



HAL
open science

Dynamical Properties of Sea Surface Microwave Backscatter at Low-Incidence: Correlation Time and Doppler Shift

Olivier Boisot, Laiba Amarouche, Jean-Claude Lalaurie, Charles-Antoine Guérin

► **To cite this version:**

Olivier Boisot, Laiba Amarouche, Jean-Claude Lalaurie, Charles-Antoine Guérin. Dynamical Properties of Sea Surface Microwave Backscatter at Low-Incidence: Correlation Time and Doppler Shift. *IEEE Transactions on Geoscience and Remote Sensing*, 2016, 54 (12), pp.7385 - 7395. 10.1109/TGRS.2016.2601242 . hal-01780159

HAL Id: hal-01780159

<https://hal.science/hal-01780159>

Submitted on 19 Feb 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Dynamical Properties of Sea Surface Microwave Backscatter at Low-Incidence: Correlation Time and Doppler Shift

Olivier Boisot, Laïba Amarouche, Jean-Claude Lalaurie, and Charles-Antoine Guérin

Abstract—We investigate some temporal properties of the microwave backscattered field from the sea surface at low incidence, namely, the decorrelation time and the Doppler shift distribution. These quantities may have an important impact on the performances of the altimeter and synthetic aperture radar systems in Ku- and Ka-bands and must be accurately evaluated. In the framework of classical analytical scattering models and for realistic sea spectra, we obtain a simple expression for the decorrelation time with respect to the main sea state parameters and the scattering geometry. We further propose an original approach based on a time-domain estimator to evaluate the distribution of instantaneous Doppler shifts and the Doppler centroid. The evolution of the latter with the sea state and scattering angles is calculated and discussed. A procedure is proposed to recover the full two-sided Doppler spectrum. We discuss the use of the Doppler shift in view of the geophysical parameter retrieval at low incidence. We find that the surface wind vector can, in principle, be well estimated from the azimuthal variation of the Doppler shift, whereas the signature of the surface current is not sufficient to allow for its estimation.

Index Terms—Correlation time, Doppler shift, low-incidence, microwave ocean remote sensing.

I. INTRODUCTION

IN THE last years, there has been a significant improvement of the capabilities of current or forthcoming altimeter missions in terms of resolution and accuracy. Some of these improvements consist in the replacement of the usual centimeter radar wavelength (C- and Ku-bands) with millimeter wavelength (Ka-band). In a conventional altimeter, such as the AltiKa mission, the use of the Ka-band allows to work at higher pulse repetition frequencies (PRFs) due to the faster decorrelation of the backscattered signal and therefore to reach an average waveform within a smaller amount of time with a correct speckle noise amplitude. In the wide-swath altimetry

based on the synthetic aperture radar (SAR) interferometry such as in the Surface Water and Ocean Topography (SWOT) mission concept, Doppler information due to the satellite motion is used to enhance the along-track resolution as it is done in SAR imaging. However, this useful Doppler quantity is affected by a Doppler anomaly due to the motion of waves. The impact of this Doppler anomaly becomes crucial to estimate, as it can impact the SAR ground cells estimated location and azimuthal resolution and, hence, the final geophysical surface estimates. In this respect, the dynamical aspects of the near-nadir backscattered field become crucial, and their impact must be carefully quantified.

There has been a certain number of studies devoted to the analysis of the Doppler signal of the microwave radar echo in the framework of analytic (e.g., [1]–[11]) and rigorous numerical scattering models (e.g., [12]–[18]). As shown in some of these works at large and grazing incidence, a relevant description of the different mechanisms at the origin of the Doppler spectrum requires a (at least weakly) nonlinear description of water waves combined with advanced scattering models that can take into account complex effects related to polarization, multiple scattering, or shadowing. At low incidence, however, the situation is much simpler since the physical optics (PO) is the reference scattering model, whereas nonlinear interactions at the surface can be neglected in the first place. This opens the way to a complete analytic description of the Doppler spectrum in view of further comparisons with experimental data.

In this paper, we propose a study of the correlation time, as well as the Doppler shift of the backscattered signal induced by wave motion at low incidence. We obtain a simple expression of the decorrelation time with respect to the main oceanic parameters. We further discuss the wave-induced Doppler shift and propose an original approach to calculate the Doppler centroid and the full two-sided Doppler spectrum, which is based on the distribution of the instantaneous Doppler shift.

II. TIME-EVOLVING LINEAR WATER SURFACE

We assume that the elevation of the sea surface about its mean plane is described by a function $z = \eta(\mathbf{r}, t)$ of the horizontal coordinate ($\mathbf{r} = (x, y)$). We adopt the classical linear picture in which the time-evolving sea surface can be written as a continuous summation of independent harmonics, i.e.,

$$\eta(\mathbf{r}, t) = \text{Re} \left\{ \int_{\mathbb{R}^2} a(\mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)} d\mathbf{k} \right\} \quad (1)$$

Manuscript received May 25, 2016; revised July 22, 2016; accepted July 22, 2016. This work was supported in part by the Centre National d'Études Spatiales and in part by the Collecte Localisation Satellite.

O. Boisot is with the Mediterranean Institute of Oceanography, University Toulon, Aix Marseille University, CNRS, IRD, MIO UM 110 La Garde, France, and also with the Office National d'Études et de Recherches Aéropatiales (ONERA), Salon-de-Provence, France (e-mail: olivier.boisot@univ-tln.fr).

L. Amarouche and J.-C. Lalaurie are with Collecte Localisation Satellite and Centre National d'Études Spatiales, 31400 Toulouse, France (e-mail: lamarouche@cls.fr; Jean-Claude.Lalaurie@cnes.fr).

C.-A. Guérin is with the Mediterranean Institute of Oceanography, University Toulon, Aix Marseille University, CNRS, IRD, MIO UM 110 La Garde, France (e-mail: guerin@univ-tln.fr).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TGRS.2016.2601242

81 where $a(\mathbf{k})$ is the random complex amplitude of the wave
 82 associated to wavenumber \mathbf{k} ; $\omega_k = \sqrt{g\|\mathbf{k}\| + \gamma_0\|\mathbf{k}\|^3}$ is the
 83 gravity–capillarity wave dispersion relationship with $g =$
 84 $9.81 \text{ m} \cdot \text{s}^{-2}$, i.e., the gravitational constant; and $\gamma_0 = 7.29 \times$
 85 $10^{-5} \text{ m}^3 \cdot \text{s}^{-2}$, i.e., the surface tension coefficient of seawater
 86 (estimated from [19] with a sea surface temperature of 10°C
 87 and a salinity of 35 PSU). The spatiotemporal surface corre-
 88 lation function ($\rho(\mathbf{r}, t) = \langle \eta(\mathbf{r}, t)\eta(\mathbf{0}, 0) \rangle$), where $\langle \cdot \rangle$ represents
 89 the ensemble average) can be written as

$$\rho(\mathbf{r}, t) = \text{Re} \left\{ \int_{\mathbb{R}^2} \Psi(\mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega_k t)} d\mathbf{k} \right\} \quad (2)$$

90 where $\Psi(\mathbf{k}) = (1/2)\langle |a(\mathbf{k})|^2 \rangle$ is the directional wavenumber
 91 spectrum. The latter is usually written in polar coordinates
 92 (k, ϕ_k) as

$$\Psi(\mathbf{k}) = \frac{\Psi_0(k)}{k} F(k, \phi_k) \quad (3)$$

93 where $\Psi_0(k)$ is the omnidirectional spectrum, and $F(k, \phi_k)$
 94 is the spreading function describing the azimuthal variation
 95 of wave energy with respect to the wind direction. The di-
 96 rectional spectrum is, in general, not centrosymmetric (i.e.,
 97 $\Psi(-\mathbf{k}) \neq \Psi(\mathbf{k})$) as waves propagating along or against the
 98 main wind direction do not have the same energy. However,
 99 the spatial variations of a frozen surface at a given time, for
 100 example, $t = 0$, are described by a “true” power spectrum,
 101 which is the symmetrized version of the directional spectrum,
 102 i.e., $\Psi_s(\mathbf{k}) = (1/2)(\Psi(\mathbf{k}) + \Psi(-\mathbf{k}))$. The symmetrized spec-
 103 trum Ψ_s is relevant for the evaluation of the normalized radar
 104 cross section (NRCS) using the classical backscattering model.
 105 A popular model is the Elfouhaily *et al.* unified spectrum [20],
 106 whose spreading function is described by a simple biharmonic
 107 function, i.e.,

$$F(k, \phi_k) = \frac{1}{2\pi} \{1 + \Delta(k) \cos [2(\phi_k - \phi_w)]\} \quad (4)$$

108 where $0 < \Delta(k) < 1$ is a contrast function ensuring a correct
 109 ratio of upwind/crosswind slopes, and ϕ_w is the direction of
 110 the wind vector with respect to the x -axis. However, such
 111 directional spectrum, which does not distinguish the upwind
 112 and downwind directions, is insufficient to describe the dynam-
 113 ics of waves. Asymmetric spreading functions have been first
 114 proposed by Longuet-Higgins *et al.* [21] and later on refined by
 115 Plant [22] in order to preserve the ratio of upwind/crosswind
 116 slopes, i.e.,

$$F(k, \phi_k) = \frac{\left[\cos \left(\frac{\phi_k - \phi_w}{2} \right) \right]^{2\gamma(k)}}{\int_{-\pi}^{\pi} \left[\cos \left(\frac{\phi_k - \phi_w}{2} \right) \right]^{2\gamma(k)} d\phi_k} \quad (5)$$

117 where

$$\gamma(k) = -\ln \left(\frac{1 - \Delta(k)}{1 + \Delta(k)} \right) / \ln 2. \quad (6)$$

118 Along this paper, we will adopt this formulation of the spread-
 119 ing function together with the omnidirectional expression of the
 120 Elfouhaily *et al.* spectrum in the numerical experiments.

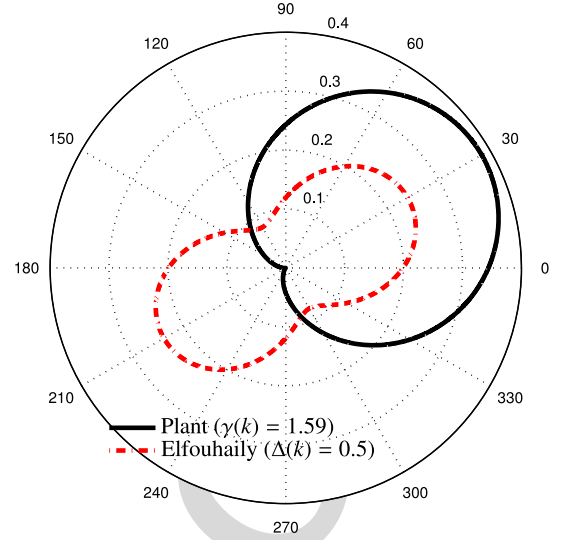


Fig. 1. Comparison of the Elfouhaily 1997 and Plant 2002 spreading functions for $\Delta(k) = 0.5 \leftrightarrow \gamma(k) = 1.59$. In this example, the wind is oriented at 30° from the origin. The dashed circles represent the values of the spreading functions F .

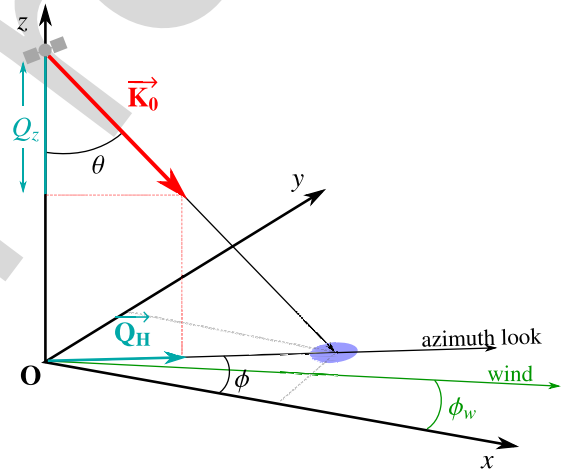


Fig. 2. Schematic of the scattering geometry.

III. TEMPORAL FIELD CORRELATION

121

A. PO Formalism

122

As it is customary, we assume the incident field on the 123
 sea surface to be a monochromatic plane wave with wave 124
 vector \mathbf{K}_0 and wavenumber K_0 (see Fig. 1). The temporal 125
 signal recorded on the receiver is proportional to the complex 126
 scattering amplitude $\mathbb{S}(t)$ of the backscattered field (see, e.g., 127
 [23]). The backscattered amplitude in the PO approximation, 128
 which is also known as the Kirchhoff approximation [24], is 129
 given by 130

$$\mathbb{S}(t) = \frac{1}{(2\pi)^2} \frac{\mathcal{K}}{Q_z} \int_{\mathbb{R}^2} e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{iQ_z \eta(\mathbf{r}, t)} d\mathbf{r} \quad (7)$$

where we have introduced the so-called Ewald vector $\mathbf{Q} =$ 131
 $-2\mathbf{K}_0$ with its horizontal projection \mathbf{Q}_H and vertical projection 132
 Q_z , and $\mathcal{K} = \mathbf{Q}^2 \mathcal{R} / 2$ is a geometric kernel, where \mathcal{R} stands for 133
 the complex Fresnel reflection coefficient at normal incidence 134
 for sea water. Fig. 2 depicts the geometry of the problem. 135

136 The Cartesian system (x, y, z) is a fixed reference frame. The
137 incidence (θ) and azimuth (ϕ) scattering angles are taken with
138 respect to the z - and x -axis, respectively. The azimuthal wind
139 direction is denoted by ϕ_w . In this configuration, the clas-
140 sical observation directions, namely, upwind, crosswind, and
141 downwind, are obtained for the azimuthal angles $\phi = \phi_w \pm \pi$,
142 $\phi = \phi_w \pm \pi/2$, and $\phi = \phi_w$, respectively. The dependence of
143 the viewing angles in expression (7) is implicit. Under the
144 assumption of a Gaussian random process for the sea surface,
145 the resulting field correlation function is given by

$$\begin{aligned} C_{\text{PO}}(t) &= \lim_{A \rightarrow \infty} \frac{4\pi}{A} \left(\langle \mathbb{S}(t)\mathbb{S}^*(0) \rangle - |\langle \mathbb{S}(0) \rangle|^2 \right) \\ &= \frac{1}{\pi} \frac{|\mathcal{K}|^2}{Q_z^2} \int_{\mathbb{R}^2} e^{-i\mathbf{Q}_H \cdot \mathbf{r}} \left[e^{-Q_z^2[\rho_0 - \rho(\mathbf{r}, t)]} - e^{-Q_z^2 \rho_0} \right] d\mathbf{r} \end{aligned} \quad (8)$$

146 where ρ_0 denotes the surface correlation at the origin (in time
147 and space). Note that the field correlation at the origin, i.e.,
148 $C_{\text{PO}}(0)$, reduces to the well-known expression of the NRCS in
149 the Kirchhoff approximation, i.e., σ_{PO}^0 . However, the numerical
150 evaluation of this correlation is, in general, more difficult than
151 the ordinary NRCS and deserves a specific numerical procedure
152 using polar coordinates and azimuthal Fourier series expansion.

153 B. GO Formalism

154 A well-known approximation of the PO, which is valid in
155 the limit of short radar wavelength, is the geometrical optics
156 (GO) approximation. Simple algebra using a spatiotemporal
157 second-order Taylor expansion of the surface autocorrelation
158 function ρ in the integrand (8) leads to the following expression
159 for the field correlation function in the GO approximation
160 (see the Appendix for detailed calculations):

$$C_{\text{GO}}(t) = \sigma_{\text{GO}}^0 \times T_{\text{GO}}(t) \times \mathbb{M}_{\text{GO}}(t). \quad (9)$$

161 Here, σ_{GO}^0 is the classical expression of the NRCS in the GO
162 approximation, i.e.,

$$\begin{aligned} \sigma_{\text{GO}}^0 &= \frac{|\mathcal{R}|^2}{2\Sigma \cos^4 \theta} \exp \left(-\frac{\tan^2 \theta}{2\Sigma^2} (s_{yy}^2 \cos^2 \phi \right. \\ &\quad \left. - 2s_{xy}^2 \sin \phi \cos \phi + s_{xx}^2 \sin^2 \phi) \right) \end{aligned} \quad (10)$$

163 with the directional mean-square slopes (the derivatives are
164 taken at the origin)

$$s_{xx}^2 = -\frac{\partial^2 \rho}{\partial x^2}, \quad s_{yy}^2 = -\frac{\partial^2 \rho}{\partial y^2}, \quad s_{xy}^2 = -\frac{\partial^2 \rho}{\partial x \partial y} \quad (11)$$

$$\Sigma^2 = s_{xx}^2 s_{yy}^2 - s_{xy}^4. \quad (12)$$

165 The second term T_{GO} is a Gaussian damping function, i.e.,

$$T_{\text{GO}}(t) = \exp(-2K_0^2 L^2 \cos^2 \theta t^2) \quad (13)$$

166 with

$$L^2 = s_{tt}^2 - \frac{s_{yy}^2 s_{xt}^2 - 2s_{xy}^2 s_{xt} s_{yt} + s_{xx}^2 s_{yt}^2}{\Sigma^2} \quad (14)$$

$$s_{tt}^2 = -\frac{\partial^2 \rho}{\partial t^2}, \quad s_{xt} = \frac{\partial^2 \rho}{\partial x \partial t}, \quad s_{yt} = \frac{\partial^2 \rho}{\partial y \partial t}. \quad (15)$$

The second-order time derivative s_{tt}^2 can be interpreted as the
167 variance of the vertical orbital velocity of waves. The spa- 168
169 tiotemporal cross-derivatives s_{xt} and s_{yt} have no such simple
170 interpretation. The third term in (9) is a complex azimuthal
171 modulation function, i.e.,

$$\begin{aligned} \mathbb{M}_{\text{GO}}(t) &= \exp \left(-i \frac{2K_0 t}{\Sigma^2} \sin \theta \right. \\ &\quad \left. \times [(s_{yy}^2 s_{xt} - s_{xy}^2 s_{yt}) \cos \phi + (s_{xx}^2 s_{yt} - s_{xy}^2 s_{xt}) \sin \phi] \right). \end{aligned} \quad (16)$$

Note that the expression of the temporal correlation in the GO
172 framework solely depends on the spatiotemporal second-order
173 derivatives of the surface correlation function at the origin.
174 These coefficients can be easily obtained using the correspond-
175 ing moments of the power spectrum. 176

C. Correlation Time 177

The correlation time τ_c of the backscattered field is typically
178 defined by 179

$$|C(\tau_c)| = a C(0) \quad (17)$$

where $|\cdot|$ stands for the absolute value (modulus), for some
180 threshold $0 < a < 1$. This quantity must be numerically evalu-
181 ated in the PO formalism but can be analytically estimated in
182 the GO formalism, with $T_{\text{GO}}(\tau_c) = a$, yielding 183

$$\tau_c = \frac{\sqrt{-\ln(a)/2}}{K_0 |\cos \theta| L}. \quad (18)$$

Note that the correlation time does not depend on the azimuth
184 angle in the GO formalism, which is confirmed by numerically
185 calculating this correlation time with the PO formalism. More-
186 over, we numerically found that the correlation times calculated
187 from the PO formalism and from the GO formalism are very
188 close in Ku- and Ka-bands, although the corresponding NRCS
189 can be significantly different (i.e., the GO formalism is not
190 valid to evaluate the NRCS at low incidence but useful to
191 evaluate the correlation time). At an incidence angle of $\theta = 20^\circ$,
192 which is in the limit of validity of the models, we found an
193 absolute difference smaller than 0.1 ms between the PO and
194 GO correlation times. 195

The simple analytic expression (18) of the correlation time
196 shows that the field decorrelation is mainly due to the motion
197 of the wave field in the line-of-sight of the radar through the
198 vertical orbital wave velocity (s_{tt}^2), which is numerically found
199 to be the dominant term in (14), i.e., $L \simeq s_{tt}$. This quantity is
200 related to the first moment of the wave spectrum and is mainly
201 contributed to by long waves, i.e., 202

$$s_{tt}^2 = \int_0^\infty \omega_k^2 \Psi_0(k) dk \simeq g \int_0^\infty k \Psi_0(k) dk. \quad (19)$$

For wind waves spectra in the gravity range, i.e., $\Psi_0(k) \sim k^{-3}$
203 and $s_{tt}^2 \simeq g k_p H_s^2 / 8$, where k_p is the peak wavenumber. Hence, 204

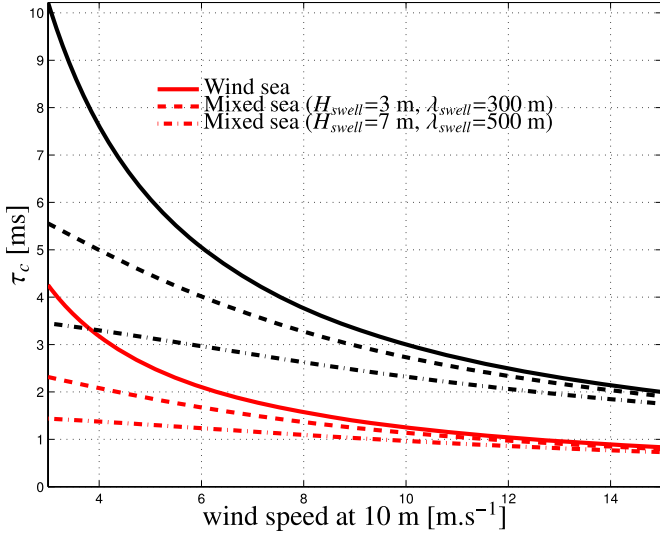


Fig. 3. Correlation time τ_c in (red lines) Ka-band ($f_0 = 36$ GHz) and (black lines) Ku-band ($f_0 = 15$ GHz) at nadir as a function of wind speed at 10 m for the decorrelation threshold $a = 1/e$.

205 the correlation time can be simply expressed in terms of the
206 main oceanic parameters k_p and H_s , i.e.,

$$\tau_c \simeq \frac{2\sqrt{-\ln(a)}}{K_0 |\cos \theta| \sqrt{gk_p H_s}}. \quad (20)$$

207 In the case of a mixed sea composed of a swell (with wavenum-
208 ber k_{swell} and significant wave height H_{swell}) and a wind sea
209 (with peak wavenumber k_p and significant wave height H_{wind} ,
210 the aforementioned formula can be easily adapted with $s_{tt}^2 \simeq$
211 $gk_p H_{wind}^2/8 + gk_{swell} H_{swell}^2/16$.

212 Fig. 3 shows a comparison of the correlation time τ_c in Ku-
213 (15 GHz) and Ka-band (36 GHz) at nadir as a function of
214 wind speed at 10 m above the sea surface, with a decorrelation
215 threshold set to $a = 1/e$. It has been calculated with both the
216 GO-based formula (18) and the PO model, with no numerical
217 difference. In addition to a pure wind sea case, a mixed sea
218 with a swell of wavelength longer than the dominant wind
219 sea wavelength (for a $15 \text{ m} \cdot \text{s}^{-1}$ wind speed, the dominant
220 wavelength is about 200 m) has been considered. For a pure
221 wind sea, the correlation time is decreased by a factor 4–5
222 from low to high wind speed. In the case of a mixed sea, the
223 correlation time is less sensitive to the wind speed and is mainly
224 driven by the swell parameters. It ranges from 0.5 to 4 ms in
225 Ka-band and from 2 to 10 ms in Ku-band. The simplified
226 expression (20) for the correlation time has been compared with
227 expression (18) and is found in excellent agreement (with at
228 most a few percent relative difference), showing that the corre-
229 lation time can be expressed in a simple and accurate manner
230 with only the peak wavenumber and the significant wave height.

231 A study of the phase of the backscattered signal from the sea
232 surface has been conducted by Chapman *et al.* [25], where au-
233 thors give measurements of the backscattered signal correlation
234 time at nadir for different microwave frequencies. Results are
235 given for a developed sea with $u_{10} \simeq 4.7 \text{ m/s}$ with negligible
236 swell. Using the classical relation for fully developed wind
237 seas, i.e., $H_s = 0.025 \times u_{10}^2$ and $k_p = 0.84^2 g/u_{10}^2$, we obtain

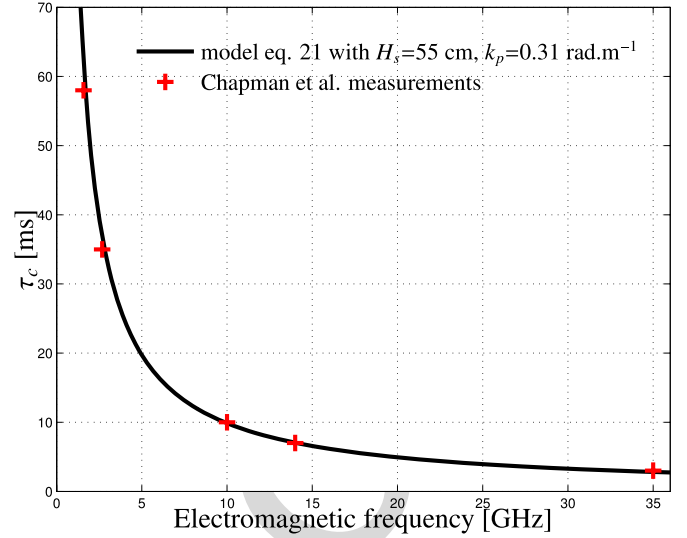


Fig. 4. Comparison of the correlation time according to model (20) and Chapman *et al.* [25] measurements as a function of the electromagnetic frequency at nadir for a decorrelation threshold $a = 1/e$.

$k_p \simeq 0.31 \text{ rad} \cdot \text{m}^{-1}$ and $H_s \simeq 55 \text{ cm}$. Fig. 4 compares the
238 measurements of the decorrelation time by Chapman *et al.*
239 and the estimation after formula (20) as a function of radar
240 frequency. An excellent agreement is obtained. 241

IV. WAVE-INDUCED DOPPLER SHIFT 242

A. Different Approaches to the Doppler Centroid 243

As it is well known, the backscattered time signal undergoes
244 a Doppler shift due to the combined motion of the platform
245 (airborne or spaceborne sensor) and the motion of waves at the
246 sea surface. The dominant contribution to the Doppler centroid
247 frequency arises from the relative velocity of the platform with
248 respect to the ground. It can be easily estimated and removed
249 by means of a geometrical model. The residual Doppler shift,
250 or Doppler anomaly, due to wave motion is much smaller but
251 induces azimuthal smearing resulting in the loss of resolution
252 in the SAR image. Here, we will try to estimate its statis-
253 tical properties as a function of sea state. There are several
254 approaches to calculate the Doppler shift of waves. The most
255 classical technique is the frequency-domain estimation (FDE)
256 based on the Doppler spectrum. For this, the Fourier transform
257 of the temporal correlation function is taken, i.e., 258

$$D(f) = \int_{-\infty}^{+\infty} e^{-2i\pi ft} C(t) dt \quad (21)$$

and the Doppler shift is defined as the mean (e.g., [7] and [8]) or
259 median (see [26]) frequency with respect to the normalized dis-
260 tribution $D(f)/\int D(f)df$. In practice, the Doppler spectrum
261 $D(f)$ is obtained through the variance of the periodogram of
262 the time signal (i.e., the sample average of $|\text{FFT}(\mathbb{S}(t))|^2$). The
263 advantage of the technique is that it gives the full shape of
264 the Doppler spectrum and discriminate positive and negative
265 frequency shifts ($D(-f) \neq D(f)$, in general). Note that the
266 estimation of the Doppler shift based on the mean or median of 267

268 the Doppler spectrum is meaningful only if the latter is essen-
 269 tially half-sided, as the negative and frequency components of
 270 the two-sided spectrum would cancel out and make the mean
 271 Doppler shift actually close to zero. The difficulty of the
 272 technique for simulation purposes is the full calculation of the
 273 temporal correlation function necessary to estimate the Doppler
 274 spectrum. To avoid this calculation, one often limits oneself
 275 to the estimation of the first two moments of the Doppler
 276 spectrum, which can be obtained with the time derivative of the
 277 signal correlation function at the origin (see, e.g., [7] and [8]).
 278 Another approach is the time-domain estimator (TDE),
 279 which has been found advantageous in the context of SAR
 280 systems (see [26]). It consists in estimating the phase of the
 281 complex signal correlation, i.e.,

$$C(t) = |C(t)| e^{i2\pi t f_c}. \quad (22)$$

282 The mean Doppler shift frequency is then simply obtained with

$$f_c = \frac{1}{t} \arg(C(t)). \quad (23)$$

283 This approach (also employed in [6] in the context of analytic
 284 scattering models) has the merits of simplicity but does not
 285 provide the dispersion around the Doppler shift.

286 An alternative TDE can be obtained using the instantaneous
 287 random signal, $\mathbb{S}(t)$ instead of its correlation function. We
 288 define the instantaneous Doppler frequency shift induced by
 289 wave motion as

$$f(t) = -\frac{1}{2\pi} \partial_t \varphi(t) \quad (24)$$

290 where $\varphi(t)$ is the scattering phase of the illuminated target (we
 291 note that with this convention, positive frequencies represent
 292 waves traveling to the radar). The scattering phase is related to
 293 the complex backscattered signal $\mathbb{S}(t)$ through

$$\varphi(t) = \arg(\mathbb{S}(t)) = \arctan \left[\frac{\text{Im}(\mathbb{S}(t))}{\text{Re}(\mathbb{S}(t))} \right] \quad (25)$$

294 from which we can infer the simple expression

$$f(t) = -\frac{1}{2\pi} \text{Im} \left[\frac{\partial_t \mathbb{S}(t)}{\mathbb{S}(t)} \right]. \quad (26)$$

295 We define the instantaneous Doppler shift distribution as
 296 the probability density function (pdf) associated to this last
 297 quantity.

298 B. Doppler Shift in the PO Formalism

299 Using expression (7) of the scattered field in the PO approx-
 300 imation, we obtain

$$f(t) = -\frac{1}{2\pi} \text{Im} \left(\frac{N_t}{D_t} \right) \quad (27)$$

301 with

$$\begin{cases} N_t = iQ_z \int_{\mathbb{R}^2} \partial_t \eta(\mathbf{r}, t) e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{iQ_z \eta(\mathbf{r}, t)} d\mathbf{r} \\ D_t = \int_{\mathbb{R}^2} e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{iQ_z \eta(\mathbf{r}, t)} d\mathbf{r}. \end{cases} \quad (28)$$

Here, a common normalization factor $2\pi\sqrt{A}$ appeared in the
 calculation and vanished to make these quantities indepen-
 dent of the illuminated area A . By virtue of the central limit
 theorem, these random surface integrals follow a centered
 complex-normal distribution. Classical two-point calculations
 on Gaussian random variables lead to the following expressions
 for their respective co- and cross-variances:

$$\begin{cases} \langle |N_t|^2 \rangle = -Q_z^2 \int_{\mathbb{R}^2} \left[Q_z^2 (\partial_t \rho(\mathbf{r}, 0))^2 + \partial_{tt} \rho(\mathbf{r}, 0) \right] \\ \quad \times e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{-Q_z^2 [\rho_0 - \rho(\mathbf{r}, 0)]} d\mathbf{r} \\ \langle |D_t|^2 \rangle = \int_{\mathbb{R}^2} e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{-Q_z^2 [\rho_0 - \rho(\mathbf{r}, 0)]} d\mathbf{r} \\ \langle N_t^* D_t \rangle = Q_z^2 \int_{\mathbb{R}^2} \partial_t \rho(\mathbf{r}, 0) e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{-Q_z^2 [\rho_0 - \rho(\mathbf{r}, 0)]} d\mathbf{r}. \end{cases} \quad (29)$$

Note that these quantities are time independent, with the second
 term equal to the NRCS in PO approximation, apart from a
 geometrical factor. The pdf associated to the phase derivative
 (27) can be obtained using recent results on the ratio of two
 correlated complex-Gaussian random variables (see [27]). It
 can be expressed as a nonstandardized Student's t -distribution

$$p(f) = \frac{\pi}{\sqrt{2}\Delta f} \left[1 + \frac{1}{2} \left(\frac{f - f_c}{\Delta f} \right)^2 \right]^{-3/2} \quad (30)$$

with the mean (central) Doppler shift frequency

$$f_c = \frac{C_i \langle |N_t|^2 \rangle^{1/2}}{2\pi \langle |D_t|^2 \rangle^{1/2}} \quad (31)$$

and the dispersion parameter

$$(\Delta f)^2 = \frac{1 - |C|^2 \langle |N_t|^2 \rangle}{8\pi^2 \langle |D_t|^2 \rangle}. \quad (32)$$

It involves the complex cross correlation of the random vari-
 ables N_t and D_t , i.e.,

$$C = \frac{\langle N_t^* D_t \rangle}{\sqrt{\langle |N_t|^2 \rangle \langle |D_t|^2 \rangle}} \quad (33)$$

which is decomposed into real and imaginary parts, i.e., $C =$
 $C_r + iC_i$. From this, we obtain a simple representation of
 the Doppler shift distribution after (24) requiring only the
 calculation of the three surface integrals in (29) involved in
 the three statistical parameters (31)–(33). Note that expression
 (31) of the mean Doppler shift is consistent with expression
 [7, eq. (IV.12)] derived from the first moment of the Doppler
 spectrum obtained with the FDE. Note, however, that the
 Doppler shift distribution (30) does not possess a finite variance,
 contrarily to the latter definition. Nevertheless, the nonstandard-
 ized Student's t -distribution possesses a dispersion parameter
 allowing to characterize the dispersion around its mean.
 We performed a numerical calculation of the mean Doppler
 shift due to wave motion (f_c) and its dispersion (Δf) with
 the sea spectrum described in Section II. An efficient numerical
 evaluation of the surface integrals in (29) has been achieved
 using an integration in polar coordinates together with an az-
 imuthal Fourier expansion of the surface correlation functions

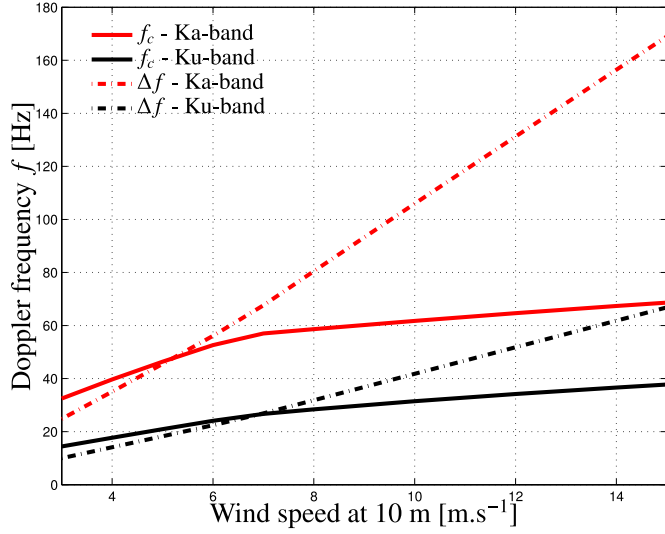


Fig. 5. Evolution of the residual Doppler shift parameters: central frequency f_c (plain) and dispersion parameter Δf (dash-dotted) as a function of surface roughness represented with the wind speed at 10 m for an incidence angle $\theta = 5^\circ$ in the upwind direction. Red plots represent the calculation in Ka-band ($f_0 = 36$ GHz), and black plots represent the calculation in Ku-band ($f_0 = 15$ GHz).

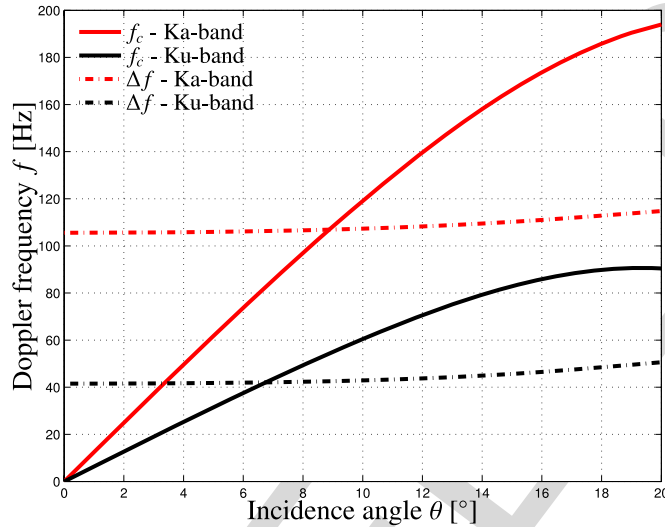


Fig. 6. Same as Fig. 5 except that parameters are plotted as a function of the incidence angle θ for a wind speed at 10 m of $10 \text{ m} \cdot \text{s}^{-1}$ in the upwind direction.

337 and related quantities. Figs. 5–7 describe the evolution of the 338 center and width of the Doppler shift distribution as a function 339 of surface roughness (see Fig. 5), incidence angle θ (see Fig. 6), 340 and azimuth angle ϕ (see Fig. 7) for the two microwave bands, 341 i.e., Ka ($f_0 = 36$ GHz) and Ku ($f_0 = 15$ GHz).

342 The mean and dispersion parameters of the residual Doppler 343 shift have a similar behavior in the two microwave bands, 344 except that absolute values are higher in Ka-band. In our 345 calculations, their ratio between the two bands is found to 346 be nearly constant and about 2 for the central frequency and 347 2.5 for the dispersion parameter regardless of the scattering 348 geometry and surface roughness. Note that the ratio of the 349 residual Doppler shift central frequencies is close but not equal 350 to the ratio of radar wavelengths (2.4). Fig. 5 shows that the

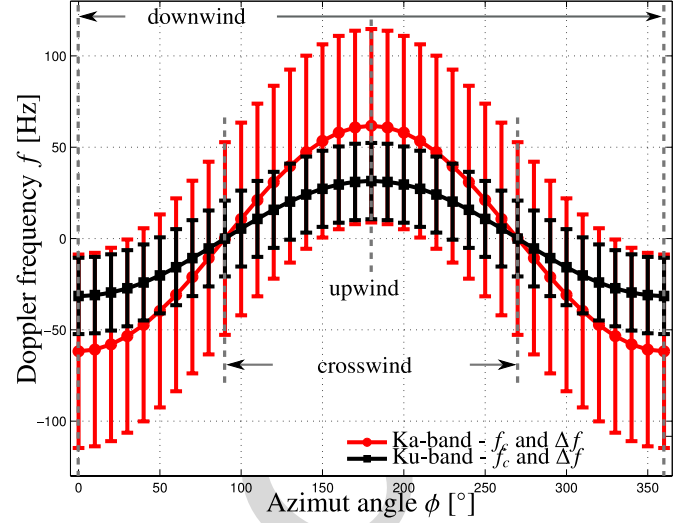


Fig. 7. Same as Fig. 5 except that parameters are plotted as a function of the azimuth angle ϕ for an incidence angle $\theta = 5^\circ$, a wind speed at 10 m of $10 \text{ m} \cdot \text{s}^{-1}$, and a wind direction $\phi_w = 0^\circ$. Dispersion parameter Δf is plotted as an error bar around its mean f_c .

central frequency f_c has a weak sensitivity to sea state, whereas 351 the dispersion parameter undergoes a dramatic increase with 352 wind speed. Hence, it seems that the width of the Doppler 353 shift distribution is a better proxy for wind speed estimation 354 than its mean value. The situation is opposite when it comes 355 to the dependence on the scattering angles: Fig. 6 shows that 356 central frequency f_c quickly increases with the incidence angle, 357 whereas the dispersion parameter remains quite constant. The 358 same qualitative behavior holds for the azimuthal dependence, 359 with a central frequency oscillating between positive (upwind 360 angular sector) and negative (downwind angular sector) values 361 and a quasi-constant dispersion parameter. A noticeable result 362 is that the central frequency follows a sinusoidal dependence 363 on the azimuthal angle as will be confirmed later on the basis of 364 theoretical considerations (see the discussion in Section VI-A). 365

V. TWO-SIDED DOPPLER SPECTRUM

366

Doppler spectra in the microwave regime exhibit, in general, 367 asymmetric components in the positive and negative frequen- 368 cies related to the velocities of waves traveling to and away 369 from the radar. The previous definition of the instantaneous fre- 370 quency (27) does not allow to differentiate the progressive and 371 regressive parts of the surface, which separately contributes to 372 the two parts of the Doppler spectrum. Hence, the instantaneous 373 Doppler shift distribution (30) is expected to be consistent 374 with normalized Doppler spectra, according to the classical 375 definition (21) for the one-sided spectra only (i.e., if all waves 376 are supposed to travel in the same direction). To be able to 377 distinguish positive and negative frequencies, we mathemat- 378 ically decompose the surface into a sum of progressive and 379 regressive waves (i.e., waves traveling to or against the radar 380 look direction) 381

$$\eta(\mathbf{r}, t) = \eta^+(\mathbf{r}, t) + \eta^-(\mathbf{r}, t) \quad (34)$$

where η^+ (respectively, η^-) is defined by the integral (1), with 382 a domain of integration restricted to wave vectors in the same 383

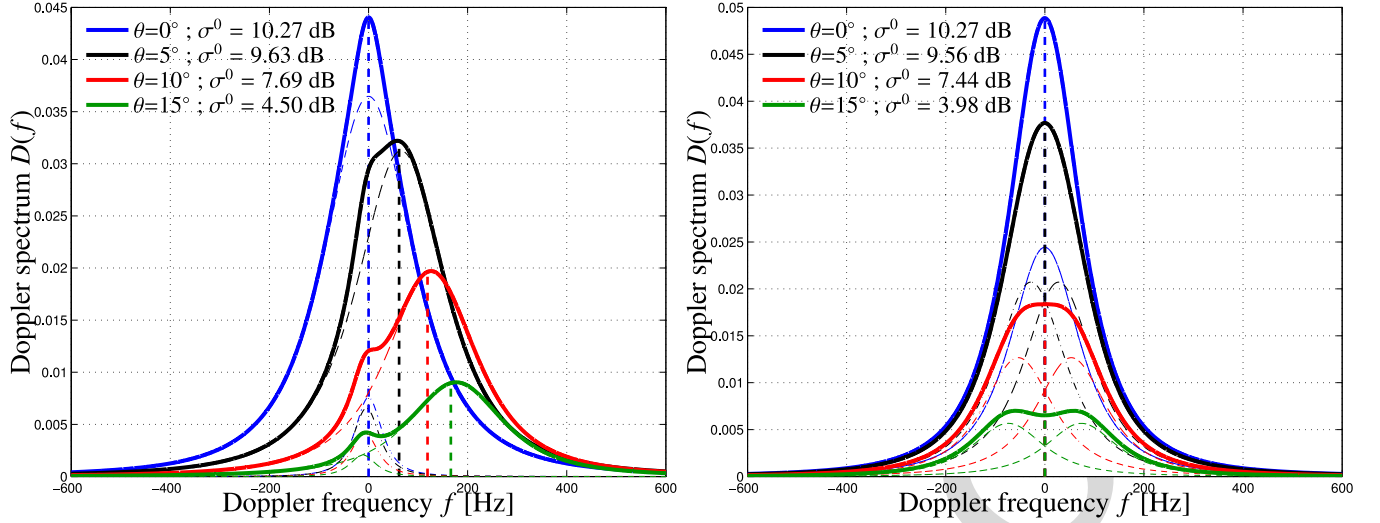


Fig. 8. Examples of the recomposed Doppler spectrum (thick lines) calculated for a wind speed at 10 m of $10 \text{ m} \cdot \text{s}^{-1}$ in Ka-band ($f_0 = 36 \text{ GHz}$) and several values of incidence angles θ . (Left) Calculation is made in the upwind direction. (Right) Calculation is made in the crosswind direction. The thin dashed lines represent the progressive part, and the thin dash-dotted lines the regressive part of the Doppler spectrum. The thick dashed lines represent the central frequency of the Doppler shift f_c .

384 half-plane (respectively, opposite half-plane) as the radar inci-
 385 dent wavenumber, i.e., $\mathbf{K}_0 \cdot \mathbf{k} > 0$ (respectively, $\mathbf{K}_0 \cdot \mathbf{k} < 0$).
 386 We accordingly decompose the surface autocorrelation function
 387 with

$$\rho(\mathbf{r}, t) = \rho^+(\mathbf{r}, t) + \rho^-(\mathbf{r}, t) \quad (35)$$

388 where, analogously, ρ^\pm is defined through the spectral integral
 389 (2) restricted to the integration domain $\pm \mathbf{K}_0 \cdot \mathbf{k} > 0$. We now
 390 assume that the Doppler spectrum is the summation of two
 391 subspectra obtained by assuming that only one category of
 392 waves is moving (progressive or regressive), whereas the other
 393 is frozen. Each subspectrum is proportional to the distribution
 394 p^\pm of instantaneous Doppler shifts f^\pm associated to the pro-
 395 gressive and regressive parts of the surface, i.e.,

$$f^\pm(t) = -\frac{1}{2\pi} \partial_t \varphi^\pm(t) = -\frac{1}{2\pi} \text{Im} \left[\frac{\partial_t \mathbb{S}^\pm(t)}{\mathbb{S}(t)} \right] \quad (36)$$

396 where it is understood that the time derivation $\partial_t \mathbb{S}^+$ is taken
 397 with respect to the progressive part of the surface only (η^+),
 398 the regressive part (η^-) being frozen, and conversely for $\partial_t \mathbb{S}^-$.
 399 We can therefore rewrite

$$D(f) = \alpha^+ p^+(f) + \alpha^- p^-(f) \quad (37)$$

400 for some weights α^\pm to be determined. The moments on the
 401 order of 0 and 1 of the Doppler spectrum must satisfy the fol-
 402 lowing consistency relation with the NRCS (σ^0) and the mean
 403 Doppler shift:

$$\begin{cases} \sigma^0 = \alpha^+ + \alpha^- \\ \sigma^0 f_c = \alpha^+ f_c^+ + \alpha^- f_c^- \end{cases} \quad (38)$$

404 where f_c and f_c^\pm are the mean frequencies associated to the
 405 Doppler spectrum and its subspectra

$$f_c = \int_{\mathbb{R}} f p(f) df, \quad f_c^\pm = \int_{\mathbb{R}} f p^\pm(f) df. \quad (39)$$

By solving this last system of equations, we obtain

$$\alpha^\pm = \pm \sigma^0 \frac{f_c - f_c^\mp}{f_c^+ - f_c^-}. \quad (40)$$

The calculation of the pdf of the phase derivatives (p^\pm) in 407
 the PO formalism is very similar to the calculation developed in 408
 Section IV-B, with the difference that the time derivation should 409
 be taken with respect to progressive or regressive waves only. 410
 The formula (28)–(33) remain similar with the only change that 411
 N_t and related quantities should be replaced by 412

$$\begin{cases} N_t^\pm = iQ_z \int_{\mathbb{R}^2} \partial_t \eta^\pm(\mathbf{r}, t) e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{iQ_z \eta(\mathbf{r}, t)} d\mathbf{r} \\ \langle |N_t^\pm|^2 \rangle = -Q_z^2 \int_{\mathbb{R}^2} \left[Q_z^2 (\partial_t \rho^\pm(\mathbf{r}, 0))^2 + \partial_{tt} \rho^\pm(\mathbf{r}, 0) \right] \\ \quad \times e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{-Q_z^2 [\rho_0 - \rho(\mathbf{r}, 0)]} d\mathbf{r} \\ \langle N_t^{\pm*} D_t \rangle = Q_z^2 \int_{\mathbb{R}^2} \partial_t \rho^\pm(\mathbf{r}, 0) e^{-i\mathbf{Q}_H \cdot \mathbf{r}} e^{-Q_z^2 [\rho_0 - \rho(\mathbf{r}, 0)]} d\mathbf{r} \end{cases} \quad (41)$$

Fig. 8 shows an example of the two-sided Doppler spectrum 413
 in Ka-band at 10 m/s wind speed for different incidence angles. 414
 At low incidence, the positive and negative Doppler spectra 415
 merged into a single wider peak. 416

VI. ESTIMATION OF SURFACE PARAMETERS 417

A. Wind Direction and Doppler Shift 418

Recent studies with coherent radars have established a clear 419
 relationship between the centroid of the Doppler anomaly 420
 and the wind vector above the sea surface (e.g., [28]–[31]) 421
 at moderate and large incidence angles ($> 20^\circ$). However, at 422
 low incidence, the central residual Doppler shift has a weak 423
 dynamic with respect to wind speed and is smaller than the 424
 dispersion parameter, as shown in Fig. 5. This makes the 425
 Doppler anomaly, when taken at fixed angles, a bad tracer for 426
 the wind vector. Nevertheless, a clear dependence with wind 427
 direction can be seen on the azimuthal variations of the Doppler 428
 shift central frequency. The sinusoidal variation observed in 429
 Fig. 7 can be well understood using GO-like developments 430

431 for the central frequency (31) expressed in the PO formalism.
 432 The main advantage of this formulation is that the central
 433 frequency is expressed as a function of statistical parameters of
 434 the surface and does not depend on the chosen wave spectrum.
 435 The expression of the central frequency is the same as the
 436 complex azimuthal modulation function of the GO formalism
 437 (16) and is expressed, in term of central frequency, i.e.,

$$f_c = -\frac{K_0 \sin \theta}{\pi \Sigma^2} \left[(s_{yy}^2 s_{xt} - s_{xy}^2 s_{yt}) \cos \phi \right. \\ \left. + (s_{xx}^2 s_{yt} - s_{xy}^2 s_{xt}) \sin \phi \right] \quad (42)$$

438 which can be rewritten as

$$f_c = -F_c \cos(\phi - \phi_w) \quad (43)$$

439 where

$$F_c = \frac{K_0 \sin \theta}{\pi \Sigma^2} \sqrt{(s_{yy}^2 s_{xt} - s_{xy}^2 s_{yt})^2 + (s_{xx}^2 s_{yt} - s_{xy}^2 s_{xt})^2} \quad (44)$$

440 is the maximum central frequency, and

$$\tan(\phi_w) = \frac{s_{xx}^2 s_{yt} - s_{xy}^2 s_{xt}}{s_{yy}^2 s_{xt} - s_{xy}^2 s_{yt}} \quad (45)$$

441 is the wind direction. The approximate formulas (42)–(45)
 442 based on GO-like expansion therefore explains the observed
 443 sinusoidal variation with the azimuthal angle observed in the
 444 PO formalism. However, they are not accurate enough to repro-
 445 duce the full dependence on the incidence angle. For example,
 446 it is shown in Fig. 6 that the evolution with the incidence angle
 447 is not merely sinusoidal as suggested by (44), although it is a
 448 good approximation at the lowest angles. This loss of accuracy
 449 in using the GO-like expression of the central frequency mainly
 450 arises from the difference between the PO and GO models in
 451 estimating the NRCS. As it is well known, the GO model has a
 452 more restricted domain of validity than the PO as the incidence
 453 angle is increased. Since the signal intensity enters in the
 454 calculation of the central frequency (see parameter $\langle |D_t|^2 \rangle$ in
 455 (29) which is proportional to the NRCS), this explains the slight
 456 discrepancy of the mean Doppler shifts calculated after these
 457 two models at a higher incidence. Note that this is not the case
 458 for the correlation time estimated in Section III, which is robust
 459 to the use of the GO model since it involves only the normalized
 460 (i.e., divided by the NRCS) spatiotemporal correlation function
 461 of the backscattered signal. An interesting consequence of the
 462 sinusoidal variation in azimuth for the central frequency is that
 463 a robust joint estimator of both the wind direction (ϕ_w), and
 464 the maximum central frequency (F_c) can be built from any
 465 azimuthal sampling of the instantaneous residual Doppler shift
 466 at a fixed incidence angle. This could be achieved, for example,
 467 using a maximum-likelihood estimator, as was done in [32] in
 468 the context of NRCS azimuthal airborne data with a strong
 469 level of noise. However, the construction and evaluation of this
 470 estimator goes beyond the scope of this paper and is left for
 471 further research.

472 B. Influence of a Constant Surface Drift

473 In the last decade, it has been demonstrated with spaceborne
 474 [5] and airborne [30] data that the Doppler shift of the radar

echo carries a clear signature of the surface current once cor- 475
 rected from the wind–wave-induced Doppler anomaly. These 476
 results are, however, limited to medium incidences and strong 477
 currents, and we will address here the issue of low incidence 478
 and small currents. The additional Doppler frequency shift 479
 induced by a surface drift of norm U oriented in the horizontal 480
 plane with an azimuth angle ϕ_U with respect to the x -axis 481
 (see Fig. 2) is of the form 482

$$f_{\text{drift}} = -\frac{2}{\lambda_0} U \sin \theta \cos(\phi - \phi_U) = -F_U \cos(\phi - \phi_U). \quad (46)$$

The resulting total Doppler shift is given by (assuming no 483
 hydrodynamical interactions between surface drift and wave 484
 motion) 485

$$F = -F_c \cos(\phi - \phi_w) - F_U \cos(\phi - \phi_U) = -F_m \cos(\phi - \Phi) \quad (47)$$

where F_m and Φ are a combination of the Doppler shift 486
 parameters associated to wave motion and surface drift 487

$$\begin{cases} F_m = \sqrt{F_c^2 + F_U^2 + 2F_c F_U \cos(\phi_w - \phi_U)} \\ \tan \Phi = \frac{F_c \sin \phi_w + F_U \sin \phi_U}{F_c \cos \phi_w + F_U \cos \phi_U} \end{cases} \quad (48)$$

The four unknown parameters F_c , F_U , ϕ_w , and ϕ_U cannot be 488
 recovered from the sole knowledge of F_m and Φ . Therefore, 489
 the inversion of the surface current magnitude and direction re- 490
 quires independent knowledge and compensation of the wave- 491
 induced surface velocity, as was done in [30] with the airborne 492
 ATI SAR data. Furthermore, even in the most favorable case, 493
 where the surface drift is oriented in the direction of the 494
 azimuthal look of the radar ($\phi_U = \phi$), we find $F_U/F_c \simeq 495$
 $0.4 \text{ s} \cdot \text{m}^{-1} U$ in Ka-band and $\simeq 0.3 \text{ s} \cdot \text{m}^{-1} U$ in Ku-band at 496
 $10 \text{ m} \cdot \text{s}^{-1}$ wind speed, regardless of the incidence angle. 497
 Hence, given the level of dispersion of the residual Doppler 498
 shift, the relative variation induced by the surface drift seems 499
 too small (4% for $U = 10 \text{ cm} \cdot \text{s}^{-1}$) to allow for an estimation 500
 of the latter in the case of moderate small-scale surface currents, 501
 even if the wave-induced Doppler anomaly is known from some 502
 other instrument or a geophysical model function (such as [28]). 503

504 C. Potential Applications to Forthcoming 505 Spaceborne Missions

This study was primarily motivated by the need to estimate 506
 the loss of resolution and ground cells shifts induced by wave 507
 motion for the SWOT Ka interferometer (Karin, [33]). In its 508
 Low Rate mode over the ocean surface, the nominal resolution 509
 is expected to be 1 km^2 after averaging of the 250-m on-board 510
 unfocused SAR resolution at a PRF of 4420 Hz with 2-ms 511
 integration time. Our analysis has shown that with such a value, 512
 the integration time remains larger than the correlation time, 513
 which has been found on the order of 1 ms so that the mul- 514
 tilook averaging process is efficient in processing independent 515
 samples. 516

However, the wave-induced Doppler shifts are not negligible 517
 and have to be considered in the algorithms used to estimate 518
 ocean surface parameters from on-board radar altimeter signals. 519

520 Indeed, if we consider the configuration of SWOT mission and,
 521 for example, an incidence angle of 3° in the upwind direction
 522 for a wind speed of $10 \text{ m} \cdot \text{s}^{-1}$ at 10 m, the mean Doppler shift
 523 is of 40 Hz (see Fig. 6). This Doppler shift, if not corrected
 524 for, induces a shift of the ground cells location of about 20 m
 525 (using a simple relationship between Doppler frequency and
 526 ground location), which may be not negligible for the ocean
 527 height estimation.

528 In the case of crosswind direction, the mean Doppler shift
 529 was found to be zero. Nevertheless, for all wind conditions, the
 530 dispersion of the Doppler shift is always not null and for a wind
 531 speed of $10 \text{ m} \cdot \text{s}^{-1}$ the ground cells location shift can reach
 532 50 m whatever the wind direction and the incidence angle.

533 It should be noted that the aforementioned conclusions have
 534 considered the mean Doppler shift and its dispersion value, and
 535 they should be moderated at least for the cases of null mean
 536 Doppler shifts. Indeed, in that case, we can consider that the
 537 dispersion value of the shift is a possible value that does not
 538 systematically occur and that the shift in the ground cells loca-
 539 tion is random. We can anticipate that the impact of this random
 540 shift on the ground cells location is an additional noise on the
 541 range and, hence, the ocean height estimate. We recommend
 542 that further assessment of the impact of the aforementioned
 543 findings on the present algorithms being designed for SWOT
 544 mission be performed.

545 In the case of the nadir Doppler altimetry in Ku-band as
 546 for CryoSat-2, Sentinel-3, and Jason-6, the incidence angle is
 547 zero; and the mean value of the Doppler shift is always zero.
 548 However, the dispersion value of the Doppler shift is, again, not
 549 null as for the SWOT mission case. For the case of Ku-band,
 550 the dispersion term is of about 40 Hz inducing a possible
 551 ground cell shift of approximately 50 m. This may be again
 552 not negligible and could increase the ocean height estimation
 553 noise. This should be carefully analyzed in the future.

554 Another conclusion of the present study is that the instan-
 555 taneous residual Doppler shift distribution at low incidence
 556 can be used as a proxy of wind speed through its dispersion
 557 parameter rather than its centroid frequency, whereas the wind
 558 direction can be, in principle, estimated from its azimuthal vari-
 559 ations. While this is certainly not the optimal way to estimate
 560 the wind vector, as compared with conventional scatterome-
 561 ters, this information could be used in complement to another
 562 sensor. For example, this supports the concept of azimuthally
 563 scanning radars in the range of 0° – 15° of incidence, such as
 564 the SWIM instrument of the CFOSAT mission [34], which has
 565 been primarily devised for wave spectra estimation but could
 566 be also used in an upgraded coherent version for simultaneous
 567 wind estimation. Another interesting potential application is the
 568 estimation of the wind vector in extreme weather conditions.
 569 We did not push the model to very high wind speed, at which
 570 the spectral surface models, as well as the PO/GO scattering
 571 models, become questionable. However, it is known that the
 572 altimeter cross section remains sensitive to wind speed even
 573 by strong sea states. If the observed trends remain true at
 574 higher wind speed, the Doppler-based wind estimation at low
 575 incidence could be a valuable tool for wind vector estimation in
 576 high wind conditions. As to the surface current, its component
 577 along the radar direction at low incidence is too weak to be esti-

mated from a Doppler shift. However, even at large incidences,
 578 where its effect is larger, the contribution of surface current to
 579 the Doppler shift is drowned in the Doppler anomaly induced
 580 by wave motion, which is the dominant contribution. Hence,
 581 the elimination of this Doppler anomaly is a first necessary step
 582 in view of any estimation of the surface current. For this, it
 583 might be interesting to combine multiple sensors at low and
 584 high incidences to better characterize the long waves and their
 585 Doppler anomaly. 586

VII. CONCLUSION

587

We have investigated the decorrelation time and the instan-
 588 taneous residual Doppler shift distribution of the sea surface
 589 backscattered signal in the framework of the PO and GO at
 590 low incidence. A simple expression has been found for the
 591 decorrelation time as a function of the main sea state parameters
 592 (peak wavenumber and significant wave height), in excellent
 593 agreement with the rare data available in the literature. For the
 594 SWOT mission, the decorrelation time is consistent with the
 595 multilook averaging process. In the framework of the PO, we
 596 have proposed a TDE for the instantaneous residual Doppler
 597 shift distribution due to wave motion resulting in a simple
 598 analytic expression in terms of a nonstandardized Student's
 599 t -distribution and the statistical parameters of the surface. The
 600 limitation of the TDE, namely, the inability to separate positive
 601 and negative Doppler shifts, is overcome to produce a full two-
 602 sided Doppler spectrum. The evolution of the mean and dis-
 603 persion of the Doppler shift has been investigated with respect
 604 to the sea states and the scattering angles. The mean Doppler
 605 shift has a weak sensitivity to wind speed but its dispersion
 606 dramatically increases with the latter, suggesting that the width
 607 of the Doppler shift distribution is a better proxy than its mean
 608 value for wind speed estimation at low incidence. The situation
 609 is opposite when it comes to the influence of the scattering
 610 geometry as the mean Doppler frequency mainly depends on
 611 the scattering angles, whereas the dispersion parameter mainly
 612 depends on the surface roughness. We have further shown that
 613 the mean Doppler frequency follows a sinusoidal variation in
 614 azimuth with respect to the wind direction, which could be
 615 used to devise a robust estimator of its direction. The influence
 616 of additional surface currents has been evaluated. Their impact
 617 on the mean Doppler shift cannot be separated from the wave-
 618 induced Doppler anomaly unless the latter is known and accu-
 619 rately compensated by some other means. However, even in this
 620 case, a systematic inversion seems out of reach as the relative
 621 contribution of the surface current to the mean Doppler shift
 622 is small, whereas the dispersion of the wave-induced Doppler
 623 shift is large. Finally, it has been recommended that the derived
 624 distribution of Doppler shift be further considered in the SWOT
 625 mission to further assess the impact on the estimated parameters
 626 and possibly improve the algorithms accordingly. 627

APPENDIX A

628

In the limit of short radar-wavelength the coherent term
 629 $e^{-Q_z^2 \rho_0}$ in integral (8) vanishes and the spatiotemporal corre-
 630 lation function of the surface (ρ) can be approximated by its
 631

$$C_{GO}(t) = e^{-\frac{Q_z^2}{2} \left[s_{tt}^2 - \frac{s_{yy}^2 s_{xt}^2 - 2s_{xy}^2 s_{xt} s_{yt} + s_{xx}^2 s_{yt}^2}{\Sigma^2} \right] t^2} \underbrace{e^{-i[Q_{Hx}x_0(t) + Q_{Hy}y_0(t)]}}_{M_{GO}(t) \text{ (16)}} \times \underbrace{\frac{1}{\pi} \frac{|\mathbb{K}|^2}{Q_z^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i[Q_{Hx}x + Q_{Hy}y]} e^{-\frac{Q_z^2}{2} [s_{xx}^2 x^2 + 2s_{xy}^2 xy + s_{yy}^2 y^2]} dx dy}_{\sigma_{GO}^0 \text{ (10)}} \quad (51)$$

second-order Taylor expansion about the origin (the dependence to the wind direction ϕ_w is implicit in the following expressions):

$$\rho(0, 0, 0) - \rho(x, y, t) \simeq s_{xx}^2 \frac{x^2}{2} - s_{xt} x t + s_{xy}^2 x y - s_{yt} y t + s_{yy}^2 \frac{y^2}{2} \quad (49)$$

where the coefficients are defined by (11) and (15). Denoting Q_{Hx} and Q_{Hy} the projections of the horizontal component of the Ewald vector (\mathbf{Q}_H) on the x - and y -axis, respectively, this leads to the following approximation of the correlation integral:

$$C_{GO}(t) = \frac{1}{\pi} \frac{|\mathbb{K}|^2}{Q_z^2} e^{-\frac{Q_z^2}{2} s_{tt}^2 t^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i[Q_{Hx}x + Q_{Hy}y]} \times \dots \dots e^{-\frac{Q_z^2}{2} [s_{xx}^2 x^2 - 2s_{xt} x t + 2s_{xy}^2 x y - 2s_{yt} y t + s_{yy}^2 y^2]} dx dy. \quad (50)$$

Using the standard formula for the Fourier transform of a noncentered 2-D Gaussian function, we obtain, (51) shown at the top of the page. leading to expression (9).

ACKNOWLEDGMENT

The authors would like to thank M. Saillard for useful comments.

REFERENCES

[1] D. R. Thompson, "Calculation of microwave Doppler spectra from the ocean surface with a time-dependent composite model," in *Radar Scattering from Modulated Wind Waves*. New York, NY, USA: Springer-Verlag, 1989, pp. 27–40.

[2] W. J. Plant, "A model for microwave Doppler sea return at high incidence angles: Bragg scattering from bound, tilted waves," *J. Geophys. Res., Oceans*, vol. 102, no. C9, pp. 21 131–21 146, 1997.

[3] V. U. Zavorotny and A. G. Voronovich, "Two-scale model and ocean radar Doppler spectra at moderate-and low-grazing angles," *IEEE Trans. Antennas Propag.*, vol. 46, no. 1, pp. 84–92, Jan. 1998.

[4] R. Romeiser and D. Thompson, "Numerical study on the along-track interferometric radar imaging mechanism of oceanic surface currents," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 1, pp. 446–458, Jan. 2000.

[5] B. Chapron, F. Collard, and F. Ardhuin, "Direct measurements of ocean surface velocity from space: Interpretation and validation," *J. Geophys. Res., Oceans*, vol. 110, no. C7, pp. 76–92, Jul. 2005.

[6] A. A. Mouche, B. Chapron, N. Reul, and F. Collard, "Predicted Doppler shifts induced by ocean surface wave displacements using asymptotic electromagnetic wave scattering theories," *Waves Random Complex Media*, vol. 18, pp. 185–196, 2008.

[7] F. Nouguier, C.-A. Guérin, and G. Soriano, "Analytical techniques for the Doppler signature of sea surfaces in the microwave regime—I: Linear surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 12, pp. 4856–4864, Dec. 2011.

[8] F. Nouguier, C.-A. Guérin, and G. Soriano, "Analytical techniques for the Doppler signature of sea surfaces in the microwave regime—II: Non-linear surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 12, pp. 4920–4927, Dec. 2011.

[9] X. Li and X. Xu, "Scattering and Doppler spectral analysis for two-dimensional linear and nonlinear sea surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 2, pp. 603–611, Feb. 2011.

[10] Y. Wang, Y. Zhang, M. He, and C. Zhao, "Doppler spectra of microwave scattering fields from nonlinear oceanic surface at moderate-and low-grazing angles," *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 4, pp. 1104–1116, Apr. 2012.

[11] F. Fois, P. Hooeboom, F. Le Chevalier, and A. Stoffelen, "An analytical model for the description of the full-polarimetric sea surface Doppler signature," *J. Geophys. Res., Oceans*, vol. 120, no. 2, pp. 988–1015, 2015. doi: <http://dx.doi.org/10.1002/2014JC010589>.

[12] J. V. Toporkov and G. S. Brown, "Numerical simulations of scattering from time-varying, randomly rough surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 4, pp. 1616–1624, Jul. 2000.

[13] J. Johnson and R. Burkholder, "Coupled canonical grid/discrete dipole approach for computing scattering from objects above or below a rough interface," *IEEE Trans. Geosci. Remote Sens.*, vol. 39, no. 6, pp. 1214–1220, Jun. 2001.

[14] A. R. Hayslip, J. T. Johnson, and G. R. Baker, "Further numerical studies of backscattering from time evolving non-linear sea surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 41, no. 10, pp. 2287–2293, Oct. 2003.

[15] G. Soriano, M. Joelson, and M. Saillard, "Doppler spectra from a two-dimensional ocean surface at l-band," *IEEE Trans. Geosci. Remote Sens.*, vol. 44, no. 9, pp. 2430–2437, Sep. 2006.

[16] J. V. Toporkov and M. A. Sletten, "Statistical properties of low-grazing range-resolved sea surface backscatter generated through two-dimensional direct numerical simulations," *IEEE Trans. Geosci. Remote Sens.*, vol. 45, no. 5, pp. 1181–1197, May 2007.

[17] C.-S. Chae and J. T. Johnson, "A study of sea surface range-resolved Doppler spectra using numerically simulated low-grazing-angle backscatter data," *IEEE Trans. Geosci. Remote Sens.*, vol. 47, no. 6, pp. 3452–3460, Jun. 2013.

[18] D. Miret, G. Soriano, F. Nouguier, P. Forget, M. Saillard, and C.-A. Guérin, "Sea surface microwave scattering at extreme grazing angle: numerical investigation of the Doppler shift," *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 11, pp. 7120–7129, Nov. 2014.

[19] M. H. Sharqawy, J. H. Lienhard V, and S. M. Zubair, "Thermophysical properties of seawater: A review of existing correlations and data," *Desalination Water Treat.*, vol. 16, pp. 354–380, Apr. 2010.

[20] T. Elfouhaily, B. Chapron, K. Katsaros, and D. Vandemark, "A unified directional spectrum for long and short wind-driven waves," *J. Geophys. Res.*, vol. 102, no. C7, pp. 15 781–15 796, Jul. 1997.

[21] M. S. Longuet-Higgins, D. E. Cartwright, and N. D. Smith, "Observations of the directional spectrum of sea waves using the motions of a floating buoy," in *Ocean Wave Spectra*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1963, pp. 111–136.

[22] W. J. Plant, "A stochastic, multiscale model of microwave backscatter from the ocean," *J. Geophys. Res., Oceans*, vol. 107, no. C9, pp. 3-1–3-21, Sep. 2002. [Online]. Available: <http://dx.doi.org/10.1029/2001JC000909>

[23] T. Elfouhaily and C.-A. Guérin, "A critical survey of approximate scattering wave theories from random rough surfaces," *Waves Random Complex Media*, vol. 14, no. 4, pp. 1–40, 2004.

[24] P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves From Rough Surfaces*. Norwood, MA, USA: Artech House, 1987.

[25] R. D. Chapman, B. L. Gotwols, and R. E. Sterner, "On the statistics of the phase of microwave backscatter from the ocean surface," *J. Geophys. Res., Oceans*, vol. 99, no. C8, pp. 16 293–16 301, 1994. doi: <http://dx.doi.org/10.1029/94JC01111>.

[26] S. Madsen, "Estimating the Doppler centroid of SAR data," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 25, no. 2, pp. 134–140, Mar. 1989.

[27] R. Baxley, B. Walkenhorst, and G. Acosta-Marum, "Complex Gaussian ratio distribution with applications for error rate calculation in fading channels with imperfect CSI," in *Proc. IEEE GLOBECOM*, Dec. 2010, pp. 1–5.

738 [28] A. A. Mouche *et al.*, “On the use of Doppler shift for sea surface wind
739 retrieval from SAR,” *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 7,
740 pp. 2901–2909, Jul. 2012.

741 [29] K. F. Dagestad *et al.*, “Wind retrieval from synthetic aperture Radar—An
742 overview,” Eur. Space Agency, ESA, Paris, France, 2013.

743 [30] A. C. H. Martin, C. Gommenginger, J. Marquez, S. Doody, V. Navarro,
744 and C. Buck, “Wind-wave-induced velocity in ATI SAR ocean surface
745 currents: First experimental evidence from an airborne campaign,”
746 *J. Geophys. Res., Oceans*, vol. 121, no. 3, p. 1640–1653, 2016.

747 [31] P. Forget, M. Saillard, C.-A. Guérin, J. Testud, and E. L. Bouar, “On the
748 use of x-band weather radar for wind field retrieval in coastal zone,”
749 *J. Atmos. Ocean. Technol.*, vol. 33, no. 5, pp. 899–917, May 2016.

750 [32] Z. Guerraou, S. Angelliaume, L. Rosenberg, and C.-A. Guérin, “Investiga-
751 tion of azimuthal variations from x-band medium grazing angle sea clut-
752 ter,” *IEEE Trans. Geosci. Remote Sens.*, vol. 54, no. 10, pp. 6110–6118,
753 Oct. 2016.

754 [33] M. Durand, L. L. Fu, D. P. Lettenmaier, D. E. Alsdorf, E. Rodriguez, and
755 D. Esteban-Fernandez, “The surface water and ocean topography mission:
756 Observing terrestrial surface water and oceanic submesoscale eddies,”
757 *Proc. IEEE*, vol. 98, no. 5, pp. 766–779, May 2010.

758 [34] C. Tison *et al.*, “Directional wave spectrum estimation by swim instru-
759 ment on CFOSAT,” in *Proc. IEEE Int. Geosci. Remote Sens. Symp.*,
760 Jul. 2009, vol. 5, pp. V-312–V-315.

Laïba Amarouche, photograph and biography not available at the time of 777
778 publication. 778



Jean-Claude Lalaurie 779 AQ7

Since 1991, he has been an Engineer with the 780
Centre National d’Études Spatiales (CNES) and 781
started his career in microwave instrument design 782
and development [MMIC components for radio 783
communication equipment and synthetic aperture 784
radar (SAR) systems]. In 1999, he started special- 785
izing in electric and magnetic field instruments for 786
plasma characterization. He was involved in the de- 787
velopment of several instruments, including Deme- 788
ter, Venus Express, and SWARM. Since 2010, he 789
has been on water surface backscattering characterization, interferometric SAR 790
height bias estimation, and acquisition of near-field and airborne Ka-band data 791
in the framework of the Surface Water and Ocean Topography project. 792



Olivier Boisot was born in Toulon, France, in 1987.
He received the M.S. degree in physics and engineer-
ing sciences and the Ph.D. degree in physics from
the University of Toulon, La Garde, France, in 2012,
respectively 2015.

He was with the Mediterranean Institute of Ocean-
ography, University of Toulon, La Garde, France,
the Centre National d’Études Spatiales (CNES),
Toulouse, France, and Collecte Localisation Satellite
(CLS), Toulouse, France, on electromagnetic scatter-
ing modelization from rough surfaces and its appli-
cations to the Surface Water and Ocean Topography mission. He is currently
a Postdoctoral Fellow with the Office National d’Études et de Recherches
Aéspatiales (ONERA), Salon-de-Provence, France, and is working on the
detection and characterization of chemical products and oil on the ocean surface
from synthetic aperture radar measurements in microwave bands.



Charles-Antoine Guérin received the B.Eng. de- 793
gree from the Ecole Nationale Supérieure de 794
l’Aéronautique et de l’Espace, Toulouse, France, in 795
1994 and the Ph.D. degree in theoretical physics 796
from the University of Aix-Marseille, Marseille, 797
France, in 1998. 798

He is currently a Professor and Researcher with 799
the Mediterranean Institute of Oceanography, Uni- 800
versity of Toulon, La Garde, France. He specializes 801
in ocean remote sensing. 802