



**HAL**  
open science

## Survival Amblard-Girard copulas

Emil Stoica

► **To cite this version:**

| Emil Stoica. Survival Amblard-Girard copulas. 2018. hal-01781311

**HAL Id: hal-01781311**

**<https://hal.science/hal-01781311>**

Preprint submitted on 30 Apr 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Survival Amblard-Girard copulas

Emil Stoica

Dragan European University of Lugoj

## Abstract

A copula is a function that completely describes the dependence structure between the marginal distributions. One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family. In practical applications this copula has been shown to be somewhat limited and a symmetric extension of this family, known as the Amblard-Girard copula, has been introduced. The goal of this note is to prove that the survival copula associated with an Amblard-Girard copula still is an Amblard-Girard copula.

A copula is a function that completely describes the dependence structure. It contains all the information to link the marginal distributions to their joint distribution. To obtain a valid multivariate distribution function, it suffices to combine several marginal distribution functions with any copula function. Thus, for the purposes of statistical modeling, it is desirable to have a large collection of copulas at one's disposal. Many examples of copulas can be found in the literature, most are members of families with one or more real parameters.

One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family defined as

$$C_{\theta}^{FGM}(u, v) = uv + \theta uv(1 - u)(1 - v), \quad u, v \in [0, 1], \quad (1)$$

where  $\theta \in [-1, 1]$ . The density function of FGM copulas is given by

$$\frac{\partial^2 C_{\theta}^{FGM}(u, v)}{\partial u \partial v} = \theta(2u - 1)(2v - 1) + 1,$$

for any  $u, v \in [0, 1]$ .

Members of the FGM family are symmetric, i.e.,  $C_{\theta}^{FGM}(u, v) = C_{\theta}^{FGM}(v, u)$  for all  $(u, v)$  in  $[0, 1]^2$  and have the lower and upper tail dependence coefficients equal to 0.

The copula given in (1) is PQD for  $\theta \in (0, 1]$  and NQD for  $\theta \in [-1, 0)$ . In practical applications this copula has been shown to be somewhat limited, for copula dependence parameter  $\theta \in [-1, 1]$ , Spearman's correlation  $\rho \in [-1/3, 1/3]$  and Kendall's  $\tau \in [-2/9, 2/9]$ , for more details on copulas see, for example, [1].

To overcome this limited dependence, several authors proposed extensions of this family.

- Theoretical issues: [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]
- Applications: [29, 30, 31, 32, 33, 34, 35, 36]

Let  $C$  be a copula introduced in [7]:

$$C(u, v) = uv + \theta\phi(u)\phi(v).$$

The parameter  $\theta$  is called the dependence parameter while the function  $\phi$  is called the generator. Given a random pair  $(U, V)$  distributed according to this copula, we define the survival copula as:

$$C^*(u, v) = P(U > 1 - u, V > 1 - v).$$

Probabilistic calculations show that

$$\begin{aligned} C^*(u, v) &= u + v - 1 + C(1 - u, 1 - v) \\ &= uv + \theta\phi(1 - u)\phi(1 - v) \\ &= uv + \theta\phi^*(u)\phi^*(v). \end{aligned}$$

Therefore,  $C^*$  is still an Amblard-Girard copula [7] with same dependence parameter  $\theta$  and with symmetrized generator  $\phi^*(\cdot) = \phi(1 - \cdot)$ .

## References

- [1] Roger B Nelsen. *An introduction to copulas*. Springer, 1999.
- [2] E. Stoica. A stability property of farlie-gumbel-morgenstern distributions. <http://hal.archives-ouvertes.fr/hal-00861234>.
- [3] E. Stoica. Convex sums of farlie-gumbel-morgenstern distributions. <http://hal.archives-ouvertes.fr/hal-00862992>.
- [4] E. Stoica. Extensions of farlie-gumbel-morgenstern distributions: A review. <http://hal.archives-ouvertes.fr/hal-00864676>.
- [5] Dong Yong-quan. Generation and prolongation of fgm copula. *Chinese journal of engineering mathematics*, 25:1137–1141, 2008.
- [6] Matthias Fischer and Ingo Klein. Constructing generalized fgm copulas by means of certain univariate distributions. *Metrika*, 65(2):243–260, 2007.
- [7] Cécile Amblard and Stéphane Girard. Symmetry and dependence properties within a semiparametric family of bivariate copulas. *Journal of Non-parametric Statistics*, 14(6):715–727, 2002.

- [8] Cécile Amblard and Stéphane Girard. Estimation procedures for a semi-parametric family of bivariate copulas. *Journal of Computational and Graphical Statistics*, 14(2), 2005.
- [9] Cécile Amblard and Stéphane Girard. A new extension of bivariate fgm copulas. *Metrika*, 70(1):1–17, 2009.
- [10] José Antonio Rodríguez-Lallena and Manuel Úbeda-Flores. A new class of bivariate copulas. *Statistics & probability letters*, 66(3):315–325, 2004.
- [11] Carles M Cuadras. Constructing copula functions with weighted geometric means. *Journal of Statistical Planning and Inference*, 139(11):3766–3772, 2009.
- [12] Fabrizio Durante. A new family of symmetric bivariate copulas. *Comptes Rendus Mathématique*, 344(3):195–198, 2007.
- [13] Fabrizio Durante, Stéphane Girard, and Gildas Mazo. Marshall–olkin type copulas generated by a global shock. *Journal of Computational and Applied Mathematics*, 296:638–648, 2016.
- [14] Fabrizio Durante, Stéphane Girard, and Gildas Mazo. Copulas based on marshall–olkin machinery. In *Marshall Olkin Distributions-Advances in Theory and Applications*, pages 15–31. Springer, 2015.
- [15] H Bekrizadeh, GA Parham, and MR Zadkarmi. A new class of positive dependent bivariate copula and its properties. In *2nd Workshop on Copula and its Applications*, page 12, 2012.
- [16] Gildas Mazo, Stephane Girard, and Florence Forbes. A flexible and tractable class of one-factor copulas. *Statistics and Computing*, 26(5):965–979, 2016.
- [17] C Amblard and S Girard. A semiparametric family of symmetric bivariate copulas. *Comptes Rendus de l'Académie des Sciences-Series I-Mathematics*, 333(2):129–132, 2001.
- [18] JS Huang and Samuel Kotz. Modifications of the farlie-gumbel-morgenstern distributions. a tough hill to climb. *Metrika*, 49(2):135–145, 1999.
- [19] Norman L Johnson and Samuel Kott. On some generalized farlie-gumbel-morgenstern distributions. *Communications in Statistics-Theory and Methods*, 4(5):415–427, 1975.
- [20] Norman L Johnson and Samuel Kotz. On some generalized farlie-gumbel-morgenstern distributions-ii regression, correlation and further generalizations. *Communications in Statistics-Theory and Methods*, 6(6):485–496, 1977.

- [21] I Bairamov, S Kotz, and M Bekci. New generalized farlie-gumbel-morgenstern distributions and concomitants of order statistics. *Journal of Applied Statistics*, 28(5):521–536, 2001.
- [22] CD Lai and M Xie. A new family of positive quadrant dependent bivariate distributions. *Statistics & probability letters*, 46(4):359–364, 2000.
- [23] Ismihan Bairamov and Samuel Kotz. On a new family of positive quadrant dependent bivariate distributions. *International Mathematical Journal*, 3(11):1247–1254, 2003.
- [24] RB Nelsen, JJ Quesada-Molina, and JA Rodríguez-Lallena. Bivariate copulas with cubic sections. *Journal of Nonparametric Statistics*, 7(3):205–220, 1997.
- [25] JJ Quesada-Molina and JA Rodríguez-Lallena. Bivariate copulas with quadratic sections. *Journal of Nonparametric Statistics*, 5(4):323–337, 1995.
- [26] Laurent Gardes and Stephane Girard. Nonparametric estimation of the conditional tail copula. *Journal of Multivariate Analysis*, 137:1–16, 2015.
- [27] Gildas Mazo, Stephane Girard, and Florence Forbes. Weighted least square inference based on dependence coefficients for multivariate copulas. *ESAIM: Probability and Statistics*, 19:746–765, 2015.
- [28] Gildas Mazo, Stephane Girard, and Florence Forbes. A class of multivariate copulas based on products of bivariate copulas. *Journal of Multivariate Analysis*, 140:363–376, 2015.
- [29] Helene Cossette, Etienne Marceau, and Fouad Marri. On the compound poisson risk model with dependence based on a generalized farlie-gumbel-morgenstern copula. *Insurance: Mathematics and Economics*, 43(3):444–455, 2008.
- [30] Margaret Armstrong and Alain Galli. Sequential nongaussian simulations using the fgm copula. *Cerna Working Paper*, 2002.
- [31] Mathieu Bargès, Hélène Cossette, Stéphane Loisel, Etienne Marceau, et al. On the moments of the aggregate discounted claims with dependence introduced by a fgm copula. *Astin Bulletin*, 41(1):215–238, 2011.
- [32] Yoon-Sung Jung, Jong-Min Kim, and Jinhwa Kim. New approach of directional dependence in exchange markets using generalized fgm copula function. *Communications in Statistics-Simulation and Computation*, 37(4):772–788, 2008.
- [33] Jong-Min Kim, Yoon-Sung Jung, Engin A Sungur, Kap-Hoon Han, Changyi Park, and Insuk Sohn. A copula method for modeling directional dependence of genes. *BMC bioinformatics*, 9(1):225, 2008.

- [34] D Long and R Krzysztofowicz. Farlie-gumbel-morgenstern bivariate densities: Are they applicable in hydrology? *Stochastic Hydrology and Hydraulics*, 6(1):47–54, 1992.
- [35] David X Li. On default correlation: A copula function approach. *The Journal of Fixed Income*, 9(4):43–54, 2000.
- [36] Dimitrios Panagiotou and Athanassios Stavrakoudis. Price dependence between different beef cuts and quality grades: a copula approach at the retail level for the us beef industry. *Journal of Agricultural & Food Industrial Organization*, 14(1):121–131, 2016.