

Towards a weak interpretation of the de Broglie-Bohm theory

Michel and Alexandre Gondran



Discussion Day
"Foundations of Quantum Physics"
28 / 08 / 2017
UTT Troyes

- 1 Introduction to de Broglie-Bohm (dBB) theory
- 2 Three randomness of preparation versus randomness of measurement
- 3 Convergence of quantum mechanics to classical mechanics
- 4 Conclusion and weak interpretation of dBB theory

- 1** Introduction to de Broglie-Bohm (dBB) theory
 - Measurement problem: Non consistency of Quantum Mechanics
 - What is dBB theory?
- 2 Three randomness of preparation versus randomness of measurement
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The problem: Non consistency of Quantum Mechanics

Non consistency of Quantum Mechanics

2 different ways to make the evolution of the wave function

1 Schrödinger equation

2 Wave function collapse

⇒ No rules in the theory to choose which one to use

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Non consistency of Quantum Mechanics

2 different ways to make the evolution of the wave function

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⇒ No rules in the theory to choose which one to use

Various views

■ Measurement problem:

1 Superposition does not exist in macroscopic scale

2 Schrödinger equation is linear and deterministic

3 Quantum mechanics is complete

■ Schrödinger cat

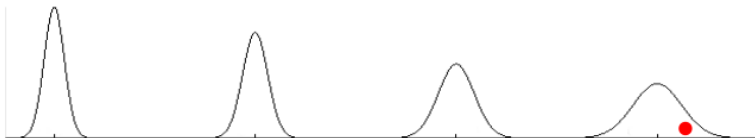
■ Interpretation of the superposition: real vs incompleteness

What is dBB theory?

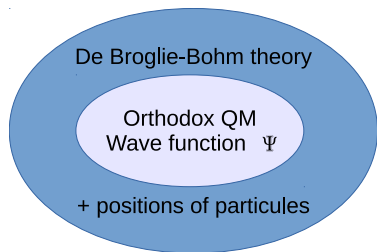
Orthodox QM
Wave function Ψ

Orthodox quantum mechanics

- Particle = wave function
- Measurement: Wave function collapse



What is dBB theory?

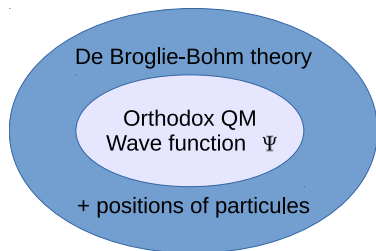


dBB theory

- Particle = wave function + **position**
- Random initial position (inside the wave packet)



What is dBB theory?



dBB theory

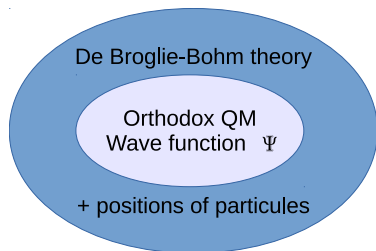
- Particle = wave function + **position**
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- The wave function guides the particle position:

$$v(x, t) = \frac{\nabla S^{\hbar}(x, t)}{m}$$

$$\text{with } \Psi(x, t) = \sqrt{\rho^{\hbar}(x, t)} e^{\frac{i}{\hbar} S^{\hbar}(x, t)}$$



What is dBB theory?



dBB theory

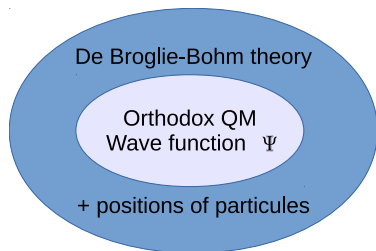
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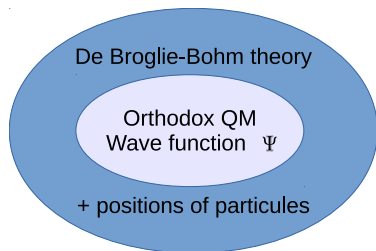
What is dBB theory?



dBB theory

- Replace the randomness of measurement by a randomness of preparation
- Solve the measurement problem
- Augmented reality: underlying model

What is dBB theory?



dBB theory

- Replace the randomness of measurement by a randomness of preparation
- Solve the measurement problem
- Augmented reality: underlying model

Orthodox quantum mechanics

- Truncation of dBB theory
- Algorithm of computation \neq theory

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Replace the randomness of measurement by randomness of preparation

Three kind of randomness of preparation

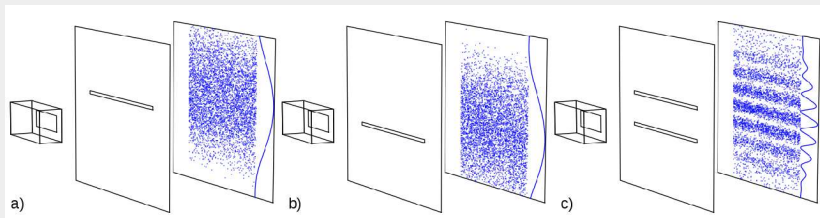
- 1 Randomness of position: dBB theory
- 2 Randomness of wave function parameters: classical statistic
- 3 Randomness of intern state: other underlying model?

Randomness of position

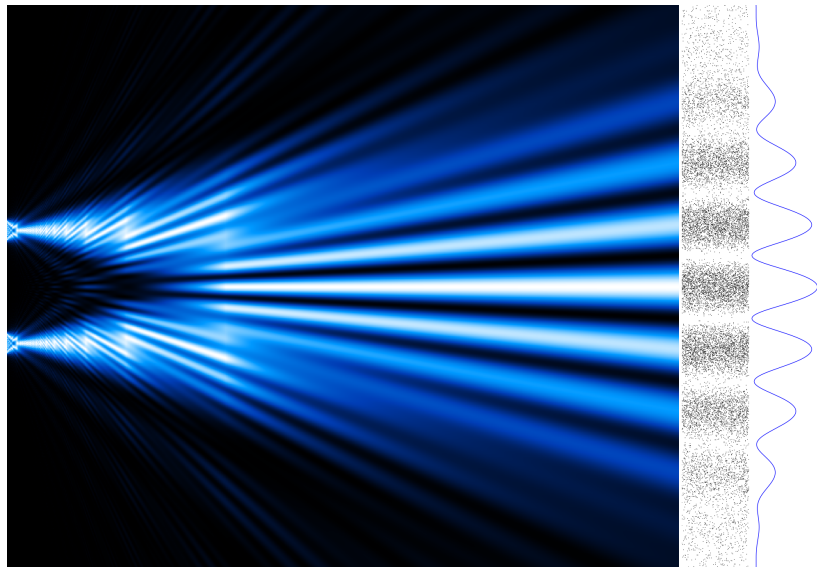
- Underlying model of dBB
- Position: unpredictable but deterministic

Double slits experiment

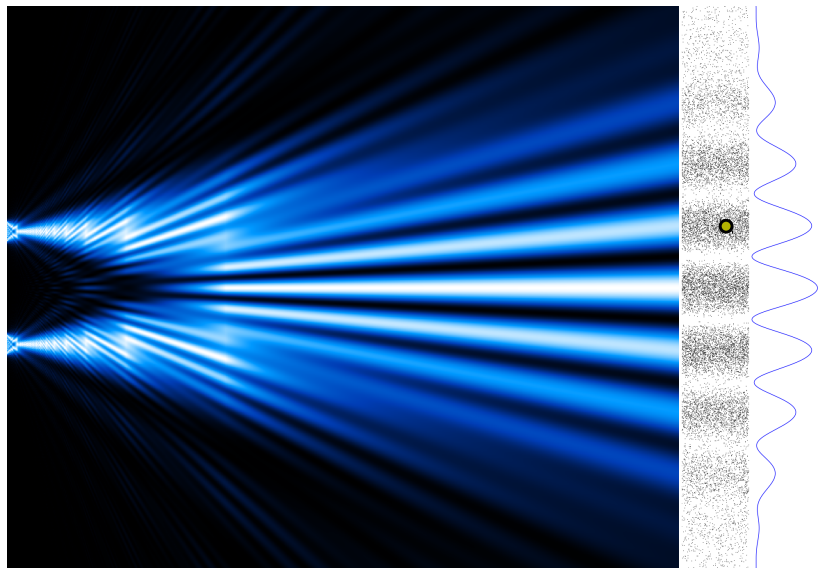
photon, electron, atoms, Bose-Einstein condensate, molecules (C_{60} , Fullerene)



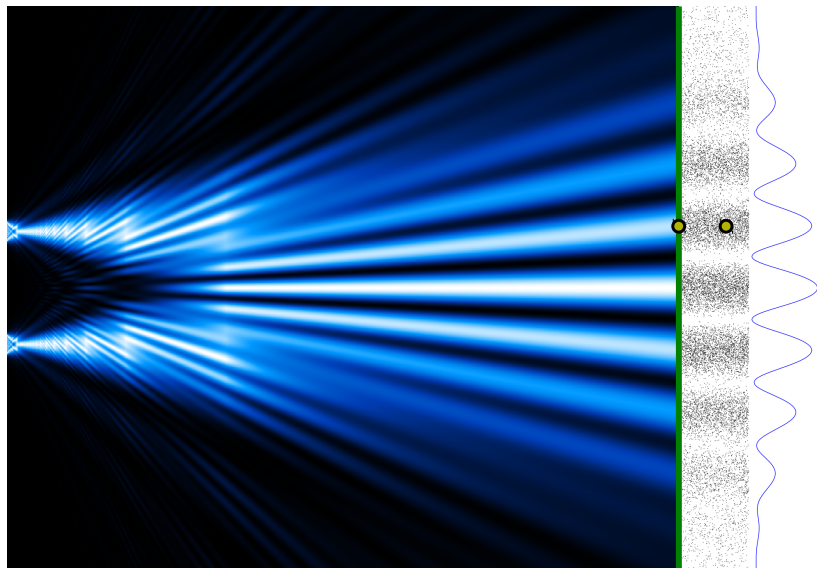
Double slits experiment: computation



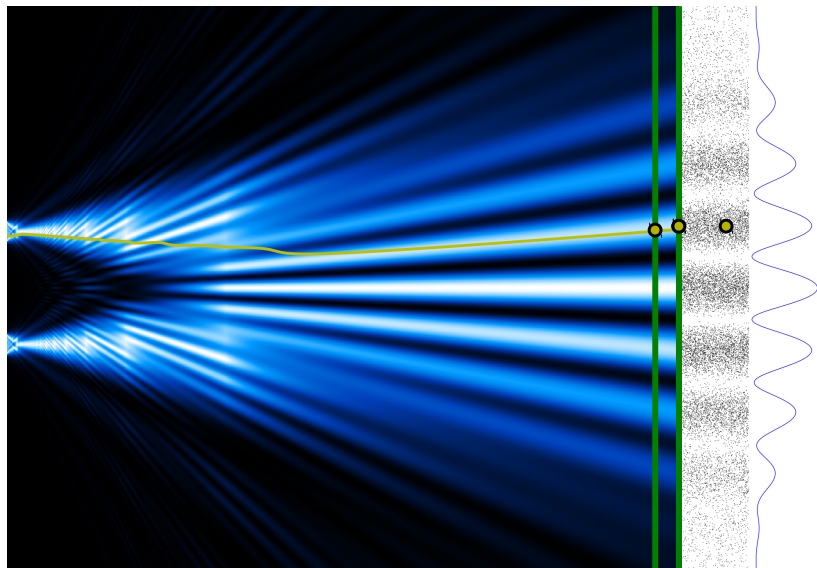
Double slits experiment: computation



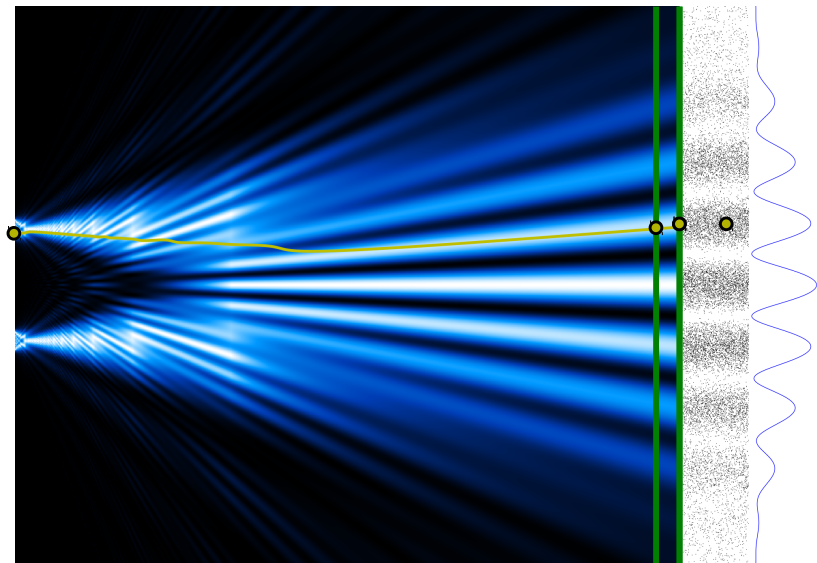
Double slits experiment: computation



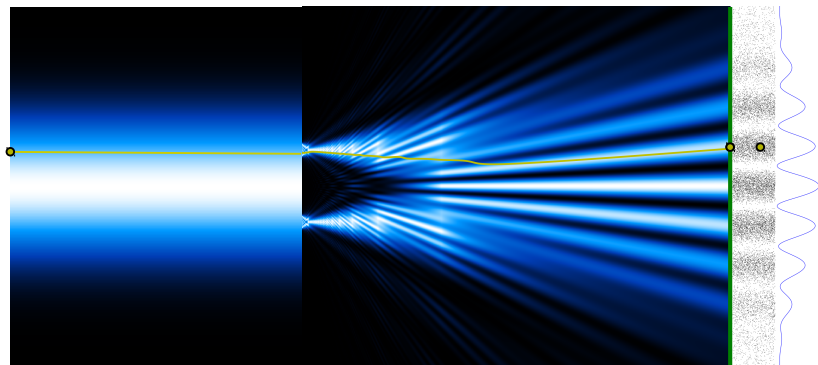
Double slits experiment: computation



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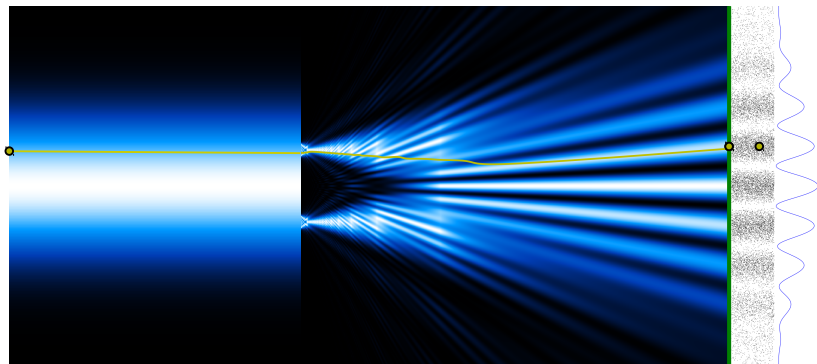


Double slits experiment: computation



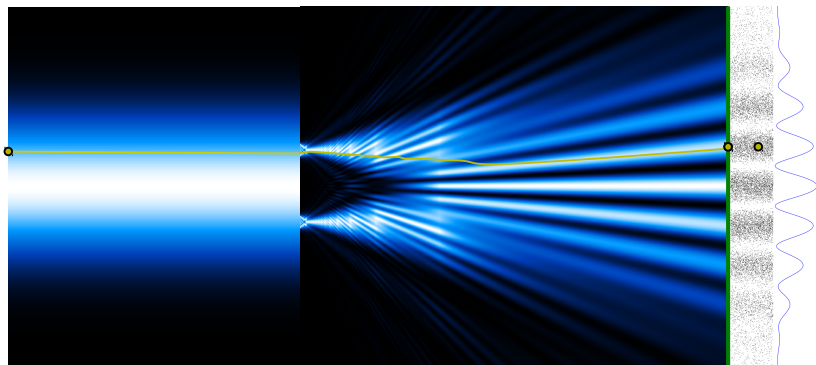
Double slits experiment: randomness of initial position

Replace the randomness of position measurement
by randomness of position preparation

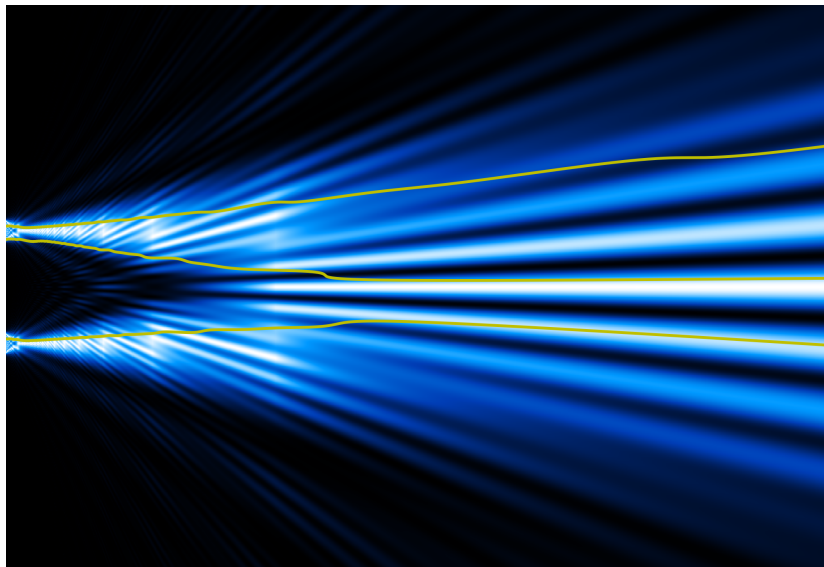


Double slits experiment: randomness of initial position

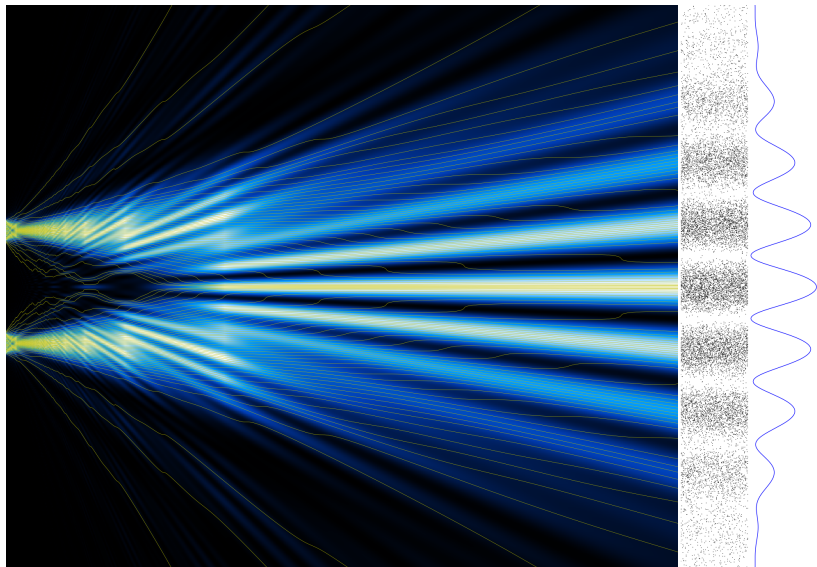
$$v(x, t) = \frac{\nabla S^{\hbar}(x, t)}{m} \quad \text{with} \quad \Psi(x, t) = \sqrt{\rho^{\hbar}(x, t)} e^{i \frac{S^{\hbar}(x, t)}{\hbar}}$$



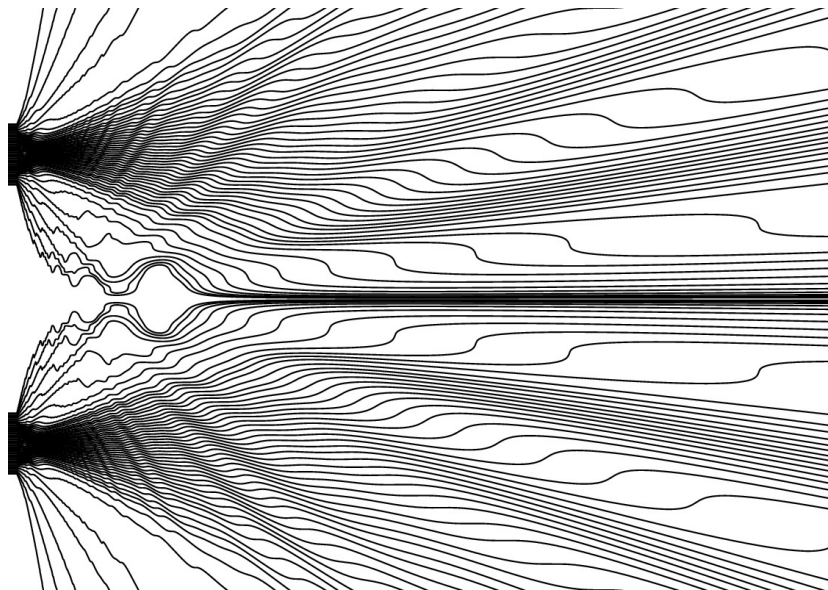
Double slits experiment: randomness of initial position



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Double slits experiment: randomness of initial position

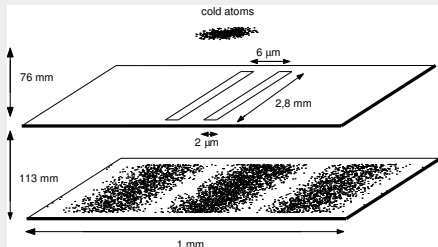


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Randomness of wave function parameters

Double slit experiment

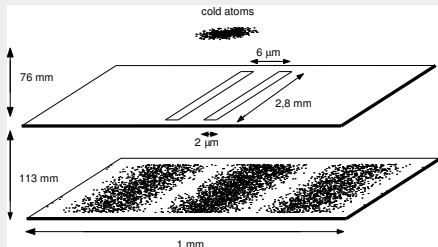
- Difference between cold atoms (Shimizu and al. 1992) and Bose-Einstein condensate (Ketterle and al. 1997)



Randomness of wave function parameters

Double slit experiment

- Difference between cold atoms (Shimizu and al. 1992) and Bose-Einstein condensate (Ketterle and al. 1997)



⇒ Initial wave function is not exactly the same for each clod atom:
different initial velocity ($\neq \mathbf{k}_{0x}$ then $v_{0x} = \frac{\nabla S}{m} = \frac{k_{0x}}{m}$)

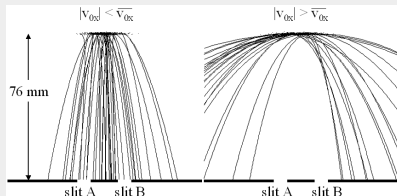
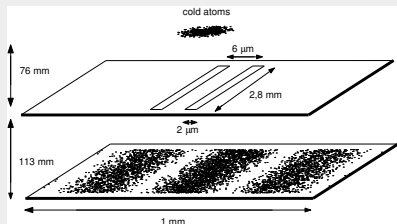
$$\Psi_0(x, k_{0x}) = (2\pi\sigma_0^2)^{-1/4} \times e^{-x^2/4\sigma_0^2} \times e^{ik_{0x}x}$$

Randomness of wave function parameters

Double slit experiment [1]

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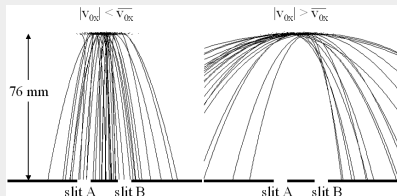
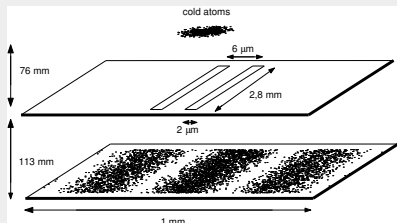


Randomness of wave function parameters

Double slit experiment [1]

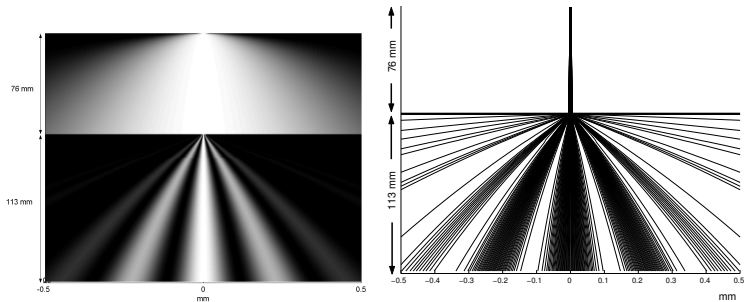
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$$\Psi_0(x, k_{0x}) = (2\pi\sigma_0^2)^{-1/4} \times e^{-x^2/4\sigma_0^2} \times e^{ik_{0x}x}$$



⇒ No interference if the source is too near of the slits

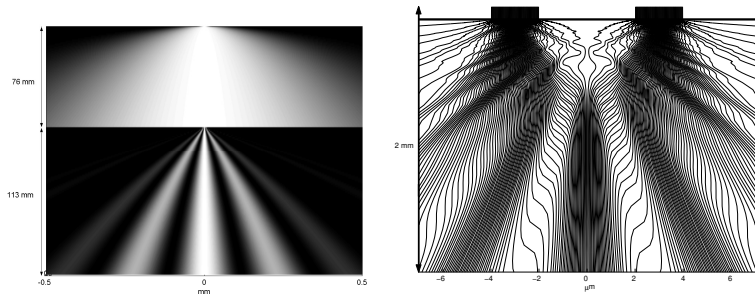
Randomness of wave function parameters



Double slit experiment

- Possibility to filter the transversal velocity \neq randomness of initial position (dBB theory)
- Controllable by the observer
- Classical randomness: integrate on all the possible velocity

Randomness of wave function parameters



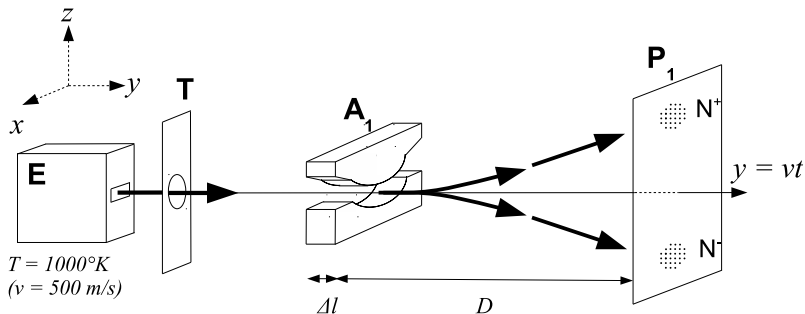
Double slit experiment

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Randomness of wave function parameters

Stern and Gerlach experiment

- Same problem: pure states vs mixed states



Randomness of wave function parameters

Stern and Gerlach experiment [2]

- Solve the Pauli equation (time-dependent)
- Initial wave function:

$$\Psi_0(z) = (2\pi\sigma_0^2)^{-1/4} \times e^{-z^2/4\sigma_0^2} \times \begin{pmatrix} \cos \frac{\theta_0}{2} e^{i\frac{\phi_0}{2}} \\ \sin \frac{\theta_0}{2} e^{i\frac{-\phi_0}{2}} \end{pmatrix}$$

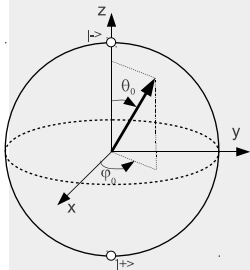
θ_0 and ϕ_0 define the initial orientation of the spinor

- Pure states: θ_0 and ϕ_0 known

⇒ only randomness of position

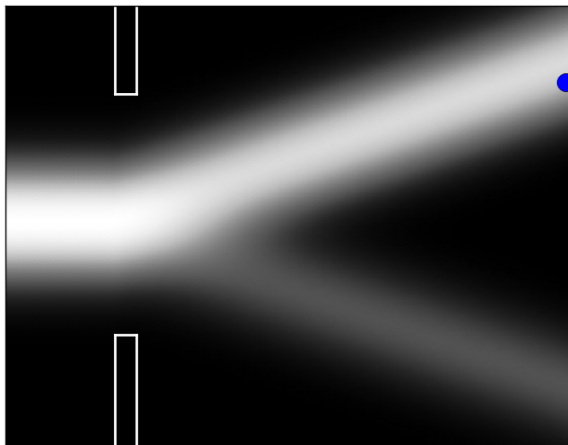
- Mixed states: θ_0 and ϕ_0 unknown therefore randomly drawn

⇒ randomness of position and of wave function parameters



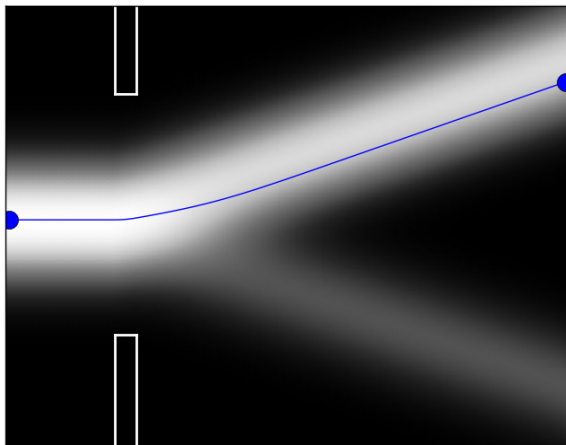
Randomness of wave function parameters

Pure state: $\theta_0 = \frac{\pi}{3}$ and ϕ_0 fixed



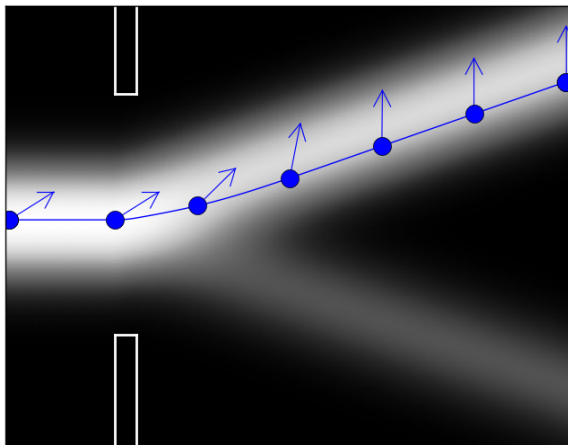
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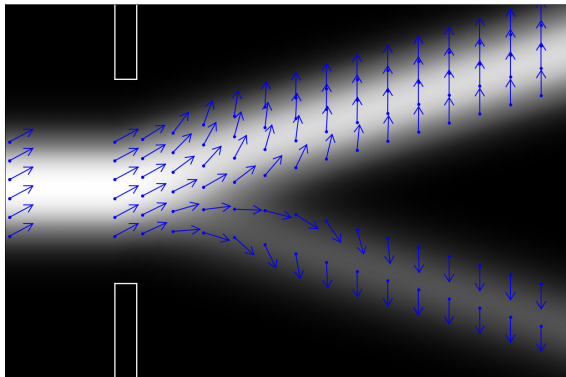
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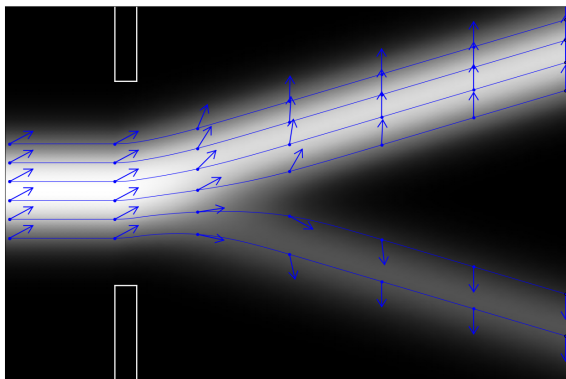
Randomness of wave function parameters

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Randomness of wave function parameters

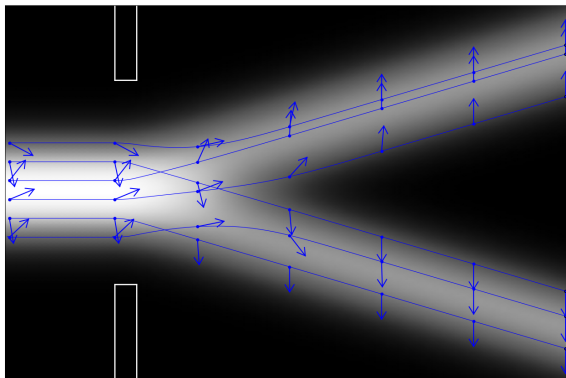
Pure state: $\theta_0 = \frac{\pi}{3}$ and ϕ_0 fixed



Randomness of wave function parameters

Mixed states: θ_0 and ϕ_0 randomly drawn

Mixed states: θ_0 randomly drawn



Stern and Gerlach experiment

- Possibility to filter the orientation of the spinner
 - Randomness of wave function parameters \neq randomness of initial position (dBB theory)
⇒ Controllable by the observer
- False idea: trajectories can cross each other in dBB theory

EPR-B experiment [3]

- Combine the both randomness: position and wave function parameter
- Not possible to control the wave function parameter

Randomness of wave function parameters

EPR-B experiment

- Unknown initial positions of entangled particles A and B
- Unknown initial orientation of the spinor θ_0^A, ϕ_0^A

⇒ $\theta_0^B = \pi - \theta_0^A$ and $\phi_0^B = \phi_0^A - \pi$ (anti-correlation)

$$\begin{aligned}\psi_0^A(\mathbf{r}^A, \theta_0^A, \phi_0^A) &= (2\pi\sigma_0^2)^{-1/4} \times e^{-z^2/4\sigma_0^2} \times \begin{pmatrix} \cos \frac{\theta_0^A}{2} e^{i\frac{\phi_0^A}{2}} \\ \sin \frac{\theta_0^A}{2} e^{i\frac{-\phi_0^A}{2}} \end{pmatrix} \\ &= f(\mathbf{r}^A) \left(\cos \frac{\theta_0^A}{2} |+_A\rangle + \sin \frac{\theta_0^A}{2} e^{-i\phi_0^A} |-_A\rangle \right) \\ \psi_0^B(\mathbf{r}^B, \theta_0^B, \phi_0^B) &= f(\mathbf{r}^B) \left(\cos \frac{\theta_0^B}{2} |+_B\rangle + \sin \frac{\theta_0^B}{2} e^{-i\phi_0^B} |-_B\rangle \right)\end{aligned}$$

Randomness of wave function parameters

EPR-B experiment

Pauli principle

$$\begin{aligned}\Psi_0(\mathbf{r}^A, \theta_0^A, \phi_0^A, \mathbf{r}^B, \theta_0^B, \phi_0^B) &= \Psi_0^A(\mathbf{r}^A, \theta_0^A, \phi_0^A)\Psi_0^B(\mathbf{r}^B, \theta_0^B, \phi_0^B) \\ &\quad - \Psi_0^A(\mathbf{r}^B, \theta_0^B, \phi_0^B)\Psi_0^B(\mathbf{r}^A, \theta_0^A, \phi_0^A) \\ &= -e^{i\phi_0^A} f(\mathbf{r}^A) f(\mathbf{r}^B) (|+_A\rangle |-_B\rangle - |-_A\rangle |+_B\rangle)\end{aligned}$$

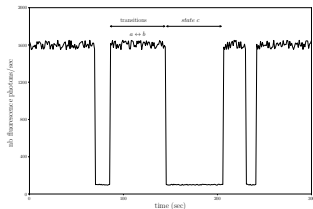
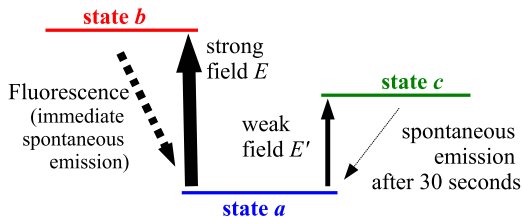
⇒ Singlet state

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Randomness of intern state

Quantum jumps

- Quantum jumps: Dehmelt experiment (1986)
- Random events: times of creation and annihilation of unique photon
- Schrödinger equation is no longer sufficient



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Euler-Lagrange action versus Hamilton-Jacobi action and MinPlus path integral

System evolution

$$\frac{d\mathbf{x}(s)}{ds} = \mathbf{u}(s), \quad \forall s \in [0, t]$$
$$\mathbf{x}(0) = \mathbf{x}_0, \quad \mathbf{x}(t) = \mathbf{x}.$$

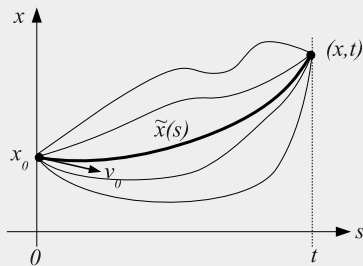
- \mathbf{x}_0 position at initial time
- \mathbf{x} position at time t
- $\mathbf{u}(s)$ velocity

Least action principle: Euler-Lagrange and Hamilton-Jacobi actions

Euler-Lagrange action: the classical action

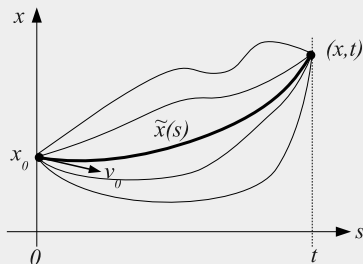
$$S_{cl}(\mathbf{x}, t; \mathbf{x}_0) = \min_{\mathbf{u}(s)} \int_0^t L(\mathbf{x}(s), \mathbf{u}(s), s) ds$$

■ $L(\mathbf{x}, \dot{\mathbf{x}}, t)$ Lagrangian of the system



Euler-Lagrange action: the classical action

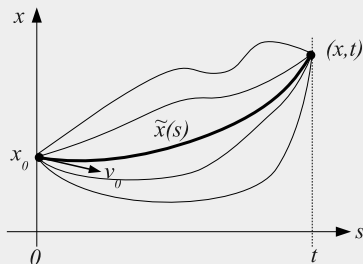
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- $L(\mathbf{x}, \dot{\mathbf{x}}, t)$ Lagrangian of the system
- the minimum is taken over all trajectories $\mathbf{x}(s)$ with velocity $\mathbf{u}(s)$ from \mathbf{x}_0 to \mathbf{x} between 0 and t .

Euler-Lagrange action: the classical action

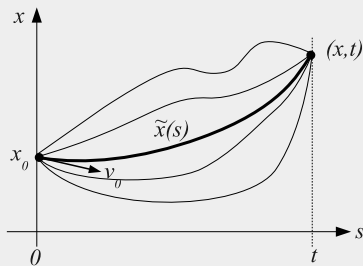
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- ⇒ The initial velocity depends on the final position!!!

Euler-Lagrange action: the classical action

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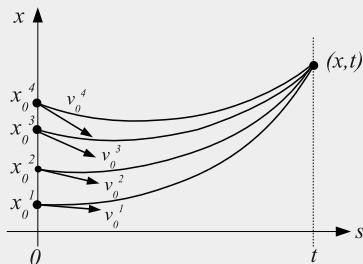


- $L(\mathbf{x}, \dot{\mathbf{x}}, t)$ Lagrangian of the system
- the minimum is taken over all trajectories $\mathbf{x}(s)$ with velocity $\mathbf{u}(s)$ from \mathbf{x}_0 to \mathbf{x} between 0 and t .
- ⇒ The initial velocity depends on the final position!!!
- ⇒ Action for the Observer (epistemological action)

Hamilton-Jacobi action, the ontological action

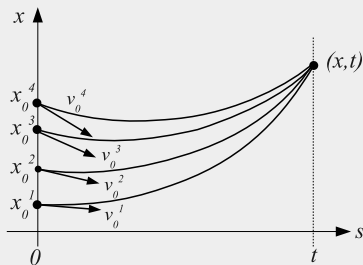
$$S(\mathbf{x}, t) = \min_{\mathbf{x}_0; \mathbf{u}(s), 0 \leq s \leq t} \left\{ S_0(\mathbf{x}_0) + \int_0^t L(\mathbf{x}(s), \mathbf{u}(s), s) ds \right\}$$

- the minimum is taken over all initial positions \mathbf{x}_0 and over the velocities $\mathbf{u}(s)$



Hamilton-Jacobi action, the ontological action

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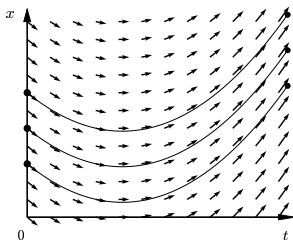


- the minimum is taken over all initial positions \mathbf{x}_0 and over the velocities $\mathbf{u}(s)$
- Initial velocity is known for all \mathbf{x}_0 :

$$\mathbf{v}_0(\mathbf{x}_0) = \frac{\nabla S_0(\mathbf{x}_0)}{m}$$

Hamilton-Jacobi action, the ontological action

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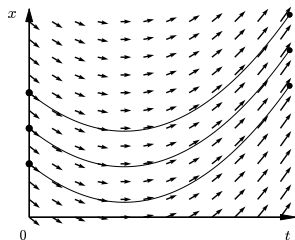
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- Initial velocity is known for all \mathbf{x}_0 :

$$\mathbf{v}_0(\mathbf{x}_0) = \frac{\nabla S_0(\mathbf{x}_0)}{m}$$

⇒ Trajectories of particles are piloted by this field: $\mathbf{v}(\mathbf{x}, t) = \frac{\nabla S(\mathbf{x}, t)}{m}$

Hamilton-Jacobi action, the ontological action

$$S(\mathbf{x}, t) = \min_{\mathbf{x}_0; \mathbf{u}(s), 0 \leq s \leq t} \left\{ S_0(\mathbf{x}_0) + \int_0^t L(\mathbf{x}(s), \mathbf{u}(s), s) ds \right\}$$



- the minimum is taken over all initial positions \mathbf{x}_0 and over the velocities $\mathbf{u}(s)$
- Initial velocity is known for all \mathbf{x}_0 :

$$\mathbf{v}_0(\mathbf{x}_0) = \frac{\nabla S_0(\mathbf{x}_0)}{m}$$

- ⇒ Trajectories of particles are piloted by this field: $\mathbf{v}(\mathbf{x}, t) = \frac{\nabla S(\mathbf{x}, t)}{m}$
- ⇒ We have to add the initial position

MinPlus path integral

MinPlus analysis

- Replace $(+, \times)$ by $(\min, +)$
- Hamilton-Jacobi equation is linear in MinPlus analysis

$$\frac{\partial S(\mathbf{x}, t)}{\partial t} + \frac{1}{2m} (\nabla S(\mathbf{x}, t))^2 + V(\mathbf{x}, t) = 0$$

with initial conditions $S(\mathbf{x}, 0) = S_0(\mathbf{x})$

MinPlus path integral

$$\begin{aligned} S(\mathbf{x}, t) &= \min_{\mathbf{x}_0; \mathbf{u}(s), 0 \leq s \leq t} \left\{ S_0(\mathbf{x}_0) + \int_0^t L(\mathbf{x}(s), \mathbf{u}(s), s) ds \right\} \\ &= \min_{\mathbf{x}_0} (S_0(\mathbf{x}_0) + S_{cl}(\mathbf{x}, t; \mathbf{x}_0)) \end{aligned}$$

⇒ Similar to Feynman path integral for classical mechanics

- 1 Introduction to de Broglie-Bohm (dBB) theory
 - Measurement problem: Non consistency of Quantum Mechanics
 - What is dBB theory?
- 2 Three randomness of preparation versus randomness of measurement
 - Randomness of position
 - Randomness of wave function parameters
 - Randomness of intern state
- 3 Convergence of quantum mechanics to classical mechanics**
 - Euler-Lagrange action versus Hamilton-Jacobi action and MinPlus path integral
 - Mathematic convergence**
- 4 Conclusion and weak interpretation of dBB theory

Mathematic convergence of quantum to classical mechanics

Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + V(\mathbf{x}, t) \Psi \quad (1)$$

$$\Psi(\mathbf{x}, 0) = \Psi_0(\mathbf{x}). \quad (2)$$

Mathematic convergence of quantum to classical mechanics

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Variable change

$$\Psi(\mathbf{x}, t) = \sqrt{\rho^{\hbar}(\mathbf{x}, t)} \exp\left(i \frac{S^{\hbar}(\mathbf{x}, t)}{\hbar}\right)$$

- the quantum density: $\rho^{\hbar}(\mathbf{x}, t)$
- the quantum action: $S^{\hbar}(\mathbf{x}, t)$

Mathematic convergence of quantum to classical mechanics

Madelung equations

$$\frac{\partial S^{\hbar}(\mathbf{x}, t)}{\partial t} + \frac{1}{2m} (\nabla S^{\hbar}(\mathbf{x}, t))^2 + V(\mathbf{x}, t) - \frac{\hbar^2}{2m} \frac{\Delta \sqrt{\rho^{\hbar}(\mathbf{x}, t)}}{\sqrt{\rho^{\hbar}(\mathbf{x}, t)}} = 0 \quad (3)$$

$$\frac{\partial \rho^{\hbar}(\mathbf{x}, t)}{\partial t} + \nabla \cdot \left(\rho^{\hbar}(\mathbf{x}, t) \frac{\nabla S^{\hbar}(\mathbf{x}, t)}{m} \right) = 0 \quad \forall(\mathbf{x}, t) \quad (4)$$

with initial conditions

$$\rho^{\hbar}(\mathbf{x}, 0) = \rho_0^{\hbar}(\mathbf{x}) \quad \text{and} \quad S^{\hbar}(\mathbf{x}, 0) = S_0^{\hbar}(\mathbf{x}). \quad (5)$$

Initial velocity known for all \mathbf{x} : $\mathbf{v}_0(\mathbf{x}) = \frac{\nabla S_0^{\hbar}(\mathbf{x})}{m}$

Mathematic convergence of quantum to classical mechanics

Conditions to convergence when $\hbar \rightarrow 0$

- 1 Initial quantum density and quantum action not depending on \hbar :

$$\rho_0^{\hbar}(\mathbf{x}) = \rho_0(\mathbf{x}) \quad \text{and} \quad S_0^{\hbar}(\mathbf{x}) = S_0(\mathbf{x})$$

- 2 $V(\mathbf{x}, t)$ can be described classically

⇒ Case of free particles beam in a quadratic potential

Mathematic convergence of quantum to classical mechanics

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Madelung equations → Statistical Hamilton-Jacobi equations

$$\frac{\partial S(\mathbf{x}, t)}{\partial t} + \frac{1}{2m} (\nabla S(\mathbf{x}, t))^2 + V(\mathbf{x}, t) = 0 \quad (6)$$

$$\frac{\partial \rho(\mathbf{x}, t)}{\partial t} + \text{div} \left(\rho(\mathbf{x}, t) \frac{\nabla S(\mathbf{x}, t)}{m} \right) = 0 \quad \forall (\mathbf{x}, t) \quad (7)$$

$$\rho(\mathbf{x}, 0) = \rho_0(\mathbf{x}) \quad \text{and} \quad S(\mathbf{x}, 0) = S_0(\mathbf{x}). \quad (8)$$

Mathematic convergence of quantum to classical mechanics

Demonstration [4]

$$\Psi(\mathbf{x}, t) = \int F(t, \hbar) \exp\left(\frac{i}{\hbar} S_{cl}(\mathbf{x}, t; \mathbf{x}_0)\right) \Psi_0(\mathbf{x}_0) d\mathbf{x}_0$$

Mathematic convergence of quantum to classical mechanics

Demonstration [4]

$$\begin{aligned}\Psi(\mathbf{x}, t) &= \int F(t, \hbar) \exp\left(\frac{i}{\hbar} S_{cl}(\mathbf{x}, t; \mathbf{x}_0)\right) \Psi_0(\mathbf{x}_0) d\mathbf{x}_0 \\ &= F(t, \hbar) \int \sqrt{\rho_0(\mathbf{x}_0)} \exp\left(\frac{i}{\hbar} (S_0(\mathbf{x}_0) + S_{cl}(\mathbf{x}, t; \mathbf{x}_0))\right) d\mathbf{x}_0\end{aligned}$$

Mathematic convergence of quantum to classical mechanics

Demonstration [4]

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Mathematic convergence of quantum to classical mechanics

Demonstration [4]

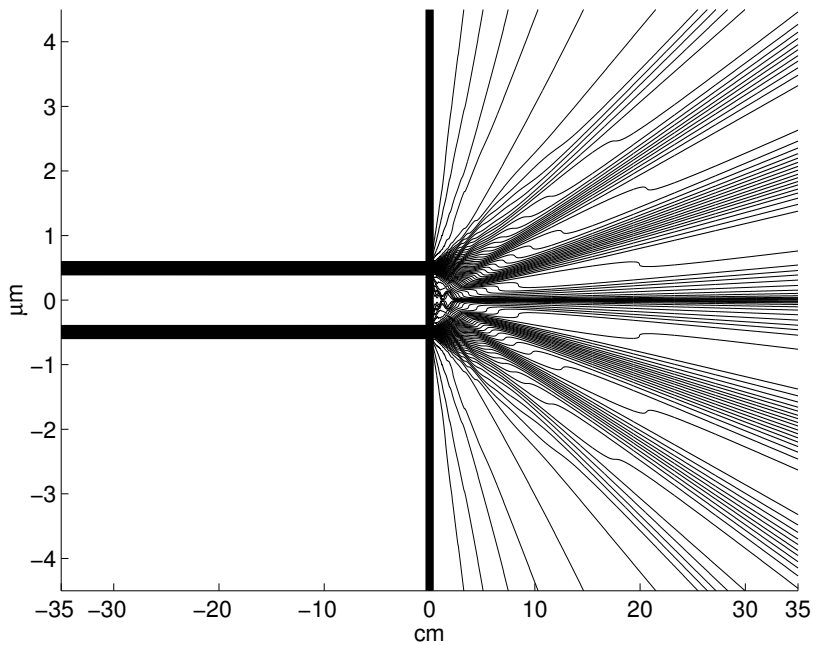
$$\begin{aligned}\Psi(\mathbf{x}, t) &= \int F(t, \hbar) \exp\left(\frac{i}{\hbar} S_{cl}(\mathbf{x}, t; \mathbf{x}_0)\right) \Psi_0(\mathbf{x}_0) d\mathbf{x}_0 \\ &= F(t, \hbar) \int \sqrt{\rho_0(\mathbf{x}_0)} \exp\left(\frac{i}{\hbar} (S_0(\mathbf{x}_0) + S_{cl}(\mathbf{x}, t; \mathbf{x}_0))\right) d\mathbf{x}_0 \\ &\underset{\hbar \rightarrow 0}{\sim} \exp\left(\frac{i}{\hbar} \min_{\mathbf{x}_0} (S_0(\mathbf{x}_0) + S_{cl}(\mathbf{x}, t; \mathbf{x}_0))\right) \\ &\underset{\hbar \rightarrow 0}{\sim} \exp\left(\frac{i}{\hbar} S(\mathbf{x}, t)\right)\end{aligned}$$

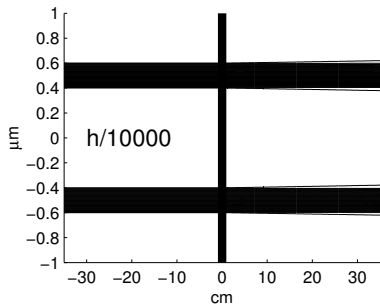
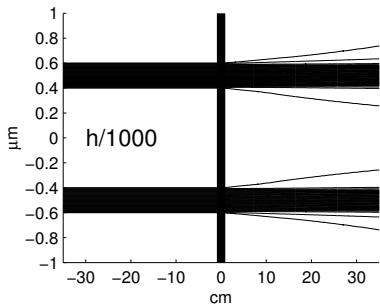
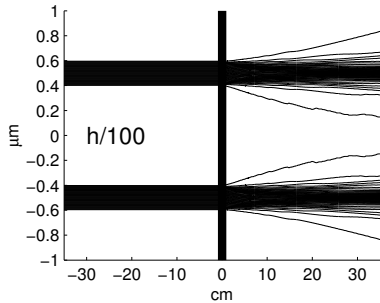
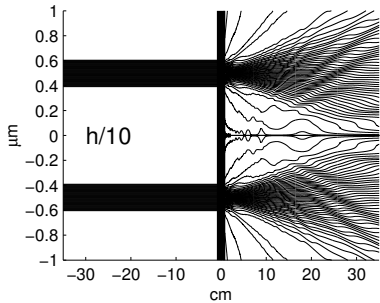
$$S^{\hbar}(\mathbf{x}, t) \xrightarrow{\hbar \rightarrow 0} S(\mathbf{x}, t)$$

H-J action

$$\rho^{\hbar}(\mathbf{x}, t) \xrightarrow{\hbar \rightarrow 0} \rho(\mathbf{x}, t)$$

classical density





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Conclusion dBB theory, consistent theory

- Replace the randomness of measurement by a randomness of preparation

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Conclusion dBB theory, consistent theory

- Replace the randomness of measurement by a randomness of preparation
- Solve the measurement problem and many paradox
- Restore causality and more realism
- Does not explain all kinds of randomness

Interpretation of the convergence theorem

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- Concern free or unbounded particles (in real 3D space not in configuration space)

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- Restriction of the pilot wave to the case of identical particles without interaction

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Weak interpretation of dBB theory

- Restriction of the pilot wave to the case of identical particles without interaction
- Extends to the particles entangled by the spin (EPR-B experiment)



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