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A probabilistic method for the estimation of ocean surface currents from short time series of HF radar data

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Abstract

We present a new method for inverting ocean surface currents from beam-forming HF radar data. In contrast with the classical method, which inverts radial currents based on shifts of the main Bragg line in the radar Doppler spectrum, the method works in the temporal domain and inverts currents from the amplitude modulation of the I and Q radar time series. Based on this principle, we propose a Maximum Likelihood approach, which can be combined with a Bayesian inference method assuming a prior current distribution, to infer values of the radial surface currents. We assess the method performance by using synthetic radar signal as well as field data, and systematically comparing results with those of the Doppler method. The new method is found advantageous for its robustness to noise at long range, its ability to accommodate shorter time series, and the possibility to use *a priori* information to improve the estimates. Limitations are related to current sign errors at far-ranges and biased estimates for small current values and very short samples. We apply the new technique to a data set from a typical 13.5 MHz WERA radar, acquired off of Vancouver Island, BC, and show that it can potentially improve standard synoptic current mapping.

Keywords: HR radar, ocean surface current, Bayesian estimation

1. Introduction

In the past four decades, High Frequency (HF) radars have routinely been used to monitor ocean surface currents in coastal regions (see, e.g., the reviews [1, 2]) and, more recently, to estimate other sea state parameters (see the recent review in [3, 4, 5]). HF radars take advantage of two specific physical mechanisms occurring in the HF electromagnetic (EM) wave regime, namely the over-the-horizon propagation of EM surface waves, for a vertically polarized electric field, and the dominance in the backscattered signal of a single resonant component of the ocean wave field, a process known as “Bragg scattering”. The estimation of ocean surface currents along the radar looking direction (referred to as “radial surface currents”) is typically based on identifying the frequency shift caused by the current in the backscattered Doppler spectrum to the well-defined Bragg frequency line. The performance of this estimation, in terms of accuracy, reliability, achievable range, and spatial resolution, depends on several parameters related to the radar system (e.g., carrier frequency, antenna system, integration time, emitted power), the environment (e.g., sea state, Radio Frequency Interferences (“RFI”), or a combination of both (signal-to-noise ratio (“SNR”)).

Even though the physics underlying the estimation of ocean surface currents through Bragg scattering has been well understood for a long time, we show in this paper that the processing of the radar signal can still be improved. To this effect, we propose a new non-spectral estimator that can recover (or invert) radial currents from complex radar time series, based on a Maximum Likelihood Bayesian optimization approach. The main principle of the new estimator is that, in the HF regime, ocean surface currents cause a slow amplitude modulation of the real and imaginary parts of the backscattered radar signal which, depending on the level and nature of the environmental noise affecting the data, may in some cases be easier to identify than a shift in the main spectral peak in the Doppler spectrum. Additionally, the estimation based on the amplitude modulation can be improved by resorting to Bayesian inference,

31 in which an *a posteriori* probability distribution is derived for the estimated
32 parameter (here the radial current), given the observations and some *a priori*
33 information. The estimation then results from maximizing the likelihood of the
34 *a posteriori* probability distribution (i.e., minimizing errors). The introduction
35 of *a priori* information on radial surface currents reduces the undesirable oc-
36 currence of outliers and the dispersion of the estimates. As a result, with the
37 new estimator, shorter time series of radar signal can in principle be used to
38 achieve a specified confidence interval on the inverted currents, than with the
39 Doppler-based estimation. These properties of the new estimator will be made
40 clear in the paper.

41 In physical studies of ocean surface currents inverted from HF radar data
42 based on the standard Doppler method, the time scales considered are typically
43 on the order of 2 to 60 minutes, depending on radar frequency, the antenna sys-
44 tem (compact antenna or large arrays), and signal-to-noise ratio. The selected
45 observation time results from a trade-off between the conflicting requirements
46 of using a large enough integration time to accurately compute the spectra
47 and achieve sufficient resolution on the currents, and a short enough time to
48 capture the temporal variability of mesoscale oceanic features. A number of
49 studies of ocean surface currents at such spatial scales have shown that an in-
50 tegration/observation time on the order of 20 minutes is in general adequate
51 to characterize most classical patterns of background oceanic currents, such as
52 rapidly evolving eddies, tidal and subtidal fluctuations, and the ocean response
53 to a changing wind stress (see references in [3, 4]). However, for some emerging
54 applications, such as the early detection of tsunamis (e.g. [6, 7, 8]) and particu-
55 larly the non-seismic tsunamis (e.g., meteotsunamis, landslide tsunamis), which
56 have shorter wavelengths, a shorter integration time is required to properly cap-
57 ture the signature of such events in ocean surface currents, whose typical period
58 is on the order of a few to a few tens of minutes [9]. Other applications such
59 as the prediction of Lagrangian transport in oil spill tracking (e.g., [10]), the
60 distribution of plankton blooms in ecological systems ([11]), or the identifica-
61 tion of small-scale coherent structures (e.g. [12, 13]), while not restricted to,

62 would clearly benefit from a higher temporal resolution in the inverted surface
63 currents. The reliable estimate of small magnitude surface currents varying at
64 minute temporal scales would also make it possible observing infragravity waves,
65 through the measurement of their orbital motion, which have a typical period
66 of 30 sec to a few minutes ([14]). Such waves, which are generated and grow in
67 parallel with large storm systems, are key contributors to the coastal flooding
68 and erosion such storms cause upon landing.

69 Estimating the Bragg frequency shift in the time domain is not new, and sev-
70 eral attempts have been made in the literature to carry it out using non-spectral
71 parametric methods (see [29] for a comprehensive review). Kahn [30] studied
72 the rapid variations (at the sub-minute time scale) of oceanic HF backscattered
73 signal in the context of target detection and sea clutter suppression. He ob-
74 served amplitude modulations of the HF radar time signal (his Figs. 2 and
75 3), which he attributed to velocity variations of ocean waves. He then pro-
76 posed tracking short term variations of the main frequency around the Bragg
77 line via an autoregressive, low-order linear, prediction model, which he showed
78 predicted well the observed modulations of the radar signal. This made it pos-
79 sible demodulating the radar signal and discriminating the target, by filtering
80 out the rapid variations of the Bragg peak. An interesting application of this
81 for surface current estimation at a rapid time scale is the interpretation of the
82 shifted Bragg frequency in terms of the complex roots of the prediction-error
83 filter on the unit circle. The idea of using an autoregressive approach in the
84 time domain for improving the estimation of surface currents was later applied
85 by [31]. An instantaneous Bragg frequency, calculated on short samples (256
86 points), was obtained through the coefficient of the autoregressive model and
87 the resulting estimated current was shown to be more realistic, in the sense that
88 it did not suffer from rapid non-physical fluctuations in time, such as observed
89 with the Doppler approach. This technique has no physical basis or constraint
90 other than that the difference between the positive and negative Bragg peak
91 frequencies must be twice the Bragg frequency. Another method dealing with
92 instantaneous variations in the Bragg peak frequency was the Instantaneous-

93 Filtering technique proposed by [32], who used it to identify and remove the
94 frequency modulation due to the ocean variability. Our probabilistic approach
95 belongs to the same family of parametric non-spectral techniques, as it accu-
96 rately estimates instantaneous frequencies. Its main difference in purpose with
97 these earlier approaches, however, is that it is not based on a filtering techniques
98 but instead constrains the signal to match a physical (albeit simplistic) analyt-
99 ical model, namely the first-order Bragg theory, while other methods proposed
100 earlier were entirely empirical. Another important feature of the new method
101 is the possibility to incorporate *a priori* information in the current estimation.
102 We did not yet perform a systematic comparison of our method with the earlier
103 alternative methods for estimating radial surface currents; this task will be left
104 out for future work.

105 In this paper, in Section 2, we first review the classical spectral estimation
106 method and summarize its main features and limitations. In Section 3 we in-
107 troduce a key simplified representation of the radar signal, within the frame of
108 first-order Bragg theory, which in Section 4 naturally leads to the formulation
109 of the new estimator of radial surface currents. The general performance of the
110 estimator is then assessed in Sections 5 and 6, first on the basis of synthetic
111 radar data. In Section 7, the new method is finally applied to an actual data
112 set of HF data, recently acquired by a WERA radar system installed in Tofino
113 (BC, Canada) to monitor ocean surface currents off of Vancouver Island. Based
114 on a few test cases, we show that the new method has the potential of both
115 better capturing the variability of surface currents at short time scales and ex-
116 tending the spatial coverage of standard synoptic current mapping. A critical
117 discussion of the method is provided in Section 8, where its potential limitations
118 and improvements are presented.

119 **2. A review of the classical estimation method**

120 Crombie [15] first identified the main principle underlying the estimation of
121 ocean surface currents on the basis of HF radar data and, subsequently, Barrick

122 [16, 17, 18, 19] provided its theoretical foundation. The latter relies on Rayleigh-
 123 Rice’s perturbation theory for a scattered electromagnetic field, which is valid
 124 when the vertical scale of elevation is much smaller than the radar wavelength.
 125 In the context of ocean remote sensing, this theory is commonly referred to as
 126 “Bragg scattering”, due to its formal similarity with the diffraction mechanism
 127 occurring in crystalline solids.

To the first-order in ocean wave steepness, the dominant contribution to the
 backscattered radar field is associated with an ocean wave of specific wavelength,
 of vector wavenumber \mathbf{K}_B , referred to as the “Bragg wave”. This wave is defined
 by the resonant condition $\mathbf{K}_B = -2\mathbf{K}_0$, where \mathbf{K}_0 is the horizontal projection
 of the EM wave vector. At grazing incidence, the Bragg wave has half the
 radar EM wavelength and propagates in either direction along the radar looking
 direction. The resulting backscattered temporal echo (i.e., the radar signal) is
 affected by a Doppler frequency shift proportional to the celerity of the Bragg
 wave. Assuming purely gravity waves in deep water, the resonant condition
 yields the well-known expression of the Bragg (circular) frequency,

$$\omega_B = 2\pi\sqrt{\frac{gf_0}{\pi c_0}} \quad (2.1)$$

128 in which f_0 denotes the radar frequency and c_0 is the celerity of light in a
 129 vacuum.

On this basis, the so-called backscattered Doppler spectrum $\sigma(\omega)$ is defined
 as the average normalized radar cross-section per unit bandwidth. Within the
 framework of first-order Bragg theory applied to a random sea surface, in the
 absence of a current, Barrick [16, 17, 18, 19] derived its expression as,

$$\sigma(\omega) = 2^6\pi K_0^4 \left\{ S_d(+\mathbf{K}_B) \delta(\omega - \omega_B) + S_d(-\mathbf{K}_B) \delta(\omega + \omega_B) \right\}, \quad (2.2)$$

130 where S_d is the directional (non-symmetrical) wave energy density spectrum and
 131 the Dirac δ functions represent two spectral lines at plus or minus the Bragg
 132 frequency.

In the presence of a stationary surface current of velocity \mathbf{U} , the two Bragg
 lines are translated in frequency by a value ω_c , resulting from the additional

Doppler shift caused by the current advection in the radar looking direction. Discarding the sign at this stage this is defined by,

$$\omega_c = \frac{4\pi |U_r|}{\lambda_o}, \quad (2.3)$$

where $U_r = \mathbf{U} \cdot \mathbf{R}$ is the radial surface current, that is the component of the current along the radar looking direction, and λ_o the radar wavelength. As was shown in [20], this result is exact to the first-order in wave steepness when U_r is the vertically-averaged current over a surface layer of thickness $\lambda_o/(2\pi)$. The first-order Doppler spectrum in the presence of a current therefore reads,

$$\sigma(\omega) = 2^6 \pi K_0^4 \left\{ S_d(+\mathbf{K}_B) \delta(\omega - \omega_B + \epsilon_c \omega_c) + S_d(-\mathbf{K}_B) \delta(\omega + \omega_B + \epsilon_c \omega_c) \right\}, \quad (2.4)$$

133 where $\epsilon_c = \pm 1$ is the sign of the additional Doppler shift induced by the current,
 134 depending on whether it is flowing towards ($\epsilon_c = +1$) or away ($\epsilon_c = -1$) from
 135 the radar. Measuring the frequency shift ω_c , between the actual and theoretical
 136 values of the Bragg lines in the radar Doppler spectrum, allows inverting for the
 137 algebraic value of the radial current U_r , using Eq.(2.3).

138 As this is well-known and not the object of this paper, we will not belab-
 139 bor here the main steps of radar signal processing, which make it possible to
 140 transform the backscattered electromagnetic field on the antenna system into
 141 a complex electric signal resolved, in both range (R) and azimuth (Φ), over a
 142 grid of “radar cells” defined on the sea surface. We will refer to, e.g., [21] for a
 143 review of the main techniques underlying the operational use of HF radars and
 144 their features and limitations.

In the following, $s(t)$ refers to the complex time series of processed backscat-
 tered radar signal measured in a given radar cell, located at a specific range R
 and azimuth Φ . Within a calibration factor, the Doppler spectrum of this signal
 is given by,

$$\sigma(\omega) = \lim_{T \rightarrow +\infty} \frac{1}{T} \left\langle \left| \int_{-T/2}^{+T/2} e^{i\omega t} s(t) dt \right|^2 \right\rangle, \quad (2.5)$$

145 where $\langle \cdot \rangle$ denotes an ensemble average, which can be computed as the mean over
 146 successive incoherent samples. The Doppler spectrum is easily calculated via

147 a Fast Fourier Transform, which can be applied to the measured backscattered
 148 signal either before or after it has been processed in azimuth by a beam-forming.
 149 In the classical technique of estimating radial currents based on measuring the
 150 shift of the main Bragg lines, according to Eq. (2.4), the order in which these op-
 151 erations are performed is not important. However, for the new method proposed
 152 here, which is based on time series of range-azimuth resolved radar signal, it is
 153 necessary that the azimuthal processing occur first, that is before any Doppler
 154 processing. Note that this implies that the new method cannot be applied to
 155 data from compact radar systems, where the azimuthal resolution is achieved
 156 in the frequency domain, and its applicability is thus limited to radar systems
 157 using beam-forming techniques.

158 Defining the accuracy of the radial current estimates is a delicate matter,
 159 which requires both physical and radar processing considerations. This accu-
 160 racy is usually related to the width of the Bragg peak, which is controlled by
 161 two main phenomena. The first one is the broadening of the main Bragg line
 162 due to signal modulations by long waves, resulting from induced orbital cur-
 163 rents. The second one is the effect of the finite integration time T used in Eq.
 164 (2.5), which results in a frequency resolution $\Delta\omega \sim 2\pi/T$ and a broadening
 165 by the same amount of the theoretical Dirac δ function for the Bragg spectral
 166 line. For short integration times, the second phenomenon, that is the finite
 167 frequency resolution, is the main limiting factor, yielding a principal resolution
 168 of the estimated radial currents of $\Delta U_r = \lambda_o/2T$. In fact, when estimating the
 169 Bragg peak frequency by the centroid method (i.e., taking the barycenter of the
 170 Doppler spectrum over a few points surrounding the maximal spectral line), one
 171 can achieve a somewhat better accuracy than the FFT frequency resolution. As
 172 was shown in [22] (see Eqs. (21-23) and the corresponding discussion), the rms
 173 error on the peak frequency is the result of a compromise between the frequency
 174 resolution $\Delta\omega$, the number of spectral lines within the half-power peak, and the
 175 number of incoherent samples used to form the Doppler spectrum. Another im-
 176 portant factor impacting the accuracy of current estimates is the noise level [23].
 177 In operational conditions, the observation time required to achieve the afore-

178 mentioned accuracy can be larger in order to produce a sufficient SNR (i.e.,
179 the ratio of the main Bragg line peak power to the mean background power).
180 However, the sample size cannot be taken arbitrary large as one must ensure
181 stationarity of the sea state within the integration time (otherwise the accuracy
182 could be reduced, e.g., by tidal variability). The choice of the observation time,
183 hence, results from a compromise made between several conflicting requirements
184 and corresponding parameters: update rate of the current estimation, effective
185 accuracy, stationarity of sea state, and noise level.

186 Commercial software available in modern radar systems, such as WERA's,
187 use complex and highly optimized algorithms to resolve the backscattered sig-
188 nal as a function of range and azimuth, which eliminate RFIs, select the Bragg
189 lines of the radar data, and apply variance reduction techniques to the Doppler
190 spectrum calculation [24, 25, 26, 27, 28]. Such optimizations are specific to
191 the implemented algorithms and, hence, it would be difficult to make a gen-
192 eral comparison between the performance of the new proposed method and
193 the best available operational surface current measurement systems based on
194 a spectral approach. Therefore, in the context of this study, we will only per-
195 form a theoretical comparison between the two signal processing methods using,
196 for the spectral approach, a basic, non-optimized, algorithm, referred to as the
197 "Doppler Method" (DM). In the DM, only the Bragg peaks in the radar spec-
198 trum that are larger than some threshold SNR are retained for the estimation of
199 the current-induced Doppler shift, ω_c . In addition, an independently estimated
200 maximum value of the radial current that can be reasonably expected to occur
201 at the considered site is used to restrict the search domain for the shifted Bragg
202 lines. These two conditions reduce the number of faulty estimates of the radial
203 current, but usually yield incomplete current maps, having lacunary data at
204 the most distant ranges. In the quantitative applications presented throughout
205 this paper, the maximum current was set to 80 cm/s, unless stated otherwise;
206 and the radial current magnitude was estimated from the mean position of the
207 positive and negative maximal spectral lines (defined from a 3-point barycenter
208 calculation around the maximal peak), whenever the two peaks have a SNR > 3

209 dB, or the most energetic peak otherwise.

210 3. Approximate expression of the complex radar signal

Applying first-order Bragg theory to the backscattered signal amplitude, rather than to its Doppler spectrum, the complex radar signal can be approximated in each radar cell by,

$$s(t) = A^- e^{-i(\omega_B - \epsilon_c \omega_c)t} e^{-i\varphi^-} + A^+ e^{+i(\omega_B + \epsilon_c \omega_c)t} e^{-i\varphi^+}, \quad (3.6)$$

for some positive coefficients A^\pm and random phase shifts φ^\pm , with,

$$s(t) = s_1(t) + i s_2(t), \quad (3.7)$$

211 where s_1 and s_2 denote the real and imaginary parts of the complex radar signal,
 212 originating from the “I” and “Q” channels of the receiver.

213 Note that, in the following analysis, it will not be necessary to know the
 214 detailed expressions of coefficients A^\pm , in terms of sea state, radar system, and
 215 coordinates of the considered radar cells, but instead only their ratio, A^+/A^- .
 216 It should be pointed out that the model (3.6), selected to represent the backscat-
 217 tered time series of radar signal, is highly idealized as it ignores some physical
 218 effects, such as second-order components in Bragg theory, and the variability
 219 of the surface current within each grid cell (which may become important at
 220 far-ranges, where the azimuthal extent of radar cells is larger). However, we
 221 will show in the following that this simplified model is sufficient to achieve a
 222 good estimation of radial surface currents and, in Section 8, we provide some
 223 justifications for why these effects can be ignored.

We first illustrate the principle of the new current estimation method in a simple way, by considering the particular case of, $\epsilon_c = 1$, $A^+ = A^- = A$, and $\varphi^+ = \varphi^- = \varphi$, which for instance occurs when the two Bragg lines are of equal magnitude, yielding,

$$\begin{aligned} s_1(t) &= 2A \cos(\omega_c t - \varphi) \cos(\omega_B t) \\ s_2(t) &= 2A \sin(\omega_c t - \varphi) \cos(\omega_B t), \end{aligned} \quad (3.8)$$

224 which represents periodic oscillations of the radar signal components at the
 225 known Bragg frequency, modulated in amplitude by another oscillation caused
 226 by the unknown Doppler frequency shift ω_c . The principle of the new current
 227 estimation method is that the Doppler shift ω_c can be inferred from this ampli-
 228 tude modulation, from which the radial current $|U_r|$ can then be calculated with
 229 Eq. (2.3) without having to compute the radar Doppler spectrum. Hence, while
 230 the classical estimation of radial currents based on the Doppler spectrum takes
 231 place in the frequency space, requiring a spectral analysis, the new estimator
 232 proposed here detects the amplitude modulation of the radar signal directly in
 233 the time domain. Key advantages of this approach, as we shall see, are a shorter
 234 integration time than required to compute a meaningful Doppler spectrum and
 235 a greater robustness of the results (i.e., the inverted currents) to noise.

Next, Eq. (3.8) is generalized, albeit in an approximate manner, to the case
 of arbitrary amplitudes A^\pm and phases φ^\pm . As detailed in Appendix, whenever
 $\omega_c \ll \omega_B$, the following are good approximations of s_1 and s_2 in the general
 case,

$$\begin{aligned} s_1(t) &\simeq \tilde{s}_1(t) = \alpha \cos(\omega_B(t - t_1)) \cos(\omega_c(t - t_1)) \\ s_2(t) &\simeq \tilde{s}_2(t) = \alpha \cos(\omega_B(t - t_2)) \cos(\omega_c(t - t_2)) \end{aligned} \quad (3.9)$$

where the reference times (t_1, t_2) and the amplitude α are defined as,

$$\begin{aligned} t_1 &= t(s_1^{max}), \quad t_2 = t(s_2^{max}), \text{ and} \\ \alpha &= \max(s_1, s_2), \quad \text{with } s_1^{max} = \max(s_1); s_2^{max} = \max(s_2) \end{aligned} \quad (3.10)$$

236 i.e., the time and magnitude of the maxima of s_1 and s_2 , respectively.

237 Eqs. (3.9) for the I and Q signals can be used to estimate the Doppler shift
 238 ω_c and, in turn, the absolute value of the radial surface current, through the
 239 estimation of the amplitude α of an observed time series of complex backscat-
 240 tered signal $s(t)$, by way of a Least Square Method (LSM); importantly, this
 241 estimation does not require *a priori* knowledge of any other parameter, since
 242 the reference phases $(\omega_c t_1, \omega_c t_2)$ and α all follow from Eq. (3.10).

We define the normalized discrepancy D between the actual signal s from

Eq. (3.6) and its approximation $\tilde{s} = \tilde{s}_1 + i\tilde{s}_2$ from Eq. (3.9) as,

$$D(s) = \frac{\|\mathbf{s} - \tilde{\mathbf{s}}\|^2}{\|\mathbf{s}\|^2}, \quad \text{with} \quad \|\mathbf{s}\|^2 = \frac{1}{T} \int_0^T |s(t)|^2 dt, \quad (3.11)$$

the mean square norm of the radar signal over an integration time T . With this normalization, D only depends on the ratio of amplitudes of the positive and negative Bragg lines, i.e., parameter,

$$\rho = \frac{\min(A^-, A^+)}{\max(A^-, A^+)} \quad (3.12)$$

and the relative phase shift,

$$\varphi = \varphi^+ - \varphi^- \quad (3.13)$$

243 For given integration time T and Doppler frequency shift ω_c , D can be
 244 evaluated in a systematic manner by inspecting the complete (ρ, φ) domain.
 245 One can numerically show that the quality of the approximation is excellent
 246 when the respective amplitudes of the positive and negative frequencies are
 247 close to each other ($\rho \simeq 1$) regardless of their relative phase φ , but deteriorates
 248 as the amplitude ratio becomes smaller. This is consistent with the fact that an
 249 amplitude modulation can only be generated by the interference of two opposite
 250 frequencies.

In the presence of environmental noise, the evaluation of the amplitude parameter α with Eq. (3.10), through the maximum of the signal, becomes less accurate as noise can cause arbitrary high spurious values in the radar signal. In this case it becomes advantageous to rely on a time average over T , provided $T \gg T_c = 2\pi/\omega_c$. Indeed, since ω_B and ω_c have very different orders of magnitude, their time-average (i.e., mean square norm) is simply,

$$\frac{1}{T} \int_0^T \cos^2(\omega_B t) \cos^2(\omega_c t) dt \simeq \frac{1}{4}, \quad (3.14)$$

yielding,

$$\alpha \simeq \sqrt{2} \|\mathbf{s}\|. \quad (3.15)$$

251 Fig. 1 shows an example of fitting the approximate Eq. (3.9) to idealized I
 252 and Q signals (Eq. 3.6), for 256 point samples ($T = 66$ s), in the presence of a

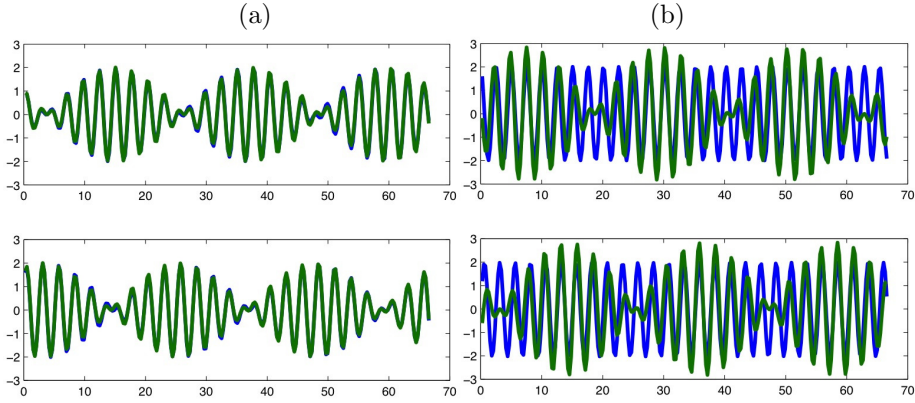


Figure 1: Time series of I and Q radar signals over a $T = 66$ s duration sample, for $U = 0.25$ cm/s, $\varphi = 2\pi/3$, and: (a) $A^\pm = 1$ ($\rho = 1$); (b) $A^+ = 2, A^- = 0$ ($\rho = 0$). The blue curves are the actual signal form first-order Bragg formula (Eq. 3.6) and the green curves its approximation (Eq. (3.9))

253 $U_r = 0.25$ cm/s current, for a relative phase shift $\varphi = 2\pi/3$ and two extreme
 254 values of the amplitude ratio: $\rho = 1$ (optimal case), and $\rho = 0$ (unfavorable
 255 case), in Figs. 1a,b, respectively. The optimal value of ω_c is found by applying
 256 a standard LSM to the joint signals (s_1, s_2) . The figures show, as expected,
 257 a perfect reconstruction in the first case but, even in the second case where
 258 there is no amplitude modulation, the model still captures well some of the
 259 main signal oscillations, which occur at the combined frequency $\omega_B + \omega_c$. This is
 260 made possible by the joint use of the I and Q signals, which imposes additional
 261 constraints on the relative phases and therefore on the value of ω_c .

The sign ϵ_c of the Doppler shift and, hence, the radial current direction (towards or away from the radar) can also be recovered by combining the signal's real and imaginary part estimates. Rewriting the approximation $\tilde{s} = \tilde{s}_1 + i\tilde{s}_2$ in terms of complex exponentials and comparing with the initial expression of s in Eq. (3.6), we obtain a set of consistency relationships. Thus, for a positive Doppler shift ($\epsilon_c = +1$), the complex function \tilde{s} should not contain any factor

$e^{-i\omega_c t}$, which implies,

$$\begin{aligned} C_A &= A^- e^{-i(\omega_B - \omega_c)t_1} + iA^+ e^{-i(\omega_B - \omega_c)t_2} = 0 \\ C_B &= A^- e^{i(\omega_B + \omega_c)t_1} + iA^+ e^{i(\omega_B + \omega_c)t_2} = 0 \end{aligned} \quad (3.16)$$

Likewise, for a negative Doppler shift ($\epsilon_c = -1$), \tilde{s} should not contain any factor $e^{+i\omega_c t}$, that is,

$$\begin{aligned} C_C &= A^- e^{-i(\omega_B + \omega_c)t_1} + iA^+ e^{-i(\omega_B + \omega_c)t_2} = 0 \\ C_D &= A^- e^{i(\omega_B - \omega_c)t_1} + iA^+ e^{i(\omega_B - \omega_c)t_2} = 0 \end{aligned} \quad (3.17)$$

Hence, an estimator of the Doppler shift sign is obtained as,

$$\epsilon_c = \text{sign}(|C_D C_C| - |C_A C_B|) \quad (3.18)$$

262 which can also be calculated based on radar signal data (see below for examples).

263 4. New estimators for the radial surface current

A new estimator of the radial current U_r can now be defined based on Eq. (3.9). Assuming the radar signal $s(t)$ is measured at N times $t_n = n \Delta t$ ($n = 1, \dots, N$), i.e., $s_n = s(t_n)$, and representing it by Eq. (3.6) with the addition of a complex Gaussian noise of standard deviation σ_N , we have,

$$s_n = A^- e^{-i(\omega_B - \omega_c)t_n} e^{-i\varphi^-} + A^+ e^{+i(\omega_B + \omega_c)t_n} e^{-i\varphi^+} + \sigma_N \mathcal{N}(X_n + iY_n), \quad (4.19)$$

where $X_n = \mathcal{N}(0, 1)$ and $Y_n = \mathcal{N}(0, 1)$ are normal random variables with zero mean and unit standard deviation. We denote by $s_{1,n}$ and $s_{2,n}$ (for $n = 1, \dots, N$) the associated discrete time series of the signal real and imaginary parts, which will be separately analyzed. To devise an estimator that is independent of any calibration factor, we first recenter and renormalize these time series in such a way that (in view of Eq. (3.15)),

$$\frac{1}{N} \sum_{n=1}^N s_{1,n} = \frac{1}{N} \sum_{n=1}^N s_{2,n} = 0 \quad (4.20)$$

and,

$$\frac{1}{N} \sum_{n=1}^N s_{1,n}^2 = \frac{1}{N} \sum_{n=1}^N s_{2,n}^2 = \frac{1}{4} \quad (4.21)$$

Next, we use the approximate Eq. (3.9), which we assume to be exact, and rewrite,

$$\begin{aligned} s_{1,n} &= \tilde{s}_1(t_n; \omega_c) + \sigma_{\mathcal{N}} X_n \\ s_{2,n} &= \tilde{s}_2(t_n; \omega_c) + \sigma_{\mathcal{N}} Y_n \end{aligned} \quad (4.22)$$

with,

$$\begin{aligned} \tilde{s}_1(t_n; \omega_c) &= \cos(\omega_B(t_n - t_1)) \cos(\omega_c(t_n - t_1)) \\ \tilde{s}_2(t_n; \omega_c) &= \cos(\omega_B(t_n - t_2)) \cos(\omega_c(t_n - t_2)), \end{aligned} \quad (4.23)$$

and $t_1 = t(s_1^{max})$, $t_2 = t(s_2^{max})$.

According to this model, the Likelihood $\mathcal{L}(U_r|s_n)$ of the absolute radial current, $U_r = \lambda_o \omega_c / 2$, given the observations s_n is given by,

$$\mathcal{L}(U_r|s_n) = \frac{1}{(2\pi\sigma_{\mathcal{N}}^2)^N} \exp\left(-\frac{D(U_r)}{2\sigma_{\mathcal{N}}^2}\right) \quad (4.24)$$

where,

$$D(U_r) = \sum_{n=1}^N \left\{ \tilde{s}_1(t_n; \omega) - s_{1,n} \right\}^2 + \sum_{n=1}^N \left\{ \tilde{s}_2(t_n; \omega) - s_{2,n} \right\}^2 \quad (4.25)$$

is the discrepancy, defined by Eq. (3.11), of the radar signal with respect to its approximation, expressed by Eq. (4.23) in the discrete case, for a trial radial current U_r . This leads to the classical Maximum (log) Likelihood Estimate (MLE) for the absolute radial current,

$$\widehat{U}_r = \arg \max \left(\log \mathcal{L}(U_r|s_n) \right) = \arg \min D(s_n; U_r) \quad (4.26)$$

In case of a weak SNR (or, equivalently, large $\sigma_{\mathcal{N}}$), the MLE can lead to non-physical estimates of the radial current, due to the occurrence of spurious maxima in the Likelihood function. This issue can partly be overcome by introducing *a priori* information on the distribution of radial currents and resorting to a Bayesian analysis. In this case, the *a posteriori* probability P of radial current, given the observations (which we simply denote by s_n , which should be understood as a generic term), is defined by,

$$P(U_r|s_n) = \mathcal{L}(U_r|s_n) P_0(U_r), \quad (4.27)$$

where P_0 is the prior distribution, that is the *a priori* probability distribution of the radial currents U_r in the absence of present observations. The knowledge of the prior distribution may result from reasonable physical assumptions or from prior measurements made over a long integration time. For instance, if a reasonable estimate of the mean radial current $\overline{U_r}$ is known, together with the maximum departure from this value, $2\sigma_{U_r}$, we may assume a normal prior distribution,

$$P_0(U_r) = \frac{1}{\sqrt{2\pi\sigma_{U_r}^2}} \exp\left(-\frac{(U_r - \overline{U_r})^2}{2\sigma_{U_r}^2}\right), \quad (4.28)$$

In this case, the absolute radial current U_r can again be estimated as the most probable value,

$$\hat{U}_r = \arg \max (\log P(U_r|s_n)) = \arg \min \left(\frac{D(s_n; U_r)}{2\sigma_{\mathcal{N}}^2} + \frac{(U_r - \overline{U_r})^2}{2\sigma_{U_r}^2} \right) \quad (4.29)$$

265 Following the standard terminology we will henceforth refer to Eq. (4.29) as
 266 the ‘‘Maximum A Posteriori Probability Estimate’’ (MAPPE). In the absence
 267 of any *a priori* information (other than the maximum physical value U_r^{max}), the
 268 prior distribution is assumed uniform and the MAPPE coincides with the MLE,
 269 given by Eq. (4.26) on the selected search interval.

One difficulty of the Bayesian approach is that it requires knowledge of the noise level standard deviation $\sigma_{\mathcal{N}}$, which is a key parameter of the *a posteriori* distribution. However, a good estimate of the noise level can be obtained by differentiating the signal, which has the effect of damping its continuous deterministic part and enhancing its white noise components. Defining the sequences $\Delta s_{1,n} = s_{1,n+1} - s_{1,n}$ and $\Delta s_{2,n} = s_{2,n+1} - s_{2,n}$, we have from Eqs. (4.22) and (4.23):

$$\langle (\Delta s_{1,n})^2 \rangle = \langle (\Delta s_{2,n})^2 \rangle = \frac{1}{4}(\omega_B^2 + \omega_c^2)\Delta t^2 + 2\sigma_{\mathcal{N}}^2 \quad (4.30)$$

For a small sampling rate ($\sqrt{\omega_B^2 + \omega_c^2}\Delta t < 1$ and a small SNR ($\sigma_{\mathcal{N}} > 1$), this yields the estimate:

$$\sigma_{\mathcal{N}}^2 = \frac{1}{4}\langle (\Delta s_{1,n})^2 + (\Delta s_{2,n})^2 \rangle \quad (4.31)$$

270 Note that the devised MLE and MAPPE estimators from Eqs. (4.26) and
 271 (4.29), only predict the absolute value of the radial current and not its sign.

272 Hence, once an estimate of the former has been obtained, the corresponding
 273 sign is inferred from Eq. (3.18). This will be made clearer in applications.

274 5. Performance assessment of the MLE

275 In the following, we assess the performance of the MLE estimator with re-
 276 spect to the DM. We do this in a simplified manner since, as pointed out before,
 277 it is difficult to make a general comparison between these methods as this de-
 278 pends on many parameters, such as radial current magnitude U_r , integration
 279 time T , noise level σ_N , and amplitude ratio ρ . Thus, we only perform basic tests
 280 of the estimator, to verify that it satisfies some minimal requirements. Here, we
 281 do not consider the performance of MAPPE, as it very much depends on the
 282 choice of the prior distribution.

We first investigated the MLE defined by Eq. (4.26) in the absence of noise
 ($\sigma_N = 0$), using a large number $N = 128$ to 512 of (measured) signal real-
 izations defined by Eq. (4.19), and sweeping a wide range of radial current
 magnitude U_r , with random phases φ^\pm and various amplitude ratios ρ . Us-
 ing the characteristics of the Tofino WERA HF radar, we performed Monte
 Carlo (MC) simulations to systematically investigate the current estimate bias,
 $\Delta U_r = \langle \widehat{U}_r \rangle - U_r$, for time series of duration $T =$ (i) 33 s (128 points); (ii) 66
 s (256 points); and (iii) 133 s (512 points). This bias was found to have little
 dependence on the value of ρ , to be approximatively linear for small currents,

$$\Delta U_r \simeq \alpha(U_{r0} - U_r), \quad U_r \leq U_{r0}, \quad (5.32)$$

283 and nearly zero for large currents, $\Delta U_r \simeq 0$, $U_r \geq U_{r0}$, where U_{r0} is some
 284 threshold, which is function of T . When U_r, U_{r0} , and ΔU_r are all expressed in
 285 cm/s, we found: (i) $T = 33$ s, $U_{r0} = 20$, $\alpha = 0.23$; (ii) $T = 66$ s, $U_{r0} = 13$, $\alpha =$
 286 0.55 ; and (iii) $T = 133$ s, $U_{r0} = 5$, $\alpha = 2.5$.

287 Next, we systematically evaluated the MLE performance for different levels
 288 of noise σ_N , and the mean and standard deviation of the estimated current
 289 were calculated for each actual value of the current. Fig. 2 shows results of such

290 estimations, made using $T = 33$ s samples (128 points), of a $U_r = 30$ cm/s radial
 291 current in a reasearch interval $[0 - 100]$ cm/s, for 3 representative values of the
 292 amplitude ratio $\rho = 0.1, 0.5$ and 1, and a noise level $\sigma_N = 0$ to 3.5, compared to
 293 similar estimations made using the DM. Results show that the DM performance
 294 strongly depends on the amplitude ratio and drastically deteriorates for small
 295 values of the latter. The MLE, however, only weakly depends on the amplitude
 296 ratio and shows a comparable performance to the DM otherwise. Even though
 297 the MLE is less accurate than the DM in the optimal case ($\rho = 1$), it is in
 298 general much more reliable when the level of noise increases. Based on the same
 299 analysis including noise, for other cases not shown here, we also found that the
 300 MLE remains an unbiased estimator ($\Delta U_r = \langle \widehat{U}_r \rangle - U_r \simeq 0$) except when both
 301 the current magnitude and the sample size N are small.

302 The quality of the estimation can be greatly improved when using *a priori*
 303 information, such as the probability distribution function of surface currents
 304 (prior distribution), as is done using MAPPE. This prevents the occurrence of
 305 outliers and considerably reduces the dispersion of estimated values. However,
 306 this can also introduce a bias if the prior distribution is centered about an
 307 erroneous value. Therefore, the use of *a priori* information must be made with
 308 caution and rests on the availability of reliable observations (such as the mean
 309 current estimated in the recent past).

310 **6. Application of the MLE to synthetic HF radar data**

311 *6.1. Rapidly changing current*

In this first application of the new MLE current estimator, we investigate its
 ability to estimate variations of the radial surface current occurring over short
 time scale (minutes), such as those induced by the propagation of very long
 waves, like infragravity waves or tsunamis. To this aim, we consider a current
 defined as a sinusoidal perturbation on top of a constant background current,

$$U_r(t) = 0.2 + 0.03 \cos \left\{ 2\pi \frac{t}{600} \right\} \quad (\text{m/s}) \quad (6.33)$$

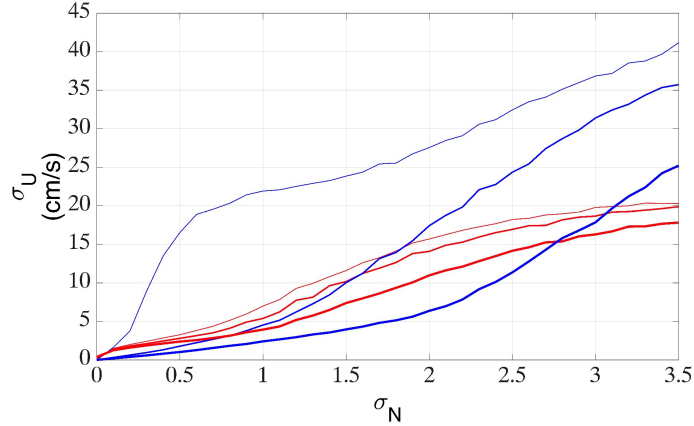


Figure 2: Standard deviation of radial surface current estimates using HF radar synthetic data ($T = 33$ s; $N = 128$), based on the MLE (red lines) or the DM (blue lines), as a function of noise level σ_N . The value of the radial surface current is $U_r = 30$ cm/s and, in the MLE, the current is searched within a 0 to 100 cm/s interval; three values of ρ are considered: $\rho = 0.1$ (thin lines), $\rho = 0.5$ (medium thick lines) and $\rho = 1$ (thick lines).

with a period of 600 s, which is typical of infragravity waves or landslide tsunamis [9]. We simulate the effect of this analytical current onto a synthetic radar time series generated with the noisy first-order Bragg model of Eq. (4.19), with $A^+ = 1$ and $A^- = 0.25$. Specifically, following the standard approach (e.g., [9]), the Doppler shift caused by this time varying current on the gravity wave dispersion relationship is simulated by replacing the phase $\omega_c t$ in Eq. (4.19) by a current “memory term” taking the form of an integral, i.e.,

$$e^{i\epsilon_c \omega_c t} \rightarrow e^{i\epsilon_c \frac{4\pi}{\lambda_o} \int_0^t U_r(\tau) d\tau}, \quad (6.34)$$

312 and calculating its discrete values for t_n ($n = 1, \dots, N$) in Eq. (4.19).

313 Based on this synthetic radar signal time series, Fig. 3 shows results for
 314 the current estimation made with the MLE and the DM in a reasearch interval
 315 $[0 - 80]$ cm/s, every $\Delta t = 33$ s, by taking sliding intervals of $N = 512$ points
 316 ($T = 133$ s). Two levels of noise are tested for, $\sigma_N = 0.1$ (low noise level) or
 317 $\sigma_N = 1.5$ (high noise level), whose results are shown in Figs. 3a,b, respectively.
 318 Results show that the two methods perform equally well for the low noise level

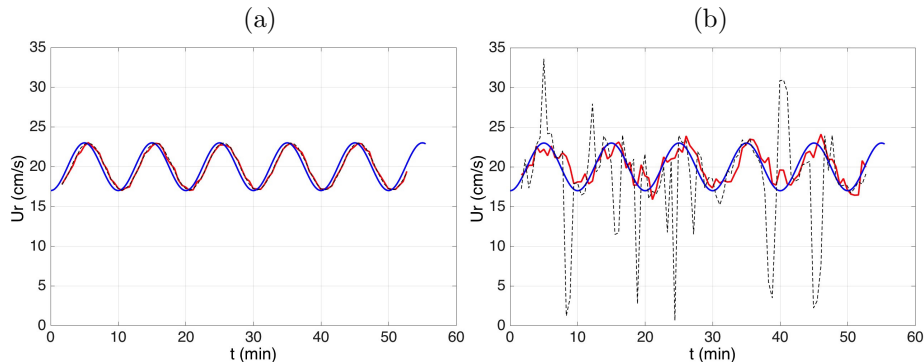


Figure 3: Time series of radial current magnitude estimated with: MLE (red lines), and DM (black dashed lines); from synthetic HF radar data with: (a) low, and (b) high noise levels. Sliding intervals of $N = 512$ points ($T = 133$ s) are used in the estimation, every 33 sec. The actual current from Eq. (6.33) is superimposed as blue lines.

319 (in fact there are little measurable differences between both), but for the high
 320 noise level, the DM exhibits erratic variations while the MLE keeps following
 321 the main trend of the actual oscillating current. Hence, in this first test case,
 322 even for very noisy data, the MLE is able to provide a fairly reliable estimate
 323 of a slowly varying current.

324 6.2. Robustness to wide-band signals

325 Here, we assess the robustness of the MLE approach for cases where the
 326 backscattered radar signal is perturbed by another deterministic broad band
 327 source. Let us assume for illustration that the available frequency band ($[-2, +2]$
 328 Hertz at a 0.26 s sampling rate) is entirely contaminated by a chirp originating
 329 from another radio source, so that the backscattered signal (4.19) includes an
 330 additional “noise” of the form $n(t) = n_0 \exp\{2i\pi(-2t + 2t^2/T)\}$, over an inte-
 331 gration time T . Such a deterministic signal strongly deteriorates the Doppler
 332 spectrum, particularly regarding the identification of shifted Bragg lines (Fig.
 333 4a). By contrast, the log-likelihood function computed with the MLE remains
 334 robust to this perturbation, showing a well-defined peak near the actual value
 335 of the radial surface current (Fig. 4b). In the example shown in Fig. 4, the
 336 signal was generated with $A^\pm = 1$, $\sigma_{\mathcal{N}} = n_0 = 5$, with a radial current value

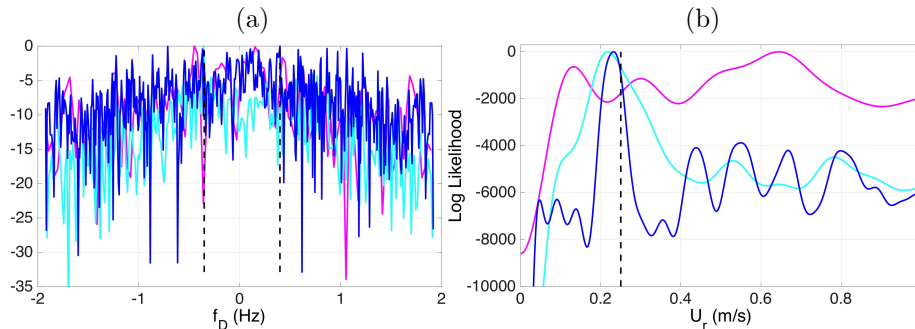


Figure 4: Doppler spectra (a) and Log-likelihood functions (b) obtained when the backscattered signal is contaminated by an external chirped source over the entire frequency band ($[-2, +2]$ Hertz) for different sample sizes $N =$ (magenta) 128 ; (cyan) 256 and (blue) 512. The black dashed lines mark: (a) the position of the expected Bragg lines (including the shift caused by the current); (b) the actual value of the radial current $U_r = 0.25$ m/s.

337 $U_r = 25$ cm/s, and different sample sizes ($N=128$ or $T=33$ sec; $N=256$ or $T=66$
338 sec; $N=512$ or $T=133$ sec). The DM is unable to provide a relevant estima-
339 tion of the radial current, while the MLE still provides an accurate estimation
340 ($U_r = 0.217$ m/s with $N=256$ and 0.232 m/s with $N=512$).

341 7. Application to the Tofino WERA HF radar data

342 To mitigate the elevated tsunami hazard along the shores of Vancouver Is-
343 land, BC (Canada), Ocean Networks Canada (ONC) has been developing a
344 Tsunami Early Warning System combining instruments deployed on the seafloor,
345 as part of their Neptune Observatory, and a shore-based WERA HF radar in-
346 stalled near Tofino (BC) (operational since April 2016). This radar has a carrier
347 frequency $f_0 = 13.5$ MHz and can detect and estimate ocean radial currents up
348 to a 85-110 km range, depending on sea state as propagation losses increase with
349 sea surface roughness. The radar sweep area is outlined in Fig. 5 and is cov-
350 ered by a grid of radar cells, within which the received radar signal is averaged.
351 There are 70 available cells in the range direction, with a radial step $\Delta R = 1.5$
352 km, for 121 sectors in the azimuthal direction, with angular opening $\Delta\phi_r = 1$
353 deg. The detection sector of the sweep area is 120 deg, implying that cells are

354 1.48 km wide at a 85 km range and narrower closer to the radar (cell area:
 355 $\Delta S = R \Delta R \Delta \phi_r$ increases with range). The orientation of the radar array of
 356 12 antennas (275 deg. from N, clockwise; centered at $49^\circ 4' 24.82''$ N, $125^\circ 46'$
 357 $11.55''$ W) is such that one side of the sweep area boundary is nearly parallel to
 358 the coastline southeast of Tofino, and the array length (110 m) allows for an ap-
 359 proximately 12 degree azimuthal resolution in the center of the sweep area, with
 360 a coarser resolution of about 20 degree near the edges of the scanned sector. In
 361 the beam forming algorithm, since the radar signal is processed for overlapping
 362 angular windows, surface currents can be estimated in a larger number of radar
 363 cells, with a 1 deg step size in azimuth. The different azimuths will be referred
 364 to by their number starting from the East.

365 We obtained a large amount of raw radar signal recorded by the Tofino
 366 HF radar system, at a sampling rate $\Delta t = 0.26$ s, and processed it in range
 367 and azimuth using software provided by Helzel Messtechnik GmbH (the radar
 368 vendor), to produce time series of complex backscattered signal $s(t)$ within each
 369 radar cell. No further processing was applied to the raw signal, in contrast
 370 with algorithms implemented in the WERA radar system, which is based on
 371 the spectral approach and applies a RFI elimination scheme in the spectrum
 372 computations. Within the available data set, a few complete days of records
 373 (referred to by their day number in year 2016) were processed, that represented
 374 different oceanic conditions; only a few generic examples are presented here. The
 375 basic recording format for the WERA coherent radar time series is the so-called
 376 “usort” file, which consists of a $T = 33$ sec duration sequence of $N = 128$ points
 377 at the said sampling rate. An example of 2 usort sequences (real and imaginary
 378 parts) is shown in Fig. 6. In the following examples, as in the previous section,
 379 we therefore consider integer multiples of these building blocks, by concatenating
 380 successive usort files (that is $N = 128, 256, 512$ points, etc).

381 Figure 6 shows examples of directly fitting the approximate backscattering
 382 model of Eq. (3.9) to radar signal time series $s(t)$ acquired at different ranges
 383 for short duration samples ($T = 66$ s). While a nearly perfect match is obtained
 384 at short range (gate 7; 10.5 km), the quality of the fit drastically deteriorates

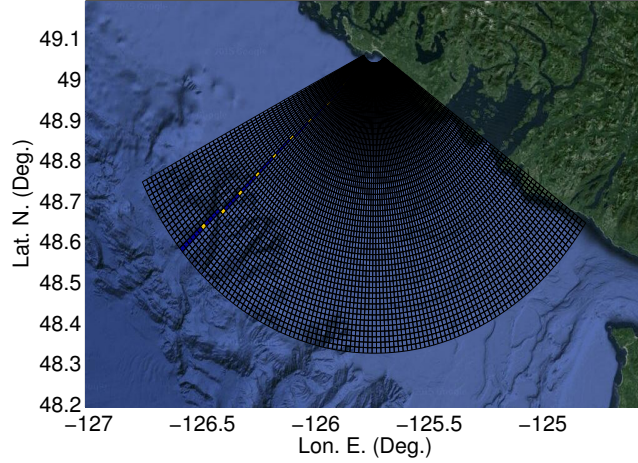


Figure 5: Grid of radar cells covering the Tofino, BC, WERA HF radar sweep area. The marked ray corresponds to azimuth 100 (40 degrees from boresight).

385 as range, and hence noise, increases (e.g., range 27; 40.5 km). As we shall see,
 386 an accurate estimation of the current based on the signal amplitude modula-
 387 tion, which is still visible at large range, remains possible with the MAPPE
 388 method, even though the instantaneous variations of the signal are no longer
 389 well represented.

390 The ability of the MLE and MAPPE methods to perform spatial current
 391 mapping over the entire Tofino HF radar sweep area is assessed next, and results
 392 are compared to those obtained with the DM of the optimized WERA radar
 393 system algorithms. For current spatial mapping using the MAPPE method, *a*
 394 *priori* information can be introduced in at least two ways: (i) using a long-time
 395 series, say 20 min, of radial currents evaluated in the recent past as a starting
 396 point for the updated estimation based on shorter time series; (ii) evaluating
 397 radial currents at increasing ranges and using results of a prior range n as input
 398 for the estimation at range $n + 1$. We tested the latter method for several days
 399 of radar data representing different oceanic conditions, and found results to be
 400 similar for the different days. Hence, in the following, we only present one day

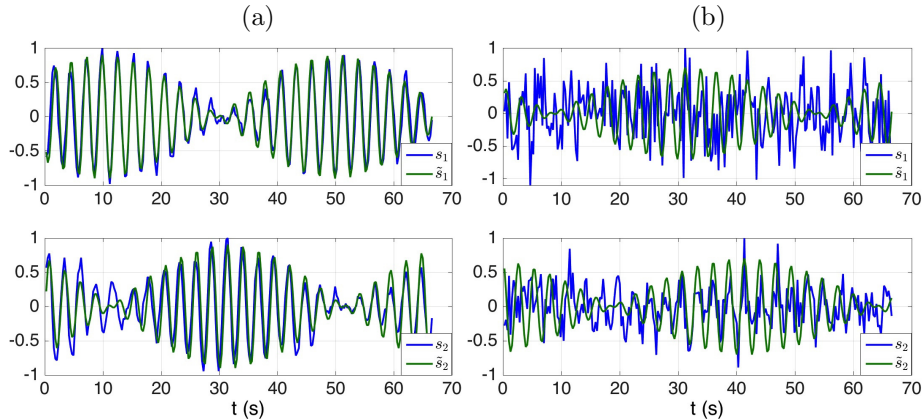


Figure 6: Data and reconstruction (tilde) with approximate Eq. (3.9) of the real and imaginary parts of the backscattered radar signal measured by the Tofino WERA HF radar on August 25th, 2016, for $T = 66$ s time series (2 usort files; $N = 256$ points) acquired at range: (a) 7 (10.5 km); (b) 27 (40.5 km).

401 of data, which can be considered as generic.

402 Figure 7 shows maps of the radial current over the Tofino WERA radar
 403 sweep area, inverted from data acquired on October 19th, 2016 (around 01.15
 404 UTC). According to records made at the nearby NOAA buoy 46087¹, this was
 405 a day with low wind speed (3 m/s), but strong swell (2.36 m wave height and
 406 12.9 s dominant wave period). These maps were calculated for two different
 407 sample sizes: $T = 33$ s, $N = 128$ points (Figs. 7a,b) and $T = 133$ s, $N = 512$
 408 points (Figs. 7c,d). Figs. 7a,c show currents estimated with the MLE method
 409 while Figs. 7b,d show currents estimated with the MAPPE method. Fig. 7e,
 410 shows currents estimated with by the WERA system optimized software, based
 411 on the DM, and using the same 133 s sampling time. For MAPPE, values of
 412 the radial current were estimated by increasing both range and azimuth, us-
 413 ing a prior normal distribution. Thus, the estimation at range n and azimuths
 414 $(m - 1, m, m + 1)$ was used as the central value for the prior distribution when
 415 estimating the current at range $n + 1$ and azimuth m , with an assumed standard

¹http://www.ndbc.noaa.gov/station_history.php?station=46087

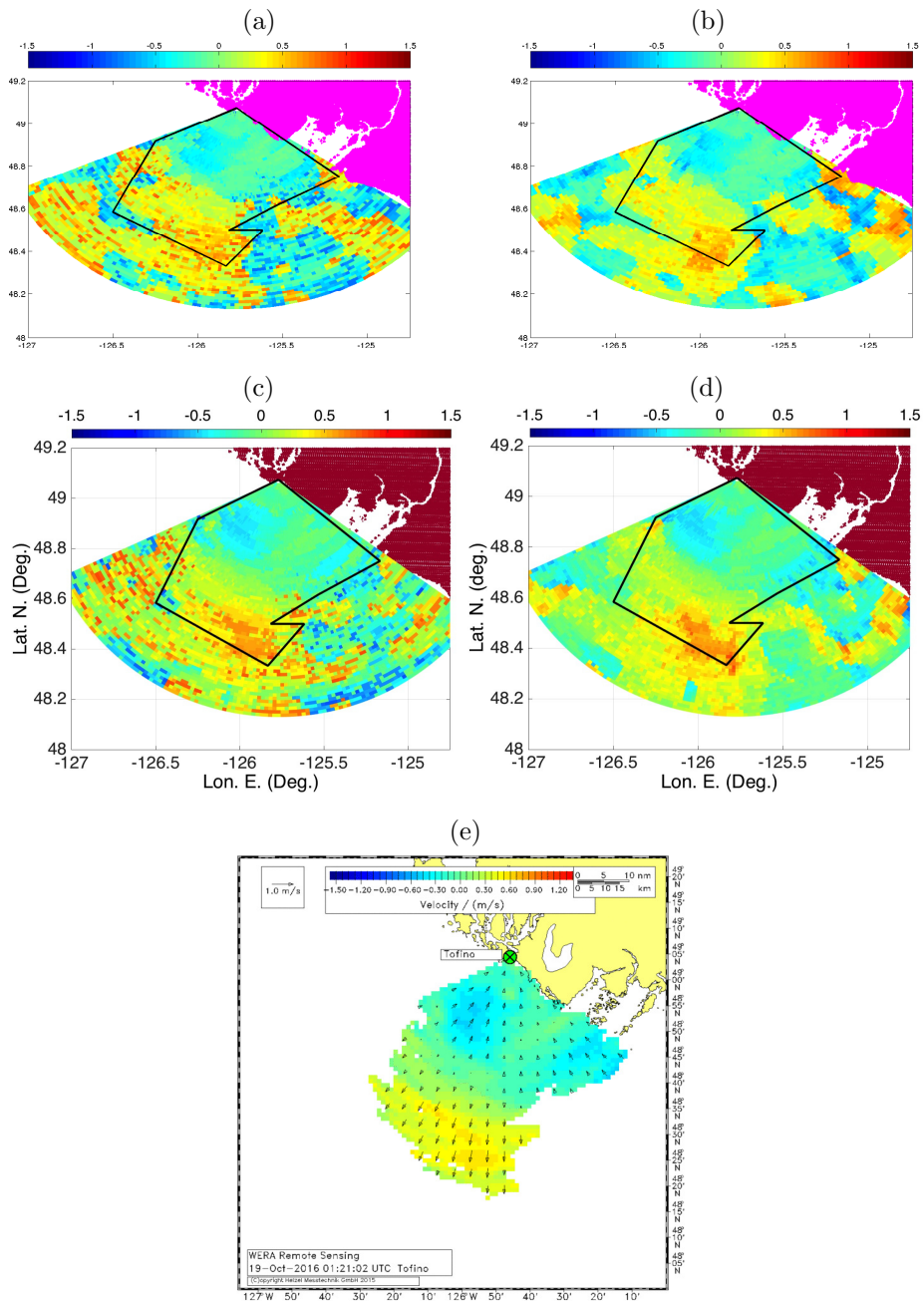


Figure 7: Maps of radial surface current U_r (color scale; m/s) inverted from Tofino WERA HF radar data acquired on Oct. 19th, 2016 (around 01:15 UTC), over the sweep area of Fig. 5, estimated with the: (a) MLE, 1 sample of $T = 33$ s ($N = 128$ points); (b) MAPPE, same sample; (c) MLE, 1 sample of $T = 133$ s ($N = 512$ points); (d) MAPPE, same sample; (e) optimized WERA's system software based on DM, using 1 sample of $T = 133$ s ($N = 512$ points) (courtesy of Helzel Messtechnik GmbH 2015). Black lines in (a,b,c,d) approximately outline the area of available data in (e).

416 deviation $\sigma_{U_r} = 0.1$ m/s (dispersion parameter). The choice of the latter pa-
417 rameter was driven by physical considerations on how much the time-averaged
418 current (here, over 2 min) can be allowed to vary between two neighboring cells.
419 To further reduce errors on estimated values, a moving average was applied,
420 over 3 cells in range and azimuth, to the signed (i.e., not the absolute value) of
421 the estimated current.

422 Fig. 7 shows that, at short ranges, results of the MLE and MAPPE methods
423 are consistent with each other, and also with WERA's system software results.
424 Using a short sample duration $T = 33$ s already yields a good current estima-
425 tion with both the MLE and MAPPE methods. At far ranges, however, the
426 MLE method yields unrealistic noisy patterns of radial currents, likely due to
427 the frequent occurrence of outliers in the estimation. Obvious current sign er-
428 rors can be seen in the rapidly alternating and very contrasted red/blue areas
429 in the farther ranges. By contrast, the MAPPE method yields a much cleaner
430 and more realistic current pattern, with still some obvious sign errors near the
431 furthest corners of the radar sweep area. Such sign errors are possibly re-
432 lated to i) the increased antenna side lobes near these corners and the related
433 increased currents variability; ii) a smaller SNR in these far ranges, which in-
434 creases the dispersion in the estimation; iii) RFIs, which introduce spurious
435 positive/negative Doppler frequencies. Since, unlike in the standard WERA
436 system software, we did not perform any specific treatment to reduce noise and
437 eliminate RFIs, we believe that there is great room for improvement of the prob-
438 abilistic method by implementing similar techniques of RFI reduction as those
439 used with the DM approach. This will be tested in future work.

440 8. A critical discussion

441 Based on the above results, we briefly discuss below some features of the
442 proposed method, including issues and limitations. Any of these might warrant
443 a more in-depth study, but these developments will be left out for forthcoming
444 work and papers.

445 *8.1. Accuracy*

446 An important concept in the framework of estimation is accuracy. In the
447 classical Doppler-based (DM) estimation, the accuracy of estimated currents is
448 characterized by the width of the first-order Bragg peak. With a probabilistic
449 method, however, assessing the accuracy of current estimates is far less obvious
450 and it is important to first properly define it. What would make sense with
451 the proposed method is establishing a confidence interval (CI) of the estimated
452 value, that is a given interval around it, with a certain high quantile of the distri-
453 bution (for example, a 95% CI). However, in the present context, accuracy very
454 much depends on noise level, as seen for instance in Fig. 2, which shows the evo-
455 lution of the rms error (standard deviation) of the estimated radial current with
456 noise level. Note, even with the DM, the theoretical accuracy of the estimation
457 based on the Bragg peak centroid only holds in the noise free case, while the
458 actual resolution deteriorates with increasing noise, as can be seen for instance
459 in Fig. 4. However, one point is clear from our results, namely that, similar to
460 the DM, the accuracy of the probabilistic method increases with sample size (as
461 long as the latter remains smaller than the temporal scale of ocean variability).
462 This is illustrated in Fig. 8a, which shows an example of MLE log-likelihood
463 function computed for a noise-free signal based on the first-order Bragg model
464 Eq. (4.19), with $U_c = 0.25$ m/s. Here, the main peak of the distribution be-
465 comes sharper and more pronounced as the number of samples N is increased
466 from 64 to 1,024. Besides N , other parameters affect the accuracy, namely the
467 noise level σ_N (here taken to be 3 times the signal amplitude in Eq. 4.19),
468 which can widen and displace the main peak of the distribution, particularly
469 for small N values (Fig. 8b), and the Bragg amplitude ratio ρ (the peak of the
470 distribution being far less contrasted in the case of a single Bragg amplitude; see
471 dashed lines in Figs. 8a,b). To summarize, the accuracy of the MLE method
472 can only be rigorously defined as a CI, with respect to some quantile, which
473 depends on integration time T (or sample size N), noise level σ_N , and Bragg
474 amplitude ratio ρ . Such a CI can only be derived through a statistical anal-
475 ysis, by performing Monte Carlo simulations over many samples covering the

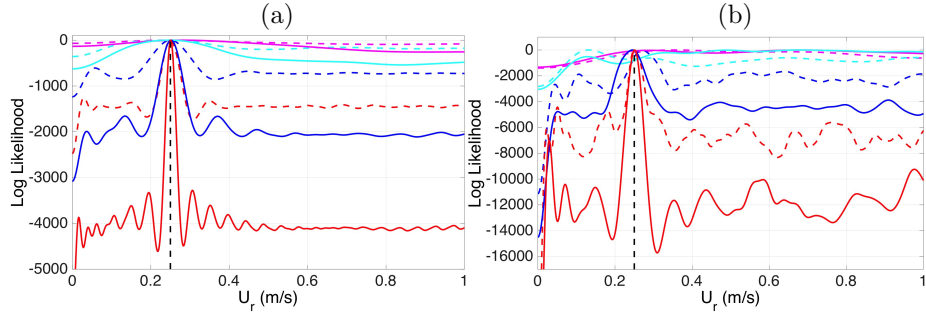


Figure 8: MLE log-likelihood function as a function of trial radial current, for a signal simulated with first-order Bragg model Eq. (4.19): (a) without; and (b) with, noise. The colored lines correspond to different sample sizes $N =$ (magenta) 64; (cyan) 128; (blue) 512; and (red) 1024, with solid lines corresponding to a Bragg ratio $\rho = 1$ and dashed lines to a $\rho = 0$. Black vertical dashed lines mark the location of the actual current value (0.25 m/s). The maximum log-likelihood values have been normalized to zero.

476 range of parameter values; hence, there is no a priori simple estimation of the
 477 method accuracy. Assessing such accuracy can thus only be achieved through a
 478 comparison with other instrument measurements and any kind of sea-truthing.

479 8.2. Current sign

480 A weakness of the MLE method is the more frequent occurrence of sign
 481 errors in the radial surface current estimates, as the noise level increases. This
 482 is illustrated by the spotty aspect of the maps of Fig. 7 in the far-ranges and
 483 can partly be compensated by averaging the sign over several cells, to prevent
 484 from unrealistic sign changes from one cell to another. Hence, when estimating
 485 the current in the far-ranges, it is useful to complement the map of algebraic
 486 radial current by a map of the absolute value of radial currents, which is more
 487 robust to noise. This shortcoming of the method is one point to be addressed in
 488 future work and we have some hope that this can be improved upon by applying
 489 an adequate RFI processing.

490 8.3. Second-order spectrum

491 The MLE method is solely based on first-order Bragg theory, with second-
 492 order effects being ignored. It is well known that such effects may become

493 important for higher frequency radars, and in higher sea states (i.e., steeper
494 waves). In the Doppler spectrum, second-order effects can produce swell peaks,
495 which in some cases can be very close to, and thus contaminate, the main Bragg
496 lines. They can also broaden the first-order peaks if the Doppler resolution is
497 insufficient. Hence, second-order effects can be an additional source of error in
498 the model and thus deserve a specific study, which is left out for further work. In
499 the meantime, our preliminary numerical tests, based on numerical simulations,
500 indicate that the MLE method is robust to the presence of swell peaks.

501 *8.4. CPU time*

502 The estimation of the radial surface currents with the DM is numerically effi-
503 cient, as it is based on FFTs which require an $O(N \log(N))$ CPU time, where N
504 is the time series length. The probabilistic MLE method requires the evaluation
505 of a cost function for every trial current and therefore has a computational cost
506 in $O(N \times N_c)$, where N_c is the number of trial values used for the surface current
507 estimation. The CPU time of the new method, hence, strongly depends on the
508 latter, which is related to the accuracy of the estimation, since increasing N_c
509 allows better resolving the main peak of the log-likelihood function. However,
510 our experience is that N_c can be kept fairly low in practice, so that computa-
511 tional time with the MLE remains small, even though it is found slightly larger
512 than for the DM (although it only takes a few seconds to generate an entire
513 current map on a laptop). Moreover, a good a priori knowledge of the surface
514 current search interval helps dramatically reduce the required number of trial
515 values and thus greatly accelerate the estimation.

516 *8.5. Radar types*

517 As already mentioned, this non-spectral method requires that the range-
518 azimuth resolved radar signal be available in the time domain. This precludes
519 using data from compact radar systems, where the azimuthal discrimination is
520 performed in the frequency domain via a direction finding algorithm.

521 9. Conclusions

522 We presented a new probabilistic method to process HF radar data for es-
523 timating oceanic radial surface currents, which provides an alternative to the
524 Doppler-based method that has been in use for more than four decades. This
525 new method is based on identifying an amplitude modulation of the Bragg fre-
526 quency in the real and imaginary parts of the complex backscattered radar
527 signal. The absolute radial current, which is responsible for this modulation,
528 is found using a Maximum Likelihood approach, for a simplified radar signal
529 model based on first-order Bragg theory. A refined estimation can be obtained
530 with a Bayesian analysis, which uses an *a posterior* probability distribution of
531 the absolute radial current, based on a reasonable *prior distribution*, with pa-
532 rameters estimated from earlier current observations or physical assumptions on
533 its range and variability in the considered region. Our investigations so far have
534 shown that this probabilistic approach can be a useful complementary method
535 for estimating the radial surface currents, particularly in cases of weak SNR
536 or for a rapid temporal variability of the current, requiring shorter observation
537 times to be captured. Some limitations of the method have been identified, such
538 as errors occurring in the estimation of the radial current sign in the most dis-
539 tant ranges and biased estimates for small current values. However, we believe
540 that this new approach has a promising range of potential applications and thus
541 deserves further validation, particularly on the basis of independently measured
542 surface currents. This will be the object of future work.

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In the following, we derive the approximate Eq. (3.9) of the radar signal. Starting from Eq. (3.6) and using some elementary trigonometric relationships, we can recast s_1 and s_2 in the form,

$$\begin{aligned} s_1(t) &= \alpha^+ \cos(\omega_B t) \cos(\omega_c t - \psi^+) + \alpha^- \sin(\omega_B t) \cos(\omega_c t - \psi^-) \\ s_2(t) &= \alpha^+ \cos(\omega_B t) \sin(\omega_c t - \psi^+) + \alpha^- \sin(\omega_B t) \sin(\omega_c t - \psi^-), \end{aligned} \quad (10.35)$$

with,

$$\begin{aligned} \alpha^\pm &= \sqrt{(A^-)^2 + (A^+)^2 \pm 2A^-A^+ \cos(\varphi^+ - \varphi^-)} \\ \tan \psi^+ &= \frac{A^- \sin \varphi^- + A^+ \sin \varphi^+}{A^- \cos \varphi^- + A^+ \cos \varphi^+} \\ \tan \psi^- &= \frac{-A^- \cos \varphi^- + A^+ \cos \varphi^+}{A^- \sin \varphi^- - A^+ \sin \varphi^+} \end{aligned} \quad (10.36)$$

If the time series is long enough it is always possible to find some t_1^\pm and t_2^\pm values such that,

$$\begin{aligned} \omega_B t_1^+ &= \omega_B t_2^+ = 0 \bmod 2\pi \\ \omega_B t_1^- &= \omega_B t_2^- = \pi/2 \bmod 2\pi \\ \omega_c t_1^\pm &= \psi^\pm \bmod 2\pi \\ \omega_c t_2^\pm &= \psi^\pm + \pi/2 \bmod 2\pi \end{aligned} \quad (10.37)$$

and therefore rewrite Eq. (10.35) as,

$$\begin{aligned} s_1(t) &= \alpha^+ \cos(\omega_B(t - t_1^+)) \cos(\omega_c(t - t_1^+)) + \alpha^- \cos(\omega_B(t - t_1^-)) \cos(\omega_c(t - t_1^-)) \\ s_2(t) &= \alpha^+ \cos(\omega_B(t - t_2^+)) \cos(\omega_c(t - t_2^+)) + \alpha^- \cos(\omega_B(t - t_2^-)) \cos(\omega_c(t - t_2^-)), \end{aligned} \quad (10.38)$$

Eq. (3.9) is exactly recovered whenever $\alpha^- = 0$ or $\alpha^+ = 0$; otherwise, it is only an approximation that can be used with,

$$\alpha \simeq \max(\alpha^-, \alpha^+) = \sqrt{(A^-)^2 + (A^+)^2 + 2A^-A^+ |\cos(\varphi^+ - \varphi^-)|} \quad (10.39)$$

and $(t_1, t_2) = (t_1^+, t_2^+)$ if $\alpha = \alpha^+$ or $(t_1, t_2) = (t_1^-, t_2^-)$ if $\alpha = \alpha^-$. For this approximate parametric identification to hold, one needs to verify that

$\max(s_1) \simeq \max(s_2) \simeq \max(\alpha^-, \alpha^+)$ and that (t_1^\pm, t_2^\pm) are the actual times of the maxima of (s_1, s_2) . This has been numerically assessed by verifying that,

$$\Delta\alpha = 1 - \frac{\max(\alpha^-, \alpha^+)}{\max(s_1)}, \quad (10.40)$$

computed as a function of $\varphi = \varphi^+ - \varphi^-$ and $\rho = \min(A^-, A^+)/\max(A^-, A^+)$ is small, and that the product,

$$P = \cos(\omega_B t_1) \cos(\omega_c t - \psi^+) \text{ or } P = \sin(\omega_B t_1) \cos(\omega_c t - \psi^-), \quad (10.41)$$

552 depending on whether, $\alpha = \alpha^+$ or $\alpha = \alpha^-$, is close to 1.

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