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An application of stochastic simulation to the study of the variability of road induced fatigue loads.

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Abstract

To adapt their products to the durability requirements, car manufacturers must possess a suitable methodology to quantify the large variety of road induced fatigue loads that vehicles' components undergo during their life. The load data acquisition process is generally based on long and costly measurement campaigns. Here an alternative load characterization methodology is proposed. It is based on stochastic modelling and simulation rather than purely on a collection of load measurements. Stochastic models, in the form of random processes, are proposed for vehicle-independent influential factors, namely road roughness and vehicle's speed. Random realizations are generated and used as inputs for multi-body simulations, describing the dynamics of any given vehicle. An arbitrarily large set of fatigue loads can therefore be obtained inexpensively, for any given elementary life situation, and subsequently, for the total life of the vehicle, using customer usage information. A study of the influence of a change in market region is performed as an illustration of the stochastic simulation methodology. For this study, existing measurements carried out by Renault within the considered market regions, are used as a source of statistical information.

Keywords: Load variability; Durability; Road roughness; Stochastic modelling; Vehicle dynamics simulation;

1. Introduction

During their life, vehicles and their components undergo a large variety of fatigue loads due to the excitation of the road. In each of their markets, car manufacturers such as Renault, seek to adapt their products to meet local and customer-related durability requirements, see [1] for an overview. Therefore the need for a suitable load characterization methodology is quite critical. The acquisition of relevant statistical information about fatigue loads imposed onto vehicles' components, is usually based on load measurement campaigns, see [2].

The constitution of sufficiently large samples through numerous measurements, using an heavily instrumented vehicle, is long and expensive. Also, the information gathered through such campaigns may be highly dependent on the characteristics of the vehicle used for the measurement. This is a serious issue since the campaign cannot be carried out with a new vehicle in early design stages, nor repeated for an important number of vehicles with different characteristics.

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The present paper proposes an alternative methodology to build a quantitative description of the variability of road induced vertical loads. Rather than collecting numerous, expensive and inevitably vehicle-dependent samples of load data from measurements, the latter are generated through simulation. For each elementary life situation, the different factors influencing the dynamics of the vehicle, mainly road roughness and vehicle speed for vertical loads, are sampled as realizations of random processes and used as inputs for a multi-body simulation (MBS) tool. Subsequently, the sets of simulated load time histories may be post-processed to quantify the variability of the fatigue loads acting on the vehicle.

The core principle underlying the proposed methodology is the following. Statistical variability is not directly studied on sampled sets of output values, namely mechanical responses or loads, but rather on input values, labelled as influential factors, which uniquely determine a set of outputs that can be obtained through simulation. If representative stochastic models can be proposed for vehicle-independent factors, and sufficiently accurate simulation tools are available, large sets of load data can therefore be created inexpensively for any given vehicle or component.

The outline of the paper is the following. Section 2 introduces the issue of the variability of road induced fatigue loads, as well as the approach commonly employed to assess the latter, namely load measurement campaigns. Subsequently, a methodology based on stochastic modelling and simulation, is proposed as an alternative approach to study the variability of road induced vertical loads. In section 3, an overview on road roughness is given and a stochastic model is chosen from literature to describe it. An efficient input estimation algorithm is proposed, in order to gather statistical information on road roughness using the measurement of a vehicle's responses. In section 4, data sets from existing load characterization campaigns carried out by Renault, are processed through the estimation algorithm and the results are used to identify the parameters of the selected stochastic models. Several campaigns are then simulated, based on that stochastic information. Eventually, in section 5, the results of the simulated campaigns are used to study the influence of a change in market region, on the variability of the fatigue life of the vehicle's components. The influence of a change in vehicle characteristics is also analysed. These studies serve as an illustration of the interest of the proposed methodology.

2. Stochastic simulation of load characterization campaigns

2.1. Characterization of load variability

The variety of fatigue loads experienced by vehicles within a population of customers is arguably very important. In this paper, the attention will be restricted to vertical excitations only, which represent for many components, the main determinant of their fatigue life. Thus, this excludes longitudinal and lateral excitations respectively due to braking, accelerating and steering. To carry out a statistical assessment of the variability of loads, *i.e.* obtain quantified information relative to the environment in which vehicles evolve, samples of load histories must be collected within the population of customers. One should note that these histories are often converted into quantities more amenable to statistical analysis, see examples in [2, 3] and details in [4–6]. The latter include rainflow matrices, load spectra or so-called pseudo-damage values defined by:

$$d_y = \sum_{i=1}^{p} \Delta y_i^{-\beta} \tag{1}$$

where d_y is the pseudo-damage value, $(\Delta y_i)_{i=1..p}$ are the amplitudes corresponding to the Rainflow ranges extracted from a load signal y, β is the exponent of a Basquin relation. A pseudo-damage value is not related to a particular fatigue life, but it represents a scalar criterion which allows to compare loads, while accounting for the damage accumulation phenomenon. Here, it is relevant to extend the definition of loads to include mechanical responses (such as displacements or accelerations) that can be related to loads through a monotonic relation [1]. For example, acceleration cycles could be associated to stress cycles in a given component of the vehicle.

The selection of load samples within the population of customer is a very complicated statistical question [1, 3]. One could randomly sample customers within the population and record a significant portion of the life of their vehicle (say a thousand kilometers). This approach may not be very time and cost-efficient as the diversity of customers will make it difficult to constitute a statistically representative sample. Studying the variability of road induced fatigue loads across the complete life of vehicles, can prove to be impractical. Among a population of customers, it is likely that the diversity of usage will significantly contribute to the overall variability of the loading history.

Consequently, it is helpful to divide the history of loads experienced by the vehicle into multiple segments corresponding to what is denoted as elementary life situations, see similar approaches in [2–4]. The load history corresponding to the whole life of the vehicle can then be reconstructed, if the relative contributions of the different elementary life situations are known. Statistics about these contributions, within a population of customer, can be collected through surveys in which potential customers are questioned about the usage of their vehicle. Table 1, gives a example of the definition of fifteen elementary life situations and the mean result of a survey carried out among approximatively 2000 customers, in a given country.

Road type	0% payload	50% payload	100% payload
City road	20.6%	21.2%	9.8%
Country road	6.7%	7.8%	6.7%
Highway	5.4%	6.3%	4.4%
Mountain road	0.8%	1.0%	1.1%
Off road	2.0%	3.2%	3.0%

Table 1. Mean customer usage and the related elementary life situations. Survey carried out among approximately 2000 customers.

In this framework, load measurement campaigns consist in acquiring statistically representative load samples for each of those elementary life situations. The generation of so-called virtual customers can then be achieved by mixing load related values with usage statistics [2]. Although more rapid and cheap than a purely random sampling of customers, this approach based on measurements is still costly and depending on a particular vehicle.

2.2. Proposed methodology based on stochastic simulation

Simply put, the proposed methodology may be seen as analogy to the (physical) load measurement campaign, although in the numerical domain. Indeed, it is based on the same idea of the division of the life of the vehicle into elementary life situations. The specificity of the methodology is that the samples used to quantify the intrinsic variability of the loads associated to each life situation, generally collected through measurements, are gathered instead through the use of vehicle dynamics multi-body simulation (MBS) and stochastic modelling of input parameters.

In the present case of the simulation of vertical loads, three factors are considered, namely road roughness (geometry of the road profile), vehicle's speed and vehicle's payload. They are subsequently denoted as influential factors, in the sense that they represent varying quantities which may influence the value of the loads acting on the vehicle's components. From the standpoint of multi-body simulation, they represent input parameters. Road roughness and vehicle's speed are continuously varying parameters and they may be described using stochastic processes representations. Different stochastic models, mostly in terms of their constituting parameters, can be assigned to different elementary situations.

As described on figure 1, random realizations are drawn from the corresponding stochastic models and used as inputs to carry out numerous multi-body simulations. This yields arbitrarily large sets of load histories, for any given vehicle and for each elementary life situation. Those histories may then be processed into fatigue related criteria, as discussed in the previous section. Eventually, the simulated load data and the usage data are combined, thus generating loading histories for the total life of vehicles, within a population of virtual customers.

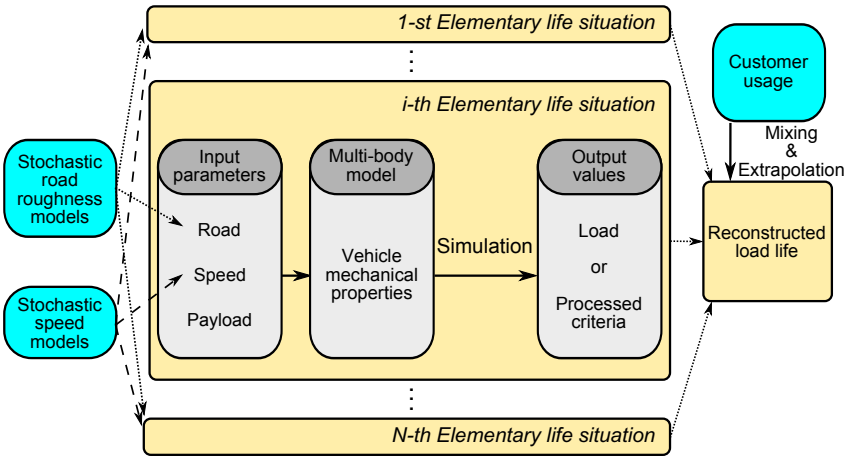


Fig. 1. Overall structure of the proposed methodology: generation of a simulated campaign.

Stochastic process theory, see *e.g.* [7], has been commonly studied and applied for the modelling of loads and fatigue life prediction, see *e.g.* [8]. Here, stochastic models are proposed for road roughness and vehicle's speed. The core conceptual idea of the proposed methodology is that the statistical variability is not directly studied on sampled sets of output values, namely mechanical responses or loads, but rather on (vehicle-independent) input values. Loads are then deduced from those random input parameters using simulation. The proposed methodology is particularly well adapted to the sub-division of the vehicle's life. This latter technique can be seen as a implementation of stratified sampling. Each elementary life situation correspond to a so-called strata and the intrinsic variability of the loads may be different from one strata to another, while the relative weight of each strata is known (in statistical terms). More importantly in the context of stochastic modelling, this makes the identification of stochastic models easier. Indeed, within each life situation, the variation of a particular influential factor may be more 'statistically stable'.

With the proposed simulation-based methodology and contrary to the use of measurements, arbitrarily large sets of load samples can be generated rapidly and at a very low cost. Additionally, the simulation can be carried out for any desired vehicle, given a representative multi-body model. This makes the methodology particularly adapted to provide information in early design stages or for vehicles with different characteristics, again at a very low cost. Eventually, focusing on input parameters rather than output values allows to consider sensitivity analysis, which is either impossible or very costly when using measurements.

3. Stochastic modelling of road roughness

3.1. Road roughness characterization and modelling

Road roughness designates geometric irregularities of the road surface that are susceptible to generate dynamic responses on vehicles. The issue of the characterization of road roughness has generated much research work, see *e.g.* [9, 10] for a detailed description and history. For the simulations considered in this paper, it will be sufficient to focus on longitudinal sections of the road surface, namely road profiles. Here, it is necessary to acquire statistical information on road profiles in order to be able to propose a model, in the form of a stochastic process, which will be subsequently used to generate random road profiles and feed numerous multi-body simulations. Such a model must constitute a representative description of the intrinsic variability of the road profiles covered by customers within a particular elementary life situation.

The use of stochastic processes to model road profiles traces back to [11], where a formal mathematical context was introduced, stating that: 'road surfaces appear amenable to representation as realizations of random processes'. Multiple models have been developed, see *e.g.* [12–14], but overwhelmingly they rely on frequency-based descriptions, either implying Gaussian or non-Gaussian processes, and stationary or

non-stationary processes. Such processes are characterized, either fully or partially, through estimates of their auto-correlation function, or equivalently in the frequency domain, of their power spectral density function (PSD), see details about stochastic process theory and inference in [7].

Although it represents a key contributor to the results of simulated campaigns, thorough discussions on stochastic modelling of road profiles are not the central focus of the present paper, and the reader may refer to [12–14] for additional details. The methodology proposed in this paper remains valid, no matter what stochastic models are chosen. Here a structure for the road model is selected from literature. A two-parameter model is adapted from work [12], and deemed sufficient to carry out the study of the influence of a change in market region, in section 5. The stochastic road model is a piecewise-stationary Gaussian process with a PSD function of the form:

$$S_u(n) = S_0 \left(\frac{n}{n_0} \right)^{-w} \tag{2}$$

$S_u(n)$ is the PSD of the road profile $u(x)$, S_0 is the ISO roughness coefficient [15], n is the spatial frequency (generally comprised between $0.01m^{-1}$ and $10m^{-1}$), $n_0 = 0.1m^{-1}$ and w is the waviness, set to $w = 2$ in the ISO standard. The non-stationary character of the process lies in the fact that realizations of road profiles are constituted by $j = 1, ..., M$ sections of identical length, here $200m$ is selected from experience, on which the process is stationary and has variance σ_j^2, such that:

$$\frac{1}{M} \sum_{j=1}^{M} \sigma_j^2 \approx \sigma^2 \tag{3}$$

where σ^2 is the variance of the complete road profile. The PSD function on each section remains proportional to the form in equation (2), and the normalized variance $r_j = \sigma_j^2/\sigma^2$ follows a Gamma distribution with shape parameter θ and mean value $E[R] = 1$. In practice, a random profile is generated with the target PSD of equation (2), and the constituting $200m$-long sections are weighted by a set of random coefficients $\left(r_j\right)_{j=1..N}$, drawn from a Gamma distribution. An example of road profile generated according to this model is shown on figure 2. Details about the random generation of realizations from this model are given in [12].

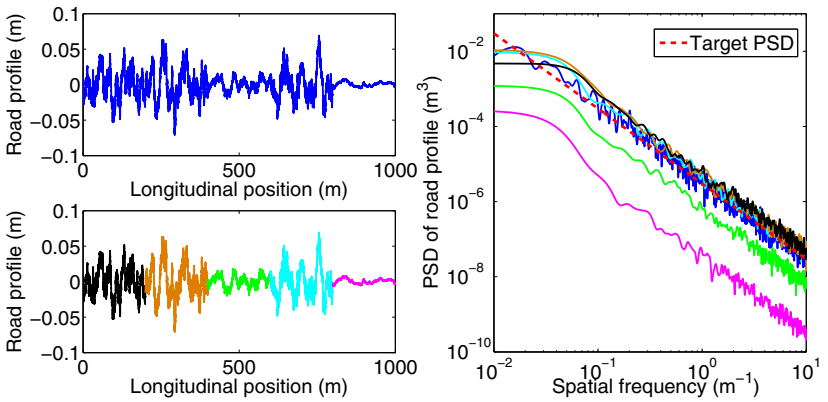


Fig. 2. Example of randomly generated road profile (left). The estimates of the PSD across the whole profile and on each section (colored) are displayed (right).

The two parameters of the model, namely the roughness coefficient S_0 and the Gamma distribution shape parameter θ, may be identified easily when observed road profiles, acquired here through the estimation technique presented in the following subsection, are available. A straightforward approach consists in calculating the PSD estimate across the whole profile and identify S_0 through a simple least-squares regression using equation (2). Normalized variances $\left(r_j\right)_{j=1..N}$ can also be calculated on each $200m$-long sections

and the gamma distribution shape parameter identified through a maximum likelihood identification. Other inference procedures, or sources of data, can be considered, see *e.g.* [16].

This relatively simple model offers the ability to account for the non-stationary character of real roads observed in practice, see *e.g.* [13]. Its small number of parameters may not be sufficient to describe very precisely every particular road profile, however, on average, this model may describe a large variety of roads that can be encountered within a given elementary life situation, which is a key component for the analysis of load variability through stochastic simulation.

3.2. An input estimation algorithm to gather statistical information on road roughness

In order to identify the parameters of the stochastic model selected above, one must possess samples of road profiles, representative of the different elementary life situations. The most straightforward approach is to carry out direct measurements of road profiles, using profile measuring equipments such as inertial profilometers or car trailers, see *e.g.* [9, 10]. In this paper, it is proposed to acquire road roughness information, not from direct measurements but rather through the processing of measured dynamic responses of a given vehicle. This allows to extract valuable information from existing measurement data sets (in the considered market regions), which contain signals recorded from sensors mounted on the vehicle, namely displacement and acceleration signals.

In the framework of the proposed methodology, this estimation method represents an interesting approach to gather statistical data on road roughness, without specific measuring equipments, thereby enabling large scale road characterization campaigns (much shorter and cheaper than the campaigns carried out to characterize load variability) using a vehicle with only a few sensors. National databases containing information on the road networks could also be considered as a source of information for the stochastic road model, see [16].

Various solutions have been proposed to address the issue of the estimation of road profiles from a vehicle's responses, see *e.g.* [17, 18]. Here an efficient data-processing method is proposed, based on linear Kalman filtering, see *e.g.* [19] for theoretical details on Kalman filtering. The search for an excitation signal, given the response of a physical system, involves the resolution of an inverse problem. The latter have a tendency to behave chaotically, and must be handled with specific mathematical care in order to get viable results, see [20] and the references therein. Kalman filtering constitutes an interesting framework to deal with inverse problems, as proposed by [21], and applied to the issue of road profile estimation in [22].

The method considered here, proposed by the authors in [23], is used to process the accelerations of the sprung and unsprung masses, the suspension displacement, as well as the vehicle's speed, all acquired on a measuring vehicle, into an estimate of the road profile imposed onto the latter. A quarter-car model of the vehicle is considered, see Appendix A. The estimation proceeds using an iterative scheme and according to the algorithm summarized in Appendix B. In a nutshell, the aumented Kalman filter represents a very fast algorithm, which efficiently combines the information originating from different sensors, using an imprecise but simple linear quarter-car model of the vehicle, while accounting for modelling errors in a controlled manner. The estimation method has been validated with experimental data in [23] and an illustration of this validation is given on figures 3 and 4.

The method provides a very good estimation of the road profile in the low and medium frequency parts of the spatial frequency spectrum (between $0.05m^{-1}$ and $1m^{-1}$) as seen on figures 3 and 4. Consequently, in section 4, where the method is applied to extract data from an existing measurement campaign, the roughness coefficient S_0 will be identified, through (2), by restraining the identification on that frequency band. The very low frequency difference between both the true and estimated profile, due to the pre-treatment of measurement data, is of minimal importance as it does not produce significant responses on the vehicle.

4. Simulation of a stochastic campaign

The proposed methodology is applied here on two practical examples. Stochastic campaigns are simulated in order to represent two different market regions and two different sets of mechanical characteristics for the vehicle.

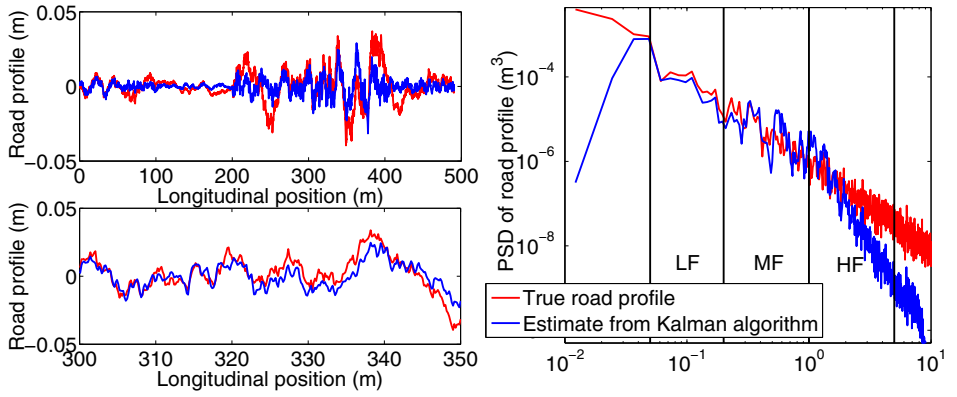


Fig. 3. Simulation based-validation. The road profile (in red) is injected in a state of the art multi-body simulation (MBS) software, comprising a described model of the vehicle and its tyres. The simulated responses are fed into the estimation algorithm and the resulting estimate (in blue) is compared with the true profile. Three frequency band of interest are defined, namely low (LF), medium (MF) and high frequency (HF).

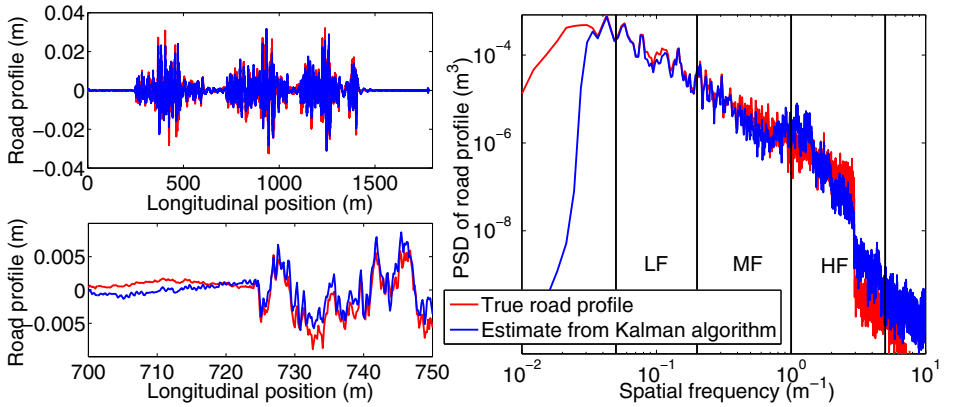


Fig. 4. Measurement based-validation. The road profile (in red) is injected through a vertical test-rig composed of 4 actuators, into the physical vehicle. The measured responses are fed into the estimation algorithm and the resulting estimate (in blue) is compared with the true profile injected into the test-rig. Three frequency band of interest are defined, namely low (LF), medium (MF) and high frequency (HF).

4.1. Acquisition of statistical information on influential factors

The road estimation algorithm described in subsection 3.2 is applied on an existing data set, acquired through a load measurement campaign, carried out by Renault in two particular market regions. A large set of road profiles (approximatively 50km-long each) is estimated in relation with each of the defined road types, see table 1, and the parameters of the corresponding stochastic road models are identified according to the inference procedure defined in subsection 3.1.

The issue of vehicle's speed is a highly complicated statistical question. In this paper, the speed of the vehicle is assumed to be constant across simulated samples. As far as the inference of a stochastic model is concerned, vehicle's speed is here simply equated to its mean value v_m, calculated on a particular recording. Statistical information about the mean speed and its distribution is extracted from the considered load measurement campaign. Let us point out that speed-related information is much more easy to acquire than road related information, as the former do not require particular sensors other than the vehicle's odometer, nor do they imply the resolution of an inverse problem.

Each recording (approximatively 50km-long) correspond to a particular value for S_0, θ and v_m. All

statistical data describing the variation of influential factors are summarized in tables 2 and 3 for the two selected regions.

	Region A		
Road type	Roughness S_0 mean value (std.dev) in $m^3 \times 10^{-6}$	Gamma distribution parameter θ mean value (std.dev)	Vehicle mean speed v_m mean value (std.dev) in km/h
City road	114.38 (44.98)	1.28 (0.46)	25.31 (4.76)
Country road	75.26 (39.54)	0.93 (0.56)	43.26 (5.74)
Highway	9.34 (4.89)	1.10 (0.51)	83.73 (14.81)
Mountain road	83.13 (27.57)	1.54 (0.55)	35.96 (3.34)
Off road	603.63 (189.02)	1.98 (0.72)	23.09 (3.14)

Table 2. Parameters of the stochastic models for the different road types. All results are obtained from the processing of the responses recorded during a real load measurement campaign in region A. Standard deviations are given for the road and speed parameters.

	Region B		
Road type	Roughness S_0 mean value (std.dev) in $m^3 \times 10^{-6}$	Gamma distribution parameter θ mean value (std.dev)	Vehicle mean speed v_m mean value (std.dev) in km/h
City road	155.54 (70.59)	0.62 (0.17)	41.24 (7.00)
Country road	62.69 (33.04)	0.80 (0.48)	63.12 (14.34)
Highway	24.99 (20.06)	1.15 (0.61)	88.09 (14.52)
Mountain road	69.02 (37.45)	1.24 (0.59)	58.49 (21.75)
Off road	793.54 (1023.8)	1.71 (0.58)	45.29 (17.53)

Table 3. Parameters of the stochastic models for the different road types. All results are obtained from the processing of the responses recorded during a real load measurement campaign in region B. Standard deviations are given for the road and speed parameters.

	Region A			Region B		
Road type	0% payload	50% payload	100% payload	0% payload	50% payload	100% payload
City road	20.6%	21.2%	9.8%	26.3%	22.7%	4.3%
Country road	6.7%	7.8%	6.7%	5.5%	15.1%	4.2%
Highway	5.4%	6.3%	4.4%	6.9%	8.1%	4.0%
Mountain road	0.8%	1.0%	1.1%	0.02%	0.05%	0.03%
Off road	2.0%	3.2%	3.0%	0.4%	1.8%	0.6%

Table 4. Mean customer usage in region A and B. Survey carried out among approximately 2000 customers for region A and 500 customer for region B.

4.2. Construction of the stochastic campaign

Either when considering the planning and analysis of a measurement campaign or a simulated one, several difficult statistical questions are to be considered, namely:

- How long should measurement records or simulated signals be?
- How many simulations should be carried out in order to construct a sample which is statistically representative of the population?
- Is load variability constant throughout the life of the vehicle for a given customer?

- If not, how can one extrapolate the load information acquired on a few dozens or hundreds of kilometres to the complete life of the vehicle, often corresponding to several thousands of kilometres for each elementary life situation?

Those statistical questions are likely to have important consequences on the estimation of load variability for the total life of the vehicle, see [1, 3] for discussions on the matter. Here, the objective is not to bring answers to all of these questions. Additionally, the answer may vary from one population to another, one type of vehicle to another or from one life situation to another. In this paper, a comparative study is proposed rather than a absolute prediction of load variability, and some of these difficulties may be somewhat circumvented. Nonetheless, they must be as seriously considered when constructing and exploiting the simulated campaign, as they would be for the planning and analysis of a measurement campaign. The methodology proposed in this paper, illustrated on figure 1, can be practically implemented in slightly different variants, depending on the particular statistical hypotheses that are considered. The following implementation will be used based on engineering insight and experience.

First the stochastic simulation of loads is realized. For each elementary life situation, 50 simulations are carried out, using a road profile with a length of $20km$. For each simulation, the road profile is drawn from the stochastic model presented in subsection 3.1, with parameters S_0 and θ. The latter are themselves realizations of a normal distribution for which the parameters are taken from tables 2 and 3. The vehicle's speed v_m is constant along the $20km$-long road profile. Within each set of fifteen simulations, corresponding to the different life situations (thus defining a simulated customer), a correlation is enforced when drawing v_m values, in order to take into account the fact that a customer with a severe behaviour is likely to be severe in all situations, and vice versa. The values v_m are drawn from a multi-normal distribution, where the marginal means as well as the diagonal terms of the covariance matrix are adapted from tables 2 and 3, and the extra-diagonal terms are set in a way that the correlation coefficient between pairs of realizations is $\rho = 0.8$ (from experience). The payload is adapted to three different levels, namely $m_{s1} = 380kg$, $m_{s2} = 420kg$ and $m_{s3} = 460kg$, simply by modifying the sprung mass in the model used for the simulation.

The model used here for the simulation is the quarter-car model described in Appendix A. This latter model is not be expected to predict the loads imposed on components very accurately, but it represents an interesting tool to gather quantitative information on load variability at a reasonable simulation cost.

Pseudo-damage values are calculated, namely for the sprung and unsprung acceleration, for all the different simulated responses and according to equation (1). The latter may be use to compare what may be thought of as the relative 'severity' of loads, both for body and chassis components. The Basquin coefficient is taken as $\beta = 6$. It must be recalled here that this selected Basquin coefficient is not exactly representative of the fatigue life of a particular component, but rather used in order to construct a scalar criterion amenable to statistical analysis and accounting for the damage accumulation phenomenon.

Secondly, for each usage arrangement taken from the usage enquiry, a simulated set of fifteen pseudo-damage values $\left(d_{yj}\right)_{j=1..15}$ (5 road types, 3 payloads) is associated at random. A target mileage is defined at $200000km$, for the present study. The mileage corresponding to the different life situations is adapted so that the proportion between life situations is respected and the total mileage corresponds to $200000km$. The pseudo-damage values corresponding to the simulations of $20km$ are extrapolated to values representing a higher mileage (in general thousands of kilometres), for each life situation. The extrapolation is made through a simple proportionality rule (thus assuming that the load variability is time-invariant throughout the life of the vehicle). Eventually, the contribution of all elementary life situations to the total damage are summed, thus constituting one simulated customer. A population as large as the product of the number of simulated customers by the number of surveyed customers, can in principle be constructed.

To sum up, the complete implementation of the stochastic simulation methodology that is employed to perform the simulated campaigns is illustrated on figure 5.

4.3. Comparison of the simulated stochastic campaigns

Here the multiple arrangements of usage are randomly associated to values taken from the 50 simulated customers. A population of 2000 virtual customers is obtained for region A and of 500 virtual customers for

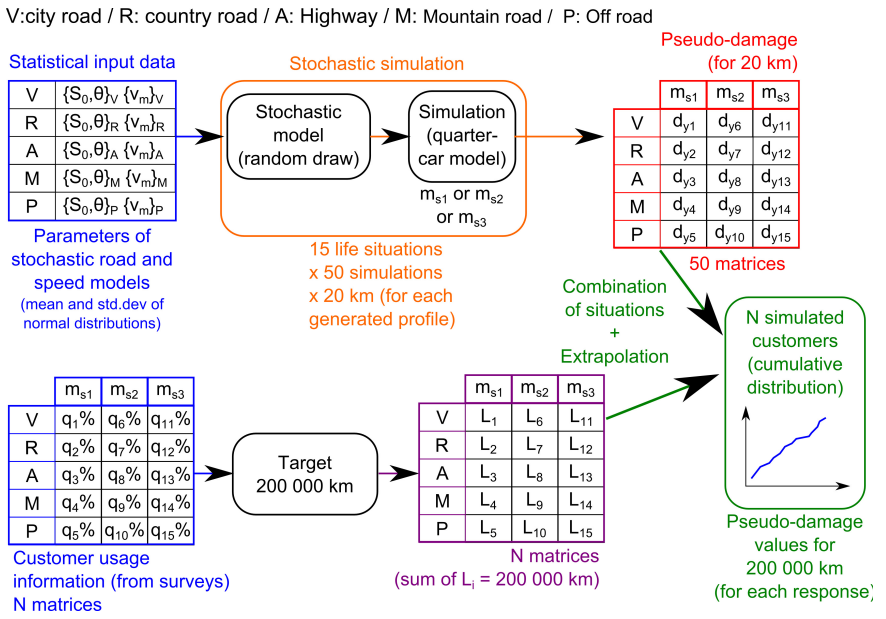


Fig. 5. Illustration of the practical implementation of the stochastic simulation methodology: generation of simulated campaigns.

region B. The populations are displayed on a distribution (quantile) plot in figure 6, for both the unsprung acceleration pseudo-damage and sprung acceleration pseudo-damage.

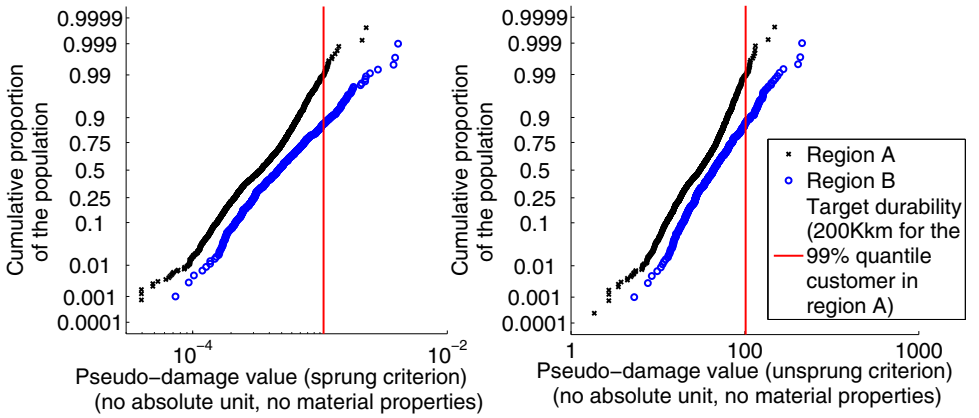


Fig. 6. Populations generated using the stochastic simulation methodology, for two different market regions.

The obtained distributions may used as the basis for multiple comparisons. In what follows, the pseudo-damage value associated with the 99% quantile of the population (for region A, in red on figure 6) is considered.

5. Influence of a change in market region or vehicle characteristics on fatigue life

Two stochastic campaigns have been simulated in view of representing two different market regions. An additional stochastic campaign is generated, using the same statistical information on influential factors as

used for the first campaign in region A, but with a different value for the tyre stiffness, namely an additional 25%.

As shown on figure 6, the result of a stochastic campaign can be a distribution of pseudo-damage values for any desired response signal. In this section, a simple calculation is considered as an illustration of the link between load variability and the fatigue life of components. The difference in terms of loads, between market regions, is analysed in terms of fatigue life. The following calculation is carried out.

An hypothetical component is studied. It is supposed that, (through means not described in this paper) the design of the latter component guarantees that its mean (average) fatigue life is exactly equal to $200Kkm$ for the customer corresponding to the 99% quantile of the population in region A. Hence the pseudo-damage value corresponding to this 99% quantile represents a target durability requirement, *i.e.* the 'severity' that the component must be able to withstand. It is displayed on figure 6. Then, assuming that for a given customer, the damage accumulation is a time-invariant function of the covered mileage, the mean fatigue life for the different customers within the population, is derived from a simple proportionality rule:

$$\bar{M}_{fl}(q) = \frac{R_t}{d_y(q)} * 200000 \tag{4}$$

where $M_{fl}(q)$ is the mean fatigue life corresponding to a quantile q of the population of customers, R_t is the mean resistance target (or durability requirement), here selected with respect to the 99% quantile of the population associated with region A, $R_t = d_y^A(99)$. $M_{fl}(q)$ is a mean value in kilometres. The calculation is done for an hypothetical component, sensitive to cycles in the sprung acceleration. The results are displayed on figure 7. Let us note that, the durability requirement selected here, which may appear very loose (on average 1% of the population will experience a failure before $200Kkm$), is set only for an illustration purpose.

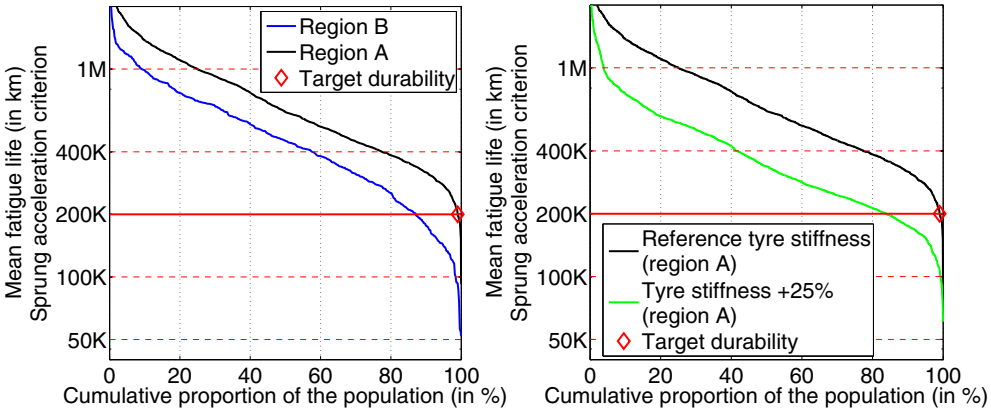


Fig. 7. Distribution of mean fatigue life across the population. The resistance of the component is set in a way that, on average, the fatigue life related to the customer corresponding to the 99% quantile of the population of region A, is 200Kkm. Comparison between region A and B (left). Comparison between 2 values of tyre stiffness (right).

As shown on figure 7, it is imperative to proceed to the acquisition of new information on loads (since the fatigue life of components may be seriously affected if no corrective action is taken), for example, when the vehicle is commercialized in a different market or when its mechanical characteristics are significantly modified. This application illustrates the strength of the proposed methodology. Indeed, the available statistical information on the variability of influential factors such as road roughness and vehicle's speed has been converted inexpensively (approximatively one hour of calculation here) into information on load variability, through simulation. Additionally, load information may be obtained for any given vehicle, and for any response signal on the vehicle (if the vehicle model is sufficiently accurate to predict it). Eventually, the

proposed methodology is a very interesting tool to carry out sensitivity analysis, *i.e.* study the influence of different factors, as has been partly done in this section.

Let us note that, in this paper, the input data on influential factors has been extracted from a measurement campaign, involving a heavily instrumented vehicle, but this is by no means the most straightforward manner to collect such data. It could be collected through more specific characterization campaigns, for example, involving the use of a lightly instrumented vehicle (only a few displacement and acceleration sensors) and the proposed road estimation algorithm.

Let us also emphasize that the central objective of this paper is to describe the proposed stochastic simulation methodology. A practical implementation (*i.e.* through the selected models) of the methodology is proposed here in order to illustrate its functioning using real data. Arguably, the models that have been selected here for the probabilistic description of road and speed profiles are quite simple and more complex models may be necessary depending on the precision that is needed when studying the loads acting on certain components. For example, the effect of occasional transients events such as bump stops and pot-holes represent a particularly complex issue, since such elements may come in multiple forms and amplitudes but also because they generally induce a strong correlation between road and speed profiles. Such modelling difficulties represent a perspective of the present work.

6. Conclusion

A methodology has been proposed in order to gather quantitative information on road induced vertical fatigue loads and their variability, within a population of customers. It is based on stochastic modelling and simulation rather than purely on a collection of load measurements from this population. The strength of this approach lies in the fact that statistical analysis effort is focused on input factors such as road roughness and vehicle's speed, rather than on output values such as vehicle responses or loads, which collection is necessarily long, expensive and dependant on the vehicle used for the measurement. Output quantities can then be obtained through simulation.

Conceptually, the methodology combines the use of stratified sampling, whereby the life of the vehicle is divided into elementary life situations and stochastic models can be proposed more easily, with the advantages of simulation. Quantitative information on the variability of loads can be derived inexpensively for any given vehicle or any response signal on such vehicle. The necessary acquisition of data on vehicle-independent influential factors is generally much cheaper (few sensors involved) than the direct measurement of load samples and the variability of such factors can be more easily and extensively described. For that purpose, the use of an estimation algorithm, which extracts data on road profiles from a vehicle's responses, has been proposed here, see [23] for details.

In this paper, simple models have been used either for road roughness, vehicle's speed or the dynamics of the vehicle. Obviously the methodology remains valid for more complex models, which may be more adapted for fatigue life predictions than simple models, more suited for relative or sensitivity analysis.

The proposed methodology constitutes a powerful tool to carry out sensitivity analysis in a statistical framework. This has been illustrated on a practical example in the present paper, where the study of both a change in market region and vehicle characteristics has been considered. The methodology yields objective information on the effect that such factors may have on loads and their variability and subsequently, on the variability of the fatigue life related to the different components of the vehicle, within a population of customers.

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Appendix A. 2-DOF, quarter car model of the vehicle

The model considered both for the simulation of stochastic campaigns, but also as a part of the road estimation algorithm, is a 2-Degrees-of-Freedom (DOF) vehicle model or quarter car model, see [24]. Its parameters are defined on figure A.8.

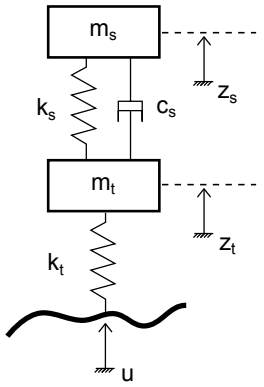


Fig. A.8. Quarter-car model of the vehicle.

The matrices associated with the selected model and measurement setting (sensors mounted on the vehicle) are defined as follows:

$$\mathbf{A} = \begin{bmatrix} -c_s/m_s & c_s/m_s & -k_s/m_s & k_s/m_s \\ c_s/m_t & -c_s/m_t & k_s/m_t & -(k_s+k_t)/m_t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 \\ k_t/m_t \\ 0 \\ 0 \end{bmatrix} \tag{A.1}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ -c_s/m_s & c_s/m_s & -k_s/m_s & k_s/m_s \\ c_s/m_t & -c_s/m_t & k_s/m_t & -(k_s+k_t)/m_t \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_t/m_t \end{bmatrix} \tag{A.2}$$

where the state vector is defined as $\mathbf{x} = [\dot{z}_s, \dot{z}_t, z_s, z_t]$ and the measurement vector is defined as $\mathbf{y} = [z_s - z_t, z_s, \ddot{z}_s, \ddot{z}_t]$.

Appendix B. Road estimation algorithm

The augmented Kalman filter seeks to estimate the time evolution of the state vector, to which the excitation to be estimated is included, and using :

- Knowledge on the dynamics of the vehicle and its mechanical characteristics (here a quarter-car model is used)
- Information provided by the different measurement sensors mounted on the vehicle
- *A priori* information about modelling errors, measurement noises or the statistical characteristics of the excitation to be estimated. This information is used to regularize the inverse problem, see [21, 23] for details.

The evolution of the state vector $\mathbf{x}_k$ is described by the equation:

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d u_k + \mathbf{w}_k \tag{B.1}$$

where $\mathbf{A}_d$ and $\mathbf{B}_d$ respectively traduce the dynamic behaviour of the system and the influence of the excitation u_k on the system. They are derived from the time discretization of the solution of the linear ODE describing the system, such that $\mathbf{A_d} = e^{\mathbf{A}dt}$ and $\mathbf{B_d} = [(e^{\mathbf{A}dt}) - \mathbf{I}]\mathbf{A}^{-1}\mathbf{B}$. $\{\mathbf{w}_k\}$ is a zero-mean Gaussian white noise process of covariance matrix $\mathbf{Q}$, which is used to account for potential discrepancies between the actual behaviour of the system and its model. Additionally, the measurement vector $\mathbf{y}_k$ taken at time t_k can be related to the state of the system through the measurement equation:

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}u_k + \mathbf{v}_k \tag{B.2}$$

where $\mathbf{C}$ and $\mathbf{D}$ depends on the system's model and on the physical quantities being measured, here, the suspension displacement and the sprung and unsprung acceleration. $\{\mathbf{v}_k\}$ is a zero-mean Gaussian white noise process of covariance matrix $\mathbf{R}$, which components are used to account for disturbances in the measurement devices or the ability of the model to exploit the measured responses.

An *a priori* evolution of the excitation process to be estimated is postulated, as a way of regularizing the inverse problem, see [21, 23]:

$$u_{k+1} = u_k + \eta_k \tag{B.3}$$

where $\{\eta_k\}$ is a zero-mean Gaussian white noise process of variance S. The state equation of the system is then:

$$\left\{ \begin{array}{c} \mathbf{x}_k \\ u_k \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A}_d & \mathbf{B}_d \\ 0 & \mathbf{I} \end{array} \right] \left\{ \begin{array}{c} \mathbf{x}_k \\ u_k \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{w}_k \\ \eta_k \end{array} \right\} \tag{B.4}$$

Introducing the augmented state vector $\mathbf{x}_k^a$, the state equation (B.4) may be written:

$$\mathbf{x}_k^a = \mathbf{A}_a\mathbf{x}_k^a + \left\{ \begin{array}{c} \mathbf{w}_k \\ \eta_k \end{array} \right\} \tag{B.5}$$

and the measurement equation (B.2) becomes:

$$y_k = [\mathbf{C} \;\; \mathbf{D}] \left\{ \begin{array}{c} \mathbf{x}_k \\ \mathbf{u}_k \end{array} \right\} + \mathbf{v}_k = \mathbf{C}_a\mathbf{x}_k^a + \mathbf{v}_k \tag{B.6}$$

The algorithm is initialized on a hypothesized state of the system $\{\mathbf{x}_0, u_0\}$ and the augmented state vector is then estimated recursively. First the estimate of the state of the system $\hat{\mathbf{x}}_{k+1|k}^a$ and its covariance matrix $\mathbf{P}_{k+1|k}$, at time t_{k+1} are predicted using the information available at time t_k:

$$\hat{\mathbf{x}}_{k+1|k}^a = \mathbf{A}_a\hat{\mathbf{x}}_{k|k}^a \tag{B.7}$$

$$\mathbf{P}_{k+1|k} = \mathbf{A}_a\mathbf{P}_{k|k}\mathbf{A}_a^T + \mathbf{Q}_a \tag{B.8}$$

where $\mathbf{Q}_a = diag(\mathbf{Q}, S)$. Secondly, an optimal correction is applied to the estimate $\hat{\mathbf{x}}_{k+1|k}^a$ and its covariance matrix $\mathbf{P}_{k+1|k}$ when the measurement $\mathbf{y}_{k+1}$ at time t_{k+1} is taken into account:

$$\hat{\mathbf{x}}_{k+1|k+1}^a = \hat{\mathbf{x}}_{k+1|k}^a + \mathbf{M}_{k+1}(\mathbf{y}_{k+1} - \mathbf{C}_a\hat{\mathbf{x}}_{k+1|k}^a) \tag{B.9}$$

$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{M}_{k+1}\mathbf{C}_a\mathbf{P}_{k+1|k} \tag{B.10}$$

where the correction gain matrix $\mathbf{M}_{k+1}$ at time t_{k+1} is:

$$\mathbf{M}_{k+1} = \mathbf{P}_{k+1|k}\mathbf{C}_a^T(\mathbf{C}_a\mathbf{P}_{k+1|k}\mathbf{C}_a^T + \mathbf{R})^{-1} \tag{B.11}$$

To sum up, the Kalman-based algorithm consists in processing the measurement vector y using equations (B.7) through (B.11). One must choose tuning values for $\mathbf{Q}$, $\mathbf{R}$ and S in order to account for modelling deficiencies and obtain an acceptable estimate of the road profile.

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