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History-Preserving Bisimulations on Reversible Calculus of Communicating Systems

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Abstract. *History- and hereditary history-preserving bisimulation (HPB and HHPB)* are equivalences relations for denotational models of concurrency. Finding their counterpart in process algebras is an open problem, with some partial successes: there exists in calculus of communicating systems (CCS) an equivalence based on causal trees that corresponds to HPB. In Reversible CSS (RCCS), there is a bisimulation that corresponds to HHPB, but it considers only processes without *auto-concurrency*. We propose equivalences on CCS with auto-concurrency that correspond to HPB and HHPB, and their so-called “weak” variants. The equivalences exploit not only reversibility but also the memory mechanism of RCCS.

Keywords: Formal semantics · Process algebras and calculi · Reversible CCS · Hereditary history-preserving bisimulation.

1 Introduction

Reversing Concurrent Computation Implementing reversibility in a programming language often requires to record the history of the execution. Ideally, this history should be *complete*, so that every forward step can be unrolled, and *minimal*, so that only the relevant information is saved. Concurrent programming languages have a third requirement: the history should be *distributed*, to avoid centralization of information. To fulfill those requirements, Reversible CCS [6,7] uses *memories* attached to the threads of a process.

Equivalences for Reversible Processes A theory of reversible concurrent computation relies not only on a syntax, but also on “meaningful” behavioral equivalences. In this paper we study behavioral equivalences defined on configuration structures [14], which are denotational models for concurrency. In configuration structures, an *event* represents an execution step, and a *configuration*—a set of events that occurred—represents a state. A forward transition is then represented as moving from a configuration to one of its superset, and backward transitions have a “built-in” representation: it suffices to move from a configuration to one of its subset. Many behavioral equivalences have been defined for configuration structures ; some of them, like *history- and hereditary history-preserving bisimulations* (HPB and HHPB), use that “built-in” notion of reversibility.

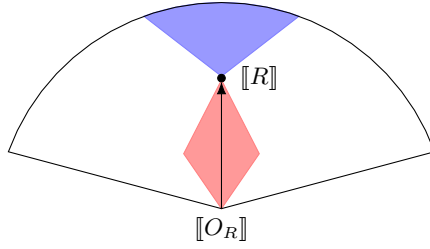


Fig. 1: The encoding of O_R , of the future and of the past of a reversible R .

Encoding Reversible Processes in Configuration Structures An ongoing research effort [1,10] is to transfer equivalences defined in denotational models, which are by construction adapted for reversibility, back into the reversible process algebra. Of course, showing that an equivalence on configuration structures corresponds to one on RCCS processes depends on the encoding of RCCS terms into configuration structures. One of them uses the fact that we are typically interested only in *reachable* reversible processes—processes R which can backtrack to a process O_R with an empty memory—called its *origin*. Then, a natural choice is to consider $\llbracket O_R \rrbracket$ —the encoding of the origin of R , using the common mapping for CCS processes [14]—, and to identify in it the configuration corresponding to the current state of the reversible process. In this set-up, the encoding of R is *one configuration*, $\llbracket R \rrbracket$, in the configuration structure $\llbracket O_R \rrbracket$: every configuration “below” is the “past” of R , every configuration “above”, its “future” (Fig. 1).

Contribution This paper improves on previous results by defining relations on CCS processes that correspond to HPB, HHPB, and their “weak function” variants. The result does not require to consider a restricted class of processes. We introduce an encoding of memories independent of the rest of the process and show that, as expected, the “past” of a process corresponds to the encoding of its memory. The memories attached to a process are no longer only a syntactic layer to implement reversibility, but become essential for defining equivalences. This result gives an insight on the expressiveness of reversibility, as the back-and-forth moves of a process are not enough to capture HHPB.

Related work The correspondence between HHPB and back-and-forth bisimulations for processes without auto-concurrency [1,10] motivated some of the work presented here. Our approach shares similarity with causal trees—in the sense that we encode only *part of the execution* in a denotational representation—where some bisimulations corresponds to HPB [8].

Outline We start by recalling the definitions of configuration structures (Sect. 2.1), of the encoding of CCS in configuration structures (Sect. 2.2), of (hereditary) history-preserving bisimulations (Sect. 2.3), of RCCS (Sect. 2.4) and of related notions. We also recall previous result on HHPB (Theorem 1). We consider the

reader familiar with CCS, in particular with its congruence relations and reduction rules.

Sect. 3 starts by defining a structure slightly richer than configuration structures, that we call “identified configuration structures” (**Sect. 3.1**), and defines basic operations on them. **Sect. 3.2** defines and illustrates with numerous examples how identified configuration structures can encode memories. Finally, **Sect. 3.3** uses this encoding to define relations on RCCS and CSS processes that are then stated to correspond to HPB and HHPB on configuration structures.

Sect. 4 concludes, and **Sect. A** gathers the proofs and establishes the robustness of the tools introduced.

2 Preliminary Definitions

We recall the definitions of configuration structures, auto-concurrency (**Sect. 2.1**), how to encode CCS processes into configuration structures (**Sect. 2.2**) and the history-preserving bisimulations (**Sect. 2.3**).

We write \subseteq the set inclusion, \mathcal{P} the power set, \setminus the set difference, Card the cardinal, \circ the composition of functions, $A \rightarrow B$ the set of functions between A and B , $A \dashrightarrow B$ the partial functions and $f|_C$ the restriction of $f : A \rightarrow B$ to $C \subseteq A$.

Let $\mathbf{N} = \{a, b, c, \dots\}$ be a set of *names* and $\overline{\mathbf{N}} = \{\overline{a}, \overline{b}, \overline{c}, \dots\}$ its *co-names*. The complement of a (co-)name is given by a bijection $[\cdot] : \mathbf{N} \rightarrow \overline{\mathbf{N}}$, whose inverse is also denoted by $[\cdot]$. We write \vec{a} for a list of names a_1, \dots, a_n . We define the sets of labels $\mathbf{L} = \mathbf{N} \cup \overline{\mathbf{N}} \cup \{\tau\}$, let $\mathcal{L} = \mathcal{P}(\mathbf{L})$, and use α (resp. λ, ν) to range over \mathbf{L} (resp. $\mathbf{L} \setminus \{\tau\}$).

2.1 Configuration Structures

Definition 1 (Configuration structures). A configuration structure \mathcal{C} is a tuple (E, C, L, ℓ) where E is a set of events, $L \subseteq \mathcal{L}$ is a set of labels, $\ell : E \rightarrow L$ is a labeling function and $C \subseteq \mathcal{P}(E)$ is a set of subsets satisfying:

$$\forall x \in C, \forall e \in x, \exists z \in C \text{ finite}, e \in z, z \subseteq x \quad (\text{Finiteness})$$

$$\forall x \in C, \forall e, e' \in x, e \neq e' \Rightarrow \exists z \in C, z \subseteq x, e \in z \iff e' \notin z \quad (\text{Coincidence Freeness})$$

$$\forall X \subseteq C, \exists y \in C \text{ finite}, \forall x \in X, x \subseteq y \Rightarrow \bigcup X \in C \quad (\text{Finite Completeness})$$

$$\forall x, y \in C, x \cup y \in C \Rightarrow x \cap y \in C \quad (\text{Stability})$$

We denote $\mathbf{0}$ the configuration structure with $E = \emptyset$, and write $x \xrightarrow{e} y$ and $x \xrightarrow{e} x$ for $x, y \in C$ such that $x = y \cup \{e\}$.

For the rest of this paper, we often omit L , let x, y, z range over configurations, and assume that we are always given $\mathcal{C} = (E, C, \ell)$ and $\mathcal{C}_i = (E_i, C_i, \ell_i)$, for $i = 1, 2$.

Definition 2 (Causality, Concurrency, and Maximality). For $x \in C$ and $d, e \in x$, the causality relation on x is given by $d <_x e$ iff $d \leq_x e$ and $d \neq e$, where $d \leq_x e$ iff for all $y \in C$ with $y \subseteq x$, we have $e \in y \Rightarrow d \in y$. The concurrency relation on x [9, Definition 5.6] is given by $d \text{ co}_x e$ iff $\neg(d <_x e \vee e <_x d)$. Finally, x is maximal if $\forall y \in C, x = y$ or $x \not\subseteq y$.

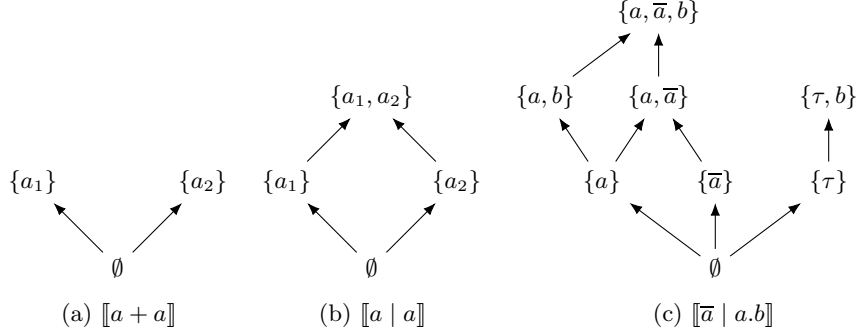


Fig. 2: Examples of configuration structures

Example 1. Consider the configuration structures of Fig. 2, where the set of events and of configurations can be read from the diagram, and where we make the abuse of notation of writing the events as their labels (with a subscript if multiple events have the same label). Note that two events with complement names can happen at the same time (Fig. 2c), in which case they are labeled with τ and called *silent transition*, as it is usual in CCS (Sect. 2.2).

Definition 3 (Category of configuration structures). We define \mathbb{C} the category of configuration structure, where objects are configuration structures, and a morphism f from \mathcal{C}_1 to \mathcal{C}_2 is a triple (f_E, f_L, f_C) such that

- $f_E : E_1 \rightarrow E_2$ preserves labels: $l_2(f_E(e)) = f_L(l_1(e))$, for $f_L : L_1 \rightarrow L_2$;
- $f_C : C_1 \rightarrow C_2$ is defined as $f_C(x) = \{f_E(e) : e \in x\}$.

If there exists an isomorphism $f : \mathcal{C}_1 \rightarrow \mathcal{C}_2$, then we write $\mathcal{C}_1 \cong \mathcal{C}_2$.

We omit the f_L part of the morphisms when it is the identity morphism.

We now recall how process algebra constructors are defined on configuration structures [13]. The definition below may seem technical, but Definition 6 should make it clear that they capture the right notion.

This definition uses the product (\times_*, p_1, p_2) of the category of sets and partial functions [13, Appendix A]: letting \star denote *undefined* for a partial function, $C^\star = C \cup \{\star\}$ for a set C , we define, for two sets A and B ,

$$A \times_* B = \{(a, \star) \mid a \in A\} \cup \{(\star, b) \mid b \in B\} \cup \{(a, b) \mid a \in A, b \in B\}$$

with $p_1 : A \times_* B \rightarrow A^\star$ and $p_2 : A \times_* B \rightarrow B^\star$.

Definition 4 (Operations on configuration structures [1,12]).

The product of \mathcal{C}_1 and \mathcal{C}_2 is $\mathcal{C}_1 \times \mathcal{C}_2 = (E_1 \times_{\star} E_2, C, \ell)$. Define the projections $\pi_i : \mathcal{C} \rightarrow \mathcal{C}_i$ and the configurations $x \in C$ such that:

$$\begin{aligned} \forall e \in E, \pi_i(e) &= p_i(e), \pi_i(\ell_i(e)) = \ell_i(\pi_i(e)) \\ \pi_i(x) &\in C_i \\ \forall e, e' \in x, \pi_1(e) = \pi_1(e') \neq \star \text{ or } \pi_2(e) = \pi_2(e') \neq \star &\Rightarrow e = e' \\ \forall e \in x, \exists z \subseteq x \text{ finite, } \pi_1(x) \in C_1, \pi_2(x) \in C_2, e \in z & \\ \forall e, e' \in x, e \neq e' \Rightarrow \exists z \subseteq x, \pi_i(z) \in C_i, e \in z &\iff e' \notin z \end{aligned}$$

The labeling function $\ell : E_1 \times_{\star} E_2 \rightarrow L_1 \cup L_2 \cup (L_1 \times L_2)$ is

$$\ell(e) = \begin{cases} \ell_1(e_1) & \text{if } \pi_1(e) = e_1 \neq \star \text{ and } \pi_2(e) = \star \\ \ell_2(e_2) & \text{if } \pi_1(e) = \star \text{ and } \pi_2(e) = e_2 \neq \star \\ (\ell_1(e_1), \ell_2(e_2)) & \text{otherwise} \end{cases}$$

The relabeling of \mathcal{C}_1 along $r : E_1 \rightarrow L$ is $r \circ \mathcal{C}_1 = (E_1, C_1, r)$.

The restriction of \mathcal{C}_1 to $E \subseteq E_1$ is $\mathcal{C}_1 \upharpoonright_E = (E, C, \ell \upharpoonright_E)$, where $x \in C \iff x \in C_1$ and $x \subseteq E$. The restriction of \mathcal{C}_1 to a name a is $\mathcal{C}_1 \upharpoonright_a := \mathcal{C}_1 \upharpoonright_{E_1^a}$ where $E_1^a = \{e \in E_1 \mid \ell(e) \notin \{a, \bar{a}\}\}$. For $\vec{a} = a_1, \dots, a_n$ a list of names, we define similarly $\mathcal{C}_1 \upharpoonright_{\vec{a}} = \mathcal{C}_1 \upharpoonright_{E_1^{\vec{a}}}$ for $E_1^{\vec{a}} = \{e \in E_1 \mid \ell(e) \notin \{a_1, \bar{a}_1, \dots, a_n, \bar{a}_n\}\}$.

The parallel composition of \mathcal{C}_1 and \mathcal{C}_2 is $\mathcal{C}_1 \mid \mathcal{C}_2 = (r \circ (\mathcal{C}_1 \times \mathcal{C}_2)) \upharpoonright_F$, with

- $\mathcal{C}_1 \times \mathcal{C}_2 = \mathcal{C}_3 = (E_3, C_3, \ell_3)$ is the product;
- $r \circ \mathcal{C}_3$ with $r : E_3 \rightarrow L_1 \cup L_2 \cup \{\perp\}$ defined as follows:

$$r(e) = \begin{cases} \ell_3(e) & \text{if } \ell_3(e) \in \{a, \bar{a}\} \\ \tau & \text{if } \ell_3(e) \in \{(a, \bar{a}), (\bar{a}, a), \tau\} \\ \perp & \text{otherwise} \end{cases}$$

- $(r \circ \mathcal{C}_3) \upharpoonright_F$, where $F = \{e \in E_3 \mid r(e) \neq \perp\}$.

The coproduct of \mathcal{C}_1 and \mathcal{C}_2 is $\mathcal{C}_1 + \mathcal{C}_2 = \mathcal{C}$, where $E = (\{1\} \times E_1) \cup (\{2\} \times E_2)$ and $C = \{\{1\} \times x \mid x \in C_1\} \cup \{\{2\} \times x \mid x \in C_2\}$. The labeling function ℓ is defined as $\ell(e) = \ell_i(\pi_2(e))$ when $\pi_1(e) = i$.

The prefixing of \mathcal{C}_1 by the name λ is $\lambda.\mathcal{C}_1 = (e \cup E_1, C, \ell)$, for $e \notin E_1$ where $x \in C \iff x = \emptyset \vee \exists x' \in C_1, x = x' \cup e$; $\ell(e) = \lambda$ and $\forall e' \neq e, \ell(e') = \ell_1(e')$.

Definition 5 (Auto-concurrency [9, Definition 9.5]). If $\forall x \in C, \forall e_1, e_2 \in x, e_1 \text{ co}_x e_2$ and $\ell(e_1) = \ell(e_2)$ implies $e_1 = e_2$, then \mathcal{C} is without auto-concurrency.

Any configuration structure where configurations have at most one event (like Fig. 2a) are without auto-concurrency. Fig. 2b, on the other hand, is a configuration structure with auto-concurrency: for $x = \{a_1, a_2\}$ we have that $a_1 \text{ co}_x a_2, \ell(a_1) = \ell(a_2)$, and yet $a_1 \neq a_2$.

2.2 CCS and its Encoding in Configuration Structures

The set of CCS processes P is inductively defined:

$$P, Q := \lambda.P \parallel P \mid Q \parallel \lambda.P + \nu.Q \parallel P \setminus a \parallel 0 \quad (\text{CCS Processes})$$

In the category of configuration structures \mathbb{C} (Definition 3) one can “match” the process constructors of CCS with the categorical operations of Definition 4.

Definition 6 (Encoding a CCS process [15, p. 57]). *Given a CCS process P , its encoding $\llbracket P \rrbracket$ as a configuration structure is built inductively:*

$$\begin{aligned} \llbracket \lambda.P \rrbracket &= \lambda.\llbracket P \rrbracket & \llbracket P \mid Q \rrbracket &= \llbracket P \rrbracket \mid \llbracket Q \rrbracket & \llbracket \lambda.P + \nu.Q \rrbracket &= \llbracket \lambda.P \rrbracket + \llbracket \nu.Q \rrbracket \\ \llbracket P \setminus a \rrbracket &= \llbracket P \rrbracket \upharpoonright_a & \llbracket 0 \rrbracket &= \mathbf{0} \end{aligned}$$

For now on we assume that all structures use the same set of labels L .

Definition 7 (Auto-concurrency in CCS). *A process P is without auto-concurrency if $\llbracket P \rrbracket$ is.*

2.3 (Hereditary) History-Preserving Bisimulations

HPB [11,10], [2, Theorem 4] and HHPB [3, Definition 1.4], [2, Theorem 1] are equivalences on configuration structures that use label- and order-preserving bijections between the events of the two configuration structures.

Definition 8 (Label- and order-preserving functions). *A function $f : x_1 \rightarrow x_2$, for $x_i \in C_i$, $i \in \{1, 2\}$ is label-preserving if $\ell_1(e) = \ell_2(f(e))$ for all $e \in x_1$. It is order-preserving if $e_1 \leq_{x_1} e_2 \Rightarrow f(e_1) \leq_{x_2} f(e_2)$, for all $e_1, e_2 \in x_1$.*

Definition 9 (HPB and HHPB). *A relation $\mathcal{R} \subseteq C_1 \times C_2 \times (E_1 \rightarrow E_2)$ such that $(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and if $(x_1, x_2, f) \in \mathcal{R}$, then f is a label- and order-preserving bijection between x_1 and x_2 and (1) and (2) (resp. (1-4)) hold is called a history- (resp. hereditary history-) preserving bisimulation between C_1 and C_2 .*

$$\forall y_1, x_1 \xrightarrow{e_1} y_1 \Rightarrow \exists y_2, g, x_2 \xrightarrow{e_2} y_2, g \upharpoonright_{x_1} = f, (y_1, y_2, g) \in \mathcal{R} \quad (1)$$

$$\forall y_2, x_2 \xrightarrow{e_2} y_2 \Rightarrow \exists y_1, g, x_1 \xrightarrow{e_1} y_1, g \upharpoonright_{x_1} = f, (y_1, y_2, g) \in \mathcal{R} \quad (2)$$

$$\forall y_1, x_1 \xrightarrow{\sim^1} y_1 \Rightarrow \exists y_2, g, x_2 \xrightarrow{\sim^2} y_2, g = f \upharpoonright_{x_1}, (y_1, y_2, g) \in \mathcal{R} \quad (3)$$

$$\forall y_2, x_2 \xrightarrow{\sim^2} y_2 \Rightarrow \exists y_1, g, x_1 \xrightarrow{\sim^1} y_1, g = f \upharpoonright_{x_1}, (y_1, y_2, g) \in \mathcal{R} \quad (4)$$

Note that the bijection on events is preserved from one step to the next. This condition can be weakened, and we call the corresponding relations the *weak-function* HPB and *weak-function* HHPB [3, Definition 1.4], [10, Definition 3.11].³

³ The names *weak-HPB* and *weak-HHPB* are more common [3,10], but can be confused with the *weak* equivalences of process algebra, which refers to ignoring τ -transitions.

Definition 10 (wfHPB). A weak-function history-preserving bisimulation between \mathcal{C}_1 and \mathcal{C}_2 is a relation $\mathcal{R} \subseteq \mathcal{C}_1 \times \mathcal{C}_2 \times (E_1 \rightarrow E_2)$ such that $(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$ and if $(x_1, x_2, f) \in \mathcal{R}$, then f is a label- and order-preserving bijection between x_1 and x_2 and

$$\begin{aligned} \forall y_1, x_1 \xrightarrow{e_1} y_1 &\Rightarrow \exists y_2, g, x_2 \xrightarrow{e_2} y_2, (y_1, y_2, g) \in \mathcal{R} \\ \forall y_2, x_2 \xrightarrow{e_2} y_2 &\Rightarrow \exists y_1, g, x_1 \xrightarrow{e_1} y_1, (y_1, y_2, g) \in \mathcal{R} \end{aligned}$$

Similarly one defines wfHHPB. If there is a HPB between \mathcal{C}_1 and \mathcal{C}_2 , we just write that \mathcal{C}_1 and \mathcal{C}_2 are HPB, and similarly for HHPB, wfHPB and wfHHPB.

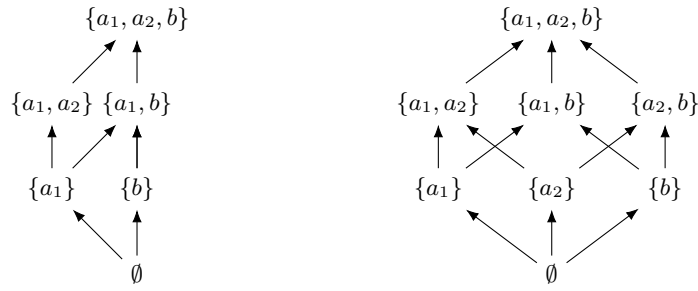


Fig. 3: There is no HHPB relation between $\llbracket a.a \mid b \rrbracket$ and $\llbracket a \mid a \mid b \rrbracket$.

Example 2. Observe that $\llbracket a.a \mid b \rrbracket$ and $\llbracket a \mid a \mid b \rrbracket$, presented in Fig. 3, are HPB, but not HHPB. Any HHPB relation would have to associate the maximal configurations $\{a_1, a_2, b\}$ of the two structures and to construct a bijection: taking $(a_1 \mapsto a_1, a_2 \mapsto a_2, b \mapsto b)$ wouldn't work, since $\llbracket a \mid a \mid b \rrbracket$ can backtrack on a_2 and $\llbracket a.a \mid b \rrbracket$ cannot. Taking the other bijection, $(a_1 \mapsto a_2, a_2 \mapsto a_1, b \mapsto b)$, fails too, since $\llbracket a \mid a \mid b \rrbracket$ can backtrack on a_1 and $\llbracket a.a \mid b \rrbracket$ cannot.

Example 3 ([9]). The processes $a \mid (b+c)+a \mid b+(a+c) \mid b$ and $a \mid (b+c)+(a+c) \mid b$ is another example of processes whose encodings are HPB but not HHPB.

Example 4. Finally, observe that $a.(b+b)$ and $a.b+a.b$ are not structurally congruent in CCS, but the encoding of the two processes are HHPB.

2.4 Reversible CCS and Coherent Memories

Let I be a set of identifiers, and i, j, k range over elements of I . The set of RCCS processes R is built on top of the set of CCS processes P (Sect. 2.1):

$$\begin{aligned} e &:= \langle i, \lambda, P \rangle && \text{(Memory Events)} \\ m &:= \emptyset \parallel \gamma . m \parallel e.m && \text{(Memory Stacks)} \\ T, U &:= m \triangleright P && \text{(Reversible Thread)} \\ R, S &:= T \parallel T \mid U \parallel R \setminus a && \text{(RCCS Processes)} \end{aligned}$$

We denote $I(m)$ (resp. $I(R)$, $I(e)$) the set of identifiers occurring in m (resp. R , e), and always take $I = \mathbb{N}$. A structural congruence \equiv can be defined on RCCS terms [1, Definition 5], the only rule we will use here is *distribution of memory*: $m \triangleright (P \mid Q) \equiv (\gamma.m \triangleright P) \mid (\gamma.m \triangleright Q)$. We also note that $R \equiv (\prod_i m_i \triangleright P_i) \setminus \vec{a}$, for any reversible process R and for some names \vec{a} , memories m_i and CCS processes P_i , writing \prod_i the i -ary parallel composition.

$$\begin{array}{c}
i \notin I(m) \frac{}{(m \triangleright \lambda.P + Q) \xrightarrow{i:\lambda} \langle i, \lambda, Q \rangle.m \triangleright P} \text{act.} \quad \frac{R \xrightarrow{i:\lambda} R' \quad S \xrightarrow{i:\bar{\lambda}} S'}{R \mid S \xrightarrow{i:\tau} R' \mid S'} \text{syn.} \\
\\
i \notin I(m) \frac{}{\langle i, \lambda, Q \rangle.m \triangleright P \xrightarrow{i:\bar{\lambda}} m \triangleright (\lambda.P + Q)} \text{act.*} \quad \frac{R \xrightarrow{i:\bar{\lambda}} R' \quad S \xrightarrow{i:\bar{\lambda}} S'}{R \mid S \xrightarrow{i:\bar{\tau}} R' \mid S'} \text{syn.*} \\
\\
i \notin I(S) \frac{R \xrightarrow{i:\alpha} R'}{R \mid S \xrightarrow{i:\alpha} R' \mid S} \text{par.} \quad \frac{R \xrightarrow{i:\alpha} R' \quad a \notin \alpha}{R \setminus a \xrightarrow{i:\alpha} R' \setminus a} \text{res.} \quad \frac{R_1 \equiv R \xrightarrow{i:\alpha} R' \equiv R'_1}{R_1 \xrightarrow{i:\alpha} R'_1} \equiv
\end{array}$$

Fig. 4: Rules of the labeled transition system

The labeled transition system for RCCS is given by the rules of Fig. 4. We use $\xrightarrow{i:\alpha}$ as a wildcard for $\xrightarrow{i:\alpha}$ (forward) or $\xrightarrow{i:\bar{\alpha}}$ (backward transition), and if there are indices i_1, \dots, i_n and labels $\alpha_1, \dots, \alpha_n$ such that $R_1 \xrightarrow{i_1:\alpha_1} \dots \xrightarrow{i_n:\alpha_n} R_n$, then we write $R_1 \xrightarrow{*} R_n$. If there is a CCS term P such that $\emptyset \triangleright P \xrightarrow{*} R$, we say that R is *reachable*, that P is the *origin* of R [1, Lemma 1] and write $P = O_R$. Similarly to what we did in Definition 7, we will write that R is without auto-concurrency if $\llbracket O_R \rrbracket$ is. An example of the execution of a reversible process is given at the beginning of Example 7.

Note that we *cannot* work up to α -renaming of the identifiers: concurrent, distributed computation is about splitting threads between independent units of computation. If one of unit were to re-tag a memory event $\langle 89, \lambda, P \rangle$ as $\langle 1, \lambda, P \rangle$, and another were to try to backtrack on the memory event $\langle 89, \bar{\lambda}, P \rangle$, then the trace of the synchronization would be lost, and backtracking made impossible. Since we don't want to keep a "global index" which would go against the benefits of distributed computation, the only option is to forbid α -renaming of identifiers.

Memory coherence [6, Definition 1] was defined for RCCS processes with less structured memory events (i.e., without identifiers), but can be adapted.

Definition 11 (Coherence relation). *Coherence, written \curvearrowright , is the smallest symmetric relation on memory stacks such that rules of Fig. 5 hold.*

Note that \curvearrowright is not reflexive, and hence not an equivalence, nor anti-reflexive. For the rest of this paper, we will just write "memory" for "memory stack".

$$\begin{array}{c}
\frac{}{\emptyset \frown \emptyset} \text{em.} \quad I(e) \notin I(m) \cup I(m') \quad \frac{m \frown m'}{e.m \frown m'} \text{ev.} \\
\\
i \notin I(m) \cup I(m') \quad \frac{m \frown m'}{\langle i, \lambda, P \rangle.m \frown \langle i, \bar{\lambda}, Q \rangle.m'} \text{syn.} \quad \frac{m \frown \emptyset}{\Upsilon.m \frown \Upsilon.m} \text{fo.}
\end{array}$$

Fig. 5: Rules for the coherence relation

Definition 12 (Coherent processes, [6, Definition 2]). A RCCS process is coherent if all of its memories are pairwise coherent, or if its only memory is coherent with \emptyset .

We require the memory to be coherent with \emptyset to make it impossible to have $\langle 1, \lambda, P \rangle.\langle 1, \nu, Q \rangle.\emptyset$ as a memory in a coherent process.

Lemma 1 ([7, Lemma 5]). If $R \xrightarrow{*} S$ and R is coherent then so is S .

Corollary 1. For every reachable $m \triangleright P$ and $i \in I(m)$, i occurs once in m .

Note that the property above holds for reversible threads, and not for RCCS processes in general: indeed, we actually *want* memory events to have the same identifiers if they result from a synchronization or a fork.

Definition 13 (Back-and-forth bisimulation). A back-and-forth bisimulation in RCCS is a relation $\mathcal{R} \subseteq R \times R$ such that if $(R_1, R_2) \in \mathcal{R}$

$$\begin{array}{l}
\forall S_1, R_1 \xrightarrow{i:\alpha} S_1 \Rightarrow \exists S_2, R_2 \xrightarrow{j:\alpha} S_2, (S_1, S_2) \in \mathcal{R} \\
\forall S_2, R_2 \xrightarrow{i:\alpha} S_2 \Rightarrow \exists S_1, R_1 \xrightarrow{j:\alpha} S_1, (S_1, S_2) \in \mathcal{R} \\
\forall S_1, R_1 \xrightarrow{i:\alpha} S_1 \Rightarrow \exists S_2, R_2 \xrightarrow{j:\alpha} S_2, (S_1, S_2) \in \mathcal{R} \\
\forall S_2, R_2 \xrightarrow{i:\alpha} S_2 \Rightarrow \exists S_1, R_1 \xrightarrow{j:\alpha} S_1, (S_1, S_2) \in \mathcal{R}
\end{array}$$

Example 5. The processes $\emptyset \triangleright (a.a \mid b)$ and $\emptyset \triangleright (a \mid a \mid b)$ are in a back-and-forth bisimulation.

Theorem 1 ([1, Theorem 2],[10]). Back-and-forth bisimulation on RCCS processes without auto-concurrency corresponds to HHPB on their encoding.

Note that this result does not hold for the processes in [Example 5](#) (see also [Example 2](#)). This does not contradict the theorem, as the processes in [Example 5](#) are with auto-concurrency.

3 Lifting the Restrictions

To define a bisimulation on RCCS (with auto-concurrency) that corresponds to HHPB we first have to encode the memories of a reversible process into a structure similar to the configuration structures, called *identified configuration structures* ([Sect. 3.1](#)). We can then define the encoding ([Sect. 3.2](#)), and the equivalences in RCCS that use this encoding of memories ([Sect. 3.3](#)).

3.1 Identified Configuration Structures

Definition 14 (Identified configuration structure). An identified configuration structure, or \mathcal{D} -structure, $\mathcal{D} = (E, C, \ell, I, m)$ is a configuration structure $\mathcal{C} = (E, C, \ell)$ endowed with a set of identifiers I and a function $m : E \rightarrow I$ such that,

$$\forall x \in C, \forall e_1, e_2 \in x, m(e_1) \neq m(e_2) \quad (\text{Collision Freeness})$$

We call \mathcal{C} the underlying configuration structure of \mathcal{D} and write $\mathcal{D} = \mathcal{C} + m$. We write $\mathbf{0}$ for the identified configuration structure with $E = \emptyset$.

For the rest of this paper, we omit I , and assume that we are always given $\mathcal{D} = (E, C, \ell, m)$ and $\mathcal{D}_i = (E_i, C_i, \ell_i, m_i)$, for $i = 1, 2$.

Example 6. **Fig. 2c**, with $m(a) = 1$, $m(\bar{a}) = 2$, $m(b) = 3$ and $m(\tau) = 4$, is a \mathcal{D} -structure. Note that it is possible to have fewer identifiers than events: take $I = \{1, 2, 3\}$ and $m(a) = m(\tau) = 1$, $m(\bar{a}) = 2$ and $m(b) = 3$.

For the following remark, we need to suppose that every configuration structure is endowed with a total ordering on its events.

Remark 1. Every configuration structure can be mapped to a \mathcal{D} -structure.

The mapping is trivial: take I to be $\{1, \dots, \text{Card } E\}$, and define $m : E \rightarrow I$ to follow the ordering on E . Note that, in this case, m is a bijection.

Definition 15 (Category of \mathcal{D} -structures). We define \mathbb{D} the category of identified configuration structure, where objects are \mathcal{D} -structures, and a morphism f from \mathcal{D}_1 to \mathcal{D}_2 is a triple (f_E, f_C, f_m) such that

- (f_E, f_C) is a morphism in \mathbb{C} from (E_1, C_1, ℓ_1) to (E_2, C_2, ℓ_2) ;
- $f_m : I_1 \rightarrow I_2$ preserves identifiers: $f_m(m_1(e)) = m_2(f_E(e))$.

We denote $\mathcal{F} : \mathbb{D} \rightarrow \mathbb{C}$ the forgetful functor.

Definition 16 (Operations on \mathcal{D} -structures).

The product of \mathcal{D}_1 and \mathcal{D}_2 is $\mathcal{D}_1 \times \mathcal{D}_2 = (E, C, \ell, m)$:

- $(E_1, C_1, \ell_1) \times (E_2, C_2, \ell_2) = (E, C, \ell)$ is the product in the category of configuration structures with projections $\pi_i : (E, C, \ell) \rightarrow (E_i, C_i, \ell_i)$;
- $m : E_1 \times_{\star} E_2 \rightarrow (I_1 \cup \{m_{\star}\}) \times (I_2 \cup \{m_{\star}\})$, for $m_{\star} \notin I_1 \cup I_2$, is defined as

$$m(e) = \begin{cases} (m_1(\pi_1(e)), m_{\star}) & \text{if } \pi_2(e) = \star \\ (m_{\star}, m_2(\pi_2(e))) & \text{if } \pi_1(e) = \star \\ (m_1(\pi_1(e)), m_2(\pi_2(e))) & \text{otherwise} \end{cases}$$

with the projections $p_i : m \rightarrow m_i \cup \{m_{\star}\}$.

Define the projections $\gamma_i : \mathcal{D}_1 \times \mathcal{D}_2 \rightarrow \mathcal{D}_i$ as the pair (π_i, p_i) .

The relabeling of \mathcal{D}_1 along $r : E_1 \rightarrow L$ is $r \circ \mathcal{D}_1 = (E_1, C_1, r, m_1)$.

The restriction of \mathcal{D}_1 to $E \subseteq E_1$ is $\mathcal{D}_1|_E = ((E_1, C_1, \ell_1)|_E + m_1|_E)$.

The parallel composition of \mathcal{D}_1 and \mathcal{D}_2 is $\mathcal{D}_1 \mid \mathcal{D}_2 = (r \circ (\mathcal{D}_1 \times \mathcal{D}_2)) \upharpoonright_F$, with
– $\mathcal{D}_1 \times \mathcal{D}_2 = \mathcal{D}_3 = (E_3, C_3, \ell_3, m_3)$ is the product of \mathcal{D} -structures.
– $r \circ \mathcal{D}_3$, with $r : E_3 \rightarrow L \cup \{\perp\}$ defined as follows,

$$r(e) = \begin{cases} \perp & \text{if } \pi_1(e) = e_1 \neq \star \wedge \pi_2(e) = e_2 \neq \star \wedge m_1(e_1) \neq m_2(e_2) \\ & \text{or if } \pi_1(e) = e_1 \neq \star \wedge \pi_2(e) = \star \\ & \quad \wedge (\exists e_2 \in E_2, \text{ s.t. } m_1(e_1) = m_2(e_2)) \\ & \text{or if } \pi_2(e) = e_2 \neq \star \wedge \pi_1(e) = \star \\ & \quad \wedge (\exists e_1 \in E_1, \text{ s.t. } m_1(e_1) = m_2(e_2)) \\ \tau & \text{if } \pi_1(e) = e_1 \neq \star \wedge \pi_2(e) = e_2 \neq \star \wedge m_1(e_1) = m_2(e_2) \\ & \text{and } \ell_3(e) = (\alpha, \bar{\alpha}) \quad (\text{Valid Synchronisations}) \\ \alpha & \text{if } \pi_1(e) = e_1 \neq \star \wedge \pi_2(e) = e_2 \neq \star \wedge m_1(e_1) = m_2(e_2) \\ & \text{and } \ell_3(e) = (\alpha, \alpha) \quad (\text{Valid Forks}) \\ \ell_3(e) & \text{otherwise} \end{cases}$$

– $(r \circ \mathcal{D}_3) \upharpoonright_F$, where $F = \{e \in E_3 \mid r(e) \neq \perp\}$.

In the definition of parallel composition, r detects the wrong synchronization (or fork) pairs: if two events occur at the same time, then they must have the same identifier. And if an event occurs alone, then no other event can occur with the same identifier.

3.2 Encoding the Memory of Reversible Processes

Definition 17 (Encoding a RCCS memory). *The encoding of the memory of a RCCS process in a \mathcal{D} -structure is defined⁴ by induction on the process:*

$$\begin{aligned} [m \triangleright P] &= [m] & [R_1 \mid R_2] &= [R_1] \mid [R_2] & [R \setminus a] &= [R] \\ [\langle i, \alpha, P \rangle . m] &= (E, C, m) & [\gamma . m] &= [m] & [\emptyset] &= \mathbf{0} \end{aligned}$$

For $[\langle i, \alpha, P \rangle . m]$, letting $[m] = (E_m, C_m, \ell_m, m_m)$ and $e \notin E_m$, we let (E, C, ℓ, m) be $E = E_m \cup e$, $C = C_m \cup \{x \cup \{e\} \mid x \in C_m \text{ is maximal}\}$, $\ell = \ell_m \cup \{e \mapsto \alpha\}$ and $m(e') = m_m(e')$ if $e' \neq e$, and $m(e) = i$ otherwise.

The memories of any RCCS process could be encoded into \mathcal{D} -structures, but we will encode only reachable (and thus coherent) processes (Definition 12).

Example 7. Consider the following transitions:

$$\begin{aligned} \emptyset \triangleright (a.b \mid c.\bar{a}) &\equiv (\gamma.\emptyset \triangleright a.b) \mid (\gamma.\emptyset \triangleright c.\bar{a}) \\ &\xrightarrow{1:c} (\gamma.\emptyset \triangleright a.b) \mid (\langle 1, c, 0 \rangle . \gamma.\emptyset \triangleright \bar{a}) \\ &\xrightarrow{2:\tau} (\langle 2, a, 0 \rangle . \gamma.\emptyset \triangleright b) \mid (\langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \gamma.\emptyset \triangleright 0) \\ &\xrightarrow{3:b} (\langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \gamma.\emptyset \triangleright 0) \mid (\langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \gamma.\emptyset \triangleright 0) \end{aligned}$$

⁴ We make the abuse of notation of writing $[\cdot]$ for the encoding of both a reversible process and a memory, into a \mathcal{D} -structure.

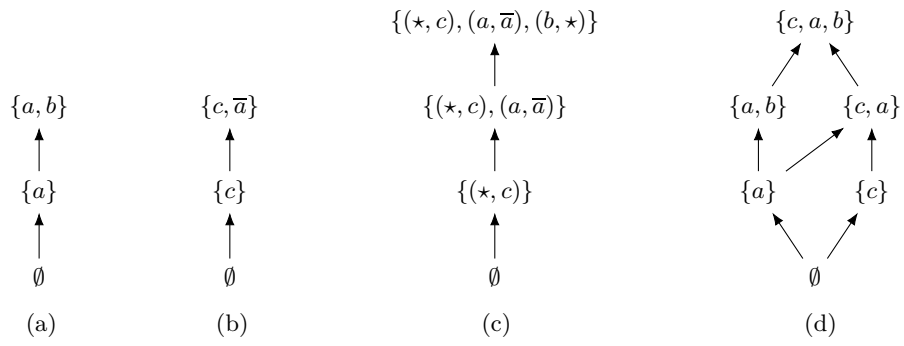


Fig. 6: (Identified) Configurations Structures of Examples 7 and 8

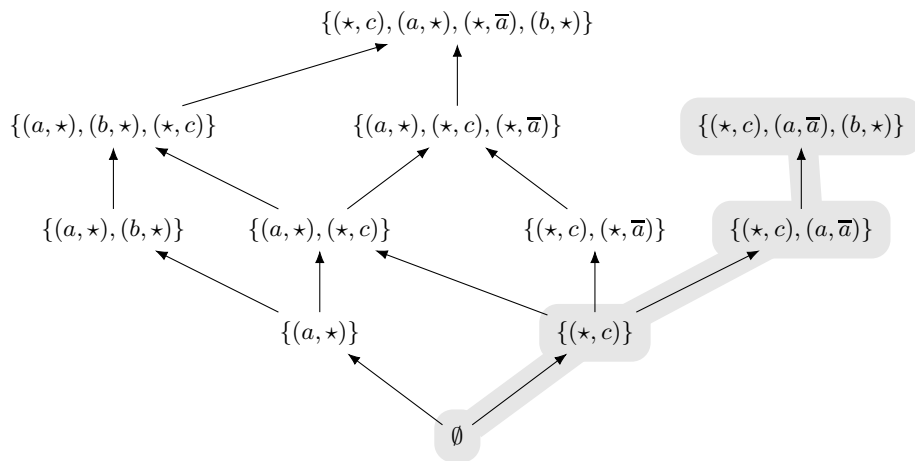


Fig. 7: Configurations Structure of Example 7 (Continued)

Now, observe that:

$$\begin{aligned} & \llbracket (\langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \Upsilon . \emptyset \triangleright 0) \mid (\langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \Upsilon . \emptyset \triangleright 0) \rrbracket \\ &= \llbracket \langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \Upsilon . \emptyset \triangleright 0 \rrbracket \mid \llbracket \langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \Upsilon . \emptyset \triangleright 0 \rrbracket \\ &= \llbracket \langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \Upsilon . \emptyset \rrbracket \mid \llbracket \langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \Upsilon . \emptyset \rrbracket \end{aligned}$$

If we let (with the obvious labeling functions $\ell(x) = x$ for $x \in \{a, \bar{a}, b, c\}$):

$$\begin{aligned} \llbracket \langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \Upsilon . \emptyset \rrbracket &= (\{a, b\}, \{\emptyset, \{a\}, \{b, a\}\}, \{a \mapsto 2, b \mapsto 3\}) \\ \llbracket \langle 2, \bar{a}, 0 \rangle . \langle 1, c, 0 \rangle . \Upsilon . \emptyset \rrbracket &= (\{c, \bar{a}\}, \{\emptyset, \{c\}, \{c, \bar{a}\}\}, \{c \mapsto 1, \bar{a} \mapsto 2\}) \end{aligned}$$

Then we respectively get the structures of Figures 6a and 6b.

The product of those two configurations results in the following set of events (where we keep naming the events after their label):

$$\begin{array}{cccccc} (a, \star) & (\text{ev. } 1) & (\star, c) & (\text{ev. } 3) & (a, c) & (\text{ev. } 5) & (b, c) & (\text{ev. } 7) \\ (b, \star) & (\text{ev. } 2) & (\star, \bar{a}) & (\text{ev. } 4) & (a, \bar{a}) & (\text{ev. } 6) & (b, \bar{a}) & (\text{ev. } 8) \end{array}$$

When doing the parallel composition, the relabeling labels with \perp events [ev. 5](#), [ev. 7](#) and [ev. 8](#), since their event identifier do not match, as well as [ev. 1](#) and [ev. 4](#), since they have the same identifiers but occur unsynchronized. Hence, we obtain the \mathcal{D} -structure of [Fig. 6c](#), with $m(\star, c) = 1$, $m(a, \bar{a}) = 2$ and $m(b, \star) = 3$.

Observe the encoding of the origin of this process, $a.b \mid c.\bar{a}$, in [Fig. 7](#): the underlying configuration of the encoding of the memory previously exposed is just a particular path in that configuration structure. This remark is made formal in [Lemma 2](#).

Example 8. Similarly, we can encode the execution

$$\emptyset \triangleright (a.b \mid c) \xrightarrow{1:c} \xrightarrow{2:a} \xrightarrow{3:b} (\langle 3, b, 0 \rangle . \langle 2, a, 0 \rangle . \Upsilon . \emptyset \triangleright 0) \mid (\langle 1, c, 0 \rangle . \Upsilon . \emptyset \triangleright 0)$$

and obtain the structure of [Fig. 7](#), with $m(c) = 1$, $m(a) = 2$ and $m(b) = 3$.

For the rest of this subsection, we assume given a coherent reversible process R whose origin is written O_R . As expected the underlying configuration of $\llbracket R \rrbracket$ is included in $\llbracket O_R \rrbracket$ (this will be a direct consequence of [Lemma 4](#)):

Lemma 2. $\mathcal{F}(\llbracket R \rrbracket) \subseteq \llbracket O_R \rrbracket$.⁵

The following lemma states that all memory events in R are either causally linked or concurrent. Therefore their encoding results in partially ordered set (poset) with one maximal element ([Definition 2](#)), linked by the subset relation. This is illustrated by [Examples 7](#) and [8](#), and proved by induction on the RCCS process.

Lemma 3. $\llbracket R \rrbracket$ is a poset with one maximal configuration.

⁵ Formally, $\mathcal{C}_1 = (E_1, C_1, \ell_1) \subseteq \mathcal{C}_2 = (E_2, C_2, \ell_2)$ iff $E_1 \subseteq E_2$, $C_1 \subseteq C_2$ and $\ell_1 \subseteq \ell_2$.

Another way to encode a reachable memory in a configuration structure [1] is to encode R as a configuration, called the *address* of R , inside $\llbracket O_R \rrbracket$ ⁶:

$$\llbracket R \rrbracket = (\llbracket O_R \rrbracket, x) \text{ where } x \text{ is a configuration in } \llbracket O_R \rrbracket.$$

Definition 18 (Generation). *Given (E, C, ℓ) and $x \in C$, the configuration structure generated by x is $x \downarrow = (x, C_x, \ell \upharpoonright_x)$, where $C_x = \{y \mid y \in C, y \subseteq x\}$.*⁷

The encoding of the memory of R , $\llbracket R \rrbracket$, is the configuration structure generated by the configuration x . In other words, the memory of R is “everything under” the address of R in $\llbracket O_R \rrbracket$. We state this formally in the following lemma:

Lemma 4. *For $\llbracket R \rrbracket = (\llbracket O_R \rrbracket, x)$, $\mathcal{F}(\llbracket R \rrbracket) \cong x \downarrow$.*

We can now make the intuitions of Fig. 1 formal, letting $R \equiv (\prod_i m_i \triangleright P_i) \setminus \vec{a}$ and O_R be its origin. Informally speaking, the memories of R , i.e. the m_i , is the “past” of R , and the process $(\prod_i P_i) \setminus \vec{a}$ is its “future”: in the configuration structure $\llbracket O_R \rrbracket$, they are respectively represented by $\mathcal{F}(\llbracket R \rrbracket)$ and $\llbracket (\prod_i P_i) \setminus \vec{a} \rrbracket$. Consider Example 7: the past of the process is the structure shown in Fig. 6c, which corresponds to the gray part in Fig. 6d. The future of this process is empty, as all actions have been consumed.

However, we cannot recover the origin process from the encoding of the past and future of a reversible process. There is a loss of information that occurs for the synchronisation events when encoding memories into identified configuration structures: we label τ such events, whereas in the memories of a RCCS process the synchronised events keep their input (or output) label.

3.3 (Hereditary) History-Preserving Bisimulations on CCS

We adapt the (hereditary) history-preserving bisimulations of Sect. 2.3 to CCS, making the bijections becomes isomorphisms between memory encodings.

Below, we let P_1 and P_2 be two CCS processes (possibly with auto-concurrency, Definition 7), and $\llbracket P_i \rrbracket = (E_i, C_i, \ell_i)$, for $i = 1, 2$.

Definition 19 (HPB and HHPB on CCS). *A relation $\mathcal{R} \subseteq \mathbb{R} \times \mathbb{R} \times (E_1 \rightarrow E_2)$ such that $(\emptyset \triangleright P_1, \emptyset \triangleright P_2, \emptyset) \in \mathcal{R}$ and if $(R_1, R_2, f) \in \mathcal{R}$ then f is an isomorphism between $\mathcal{F}(\llbracket R_1 \rrbracket)$ and $\mathcal{F}(\llbracket R_2 \rrbracket)$ and (6) and (7) (resp. (6–9)) hold is called a*

⁶ By abuse of notation, $\llbracket \cdot \rrbracket$ denotes two different encodings, on CCS and RCCS processes.

⁷ For the reader familiar with event structures, a configuration x defines an event structure (x, \leq_x, ℓ) for \leq_x the causality in x . The construction here mirrors the transformation from an event structures to a configuration structure [15].

history- (resp. hereditary history-) preserving bisimulation between P_1 and P_2 .

$$\forall S_1, R_1 \xrightarrow{i:\alpha} S_1 \Rightarrow \exists S_2, g, R_2 \xrightarrow{j:\alpha} S_2, g \upharpoonright_{\mathcal{F}(\lfloor S_1 \rfloor)} = f, (S_1, S_2, g) \in \mathcal{R} \quad (6)$$

$$\forall S_2, R_2 \xrightarrow{i:\alpha} S_2 \Rightarrow \exists S_1, g, R_1 \xrightarrow{j:\alpha} S_1, g \upharpoonright_{\mathcal{F}(\lfloor S_1 \rfloor)} = f, (S_1, S_2, g) \in \mathcal{R} \quad (7)$$

$$\forall S_1, R_1 \overset{i:\alpha}{\rightsquigarrow} S_1 \Rightarrow \exists S_2, f, R_2 \overset{j:\alpha}{\rightsquigarrow} S_2, g = f \upharpoonright_{\mathcal{F}(\lfloor S_1 \rfloor)}, (S_1, S_2, g) \in \mathcal{R} \quad (8)$$

$$\forall S_2, R_2 \overset{i:\alpha}{\rightsquigarrow} S_2 \Rightarrow \exists S_1, g, R_1 \overset{j:\alpha}{\rightsquigarrow} S_1, g = f \upharpoonright_{\mathcal{F}(\lfloor S_1 \rfloor)}, (S_1, S_2, g) \in \mathcal{R} \quad (9)$$

Definition 20 (wfHPB on CCS). A weak-function history-preserving bisimulation between P_1 and P_2 is a relation $\mathcal{R} \subseteq \mathbb{R} \times \mathbb{R} \times (E_1 \rightarrow E_2)$ such that $(\emptyset \triangleright P_1, \emptyset \triangleright P_2, \emptyset) \in \mathcal{R}$ and if $(R_1, R_2, f) \in \mathcal{R}$, then f is an isomorphism between $\mathcal{F}(\lfloor R_1 \rfloor)$ and $\mathcal{F}(\lfloor R_2 \rfloor)$ and

$$\forall S_1, R_1 \xrightarrow{i:\alpha} S_1 \Rightarrow \exists S_2, g, R_2 \xrightarrow{j:\alpha} S_2, (S_1, S_2, g) \in \mathcal{R}$$

$$\forall S_2, R_2 \xrightarrow{i:\alpha} S_2 \Rightarrow \exists S_1, g, R_1 \xrightarrow{j:\alpha} S_1, (S_1, S_2, g) \in \mathcal{R}$$

Of course, wfHHPB on CCS is defined similarly.

Note that the definitions above reflect definitions 9 and 10: the condition $(\emptyset \triangleright P_1, \emptyset \triangleright P_2, \emptyset) \in \mathcal{R}$ is intuitively the counterpart to the condition that $(\emptyset, \emptyset, \emptyset)$ have to be included in the relation on configuration structures. Also, f shares similarity with the label- and order-preserving bijection. Finally, note that, to keep the definitions concise, we wrote \mathbb{R} , but one needs only to consider the reachable processes from $\emptyset \triangleright P_1$ and $\emptyset \triangleright P_2$.

Theorem 2 (Main result). P_1 and P_2 are HHPB (resp. wfHHPB, HPB, wfHPB) iff $\llbracket P_1 \rrbracket$ and $\llbracket P_2 \rrbracket$ are.

4 Conclusion

In this paper, we recalled how the previous attempt to characterize syntactically HHPB gave partial result (Theorem 1). Then, we defined a series of bisimulations on CCS processes that corresponds to HPB, HHPB, and their “weak function” variants, on configuration structures. We managed therefore to define an equivalence on CCS which distinguishes for instance $a.a \mid b$ from $a \mid a \mid b$, whose encodings are not HHPB. We would like to conclude with three observations.

Fist, we should stress that our relations are defined in terms of CCS processes: on the surface, this paper offers a new result on *non-reversible* CCS, using tools stemming from the study of reversible computation. We believe this is an interesting contribution, that witnesses the relevance of studying concurrent reversible computation.

Our paper also introduces a natural technical tool, identified configuration structure, to encode information on memory events. It should be noted that a memory event is made of *three* elements, $\langle i, \alpha, Q \rangle$, and that configuration structures encode names α , and \mathcal{D} -structures furthermore encode identifiers i . We

could take a step further and also enclose information about the residual process Q in a sum. Representing the “whole memory” in denotational model could lead to interesting new bisimulation on processes, but we leave this for a future possible extension.

Other CCS bisimulations such as the *pomset* bisimulations [5] or the *localities* bisimulations [4] are known to be different from the history-preserving ones. However, these bisimulations add some information on auto-concurrent events, that can be used to distinguish them. A possible direction of future work is then to adapt these bisimulations to the reversible setting to maybe capture HHPB.

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A Appendix

The appendix is divided in three subsections.

Sect. A.1 gathers results about identified configuration structures, introduced in **Sect. 3.1**. They are not required to understand the rest of the technical development, but illustrates why we believe this notion is “solid”, and gives some insights on how to manipulate it.

The main purpose of **Sect. A.2** is to prove **Lemma 4**, stated in **Sect. 3.2**. This apparently intuitive result actually requires a heavy machinery to be proven: not only do we prove **Lemma 3**, but we also state and prove some intermediate lemmas.

The immediate advantage of **Lemma 4** is that it makes the proof of **Theorem 2**, in **Sect. A.3**, almost straightforward.

Before doing so, we need to prove the only new result of **Sect. 2**:

Proof (Corollary 1, page 9). Since $m \triangleright P$ is reachable, $m \triangleright P$ is coherent by **Lemma 1**, and hence $m \frown \emptyset$ by **Definition 12**. Looking at **Definition 11**, the only way to derive $m \frown \emptyset$ is to start with em. , and then to apply ev. to “stack” the memory events. If an identifier were to appear in two memory event in m , we would not be able to use ev. to add the second memory event, since the side-condition would forbid it. \square

A.1 On the Robustness of Identified Configuration Structures

This subsection gathers elementary results on the structures introduced in **Sect. 3.1**. These results, e.g., that the category of \mathcal{D} -structures of **Definition 15** is indeed a category (**Lemma 5**) or that the operations on \mathcal{D} -structures of **Definition 16** are well-defined (**Lemma 7**), were not stated in the body of the document, but are implicitly used.

Lemma 5. *Identified configuration structure and their morphisms (Definition 15, page 10) form a category .*

Proof. Identity For every \mathcal{D} -structure $\mathcal{D} = (E, C, \ell, \text{I}, m)$, $\text{id}_{\mathcal{D}} : \mathcal{D} \rightarrow \mathcal{D}$ is defined to be the identity on the underlying configuration structure $\text{id} : (E, C, \ell) \rightarrow (E, C, \ell)$ from \mathbb{C} , that trivially preserves identifiers. For any morphism $f : \mathcal{D}_1 \rightarrow \mathcal{D}_2$, $f \circ \text{id}_{\mathcal{D}_1} = f = \text{id}_{\mathcal{D}_2} \circ f$ is trivial.

Associativity for $f : \mathcal{D}_1 \rightarrow \mathcal{D}_2$, $g : \mathcal{D}_2 \rightarrow \mathcal{D}_3$ and $h : \mathcal{D}_3 \rightarrow \mathcal{D}_4$, $h \circ (g \circ f) = (h \circ g) \circ f$ is inherited from the associativity in \mathbb{C} , and since f , g and h all preserves identifiers.

Hence \mathbb{D} is a category. \square

Lemma 6. *The product of \mathcal{D} -structures of Definition 16, page 10, is the product in \mathbb{D} .*

Proof. First note that $\mathcal{D}_1 \times \mathcal{D}_2$ is a \mathcal{D} -structures as it is a configuration structure and from the definition of m every event in a configuration has a unique label. Secondly, it is easy to show that the projections are morphisms. Lastly to show that the structure $\mathcal{D}_1 \times \mathcal{D}_2$ has the universal property, we proceed in two steps:

- the underlying configuration structure is the product of the underlying configuration structures, by definition:

$$\mathcal{F}(\mathcal{D}_1 \times \mathcal{D}_2) = \mathcal{F}(\mathcal{D}_1) \times \mathcal{F}(\mathcal{D}_2);$$

- for any \mathcal{D}' which projects into \mathcal{D}_1 and \mathcal{D}_2 , then $\mathcal{F}(\mathcal{D}')$ projects into $\mathcal{F}(\mathcal{D}_1)$ and $\mathcal{F}(\mathcal{D}_2)$ and therefore there exists a unique morphism $h : \mathcal{F}(\mathcal{D}') \rightarrow \mathcal{F}(\mathcal{D}_1 \times \mathcal{D}_2)$. It is easy to show that since the projections preserve identifiers, then so does h which concludes our proof.

This lemma also follows from [15, Proposition 85]. □

Lemma 7. *The operations of Definition 16 (product, relabeling, restriction and parallel composition), page 10, preserve \mathcal{D} -structures.*

Proof. Let us note that (i) the product, relabeling, restriction, and parallel composition on configuration structures from Definition 4 preserve configuration structures and that (ii) any configuration structure endowed with a valid identifier function (i.e., such that no two events in the same configuration have the same identifier, cf. Collision Freeness) is a valid \mathcal{D} -structure.

For the product, it follows trivially from Lemma 6.

Relabeling does not change anything but the labels, so we have nothing to prove.

The restriction only removes events in configurations and keeps the identifier function intact. Hence if the initial structure has a valid identifier function, then the identifier function of the new structure is a valid one by assumption.

Let us now consider the parallel composition of two \mathcal{D} -structures, denoted $\mathcal{D}_1 \mid \mathcal{D}_2 = (E, C, \ell, m)$. Proving that the identifier function is valid follows from a case analysis. Given a configuration $x \in C$ and two events $e, e' \in x$, these are the possible cases:

- $\pi_2(e) = \star$ and $\pi_2(e') = \star$. In this case, looking at the definition of the product in \mathcal{D} -structures, $m(e) = (m_1(\pi_1(e)), m_\star)$ and $m(e') = (m_1(\pi_1(e')), m_\star)$. If $m(e) = m(e')$, then $m_1(\pi_1(e)) = m_1(\pi_1(e'))$ in the configuration $\pi_1(x)$ in \mathcal{D}_1 . But that's a contradiction, since $\pi_1(e)$ and $\pi_1(e')$ are in the same configuration and the identifier function of \mathcal{D}_1 is valid.
- $\pi_1(e) = \star$ and $\pi_1(e') = \star$. This case is similar as the previous one, except that it uses that the identifier function of \mathcal{D}_2 is valid.
- $\pi_1(e) \neq \star$ and $\pi_2(e') \neq \star$ (with either $\pi_1(e') = \star$ or $\pi_1(e') \neq \star$). If $m(e) = m(e')$, then $m_i(\pi_i(e)) = m_i(\pi_i(e'))$ for $i = 1, 2$. Then in this case,
 - either one of them, say, e , is a synchronisation or a fork: in this case, $m_1(\pi_1(e)) = m_2(\pi_2(e)) = m_2(\pi_2(e'))$, and e' was relabeled \perp at the relabeling stage of the parallel composition, and then removed during the restriction. Hence a contradiction: e and e' can't be two events in the same configuration.
 - or none of them is a synchronisation, in which case both events were removed by the restriction. Hence, again, a contradiction: e and e' can't be two events in the same configuration.

The symmetric case (where $\pi_2(e) \neq \star$ and $\pi_1(e') \neq \star$) is similar. □

The following lemma makes more formal the intuition of [Remark 1](#), page 10. Remember that we assumed that for every configuration structure (E, C, ℓ) , E was endowed with a total order, that we write \preceq .

Lemma 8. $\mathcal{F} : \mathbb{D} \rightarrow \mathbb{C}$, defined by

- $\mathcal{F}(E, C, \ell, I, m) = (E, C, \ell)$
- $\mathcal{F}(f_E, f_C, f_m) = (f_E, f_C)$

and $\mathcal{S} : \mathbb{C} \rightarrow \mathbb{D}$, defined by

- $\mathcal{S}(E, C, \ell) = (E, C, \ell, I, m)$, where $I = \{1, \dots, \text{Card } E\}$ and

$$m(e) = \begin{cases} 1 & \text{if } \forall e', e' \preceq e \\ i + 1 & \text{if } \exists e', e' \preceq e, m(e') = i \text{ and there is no } e'' \text{ s.t. } e' \preceq e'' \preceq e \end{cases}$$

- For $(f_E, f_C) : (E_1, C_1, \ell_1) \rightarrow (E_2, C_2, \ell_2)$, $\mathcal{S}(f_E, f_C) = (f_E, f_C, f_m)$, where $f_m(m_1(e)) = m_2(f_E(e_2))$.

are functors.

Proof. Proving that \mathcal{F} is a functor is immediate.

Proving that $\mathcal{S}(\mathcal{C})$ is a \mathcal{D} -structure is immediate, since our construction of m trivially insures [Collision Freeness](#). For $(f_E, f_C) : \mathcal{C}_1 \rightarrow \mathcal{C}_2$, proving that $\mathcal{S}(f_E, f_C)$ is a morphism between $\mathcal{S}(\mathcal{C}_1)$ and $\mathcal{S}(\mathcal{C}_2)$ is also immediate. For the preservation of the identity, we compute:

$$\begin{aligned} \mathcal{S}(\text{id}_{\mathcal{C}}) &= \mathcal{S}(\text{id}_E, \text{id}_C) \\ &= (\text{id}_E, \text{id}_C, f_m) \end{aligned}$$

$$\begin{aligned} \text{where } f_m(m(e)) &= m(\text{id}_E(e)) = m(e), \text{ hence } f_m = \text{id}_I : I \rightarrow I, \\ &= (\text{id}_E, \text{id}_C, \text{id}_I) \\ &= \text{id}_{\mathcal{S}(\mathcal{C})} \end{aligned}$$

For the composition of morphisms, given $f = (f_C, f_E) : \mathcal{C}_1 \rightarrow \mathcal{C}_2$ and $g = (g_C, g_E) : \mathcal{C}_2 \rightarrow \mathcal{C}_3$, we write $\mathcal{S}(\mathcal{C}_i) = (E_i, C_i, \ell_i, I_i, m_i)$ and we compute:

$$\begin{aligned} \mathcal{S}(g) \circ \mathcal{S}(f) &= (g_C, g_E, g_m) \circ (f_C, f_E, f_m) \\ &= (g_C \circ f_C, g_E \circ f_E, g_m \circ f_m) \end{aligned}$$

where, for all $e \in E_1$, we compute:

$$\begin{aligned} (g_m \circ f_m)(m_1(e)) &= g_m(f_m(m_1(e))) \\ &= g_m(m_2(f_E(e))) \\ &= m_3(g_E(f_E(e))) \end{aligned}$$

And hence we can conclude:

$$\mathcal{S}(g) \circ \mathcal{S}(f) = \mathcal{S}(g \circ f)$$

□

A.2 Proofs for Sect. 3.2

In the following, we start by observing that [Lemma 2](#) follows from [Lemma 4](#). [Lemma 4](#), on its side, requires a bit of work: on top of proving [Lemma 3](#), we state and prove some intermediate lemmas ([Lemma 9](#), [10](#) and [11](#)) needed to obtain it.

Proof ([Lemma 2](#), page 13). Follows from [Lemma 4](#). □

Lemma 9. *For all RCCS process R , letting $\lfloor R \rfloor = (E, C, m)$, for all $e_1, e_2 \in E$, $m(e_1) = m(e_2)$ implies $e_1 = e_2$.*

Proof. We proceed by structural induction on R . From [Corollary 1](#) the only interesting case is the parallel composition, i.e. $R = R_1 \mid R_2$. From the definition of parallel composition in \mathcal{D} -structures ([Definition 16](#)), it follows that $m(e_1) = m(e_2)$ implies $e_1 = e_2$. □

Proof ([Lemma 3](#), page 13). We proceed by induction on R .

If R is $m \triangleright P$, we prove that $\lfloor m \rfloor$ is a poset with one maximal element by induction on m . The base case, if m is \emptyset , is trivial, since $\lfloor \emptyset \rfloor = \mathbf{0}$ is a poset with one maximal element. If m is $\Upsilon.m'$, then it follows by induction hypothesis, since $\lfloor \Upsilon.m' \rfloor = \lfloor m' \rfloor$. If m is $\langle i, \alpha, P \rangle.m'$, then by induction hypothesis, $\lfloor m' \rfloor$ is a poset with one maximal element, and the construction of $\lfloor \langle i, \alpha, P \rangle.m' \rfloor$ detailed in [Definition 17](#) preserves that property.

If R is $R' \setminus a$, then it trivially follows by induction hypothesis.

Finally, if R is $R_1 \mid R_2$, then by induction hypothesis we get that $\lfloor R_1 \rfloor$ and $\lfloor R_2 \rfloor$ are both posets with a maximal configuration. We also know by [Lemma 9](#) that they have disjoint identifiers. Looking at the definition of parallel composition for \mathcal{D} -structures ([Definition 16](#)), we may observe that $\lfloor R \rfloor = \lfloor R_1 \mid R_2 \rfloor = \lfloor R_1 \rfloor \mid \lfloor R_2 \rfloor = (r \circ (\lfloor R_1 \rfloor \times \lfloor R_2 \rfloor)) \upharpoonright_F$.

We show that there exists more than one maximal configurations in $\lfloor R_1 \rfloor \times \lfloor R_2 \rfloor$ and that all but one are removed by the restriction.

We show this by first showing that there exists more than one maximal configurations in $\mathcal{F}(\lfloor R_1 \rfloor) \times \mathcal{F}(\lfloor R_2 \rfloor)$, denoted here with \mathcal{C} . From the definition of product ([Definition 4](#)), we have that there exists y_1, \dots, y_n maximal configurations in \mathcal{C} such that $\pi_1(y_i)$ and $\pi_2(y_i)$ are maximal in \mathcal{C}_1 and \mathcal{C}_2 , respectively. As $\lfloor R_1 \rfloor \times \lfloor R_2 \rfloor = (\mathcal{F}(\lfloor R_1 \rfloor) \times \mathcal{F}(\lfloor R_2 \rfloor)) + m$ it follows that the maximal configurations of \mathcal{C} are preserved in $\lfloor R_1 \rfloor \times \lfloor R_2 \rfloor$.

A second step is then to show that the restriction keeps only one maximal configuration. Let y_i, y_j be two maximal configurations. As they are maximal it implies that $y_i \cup y_j \notin \mathcal{C}$, for $i \neq j \leq n$. In turn, this implies that there exists $e_i \in y_i$ and $e_j \in y_j$ such that $\pi_1(e_i) = \pi_1(e_j)$ and $e_i \neq e_j$, as otherwise $y_i \cup y_j$ would be defined. Here we assume that $\pi_1(e_i) = \pi_1(e_j)$ but we could also take $\pi_2(e_i) = \pi_2(e_j)$ and the argument still holds.

Let us now take d an event in \mathcal{C}_1 and take e_1, \dots, e_m the subset of events in E where $\pi_1(e_i) = d$. The restriction in the parallel composition of $\lfloor R_1 \rfloor \mid \lfloor R_2 \rfloor$ keeps only one such event e_i and removes the rest. Therefore, from all maximal configurations y_1, \dots, y_m such that $e_i \in y_i$, $i \leq m$, only one remains.

By applying the argument above to all events in $\lfloor R_1 \rfloor$ (and $\lfloor R_2 \rfloor$), we have that the restriction removes all but one y_i , which is then the maximal configuration in $\lfloor R_1 \rfloor \mid \lfloor R_2 \rfloor$. \square

Let us write x_{MAX} the maximal configuration from [Lemma 3](#).

For the following proof, we need to introduce the causality relation on memory events and on transitions from [\[7\]](#). We write $i_1 <_R i_2$ for R a proces and $i_1, i_2 \in I$ if there exist a memory stack $m.\langle i_1, \alpha_1, P_1 \rangle.m'.\langle i_2, \alpha_2, P_2 \rangle.m''$ in R . We write $i_1 \leq_R i_2$ for the transitive closure of $<$ over all memories of R .

Lemma 10. *Let R be a reversible process and $\lfloor R \rfloor = (E, C, \ell, m)$ be the encoding of its memory, with x_{MAX} the maximal configuration in C . Let $\langle i_1, \alpha_1, P_1 \rangle$ and $\langle i_2, \alpha_2, P_2 \rangle$, $i_1 \neq i_2$ be two memory events in R and let $e_1, e_2 \in E$ be the two corresponding events, i.e. $m(e_j) = i_j$, $j \in \{1, 2\}$. Then $i_1 \leq_R i_2 \iff e_1 \leq_{x_{\text{MAX}}} e_2$.*

Proof. Follows by a structural induction on R .

We also import from [\[7, Definition 1\]](#) the definition of causality on transitions t_1 and t_2 , of a trace θ , that we denote here with $t_1 \leq_\theta t_2$. We do not give the definition formally, as it is lengthy, but intuitively, it is the causality relation on memory events lifted to transitions.

Let R be an RCCS process and $\theta : O_R \xrightarrow{\star} R$ a trace. Also, let $\llbracket O_R \rrbracket = (E, C, \ell)$ and let x be a configuration such that $\llbracket R \rrbracket = (\llbracket O_R \rrbracket, x)$. We define a bijection from transitions in θ to events in x , that we write TE . We define the bijection by induction on the trace θ using the fact for each transition $t : R' \xrightarrow{i, \alpha} R''$ in θ , there exists x' a configuration and e' an event in $\llbracket O_R \rrbracket$ such that $\llbracket R' \rrbracket = (\llbracket O_R \rrbracket, x')$ and $\llbracket R'' \rrbracket = (\llbracket O_R \rrbracket, x' \cup \{e'\})$. Then $\text{TE}(t) = e'$.

Lemma 11. *Let t_1 and t_2 be two transitions of a trace $\theta : O_R \xrightarrow{\star} R$ and let $\llbracket R \rrbracket = (\llbracket O_R \rrbracket, x)$. Then $t_1 \leq_\theta t_2 \iff \text{TE}(t_1) \leq_x \text{TE}(t_2)$.*

Proof. Follows by induction on the trace θ .

Proof (Lemma 4, page 14). We reformulate the hypothesis and show a stronger (in the sense that it is more specific) result from which [Lemma 4](#) follows.

As R is reachable there exists a forward-only trace $O_R \xrightarrow{i_1, \alpha_1} \dots \xrightarrow{i_n, \alpha_n} R$ [\[1, Lemma 1\]](#), denoted by θ . We write $\lfloor R_j \rfloor = (E_j, C_j, \ell_j, m_j)$ for R_j a process in the trace above and $\theta_j : O_R \xrightarrow{i_1, \alpha_1} \dots \xrightarrow{i_j, \alpha_j} R_j$ a subtrace of θ .

Let $\llbracket O_R \rrbracket = (E, C, \ell)$. From [\[1\]](#) we have that $\llbracket R_j \rrbracket = (\llbracket O_R \rrbracket, x_j)$ for $x_j \in C$. We show that there exists three bijections:

- TE_j between transitions in the trace θ_j and events in $\llbracket O_R \rrbracket$, i.e. $\text{TE}_j : \theta_j \rightarrow E$;
- MT_j between events in $\lfloor R_j \rfloor$ and transitions, i.e. $\text{MT}_j : E_j \rightarrow \theta_j$;
- ME_j between events in $\lfloor R_j \rfloor$ and events in $\llbracket O_R \rrbracket$, i.e. $\text{ME}_j : E_j \rightarrow E$, such that $\text{ME}_j = \text{TE}_j \circ \text{MT}_j$.

Moreover, all three bijections preserve the labels and the causality relations. In particular, ME is label and order preserving for all events in x_{MAX} , the maximal configuration in $\llbracket R_j \rrbracket$:

$$\text{ME}_j(x_{\text{MAX}}) = x_j \quad (10)$$

$$e_1 \leq_{x_{\text{MAX}}} e_2 \iff \text{ME}_j(e_1) \leq_{x_j} \text{ME}_j(e_2). \quad (11)$$

From [Definition 18](#), $x \downarrow = (x, C_x, \ell \upharpoonright_x)$, for $C_x = \{y \mid y \in C, y \subseteq x\}$. We can write $\llbracket R_j \rrbracket = x_{\text{MAX}} \downarrow$, and therefore have $\mathcal{F}(\llbracket R_j \rrbracket) \cong (\text{ME}_j(x_{\text{MAX}})) \downarrow$.

We proceed by induction on the trace θ , and at each step we extend the three bijections above.

For a transition $R_j \xrightarrow{i, \alpha} R_{j+1}$ and a trace θ_j we have by induction that

- $\mathcal{F}(\llbracket R_j \rrbracket) \cong x_j \downarrow$, for $\llbracket R_j \rrbracket = (\llbracket O_R \rrbracket, x_j)$;
- there exists $\text{TE}_j : \theta_j \rightarrow E$, $\text{MT}_j : E_j \rightarrow \theta_j$ and $\text{ME}_j : E_j \rightarrow E$

as defined above.

There is an operational correspondence between R_j and its encoding [[1](#), Lemma 6], and therefore there exists e an event in $\llbracket O_R \rrbracket$ such that

$$(\llbracket O_R \rrbracket, x_j) \xrightarrow{e} (\llbracket O_R \rrbracket, x_j \cup \{e\}) \quad (12)$$

with $\ell(e) = \alpha$ and where we write $x_{j+1} = x_j \cup \{e\}$.

We have to show that $\mathcal{F}(\llbracket R_{j+1} \rrbracket) \cong x_{j+1} \downarrow$. More specifically we show that $E_{j+1} \setminus E_j = \{e_{j+1}\}$ and that we can extend the bijections such that

$$\text{TE}_{j+1} = \text{TE}_j \cup \{(R_j \xrightarrow{i, \alpha} R_{j+1}) \mapsto e\} \quad (13)$$

$$\text{MT}_{j+1} = \text{MT}_j \cup \{e_{j+1} \mapsto (R_j \xrightarrow{i, \alpha} R_{j+1})\} \quad (14)$$

$$\text{ME}_{j+1} = \text{ME}_j \cup \{e_{j+1} \mapsto e\} \quad (15)$$

As ME_{j+1} is a label and order preserving bijection on x_{MAX} , the maximal configuration in $\llbracket R_{j+1} \rrbracket$, it follows that $\text{ME}_{j+1}(x_{\text{MAX}}) = x_{j+1}$. As $\llbracket R_{j+1} \rrbracket \cong x_{\text{MAX}} \downarrow$ it follows $\mathcal{F}(\llbracket R_{j+1} \rrbracket) \cong x_{j+1} \downarrow$.

We now proceed by cases on the transition $R_j \xrightarrow{i, \alpha} R_{j+1}$. We distinguish two cases: α is an unsynchronized input or output, and $\alpha = \tau$.

- Let us suppose w.l.o.g. that $\alpha = a$. Then we can rewrite the transition as follow:

$$R_j = (\vec{b})(S \mid m \triangleright a.P + Q) \xrightarrow{i, \alpha} R_{j+1} = (\vec{b})(S \mid \langle i, \alpha, Q \rangle.m \triangleright P)$$

Let us also define the following projection on events: $\pi_S(e) = e_s$ if there exists $e_s \in [S]$ such that e, e_s have the same identifiers and undefined otherwise. Similarly define π_m for the projections of events from $\llbracket R_i \rrbracket$ to $\llbracket m \rrbracket$.

Note that we can extend TE_{j+1} as in [Equation 13](#). We show that there exists an event in $\llbracket R_{j+1} \rrbracket$ which corresponds to the transition $R_j \xrightarrow{i, \alpha} R_{j+1}$. Let

us unfold the encoding of the two processes above:

$$\begin{aligned} \llbracket R_j \rrbracket &= \llbracket (\vec{b})(S \mid m \triangleright a.P + Q) \rrbracket = (r \circ (\llbracket S \rrbracket \times \llbracket m \rrbracket)) \upharpoonright_{\perp} \upharpoonright_{\vec{b}} \\ \llbracket R_{j+1} \rrbracket &= \llbracket (\vec{b})(S \mid \langle i, \alpha, Q \rangle.m \triangleright a) \rrbracket = (r \circ (\llbracket S \rrbracket \times \llbracket \langle i, \alpha, Q \rangle.m \rrbracket)) \upharpoonright_{\perp} \upharpoonright_{\vec{b}}. \end{aligned}$$

Let us write $\llbracket m \rrbracket = (E_m, C_m, \ell_m, m_m)$. From [Lemma 3](#) we have that there exists a single maximal configuration in C_m , denoted with x_{MAX}^m . Using [Definition 17](#) we can unfold $\llbracket \langle i, \alpha, Q \rangle.m \rrbracket$ and write

$$\begin{aligned} \llbracket R_j \rrbracket &= (r \circ (\llbracket S \rrbracket \times (E_m, C_m, \ell_m, m_m))) \upharpoonright_{\perp} \upharpoonright_{\vec{b}} \\ \llbracket R_{j+1} \rrbracket &= (r \circ (\llbracket S \rrbracket \times \\ &\quad (E_m \cup \{e_m\}, C_m \cup (x_{\text{MAX}}^m \cup \{e_m\}), \\ &\quad \ell_m \cup \{e_m \rightarrow \alpha\}, m_m + \{e_m \rightarrow i\}))) \upharpoonright_{\perp} \upharpoonright_{\vec{b}} \end{aligned}$$

for some event $e_m \notin E_m$. From rules act. and par. of [Fig. 4](#), i is not in the domain of m_m . Therefore all synchronisations in $\llbracket S \rrbracket \times \llbracket \langle i, \alpha, Q \rangle.m \rrbracket$ of the form (e_s, e_m) , with $e_s \in \llbracket S \rrbracket$, are relabeled \perp and removed by the first restriction. The event (\star, e_m) is preserved by the first restriction. It is not removed by the second restriction as $a \notin \vec{b}$. Remember that from [Lemma 3](#) we have that there exists a single maximal configuration in $\llbracket R_{j+1} \rrbracket$ and from the definition of the parallel composition there is only one event in x_{MAX} with the first projection equal to e_m , denoted e_{j+1} : $e_{j+1} \in x_{\text{MAX}}$ with $\pi_m(e_{j+1}) = e_m$. It follows that $E_{j+1} = E_j \cup \{e_{j+1}\}$. We extend then the bijections as in [Equations 14](#) and [15](#). Moreover, $\ell(e_{j+1}) = \ell_m(e_m)$, and from [Equation 12](#) it follows that ME_{j+1} is label preserving.

The last part is to show [Equation 11](#). We only have to show that

$$e' \leq_{x_{\text{MAX}}} e_{j+1} \iff \text{ME}_{j+1}(e') \leq_{x_{j+1}} \text{ME}_{j+1}(e_{j+1}), \quad (16)$$

as the rest follows by induction on ME_j and from [Equation 15](#). To show [Equation 16](#), consider $e' \leq_{x_{\text{MAX}}} e_{j+1}$. From [Definition 17](#) there exists a memory event $d = \langle i', \alpha', P' \rangle$ in $\llbracket R_{j+1} \rrbracket$ such that $m_j(e') = i'$. From [Lemma 10](#) we have then that $i' \leq_R i$ and using the definition of causality on transitions ([\[7, Definition 1\]](#)), $\text{MT}_{j+1}(e') \leq_{\theta} \text{MT}_{j+1}(e_{j+1})$.

We conclude using [Lemma 11](#) which shows that $\text{TE}_{j+1}(\text{MT}_{j+1}(e')) \leq_{x_{j+1}} \text{TE}_{j+1}(\text{MT}_{j+1}(e_{j+1}))$.

Similarly, we reason for e' concurrent with e_{j+1} . Lastly, note that there are no events in conflict with e_{j+1} (or with e in $x_{j+1} \downarrow$), as there is a single maximal configuration in both $\llbracket R_{j+1} \rrbracket$ and in $x_{j+1} \downarrow$.

– Suppose that $\alpha = \tau$ and let us write the transitions as follows:

$$\begin{aligned} R_j &= (\vec{b})(S \mid m_1 \triangleright a.P_1 + Q_1 \mid m_2 \triangleright \bar{a}.P_2 + Q_2) \\ &\xrightarrow{i, \tau} R_{j+1} = (\vec{b})(S \mid \langle i, a, Q_1 \rangle.m_1 \triangleright P_1 \mid \langle i, \bar{a}, Q_2 \rangle.m_2 \triangleright P_2) \end{aligned}$$

We are assuming here, for simplification, that both thread involved in the synchronisation are under the same set of restricted names. The more general case, does not change the reasoning here, just adds in technicality.

We show that the transition adds a single event e_{j+1} in $\llbracket R_{j+1} \rrbracket$ and that the bijection ME defined on $\llbracket R_j \rrbracket$ extends to e_{j+1} such that it remains a label and order preserving bijection between the maximal configuration in $\llbracket R_{j+1} \rrbracket$ and x_{j+1} . The proof follows the reasoning above. \square

A.3 Proof of Theorem 2

Before proving the main theorem, let us make the following observation. Let $f : (E_1, C_1, \ell_1) \rightarrow (E_2, C_2, \ell_2)$ be a morphism, which is defined as function on events: $f = (f_E, f_C)$. As f_C is defined by f_E , we can w.l.o.g. write $f = f_E$. Saying that f is an isomorphism is equivalent then to saying that $f : E_1 \rightarrow E_2$ is a label- and order-preserving bijection. Therefore the functions f of Definition 9 and Definition 20 are of “the same nature”.

Proof (Theorem 2, page 15). Let us prove the HHPB case, the other three cases being similar, and actually simpler.

\Rightarrow Let $\mathcal{R}_{\text{RCCS}}$ be a HHPB between $\emptyset \triangleright P_1$ and $\emptyset \triangleright P_2$. We show that the following relation

$$\begin{aligned} \mathcal{R} = \{ (x_1, x_2, f) \mid x_1 \in \llbracket P_1 \rrbracket, x_2 \in \llbracket P_2 \rrbracket, \exists R_1, R_2 \text{ s.t. } O_{R_1} = P_1, \\ O_{R_2} = P_2, (R_1, R_2, f) \in \mathcal{R}_{\text{RCCS}} \text{ and} \\ \llbracket R_1 \rrbracket = (\llbracket P_1 \rrbracket, x_1), \llbracket R_2 \rrbracket = (\llbracket P_2 \rrbracket, x_2) \} \end{aligned}$$

is a HHPB between $\llbracket P_1 \rrbracket$ and $\llbracket P_2 \rrbracket$.

First note that $(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$: indeed $(\emptyset \triangleright P_1, \emptyset \triangleright P_2, \emptyset) \in \mathcal{R}_{\text{RCCS}}$ and $\llbracket \emptyset \triangleright P_i \rrbracket = (\llbracket P_i \rrbracket, \emptyset)$, for $i \in \{1, 2\}$.

Let us suppose that $(x_1, x_2, f) \in \mathcal{R}$ for $\llbracket R_i \rrbracket = (\llbracket P_i \rrbracket, x_i)$, for $i \in \{1, 2\}$ and $f : x_1 \rightarrow x_2$ an isomorphism. Moreover, note that $\mathcal{F}(\llbracket R_i \rrbracket) \cong x_i \downarrow$, from Lemma 4, and that $\llbracket R_i \rrbracket \cong x_i \downarrow + m$, for some function m , from Definition 14. To show that \mathcal{R} is a HHPB we have to show that if $x_1 \xrightarrow{e_1} y_1$ (or $x_1 \xrightarrow{e_1} y_1$) then there exists y_2 such that $x_2 \xrightarrow{e_1} y_2$ (or $x_2 \xrightarrow{e_2} y_2$ respectively) and such that $(y_1, y_2, f') \in \mathcal{R}$ for some f' .

Let $x_1 \xrightarrow{e_1} y_1$, hence by definition, $y_1 = x_1 \cup \{e_1\}$. From the correspondence between RCCS and their encodings (from [1, Lemma 6]), it follows that $R_1 \xrightarrow{i:\alpha} S_1$ such that $\llbracket S_1 \rrbracket = (\llbracket P_1 \rrbracket, y_1)$. We therefore deduce that $\llbracket S_1 \rrbracket \cong y_1 \downarrow + (m \cup \{e_1 \mapsto i\})$.

As $(R_1, R_2, f) \in \mathcal{R}_{\text{RCCS}}$ and as $R_1 \xrightarrow{i:\alpha} S_1$, it follows that there exists a transition $R_2 \xrightarrow{j:\alpha} S_2$ with $f = f' \upharpoonright_{\mathcal{F}(\llbracket R_1 \rrbracket)}$ and $(S_1, S_2, f') \in \mathcal{R}_{\text{RCCS}}$.

Again from the correspondence between R_2 and $\llbracket R_2 \rrbracket$ we have that $x_2 \xrightarrow{e_2} y_2$ such that $y_2 = x_2 \cup \{e_2\}$ and $\llbracket S_2 \rrbracket = (\llbracket P_2 \rrbracket, y_2)$. Then we have that $(y_1, y_2, f') \in \mathcal{R}$.

We treat similarly the cases where x_2 does a transition, or when the transitions are backwards.

\Leftarrow Let $\mathcal{R}_{\text{CONF}}$ be a HHPB between $\llbracket P_1 \rrbracket$ and $\llbracket P_2 \rrbracket$. We show that the following relation

$$\mathcal{R} = \{(R_1, R_2, f) \mid O_{R_1} = P_1, O_{R_2} = P_2 \text{ and } \llbracket R_1 \rrbracket = (\llbracket P_1 \rrbracket, x_1), \\ \llbracket R_2 \rrbracket = (\llbracket P_2 \rrbracket, x_2), \text{ with } (x_1, x_2, f) \in \mathcal{R}_{\text{CONF}}\}$$

is a HHPB between $\emptyset \triangleright P_1$ and $\emptyset \triangleright P_2$.

We have that $(\emptyset \triangleright P_1, \emptyset \triangleright P_2, \emptyset) \in \mathcal{R}$ as $(\emptyset, \emptyset, \emptyset) \in \mathcal{R}_{\text{CONF}}$ and $\llbracket \emptyset \triangleright P_i \rrbracket = (\llbracket P_i \rrbracket, \emptyset)$, for $i \in \{1, 2\}$.

We suppose now that $(R_1, R_2, f) \in \mathcal{R}$, with $f : \mathcal{F}(\lfloor R_1 \rfloor) \rightarrow \mathcal{F}(\lfloor R_2 \rfloor)$. It implies that $\llbracket R_i \rrbracket = (\llbracket P_i \rrbracket, x_i)$, $i \in \{1, 2\}$, we have that $(x_1, x_2, f) \in \mathcal{R}_{\text{CONF}}$. As $\mathcal{F}(\lfloor R_i \rfloor) \cong x_i \downarrow$, from [Lemma 4](#), f is also defined from x_1 to x_2 .

To show that \mathcal{R} is a HHPB we have to show that if $R_1 \xrightarrow{i:\alpha} S_1$ (or $R_1 \overset{i:\alpha}{\rightsquigarrow} S_1$) then there exists S_2 such that $R_2 \xrightarrow{j:\alpha} S_2$ (or $R_2 \overset{j:\alpha}{\rightsquigarrow} S_2$ respectively) and such that $(S_1, S_2, f') \in \mathcal{R}$ for some f' .

Let $R_1 \xrightarrow{i:\alpha} S_1$. We use again the correspondence between RCCS and their encodings (from [\[1, Lemma 6\]](#)) from which we have that there exists e_1 and $y_1 = x_1 \cup \{e_1\}$ such that $x_1 \xrightarrow{e_1} y_1$ and $\llbracket S_1 \rrbracket = (\llbracket P_1 \rrbracket, y_1)$. As $(x_1, x_2, f) \in \mathcal{R}_{\text{CONF}}$ it implies that there exists e_2, y_2 and f' such that $x_2 \xrightarrow{e_2} y_2$ and $(y_1, y_2, f') \in \mathcal{R}_{\text{CONF}}$. Again, from the correspondence between RCCS and configuration structures we have that from $x_2 \xrightarrow{e_2} y_2$, there exists S_2 and some j such that $R_2 \xrightarrow{j:\alpha} S_2$ with $\llbracket S_2 \rrbracket = (\llbracket P_2 \rrbracket, y_2)$. Hence $\mathcal{F}(\lfloor S_2 \rfloor) = y_2 \downarrow$. We conclude therefore that $(S_1, S_2, f') \in \mathcal{R}$.

Similarly we show the cases where R_1 does a backward transition, or if R_2 does a forward or backward transition.

□