilon_b^{\triangleq}$ are defined for a structuring function thanks to the LIP-addition law $\triangleq$. Their expressions are related to the "classical" dilation δ_b and ε_b facilitating their implementation. As both operators form an adjunction, a logarithmic-opening and closing are defined. The logarithmic-MM is compared to the "classical" MM based on an additive structuring function through several examples. Results show that Logarithmic-MM operators are particularly efficient to detect contrast variations in the dark parts (and also in the bright parts) of images caused by different illumination conditions. In future, similarly to [24], logarithmic operations will be studied for pattern recognition tasks and will be extended to colour and multivariate images.

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