# Analyse bi-objectif pour la couverture de cibles par des drones

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Dans cet article, nous étudions le problème de couverture de cibles au sol par des drones. Le but est de minimiser le nombre de drones déployés et de maximiser la qualité des communications radio afin de collecter efficacement l'information des cibles au sol jusqu'à une station de base centrale. Nous modélisons ce problème à l'aide d'un programme linéaire multi-objectif dans lequel nous étudions le compromis entre le coût de déploiement et l'altitude des drones assurant une communication air-sol efficace. Nous proposons une solution équitable permettant de limiter l'ajout de drones nécessaires à l'optimisation des transmissions pour la collecte de données.

## 1 Introduction

Unmanned Aerial Vehicles (UAVs) with air-to-ground and air-to-air communications using directional antennas provide potential solution to anytime and anywhere network access. Deploying UAVs to cover targets is a complex problem. Indeed, the deployment should minimize the number of UAVs to reduce the cost. UAVs should be optimally placed to cover all the targets and gather the monitored information to a central entity for analysis. Many research focus on UAVs or robots deployment in the literature such as in [MB06, DKK03, WXTH06]. In these works related to UAV location problem, the authors assume that the devices evolve in a 2D space. Therefore their problem is simplified because the coverage radius is fixed for each mobile device. In [ZDPPRG16], authors considered the same type of problem as the one we want to tackle. However they do not ensure connectivity among the UAVs, and their primary focus is energy consumption. Ensuring UAV connectivity with a fixed base station is very important since it ensures uninterrupted bi-directional information exchanges between the base station and the targets. In [MSBD16], authors consider maximizing the total coverage area of the UAVs and their lifetime. But in their model, all UAVs are assumed to be placed at the same altitude and thus do not consider possible coverage quality. Moreover, they do not ensure connectivity between each UAV.

In this paper, we consider the following problem : Given a set of targets deployed on the ground, the goal is to cover all the targets at minimum cost with drones flying as low as possible. Moreover, we ensure that the UAVs form a connected graph with a fixed base station in order to collect and analyze efficiently the information. We show that it is possible to develop an efficient model that computes optimal positions of the UAVs. We also show that coverage problem (minimizing the deployment cost and the UAVs altitude) and connectivity should be optimized using Pareto optimality concept. And finally we show that the connectivity constraint have a non negligible effect on the number of deployed UAVs.

## 2 Problem definition and formulation

Let $\mathcal{P}$ be the set of possible positions for the UAVs in the three dimensional space, and $\mathcal{N}$ be the set of targets that has to be monitored. Each target $n \in \mathcal{N}$ is assumed to be fixed, located at position $(x_n, y_n)$ on the 2D plane. Let $u = (x_u, y_u, h_u) \in \mathcal{P}$ be respectively the position $(x_u, y_u)$ of UAV $u$ in the 2D plane, and $h_u$ its altitude. We derive the observation radius $r_u^h$ of UAV $u$ in function of $h_u$ and its directional antenna half

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beamwidth $\theta$. The coverage area of UAV $u$ on the 2D plane is represented by a disk of radius bounded by : $r_u^h \leq h_u \cdot \tan(\frac{\theta}{2})$. We say that an UAV $u \in \mathcal{P}$ covers target $n \in \mathcal{N}$ if the distance between its projection on the 2D plane and the target $d_{un} = \sqrt{(x_u - x_n)^2 + (y_u - y_n)^2}$ is below the observation radius $r_u^h$ of $u$. Similarly, for air-to-air communications, an UAV $u$ can communicate with another UAV $v$ if their distance on the $3D$ plane $D_{uv} = \sqrt{(x_u - x_v)^2 + (y_u - y_v)^2 + (h_u - h_v)^2} \leq R_u$, where $R_u$ is the communication range of UAV $u$. For efficient data collection, we enforce the deployed UAVs to be connected with each other and with a fixed base station $b$ located at coordinates $(X_b, Y_b, 0)$. We also associate a cost $c_u$ with the deployment of an UAV in position $u \in \mathcal{P}$. The goal is to deploy UAVs and choose their position and altitude in $\mathcal{P}$ so that :

- all the mobile targets are covered by at least one UAV ;
- all the UAVs are connected with each other and to the base station to efficiently collect information through multi-hop communications.

Let $z_u$ indicate if an UAV is deployed in position $u \in \mathcal{P}$, and $\chi_n^u$ state if UAV $u$ covers target $n$. For connectivity, we use variables $f_{uv} \in \mathbb{R}$ modeling the amount of flow sent between UAVs $u$ and $v$. The goal is to ensure the existence of a flow between the base station and all the deployed UAVs.

$$\begin{cases} \text{(i)} \ \min f^1 = \sum_{u \in \mathcal{P}} c_u \cdot z_u \\ \text{(ii)} \ \min f^2 = \max_{u \in \mathcal{P}} h_u \cdot z_u \end{cases} \tag{1}$$

$$\sum_{u \in \mathcal{P}} \chi_n^u \geq 1, \forall n \in \mathcal{N} \tag{2}$$

$$\chi_n^u \leq z_u \cdot \frac{r_u^h}{d_{un}}, \forall u \in \mathcal{P}, n \in \mathcal{N} \tag{3}$$

$$\sum_{v \in \mathcal{P}, v \neq u} f_{uv} - \sum_{v \in \mathcal{P}, v \neq u} f_{vu} = \begin{cases} \sum_{v \in \mathcal{P}} z_v & \text{if } u = b \\ -z_u & \text{if } u \neq b \end{cases}, \forall u \in \mathcal{P} \cup \{b\} \tag{4}$$

$$f_{uv} \leq z_u \cdot \frac{R_u}{D_{uv}} \cdot |\mathcal{P}|, \quad \forall u, v \in \mathcal{P} \tag{5}$$

$$f_{uv} \leq z_v \cdot \frac{R_u}{D_{uv}} \cdot |\mathcal{P}|, \quad \forall u, v \in \mathcal{P} \tag{6}$$

Constraints (2) and (3) verify that all the targets are covered by at least one deployed UAV that is located at distance smaller than the transmission radius of the UAV. To model the connectivity constraints, we use equations related to the existence of a flow in the graph induced by the UAVs. Since we don't know exactly how many UAVs are deployed to cover the targets, the amount of flow sent by the base station to communicate with the UAVs is equal to $\sum_{u \in \mathcal{P}} z_u$ (Constraints (4)) corresponding to the number of deployed UAVs in the solution. Also, flow can be sent between two UAVs if they are within communication range of each other (Constraints (5) and (6)). We analyze here the trade-off between two antagonistic objectives : minimizing the deployment cost (Equation (1)-(i)) and the maximum UAV's altitude (Equation (1)-(ii)). Indeed, the higher the altitude, the larger the coverage. And so we need less drones to cover all the targets. But we also want to minimize the altitude in order to ensure good wireless communications quality. So limiting the altitude inextricably increases the number of deployed UAVs. Since the two objectives are in conflict, it is not relevant to combine them in an effective optimization problem. Consequently, the concept of Pareto was introduced to study the trade-off between the metrics (1)-(i) and (ii). The goal is to find out all the possible non-dominated solutions of the problem. If a solution is non-dominated, it is not possible to improve one of the metrics without worsening the other. Each multi-objective problem has a set of Pareto-optimal solutions defined as the set of non-dominated solutions. In order to generate Pareto solutions we use the $\varepsilon$-constraint method that transforms the bi-objective problem into a sequence of parametrized single objective problems such that the optimum of each problem corresponds to a Pareto-optimal solution [BGP09].

## 3 Results

The model has been implemented in Java language and solved using IBM Cplex solver 12.7.1. Instances are deployed in a square area of $100m \times 100m$. We choose randomly the 2D coordinates of the targets. For each random set of targets, we divide the area into equal squares in which one possible point is located in

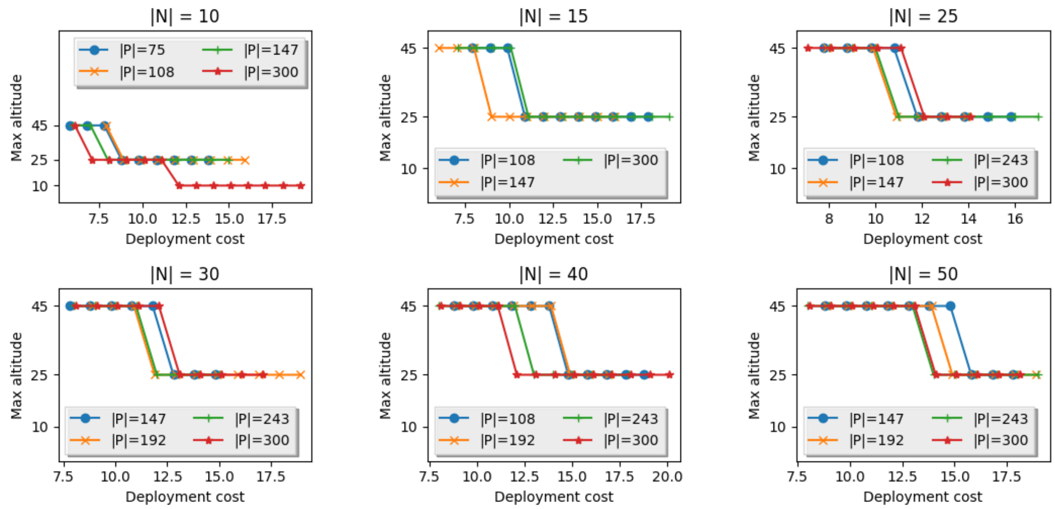



FIGURE 1: Pareto solutions for different sizes of $\mathcal{N}$ and $P$.

the center of the square. In this way, the candidate sites for placing an UAV form a regular grid. For each point of coordinate $(x_u, y_u)$, we set the allowed altitudes to $\{10m, 25m, 45m\}$. The base station is placed at coordinates $(0,0,0)$. We generate instances of size between 5 and 50 targets, and between 50 and 300 possible points for the UAVs. For each UAV, the deployment cost is set to 1, the visibility angle to 60 degrees, and the communication range to $30m$.

We first solve twice our model with one objective each. We obtain $\overline{f^1}$ the minimum deployment cost without any constraint on altitude, and $\overline{f^2}$ the smallest altitude for which the set of UAVs is connected and covers the targets, without any constraint on the cost. Then, we iteratively solve the mono-objective problem minimizing (1)-(ii) with constraint $\sum_{u \in \mathcal{P}} c_u \cdot z_u \leq \varepsilon$, $\varepsilon$ varying from the cost of $\overline{f^1}$ to the one of $\overline{f^2}$.

## 3.1 Analysis of Pareto solutions

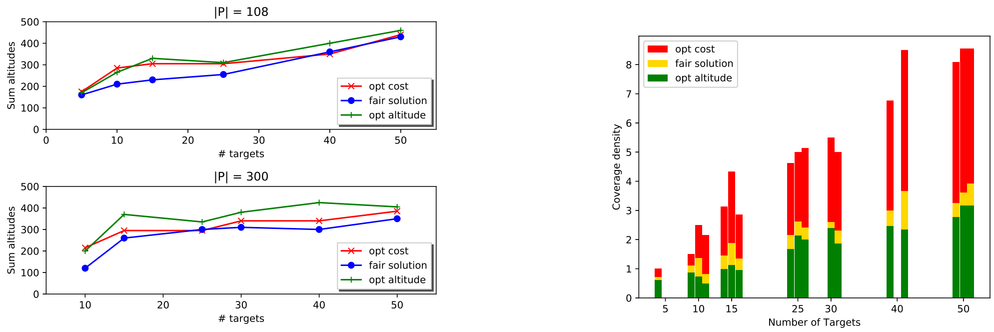
We depict in Figure 1 the trade-off between altitude and cost. In the optimization problem, it can happen that the altitude is equal for $\overline{f^1}$ and $\overline{f^2}$. In such cases (not represented in the figure), the optimal solution $\overline{f^1}$ minimizing the cost (1)-(i) is optimal in terms of cost and altitude. In our scenarios, it is possible to decrease the maximum altitude by adding additional UAVs as depicted in the Pareto solutions (Figure 1). Since the objective function (1)-(ii) minimizes the maximum altitude, we observe that the solutions quickly decrease to $\overline{f^2}$. It is thus possible to attain the minimum altitude with a low number of deployed UAVs. Figures 1 show that the minimum altitude is attained with at most 6 additional UAVs, compared to the optimum of objective function $\overline{f^1}$. For topologies with a low number of ground targets (i.e. 10 and 15 targets), the maximum is 3. This is a good compromise between deployment cost and communication quality as expected for our problem. From these observations, we derive a definition of *the best trade-off solution* :

**Definition 1.** *The fair optimal solution of the target covering problem with connectivity constraints among the UAVs corresponds to the solution* $(c^*, h^*_{max})$ *such that :*

- $h^*_{max} = \overline{f^2}$ *is the optimal solution of objective (1)-(ii) ;*
- $c^*$ *is the minimum deployment cost obtained when attaining* $\overline{f^2}$.

## 3.2 Validation of the fair optimal solution

The fair optimal solution allows us to propose one optimal solution in terms of altitude with minimum associated deployment cost. We first claim that the fair optimal solution is efficient in terms of altitude. The sum of all the altitudes of the fair solution, compared to the solutions $\overline{f^1}$ and $\overline{f^2}$, provides an UAV backbone at lower altitude (Figure 2a). Indeed, $\overline{f^1}$ (named as *opt cost*) does not optimize altitude at all,




(a) Sum of altitudes of deployed UAVs

(b) Mean number of targets covered by the deployed UAVs for $|P| = 108, 147, 300$.

FIGURE 2: Analysis of solutions in terms of altitude and coverage.

but deploys the smallest number of UAVs (at high altitude). Solution $\overline{f^2}$ (called *opt altitude*) optimizes the altitude, but gives a solution with more UAVs with a larger value of the altitude's sum. Therefore the fair optimal solution provides a better solution since the sum of the altitudes is lower than the two mono-objective solutions. Another metric validating the choice of the fair optimal solution is the *coverage density*. It corresponds to the mean number of covered targets by the deployed UAVs. We present in Figure 2b, for a given number of targets, a maximum of three bars representing the coverage density for respectively 108, 147, and 300 3D locations. We remark that the coverage density of the fair optimal solution has value between the density of $\overline{f^2}$ and $\overline{f^1}$. It cannot be lower than $\overline{f^2}$ because this solution deploys more UAVs at low altitude, thus balancing the target's coverage. On the contrary, $\overline{f^1}$ minimizes the number of deployed UAVs, each one covering more targets. But the fair optimal solution has a low coverage density, usually closed to the one of $\overline{f^2}$. This is a good result since it increases the air-to-ground communication quality while ensuring the lowest altitude for the deployed UAVs.

# 4 Conclusion

In this work, we address the problem of deploying UAVs to monitor ground targets, and forming a connected backbone to collect efficiently information from the ground. We present an optimal bi-objective linear program to model the problem and obtain Pareto-optimal solutions in reasonable time for real size instances. A study of the Pareto solutions help analyzing the trade-off between deployment cost and altitude. We observe that our bi-objective model allows us provide a *fair optimal solution* minimizing the maximum altitude with a few number of additional UAVs than the optimum deployment cost solution. This solution thus balances efficiently deployment cost and communication quality.

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