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## - To cite this version:

El-Awady Attia, Philippe Duquenne, Jean-Marc Le Lann. Considering skills evolutions in multi-skilled workforce allocation with flexible working hours. International Journal of Production Research, 2014, vol. $52\left(\mathrm{n}^{\circ} 15\right)$, pp. 4548-4573. 10.1080/00207543.2013.877613 . hal-01772667

HAL Id: hal-01772667
https://hal.science/hal-01772667
Submitted on 24 Apr 2018

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To link to this article: DOI: 10.1080/00207543.2013.877613
URL : http://dx.doi.org/10.1080/00207543.2013.877613

To cite this version : Attia, El-Awady and Duquenne, Philippe and Le Lann, Jean-Marc Considering skills evolutions in multi-skilled workforce allocation with flexible working hours. (2014) International Journal of Production Research, vol. 52 ( $\mathrm{n}^{\circ} 15$ ). pp. 4548-4573. ISSN 0020-7543

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# Considering skills evolutions in multi-skilled workforce allocation with flexible working hours 

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#### Abstract

The growing need of responsiveness for manufacturing companies facing market volatility raises a strong demand for flexibility in their organisation. Since the company personnel are increasingly considered as the core of the organisational structures, a strong and forward-looking management of human resources and skills is crucial to performance in many industries. These organisations must develop strategies for the short, medium and long terms, in order to preserve and develop skills. Responding to this importance, this work presents an original model, looking at the line-up of multi-period project, considering the problem of staff allocation with two degrees of flexibility. The first results from the annualising of working time, and relies on policies of changing schedules, individually as well as collectively. The second degree of flexibility is the versatility of the operators, which induces a dynamic view of their skills and the need to predict changes in individual performance as a result of successive assignments. We are firmly in a context where the expected durations of activities are no longer predefined, but result from the performance of the operators selected for their execution. We present a mathematical model of this problem, which is solved by a genetic algorithm. An illustrative example is presented and analysed, and, the robustness of the solving approach is investigated using a sample of 400 projects with different characteristics.


Keywords: project scheduling; workforce allocation; multi-skilled; dynamic skills; annualised hours; genetic algorithms

## 1. Introduction

Since companies are looking for reactivity as a challenging argument towards their competitors, human resources have become a reservoir for business flexibility - and this reservoir is all the more important that some of these resources are polyvalent. This is especially true for industrial fields where human labour is important with low automation rates: industrial structures in these domains should consider long-term strategies in order to enhance multi-skills of their workforce, as, for instance, cross-training programmes (K.K. Yang 2007; Davis, Kher, and Wagner 2009). Furthermore, the ability of an operator in a given skill is an evolving parameter that results from his experience. If workers are consistently assigned to various jobs where each of them will always practice the same skill, it can be considered as an optimal short-term strategy in terms of economic workforce allocation: they will increase their productivities through practice. As a consequence, this policy will not help to extend polyvalence in the company - indeed, it will result in a decrease of effectiveness for all operators in the skills they will not apply. Recent works in project management (Bellenguez-Morineau and Néron 2007; Edi 2007; Drezet and Billaut 2008; Li and Womer 2009; Valls, Perez, and Quintanilla 2009), in production and operation management (Franchini et al. 2001; K.K. Yang 2007; Davis, Kher, and Wagner 2009) led authors to strongly recommend versatility in companies. The price to pay for the development of this versatility may be considered as an investment for the firms' internal structure in order to gain flexibility and responsiveness (Hitt, Keats, and DeMarie 1998; Edi 2007). Another motivation to promote the versatility in companies is the risk of reducing or losing some of their core competencies. While keeping always the same operators allocated to the same jobs, these companies become dependent on a small number of individuals, and may be forced to outsource missions within their core competencies.

The importance of the workforce skill's factor motivates researchers to consider the employees' performance. Relying on the work of Yannibelli and Amandi (2012), there are two ways of considering employees' performance: homogeneous or heterogeneous productivity. One should differentiate between the worker-based homogeneous productivity and the skill-based homogeneous productivity. In the first, each worker can master a set of different skills with equal productivity rates, whatever the skills. In skill-based homogeneous productivity, all workers show the same performance rates in practising a given skill. The homogeneous vision can be found in the works such those of

[^0]Bellenguez-Morineau and Néron (2007), Drezet and Billaut (2008), Li and Womer (2009) or Kazemipoor et al. (2013). The case where the two workforce productivities, the 'worker-based' and the 'skill-based', one is homogeneous and the other is heterogeneous, we called it 'mixed performance'. One can find this case in the hierarchical classification of workforce productivities, e.g. senior, standard and junior classification that was adopted by Yoshimura et al. (2006), Valls, Perez, and Quintanilla (2009) and Shahnazari-Shahrezaei, Tavakkoli-Moghaddam, and Kazemipoor (2013). The workforce productivities were considered as heterogeneous in the case where neither the worker-based productivity, nor the skill-based productivity is homogeneous. According to the best of our knowledge, the first description of this heterogeneous characteristic is the work of Nelson (1970) in a dual resource constrained job shop. This heterogeneous nature was then adopted in many applications, e.g. the works of Attia, Edi, and Duquenne (2012), Kolisch and Heimerl (2012) or Yannibelli and Amandi (2012). Here also, the heterogeneous nature will be considered in addition to the evolutions of productivity as a function of previous workforce allocation (i.e. the dynamic aspect of productivity).

Additionally, the workforce flexibility also relies on a temporal concern that plays a vital role in this subject. This flexibility can be developed along two main axes: the traditional overtime working hours and the working time modulation under annualised hours (AH). AH strategy is the possibility to spread irregularly a number of working hours that is predefined on a specified period (often one year), provided some constraints are respected. According to Corominas, Lusa, and Pastor (2002), it provides a great flexibility with a reasonable cost, but on the other side it can lead to a worsening of the staff's working conditions. To minimise these negative consequences, this strategy of working time has to be negotiated and may be accompanied by some kind of reward or incentive. Simultaneously, legal constraints or collective agreements must be respected to avoid excessively overburdening workers during long, high-demand periods. Responding to the importance of such flexibility, numerous works were conducted; Hung (1999) proposed different scenarios of weekly manpower planning under AH. Grabot and Letouzey (2000) presented a software prototype to check the feasibility between the required workload and workforce capacities for discrete manufacturing short-term planning and scheduling. Corominas, Lusa, and Pastor (2002) presented a mixed-integer linear programme to the planning of workforce all over the year. Azmat, Hürlimann, and Widmer (2004) adopted the problem of single-shift workforce scheduling under AH with Swiss legal constraints. Corominas and Pastor (2010) proposed a reactive re-planning approach of the short-term workforce allocation.

Due to this importance, this paper presents two main contributions. The first one presents a mathematical modelling of the staff allocation problem on industrial activities. The proposed model will be developed following three axes; the first is the policy of working time modulation under AH. The second axis is the flexibility resulting from the heterogeneous versatility: each operator may master one skill or more in addition to his core competence, with acceptable cost and quality standard. The last axis is a dynamic vision of the workers' efficiencies (productivities), which refer to the relationship between efficiency and experience.

As a result of flexible work schedules and of the variety of the operators' efficiencies, the durations of the activities are not predetermined, and will depend on the performance of those responsible for executing them. This paper expresses the intention to consider any kind of industrial activities, regardless they belong to a project (a set of unique and original tasks) or a production programme (a manufacturing route of repetitive and well-defined jobs); in the same spirit, we will often refer to human resources as actors: an actor may be an operator in a factory as well as an appointed expert in a design office.

The other contribution is the platform software that we propose to solve the problem. Starting from the MSPSP (multi-skill project scheduling problem), which is so far a difficult problem in operations research (Bellenguez-Morineau and Néron 2007), considering the actors' efficiencies greatly increases the complexity of a non-linear model either with homogeneous (Li and Womer 2009) or heterogonous actors' productivity rates (Edi 2007; Valls, Perez, and Quintanilla 2009; Attia, Edi, and Duquenne 2012). Moreover, taking into account their dynamic productivities and working time flexibility, we referred to a method based on metaheuristics (genetic algorithms) to solve this problem.

Our contributions and the model we present are likely to be of concern to a wide range of stakeholders. For example, as far as we know, no existing commercial software in the field of project management and resources planning considers altogether these three axes. Some applications can be raised in workforce allocation in aircraft manufacturing for instance. Such production environments, where each task requires a set of highly skilled and qualified workers, depend mainly on the workforce, for sub-assemblies, final assembly of the whole aircraft, testing, repairing if any and painting. According to Démery-Lebrun (2005), the workforce flexibility is highly appreciated in such industrial sectors. It is especially true since the concept of learning at workstation was originally presented in aircraft manufacturing by Wright (1936). Furthermore, with relaxing of some constraints, the model can be adopted in planning and scheduling of manual assembly lines of vehicles, electric transformers, etc. It can be useful too in planning of maintenance works where each task or mission requires a specified teamwork with a specified skill for each actor. Due to the diversity of the likely applications of this work, we used 'programme of activities' to name these applications.

Besides its classical use as a scheduling tool, the model may also be used as a simulator enabling to test the robustness of a schedule: it may point out the difficulty or the easiness of finding alternative solutions when the chosen baseline schedule faces unexpected events.

Moreover, provided a company has long-term visions of its future activities, this model can be used as a decision-making tool for skills management. It offers the opportunity to review different policies of skills development and to point out for each of them the economic efforts it requires vs. its ability to meet future demands.

The following of this paper is organised as follows: next section presents the previous works related to the current problem. Section 3 discusses the main characteristics of the problem and Section 4 presents the mathematical model. Section 5 describes the approach proposed to solve the problem. An illustration example is presented in Section 6. Section 7 presents the investigation on the performance of the resolution approach we adopted. Section 8 concludes the paper and introduces the perspectives of this work.

## 2. State of the art

The problems of staff allocation were classified by Hlaoittinun (2009) into four categories according to two criteria, the period of assignment (PoA) and the modelling of skills (MoS). The PoA was split in two: single-period (we call it allocation on fixed work schedule (FWS)) and multi-period (we call it allocation on variable work schedule (VWS)). The FWS deals with allocation of staff to a predefined work schedule (shift, day and week). In this type, the ordering constraints between tasks have no impact on actors' allocation since they are already taken into account. In contrast, allocation on VWS is defined according to both manpower allocation constraints as well as activities ordering constraints - which take into account the temporal relationships between tasks. The second criterion, the MoS is split too into two families: the static models (SM) for which the actors' skills remain unchanged over time, whereas the dynamic models (DM) consider the efficiency increase with experience (learning effect: LE), and/or the efficiency depreciation due to interruption periods (forgetting effect: FE).

We added two additional criteria: the first refers to the working time policy, which can be 'classical' (Cls) or 'flexible' (Flx). The classic policy refers to fixed working time for each day or week (possibly taking into account overtime if necessary). The flexible strategy allocates to each actor a fixed annual work, with some constraints on the distribution of this annual amount on shorter periods (days and weeks).

The second criterion is the dependency between the duration of tasks and the efficiency of the actors who will be appointed to perform it; durations and skills may be independent (Rigid Task: RT, the tasks durations are fixed) or dependent (Elastic Task: ET): in this case, a given job will require more or less working time, depending on the qualification level of the allocated resources.

Table 1 displays how previous works can be classified according to the four criteria introduced above: as far as we know, the present work is a first contribution considering simultaneously flexible working time, actors' versatility, influence of resources on job execution and evolution of workers' efficiencies.

## 3. The characteristics of the model

### 3.1 Polyvalent workforce

Each actor masters a core competence, and may also practice an additional set of skills, each of which being characterised by a quantification of the actor's performance. The performance of an actor $a$ in the practice of a competence $k$ will then be expressed by a variable called efficiency $\theta_{a, k}$ (Duquenne, Edi, and Le Lann 2005). If ( $\omega_{a, i, k}$ ) is the total work needed for the actor $a$ to complete the task $i$ in the competence $k$, and $\left(\Omega_{i, k}\right)$ is the standard workload that characterises this task, then this efficiency will be the ratio $\left(\theta_{a, k}=\Omega_{i, k} / \omega_{a, i, k}\right)$ - which should be in the range $[0,1]$. If the actor is considered as an expert in that competence, he will perform the task within a processing time corresponding to the estimated 'standard', and we will consider his efficiency as nominal $\left(\theta_{a, k}=1\right)$. The time difference between the standard and actual processing times ( $\Delta_{a, i, k}=\omega_{a, i, k}-\Omega_{i, k}$ ) expresses the penalty induced by the use of polyvalence of workforce.

### 3.2 The flexibility of working time

According to the modulation schedules adopted by the company, each individual has his own calendar, which may vary on daily or weekly basis. These variations must comply with some limitations, such as minimum or maximum values over given periods. Moreover, a pre-specified amount of working hours per year should be respected (Corominas and Pastor 2010; Attia, Edi, and Duquenne 2012). This flexibility allows the company to get a variable working capacity from a constant staff (Grabot and Letouzey 2000).

Table 1. Classification of related works.

|  | PoA |  | MoS |  |  | Working time |  | Task duration |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | FWS | VWS | SM | DM |  | Cls | Flx | RT | ET |
|  |  |  |  | LE | FE |  |  |  |  |
| Current work |  | $\times$ |  | $\times$ | $\times$ |  | $\times$ |  | $\times$ |
| Attia, Edi, and Duquenne (2012) |  | $\times^{\text {G }}$ | $\times$ |  |  |  | $\times$ |  | $\times$ |
|  | $\times$ |  | - | - | - |  |  | - | - |
| Drezet and Billaut (2008) |  | $\times^{T}$ | $\times$ |  |  |  | $\times^{\text {D }}$ | $\times$ |  |
| Eitzen, Panton, and Mills (2004) | $\times$ |  | $\times$ |  |  | $\times^{*}$ |  | - | - |
| Gutjahr et al. (2008) |  | $\times^{T}$ |  | $\times$ | $\times$ | $\times^{* *}$ |  |  | ${ }^{\times}$ |
| Heimerl and Kolisch (2010) | $\times{ }^{\text {ST }}$ |  |  | $\times$ | $\times$ | $\times$ |  |  | $\times^{\mathrm{Ob}}$ |
| Hertz, Lahrichi, and Widmer (2010) | $\times$ |  | - | - | - |  | $\times^{\text {SH }}$ | - | - |
| Hlaoittinun (2009) | $\times$ | $x^{\text {T-Pre }}$ |  | $\times$ | $\times$ | $\times{ }^{* *}$ |  |  | $\times$ |
| Li and Womer (2009) |  | $\times{ }^{\text {G }}$ | $\times$ |  |  | $\times^{* *}$ |  | $\times$ | - |
| Bellenguez-Morineau and Néron (2007) |  | $\times^{\text {T }}$ | $\times$ |  |  | $\times^{*}$ |  | $\times$ |  |
| Nembhard (2001) | $\times^{\mathrm{EV}}$ |  |  | $\times$ |  | - | - | - | $\times$ |
| Perron (2010) | $\times$ |  | $\times$ |  |  | $\times$ |  | $\times$ |  |
| Sayin and Karabati (2007) | $\times$ |  |  | $\times$ | $\times$ | - | - | - | - |
| Valls, Perez, and Quintanilla (2009) |  | $\times{ }^{\text {G }}$ | $\times$ |  |  | - | - |  | $\times$ |
| Vidal (2000) |  | $\times^{\text {T }}$ | - | - | - | $\stackrel{{ }_{*}^{* *}}{ }$ |  |  | $\times^{\text {C }}$ |
| Wu and Sun (2006) | $\times$ |  |  | $\times$ |  | $\times^{* *}$ |  |  | $\times$ |

$x^{*}$ : Resources with binary availabilities: 0 or 1 .
$\times^{* *}$ : Maximum capacity of work per each scheduling period without considering overtime.
$x^{\mathrm{C}}$ : Concerns the cycle time of a workstation centre.
$\times^{\mathrm{D}}$ : the flexibility of working with maximum and minimum working periods per day, without overtime.
$x^{\mathrm{EV}}$ : evaluation of a heuristic policy for assigning workers to activities.
$\times^{\mathrm{G}}$ : multi period project scheduling with generalized temporal relations.
$x^{\mathrm{Ob}}$ : the time required to realise an aggregated work-package as an objective.
$x^{\mathrm{ST}}$ : strategic level to obtain a given company skill level target.
$x^{\text {SH }}$ : flexibility of working time without overtime.
$\times^{\mathrm{T}}$ : multi period project scheduling with traditional finish-start constraints.
$\times^{\mathrm{T}-\mathrm{Pre}}$ : with the consideration of: "the project breakdown and its tasks schedule over the project periods are data entry".

### 3.3 Evolution of efficiencies

### 3.3.1 Learning phenomenon

The time needed to complete a task is reduced by a given percentage each time the task is reproduced: this phenomenon was first described by Wright (1936) in the domain of aircraft manufacturing. The most common representation of experience curves is the log-linear curve of Wright, and it can be used to estimate the progress function relying either on the unit production or on the bulk quantity produced. Moreover, it can provide a good 'fits' of empirical data for many products and processes (Smunt 1999). Relying on Wright's formula, the evolution of the additional cost (working time) with the number of work repetitions ( $n$ ) can be expressed as:

$$
\begin{equation*}
\Delta_{a, i, k}(n)=\Delta_{a, i, k}^{(1)} \times(n)^{b} \tag{1}
\end{equation*}
$$

In this equation, the extra cost $\Delta_{a, i, k}(n)=\omega_{a, i, k}(n)-\Omega_{i, k}$ is represented for an actor whose efficiency is $\theta_{a, k}(n)$, and who is allocated for his competence $k$ on a task $i . \Delta_{a, i, k}^{(1)}=\Omega_{i, k} / \theta_{a, k}^{i n i}-\Omega_{i, k}$ is the extra cost found at the first time this actor is required for this skill: he will perform it with his initial (i.e. minimal) efficiency. The parameter $b$ can be expressed as: $b=\log \left(r_{a, k}\right) / \log (2)$, where $\left(r_{a, k}\right)$ expresses the learning rate of the actor $a$ in the competence $k$. This rate may vary from one actor to another and from a competence to another. As a result, the evolution of the processing time for a given operator evolves with ( $n$ ) according to the equation:

$$
\begin{equation*}
\omega_{a, i, k}(n)=\Omega_{i, k}+\left(\Omega_{i, k} / \theta_{a, k}^{i n i}-\Omega_{i, k}\right) \times(n)^{b} \tag{2}
\end{equation*}
$$

This formula is similar to the model of DeJong (1957), which involves an incompressible runtime ( $\Omega_{i, k}$ in our case) corresponding to an optimal execution of work. We can then derive the evolution of the efficiency:

$$
\begin{equation*}
\theta_{a, k}(n)=\frac{1}{1+\left(1 / \theta_{a, k}^{i n i}-1\right) \times(n)^{b}} \tag{3}
\end{equation*}
$$

Three factors are essential to estimate the work of an actor to carry out a given workload: his learning speed $\left(r_{a, k}\right)$, his initial efficiency $\left(\theta_{a, k}^{i n i}\right)$ : we assume here that these two factors, deduced from the record of past activities, are part of the data-set of the simulation. The third factor is related to the interpretation of the repetitions of work ( $n$ ). Here, 'repetition' does not refer to the repeated execution of a given task, but to the constant practice of the same skill on jobs that may differ: ( $n$ ) reflects the duration of uninterrupted practice of the relevant skill during previous assignments, and it is expressed in standard working days lasting 7 h each. However, the application to repetitive tasks is of course conceivable. In all cases, the efficiency is assumed to remain constant for one given allocation, from the beginning to the end of a same task, and equal to the efficiency calculated at the beginning of the task.

### 3.3.2 Unlearning phenomenon

The efficiencies of actors increase by practising, due to the 'learning-by-doing' effect. Contrary, this efficiency is degraded when the actors have to stop working, or have to work on other skills, as shown by Figure 1. We now come to consider a model of weakening of competences, a phenomenon caused by oblivion. The literature provides results especially when interruptions of production influence the effect of forgetting (Globerson, Levin, and Shtub 1989; Jaber and Bonney 1996). In our view, an interruption occurs when an actor is assigned to work on another skill - or is not assigned at all. We adopted the model presented by Jaber and Bonney (1996) and Hlaoittinun (2009), inspired by Wright's curve rather than the 'hyperbolic model with two variables' of Mazur and Hastie (1978) that was modified into the 'Recency model' used by Nembhard (2001); according to this exponentially decreasing representation, we can model a depreciation of efficiency as:

$$
\begin{equation*}
\Delta_{a, k}^{f}=\Delta_{a, k}^{f(1)} \times\left(\lambda_{a, k}\right)^{f} \tag{4}
\end{equation*}
$$

where $\Delta_{a, k}^{f}$ represents the extra costs that will be produced if the actor $a$ is assigned to work with his skill $k$ after an interruption period ( $d I P$ ) expressed in equivalent virtual number $\left(\lambda_{a, k}\right)$ of standard working days lasting 7 h each. $\Delta_{a, k}^{f(1)}$ is a virtual extra cost that can be found at the first execution relying only on the forgetting curve. In the same way as Jaber and Bonney (1996), we can express the evolution of the actor's efficiency, depending on $\left(\lambda_{a, k}\right)$ and on the number of previous equivalent work repetitions $\left(n_{e q}\right)$ :


Figure 1. The effect of learning and forgetting on the working efficiency of operators.

$$
\begin{equation*}
\theta f_{a, k}^{d I P}=\frac{1}{1+\left(1 / \theta_{a, k}^{i n i}-1\right) \times\left(n_{e q}\right)^{b-f} \times\left(n_{e q}+\lambda_{a, k}\right)^{f}} \tag{5}
\end{equation*}
$$

where $\theta f_{a, k}^{d I P}$ is the actor's efficiency after a period of interruption $d I P$ and $f$ is the slope of the 'unlearning curve'. The curve parameters ( $n_{e q}$, and $f$ ) will be determined for each individual during simulated periods and for the specified skill; $n_{e q}$ can be computed after equalling the right sides of Equations (3) and (5), and take ( $n=n_{e q}$ ). By assuming that there is a maximum interruption period (Tb) after which, the efficiency will have decreased back to its initial value, the slope of the forgetting curve $f$ can be calculated as:

$$
\begin{equation*}
f=-b \times(b+1) \times \log \left(n_{e q}\right) / \log (\xi+1) \tag{6}
\end{equation*}
$$

$\xi=T b / T a$ is the ratio between the two periods: a learning period $T a$ of uninterrupted practice of a given competence and the interruption period ( Tb ) described above. The parameters involved in Equations (1)-(6) are difficult to appreciate finely in industry. Works such as those of Dar-El, Ayas, and Gilad (1995) and NASA (2007) provide values for some specific industries - values that can be considered as averages. More precise, and especially individual, determinations will have to be implemented.

## 4. Representation of the problem

## Indices

| $a$ | indicates worker |
| :--- | :--- |
| $i, c$ | indicates tasks |
| $j$ | indicates days |
| $k$ | indicates skills |
| $n$ | work repetition |
| $s$ | indicates working week |

## Given parameters

| A | Set of the actors, also used as the cardinality of this set (integer): $A=\{1,2, \ldots, a, \ldots, A\}$ |
| :---: | :---: |
| $C_{S 0}$ | Standard number of working hours per week (integer) |
| $D_{i}$ | Standard duration for the task $i \in I$, in days (integer) |
| $D_{i}^{\min }, D_{i}^{\max }$ | Minimum and maximum durations for the task $i$, in days (integer) |
| DMax $12 S$ | Maximum value of the average weekly hours worked over a period of 12 consecutive weeks (integer) |
| DMaxJ | Maximum duration of daily work, in hours (integer) |
| DMaxMod | Normal weekly work set by the collective agreement, in hours (integer) |
| DMaxS | Maximum duration of weekly work, in hours (integer) |
| DSA | Maximum annual work for one individual, in hours (integer) |
| $E_{S S}-E_{S F}-$ |  |
| $E_{F S}-E_{F F}$ | Set of temporal relations between pairs of tasks; here the letter $S$ means the start date of task and $F$ means the finish date of task |
| HSA | Maximum annual overtime for an actor, in hours (integer) |
| I | Set of tasks in the project, also its cardinality: $I=\{1,2, \ldots, i, \ldots, I\}$ |
| $I P$ | Size of the initial population (integer) |
| K | Set of required skills, or the total number of skills (integer): $K=\{1,2, \ldots, k, \ldots, K\}$ |
| $L$ | Contractual duration of the project, in days (integer) |
| $N A_{k}$ | Set of the actors mastering skill $k$, also used to present its cardinality |
| NJS | Number of days worked per week, identical for all workers (integer) |
| $n k_{a}$ | Set of the skills mastered by the actor $a$ - it also means its cardinal (integer) |
| $n k_{i}$ | Set of the skills needed to perform the task $i$ - also used as its cardinal (integer) |
| $S_{S W}$ | The start week of project |
| $u$ | Multiplicative factor applied to the standard hourly cost $U_{a}$ for overtime, dimensionless (real number) |


| $U_{a}$ | Standard hourly cost of the actor $a$, in monetary units (real number) |
| :---: | :---: |
| $U F_{a}$ | Virtual value associated with the residual flexibility of the actor $a$, in monetary units (real number) |
| $U_{k}$ | Virtual value associated with the development of the actors' efficiency in competence $k$, in monetary units (real number) |
| UL | Daily rate of delay penalties, in monetary units (real number) |
| $\beta$ | Grace period without delay penalties, in days (integer) |
| $\gamma$ | A pre-specified weight associated to each objective within the set $\left\{f_{1}, f_{2}, \ldots, f_{6}\right\}$, dimensionless, real number $\in[0,1]$ |
| $\theta_{a, k}^{\text {ini }}$ | Efficiency initial level of actor $a$ on competence $k$, dimensionless (real number $\in[0,1]$ ) |
| $\theta_{k}^{\text {min }}$ | Minimum level of efficiency required to practice the competence $k$, dimensionless (real number $\in[0,1]$ ) |
| $\theta_{a, k}\left(n_{e q}^{S P}\right)$ | Efficiency of the actor $a$ in competence $k$, at the start date of the project, dimensionless (real number $\in$ [0, 1]) |
| $\tau_{j}$ | Daily discount rate, dimensionless (real number) |
| $\Omega_{i, k}$ | Standard workload defined for the task $i$, in the competence $k$, in hours (real number) |
| $\omega p_{a}$ | Work already performed by an actor $a$ on the current year on previous projects, in hours (real number) |

## Decision and auxiliary variables

| $d F$ | Finish date, integer number <br> Make-span for the task $i$, in days (integer) |
| :--- | :--- |
| $d_{i}$ | Continuous execution time of the workload $\Omega_{i, k}$ (integer) |
| $d_{i, k}$ | Start date, integer number |
| $d S$ | The set of actual workforce allocated to a given workload $\Omega_{i, k}$ during the period $j$ - also used as its |
| cardinal (integer) |  |

The problem of flexible assignment of resources can be presented as follows: each project is a group of $I$ tasks; a task $i \in I$ is characterised by the set of all the skills $\left(n k_{i}\right)$ required to complete it and by the corresponding workloads $\Omega_{i, k}$ (we sometimes refer to them as 'jobs').

Our resources are the set $A$ of all the human resources in the company. Their schedules may vary on daily or weekly basis. Each individual can be characterised by his ability to perform one or more skill(s) $n k_{a}$, with for each one a determined performance $\theta_{a, k}(n)$ varying with time. We introduced for each skill a lower limit of effectiveness $\theta_{k}^{\min }$ below which an assignment will not be considered acceptable, for economic or quality reasons: $\theta_{a, k}(n) \in\left[\theta_{k}^{\min }, 1\right]$.

We define $\sigma_{a, i, k, j}$ as the allocation decision of the actor $a$ for his skill $k$ on the activity $i$ and at the time instance $j$ : $\sigma_{a, i, k, j}=1$ if this actor is assigned under these conditions, and $\sigma_{a, i, k, j}=0$ otherwise. We can see that there is a relationship between these staff allocation decisions, and the resulting job durations $d_{i, k}$. This relationship is not linear because some tasks may require more than one actor for their completion. Moreover, each actor has his own effectiveness which evolves from job to job during a same simulation - even if we consider that it remains constant during each given job execution.

We aim at different macro-goals simultaneously. First, to balance the workloads and the availability of the operators required; then, the compliance with regulatory and temporal constraints that frame the programming of activities (scheduling, durations, etc.); and finally, we look for a compromise between the execution cost and the development of skills. The mathematical model is presented by the following equations.

## The objective function

First the objective function we wish to minimise (equation 7 ) is the sum of six terms $\left(f_{1}, \ldots, f_{6}\right)$. Each one is assigned a weight $\gamma$.

$$
\begin{equation*}
\text { Minimise: } F=\gamma_{1} \times f_{1}+\gamma_{2} \times f_{2}+\gamma_{3} \times f_{3}+\gamma_{4} \times f_{4}-\gamma_{5} \times f_{5}+\gamma_{6} \times f_{6} \tag{7}
\end{equation*}
$$

The first term $\left(f_{1}\right)$ represents the actual cost of work executed during 'normal opening' hours, i.e. with a standard hourly cost $U_{a}$.

$$
\begin{equation*}
f_{1}=\sum_{a=1}^{A}\left[U_{a} \times \sum_{s=S_{S W}}^{S_{F W}} \omega s_{a, s}\right] \tag{7a}
\end{equation*}
$$

The second term $\left(f_{2}\right)$ is the cost of overtime; it is determined by applying a multiplier $u$ to standard hourly cost:

$$
\begin{equation*}
f_{2}=\sum_{a=1}^{A}\left[U_{a} \times u \times \sum_{s=S_{S W}}^{S_{F W}} H S_{a, s}\right] \tag{7b}
\end{equation*}
$$

The third term $\left(f_{3}\right)$ is a virtual cost associated with the loss of future temporal flexibility (working capacity) of the actors at the end of the simulation. This loss derives from the weekly occupancy rates of the actors, compared to the standard duration of work per week $C_{S 0}$.

$$
\begin{equation*}
f_{3}=\sum_{a=1}^{A} U F_{a} \times\left(\sum_{s=S_{S W}}^{S_{F W}} \omega s_{a, s} /\left(N W \times C_{S 0}\right)-1\right), \quad \text { where } N W=\left|\left\{S_{S W}, \ldots, S_{F W}\right\}\right| \tag{7c}
\end{equation*}
$$

The term $\left(f_{4}\right)$ represents a penalty associated to the completion of a work outside of its tolerance range $[L-\beta, L+\beta]$ (Vidal 2000). This cost may result from storage costs if the products are completed too early or from payment of penalties for late otherwise.

$$
f_{4}= \begin{cases}{\left[f_{L}\right] \times\left(\left(1+\tau_{j}\right)^{(L-L V-\beta)}-1\right)} & \text { if, } L V<L-\beta  \tag{7d}\\ U L \times(L V-(L+\beta)) & L V>L+\beta \\ 0 & L-\beta \leq L V \leq L+\beta\end{cases}
$$

When $L V<L-\beta$, $f_{4}$ may figure the cost of the resulting financial immobilisation or a storage cost (Attia, Edi, and Duquenne 2012). Thus, the penalty can be formulated as a function of the project's costs (we only considered the labour $\operatorname{cost} f_{L}=f_{1}+f_{2}$ ) and of a daily discount rate $\tau_{j}$.

But, if the real completion date of the activity exceeds the tolerance $(L V>L+\beta)$, the resulting penalty is calculated with a daily rate $U L$.

The term $\left(f_{5}\right)$ is a fictitious profit related to the overall evolution of the average skill of the actors: it will promote solutions leading to an overall improvement of skills during the simulation period. The constant $\left(U_{k}\right)$ reflects the economic value associated to this factor in terms of profit (or loss) for the company.

$$
\begin{equation*}
f_{5}=\sum_{k=1}^{K} \frac{U k}{K \times N A_{k}} \times\left[\frac{\sum_{a=1}^{A}\left[\theta_{a, k}\left(n_{e q}^{F P}\right)-\theta_{a, k}\left(n_{e q}^{S P}\right)\right]}{\sum_{a=1}^{A} \theta_{a, k}\left(n_{e q}^{S P}\right)}\right] \tag{7e}
\end{equation*}
$$

Finally, the term $\left(f_{6}\right)$ represents the penalties associated to the constraints violation if any. $P_{O C}$ and $P_{A C}$ are, respectively, operational constraints $(O C)$ including all constraints related to the project and its execution period, and
annual constraints $(A C)$, related to the working time quotas. And $v$ is a Boolean variable: $v=1$ for constraint violation and $v=0$ for the constraint satisfaction.

$$
\begin{equation*}
f_{6}=\sum_{O=1}^{O C} P_{O C} \times v_{O}+\sum_{\vartheta=1}^{A C} P_{A C} \times v_{\vartheta} \tag{7f}
\end{equation*}
$$

The problem constraints
Constraints on actors' allocation: The set of constraints (8) ensures that any actor $a$ can be assigned to only one task $i$, for only one skill $k$, on the same day $j$ :

$$
\begin{equation*}
\sum_{i=1}^{I} \sum_{k \in n k_{a}} \sigma_{a, i, k, j} \leqslant 1, \quad \forall a \in A, \forall j \tag{8}
\end{equation*}
$$

where $\sigma_{a, i, k, j}=1$ if this actor is assigned, and $\sigma_{a, i, k, j}=0$ otherwise.
Constraints on availability of resources: For the set $\rho_{j}$ of all the tasks that are under progress at the date $j$, the staff required to perform the work corresponding to the competence $k$ are always lower than or equal to the overall capacity of the qualified personnel $\left(A_{k}\right)$.

$$
\begin{equation*}
\sum_{i \in \rho_{j}} E R_{i, k, j} \leqslant A_{k}, \quad \forall j, \forall k \in K \tag{9}
\end{equation*}
$$

$E R$ is the number of the people assigned, regardless their efficiencies - i.e. $E R$ is an integer number.
Temporal relationships between tasks: Constraints Equations (10)-(13) indicate the sequencing relationships between tasks:

$$
\begin{array}{ll}
d S_{i}+S S_{i, c}^{\min } \leqslant d S_{c} \leqslant d S_{i}+S S_{i, c}^{\max }, & \forall(i, c) \in E_{S S} \\
d S_{i}+S F_{i, c}^{\min } \leqslant d F_{c} \leqslant d S_{i}+S F_{i, c}^{\max }, & \forall(i, c) \in E_{S F} \\
d F_{i}+F S_{i, c}^{\min } \leqslant d S_{c} \leqslant d F_{i}+F S_{i, c}^{\max }, & \forall(i, c) \in E_{F S} \\
d F_{i}+F F_{i, c}^{\min } \leqslant d F_{c} \leqslant d F_{i}+F F_{i, c}^{\max }, & \forall(i, c) \in E_{F F} \tag{13}
\end{array}
$$

These constraints express dependency relationships of various types, with minimum/maximum 'advance' or 'delay' offsets, or more simply the traditional 'finish-start' precedence relations. The values of time lags can be represented either as time durations or percentage of work progress; according to Valls, Perez, and Quintanilla (2009) and Alfieri, Tolio, and Urgo (2010), the percentage of work progress shows better results in modelling an actual situation, when the task durations is not known in advance but depends on the resources that are selected to perform them. We assume that the temporal relationships between tasks are valid, and that the project graph presents no loops of strictly positive length.

Skills satisfaction constraints: No actor can be allocated on a workload without the minimum level of qualification $\theta_{k}^{\min }$.

$$
\begin{equation*}
\theta_{k}^{\min } \times \sigma_{a, i, k, j} \leqslant \theta_{a, k}\left(n_{e q}\right), \quad \forall a \in A, \forall i \in I, \forall k \in K, \forall j \tag{14}
\end{equation*}
$$

Workloads satisfaction constraints: The hours provided by the qualified actors, taking into account their efficiencies, are sufficient to balance the workload required for each skill:

$$
\begin{equation*}
\sum_{j=d S_{i, k}}^{d F_{i, k}-1} \sum_{a \in E R_{i, k, j}}\left(\omega_{a, i, k, j} \times \sigma_{a, i, k, j} \times \theta_{a, k}\left(n_{e q}\right)\right) \geqslant \Omega_{i, k}, \quad \forall i, \forall k \tag{15}
\end{equation*}
$$

The term $E R_{i, k, j}$, is the set of individuals allocated to achieve the workload $\Omega_{i, k}$ during period $j$; if one considers the effectiveness of each individual, the whole becomes an equivalent workforce $E E_{i, k, j}=\sum_{a \in E R_{i, k, j}} \theta_{a, k}\left(n_{e q}\right)$.

Constraints on tasks durations: For each job, the uninterruptible duration ( $d_{i, k}=d F_{i, k}-d S_{i, k}$ ) results from actors' allocations and their daily working decisions. This duration must be within the temporal limits defined for the corresponding task $i: d_{i, k} \in\left[D_{i}^{\text {min }}, D_{i}^{\text {max }}\right]$ :

$$
\begin{equation*}
D_{i}^{\min } \leqslant d_{i, k} \leqslant D_{i}^{\max }, \quad \forall i \in I, \forall k \in K: \Omega_{i, k} \neq 0 \tag{16}
\end{equation*}
$$

The time required to complete the task $i$ is $d_{i}=\operatorname{Max}_{k=1}^{K}\left\{d_{i, k}\right\}, \forall i \in I: \Omega_{i, k} \neq 0$.
Regulatory constraints on working time: For a period of one day: The maximum daily working time of any actor must be lower than or equal to a maximum amount of daily work (DMaxJ), knowing that the variables $\omega_{a, i, k, j}$ are amongst the model decisions.

$$
\begin{equation*}
\sum_{i=1}^{I} \sum_{k=1}^{K} \sigma_{a, i, k, j} \times \omega_{a, i, k, j} \leqslant D M a x J, \quad \forall a \in A, \forall j \tag{17}
\end{equation*}
$$

For a period of one week: The actors' weekly working hours $\omega s_{a, s}$ must always be lower than or equal to the maximum amount of weekly work DMaxS:

$$
\begin{gather*}
\omega s_{a, s}=\sum_{j=N J S \times(s-1)+1}^{N J S \times s}\left(\sum_{i=1}^{I} \sum_{k=1}^{K} \sigma_{a, i, k, j} \times \omega_{a, i, k, j}\right), \quad \forall a, \forall s  \tag{18}\\
\omega s_{a, s} \leqslant D M a x S, \quad \forall a \in A, \forall s \in\left\{S_{S W}, \ldots, S_{F W}\right\} \tag{19}
\end{gather*}
$$

For a rolling period of 12 consecutive weeks (reference period): The average weekly work, calculated on a floating horizon of 12 consecutive weeks (as a reference period in France), is also subject to regulatory constraints DMax $12 S$.

$$
\begin{equation*}
\frac{1}{12} \times\left(\sum_{p=s-11}^{s} \omega s_{a, p}\right)_{p \geqslant 1} \leqslant D M a x 12 S, \quad \forall a \in A, \text { and } \quad \forall s \in\left\{S_{S W}, \ldots, S_{F W}\right\} \tag{20}
\end{equation*}
$$

For a period of one year: For each actor, the total number of working hours for the current activity is always below his yearly quota $D S A$ :

$$
\begin{equation*}
\sum_{s=S_{S W}}^{S_{F W}} \omega s_{a, s} \leqslant D S A-\omega p_{a}, \quad \forall a \in A \tag{21}
\end{equation*}
$$

Here $\omega p_{a}$ represents the working hours of the actor $a$ on other previous activities during the considered year, and DSA is the maximum number of annual working hours of each actor.

Constraints on overtime: finally, we compute the weekly overtime $H S_{a, s}$ for each actor $a$ during week $s$ :

$$
H S_{a, s}=\left\{\begin{array}{ll}
\omega s_{a, s}-D M a x M o d & \text { if } \omega s_{a, s} \geqslant D M a x M o d  \tag{22}\\
0 & \text { otherwise }
\end{array}, \quad \forall a \in A, \forall s \in\left\{S_{S W}, \ldots, S_{F W}\right\}\right.
$$

DMaxMod represents the maximum weekly standard work (i.e. non-overtime), according to internal modulation adopted by the company. These overtime hours verify that $H S_{a, s} \in[0, D M a x S-D M a x M o d]$. Equation (23) cheques that overtime always respect an annual limit $H S A$ for each actor:

$$
\begin{equation*}
\sum_{s=S_{S W}}^{S_{F W}} H S_{a, s} \leqslant H S A-H S P_{a}, \quad \forall a \in A \tag{23}
\end{equation*}
$$

## 5. Resolution of the problem

### 5.1 Heuristics

The implementation of a dynamic vision of the actors' skills, in addition to the elastic durations of tasks, leads us to a highly non-linear model with mixed integer variables. Therefore, solving it with mathematical programming is difficult due to the huge numbers of constraints and variables which produce a combinatorial explosion. The traditional RCPSP proposed a great challenge in the arena of operational research due to its NP-hard complexity nature (Brucker and Knust 2012, 34). This complexity was increased, first by adopting the discrete time-cost trade-off, next by considering multi-skilled workforce (Bellenguez-Morineau and Néron 2007), and then with the addition of working time constraints (Attia, Edi, and Duquenne 2012). Moreover here, evolving productivities increase the difficulty to solve the problem optimally. The vast development performed in metaheuristics enhanced their capacities to reduce the gap between their solution and the optimal one (Hansen and Mladenović 2003). We therefore directed towards approximate solutions that can be obtained with metaheuristics. Genetic algorithms are one of the most used, and have proven effective in providing solutions more than adequate in a timely manner for many industrial applications (Gen and Cheng 2000), or to solve extremely difficult problems (Munawar et al. 2011).

Implementation of GAs' requires the definition of procedures intended to simultaneously explore as comprehensively as possible the space of solutions, while guiding this exploration to better solutions: many parameters of this progression must be worked out (Goldberg 1989), starting with an encoding of the problem variables.

### 5.2 Decisions-based GAs

### 5.2.1 Chromosome representation

We opted for priority-based encoding GAs (we call it decisions-based GAs): We encode processing priorities instead of descriptions of the solutions, similar to the random key representation (see e.g. Lancaster and Ozbayrak 2007). A chromosome brings three decisions: which task will be processed first? Which actor(s) will be assigned to perform this task? And what will be the daily work allowed to the operator(s) to complete it?


Figure 2. Chromosome representation.

Therefore, each chromosome contains three parts (shown by Figure 2); the first one presents the priority for realising the tasks. The locus of the gene is the task number, and the allele represents its priority. According to this part, we can build a tasks' priority list, by sorting tasks in a decreasing priority order.

The second sub-chromosome holds the actors' priorities for the allocation process. As previously, each gene's locus represents an actor number and holds his priority for the allocation process as its allele. Here again, we can construct the actors' priority list for the tasks executions.

The third part represents priorities of working time strategies. From the working time regulatory constraints, we have five intervals (expressed in daily hours), which can be described as follows, according to French regulations:
$\left[X, C_{0 s} / N J S\right]$ : Represents the daily working time within the standard weekly hours $C_{0 s}$ limits (thus $C_{0 s} / N J S$ ). Here, $X$ represents a minimum number of working hours per day; this translates a willing to avoid calling an operator on duty if his working time is not worth making the journey from home.

The second interval $\left.] C_{0 s} / N J S, D M a x M o d / N J S\right]$ is limited by the constraints of the company's internal modulation of weekly working time - relative to one day, $D M a x M o d / N J S$.

The next interval $] D M a x M o d / N J S$, $D M a x 12 S / N J S]$ will then be limited by the constraints of the maximum average weekly working time for a period of 12 successive weeks.

Then the fourth interval ]DMax12S/NJS, DMaxS/NJS] will integrate the maximum number of working hours per week.

The last interval $] D M a x S / N J S, D M a x J]$ considers the daily constraint of maximum working time DMaxJ.
This way of encoding brings two benefits: first it produces much shorter chromosomes than a direct encoding of solutions - a problem of ( 30 tasks, 83 actors and 4 skills) leads to chromosomes having 3879 genes, whereas with 'decision-based encoding' it drops down to 118 genes. Another reason is the presence of 'epitasis' (Gibbs et al. 2006): some of the chromosome's genes are interrelated, increasing the risk of engendering unfeasible solutions in a random process: considering progressively planning priorities allows an immediate detection of any aberrant solution. Classically a group of individuals can be built to form an initial population. These individuals are then decoded and evaluated by reference to the objective function to calculate for each chromosome its fitness.

### 5.2.2 Chromosome decoding

Three lists are read from each chromosome: a list of tasks, a list of actors and the list of working time policies. For each allocation period (day), the temporal relations between tasks will be checked, to get the eligible set of tasks, the most prioritised task will be considered first. At the beginning, the actors' efficiencies are updated from their previous work assignments, as discussed in Section 3.3. Then the actors' availabilities and their abilities to perform the workloads related to this task will be checked. If the result is not satisfactory, we look for another task. If it is valid, a detailed actors' allocation will be activated. First, we start to withdraw one of the task's workloads according to the loading rate (the ratio between the demand and availability) of skills; afterwards a load-capacity balance process is started in order to assign a value to each unknown variable $\left(\omega_{a, i, k, j}, E E_{i, k, j}, d_{i, k}\right)$.

The load-capacity balance starts by picking out the most prioritised and qualified operators from the actors' list; then sequentially a load-matrix is constructed: rows are the assigned actors and the columns are the different possible durations $d_{i, k}$, starting from $D_{i}^{\min }$ until $D_{i}^{\max }$. The number of rows corresponds exactly to the number of actor(s) selected to provide the workload, with the equivalent production capacity $E E_{i, k, j}$. Each element within this load-matrix represents the corresponding value of the variable $\left(\omega_{a, i, k, j}\right)$, calculated as: $\omega_{a, i, k, j}=\Omega_{i, k} /\left(E E_{i, k, j} \times d_{i, k}\right)$.

If we find a value of $\omega_{a, i, k, j}$ that would match the most prioritised daily work (carried by the third part of the chromosome) and would satisfy the working time constraints, we can freeze both the actors' assignments and the resulting $d_{i, k}$. If we cannot find any, the algorithm assigns the next prioritised actor in the list. Corresponding to the new $E E_{i, k, j}$, the algorithm constructs a new row in the load-matrix to represent new values of the variables ( $\omega_{a, i, k, j}$ ) and then investigates the working time constraints. If it fails, it then tries to increase $E E_{i, k, j}$, and repeats the procedures until it can match suitable values for the three variables $\omega_{a, i, k, j}, E E_{i, k}$ and $d_{i, k}$.

These procedures for actors' allocation are repeated for each workload required for task $i$. If the operators are not sufficient to undertake the current task, the model searches again for another feasible task to be scheduled. During the current day $j$, if none of the tasks to be done may find enough resources to be performed, then we look for another allocation period, where actors would be released by ending missions, and the checking procedures are repeated for the tasks in progress and the eligible one(s). This actors-to-tasks allocation process will be continued until all tasks are assigned to their sufficient workers. In the worst case, if at least one task cannot be performed at any time due to resources scarcity, then the infeasibility of the project will be rapidly concluded.

### 5.2.3 Fitness function calculation

The different objectives can be calculated, as Equation (7), after the chromosomes decoding procedures and the construction of the project schedule. Before determining the fitness of each chromosome, the functions $\left(f_{i}\right)$ seen above have to be normalised (Konak, Coit, and Smith 2006). The normalisation is intended to standardise the order of magnitude of the objectives, in order to project the value of each objective over a given interval: $[0,1] \times C_{\max }$, where $C_{m a x}$ is a pre-specified constant.

First, the labour cost is the sum of the cost of standard hours $\left(f_{1}\right)$ plus the cost of extra hours $\left(f_{2}\right)$ and can be expressed by $f_{L}=f_{1}+f_{2}$; this labour cost will be minimal ( $f_{L}^{\text {min }}$ ) if all the missions required in the project can be performed by fully qualified persons $(\theta=1)$ during standard hours; oppositely, the maximum labour cost ( $f_{L}^{\max }$ ) will be encountered if unfortunately all the jobs have to be undertaken by beginners $\left(\theta=\theta^{\mathrm{min}}\right)$ during their extra hours. The labour cost can then be normalised to the new function $f_{L}^{\prime}=\left(f_{L}-f_{L}^{\min }\right) /\left(f_{L}^{\max }-f_{L}^{\min }\right)$. In a similar way, the occupation of any operator may be bounded: by zero if he is not appointed at all during the project and by [ $D M a x S / C_{S 0}$ ] if he is required as much as possible. So $\left(f_{3}\right)$ can be normalised as $f_{3}^{\prime}=\left(f_{3}\right) /\left(f_{3}^{m a x}\right)$. The objectives functions $\left(f_{4}, f_{5}, f_{6}\right)$ were rescaled by simply dividing each of them by its maximum value ( $C_{4}^{\max }, C_{5}^{\max }$, and $C_{6}^{\max }$, respectively). After the normalisation of all objectives $\left(f_{i}^{\prime}\right)$, and assuming that: $\gamma_{1}=\gamma_{2}=\gamma_{L}$, we get the fitness function as:

$$
\begin{equation*}
\text { fitness }(\varepsilon)=\gamma_{L} f_{L}^{\prime}+\gamma_{3} f_{3}^{\prime}+\gamma_{4} f_{4}^{\prime}-\gamma_{5} f_{5}^{\prime}+\gamma_{6} f_{6}^{\prime}, \quad \varepsilon \in \text { Population } \tag{24}
\end{equation*}
$$

where the $\gamma_{i}$ are the objectives' weights; the sum of all these weights is equal to unity. Using this normalisation method enables us first to favour the feasible solutions with zero penalties, and then to monitor the compromise between execution costs and skills development. In order to overcome the problem of negative sign, if any, this minimisation problem was converted to a maximisation one, by using the constant $C_{\max }$ as: $f_{a b}(\varepsilon)=C_{\max }(1-\mathrm{fitness}(\varepsilon))$. We refer to it as the 'individual absolute force'. Based on this absolute force, we can perform the different selection procedures that will build the next generation.

### 5.2.4 Construction of the next generation

The scheme we used for the construction of the next generation is similar to those of (Mendes, Gonçalves, and Resende 2009; Attia, Edi, and Duquenne 2012), as shown in Figure 3. It is based on four groups of individuals: the first is the group of 'survivors', who have been selected upon their fitness through an elitist selection. The size of this group is predefined as: $10 \% I P$.

The second group is the 'children group', which results from crossover between parents. In order to perform the crossover, we construct the mating pool containing two sets of parents: the first one has a size of $I P$, but its individuals are selected from this population via a roulette-wheel selection (Goldberg 1989). The second one is the group of 'survivors' described above. After that we applied the parameterised uniform crossover discussed in Mendes, Gonçalves, and Resende (2009), in which two parents are selected randomly: one amongst the 'survivors' and the other from the group selected by the roulette-wheel. As shown by Figure 4, for each gene a random number between [0,1] is generated. The child will inherit from the first parent's allele if this random number is lower than a given crossover rate $(\mathrm{Pc}=0.7)$, and from his second parent's allele otherwise. The resulting child is then directly introduced into the new generation. We set the size of the children group exactly as the crossover rate: $70 \% I P$.


Figure 3. The next generation reproduction scheme.


Figure 4. Representation of the parameterised uniform crossover.
In order to preserve the diversity of the population, the individuals of the third group are generated randomly. We adopted the 'random immigration scheme' of S. Yang (2007). The size of this group is predefined to be $20 \% I P$, minus one: the last group consists of only one individual: the best one ever found since the very beginning of the search.

Then, when the population is built, the mutation procedure takes place to develop some of the population genotypes with the uniform mutation, (Davis 1996); the mutation probability is set to be ( 0.01 ). After bringing out each generation, the process of creating schedules, evaluating individuals and building new generations is repeated (Figure 5) until one of the stopping criteria halts the process.

### 5.2.5 Stopping criteria

Two termination criteria are checked after each generation. First, when the average fitness for a fixed number of the best individuals shows no significant change over a given number of successive generations: we then consider that the search has converged to an acceptable solution. The second simply depends on the total number of generations that were computed since the beginning of the search: this stops a search which does not seem to be successful.

## 6. Application to an example

The model was validated by solving and analysing the output of some problems (Attia, Duquenne, and Le-Lann 2012); by the same time, the GA's parameters were tuned. Here, we computed a simple project of 10 tasks, 10 actors and 4 skills, as shown in Table 2 (Edi 2007).

Table 3 provides initial data concerning the actors; The minimal efficiency $\theta_{a, k}^{i n i}$ is set at 0.4 , and the learning rates $r_{a, k}$ are 0.8 for all actors (Wright 1936; Mccreery and Krajewski 1999). In Equation (6), $\xi$ is set at 3.

```
The Genetic algorithm:
{
Generate the initial population g=1, numbering IP individuals
Apply the actors' allocation procedures to the current population g=1,
Evaluate the population g=1
IF none of the stopping criteria is fulfilled, repeat
    { Select some individuals from g}\mathrm{ for copy to g+1
            Crossover some individuals from g}\mathrm{ and copy the result to }\mathbf{g}+\boldsymbol{1
            Regenerate some new individuals to be added into g+1
            Mutate some individuals taken from g+1
            Apply the actors' allocation procedures to g+l,
            Evaluate the population g+1
            g=g+1
    }
}
```

Figure 5. The genetic algorithm.

Table 2. Project data.

| Tasks | Durations (days) |  |  | $\Omega_{i, k}(\mathrm{~h})$ |  |  |  | Successors |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $D_{i}$ | $D_{i}^{\text {min }}$ | $D_{i}^{\text {max }}$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |  |
| 1 | 4 | 2 | 6 | 0 | 60 | 0 | 50 | 2, 3, 4 |
| 2 | 5 | 3 | 7 | 45 | 68 | 0 | 0 | 3, 5, 7 |
| 3 | 4 | 3 | 7 | 0 | 63 | 45 | 35 | 5, 6 |
| 4 | 7 | 5 | 10 | 53 | 0 | 60 | 0 | 6, 9 |
| 5 | 4 | 2 | 6 | 0 | 65 | 0 | 60 | 7, 8 |
| 6 | 3 | 1 | 5 | 60 | 0 | 35 | 0 | 8, 9 |
| 7 | 5 | 3 | 7 | 35 | 56 | 0 | 40 | 10 |
| 8 | 5 | 3 | 8 | 0 | 0 | 47 | 50 | 10 |
| 9 | 4 | 2 | 5 | 0 | 45 | 26 | 0 | 10 |
| 10 | 3 | 2 | 4 | 35 | 30 | 35 | 30 | - |

Table 3. Initial efficiencies of actors.

|  | $\theta_{a, k}\left(n_{e q}^{S P}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Actors | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
| 1 | 0.8 | 1.0 | 0.0 | 0.5 |
| 2 | 1.0 | 0.0 | 0.8 | 0.0 |
| 3 | 0.0 | 0.6 | 0.0 | 1.0 |
| 4 | 0.7 | 0.0 | 1.0 | 0.6 |
| 5 | 0.0 | 1.0 | 0.7 | 0.0 |
| 6 | 0.9 | 0.0 | 0.0 | 1.0 |
| 7 | 1.0 | 0.8 | 0.0 | 0.6 |
| 8 | 0.0 | 0.7 | 1.0 | 0.0 |
| 9 | 1.0 | 0.8 | 0.0 | 0.5 |
| 10 | 0.0 | 0.9 | 1.0 | 0.0 |

Company data (working time regulations and costs) are shown in Table 4. The last set of data (Table 5) provides the parameters of the genetic algorithm - they were fixed after an exploration intended both to validate the model and to define reliable values.

To solve the problem, we must define the management priorities: they can aim at a minimisation of the working hours required by the industrial programme (thus making the maximum use of the most competent resources, and therefore develop a culture of mono-skilled operators); they can as well try to expand the versatility of the actors (with the inflation of hours, hence of costs, that entails), or seek a compromise between these two extremes.

The algorithm was encoded with C++ using 'Microsoft visual studio 2010' on Intel ${ }^{\circledR}$ Pentium ${ }^{\circledR}$ Dual Processor T3400: 2.16 GHz with 'Windows 7 ' as an operating system. First, the problem was solved seeking minimum cost with $\left(\gamma_{i}=\{0.6 ; 0.1 ; 0.1 ; 0.1 ; 0.1\}\right)$, as shown by Figure 6 ; we can observe that after a number of 369 generations ( 53 s as shown by Figure 6(a)), the GA were able to reduce the project labour cost by $6.88 \%$ (as shown by Figure 6(b)), from the best random initial situation; this represents a surcharge of only $0.39 \%$ compared to an ideal cost of $f_{o}=12,408$

Table 4. Company data.

| Maximum average weekly work over a period of 12 consecutive weeks | DMax12S $=44^{*} \mathrm{~h}$ |
| :--- | :--- |
| Maximum daily work | DMaxJ $=10^{*} \mathrm{~h}$ |
| Normal weekly work set by the collective agreement | DMaxMod $=33 \mathrm{~h}$ (we assumed it) |
| Maximum weekly work | DMaxS $=48^{*} \mathrm{~h}$ |
| Maximum yearly work | $D S A \approx 1600^{*} \mathrm{~h}$ |
| Maximum yearly overtime | $H A S=180 \mathrm{~h} /$ year (we assumed it) |
| Number of weekly working days | $N J S=5$ days (we assumed it) |
| Over cost related to the overtime working hours | $u=0.25$ |

[^1]Table 5. Exploration data.

| Population size | 100 individuals |
| :--- | :--- |
| Crossing probability | 0.7 |
| Mutation probability | 0.01 |
| Fraction of immigrated individuals in the new generation | 0.2 |
| Max. no. of generations | 800 generations |
| Number of generations without convergence | 100 generations |
| Number of the best individuals on which convergence is computed | 10 individuals |

Currency Units $(C U)$, assuming that all jobs are completed within standard hours. On the other hand, the company loses an average of $(-2.895 \%)$ of the secondary skills of its operators, due to the unlearning effect. This is illustrated in Figure 7, which shows the evolution of actors' efficiencies during the project horizon ( 30 days).

Figure 7 reveals that all of the actors have experienced a degradation of their secondary skill(s) efficiencies, except actors \#6 and \#10. First the degradation effect was produced due to the optimal economic assignment of actors \#\{1, 2, $3,4,5,7,8,9,10\}:$ they were appointed to work with their principal skills, in order to avoid the direct over costs associated to the use of non-optimal productivities. For actors \#6 and \#10, this effect of degradation (about $0.11 \%$ ) is not that visible, due to their high initial efficiencies in their secondary skills, as shown in Table $3\left(\theta_{6,1}=\theta_{10,2}=0.9\right)$; the loss of competence resulting from a lack of practice has an higher effect on beginners than on experts. This conclusion can also be deduced from actor \#2 in skill \#3, actor \#7 in skill \#2 or actor \#9 in skill \#2. Although, actor \#6 was selected to work in task \#2 using his secondary skill \#1, the evolution is also tiny. As a conclusion, we check here that, as expressed by Equations (4)-(6), if an actor has a high efficiency level, the evolutions of his skill, whatever the way, increase or decrease, will be slow and non-remarkable, so that his operational flexibility can be used periodically without risk. We can conclude that the algorithm is capable to find an economic solution as expressed by the objectives weights.

Another management strategy would be to expand the actors' flexibility: we then adopted new objectives weights $\left(\gamma_{i}=\{0.1,0.1,0.1,0.6,0.1\}\right)$ in order to enhance the actors' skills $\left(f_{5}^{\prime}\right)$. The search procedures stopped after 175 generations ( 26 s of running time). It stopped due to convergence of the average fitness computed on a specified set of individuals ( 10 of the best individuals). This exploration procedure succeeded in increasing the actors' average efficiency by about ( $+0.94 \%$ ). In addition, it reduced the best fitness (i.e. the lowest ones) of the feasible schedules by about $111.89 \%$; this value exceeds $100 \%$, since the fitness function may be negative (Equation 24, especially with high values of $\gamma_{5}$ ), which will likely be the case for the best individuals compared to the best individual in the initial population. By the same time, the labour cost was increased by about $7.42 \%$, which represents an over costs of about $36.94 \%$ compared to the optimal labour cost. As shown by Figure 8, these over costs result from the intense use of actors' secondary skills in order to develop their versatility; this evolution can be shown for actors \#1, \#3, \#4, \#5, \#7, \#8 and \#9. But for actors with high levels of secondary skills such as actors \#2, \#6 and \#10, we can see that the model prefers to use their functional flexibility (actor \#2 and actor \#10), or preserve their future temporal flexibility by reducing their involvement and put them a aside (actor \#6), and assign the ones with low efficiency levels. Indeed, this strategy looks expensive, and it may make no sense to pay about $37 \%$ more than the project ideal cost for an average


Figure 6. Evolution of the fitness function and objectives functions during exploration.


Figure 7. Best cost strategy: the evolution of actors' skills during the project horizon.
gain of about only $+0.9 \%$ of actors efficiency. However, it can be concluded here also that the proposed algorithm is capable to find a solution to develop the actors' versatilities that reflects the predefined weights. The previous results illustrate that a compromise should be investigated between the labour cost and the actors benefits from learning by doing.

Before changing the weights $\gamma_{i}$ 's of the fitness function, we should wait and introduce the main reasons that can reduce the extra costs and lead us to extract the managerial benefits. The first reason is the fraction of the actors whose secondary skills are within the transition interval (roughly estimated $\left[\theta_{a, k}^{i n i}, 0.8\right]$ ). As shown in Figure 8 we can find 7 actors out of 10 (actors \#1, \#3, \#4, \#5, \#7, \#8, \#9). A high percentage of poorly skilled workforce inflates the cost of


Figure 8. Skills development strategy: the evolution of actors' skills during the project horizon.
versatility. The second reason is the number of skills under development within the transition interval for the same actor (as for actor \#4). These two reasons make the procedure of searching a compromise notably difficult, especially for dissimilar actors' skills. We tried to get a compromise by changing the weights to the values $\gamma_{i}=\{0.35,0.1,0.1,0.35$, $0.1\}$, assigning equal weights to the labour cost and to the skills development. Afterwards the algorithm was run 10 times: for each run Table 6 displays the best solution found; most of these solutions suffer from skills depreciation as well as extra cost: finding such a compromise is not self-evident. From results in Table 6, we can see both $\left(\%\left(f_{L}-f_{o}\right) / f_{o}\right)$
and $\left(\% f_{5}\right)$ are located between the two extremes of the previous strategies. This indicates the response of the algorithm to the managerial strategies expressed by the objectives weights.

In order to lessen both extra cost and skills depreciation, we reduced the fraction of actors whose efficiencies in their secondary skills are within the interval $\left[\theta_{a, k}^{i n i}, 0.8\right]$ from $70 \%$ to only $20 \%$; additionally, we avoided to develop more than one secondary skill at a time for the same operator. Table 7 displays these changes.

After running the algorithm 10 times with the same previous weights $\gamma_{i}=\{0.35,0.1,0.1,0.35,0.1\}$, we found that all the resulting schedules have a positive value for $\left(f_{5}\right)$ as shown in Table 8, i.e. there is an average development of actors' secondary skills, of course with the associated extra costs.

For a detailed analysis, we can consider the schedule of minimum cost (exploration 6 in Table 8), Figure 9 displays the actors' skills evolutions during the project execution. This time, the effect of skills depreciation is annihilated, for individuals as well as for the whole population - and the actors' secondary skills have developed.

In order to evaluate the impact of the previously discussed factors in the associated over costs, and to find the best compromise between the two objectives, Figure 10 displays a graphical comparison between the results from Tables 6 and 8 . As we see, companies must accept extra costs just to preserve the productivity level of their actors (i.e. $\% f_{5}=$ 0.0 ). But these extra costs are all the more important as the number of actors enrolled in the development programme increases. In other words, with $70 \%$ of actors following the programme of multi-skill development (PMSD), this over cost can be estimated at about $21 \%$ over the optimal labour cost, whereas with only $20 \%$ of actors involved, it should drop down to about $2.3 \%$. According to this model, the number of the actors engaged in a PMSD can be optimised; moreover, the associated additional costs can be estimated.

Figure 10 indicates a linear growth of labour costs vs. skills evolution, whatever the percentage of weakly qualified actors that can be assigned on activities. The $y$-intercepts of these lines, all positive here, may result from the deliberate choice of the company to allocate non-ideally qualified workforce. What is more significant is that the two sets of data in Figure 10 lead to the same slopes for the curves expressing the impact of skills average change on labour cost.

Some reasons for satisfaction rise from results that show to be consistent regarding the way we modelled facts published in literature: these results always provide feasible schedules (sometimes with lateness penalties), without any constraint violation, they also express a logical relationship between the use of versatility and the cost inflation; tuning the set of parameters $\left\{\gamma_{i}\right\}$ has always resulted in the expected behaviour. These points witness a good transcription of the published observations on which the model is based: they help the user to feel more comfortable towards the model's reliability.

Table 6. Exploration results related to labour costs $f_{L}$ and the skills development $f_{5}$.

|  | Exploration number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\%\left(f_{L}-f_{o}\right) / f_{o}$ | 10.79 | 23.29 | 14.95 | 14.10 | 20.50 | 12.86 | 11.86 | 8.13 | 22.63 | 25.10 |
| \% $f_{5}$ | -1.19 | 0.10 | -0.58 | -0.70 | -0.07 | -0.94 | -1.13 | -1.51 | 0.20 | 0.39 |

Table 7. Initial efficiencies of actors after modification.

|  | $\theta_{a, k}\left(n_{e q}^{S P}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Actors | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
| 1 | 0.8 | 1.0 | 0.0 | 0.5 |
| 2 | 1.0 | 0.0 | 0.8 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 | 1.0 |
| 4 | 0.0 | 0.0 | 1.0 | 0.6 |
| 5 | 0.0 | 1.0 | 0.0 | 0.0 |
| 6 | 0.9 | 0.0 | 0.0 | 1.0 |
| 7 | 1.0 | 0.8 | 0.0 | 0.0 |
| 8 | 0.0 | 0.8 | 1.0 | 0.0 |
| 9 | 1.0 | 0.9 | 1.0 | 0.0 |
| 10 | 0.0 |  | 0.0 |  |

Table 8. Exploration results related to labour costs $f_{L}$ and skills development $f_{5}$ after modification.

|  | Exploration number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \% ( $\left.f_{L}-f_{o}\right) / f_{o}$ | 15.44 | 15.98 | 15.61 | 13.61 | 15.39 | 12.98 | 15.69 | 15.61 | 15.53 | 14.98 |
| \% $f_{5}$ | 1.48 | 1.50 | 1.46 | 1.23 | 1.43 | 1.22 | 1.47 | 1.50 | 1.50 | 1.44 |

Some managerial evidences can also be confirmed from this work: The firms seeking for reactivity should accept an amount of extra cost to develop their operators' multifunctional flexibility. In order to enhance the acquisition of secondary skills, the operators in question should be regularly practising their new skills, and the preservation of their future temporal flexibility should be avoided during the period of skills acquisition. Moreover, firms should allocate these operators to work with their new skills until they have reached a sufficient degree of mastering that can protect them from the loss of learning produced during the use of multifunctional flexibility. The same recommendation may be formulated for economic reasons: the effort of growing up a new skill is costly and must be continued until this skill is consolidated. The number of operators who will follow a PMSD should be optimised in order to find a good compromise between the over costs induced and the overall average skills developments. The surplus of developing the actors' secondary skills can be misleading or become out of control especially for non-similar skills. According to Jaber, Kher, and Davis (2003), the worker who is being trained on similar tasks is likely to experience relatively less forgetting as compared to those being trained on very dissimilar tasks. Developing an operator with more than one secondary skill beside his basic one is very difficult and costly, especially for completely different skills.

## 7. Performance analysis

In the following, we investigate the performance of the solving method facing data changes. The analysis relies on a sample of 400 projects adopted randomly from the project scheduling library (PSPLib 1996). This sample was adapted to the current model by randomly generating the missing data around the original ones. These projects are divided into four groups of 100 projects each, the groups gathering projects of $30,60,90$ and 120 tasks, respectively. Each task is characterised by a specified workloads requiring up to four skills. These workloads, expressed in working hours, vary from 4389 to $111,041 \mathrm{~h}$. The workforce varies from 41 to 199 persons. Some of these workers are unary-skilled, and others are multi-skilled with heterogeneous efficiencies. The projects networks are of different complexities expressed by the restrictiveness measure $R T$ of Schwindt (1995). This measure was originally introduced by Thesen (1977) as a measure of networks reflecting the degree to which the imposition of precedence relations eliminates possible scheduling sequences. The relation of the required workloads to the available resources showed also variations relying on the average obstruction factor ' $O F$ ' developed by Davis (1975). For each instance, the number of independent variables was calculated, it varies from 2263 to 73,440 variables; these variables represent the workers allocation, tasks start dates, and workers daily work. Figure 11 displays these characteristics.

For these simulations, the maximum number of generations was adjusted at 8000 ; the objectives weights were adjusted to $\gamma_{i}=\{0.35,0.1,0.1,0.35,0.1\}$. We conducted three simulations for each instance, as a total of 1200 simulations. The results with either the minimum or the maximum running time were discarded. We selected the medium running time in order to investigate the average performance not the best one.

The adopted performance criteria are: the running time; the convergence (expressed by the number of generations), the increase of working hours due to multi-skilled flexibility; the ratio of the overtime to the total workload; the loss of future temporal flexibility; the project penalties; and the evolution of the workforce's experience. These performance measures were investigated in instances parameters, such as: number of tasks, required workload, available workforce, $R T$ and $O F$. The variances of the performance criteria among the four groups of projects were investigated using $A N O V A$ test. The linear relation was investigated based on correlation and regression analysis in function of instances parameters.

Concerning the running time, we considered the computational time per generation $C T$. The results showed that, respectively, the workload, number of tasks, $R T$ and $O F$ are the best predictors of $C T(F$-value $=1508.67$ and $R^{2}=0.950$ ). As shown by Figure 12(a), the $C T$ increases linearly with the projects groups. We also investigated the variation of the number of generations $G N$ relative to the four groups. The results show a very small variation between $G N$ of the groups, $\left(F\right.$-value $=5.63$ and $\left.R^{2}=0.409\right)$. As shown by Figure $12(\mathrm{~b})$, the process for all of the instances converged to a solution before reaching the maximum number of generations (8000). Moreover, the four groups share a


Figure 9. Evolution of actors' skills during the project, after modification of actors' efficiencies.
similar confidence interval for mean value of $G N$, respectively [517; 590] for the first group, [584; 662] for the second, [ $611 ; 688$ ] for the third and $[556 ; 612]$ for the last.

As discussed earlier, using multi-skilled flexibility increases the real working hours to perform the project. For each instance, we computed this work excess fraction relative to the required workload $\left(\%\left(f_{L}-f_{o}\right) / f_{o}\right)$. The results in Figure 12(c) show small differences between the four groups ( $F$-value $=22.74$ and $R^{2}=0.147$ ). This excess in work can be explained by comparing it with the workforce overall average efficiency at the projects' start date (displayed on Figure 12(d)): $\Theta=\frac{\sum_{a=1}^{A} \sum_{k=1}^{K} \theta_{a, k}\left(n_{e q}^{S P}\right)}{A \times K}$.


Figure 10. Cost of skills betterment.


Figure 11. Instance variation with respect to the workload, workforce and the network topology.

As shown by Figure 12(c), the excess in labour work for the four groups is distributed around a value of $25 \%$, while the overall average efficiency (Figure 12(d)) is distributed around $75 \%$. This reflects the robustness of the approach to reduce the labour working hours and hence the excess in standard labour working costs, i.e. the excess in working hours is mainly produced due to the data nature.

We also considered the overtime working hours as one of the performance criteria. For each of the instances within the groups, we calculated the ratio of the overtime hours to the required workload, the results (Figure 12(e)) show a small variation between groups ( $F$-value $=6.02$ and $R^{2}=0.436$ ).

The loss of future working capacity (temporal flexibility) $f_{3}$ was analysed vs. instances' characteristics. We found that the best predictors of this criterion are the total workload $W$, number of workers $A$ and $O F$, with the computed linear function: $f_{3}^{*}=0.0101 W+10.5 A+1744 O F-622$, at $R^{2}=91.7 \%$. The relation between $f_{3}^{*}$ and $f_{3}$ is represented graphically using Figure 12(f).


Figure 12. Illustration of the performance criteria with respect to the different projects groups.

When investigating the objective function $f_{4}$, we did not find any storage costs, all the non-zero values of $f_{4}$ are produced due to lateness penalties resulting from resources shortage. This relation with resources shortage can be illustrated by $O F$ (Figure $12(\mathrm{~g})$ ). To confirm this hypothesis, we searched a critical value of $O F$, ' $O F_{\text {critical }}$ ', above which lateness penalties would appear; we could determine this threshold, and set its value at $O F_{\text {critical }}=0.32$. Figure $12(\mathrm{~h})$ illustrates that this value of the resources shortage index $(\operatorname{Max}(O F-0.32,0.0))$ explains the variance in $f_{4}$ with $R^{2}=83.4 \%$. This finding validates also the performance of the proposed approach.

The workforce's experience evolution was analysed too. The distribution of this evolution over the four groups of projects is shown by Figure $12(\mathrm{i})\left(F\right.$-value $=9.92$ and $\left.R^{2}=0.699\right)$. The regression analysis of $f_{5}$ presents $\{A, O F, R T, W$, and $\Theta\}$ as the suitable predictors of this objective: $\left(f_{5}^{*}=0.0749 A+19.1 O F+6.39 R T-4.5 E-5 W+28.7 \Theta-40.6\right.$, with $R^{2}=59.87 \%$ ).

But as shown, almost all instances suffer from experience degradations $\left(f_{5}<0\right)$. As discussed in the illustration example and also presented by the current authors in Attia, Dumbrava, and Duquenne (2012), there are many other factors that manage the development of workforce experience. These factors include: the number of skills under development for each worker - the number of workers following the PMSD, learning/forgetting speeds, degrees of similarities between skills, minimum accepted efficiency $\theta_{k}^{\min }$ and the conservation policy of the future working flexibility.

## 8. Conclusions

In this work, we modelled the problem of multi-period staff allocation, with flexible human resources taking into account the flexibility of working time, versatility, learning and unlearning phenomena. This model was then used to elaborate an algorithm intended to solve project planning problems, and based on a representation of sets of decisions by genetic algorithm. In this approach, each chromosome holds three series of priority decisions monitoring the search for a solution: the solution method uses the decisions about human resources allocation as a major parameter to manage and develop skills in the company.

The results obtained on a small example clearly illustrate that if the company is constantly oriented towards minimising the project cost by systematically affecting the actors on their core competence, a depreciation of the overall range of skills, and thus of the manpower flexibility, will result. Therefore, the companies that wish to develop their responsiveness through staff versatility should accept the additional labour cost linked to the work of non-ideally skilled operators: this additional cost of industrial activities can be considered as an investment in human resources and in the company's future adaptability. The amount of these additional costs can be optimised either in function of some labour-associated or skills-associated parameters.

The performance of this approach was investigated, relying on a set of 400 projects with different characteristics. The results showed the stability and the robustness of the approach regarding the different performance criteria.

As perspectives, we will first work to apply this model to case studies of decision support in terms of competence management. Finally, as an attempt to integrate the social factors such as workers' preference, a future work will consist in developing and testing a multi-criteria approach to this model.

## Funding

This research was supported by the Ministry of Higher Education \& Scientific Research, Arab Republic of Egypt/Cultural Affairs \& Missions Sector [grant number CAM-75-fm-01/06].

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[^1]:    *According to the French working laws.

