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Violation of the gyrotropic pressure closure due to a velocity shear in a magnetised plasma

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Résumé. Processus cinétiques liés aux déformations quadripolaires de la fonction de distribution d'un plasma, notamment les mécanismes d'anisotropisation de pression, peuvent être décrits dans un modèle fluide qui inclut l'évolution du tenseur de pression complet [1]. Dans ce contexte, nous montrons que l'anisotropie en moment dans un écoulement fluide avec cisaillement de vitesse peut se transférer dans une anisotropie de pression, à cause de l'action du tenseur des stress (en particulier, à cause de sa partie symétrique) sur le moment fluide d'ordre deux (tenseur de pression) de l'équation de transport [2]. Ce mécanisme, purement dynamique, induit l'anisotropisation d'un tenseur de pression initialement isotropique sur des temps de l'ordre de l'inverse de l'échelle caractéristique des gradients de vitesse, quand cette échelle devient non négligeable par rapport à la fréquence cyclotronique. L'anisotropie générée par ce mécanisme est soit gyrotropique que non-gyrotropique et peut expliquer des observations dans le vent solaire ou dans simulations de turbulence Vlasov [3]. En particulier, le cisaillement de vitesse associé aux nappes de vortacité, permet aussi d'interpréter la corrélation, observée dans les simulations, entre anisotropie de pression et vortacité fluide [4]. La génération d'anisotropie non-gyrotropique correspond à une perte de conservation du moment magnétique moyen des particules, pour lequel on écrit l'équation d'évolution dans une description fluide.

Abstract. Kinetic processes related to quadrupolar deformations of a plasma distribution function, notably mechanisms of pressure anisotropisation, can be described by a fluid model which retains the full pressure tensor dynamics [1]. In this framework we show that the momentum anisotropy in a shear flow can be transferred to a pressure anisotropy due to the action of the stress tensor (and in particular of its symmetric part) on the second order velocity moment of the transport equation (*i.e.*, the pressure tensor) [2]. This purely dynamical mechanism induces the anisotropisation of an initially isotropic pressure tensor on a time scale of the order of the inverse of the velocity gradients, when this becomes non-negligible with respect to the cyclotron frequency. Pressure anisotropy this way generated is both gyrotropic and non-gyrotropic and can explain direct observations made in the solar wind or in simulations of Vlasov turbulence [3]. In particular, the velocity shear associated to vorticity sheets allows us to interpret [4] the correlation between pressure anisotropy and fluid vorticity which has been observed in numerical simulations. The generation of non-gyrotropic anisotropy corresponds to a loss of conservation of the average particle magnetic moment, for which we provide an evolution equation in a fluid description.

1 Introduction

We consider a fluid, collisionless plasma model obtained by integrating the first three velocity moments of Vlasov equation for each species α (see Ref.[1], Appendix A). The equations read

$$\frac{\partial n^\alpha}{\partial t} + \nabla \cdot (n^\alpha \mathbf{u}^\alpha) = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}^\alpha}{\partial t} + \mathbf{u}^\alpha \cdot \nabla \mathbf{u}^\alpha = \Omega_\alpha \left(\frac{c\mathbf{E}}{B} + \mathbf{u}^\alpha \times \mathbf{b} \right) - \frac{1}{m^\alpha n^\alpha} \nabla \cdot \mathbf{\Pi}^\alpha, \quad (2)$$

$$\frac{\partial \mathbf{\Pi}^\alpha}{\partial t} + \nabla \cdot (\mathbf{u}^\alpha \mathbf{\Pi}^\alpha) + (\nabla \mathbf{u}^\alpha) \cdot \mathbf{\Pi}^\alpha + ((\nabla \mathbf{u}^\alpha) \cdot \mathbf{\Pi}^\alpha)^T + \nabla \cdot \mathbf{Q}^\alpha = \Omega_\alpha (\mathbf{\Pi}^\alpha \times \mathbf{b} + \mathbf{b} \times \mathbf{\Pi}^\alpha). \quad (3)$$

where, for each specie of charge q^α ($q^\alpha = -e$ for electrons) and mass m^α , $\Omega_\alpha \equiv q^\alpha B / (m^\alpha c)$ are the instantaneous cyclotron frequencies, c being the light velocity and B expressing the magnetic field intensity

according to $\mathbf{B} = B\mathbf{b}$, and the density, fluid velocity and full pressure tensor are respectively n^α , \mathbf{u}^α , $\mathbf{\Pi}^\alpha$. Here each pressure tensor $\mathbf{\Pi}^\alpha$ is defined with respect to the random particle velocity in each own species rest frame. Matrix transpose is indicated by T . Whenever needed, the heat flux tensor \mathbf{Q}^α is for simplicity “closed” by making the simplifying assumption $\nabla \cdot \mathbf{Q}^\alpha = 0$. Even if this restriction is *a priori* not justified, it is reasonable in the limit of spatial gradients perpendicular to the magnetic field, which we mostly consider in the following, and it does not affect (unless specified, next) the reasoning that we are going to develop [1,2]. The electric field \mathbf{E} in Eq.(2) is coupled to the magnetic field \mathbf{B} and to the current density $\mathbf{J} \equiv q^e n^e \mathbf{u}^e + q^i n^i \mathbf{u}^i$ by Maxwell’s equations, for which we assume the quasi-neutrality condition, $q^e n^e + q^i n^i = 0$, in turn consistent with neglecting the displacement current in Ampère’s law :

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}. \quad (4)$$

In the cold electron massless limit considered in Refs.[1,2,4], by neglecting $m^e/m^i \ll 1$ corrections and by dropping the i index for ion fluid quantities, equations (2) can be combined to respectively give the momentum equation of MHD and, by using the first of Eqs.(4), the Hall-MHD induction equation :

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \nabla \cdot \mathbf{u} = \Omega_i \frac{\mathbf{J} \times \mathbf{b}}{nq^i c} - \frac{1}{m^i n} \nabla \cdot \mathbf{\Pi}, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\left(\mathbf{u} - \frac{\mathbf{J}}{ne} \right) \times \mathbf{B} \right). \quad (5)$$

The ion pressure tensor in the first of Eqs.(5) evolves according to Eqs.(3) for $\alpha = i$.

2 Double adiabatic limit and first order corrections for $\omega/|\Omega_\alpha| \ll 1$

A formal closure around a double-Maxwellian distribution for the temperatures parallel and perpendicular to the magnetic field can be obtained by restricting to small frequencies with respect to $|\Omega_\alpha|$. This leads to the well-known “*double adiabatic*” or CGL-closure (after Chew, Goldberger and Low [5]), in which, using a tensor notation with subscripts $i, j, k = x, y, z$, a form $\Pi_{ij}^{\alpha,0} = P_\perp^{\alpha,0} \delta_{ij} + (P_\parallel^{\alpha,0} - P_\perp^{\alpha,0}) b_i b_j$ of each pressure tensor is maintained for the lowest order solution of a power expansion of Eq.(3) in terms of $\omega/|\Omega_\alpha| \ll 1$, that is $\Pi_{ij}^\alpha = \Pi_{ij}^{\alpha,0} + \Pi_{ij}^{\alpha,1} + \dots$ with $|\Pi_{ij}^{\alpha,n+1}|/|\Pi_{ij}^{\alpha,n}| \sim \omega/|\Omega_\alpha|$. By also introducing for the heat flux tensors \mathbf{Q}^α (meant to evolve according to some further dynamics equation –see, *e.g.*, [6]) the gyrotopic form $Q_{ijk}^\alpha = q_\perp^\alpha (\tau_{ij} b_k + \tau_{jk} b_i + \tau_{ki} b_j) + q_\parallel^\alpha b_i b_j b_k$ with $\tau_{ij} = \delta_{ij} - b_i b_j$, and by defining the total time derivative comoving with each specie bulk velocity, $d/dt^\alpha \equiv \partial/\partial t + \mathbf{u}^\alpha \cdot \nabla$, the general form of the double-adiabatic equations for each specie obtained from (3) can be written as (see, *e.g.*, [7], Appendix A) :

$$\frac{dP_\parallel^\alpha}{dt^\alpha} + P_\parallel^\alpha \nabla \cdot \mathbf{u}^\alpha + 2P_\parallel^\alpha \nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b} = -\nabla(q_\parallel^\alpha \mathbf{b}) + 2q_\perp^\alpha \nabla \cdot \mathbf{b}, \quad (6)$$

$$\frac{dP_\perp^\alpha}{dt^\alpha} + 2P_\perp^\alpha \nabla \cdot \mathbf{u}^\alpha - P_\perp^\alpha \nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b} = -\nabla(q_\perp^\alpha \mathbf{b}) - q_\perp^\alpha \nabla \cdot \mathbf{b}. \quad (7)$$

When ideal MHD is assumed (no \mathbf{J} contribution in the second of Eqs.(5)) and heat fluxes are neglected, in the cold electron limit with $m_e/m_i = 0$ and dropping again the α apex, the ion pressure equations can be combined with Eqs.(5) to get the more customary form [5] :

$$\frac{d}{dt} \left(\frac{P_\parallel B^2}{n^3} \right) = 0, \quad \frac{d}{dt} \left(\frac{P_\perp}{nB} \right) = 0. \quad (8)$$

By naming $\tilde{P}_\parallel^\alpha$ and \tilde{P}_\perp^α the scalar pressure components defined with respect to the average bulk plasma $\mathbf{E} \times \mathbf{B}$ -velocity \mathbf{U} and by retaining warm electrons in an ideal MHD closure while neglecting heat fluxes again, identical equations to (8) can be shown to hold for $\tilde{P}_\parallel^e + \tilde{P}_\parallel^i$ replacing P_\parallel , for $\tilde{P}_\perp^e + \tilde{P}_\perp^i$ replacing P_\perp , and for $d/dt_U \equiv \partial/\partial t + \mathbf{U}^\alpha \cdot \nabla$ replacing d/dt (see, *e.g.*, [8]). Eqs.(8) respectively express the average fluid conservation of two adiabatic invariants : the “action” of an infinitesimal fluid element of extension

δl along a magnetic line constituted by particles moving with velocity v_{\parallel} parallel to \mathbf{B} , and the average magnetic moment $\langle \mu \rangle$ (see again Ref.[8] for a discussion).

A ‘‘generalization’’ of Eqs.(8), valid for each specie in non-ideal MHD, can be obtained by rewriting Eqs.(6-7) while eliminating $\nabla \cdot \mathbf{u}^\alpha$ by means of the continuity equations (1) and the $\nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b}$ contribution by means of the equations for the evolution of B and of \mathbf{b} , which can be obtained from the curl of Eqs.(2). In particular, for $\alpha = e, i$ we find :

$$\frac{dB}{dt^\alpha} = B(\nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b}) - B\nabla \cdot \mathbf{u}^\alpha + \mathbf{b} \cdot \mathbf{F}^\alpha, \quad \frac{d\mathbf{b}}{dt^\alpha} = \mathbf{b} \cdot \nabla \mathbf{u}^\alpha - \mathbf{b}(\nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b}) + (\mathbf{F} - \mathbf{b}(\mathbf{b} \cdot \mathbf{F})), \quad (9)$$

where

$$\mathbf{F}^\alpha \equiv -\frac{B}{\Omega_\alpha} \mathbf{f}^\alpha \equiv -\frac{B}{\Omega_\alpha} \nabla \times \left(\frac{d\mathbf{u}^\alpha}{dt^\alpha} + \frac{\nabla \cdot \mathbf{\Pi}^\alpha}{n^\alpha m^\alpha} \right). \quad (10)$$

From the first of Eqs.(9), we write

$$(\nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b}) = \frac{1}{B} \frac{dB}{dt^\alpha} - \frac{1}{n^\alpha} \frac{dn}{dt^\alpha} + \frac{\mathbf{b} \cdot \mathbf{f}^\alpha}{\Omega^\alpha}, \quad (11)$$

which, substituted in Eqs.(6-7), gives

$$\left[\frac{d}{dt^\alpha} + 2 \frac{\mathbf{b} \cdot \mathbf{f}^\alpha}{\Omega^\alpha} \right] \left(\frac{P_{\parallel}^\alpha B^2}{(n^\alpha)^3} \right) = \frac{B^2}{(n^\alpha)^3} \left(-\nabla(q_{\parallel}^\alpha \mathbf{b}) + 2q_{\perp}^\alpha \nabla \cdot \mathbf{b} \right), \quad (12)$$

$$\left[\frac{d}{dt^\alpha} - \frac{\mathbf{b} \cdot \mathbf{f}^\alpha}{\Omega^\alpha} \right] \left(\frac{P_{\perp}^\alpha}{n^\alpha B} \right) = \frac{1}{n^\alpha B} \left(-\nabla(q_{\perp}^\alpha \mathbf{b}) - q_{\perp}^\alpha \nabla \cdot \mathbf{b} \right). \quad (13)$$

These equations, which are just a rewriting of the double adiabatic equations (6-7), state the evolution of CGL-like adiabatic invariants of each specie when heat fluxes at the zeroth $\omega/|\Omega^\alpha|$ contributions are retained (r.h.s. terms) and full-two fluid effects are considered (the $\sim \mathbf{b} \cdot \mathbf{f}^\alpha/\Omega^\alpha$ l.h.s. terms). These latter contributions, when written by using (10), must be consistently power-expanded in order to retain the proper Finite-Larmor-Radius (FLR) corrections. At first order in $\omega/|\Omega^\alpha|$, FLR effects related to the components of the next order solution $\mathbf{\Pi}^{\alpha,1}$ of the power expansion of Eq.(3) enter in the equations for P_{\perp} only (7,13) –see, e.g., Eqs.(A15-A20) of [7]. In this notation, Eq.(A15) of Ref.[7] rewrites as Eq.(13) to which a further r.h.s. term $(\nabla \mathbf{u}^\alpha : \mathbf{\Pi}^{\alpha,1})/(n^\alpha B)$ must be added, which expresses first order FLR corrections that depend on the components of the strain tensor $\nabla \mathbf{u}^\alpha$ (see also Sec.3.2.1 of Ref.[4]).

2.1 Comparison with magnetic moment conservation in gyrokinetic theory

In order to make connection with gyrokinetic theory, let us focus on the equation for P_{\perp}^α and to the associated fluid adiabatic invariant, $\langle \mu \rangle$. We can relate it to the single particle magnetic moment

$$\mu \equiv \frac{(\mathbf{v}_{\perp}^\alpha - \mathbf{V}_{\perp}^\alpha)^2}{2B}, \quad (14)$$

where \perp stands again for the component perpendicular to \mathbf{B} , $\mathbf{v}_{\perp}^\alpha$ is the total particle velocity and $\mathbf{V}_{\perp}^\alpha$ is some reference velocity. Depending on whether $\mathbf{V}_{\perp}^\alpha$ is zero, it is the guiding-center velocity, the $\mathbf{E} \times \mathbf{B}$ speed or the fluid velocity $\mathbf{u}_{\perp}^\alpha$, ‘‘different kinds’’ of magnetic moment can be defined. In the case of single particle motion, their differences and conservations have been recently discussed in [9]. In the fluid description discussed here we are interested in three cases, which we respectively label : μ_{tot}^α when $\mathbf{V}_{\perp}^\alpha = 0$; μ_T^α when $\mathbf{V}_{\perp}^\alpha = \mathbf{u}_{\perp}^\alpha$ and the magnetic moment is thus related to perpendicular thermal contributions only; and μ_D^α when $\mathbf{V}_{\perp}^\alpha = \mathbf{E} \times \mathbf{B}/B^2$. When \mathbf{B} is constant and uniform and \mathbf{E} does not depend on space, μ_D^α coincides with the definition of μ_G^α referred to the gyrocenter velocity and used in gyrokinetic theory [9]. Introducing now the brackets $\langle \dots \rangle$ to express average over the particle velocity according to $\langle A \rangle \equiv (1/n^\alpha) \int A f^\alpha d^3 v^\alpha$, we can relate the averages of the magnetic moments above defined to P_{\perp}^α :

$$\frac{P_{\perp}^\alpha}{2n^\alpha B} = \langle \mu_T^\alpha \rangle = \langle \mu_{tot}^\alpha \rangle - \frac{m^\alpha (u_{\perp}^\alpha)^2}{2B}. \quad (15)$$

Then, when a sufficiently strong magnetic field is considered, consistently with the $\omega/|\Omega_\alpha| \ll 1$ assumption, to the lowest order of the $\omega/|\Omega_\alpha|$ power expansion we have $\mathbf{u}_\perp^\alpha = \mathbf{E} \times \mathbf{B}/B^2$ for both $\alpha = e, i$, and therefore $\langle \mu_T^\alpha \rangle = \langle \mu_D^\alpha \rangle$. In this latter case $d/dt^\alpha \simeq d/dt^i$ and $\mathbf{b}^\alpha \cdot \mathbf{f}^\alpha/|\Omega_\alpha| \simeq \mathbf{b} \cdot (\nabla \cdot \mathbf{\Pi}^\alpha)/(n^\alpha m^\alpha \Omega_\alpha)$ for both $\alpha = e, i$. From Eqs.(7,13) one then recognizes the contributions of two-fluid non-ideal effects to the non-conservation of the averaged magnetic moments considered above. In the following we focus on the violations of the conservation of $\langle \mu_{\dots} \rangle$, which are due to the gradients of the fluid velocity \mathbf{u}^α . In the reduced double adiabatic theory at $\omega/|\Omega_\alpha| \ll 1$ they enter as $O(\omega/|\Omega_\alpha|)$ contributions related to the first order FLR corrections to Eqs.(7,13), which violate the gyrotropic symmetry of the pressure [10].

We now discuss in deeper detail this issue at the varying of $\omega/|\Omega_\alpha|$, when the full pressure tensor equation (3) is considered instead of its lowest order solution for $\omega/|\Omega_\alpha| \ll 1$. We emphasize in this regard the radical difference which has been evidenced for the modelling of both some linear modes [1] and pressure-driven instabilities [11] between a description which retains the full pressure tensor dynamics and a reduced description in which the non-gyrotropic degrees of freedom in the evolution of $\mathbf{\Pi}^\alpha$ are suppressed because of a gyrotropic or polytropic assumption. This difference reflects in the improper dispersion provided by FLR corrections to a CGL closure for the perpendicular propagation of magnetoacoustic waves (see discussion in Ref.[1] and references therein), and in spurious threshold conditions for the onset of Weibel-type modes when polytropic closures for a diagonal pressure tensor are assumed instead of letting all the pressure tensor components evolve according to Eqs.(3) [11].

3 Full pressure tensor dynamics and role of shear flows

In Ref.[2] the deformations induced by the different terms acting on $\mathbf{\Pi}^\alpha$ have been discussed. Beside of rigid rotations around the local direction of \mathbf{B} which are due to the r.h.s. term of Eq.(3), three kinds of deformation of the pressure tensor can be induced by the gradients \mathbf{u}^α : isotropic compressions/expansions related to $\nabla \cdot \mathbf{u}^\alpha$, rigid rotations around the axis of the local vorticity $\boldsymbol{\omega}^\alpha = \nabla \times \mathbf{u}^\alpha$, and volume-preserving deformations without rotations due to the traceless rate of shear \mathbf{D}^α , which in a full 3D geometry is defined as $\mathbf{D}^\alpha \equiv (1/2)(\nabla \mathbf{u}^\alpha + (\nabla \mathbf{u}^\alpha)^T) - (\nabla \cdot \mathbf{u}^\alpha)\mathbf{I}/3$. All of them are contributed by the components of the gradient tensors $\nabla \mathbf{u}^\alpha$. Naming P_1^α, P_2^α and P_3^α the eigenvalues of $\mathbf{\Pi}^\alpha$, which identify the length of its principal axes, we see that \mathbf{D}^α only can modify their relative amplitude being therefore capable to make an initially isotropic pressure tensor anisotropic. This anisotropisation is, in the most general case, agyrotropic, in the sense that all three eigenvalues P_1^α, P_2^α and P_3^α can change their value independently. This, in general, occurs at a rate $\tau_{an}^{-1} \sim |\nabla \mathbf{u}^\alpha|$ and the extent of the attained anisotropisation depends in a non trivial way (also due to the nonlinear plasma response to the deformation, which is described by the full set of Eqs.(1-4)) on the amplitude of $\omega/|\Omega_\alpha|$, the limit $\omega/|\Omega_\alpha| \rightarrow 0$ converging of course to the double-adiabatic, gyrotropic, solution $\mathbf{\Pi}^{\alpha,0}$ previously discussed. In this regard, we note that the gyrotropic anisotropy allowed by the different evolution of $P_{||}^\alpha$ and P_\perp^α in the double adiabatic closures (6-7) is also related to the action of \mathbf{D}^α . In this case, however, it depends on the compression contribution only, $\nabla \cdot \mathbf{u}^\alpha$, contained in the $\nabla \mathbf{u}^\alpha : \mathbf{b}\mathbf{b}$ terms which are differently weighed for $P_{||}^\alpha$ and P_\perp^α (see Ref.[4]). It is worth stressing that the gyrotropic anisotropisation in a CGL-type closure is here permitted by the rupture of the spatial isotropy of the charged particle motion, which is determined by the presence of a magnetic field, but *it is not caused* by it: it is the different fluid compression parallel and perpendicular to \mathbf{B} which in a CGL closure determines the gyrotropic anisotropy. When the full pressure tensor evolution is retained, it is then the rate of shear \mathbf{D}^α which in general causes both a non-gyrotropic and a gyrotropic anisotropy with respect to the principal axes of the matrix \mathbf{D}^α .

3.1 Agyrotropisation induced by shear flows : evolution of the gyrotropic and non-gyrotropic anisotropy in a 2D coordinate dependence

It is possible to provide a relatively simple, rigorous analytical treatment of this anisotropisation mechanism [2] in a geometrical setting in which, in the mass-less electron limit $m_e/m_i \rightarrow 0$, the ion vorticity vector is locally aligned to the magnetic field at any time, $\boldsymbol{\omega}^i \times \mathbf{B} = 0$. This occurs in a 2D

geometry in which an initial magnetic field is aligned, say, along z , and all fluid components depend just on x , y , and time. The need to restrict to the $m_e/m_i = 0$ case, which in this geometry prevents the generation of in-plane magnetic fluctuations (cf. Eqs.(2,5)), is due to the fact that, for the moment, a linear analysis of the system equations (1-4) has been carried out and checked against Vlasov-Maxwell theory only for this case, in which a spurious branch has been evidenced for velocity fluctuations along the magnetic field component [1]. The condition $\boldsymbol{\omega}^i \times \mathbf{B} = 0$ allows to get rid of these problems. A more complete linear analysis for an arbitrary propagation angle is in course of development. For simplicity, we also close now the heat flux tensor by assuming $\nabla \cdot \mathbf{Q}^i = 0$. Even though the heat fluxes are expected to oppose to the pressure anisotropisation, their role is assumed to be secondary in this geometry in which there are no gradients along the magnetic field, and this ansatz is comforted by the fact that the results obtained under all the hypotheses above have proven (see Ref.[4]) to be in quite good agreement with kinetic and hybrid kinetic simulations [3]. We finally note that these hypotheses on the geometry are also consistent with a fluid description of the drift-kinetic electrostatic turbulence in tokamaks, for which a reduced gyrokinetic Vlasov-Poisson modelling is usually adopted (see, *e.g.*, Ref.[12]).

Let us drop again the apex $\alpha = i$. We assume the initial pressure tensor to be diagonal with respect to the cartesian axes. In this 2D geometry we can then focus only on the evolution of the in-plane components of $\mathbf{\Pi}$, which we indicate with the 2×2 matrix $\mathbf{\Pi}_\perp$, and on the evolution of the parallel component to \mathbf{B} , $\Pi_{zz} = P_\parallel$. By construction, also the gradient tensor is now a 2×2 matrix, as well as it is the strain rate tensor, which in this case is defined as $\mathbf{D}_\perp \equiv (1/2)(\nabla \mathbf{u}_\perp + (\nabla \mathbf{u}_\perp)^T - (\nabla \cdot \mathbf{u}_\perp)\mathbf{I}_\perp)$, the \perp index denoting here the components in the x, y plane for both matrices and vectors. By using a polar coordinate representation [2] we introduce the angles θ and ϕ which measure the instantaneous orientation of the principal axes of $\mathbf{\Pi}_\perp$ and of \mathbf{D}_\perp with respect to the cartesian axes x and y , defined so that $\Pi_{xy} = ((P_1 - P_2)/2) \sin 2\theta$ and $D_{xy} = D_\perp \sin 2\phi$. Here $\pm D_\perp$ are the eigenvalues of \mathbf{D}_\perp , and A^{ng} provides a measure of the local agyrotropy, that is of the non-gyrotropic pressure anisotropy, which is related to the eigenvalues P_1 and P_2 of $\mathbf{\Pi}_\perp$ (when the principal axes are chosen so that $P_3 = P_\parallel$) according to

$$A^{ng} \equiv \frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{(\text{tr}[\mathbf{\Pi}_\perp])^2 - 4 \det[\mathbf{\Pi}_\perp]}}{\text{tr}[\mathbf{\Pi}_\perp]}. \quad (16)$$

The amount of gyrotropic pressure anisotropy is then quantified as

$$A^{gyr} \equiv \frac{2P_3}{P_1 + P_2} = \frac{2P_\parallel}{\text{tr}[\mathbf{\Pi}_\perp]}. \quad (17)$$

It can be shown [2,4] that the two anisotropies evolve according to

$$\frac{dA^{ng}}{dt} = 2D_\perp [(A^{ng})^2 - 1] \cos[2(\theta - \phi)], \quad \frac{dA^{gyr}}{dt} = 2D_\perp A^{gyr} A^{ng} \cos[2(\theta - \phi)], \quad (18)$$

meaning that the rate of shear acts as a source of agyrotropy, which is generated with a maximum rate $\tau_{an}^{-1} \sim D_\perp$ when the principal axes of $\mathbf{\Pi}_\perp$ and \mathbf{D}_\perp are dephased by an angle $\pi/2$ (first equation), and that with a similar mechanism a gyrotropic anisotropy develops as soon as a non-zero agyrotropy is generated (second equation).

3.2 Non conservation of the average magnetic moment induced by planar shear flows

We can relate the results above to the evolution of the average magnetic moments defined by (15). To this purpose we can define an instantaneous ‘‘perpendicular pressure’’ as $P_\perp = (P_1 + P_2)/2 = \text{tr}[\mathbf{\Pi}_\perp]/2$, whose evolution is obtained by taking the trace of (3) after projection on the x, y plane. Using $\text{tr}[\mathbf{D}_\perp \mathbf{\Pi}_\perp] = 2A^{ng} D_\perp P_\perp \cos[2(\theta - \phi)]$ (see also Ref.[4]) we write :

$$\frac{dP_\perp}{dt} = -2A^{ng} D_\perp P_\perp \cos[2(\theta - \phi)] - 2(\nabla_\perp \cdot \mathbf{u}_\perp) P_\perp. \quad (19)$$

The equation for $\langle \mu_T \rangle$ in this geometry is obtained by substituting the (first) definition of Eq.(15) into (19). We can then eliminate dn/dt by using (1) and dB/dt with the equation obtained from the curl of Eq.(5) in the $m_e/m_i = 0$ limit. In this 2D configuration in which $\mathbf{B} = B\mathbf{e}_z$, the latter equation reads

$$\frac{dB}{dt} + B\nabla_{\perp} \cdot \mathbf{u}_{\perp} + \frac{c}{4\pi e} \left(\frac{B}{n^2} (\nabla_{\perp} \times \mathbf{B}) \cdot \nabla_{\perp} n - \frac{1}{n} (\nabla_{\perp} \times \mathbf{B}) \cdot \nabla_{\perp} B \right) = 0, \quad (20)$$

which corresponds to the 1-fluid rewriting of $dB/dt^e = -B\nabla_{\perp} \cdot \mathbf{u}_{\perp}^e$. We finally get

$$\frac{d\langle \mu_T \rangle}{dt} = \langle \mu_T \rangle \left\{ -2A^{ng} D_{\perp} \cos[2(\theta - \phi)] + \frac{c}{4\pi e} \left[\frac{1}{n^2} (\nabla_{\perp} \times \mathbf{B}) \cdot \nabla_{\perp} n - \frac{1}{nB} (\nabla_{\perp} \times \mathbf{B}) \cdot \nabla_{\perp} B \right] \right\}. \quad (21)$$

The contributions in brackets are due to inhomogeneities of the plasma density and of the magnetic field (they come from the Hall-term in Ohm's law, *i.e.*, from the $\nabla \times (\mathbf{J}/(ne) \times \mathbf{B})$ contribution of the second of Eqs.(5)), and are not related to the shear-induced anisotropization process. Instead, the generation of plasma agyrotropy stated by Eq.(18) determines a corresponding violation of the conservation of the averaged thermal magnetic moment, which is described by the first contribution in curl parantheses. Using this equation, analogous equations can be deduced for the other definitions of $\langle \mu_{...} \rangle$ given by (15) or by considering further possible definitions of the particle magnetic moment (see *e.g.*, Ref.[9]). This result is relevant to studies of 2D turbulence in which relatively large values of the shear rate, $|D_{\perp}| \sim |\omega_z|$, are encountered near vorticity sheets [4] generated by the nonlinear dynamics.

4 Conclusion

In reviewing some recent results about the mechanism which, in a fluid framework, generates pressure anisotropy because of the action of the rate of shear on the full pressure tensor components [2], we have discussed the role played by non-ideal effects on the evolution of the double-adiabatic invariants in a CGL-type closure. By introducing an average particle magnetic moment related to the perpendicular fluid adiabatic invariant, $P_{\perp}/(nB)$, we have provided an equation for its evolution, which applies to 2D slab geometries relevant to the interpretation of kinetic simulations of plasma turbulence [4]. In particular, this analysis is of potential interest to studies of electrostatic drift-kinetic turbulence in a 2D slab geometry perpendicular to a background magnetic field. This extended fluid description may provide an interesting complementary description to a Vlasov modelling by making it possible to keep trace of nonlinear non-gyrotropic effects, usually excluded from a standard gyrokinetic approach.

Références

1. D. Del Sarto, F. Pegoraro, A. Tenerani, Plasma Phys. Control. Fusion **59**, 045002 (2017).
2. D. Del Sarto, F. Pegoraro, F. Califano, Phys. Rev. E **93**, 053203 (2016).
3. S. Servidio, F. Valentini, F. Califano *et al.*, Phys. Rev. Lett. **108**, 045001 (2012); L. Franci, P. Hellinger, L. Matteini *et al.*, AIP Conf. Proc., **1720**, 040003 (2016); Y. Yang, W.H. Matthaeus, T.N. Parashar *et al.*, Phys. Rev. E **95**, 061201 (2017).
4. D. Del Sarto, F. Pegoraro, MNRAS **475**, 181 (2018).
5. G.F. Chew, M.L. Goldberger, F.E. Low, Los Alamos Lecture Notes on Physics of Ionized Gases, LA-2055 (1955).
6. P.-L. Sulem, T. Passot, J. Plasma Phys. **81**, 325810103 (2015).
7. S.S. Cerri, P. Henri, F. Califano *et al.*, Phys. Plasmas **20**, 112112 (2013).
8. R.M. Kulsrud, MHD description of plasmas, in Handbook of Plasma Physics, Eds. M.N. Rosenbluth, R.Z. Sagdeev, North-Holland Publ. Company, Vol. 1 (1983).
9. C.D. Stephens, R.W. Brzozowski, F. Jenko, Phys. Plasmas **24**, 102517 (2017).
10. A.N. Kaufman, Phys. Fluids **3**, 610 (1960).
11. M. Sarrat, D. Del Sarto, A. Ghizzo, EuroPhys. Lett. **115**, 45001 (2016); M. Sarrat, D. Del Sarto, A. Ghizzo, J. Plasma Phys. **83**, 705830103 (2017).
12. A.J. Brizard, T.S. Hahm, Rev. Mod. Phys. **79**, 421 (2007).