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Rotation of a magnetized plasma

B. M. Annaratone, a) A. Escarguel, b) T. Lefevre, C. Rebont, N. Claire, and F. Doveil

PIIM, UMR6653 CNRS/Université de Provence, Campus St. Jérôme, Case 321, F-13397, Marseille, France

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The plasma rotation in the axial magnetic field of the linear machine Mistral [A. Escarguel, Eur. Phys. J. D 56, 209 (2010)] is well described by the assumption that the electrons injected from the source exit radially from the central column and are subject to the Lorentz force. Electrons and ions rotate together by ambipolarity. The solution of the momentum equations foresees correctly the observed radial dependence of the ionic radial velocity measured by laser induced fluorescence. The resolution of these equations is also in good agreement with the measured dependence of the rotation frequency on the applied magnetic field and on the background pressure. © 2011 American Institute of Physics. [doi:10.1063/1.3566004]

Plasma transport in a magnetic field is “anomalous” in the sense that it is still not completely predictable in new configurations. From this derives the need of studying the physics of magnetized plasmas not only in fusion devices but also in laboratory machines where parameters can be better controlled.  

Mistral is a typical linear machine to study the plasma instabilities developing across the magnetic field lines. It addresses the low temperature plasma physics near the limiter and its shadow, in tokamaks, or other reactors aiming at controlled atomic fusion for energy purposes. Several papers have studied the plasma instabilities in Mistral; see for example, Refs. 6 and 7 and the references therein; other papers present the results of diagnostic investigation.  

A similar regime of magnetization and geometry have been dealt in Refs. 10 and 11 in a different context, i.e., Hall thrusters. No effort has been attempted, in the authors’ knowledge, to unravel the dependence on the frequency of the observed turbulences. With increasing magnetic field, the plasma, initially uniformly spread, concentrates in an internal plasma column from which one (mode m=1) secondary plasma, here also called ‘plasma arm’, uniform in the axial coordinate, attempts to reach the walls. In doing so, the ‘arm’ rotates around the internal plasma column. At higher magnetic fields more ‘arms’ appear, (higher m modes). In this paper, we disclose the theoretical analysis of the rotation frequency of the m=1 mode. The theory correctly predicts the experimental dependences on the magnetic field and on the background gas pressure; it provides also the radial velocity of ions, in good agreement with the laser induced fluorescence measurements.

I. THE EXPERIMENTAL CONFIGURATION

The experimental device has been described elsewhere,  

see also Fig. 1. It consists of a source chamber in which plasma is produced by the thermionic emission of electrons accelerated to a large anode. Energetic electrons can overcome a grid kept at ~25 eV and enter in the magnetized plasma column. They are reflected by the floating end plate and undergo several small angle elastic collisions before colliding in ionizing or exciting collisions. The electron energy distribution shows clearly, see Ref. 8, a directional part, “primary” electrons, and cold locally generated electrons. The primary electrons ensure the uniformity of plasma parameters all along the axial z direction. The space potential of the plasma column settles down to a negative voltage with respect to ground and this difference in potential forces electrons to travel perpendicular to the magnetic field lines, B=Bz, in order to close the path of the current to the grounded walls. A laser induced fluorescence (LIF) diagnostic is used to measure the velocity distribution function of argon ions in the ‘plasma arm’.  

II. THE MODEL

Our model is based on two basic assumptions. The first is that the plasma arm rotating around the central column is quasineutral. It is much larger than several Debye lengths (typically $4 \times 10^{-4}$ m) and fluctuations frequencies are lower than $\omega_{pi}$ (1 MHz). The second assumption is that ions lose momentum by collisions with the neutrals while the electrons collide rarely (the electron mean free path, $\lambda_{pe}$, is 2 m at $P=10^{-2}$ Pa, much larger than the device radius) and there is only a small loss of momentum per collision because of the small mass. Moreover, we neglect the ionization in the plasma arm in comparison with the fluxes of ion and electrons, i.e., $\partial(nu_r)/\partial r = 0$ with $n$ the plasma density and $u_r$ the particles’ radial velocity. This is justified because the ionization rate, and the volume, of the central column are much higher than those of the ‘arm’. The electron cloud, traveling radially toward ground experiences an averaged density of Lorentz force, $F_L$, in the azimuthal direction, $\varphi:F_L=neu_\varphi B$, with $e$ the charge of the electron and $u_\varphi$ the electron drift velocity in the radial direction. The presence of an azimuthal electric field is probably the origin of the instability that controls the rotation of the plasma arm around the central column. The stationary momentum equations for the ions and the electrons in the azimuthal direction are, respectively.
different, then the azimuthal velocities of ions and electrons are not too large compared to the fluid velocity. LIF results,13,16 respectively:

\[ \frac{\partial v_{\varphi}}{\partial t} + v_r \frac{\partial v_{\varphi}}{\partial r} + v_\varphi = 0 \]  

(1)

\[ \frac{\partial v_{\varphi}}{\partial t} + v_r \frac{\partial v_{\varphi}}{\partial r} + v_\varphi = 0 \]  

(2)

with \( M \) the mass of the ions, \( Z \) the ionization rate of argon, \( v_r \) and \( v_\varphi \) (resp. \( v_{r_{i}} \) and \( v_{\varphi_{i}} \)) the ions and electrons radial (resp. azimuthal) velocity, and \( F_{e} \) and \( F_{\varphi} \) the density of frictional forces in the azimuthal direction of the ions and the electrons, respectively. The addition of Eqs. (1) and (2) gives the azimuthal momentum equation for the plasma, for which several terms can be neglected: the much larger mass of ions compared to electrons allows to neglect the electronic convective term compared to the ionic one. This ionic term is also larger than the ionization forces. Indeed, we can express \( \frac{\partial v_{\varphi}}{\partial t} + v_r \frac{\partial v_{\varphi}}{\partial r} + v_\varphi \approx 0 \). Then, for our experimental conditions, \( M Z n v_{\varphi} / (M n ow v_{\varphi}) < 0.07 \). Turning around the central column ions collide and loose momentum, \( M v_0 \), in charge exchange collisions, and part of the momentum in elastic collisions with neutrals, the density of which is \( \rho_0 \). The drag force density is then given by: \( F_{\varphi} = m v_{\varphi} / \sigma_0 \), with \( \sigma_0 = \rho_0 / \rho_0 \). The cross section \( \sigma_0 \) represents the frictions processes for the ions. These are mainly Ar/Ar^+ charge exchange collisions and Ar/Ar elastic collisions. For our conditions, \( \sigma_0 \approx 8 \times 10^{-10} \) m^2.17 The similar expression for the electrons is:

\[ F_{e} = m n_{i} v_{e} / \sigma_{e} \]  

(3)

The difference \( v_{\varphi_{i}} - v_{r_{i}} \), appearing in the Larmor force term of Eq. (3), can be derived considering the electron current density \( j_{e} \), inclusive of the electrons injected from the source, \( j_0 \), as well as the density of current of the electrons produced by the ionization in the central column. This latter density of current equates the ion density of current: \( en v_{\varphi_{i}} \), so that we have \( v_{\varphi_{i}} = \frac{j_0}{en} \), with \( j_0 = j_{injected} / S \). \( S = 1 / \alpha, \) is the length of the plasma arm along \( z \) and \( \alpha (r) \) is the angle that represents the shape of the plasma arm, see also Fig. 2. We can then write \( v_{\varphi_{i}} - v_{r_{i}} = v_s \) at any radius. The hypothesis of uniform plasma density has been validated by Ref. 9. It could also be intuitively accepted because the arm is unbounded at his lateral sides and can then mold to avoid radial gradients of density. In steady state rotation, \( v_{\varphi_{i}} = \omega r \). Moreover, we suppose that the \( z \) component of ion velocity is negligible in comparison to the radial and azimuthal ones. Eq. (3) becomes:

\[ 2v_{r_{i}} - \frac{\omega_r}{\omega} v_{S} + \frac{r}{\lambda_{fi}} v_{r_{i}} - \frac{r^2}{\lambda_{fi}^2} = 0 \]  

(4)

with \( \omega_r = eB / M \). Ions flow in the same direction as the electron produced by ionization, to recombine on the walls. The radial equilibrium equations for circulating electrons and ions, divided by the factor \( "nM" \), here follow:

\[ m^2 \left( \frac{v_{\varphi_{e}}}{v_{r_{e}}} - \frac{v_{\varphi_{e}}^2}{v_{r_{e}}^2} \right) + \frac{eB v_{r_{e}}}{M} + \frac{eE_{r}}{M} + \frac{F_{e_{e}}}{nM} = 0 \]  

(5)

\[ v_{r_{e}} \frac{\partial v_{\varphi_{e}}}{\partial r} + \frac{v_{r_{e}}^2}{v_{r_{e}}^2} - \frac{eB v_{r_{e}}}{M} - \frac{eE_{e}}{M} + \frac{F_{r_{e}}}{nM} = 0 \]  

(6)

with \( m^2 = m / M \). It is experimentally shown that the electron density \( n \) is approximately radially constant in the ejected plasma.\(^{9}\) Then, the pressure gradient term \( \nabla P \) has been neglected in the fluid Eqs. (5) and (6). The two first terms of Eq. (5) can be neglected because of the “mass” effect \( m^2 \). Then, we can write:

\[ \frac{eE_{r}}{M} + \frac{\omega_r v_{r_{e}}}{\lambda_{fe}} = 0 \]  

(7)
III. RADIAL EVOLUTION OF $v_{ir}$

The radial evolution of $v_{ir}$ has been measured with the LIF diagnostics.\cite{13,19} The experimental result shown on Fig. 3 indicates that $v_{ir}$ increases linearly with $r$. Then, we have calculated the radial evolution of $v_{ir}$ by replacing $\partial v_{ir}/\partial r$ by $v_{ir}/r$ in Eq. (9). The result is also shown on Fig. 3, by considering the experimental conditions for the LIF measurements ($B=160$ G; $\omega=3.1 \times 10^8$ rad/s). The value of $\lambda_{ji}$ is the one on the next paragraph. The agreement between experiment and theory is good.

\begin{equation}
\frac{\partial v_{ir}}{\partial r} - \frac{\omega^2 r}{M} - \omega_t v_{ir} + \frac{\sqrt{v_{ir}^2 + \omega^2 r^2}}{\lambda_{ji}} = 0
\end{equation}

where $\lambda_i = 1/n_\sigma$ with $\sigma$ the total momentum transfer cross section. Equations (7) and (8) can be combined to eliminate the electrical forces,

\begin{equation}
\frac{\partial v_{ir}}{\partial r} - \frac{\omega^2 r}{M} - \omega_t v_{ir} + \frac{m' v_{ir} |\vec{E}_r|}{\lambda_{fe}} + \frac{\sqrt{v_{ir}^2 + \omega^2 r^2}}{\lambda_{ji}} = 0
\end{equation}

with $\Delta v = v_e - v_i$. For typical experimental conditions, the third term “$\omega_t v_{ir}$” can be neglected compared to the second term “$\omega^2 r$,” if we suppose that the ions and electrons velocities are different but have the same magnitude. In this equation, the fourth term in $m'$ is much smaller than the others and therefore will be ignored in the following; physically this means that the electrical forces are negligible for the ion radial motion with respect to the convective acceleration (while they are the main driver for the electron radial motion). In the next paragraph, $v_{ir}(r)$ is calculated from the resolution of Eq. (9) and compared to experimental LIF results. Then, the couple of Eqs. (4) and (9) have been solved simultaneously to derive the dependence of $\omega$ on the magnetic field $B$ and the gas pressure $P$.

IV. DEPENDENCE ON $B$

The model has been validated recording the rotation frequency of the mode $m=1$ against increasing magnetic field in an argon pressure of $7 \times 10^{-4}$ mbars, see Fig. 4. The dependence of the theoretical frequency of rotation against the magnetic field obtained from the solution of the system of Eqs. (4) and (9) is also shown. The azimuthal velocity of ions turning around the central column is comparable with their thermal speed, of the order of $10^2$–$10^3$ m/s. This is comparable to the involved radii so that we are effectively in a regime of weak collisionality. For the primary electrons, $\sigma_{e,1}=0.17 \times 10^{-19}$ m$^2$ with $\lambda_{e,s}=1.2$ m and for the thermal (secondary) electrons, $\sigma_{e,2}=4.4 \times 10^{-20}$ m$^2$ with $\lambda_{e,s}=0.3$ m. Another unknown in Eqs. (4) and (9) are the electron velocities $v_e$ and $v_{ir}=v_e+v_s$ associated to the density of current $j_e$ and $j_{ir}$. These are derived from the experimental measurement of two currents, $I_r$, is the maximum current to a Langmuir probe biased at 16 V and placed in the arm swept volume. The probe voltage is well over the space potential that is slightly negative with respect to ground, so the probe is in the electron saturation regime and we assume the current to be roughly proportional to the plasma density. The other current is $I_r$, the current on the walls to the ground. This latter current is also representative of the electron current injected in the magnetized plasma column and of the excess of electron current, with respect to the ion current, in the ‘arm’. Then, $I_r = S \cdot e \cdot n \cdot v_e$ with $S(\alpha)=l \cdot r \cdot \alpha$, where $\alpha = \pi/2$ being the angle covered by the plasma arm at the radius corresponding to the exit of the central column. Eliminating $n$, $v_e$ is then proportional to the ratio of the above mentioned input currents: $v_e = K (I_r/I_p)$. In practice, the theoretical curve in Fig. 4 has been obtained fitting the value of $K$ for the point $B = 160$ G and $f=5.9$ kHz($K=21$); the points at other values of $B$ have been obtained using the same fitting value to validate the theory comparing the slopes. The system of Eqs. (4) and (9) then provides the theoretical curve of Fig. 4.

V. DEPENDENCE ON PRESSURE

Figure 5 shows the matching of the theoretical prediction to the experimental rotation frequency when the argon gas pressure is varying, for two different magnetic field values.
In our case, the limiting pressure is $P = 2 \times 10^{-5}$ mbars below which instabilities and poor reproducibility has been observed. The low $B$ case is in very good agreement with the experimental data, whereas the high $B$ case can only reproduce the general trend.

VI. CONCLUSION

The $m=1$ instability of the Mistral magnetized plasma is interpreted by a global model, which is able to reproduce closely the experience in a large range of pressures and applied magnetic fields. In the light of the present model, we can explain the transition to higher $m$ modes, which derive from the limits set by ambipolarity and geometrical considerations. This will form the basis of a forthcoming paper. The model also provides prediction over the radial velocity of the ions that allows us to characterize the plasma in the rotating arm and to validate the theory of transport in magnetic field.

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