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MRP parameter evaluation under fuzzy lead times

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Abstract: One of the main problems in supply chains is the supply planning of an actor (customer) in function of others network actors (suppliers). In this context, the knowledge of the supplier lead times uncertainty is essential in the client's risk management process. In this article we are particularly interested in the management of risks by an actor responsible of assembly activity. We suppose that each supplier is able to give his uncertain lead time by the meaning of intervals. In this collaborative context, we search to determine robust assembly process. In this study, we treat the case of a single-product and regular launches of the assembly. For this, we will propose a method based on MIP formulation and relaxation constraints approach to determine robust optimal suppliers' lead times which minimize the both risk of assembly delay and storage of components.

Keywords: Uncertainty, Fuzzy number, MRP, Lead-time.

1. INTRODUCTION AND RELATED WORKS

Faced to uncertainty, supply chains have implemented processes to mitigate risks. Planners plan their production using uncertain data as procurement plan (from customer to suppliers), lead times (from suppliers to customer).

An important factor of uncertainty is the difference between suppliers planned lead times (information flow) and suppliers real lead times (material flow). Indeed, this lead time is often uncertain and variable and it depends on several constraints: transportation time, prices, machine breakdowns, capacity constraints.

Several works in the literature (Tang and Musa, 2011, Koh et al., 2002, Guide and Srivasta, 2000) review identify risks in supply chain and proposed several techniques to establish a client's risk management process.

Logistics, demand volatility, supply product-monitoring, supplier selection, quality and price are identified and classified by Tand and Musa (2011) as potential risks in a supply chain. Another classification is done by Wazed et al. (2009) which consider demand, capacity and lead time uncertainties as the largest factors of risks in a manufacturing environment.

In the literature, risks associated to the suppliers' lead times appear to be insufficiently studied favouring identifying risks issues as demand volatility, quality, excess inventory (Tand and Musa, 2011). Safety stocks, safety lead times and other measures are used by planners to control uncertainties in supply and demand (Van Kampen Tim et al., 2010).

In the field of MRP (Material Requirement Planning), Dolgui et al., (2013), Damand et al. (2011) and Dolgui and Prodhon (2007) had studied the MRP parameterization under risks and classified the important techniques to reduce it. Several works made clear that safety lead times are a main parameter to cope the variability of suppliers lead time. Dolgui et al. (2008) are considered two-level assembly systems with random component procurement times. They concluded that in the literature, the demand variability seems to be more studied than the lead time uncertainties.

Recently, Ben Ammar et al. (2013a, 2013b, 2014) have study the case under stochastic lead times. Nevertheless, in some cases, it seems be difficult to access to the probability distribution of the lead time.

The possibility distribution is often used to model uncertainty in the domain of supply chain management (Peidro et al., 2009, Díaz-Madroñero et al., 2014). In this paper we propose to model the uncertainty of the lead time using possibility distribution (Dubois and Prade, 2006) since this model requires less information than probability distribution (only the mode, the maximal and minimal value of the lead time).

To deal with the possibility distribution we can distinguish several different approaches. The first one consists in using a function aiming at ranking fuzzy numbers in order to allow the decision maker to defuzzify imprecise values. Peidro et al. (2009), Liang (2008) and Liang and Cheng (2009) apply this approach for uncertainty on demand quantity. After the defuzzification process, the result is a classical linear optimization system.

A second approach, based on the fact that a possibility distribution can be seen as a set of probability distributions, consists in choosing one of those probability distributions, according to the attitude of the decision maker (e.g. pessimistic using necessities, optimistic through possibilities, etc.). Gao-Ji and Yan-Kui (2008) apply this approach on demand uncertainty. Once this choice has been made, it is possible to use a stochastic optimization model. So, the decision maker chooses for which probability distribution his solution will be optimal.

A third approach is possibilistic optimization, which tries to find the solution which minimizes the cost and maximizes the possibility level of a considered scenario. In other terms, this approach finds a "possibly optimal" solution; the level of possibility that the considered scenario will happen is therefore maximal. Mula et al. (2007) apply this approach for fuzzy constraints and demand quantity.

The fourth approach is a generalisation of the "robust optimization" to the case of fuzzy cost function. The objective is to minimize the necessity level that a cost function is greater than a given level. This approach has been applied to the case of periodic demand uncertainty (Guillaume et al. 2012) and cumulative demand uncertainty (Guillaume et al. 2013). But under our knowledge, this approach does not have been applied to the lead-time uncertainty.

Another one is to propose a decision support approach which consists in showing all possible solutions due to the uncertainty. In other worlds, it propagates the uncertainty thought the MRP and computes all possible released quantities. This approach has been developed firstly for MRP under uncertainty on quantity of demand (Grobot et al. 2005). Then it has been generalized to take into account the uncertainty on the date of the demand (Guillaume et al. 2011a) on quantity and finally has been applied for uncertain lead time (Guillaume et al. 2011b).

In this paper, the main target is to help the decision maker to choose a solution under uncertainty and not to compute one. The question is how to determine the planned lead time parameter in function of uncertainties and of the variability of the real lead time.

To apply the fourth approach aforementioned, the first step involves computing the possibility distributions of cost under uncertainty on lead-time. The case of a single product and regular launches of the assembly are considered. The final product is assembled using several types of components. Each type of components is delivered by a given supplier. The suppliers' lead time uncertainty is supposed to be shared by the supplier and the customer and varies in a fuzzy interval.

The rest of paper is organized into five sections. In section 2, the problem is described and the background is presented. Section 3 shows how the maximal-minimal fuzzy costs are estimated. The introduced method is illustrated with a numerical example in section 4. At the conclusion of the study (section 5), some future perspectives are detailed.

2. PRESENTATION OF THE PROBLEM AND BACKGROUND

In this section, the considered problem and the related background on MRP and uncertainty models are presented.

2.1 Description of the considered problem

In this paper, we suppose that the customer orders components from suppliers. The MRP perform depends on the parametrization of the suppliers planned lead times which determines the level of inventory or the level of backordering. However, the lead times of some components depend on suppliers and its uncertainty could increase instability in the supply chain.

In addition, we suppose that it seems to be difficult for the supplier to know the real lead time with precision. Nevertheless, the supplier has knowledge of the lead time uncertainty which can be shared with the customer.

In this context, the main idea is to help the customer planners to choose the appropriate planned lead times of the MRP which minimize the risks of both backordering of the final product and inventory of components. In this study the demand of the final product is supposed constant and known.

So, in this paper a method is proposed to help planners to parametrize the lead times of suppliers under some hypothesis. More precisely, we suppose that:

- A given component is only used for a final product and does not appear at several levels of the bill of materials
- The demand is constant and known over the horizon
- The component inventory is always preferred to the product inventory (added value in the assembly process)

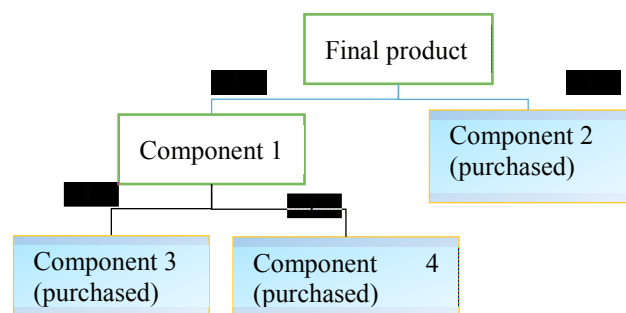


Fig. 1. The bill of materials where the final product and the components 1 are produced by the customer and the components 2, 3 and 4 by different suppliers.

2.2 Material Requirement Planning (MRP)

In supply chains, each actor uses the MRP process to compute both production and procurement plans. In this section, we present a linear model of MRP. Nevertheless, a set of other possible formulations of this problem exists. For example it depends on the planning rules, set up cost or capacity constraints. The presented model uses an unlimited capacity and a Lot-for Lot policy.

Let's consider \mathbb{P} the set of final products and \mathbb{C} the set of components (produced or supplied). Backordering cost for the final products ($p \in \mathbb{P}$) and inventory holding costs for components ($c \in \mathbb{C}$) are considered. Only the external demand for the final product is introduced.

Let the following parameters:

- $D_{p,t}$: the demand of final product $p \in \mathbb{P}$, for the period $t \in \mathbb{T}$
- Ld_i : the planned lead time of product $i \in \mathbb{P} \cup \mathbb{C}$
- $R_{c,i}$: the quantity of component $c \in \mathbb{C}$ required to produce one product $i \in \mathbb{P} \cup \mathbb{C}$
- h_c : cost of handing the component $c \in \mathbb{C}$
- b_p : cost of backordering the final product $p \in \mathbb{P}$

Let the following variables:

- Decision variables:
 - $Pr_{i,t}$: the production (or supply) quantity of product $i \in \mathbb{P} \cup \mathbb{C}$, for the period $t \in \mathbb{T}$
- Dependent variables:
 - $I_{c,t}$: inventory of component $c \in \mathbb{C}$, at the end of the period $t \in \mathbb{T}$
 - $B_{p,t}$: backordering of final product $p \in \mathbb{P}$, at the end of the period $t \in \mathbb{T}$

In this context the objective function is to minimize the cost which is the sum of the inventory holding costs for components and the backlogging costs for the final products under constraint:

$$\min \sum_{t \in \mathbb{T}} \left(\sum_{c \in \mathbb{C}} I_{c,t} \times h_c + \sum_{p \in \mathbb{P}} B_{p,t} \times b_p \right) \quad (1)$$

With:

- Production constraints

$$Pr_{i,t} \geq 0 \quad \forall i \in \mathbb{P} \cup \mathbb{C}, t \in \mathbb{T} \quad (2)$$

- Material flow constraints for the final products

$$B_{p,t} = \sum_{\tau=1}^t D_{p,\tau} - \sum_{\tau=1}^{t-Ld_p} Pr_{p,\tau} \quad \forall p \in \mathbb{P}, t \in \mathbb{T} \quad (3)$$

- Backordering constraints for the final products

$$B_{p,t} \geq 0 \quad \forall p \in \mathbb{P}, t \in \mathbb{T} \quad (4)$$

- Material flow constraints for the components

$$I_{c,t} = \sum_{\tau=1}^t Pr_{c,\tau-Ld_c} - \sum_{\tau=1}^t \sum_{j \in \mathbb{P} \cup \mathbb{C}} (Pr_{j,\tau} \times R_{c,j}) \quad (5)$$

$$\forall c \in \mathbb{C}, t \in \mathbb{T}$$

- Inventory constraints for components (the demand is the requirement at the period t of the next product p of the bill of material):

$$I_{c,t} \geq 0 \quad \forall c \in \mathbb{C}, t \in \mathbb{T} \quad (6)$$

2.3 Model of uncertainty

In this paper we suppose that the lead time shared by the supplier to the customer is known with uncertainty (since it depends on the capacity and demand of supplier). We propose here that the supplier sends a fuzzy lead time to the customer.

So this lead time is modelled by fuzzy intervals: $[F_c(\alpha), F_c(\alpha) + \Delta_c(\alpha)]$, $\forall \alpha \in [0,1]$ (see Fig.2), where $F_c(\alpha)$ is the lower value of the lead time for the possibility α of component c and $\Delta_c(\alpha)$ the uncertainty of the lead time for the possibility α . If possibility of a value is 1, it means that it is the most possible value of lead time. Otherwise, if the possibility of this value is 0, it means that it is an impossible value of lead time. A trapezoidal possibility distribution can be built by decision maker by giving the smallest interval in which he/she thinks the value of lead time will be.

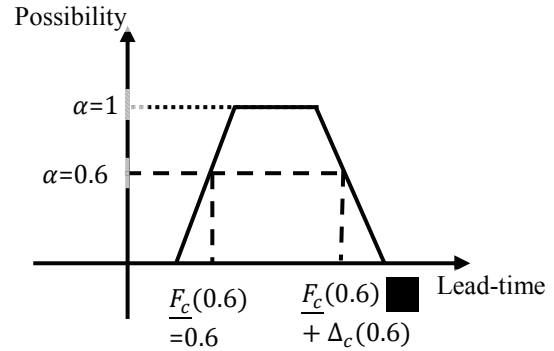


Fig. 2. Representation of a trapezoidal fuzzy interval.

3. ESTIMATION OF MAXIMAL FUZZY COST OF A LEAD TIME PARAMETRIZATION

In this section, we present the formulation of the problem to estimate the maximal and the minimal possible impacts of lead time uncertainty on different costs. First we solve the problem for a given value of $\alpha = \alpha^*$, so this problem is a problem under classical intervals $[F_c(\alpha^*), F_c(\alpha^*) + \Delta_c(\alpha^*)]$. Then, it is possible to obtain both maximal and minimal possible costs for a possibility degree α^* . Hence, to build the possibility distribution of cost for a given lead time, we need to solve the interval problem for a set of $\alpha^* \in \{\varepsilon, 2\varepsilon, \dots, 1\}$ with ε the estimation step.

3.1 Formulation of problems

Let's define, $Lr_{i,t}$ the real lead time of product i released at period t . More over a product i , $\forall i \in \mathbb{P} \cup \mathbb{C}$, is assembled from components $c \in \mathbb{C}$, themselves are produced ($Pred(c) \neq \emptyset$) or are ordered from suppliers ($Pred(c) = \emptyset$). In the same way, $Succ(c)$ is the final product or the component which needs the component c .

The maximal and minimal cost over all possible scenarios of lead time can be formulated as an optimization problem with respectively the objective (7) and (8):

$$\max_{Lr_c \in [\underline{F}_c(\alpha^*), \underline{F}_c(\alpha^*) + \Delta_c(\alpha^*)]} \sum_{t \in \mathbb{T}} \left(\sum_{c \in \mathbb{C}} I_{c,t} \times h_c + \sum_{p \in \mathbb{P}} B_{p,t} \times b_p \right) \quad (7)$$

$$\min_{Lr_c \in [\underline{F}_c(\alpha^*), \underline{F}_c(\alpha^*) + \Delta_c(\alpha^*)]} \sum_{t \in \mathbb{T}} \left(\sum_{c \in \mathbb{C}} I_{c,t} \times h_c + \sum_{p \in \mathbb{P}} B_{p,t} \times b_p \right) \quad (8)$$

This modification affects the constraints of the MRP model (the first 6 equations) by taking into account the difference between the planned lead time LD_c and the real lead time Lr_{ct} .

Let the following variables:

- Dependent variables:
 - $Adr_{i,t}$: quantity of product $i \in \mathbb{P} \cup \mathbb{C}$ which can be assembled at period $t \in \mathbb{T}$
 - $A_{i,t,t'}$: quantity of product $i \in \mathbb{P} \cup \mathbb{C}$ which arrives at period $t \in \mathbb{T}$ and which has been released at period t'
 - $Ar_{c,t}$: quantity of component $c \in \mathbb{C}$ which arrives at period $t \in \mathbb{T}$
 - $ARR_{i,t}$: quantity of product $i \in \mathbb{P} \cup \mathbb{C}$ which is really assembled at period $t \in \mathbb{T}$
- Decision variables:
 - $Pr_{c,t}$: planned production of component , $\forall c \in \mathbb{C}$ at the period $t \in \mathbb{T}$

The constraints become:

$$I_{c,t} = \sum_{\tau=1}^t Ar_{c,\tau} - \sum_{\tau=1}^t R_{i,c} \times ARR_{i,\tau} \quad (9)$$

$$i = Succ(c), \forall c \in \mathbb{C}, \forall t \in \mathbb{T}$$

$$B_{p,t} = \sum_{\tau=1}^t D_{p,c} - \sum_{\tau=1}^t Ar_{p,t} \quad \forall p \in \mathbb{P}, \forall t \in \mathbb{T} \quad (10)$$

$$A_{c,t+Lr_{c,t}} = Pr_{c,t} \quad (11)$$

$$\forall c \in \{\mathbb{C}: Pred(c) = \emptyset\}, \forall t \in \mathbb{T}$$

$$A_{c,t+Lr_{c,t}} = ARR_{i,t} \quad (12)$$

$$\forall c \in \{\mathbb{C}: Pred(c) \neq \emptyset\}, \forall t \in \mathbb{T}$$

$$Ar_{i,t} = \max \left(\sum_{\tau=1}^t A_{i,t,\tau}; 0 \right) \quad \forall i \in \mathbb{P} \cup \mathbb{C}, \forall t \in \mathbb{T} \quad (13)$$

$$\sum_{\tau=1}^t Adr_{i,t} = \max_{c \in Pred(i)} \left(\sum_{\tau=1}^t Ar_{c,\tau} \div R_{i,c} \right) \quad (14)$$

$$\forall i \in \mathbb{P} \cup \mathbb{C}, \forall t \in \mathbb{T}$$

$$\sum_{\tau=1}^t ARR_{i,t} = \min \left(\sum_{\tau=1}^t Adr_{i,t}; \sum_{\tau=1}^t Pr_{i,t} \right) \quad (15)$$

$$I_{c,t} \geq 0 \quad \forall i \in \mathbb{P} \cup \mathbb{C}, \forall t \in \mathbb{T} \quad (16)$$

$$\forall c \in \mathbb{C}, t \in \mathbb{T}$$

$$B_{p,t} \geq 0 \quad \forall p \in \mathbb{P}, \forall t \in \mathbb{T} \quad (17)$$

This formulation is not computable since decision variables are indexed. In the next section, we propose a mixed integer formulation of both maximization and minimization of cost problems under uncertain lead time.

3.2 Evaluation of maximal and minimal cost under uncertain lead time

Firstly we will show that under the hypothesis of known and constant demand, we can consider only the single period problem. Since, the best and the worst case for each demand of the horizon is the same.

For the best case it is trivial, since the less costly lead time cannot be inflated by the previous or the next period. For the worst case we have two cases, if the worst case is to be late for a given period, this worst case will be the same for all periods; and if all components are late there are no compensation between previous and next periods. Otherwise, if the worst case is to be early for the demand it is the same reasoning.

To take off the lead time in the index of constraints (4) and (5), three decisions variables are introduced:

- $\gamma_c \in [0,1]$: variable which indicates where we are in the interval of lead time (0 means lower bound and 1 upper bound) of component $c \in \mathbb{C}$
- d_c : the date of availability of component $c \in \{\mathbb{C}: Pred(c) \neq \emptyset\}$
- dt_i : the real date of assembly of product $i \in \mathbb{P} \cup \mathbb{C}$

We add two parameters:

- a_c : the number of component $c \in \mathbb{C}$ required to satisfy the demand
- l_i : the planned date to order the component $i \in \mathbb{P} \cup \mathbb{C}$

Now, the objective function does not depend on planning horizon. It can be expressed as:

$$\max_{\gamma_c, \forall c \in \mathbb{C}} \sum_{c \in \mathbb{C}} I_c \times h_c + \sum_{p \in \mathbb{P}} B_p \times b_p \quad (18)$$

Or

$$\min_{\gamma_c, \forall c \in \mathbb{C}} \sum_{c \in \mathbb{C}} I_c \times h_c + \sum_{p \in \mathbb{P}} B_p \times b_p$$

The constraints are:

- Precedence constraints:

$$d_i = \max_{c \in Pred(i)} \left(dt_c + \underline{F}_c(\alpha^*) + \gamma_c \Delta_c(\alpha^*) \right) \quad (19)$$

$$\forall i \in \{\mathbb{P} \cup \mathbb{C}: Pred(i) \neq \emptyset\}$$

- Ready dates constraints: a product i , $\forall i \in \mathbb{C}$, cannot be assembled before its planned date:

$$dt_i = \max(d_i, l_i) \quad \forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(i) \neq \emptyset\} \quad (20)$$

$$dt_c = l_c \quad \forall c \in \{\mathbb{C} : Pred(c) = \emptyset\} \quad (21)$$

- Inventory constraints for components:

$$I_c \geq 0 \quad \forall c \in \mathbb{C} \quad (22)$$

$$I_c = a_c \left(dt_i - (dt_c + \underline{F}_c(\alpha^*) + \gamma_c \Delta_c(\alpha^*)) \right) \quad (23)$$

$$i = Succ(c), \forall c \in \mathbb{C}$$

- Backordering constraint for the final products:

$$B_p \geq 0 \quad \forall p \in \mathbb{P} \quad (24)$$

- Backordering constraint of final product p for the horizon:

$$B_p = D_p(dt_p - l_p) \quad \forall p \in \mathbb{P} \quad (25)$$

In the case of minimization, the function max of constraints (19) and (20) can be easy linearized using \geq relation:

$$d_i \geq dt_c + \underline{F}_c(\alpha^*) + \gamma_c \Delta_c(\alpha^*) \quad (26)$$

$$\forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(i) \neq \emptyset\}, \forall c \in Pred(i)$$

$$dt_i \geq d_i \quad \forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(c) = \emptyset\} \quad (27)$$

$$dt_i \geq l_i \quad \forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(c) = \emptyset\} \quad (28)$$

Unfortunately for the maximization problem, we need to add binary decision variables to linearize the constraints (19) and (20).

So the constraints (19) can be reformulated using 3 constraints (29), (30) and (31). Let M a big value and δ_c a binary variable thus that $\delta_c = 1$ if the maximum is reached for the component c zero otherwise:

$$d_i \leq dt_c + \underline{F}_c(\alpha^*) + \gamma_c \Delta_c(\alpha^*) + (1 - \delta_c)M \quad (29)$$

$$\forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(i) \neq \emptyset\}, c \in Pred(i)$$

$$d_i \geq dt_c + \underline{F}_c(\alpha^*) + \gamma_c \Delta_c(\alpha^*) \quad (30)$$

$$\forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(i) \neq \emptyset\}, c \in Pred(i)$$

$$\sum_{c \in Pred(i)} \delta_c \geq 1 \quad \forall i \in \{\mathbb{P} \cup \mathbb{C} : Pred(i) \neq \emptyset\} \quad (31)$$

In the same way the constraints (20) can be reformulated using 3 constraints (32), (33) and (34) and two binaries variables β_c^d and β_c^l :

$$d_c + (1 - \beta_c^d)M \geq dt_c \geq d_c \quad (32)$$

$$\forall c \in \{\mathbb{P} \cup \mathbb{C} : Pred(c) = \emptyset\}$$

$$l_c + (1 - \beta_c^l)M \geq dt_c \geq l_c \quad (33)$$

$$\forall c \in \{\mathbb{P} \cup \mathbb{C} : Pred(c) = \emptyset\}$$

$$\beta_c^d + \beta_c^l \geq 1 \quad \forall c \in \{\mathbb{P} \cup \mathbb{C} : Pred(c) = \emptyset\} \quad (34)$$

4. NUMERICAL EXAMPLE

In this example, we consider the bill of material presented in figure 2.

The demand of final product p is equal to 100 for the date 12. The lead time of components 2, 3 and 4 are uncertain:

- For component 2, it is triangular fuzzy interval:

$$\underline{F}_2(0) = 1, \Delta_2(0) = 3, \underline{F}_2(1) = 2, \Delta_2(1) = 0$$

- For component 3, it is trapezoidal fuzzy interval:

$$\underline{F}_3(0) = 2, \Delta_3(0) = 4, \underline{F}_3(1) = 3, \Delta_3(1) = 1$$

- For component 4, it is triangular fuzzy interval:

$$\underline{F}_4(0) = 1, \Delta_4(0) = 4, \underline{F}_4(1) = 1, \Delta_4(1) = 0$$

The lead times of the final product and the component 4 are the crisp value 1.

The backlogging cost b_p for the final product is equal to 50. The inventory costs for components are: $h_1 = 10$, $h_2 = 8$, $h_3 = h_4 = 5$.

The Decision Maker wants to evaluate the impact of uncertainty of two possible parametrizations of lead times: $c = 2 \Rightarrow 2$, $c = 3 \Rightarrow 4$ and $c = 4 \Rightarrow 2$ or $c = 4 \Rightarrow 1$. So, $\forall c \in \mathbb{C}$, $l_c = \{11, 10, 8, 9\}$ for the first parametrization and $l_c = \{11, 10, 8, 10\}$ for the second one. The possible distributions of possible cost are represented in Fig.3. The second one seems to be best for the most possible values. Nevertheless, for $\alpha \geq 0.9$, the first parametrization is better than the second one for the upper bound. Both parameterizations are equivalent for the lower bound for $\alpha \geq 0.7$.

In conclusion, the first parametrization is most robust and less subject to uncertainty than the second one.

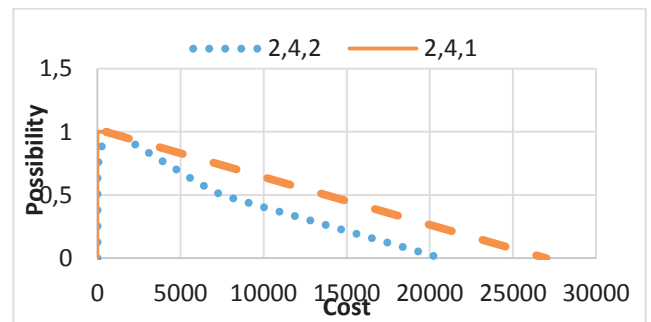


Fig. 3. Representation of possibility distribution of cost for two possible parametrization.

5. CONCLUSION AND PERSPECTIVES

In this paper, we are interested in a supply chain where the customer orders components from suppliers. However, the lead times of some components depend on suppliers and could increase instability in the supply chain. In other words, in the case of lead time uncertainty the MRP performance of the customer's ERP depends on the parametrization of the suppliers planned lead times to parameterize the level of inventory or the level of backordering.

In addition, we supposed that it seems to be difficult for the supplier to know the real lead time with precision. Nevertheless, the supplier has knowledge of the lead time uncertainty which is shared with the customer. In this context, the main idea is to help the customer planners to choose the appropriate planned lead times of the MRP which minimize the risks of both backordering of the final product and inventory of components. In this study the demand of the final product is supposed constant and known.

Our future work will focus on the parameterization of the MRP system under uncertain lead times, limited capacity and variable demand.

The main objective will be to parameterize MRP system under several uncertainties as lead times, demand and capacity.

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