



HAL
open science

Percolation approach to simulation of a sustainable network economy structure

Mira A. Kantemirova, Zaur L. Dzakojev, Zara R. Alikova, Sergei R. Chedgemov, Zarina V. Soskiewa

► **To cite this version:**

Mira A. Kantemirova, Zaur L. Dzakojev, Zara R. Alikova, Sergei R. Chedgemov, Zarina V. Soskiewa. Percolation approach to simulation of a sustainable network economy structure. *Entrepreneurship and Sustainability Issues*, 2018, 5 (3), pp.502 - 513. 10.9770/jesi.2018.5.3(7) . hal-01773587

HAL Id: hal-01773587

<https://hal.science/hal-01773587>

Submitted on 22 Apr 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Publisher

PERCOLATION APPROACH TO SIMULATION OF A SUSTAINABLE NETWORK ECONOMY STRUCTURE

Mira A. Kantemirova ^{1,2}, Zaur L. Dzakojev ³, Zara R. Alikova ², Sergei R. Chedgemov ^{2,3},
Zarina V. Soskiewa ^{2,4}

¹ Gorsky State Agrarian University,
37 Kirova Str., Vladikavkaz, 362040, Russian Federation

² North Ossetian State Medical Academy
40 Pushkinskaya Str., Vladikavkaz, 362007, Russian Federation

³ North Ossetian State University,
46 Vatutina Str., Vladikavkaz, 362025, Russian Federation

⁴ Gorsky State Agrarian University,
37 Kirova Str., Vladikavkaz, 362040, Russian Federation

E-mails: kantemirova.mira@mail.ru; dzl200@rambler.ru; alikova_zr@mail.ru;
srchedgemov@mail.ru; turandot2004@mail.ru

Received 16 September 2017; accepted 10 January 2018; published 30 March 2018

Abstract. This study is aimed at the application of the percolation theory to simulation of a sustainable network organization of the economy in conditions of high uncertainty of the external environment. The methods for investment and cost recovery efficiency calculation in order to achieve synergy are used in the course of networks formation. The methods of graph theory and one-dimensional percolation are used herein. The conceptual content of the modified percolation approach to the analysis and simulation of network structures is specified. The controlled process of network formation offers the possibility to form the percolation cluster on the basis of minimization of its length (the shortest path). The formation regularities of two types of a percolation cluster (internal and cross-border) as the basis for the creation of the appropriate network structures are revealed. The examples of the applied problems, which study the percolation based on lattice cells (lattice coupling problem), are considered herein. The results of empirical approbation of the proposed approach in the field of services with the description of the algorithm for the networks and a cluster formation are presented. The transition from the random Bernol's percolation (based on random selection of cells) in favor of the correlated percolation is justified.

Keywords: percolation, percolation theory, cluster, networks.

Reference to this paper should be made as follows: Kantemirova, M.A.; Dzakojev, Z.A.; Alikova, Z.R.; Chedgemov, S.R.; Soskiewa, Z.V. 2018. Percolation approach to simulation of a sustainable network economy structure, *Entrepreneurship and Sustainability Issues* 5(3): 502-513. [https://doi.org/10.9770/jesi.2018.5.3\(7\)](https://doi.org/10.9770/jesi.2018.5.3(7))

JEL Classifications: L14, D85

1. Introduction

The current stage of economy development is characterized by the ever-increasing application of network structures, reformatting the economic space and significantly influencing all aspects of its life activity (Badzho and Sheresheva, 2014). The formation of a networked economy means a phase transition of a system state, the parameters of which change abruptly. Such a transition requires the availability of appropriate analysis and simulation tools.

Networks become not only one of the most common forms of interaction between the economic entities, a means of communication in all spheres of society, but also the main force of business, creating a new system of economic relations. The networked economy has developed as an effective tool for market superiority achievement, based on the application of integration effects, synergy, the combination of freedom of choice and rational regulation. The phenomenon of the worldwide networks development allowed V. Wellman to assert a while back that “the world consists of networks” (Wellman, 1983). The networked organization of the economy has its distinctive characteristics compared to the traditional forms reflected in a number of studies (Dronov, 2012; Castells, 2000; Erznkian and Agafonov, 2011), demanding certain conditions to achieve the desired result. A distinctive feature of networks is the adaptability of their parameters adjustment to the specific requirements and capabilities of the participants, taking into account the socio-economic conditions of the territory of operation.

In the networked economy, the fundamentally new forms of organization are established, based on horizontal-partner interactions, replacing the traditional hierarchical entities and changing the principles of the market game. The processes of globalization and the all-embracing application of networked structures strongly require a scientific justification. That is why this study is aimed at the application of the percolation theory to creation of the models, offering the possibility to analyze and forecast the regularities of the dynamics of the networked organization of the economy.

2. Literature review

Percolation is a phenomenon that reveals the nature of critical transitions in the processes. Percolation models are widely used in various sciences (mathematical, physical, chemical, material sciences, etc.) for research and analysis of the objects and processes that have related domains (Wang, et al., 2017; Broadbent and Hammersley J.M., 2008), boundaries of interactions (Piraveenan. et al., 2017), resulting in the changes in the forms of organizations (Lütz, et al., 2017), configurations (Li and Zhang, 2014), composition of elements (Bianconi and Radicchi, 2016), textures (Li, et al., 2015), fractal formation (Hassan and Rahman, 2016), etc.

The percolation theory proposed by Broadbent and Hammersley (Broadbent and Hammersley, 1957), aimed at studying the properties of macroscopically disordered physical media, their phase transitions, has been applied in various sciences and in solving the applied problems. The most common problems of the percolation theory are associated with lattices, vertices, nodes, links, clusters, which in many respects is similar to the components of the networks (graphs, nodes, edges, links, a cluster).

In the conventional sense, the percolation is considered in the form of an abrupt change in the properties of the matter or processes in the case of exceeding a certain threshold of impact (Tarasevich, 2002). The use of percolation requires the use of mathematics and computer simulation (Menshikov, et al., 1986; Nazarov, 2011).

To solve the applied problems in the oil and coal industry, the fluid (oil, water) flow models in the formation and a phase transition in solid materials are used (Yapparova and Mayakova, 2010; Shirochin, et al., 2005). The

percolation is used in physics together with the Ising and Potts models, for example, in the study of crystal clusters (Ising, 1925; Potts, 1952; Dmitriev, et al., 2004).

The percolation theory is used in the simulation of dynamic information networks, for example, on the basis of structures of connected components (Prokoshev, et al., 2013), in approaches to image quality improvement (Davies, et al., 2011), analysis of fractals (Feder, 1991), etc. The hypothesis of a change in the dimension of the percolation cluster as it approached the lattice boundaries was proposed by R. Cafiero, G. Caldarelli, and A. Gabrielli (Cafiero, et al., 1997; Gabrielli, et al., 1998).

The percolation theory not only broadens the understanding of the phenomena in material objects, but also allows a more accurate simulation of economic and managerial processes. In particular, it offers the opportunity to withdraw from the predominantly sectoral analysis of the network formation regularities to the consideration of essential processes in their dynamics. One can argue that the processes of network formation in the economy take place in an artificial controlled environment. However, actually, the modern economic environment is characterized by a high degree of disorder, being under the influence of various factors of competition, risks, uncertainty, behavior of economic entities, etc. Therefore, the processes of network structures formation, especially at the initial stage of it, take place in conditions of a disordered environment.

At the same time, the analysis of networks and the study of patterns of their distribution in the economy are still based on methods used primarily for hierarchical structures, which is not always methodologically correct and expedient. In this regard, the foundations of the percolation approach to simulation the sustainable development of the networked economy are further suggested.

3. Methods

The formation of a networked economy can be viewed as a process of gradual spread (or percolation) of the networks among the economic entities (producers and consumers) to a certain level of saturation. The process of diffusion of networks violates the property of invariance of economic space organization, based on hierarchical structures.

The percolation model, reflecting the replacement of the existing hierarchical forms of organization with new network structures, the transformation of captured areas, characterizes the process of percolation, making it possible to monitor the movement of the interface (front) of the substituted and substituting organizations. The formation space of the network, which is visually represented as a grid for the analysis purposes, is understood as the industry, sphere, territory, area of economic or social activity, in which the elements and the prerequisites for the creation of networks (economic, social, innovative, etc.) are being formed.

It is assumed that the network existence medium is permeable (“porous”), and the investigated network formation space has the form of a lattice with the number of cells N (sides $L \times L$). Some part of the grid is capable of creating network structures.

A cell is a conditional interpretation of a subject (node, actor, physical or legal person), carrying out a social or economic activity. The size of a cell can reflect the fraction of the occupied space (or the scale of activity) by the subject, so their number in the lattice can vary significantly. The cells are conditionally separated from each other by borders, but they are open from the point of view of activities that require cooperation, links, resource flows, etc. The cells, interconnected by the close interaction, form the aggregations (associations) in the form of clusters.

Clustering is a local characteristic of both the percolation process and the network. It reflects the degree of partnership of this cell with its closest cells. The clustering coefficient of this cell shows the probability that the two nearest cells are in interaction with it. If the cell j has q_j neighboring cells with the number of t_j connections between them, then the value of the local clustering coefficient is defined as:

$$C_j(q_j) = \frac{h_j}{q_j(q_j - 1)/2}, \quad (1)$$

where h_j is the total number of triangles attached to cell j ; $q_j(q_j - 1)/2$ is the maximum possible number of possible triangles.

In the case of complete interconnection of this cell j with all adjacent cells, $C_j = 1$. In the absence of such interconnections, $C_j = 0$.

There are conditions for transferring the elements of the network structure to a new location (with the probability of percolation $P_\infty(p)$), as well as the presence of at least one path of their transfer and location. The probability of percolation $P_\infty(p)$ means the probability that the transfer of a good or resource (economic, social, etc.) having started the transfer in one, randomly marked, element of the network, will continue to move across all other elements related to the network. This probability is equal to the ratio $P_\infty(p)/p$.

The front for displacement of the non-network structures by the network forms is very unsustainable, bearing an element of randomness. If for the traditional process of percolation in physical media, the element of randomness is natural and, therefore, indisputable, then the socio-economic phenomena require the possibilities of external regulation to reduce its influence. At the same time, the level of randomness of the elements of the percolation process under the influence of the regulatory impact can be reduced, which will increase the likelihood of achieving the desired result – the creation of a specific socio-economic network and the achievement of the goals of its operation.

The probability of percolation of the networked forms without regulation can be reflected as:

$$P_\infty(p) = \lim_{N \rightarrow \infty} P_N(p) \quad (2)$$

This limit value is understood as the percolation threshold. In the processes of networked structures development, one of the regulatory objectives is to reduce the level of the percolation threshold $P_\infty(p) \rightarrow \min$. The influence of the regulation on the process of network formation results in the emergence of a directional percolation, which increases the likelihood of the formation of the network and the required indicators of its activity.

Graphically, the lattice can be represented in the form of black cells with conductivity – the conditions and resources for the network formation and white cells – with no conductivity, which is caused by the absence or insufficiency of sustainable prerequisites for the network elements creation (capabilities, conditions, connections, resources). The form of the lattice at the conditional instant of time t_0 , reflecting the beginning of the process of percolation through the cells (network formation), can be reflected in the following form (Fig. 1).

Direction of network formation

	↓				↓	↓		↓	
0	0	2	0	0	0	1	1	0	1
0	1	2	0	0	1	1	0	0	0
0	0	2	0	1	0	0	1	0	0
0	0	2	0	1	1	0	0	0	1
0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0

Legend: 0 – no percolation; 1 – availability of prerequisites for network formation; 2 – the elements of a sustainable network formation (the future percolation cluster)

Fig. 1. Scheme of network formation lattice in the time period t_0

According to the figure, the networked structures are being created in 4 directions, but none of the networks extends over the entire investigated space (lattice), and none of the possible clusters is also formed. As the percolation process develops, the lattice cells are filled with the elements of network structures, acquiring the following form at time $t + n$ (Fig. 2).

Direction of network formation

	↓				↓	↓		↓	
0	0	2	0	0	0	1	1	0	1
0	1	2	0	0	1	1	0	0	0
0	0	2	0	1	0	0	1	0	0
0	0	2	0	1	1	0	0	0	1
0	1	2	0	0	0	0	1	0	0
1	0	2	2	2	1	0		1	1
0	0	1	0	2	0	0	1	1	
1	1	0	0	2	0	0	0	1	1
0	0	0	0	2	2	2	0	0	0
1	1	0	1	1	0	2	2	1	1
0	1	0	0	0	0	0	2	0	0

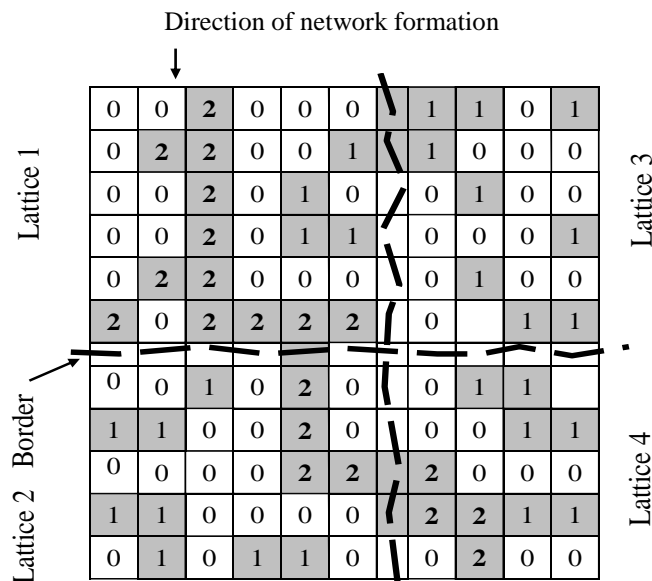
Legend: 0 – no percolation; 1 – availability of prerequisites for network formation; 2 – the elements of a sustainable network formation (the future percolation cluster)

Fig. 2. Scheme of network formation lattice in the time period $t+n$

Of the 4 network development directions, the first one is the most developed, which resulted in the creation of a through-going path for percolation of the network elements from the entrance to the exit of the lattice. The problem of creation and promotion of the network elements is to find the concentration of the black cells

determining the path from the starting position through the entire lattice to the final cell, forming a percolation cluster that actually is the network.

For each given lattice cell, there is a probability P_{∞} to belong to this cluster (or network). If all lattice cells belong to the cluster, it means that all subjects are included in the network. The process of networks formation often covers a number of neighboring lattices with the borders between them. In this case, the process of percolation can be of an inter-boundary nature, facilitating the formation of an inter-boundary percolation cluster spanning several lattices (Fig. 3).



Legend: 0 – no percolation; 1 – network (internal cluster); 2 – network (cross-border percolation cluster)

Fig. 3. The cross-border nature of the percolation process

The formation of the cross-border percolation cluster means the creation of an appropriate network, which is interregional, intersectoral in nature.

In the network, due to the emergence of an integrated potential and synergy, the interconnected subjects orient their goals of interactions to achieve a common result. Theoretically, each network (cluster) is able to create a cumulative result proportional to the number of its participants. However, within the framework of the percolation process, the actors forming the different networks do not interact with each other.

In the course of the study, it was revealed that the process of network elements transfer in space (by the lattice cells) can be terminated if the number of their formation is less than a certain threshold value. This means the absence of the percolation cluster, ensuring the percolation of the appropriate resources through it for the network structure formation.

The exact value of the percolation threshold for the network structures propagation problem has not yet been determined and requires a large number of experiments and calculations. Therefore, the threshold value equal to 0.59275, determined by the Watson and Lis experiment, is used in the simulation. The modified display of simulation results of percolation probability is shown in Fig. 4.

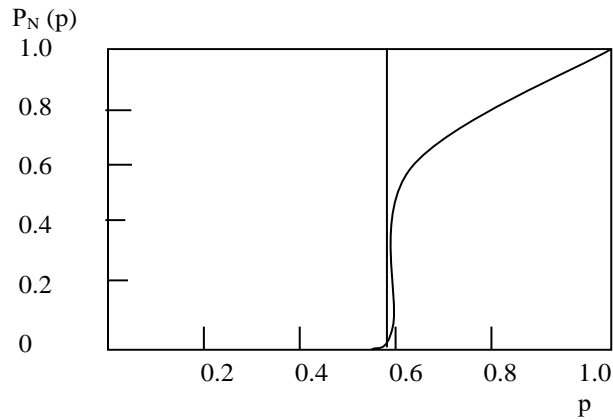


Fig. 4. Modified display of the results of percolation probability simulation

To analyze the parameters of the process of percolation of the network dynamics, a number of additional indicators should be used. The proportion of the cells coverage (subjects, participants) by the network interaction (D_s) is defined as:

$$D_s = \frac{ns}{N},$$

where ns is the number of cells participating in the network interaction; N is the total number of cells in the given lattice (the subjects in the space under study).

If $D_s \rightarrow 1$, the probability of percolation P_N increases linearly, which contributes to the emergence of the largest cluster. The size of the cluster is determined by the number of cells in it. The speed of the propagation of the percolation process (network formation) is defined as the number of cells covered by the network interaction per unit time:

$$s = \frac{ns}{t},$$

where ns is the number of cells-new participants in the network interaction; t is the time period.

4. Results and discussion

The approbation of the network structure formation on the basis of percolation presupposed the replacement of the existing hierarchical forms of the organization with the integrated firms capable of performing competitive and efficient activity in an unfavorable environment.

The space for the future network formation (for example, the service sector) for the purposes of planned actions was conditionally considered as the percolation lattice with the N number of cells in which the separate elements and conditions for a network structure creation occurred. The analysis showed that some parts (cells) of the lattice can create network structures. Based on them, in the service sector of the region the priority zones for small business development were identified, in which 7 private firms operated, including two firms (AB), closely cooperating with each other and acting as the nucleus for a percolation cluster creation (Fig. 5, a, provided below).

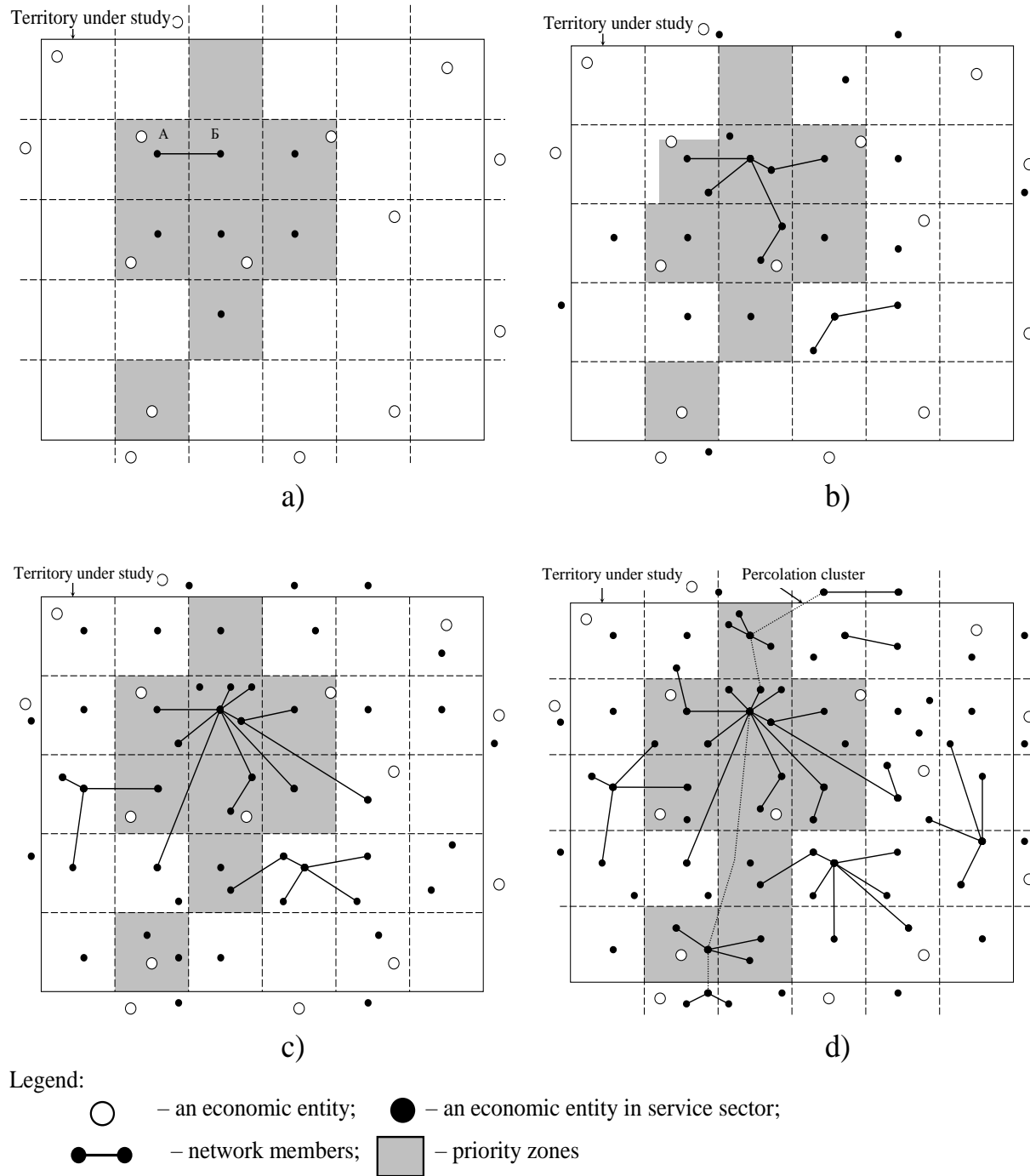


Fig. 5. Discrete visualization of the stages of network formation process

A well-founded choice of priority zones means the rejection of Bernoulli's random percolation (based on a random selection of cells) in favor of the correlated percolation.

The networks formation occurred using the methods of creation (new firms), mergers and acquisitions of the operating companies. It was assumed that the percolation cluster will consist of one or more networks of firms, approaching the opposite boundaries of the lattice, which means the onset of the percolation effect and the possibility of transfer of the network formation activity to other (neighboring) lattices.

At the stage of choosing the size of the lattice, which actually shows a certain part of the service sector, the problems of the optimal number of economic subjects, the number of firms in the network, as well as the number of networks in the cluster (not considered herein). The general approach to the solution of such problems was adopted as follows: the number of networks (and firms) should be such as the number of consumers served by them is as large as possible:

$$S = \frac{F \times Z}{\sum Z}, \quad (3)$$

where S is the number of networks; F is the number of firms; Z is the service zone of one firm (or the number of customers).

Obviously, in order to cover the demand for the services within the grid, it is required to reject the random location of the business entities across its cells.

By approbation, in 2 years the lattice acquired the form (Fig. 5, b) showing the activity of the network formation process in its various cells. Moreover, some firms were created outside of this lattice as a platform for possible further development. Over the next 2 years, the contours of three network structures emerged that could potentially form the basis of a percolation cluster (Fig. 5, c). However, two relatively small networked entities did not join the network and preferred to operate outside the priority zones.

The formation of small networks leads to the formation of small-sized appropriate clusters. In proportion to addition of more objects and increase in their concentration, they also join other clusters. A merger and consolidation of clusters take place with a decrease in their total number. Moreover, the distance between the clusters also decreases, which potentially creates the prerequisites for the development of the corresponding percolation cluster.

In this regard, it took 2 more years to clarify the priority zones and create new networks that allowed the formation of a percolation cluster and connection of the opposite boundaries of the lattice (Fig. 5, d). This cluster belongs to the number of large ones and consists of a number of small clusters and networks.

The controlled network formation process allows forming the percolation cluster on the basis of minimization of its length (the shortest path). This makes it possible to reduce the time and the costs for networks creation, increasing the coverage of the customer service areas.

The methods for investment and cost recovery efficiency calculation in order to achieve synergy are used in the course of networks formation. In general, the synergistic effect achieved by the use of the networks is defined as follows:

$$SE_{a-n} = PV_{a-n} - (PV_a + PV_b + \dots + PV_n) > 0, \quad (4)$$

where SE_{a-n} is the network containing the firms from a to n ; PV is the value of the current cost of the relevant firm.

The expenditures for the creation and operation of the network were determined by the alternative options based on the known formula for reduced costs (C_r):

$$C_r = CP + ECI, \quad (5)$$

where CP is the cost price; CI is the capital investments; E is the efficiency coefficient.

For the purposes of network creation, the option was selected, under which $C_r \rightarrow \min$ (given other conditions being equal).

Thus, the main stages of the process of network formation based on the percolation approach are applicable under the following conditions:

- (1) the boundaries of the development of the network-based economic activity are defined in the form of a lattice;
- (2) the existence of a support system for a comparatively small number of firms distributed randomly in the investigated field;
- (3) the determination of the required concentration of firms to overlap (or percolate) the surveyed customer service area;
- (4) the identification of priority zones that are promising for the creation of networks and a percolation cluster;
- (5) the introduction of a certain number of additional firms into the inter-firm and intercluster intervals of the reference system to create the networks and a continuous percolation path with a minimum length with given directions;
- (6) the attraction of additional firms in the network can be ensured by using the creation procedures, mergers and acquisitions taking into account the necessary performance and competitiveness indicators;
- (7) the stages of networking should be manageable.

Conclusion

The networked economy is a way to achieve the necessary market superiority, based on the use of integration effects, synergy, the combination of freedom of choice and the rational regulation. The formation of the networked economy is considered in this study as a process of gradual distribution of the networks among the economic entities (producers and consumers) to a certain level of saturation.

The results of the approbation of the network structure formation on the basis of the percolation theory, which involves the replacement of the existing hierarchical forms of the organization with the integrated firms, show that: some parts (cells) of the lattice are capable of creating the network structures; and the process of formation of the percolation cluster (internal and cross-border) as the basis for the creation of the appropriate network structures has a certain regularity.

Thus, the process of network formation requires the controlled percolation taking into account the conditions and the factors of the external environment dynamics. It is determined that the network formation is a percolation of the network properties, forms and resources in the socio-economic environment. This article proposes a modified approach to the study of the regularities of the network structures development in the economy based on the percolation theory.

References

- Badzho, R.; Sheresheva, M. Yu. 2014. Network Approach in Economics and Management: Interdisciplinary Nature, *Vestnik moskovskogo universiteta. Seriya 6. Ekonomika* 2: 3–21.
- Bianconi, G.; Radicchi, F. 2016. Percolation in real multiplex networks, *Journal reference: Phys. Rev. E* 94, 060301. <https://doi.org/10.1103/PhysRevE.94.060301>
- Broadbent, S. R.; Hammersley, J. M. 1957. Percolation processes. *Proceedings of the Cambridge Philosophical Society* 53(3): 629–633. <http://dx.doi.org/10.1017/S0305004100032680>
- Broadbent, S. R.; Hammersley, J. M. 2008. Percolation processes, *Mathematical Proceedings of the Cambridge Philosophical Society* 53 (03): 629. <https://doi.org/10.1017/S0305004100032680>
- Cafiero, R.; Caldarelli, G.; Gabrielli, A. 1997. Surface effects in Invasion Percolation, *Physical Review E* 56(2): R1291-R1294. <http://dx.doi.org/10.1103/PhysRevE.56.R1291>
- Castells, M. 2000. *Information Age: Economy, Society and Culture* (Trans. from English). Moscow.
- Davies, L.; Wittich, O.; Langovoy, M. 2011. *Detection of Objects in Noisy Images Based on Percolation Theory*. Cornell University, 41–42.
- Dmitriev, A. A.; Katrakhov, V. V.; Kharchenko, Yu. N. 2004. *Root Transfer Matrices in Ising Models*. Moscow: Nauka.
- Dronov, N. L. 2012. *Network Interaction of the Enterprises*. Moscow.
- Efros, A. L. 1982. *Physics and Geometry of Disorder*. Moscow: Nauka.
- Erznkian, B. A.; Agafonov, V. A. 2011. The Network Nature of the Cluster System and the Main Directions for the Formation of the Cluster Strategy of Mesoeconomic Development in Russia, *Ekonomicheskaya nauka sovremennoi Rossii* 1: 39–48.
- Feder, E. 1991. *Fractals* (Trans. from English). Moscow: MIR.
- Gabrielli, A.; Cafiero, R.; Caldarelli, G. 1998. Theory of Boundary Effects in Invasion Percolation, *Journal of Physics A: Mathematical and General* 31(37): 11–14. <http://dx.doi.org/10.1088/0305-4470/31/37/006>
- Hassan, M. K.; Rahman, M. M. 2016. Universality class of site and bond percolation on multifractal scale-free planar stochastic lattice, *Phys. Rev. E* 94, 042109. <https://doi.org/10.1103/PhysRevE.94.042109>
- Ising, E. 1925. Beitrag zur Theorie des Ferromagnetismus, *Zeitschrift für Physik* 31: 253–258. <http://dx.doi.org/10.1007/BF02980577>
- Li, C.; Zhang, Y. 2014. Percolation on the institute-enterprise R&D collaboration networks, *Open Physics* 13(1). <https://doi.org/10.1515/phys-2015-0008>
- Li, D.; Fu, B.; Wang, Y.; Lu, G.; Berezin, Y.; Stanley, H.E.; Havlin S. 2015. Percolation transition in dynamical traffic network with evolving critical bottlenecks, *PNAS* 112 (3): 669–672. <https://doi.org/10.1073/pnas.1419185112>
- Lütz, A. F.; Cazaubiel, A.; Arenzon, J. J. 2017. Cyclic Competition and Percolation in Grouping Predator-Prey Populations, *Games* 8(1), 10. <https://doi.org/10.3390/g8010010>
- Menshikov, M. V.; Molchanov, S. A.; Sidorenko, A. F. 1986. Percolation Theory and Some Applications, *Itogi nauki. Seriya "Teoriya veroyatnostei. Matematicheskaya statistika. Teoreticheskaya kibernetika"* 24: 53–110.
- Nazarov, A. V. 2011. Computer Simulation of Percolation Processes in Homogeneous Structures, *Trudy MAI* 49: 67.
- Piraveenan, M.; Prokopenko, M.; Hossain, L. 2017. Percolation Centrality: Quantifying Graph-Theoretic Impact of Nodes during Percolation in Networks, *PLoS ONE* 8(1): e53095. <https://doi.org/10.1371/journal.pone.0053095>

Potts, R. B. 1952. Some Generalized Order-Disorder Transformations, *Mathematical Proceedings of the Cambridge Philosophical Society* 48(1): 106–109. <http://dx.doi.org/10.1017/S0305004100027419>

Prokoshev, V. V.; Sklyarenko, V. A.; Shamin, P. Yu. 2013. Experience of Using the Percolation Type Models for Analyzing the Signal Transmission Process in Large Ensembles of Moving Objects, *Vestnik NGU. Seriya: Informatsionnye tekhnologii* 11(2): 73–81.

Shirochin, D. L.; Podynogina, A. V.; Minaev, V. I.; Belyi, A. A.; Astakhov, A. V. 2005. Application of the Fractal Approach and the Theory of Percolation in Studies of Fossil Coals, *Gornyi informatsionno-analiticheskiy byulleten* 3: 5–9.

Tarasevich, Yu. Yu. 2002. *Percolation: Theory, Applications, Algorithms*. Moscow: Editorial URSS.

Wang, J.; Wang, J.; Qi, S.; Sun, Y.; Tian, G.; Wu D. 2017. Investigation of percolation theory and permittivity model with one-dimensional fillers. *EPL* 1(117). <https://doi.org/10.1209/0295-5075/117/17001>

Wellman, B. 1983. Network Analysis: Some Basic Principles, *Sociological Theory* 1: 155–200. <http://dx.doi.org/10.2307/202050>

Yapparova, A. A.; Mayakova S. A. 2010. Parallelization of Algorithms for Numerical Simulation of Percolation Processes with Displacement, *Vestnik UGATU* 4(39): 160-165.

Mira A. KANTEMIROVA, Doctor of Economics Sciences, Professor at the Economic Theory and Applied Economics Department, Gorsky State Agrarian University; Professor at the Department of Social, Humanitarian and Economic Sciences, North Ossetian State Medical Academy.

ORCID ID: orcid.org/0000-0003-3704-144X

Zaur L. DZAKOEV, Candidate of Economic Sciences, Associate Professor at the Management Department, North Ossetian State University.

ORCID ID: orcid.org/0000-0002-7794-745X

Zara R. ALIKOVA, MD, Head of the Social, Humanitarian and Economic Sciences Department, North Ossetian State Medical Academy.

ORCID ID: orcid.org/0000-0002-8186-2424

Sergei R. CHEDGEMOV, Doctor of Pedagogical Sciences, Professor at the Theory and History of State and Law Department, North Ossetian State University; Professor at the Social, Humanitarian and Economic Sciences Department, North Ossetian State Medical Academy.

ORCID ID: orcid.org/0000-0003-2260-3016

Zarina V. SOSKIEVA, Candidate of Economic Sciences, Associate Professor at the Economic Theory and Applied Economics Department, Gorsky State Agrarian University; Senior Lecturer at the Social, Humanitarian and Economic Sciences Department, North Ossetian State Medical Academy.

ORCID ID: orcid.org/0000-0002-2590-4327

Register for an ORCID ID:

<https://orcid.org/register>