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Spontaneous magnetic reconnection of thin current sheets

Daniele Del Sarto¹, Maurizio Ottaviani², Fulvia Pucci³, Anna Tenerani⁴ & Marco Velli⁴

¹ Institut Jean Lamour, UMR 7198 CNRS - Université de Lorraine, Vandoeuvre-lés-Nancy, France

² CEA, IRFM, Saint-Paul-lez-Durance, France

³ NIFS, NINS, Toki 509-5292, Japan and PPPL, Princeton University, Princeton, NJ, USA

⁴ University of California Los Angeles, USA University of California Los Angeles, USA

`daniele.del-sarto@univ-lorraine.fr`

Résumé. Nous adressons le problème des taux de reconnexion magnétique spontanée sur couches de courants minces, en présentant une généralisation de la notion de renormalisation des échelles des taux de croissance pour les instabilités du type “tearing mode”. Nous révisons ainsi des résultats récents sur l’application de ces notions aux processus de reconnexion non linéaire [1,2,3] et de reconnexion des couches minces en évolution rapide [1,4,10].

Abstract. We address the problem of magnetic reconnection rates on large aspect ratio current sheets by presenting a generalisation of the notion of rescaling of the growth rates of tearing-type instabilities. In this, we review some recent results on the application of these notions to nonlinear reconnection processes [1,2,3] and to reconnection on rapidly evolving current sheets [1,4,10].

1 Introduction

Several plasma phenomena, ranging from solar eruptions to disruptive processes in tokamak, display an abrupt release of magnetic energy in the form of particle acceleration and heating induced by spontaneous magnetic reconnection events. The most known example of spontaneous reconnection is provided by tearing-type modes [5], which develop in a 2D geometry from harmonic oscillations along current sheets \mathbf{J} corresponding to sufficiently strong spatial gradients across the sheets of the magnetic field components of the \mathbf{B} components parallel to the sheet. These linear instabilities generate the characteristic “magnetic islands” structures (also named “magnetic vortices” or “plasmoids”) along a neutral line. These modes can be described in the framework of the Magnetohydrodynamic (MHD) theory, in which the fluid equation for the bulk plasma are coupled to the non-relativistic Maxwell’s equations by means of the so-called generalized Ohm’s law. This equation describes the electron response to the electromagnetic forces :

$$\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B} = \sum_i O(\varepsilon_i) \mathbf{f}_i(\nabla \mathbf{B}, \nabla \mathbf{u}, \nabla \mathbf{J}, \nabla \cdot \mathbf{\Pi}), \quad (1)$$

where coefficients of the order of infinitesimally small parameters “ ε_i ” weigh vector functions \mathbf{f}_i which depend on the gradients of the fluid components. Magnetic reconnection violates topological conservation related to the “ideal” form of Eq.(1), which combined with Faraday’s law implies the Lagrangian transport of distinct magnetic lines by the flow \mathbf{u} in an MHD plasma [6]. The violation occurs locally when the magnitude of the spatial gradients at r.h.s. of Eq.(1) compensates the smallness of the $O(\varepsilon_i)$ coefficients, which depend on effects neglected in the ideal MHD limit, such as plasma resistivity, a finite electron inertia (finite m_e/m_i corrections) or pressure anisotropy. Here, we provide examples for the first two effects only, which we respectively label with $i = \eta$ (resistivity) and $i = m_e$ (finite electron inertia) and which would enter at r.h.s. of Eq.(1) as “ $+\eta \mathbf{J}$ ” and “ $+m_e/(ne^2)(\partial_t \mathbf{J} + \mathbf{u} \cdot \nabla \mathbf{J})$ ” respectively, but our presentation can in principle be generalised to include the neglected effects.

2 Theory of linear reconnecting instabilities revised

The classical linear theory of the tearing mode instability [5], assumes a slab geometry configuration (all vector components in the plane x, y depend just on time and on the x and y coordinates) in which an

equilibrium magnetic field $\mathbf{B}_{eq} = (0, B_y^{eq}(x), B_z^{eq})$ with $B_z^{eq} \gg B_y^{eq}$ and with $B_y^{eq}(x) \sim x/a$ for $|x/a| \ll 1$, is sheared over a characteristic scale length “ a ”. This length scale therefore defines the characteristic width of the current profile, \mathbf{J}_{eq} , on which the linear stability of modes $\sim f(x)e^{i(ky-\omega t)}$ with $\omega \equiv \omega_R + \gamma$ and $k \equiv 2\pi m/L$, corresponding to m oscillations on a periodical interval of length L , is investigated by means of a boundary layer approach : at $|x/a| \gtrsim 1$ ideal MHD stability is assumed, whereas non-ideal terms are considered in the inner reconnecting layer (see Fig.1, left). Since a is here the only macroscopic spatial scale, it is the natural reference length to be assumed as a normalisation length $L_0 = a$. The Alfvén crossing time in the ideal region, defined as $\tau_A = a/c_a$ in terms of a reference in-plane Alfvén velocity $c_A \equiv B_{eq}/\sqrt{4\pi n_0 m}$, where $B_{eq} \equiv B_y^{eq}(x_0)$ is evaluated in some point x_0 of the “ideal” region where also n_0 is measured, provides the natural normalisation time of the system, $\tau_0 = \tau_A$.

Regardless of the non-ideal parameter ε_i at play, different regimes can be characterised in terms of the well-known instability parameter $\Delta'(ka)$ expressing the logarithmic derivative of the eigenmode across the reconnecting layer [5]. The instability condition $\Delta'(ka) > 0$ fixes the range of unstable wavenumbers for each equilibrium profile and, for a fixed finite L , the maximum number of magnetic islands that can be observed along the neutral line. Three main regimes can be thus identified in terms of the characteristic width of the reconnecting layer, $\delta(\varepsilon_i, ka)$, which identifies the thickness of the current sheet generated by the reconnecting mode during the linear stage (cf. Figs.1) :

i) the constant- ψ regime of the properly named “tearing mode” [5], for $\Delta'(ka)\delta(\varepsilon_i, ka) < 1$, that gives a dispersion relation of the kind

$$\gamma_{TM}\tau_A \sim \varepsilon_i^{a_1}(ka)^{a_2}(\Delta'a)^{a_3} \quad (2)$$

for some rational a_1, a_2, a_3 with $1 > a_1 > 0$;

ii) the large- Δ' regime, characterising the so-called “internal kink mode” for $\Delta'(ka)\delta(\varepsilon_i, ka) \gg 1$ [7], that gives a dispersion relation of the kind

$$\gamma_{IK}\tau_A \sim \varepsilon_i^{b_1}(ka)^{b_2} \quad (3)$$

for some rational b_1, b_2 with $1 > b_1 > 0$;

iii) the fastest growing mode ([5], Appendix D) in a continuum wavenumber spectrum, which corresponds to the condition $\Delta'(ka)\delta(\varepsilon_i, ka) \simeq 1$ and therefore gives

$$\gamma_M\tau_A \sim \varepsilon_i^{c_1} \quad (4)$$

for a $k_M\tau_A \sim \varepsilon_i^{d_1}$ with $1 > c_1 > 0$ and $d_1 > 0$. The explicit dependence of γ_M on k_M is always eliminated by deducing a dependence from k_M on ε_i thanks to the condition $\gamma_M \equiv \gamma_{TM} = \gamma_{IK}$, while c_1 depends on the particular form of the equilibrium profile and can be deduced by estimating the power dependence of Δ' on ka for $ka \ll 1$ [2]. It is important to underline that in all these cases $\gamma(\varepsilon_i)\tau_A \rightarrow 0$ as $\varepsilon_i \rightarrow 0$ by construction and consistently with the notion of non-ideal instability.

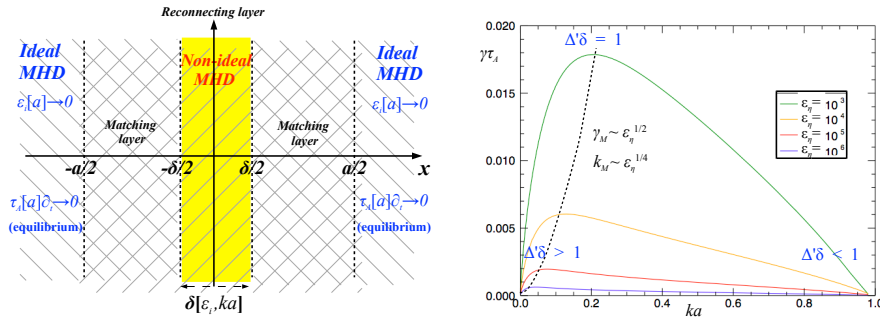


Figure 1. Sketch of the boundary layer approach (left frame) and example of tearing-type mode dispersion function for the resistive case, with the three asymptotic regimes (i-iii) highlighted (right frame).

Even if these modes were first identified in the MHD resistive regime, where $\varepsilon_\eta = \tau_A \eta c / (4\pi a^2) \equiv S^{-1}$ is the resistive Lundquist number, they were later generalized to other semi-collisional and collisionless regimes and frequency ranges (see references in [3]). Here we also consider the MHD inertia-driven regime where $\varepsilon_m = c^2 m_e / (4\pi n e^2 a^2) \equiv (d_e/a)^2$ and d_e is the electron skin-depth.

Due to the ideal MHD assumption at the scales a and τ_A , any microscopic space-depending and time-depending quantity which enters in the small non-ideal parameters ε_i is necessarily normalised so that each of the ε_i scale as some *positive* power of $(1/a)$ and/or as some *positive* power of $\tau_A = (a/c_A) \sim (a/B_0)$,

$$\varepsilon_i[a, B_0] \sim \left(\frac{1}{a}\right)^q \left(\frac{a}{B_0}\right)^r, \quad q, r \geq 0. \quad (5)$$

This introduces a non trivial, implicit dependence on the normalization scales of the growth rates γ_{τ_A} : indeed, in the classical tearing mode theory first devised for reconnecting modes in tokamaks, the unstable wave number is fixed by the integer poloidal number m associated with the resonant surface of radius r_r on which the mode is destabilised, $L_0 = a$ is of the order of the tokamak major radius, and the equilibrium current layer is by construction periodic and of length $L = 2\pi r_r \sim L_0$. Instead, in most astrophysical environments, such as coronal loops or planetary magnetotails, as well as in the nonlinear evolution of primary reconnecting modes or in MHD turbulence, large aspect ratio current sheets with $L/a \gg 1$ are encountered. In these cases, while a may be small relative to $L_0 \sim L$ it remains large enough to allow the application of a boundary layer theory ($a \gg \delta(\varepsilon_i)$).

Two cases of rescaling can be then considered for reconnection on large aspect ratio current sheets. The first one rescales only lengths, while the reference magnetic field amplitude B_0 is fixed as the reference magnetic field B_{cs} on the current sheet, *i.e.*, $B_{cs} = B_y^{eq}(x_0)$ of the classical tearing mode analysis. In this case we neglect the explicit dependence on B_0 and, from Eq.(5),

$$\tau_A[a] = \tau_A[L_0] \left(\frac{a}{L_0}\right), \quad \varepsilon_i[a] = \varepsilon_i[L_0] \left(\frac{L_0}{a}\right)^{q-r}, \quad q, r \geq 0. \quad (6)$$

The second case, which generalises the previous one, is when both the reference length scale L_0 and the magnetic field amplitude B_0 differ from a and B_{cs} . In this case, using again Eq.(5):

$$\tau_A[a, B_{cs}] = \tau_A[L_0, B_0] \left(\frac{a}{L_0}\right) \left(\frac{B_0}{B_{cs}}\right), \quad \varepsilon_i[a, B_{cs}] = \varepsilon_i[L_0, B_0] \left(\frac{L_0}{a}\right)^{q-r} \left(\frac{B_0}{B_{cs}}\right)^r, \quad q, r \geq 0. \quad (7)$$

2.1 Application to thin current sheets

Considering for example the simplest rescaling of Eq.(6), it generally follows that for some $s > 0$ which depends on q , r and on the specific scaling of $\gamma[a]$ with $\varepsilon_i[a]$,

$$\gamma[L_0] \tau_A[L_0] \sim (\gamma[a] \tau_A[a])|_{a \rightarrow L_0} (L_0/a)^s. \quad (8)$$

Here $(\gamma[a] \tau_A[a])|_{a \rightarrow L_0}$ stands for the scaling of the mode obtained in the standard tearing theory (any of the three of Eqs.(2-4)) in terms of ε_i , k and Δ' , which this time are normalised to L_0 . For the fastest growing mode, for example, using Eq.(4) we find $s = c_1(q-r) + 1$ and Eq.(8) becomes

$$\gamma_M[L_0] \tau_A[L_0] \sim (\varepsilon_i[L_0])^{c_1} (L_0/a)^{c_1(q-r)+1}. \quad (9)$$

The growth rate of the fastest growing mode (4,9) is representative of the overall reconnection rate on \mathbf{J}_{eq} when a sufficiently large aspect ratio current sheet is considered, so that an almost continuum spectrum of wavenumbers can be destabilised. In particular, it was shown [8] that an aspect ratio $L/a \gtrsim 20$ is sufficient to destabilize a mode whose growth rate is well approximated by (4). In this case the amplitude of $(L_0/a)^{c_1(q-r)+1}$ may so compensate the smallness of $\varepsilon_i[L_0]$. The consequence of this has been discussed in [4] by noticing that, when $L_0 = L$, a maximum growth rate exists for spontaneous reconnecting

modes, which can not trespass the order of magnitude $\gamma[L]\tau_A[L] \sim O(1)$ because of the ‘‘causality prescription’’ imposed by fact that in MHD energy is mediated by Alfvén waves. This leads to the notion of ‘‘*ideal tearing*’’ regime first introduced in resistive MHD in [4] and then generalized in [2], that is, of a $\Delta'\delta \sim 1$ tearing mode which develops with an Alfvénic growth rate $\gamma[L_0] \sim \tau_A[L_0]^{-1}$, independent of the macroscopic parameter ε_i allowing reconnection, when the aspect ratio becomes comparable to the threshold value

$$\left(\frac{L}{a}\right)_{B_{cs} \sim B_0} \sim (\varepsilon_i[L])^{-\alpha}, \quad \alpha = \frac{c_1}{c_1(q-r) + 1}. \quad (10)$$

For example, in the purely resistive regime where $\varepsilon_\eta[a] = S_a^{-1}$, so that $q = 2$, $r = 1$, and for an Harris-pinch equilibrium $B_{cs}(x) = B_0 \tanh(x/a)$ for which $c_1 = 1/2$, we recover the threshold scaling $\alpha = 1/3$ of Ref.[4]. Slightly different values in the range $1/4 < \alpha < 1/2$ [2,9] can be recovered for different dependences of \mathbf{J}_{eq} on the x coordinate, which imply, as shown in [2], different values of c_1 . As a second example, an analogous threshold $\alpha = 1/3$ [2] is recovered in the inertia-driven regime, where $\varepsilon_d[a] \equiv (d_e/a)^2$ so that $q = 2$, $r = 0$, and for a $B_{cs}(x) = B_0 \tanh(x/a)$, for which this time $c_1 = 1$.

It is worth stressing the physical relevance of the *ideal tearing* solution : despite all the above estimates are obtained in the framework of an asymptotic theory, for which $\varepsilon_i[a], \varepsilon_i[L_0] \rightarrow 0$, the geometric threshold condition stated by (10) has proven by numerical analysis to be well satisfied by small yet finite values of $\varepsilon_i[L_0]$ (say $\varepsilon_i[L_0] \lesssim 10^{-7}$), which are relevant to a variety of reconnection processes in nature and laboratory [1,2,4,10].

The more complex scenario in which rescaling (7) must be performed because of $B_0 \neq B_{cs}$ requires further ansatz. A case we will consider here is that of ‘‘embedded current sheets’’ [11], in which the hypothesis that the amplitude of the equilibrium current sheet, $|\mathbf{J}_{eq}| = J_{eq} \sim B_{cs}/a$, be comparable to some reference current density amplitude $J_0 \sim B_0/L_0$, allows to express the scaling of B_{cs} with respect to B_0 as a geometrical scaling with a/L_0 : from $J_0 \sim J_{eq}$ one obtains

$$\frac{B_{cs}}{B_0} \sim \left(\frac{a}{L_0}\right). \quad (11)$$

When applied to tearing-type modes [3] on large aspect ratio current sheets, Eq.(9) becomes

$$\gamma_M[L_0]\tau_A[L_0] \sim (\varepsilon_i[L_0])^{c_1} (L_0/a)^{c_1 q}. \quad (12)$$

Repeating the argument of Ref.[4] for $L_0 = L$, one finds in this case a threshold aspect ratio for the onset of a reconnection independent on ε_i , given by :

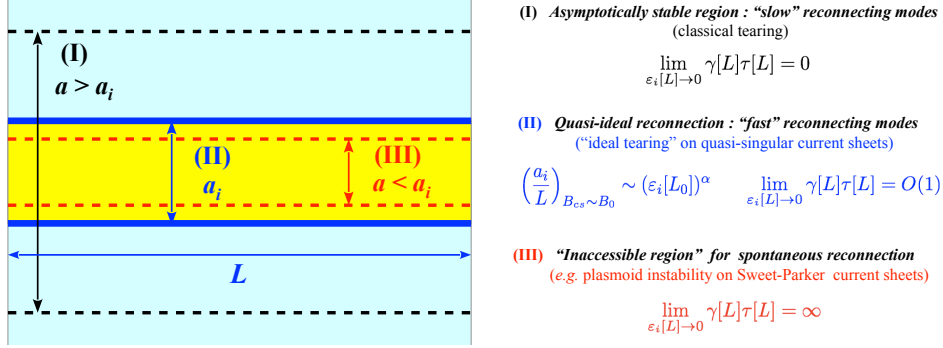
$$\left(\frac{L}{a}\right)_{J_{cs} \sim J_0} \sim (\varepsilon_i[L])^{-\alpha}, \quad \alpha = \frac{1}{q}. \quad (13)$$

Interestingly, this threshold condition *only depends on the spatial dependence of the microscopic parameter ε_i but not on its time-dependence and not on the growth rate scaling (4) of the fastest growing mode, nor, therefore, on the magnetic equilibrium profile*. In both the resistive [3] and inertia-driven regimes one then finds $\alpha = 1/2$.

3 Discussion : reconnecting instabilities on evolving current sheets

When one wants to apply the estimate of the previous section for a tearing-type growth rate $\tilde{\gamma}$ to current sheets which are evolving on a time scale τ_{cs} , one should formally stick to the condition $\tau_{cs}^{-1} \ll \tilde{\gamma}$, so to consider the evolving current sheet as a relative equilibrium profile on which to perform a linear analysis. However, even when one considers current sheets evolving on a time scale $\tau_{cs} \sim \tau_A[L_0]$, if $L_0 = L$, the threshold aspect ratio scalings of (10),(13) provide a *necessary condition* for the disruption of the current sheet by spontaneous reconnecting modes. It is indeed by recognising the geometrical threshold condition of Eqs.(10,13) that, for $B_0 = B_{cs}$, a distinction was first made [4] between modes classified as slow (*i.e.*,

ideally stable), fast (*i.e.*, (quasi-)ideally unstable), and violently unstable (*i.e.*, with diverging growth rates while approaching the ideal limit) : this made it possible to explain the paradox of the diverging growth rate obtained for the *plasmoid instability* [12] as $\varepsilon_\eta[L] \rightarrow 0$, as due to the un-realizability of the Sweet-Parker steady configuration, since its aspect ratio $L/a \sim (\varepsilon_\eta[L])^{-1/2}$ obtained for $\tau_A[L_0]/\tau_{cs} \simeq 0$ is much larger than the threshold value of (10), that is, larger than current sheets which would be disrupted at ideal time scales. The argument generalizes to collisionless steady reconnection regimes [2] (see Fig.2). The



interesting question then arises about current sheets for which $\tau_A[L_0]/\tau_{cs} \neq 0$. This requires knowledge of the dynamics of the current sheet. Here we distinguish two major cases : current sheets generated by primary reconnecting modes growing on current profiles $J_0 \sim B_0/L_0$ with rates $\gamma_0[L_0]$ and for which we can assume $\tau_{cs}^{-1} \sim \gamma_0[L_0]$, and current sheets generated by convective motions at Alfvénic scales, such as, *e.g.*, in turbulence. In the first case we can speak of secondary reconnecting instabilities, and they have been considered candidates responsible for a nonlinear reconnection rate increase relative to primary modes since the 1990s (see [3], App. B for a review). Secondary instabilities may be also of fluid type [13], which may nonlinearly enhance the reconnection rate via secondary turbulent reconnection processes, though here we focus on secondary tearing-type modes only.

Consider first modes developing on a current sheet born of the collapse of an X -point into two Y -points [14], shown to occur during the nonlinear evolution of large- Δ' “slow” modes ($\gamma_0[L_0, B_0]\tau_A[L_0, B_0] \ll 1$), both resistive and collisionless [15]. We consider in particular a primary purely resistive mode (simulations in the inertia-driven regime evidence the dominant role played by flows and the onset of secondary fluid instabilities rather than tearing modes [13]) for which Eq.(3) specialises to $\gamma_0[L_0, B_0]\tau_A[L_0, B_0] \sim (\varepsilon_\eta[L_0])^{1/3}(kL_0)^{2/3}$. This case has been studied in Ref.[3] by noting that the current sheet attains at the end of the linear stage of the primary mode a length $L \sim L_0$, a thickness a which can be (over-)estimated using the reconnecting layer width $\delta_0(\varepsilon_\eta[L_0, B_0])$ of the primary mode, and an amplitude $J_{cs} \sim J_0$, so that condition (11) holds. Heuristically assuming that this condition still holds when a sufficiently rapid secondary tearing mode disrupts \mathbf{J}_{cs} , by using (12) one finds that the fastest growing mode satisfies this condition, since it is destabilised in a few e -folding times of the primary linear instability, and since its growth rate $\tilde{\gamma}[L_0, B_0]\tau_A[L_0, B_0] \sim (\varepsilon_\eta[L_0, B_0])^{1/6}$ asymptotically satisfies $\tilde{\gamma}_M[L_0, B_0] \gg \gamma_0[L_0, B_0] \sim \tau_{cs}^{-1}$. Considering then that a transition from a resistive to a collisionless regime occurs when $\tilde{\gamma}[L_0, B_0]\tau_A[L_0, B_0] \gtrsim \nu_c[L_0, B_0]\tau_A[L_0, B_0]$ with ν_c being the electron-ion collision frequency, using $\nu_c[L_0, B_0]\tau_A[L_0, B_0] \sim \varepsilon_\eta[L_0, B_0]/\varepsilon_{de}[L_0, B_0]$, which corresponds to the non-negligibility of the $m_e/(ne^2)\partial_t \mathbf{J}$ term with respect to the $\eta \mathbf{J}$ term at r.h.s. of Ohm’s law, one finds that collisionless physics dominates the secondary mode when

$$\tilde{\gamma}[L_0, B_0]\tau_A[L_0, B_0] \gtrsim \frac{\varepsilon_\eta[L_0, B_0]}{\varepsilon_m[L_0, B_0]} = \frac{S_{L_0}^{-1}}{(d_e/L_0)^2}. \quad (14)$$

For a primary resistive large- Δ' mode in the reduced MHD framework in which Hall effects in Ohm's law are neglected, this transition occurs at $\varepsilon_\eta[L_0, B_0] \sim (\varepsilon_m[L_0, B_0])^{12/5}$, whereas secondary Alfvénic reconnection rates, yet not –by properly speaking– in the *ideal tearing* regime, can be attained when $\varepsilon_\eta[L_0, B_0] \sim (\varepsilon_m[L_0, B_0])^3$ (see Ref.[3]).

In order to study the nonlinear behaviour of reconnection starting from a quasi-equilibrium state, it was chosen in [1] to simulate current sheet collapse via a series of equilibria evolving on a time-scale such that $\tau_A[L]/\tau_{cs}[L] \gtrsim 1$, in order to reduce the length of the simulations while correctly capturing the onset of fast tearing. The authors of Ref.[1] showed that reconnecting instabilities satisfying the *ideal tearing* threshold condition develop on quasi-singular current sheets (for $B_{cs} = B_0$ and $L_0 = L$, Eq.(11) implies $J_{cs}/J_0 \sim L/a \gg 1$). It was thus demonstrated that a fractal-like cascade of *ideal tearing* modes develops from the recurrent X -point collapse of recurrent thinning sheets developing from the initial *ideal tearing*. As a result, the current sheet completely disrupted immediately after reaching the *ideal* threshold and could not thin much further.

We conclude by noting that later attempts to follow the linear growth of modes from an arbitrarily perturbed evolving equilibrium in a semi-analytic fashion [16] lead to a logarithmic correction in critical aspect ratios; computed by assuming a contemporary thinning of the equilibrium and growth of perturbations up until island widths and current sheet singular layer thicknesses are comparable, the results seem questionable for a number of reasons : first, the final result explicitly depends on the initial perturbation amplitude, which is forced to vanish in the limit of large Lundquist numbers by an inequality used to derive the result itself; second, the evolution was calculated as though any initial perturbation could be expanded in terms of purely growing modes, *i.e.* neglecting any transients, though growing transients have been shown to enhance the instability properties of thin sheets [17]. In summary, our opinion is that such attempts at a so-called “general” theory of the plasmoid instability are neither general nor correct. This point, as well as inclusion of further non-ideal effects in Ohm's law and a more accurate account of the role of flows deserve further dedicated studies and will be addressed in forthcoming articles.

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