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► **To cite this version:**

Emil Stoica. Using rotations to build non symmetric extensions of Amblard-Girard copulas. 2018.
hal-01781307

HAL Id: hal-01781307

<https://hal.science/hal-01781307>

Preprint submitted on 30 Apr 2018

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Using rotations to build non symmetric extensions of Amblard-Girard copulas

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Abstract

A copula is a function that completely describes the dependence structure between the marginal distributions. One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family. In practical applications this copula has been shown to be somewhat limited and a symmetric extension of this family, known as the Amblard-Girard copula, has been introduced. Basing on rotations, we propose a new non symmetric extension of this family.

A copula is a function that completely describes the dependence structure. It contains all the information to link the marginal distributions to their joint distribution. To obtain a valid multivariate distribution function, it suffices to combine several marginal distribution functions with any copula function. Thus, for the purposes of statistical modeling, it is desirable to have a large collection of copulas at one's disposal. Many examples of copulas can be found in the literature, most are members of families with one or more real parameters.

One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family defined as

$$C_{\theta}^{FGM}(u, v) = uv + \theta uv(1 - u)(1 - v), \quad u, v \in [0, 1], \quad (1)$$

where $\theta \in [-1, 1]$. The density function of FGM copulas is given by

$$\frac{\partial^2 C_{\theta}^{FGM}(u, v)}{\partial u \partial v} = \theta(2u - 1)(2v - 1) + 1,$$

for any $u, v \in [0, 1]$.

Members of the FGM family are symmetric, i.e., $C_{\theta}^{FGM}(u, v) = C_{\theta}^{FGM}(v, u)$ for all (u, v) in $[0, 1]^2$ and have the lower and upper tail dependence coefficients equal to 0.

The copula given in (1) is PQD for $\theta \in (0, 1]$ and NQD for $\theta \in [-1, 0)$. In practical applications this copula has been shown to be somewhat limited, for copula dependence parameter $\theta \in [-1, 1]$, Spearman's correlation $\rho \in [-1/3, 1/3]$ and Kendall's $\tau \in [-2/9, 2/9]$, for more details on copulas see, for example, [1].

To overcome this limited dependence, several authors proposed extensions of this family.

- Theoretical issues: [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]
- Applications: [29, 30, 31, 32, 33, 34, 35, 36]

Let C be a copula introduced in [7]:

$$C(u, v) = uv + \theta\phi(u)\phi(v).$$

The parameter θ is called the dependence parameter while the function ϕ is called the generator. Given a random pair (U, V) distributed according to this copula, we define two rotated copulas:

$$\begin{aligned} C_1(u, v) &= P(U > 1 - u, V \leq v) \\ C_2(u, v) &= P(U \leq u, V > 1 - v). \end{aligned}$$

Probabilistic calculations show that

$$\begin{aligned} C_1(u, v) &= v - C(1 - u, v) = uv - \theta\phi(1 - u)\phi(v) \\ C_2(u, v) &= u - C(u, 1 - v) = uv - \theta\phi(u)\phi(1 - v). \end{aligned}$$

Therefore, C_1 and C_2 can be considered as non-symmetric extensions of [7].

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