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# Forecasting sovereign CDS volatility: A comparison of univariate GARCH-class models

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## Abstract

Initially overlooked by investors, the sovereign credit risk has been reassessed upwards since the 2000's which has contributed to awaken the interest of speculators in sovereign CDS. The growing need of accurate forecasting models has led us to fill the gap in the literature by studying the predictability of sovereign CDS volatility, using both linear and non-linear GARCH-class models. This paper uses data from 38 worldwide countries, ranging from January 2006 to March 2017. Results show that the CDS markets are subject to periods of volatility clustering, nonlinearity, asymmetric leverage effects and long-memory behavior. Using 7 heteroskedastic and no heteroskedastic-robust statistic criteria, results show that the fractionally-integrated models outperform the basic GARCH-class models in terms of forecasting ability and that allowing flexibility regarding the persistence degree of variance shocks significantly improves the model's suitability to data. Despite the divergence in the economic status and geographical positions of the countries composing our sample, the FIGARCH and FIEGARCH models are mainly found to be the most accurate models in predicting credit market volatility.

**JEL Classification:** G15, G17, C58.

**Keywords:** CDS volatility, Predictability, Forecasting models, Loss functions criteria.

# 1 Introduction

Understanding the fluctuations' dynamic of financial assets has always been of a particular interest in the academic and non-academic spheres. The considerable number of studies focusing on the stock prices' mechanism point out several stylized facts characterizing the financial markets such as: the volatility clustering, the non-stationarity... (See for example [Niu and Wang \(2013\)](#) for a study of the statistical behaviors of the Shanghai Composite Index and Hang Seng Index). Besides the stock markets widely studied, analyzing the characteristics of the credit market, and particularly the sovereign CDS market, is likewise interesting especially when it comes to investigating the impact of financial properties on the suitability of the CDS volatility modeling and forecasting ability.

The curious increase in the empirical studies dealing with modeling CDS data during the last decade can be explained by several reasons: (i) the constantly evolving outstanding amount of the CDS contracts reaching its highest values during the crisis periods, (ii) the need of more clear understanding of the role played by this market in the spread of crises and (iii) and the requirement of identifying the main explaining factors of credit risk. Furthermore, the use of CDS contracts no more as hedging instruments but rather as diversification, trading and speculation instruments has legitimized the usefulness of CDS volatility forecasting to investors for both risk management and portfolio management.

Despite the relevance of the volatility forecasts particularly in the decision process and considering the grown interest in predicting credit spreads, the nonexistence of papers in the literature of CDS spreads dealing with the ability of GARCH models to accurately forecast the volatility of the CDS is completely outrageous<sup>[1]</sup>. The literature on CDS is mainly composed by studies that focus on the determinants of these credit spreads ([Oliveira et al., 2012](#); [Costantini et al., 2014](#); [Fontana and Scheicher, 2016](#)) or the Granger Causal relationship between CDS markets and related markets [Coudert et al. \(2010\)](#); [Longstaff et al. \(2011\)](#); [Coudert and Gex \(2013\)](#); [Sabkha et al. \(2017\)](#). The very few papers that investigate the forecasts of CDS spreads ([Krishnan et al., 2010](#); [Sharma and Thuraiamy, 2013](#); [Avino and Nneji, 2014](#); [Srivastava et al., 2016](#)) only focus on the first moment order, while the predictability of the CDS volatility remains understudied. Yet, these studies try to forecast the CDS spreads based on the commonly known economic and financial determinants and not based on the predictive ability of the econometric models. Considering the foregoing gaps, this study aims to extent the literature by investigating the forecasting performance of 9 GARCH-class models in the sovereign CDS markets from January 2<sup>nd</sup>, 2006 to March 31<sup>st</sup>, 2017.

Our study contributes to the existing literature in several ways: first, as far as we are concerned, none of the previous studies has focused on the predictability of CDS volatility, especially when it comes to the sovereign market. Second, our paper contributes as well to the literature by implementing a larger set of statistical loss function criteria -taking into account the nonzero mean and the heteroscedasticity of the forecast errors - to assess the out-of-sample predictive ability of the models in comparison with existing forecasting papers on financial assets. Third, the comparative study between linear and non-linear ARCH-class

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<sup>[1]</sup>The majority of papers dealing with the predictive power of GARCH models, only focus on the major stock indexes and exchange rates ([Poon, 2005](#))

models provides a better and clearer comprehension of the in-sample and out-of-sample fit of the CDS data. Finally, our data set allows us to draw more robust and worldwide conclusions, as it is composed by CDS spreads for 38 countries from all over the world covering the recent two economic and financial crises when the volatility of asset prices have reached their highest unexpected levels.

Our empirical findings show that the sovereign CDS market is characterized by the same stylized facts as the stock market: volatility clustering, leverage effects and long memory behavior. The results of the diagnostic tests on the in-sample modeling generally show that no model outperforms all the others in terms of fitting. Based on the results the 7 loss functions, the predictive performance of the fractionally-integrated models seems to be more accurate, emphasizing the importance of taking into account the long-range memory and the nonlinear behavior of CDS spreads while forecasting volatility. Among the fractionally-integrated models, our results show that the FIGARCH and the FIEGARCH are the most accurate models, providing the best out-of-sample performances in most cases.

The rest of the paper is organized as follows. A brief literature review of the previous studies predicting financial assets is section 2. Section 3 presents the sample and data used to compare the predictive ability of the models and displays the 9 volatility forecasting models studied. Results of the in-sample and out-of-sample analysis are reported in Section 4. Section 5 concludes the paper.

## 2 Literature review

Investigating the degree to which financial time series can be accurately forecast has always been in the limelight of researchers' issues. The empirical literature on the modeling and predicting volatility processes is extensive and takes into account more and more financial markets properties. Engle (1982) is the first researcher to model financial data through a time-varying stochastic process characterized by a nonconstant correlated variance so-called ARCH model. A generalization of this Autoregressive Conditional Heteroscedasticity model is then proposed by Bollerslev (1986) with more parsimonious and less overparametrization and biasedness in the estimates. Some extensions of this model are afterwards proposed, taking into account more stylized facts of the financial markets: leverage effects (Nelson, 1991; Glosten et al., 1993), stationarity issues (Engle and Bollerslev, 1986), long memory Ding et al. (1993); Baillie et al. (1996); Bollerslev and Mikkelsen (1996); Tse (1998); Davidson (2004)...<sup>[2]</sup>.

These GARCH-class volatility models have been widely used to forecast various financial data, based on their predictive power. The great focus in these studies has been primarily given to stock returns (Keim and Stambaugh, 1986; Poon, 2005; Guidolin et al., 2009; Ferreira and Santa-Clara, 2011; Niu and Wang, 2013), in which recent past information is found to help forecast the future variance. Similar studies are conducted using commodity market data, especially oil data (Agnolucci, 2009; Wei et al., 2010; Chkili et al., 2014; Charles and Darné, 2017). Generally, these studies show no model outperforms all the others in capturing the time series financial and statistical features, while the non-linear GARCH-class models

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<sup>[2]</sup>For an exhaustive survey of the proposed ARCH-class models, see Poon (2005)

are found to be more relevant in terms of forecasting accuracy<sup>[3]</sup>.

Unlike stock markets, exchange rates and oil market data, not many studies have been conducted to assess the predictive performance of the volatility GARCH-type models using CDS data. Despite [Krishnan et al. \(2010\)](#), [Sharma and Thuraisamy \(2013\)](#), [Avino and Nneji \(2014\)](#) and [Srivastava et al. \(2016\)](#) whose aim is to predict the future changes in the CDS spreads based on some macroeconomic and marketwide variables, the literature on CDS spreads focuses generally on the key drivers and determinants of these credit spreads ([Oliveira et al., 2012](#); [Costantini et al., 2014](#); [Fontana and Scheicher, 2016](#)) or rather on the interaction and comovement between CDS markets and the other related financial markets ([Coudert et al., 2010](#); [Longstaff et al., 2011](#); [Coudert and Gex, 2013](#); [Sabkha et al., 2017](#)).

Among the first authors who are interested in the prediction of credit spreads, [Krishnan et al. \(2010\)](#) construct credit-spread curves, based on several macroeconomic and firm-specific variables, for 241 highly and lowly credit-risky firms from 1990 to 2005. Results show that only the information contained in the riskless yield curve significantly improve the out-of-sample forecasts. Focusing more precisely on the CDS as proxy for the credit risk level, [Sharma and Thuraisamy \(2013\)](#) investigates the forecastability of the CDS spreads of 8 Asian sovereign from 2005 to 2012. In-sample and out-of-sample evidences reveal that the oil price uncertainty provides valuable information for predicting the future fluctuations in the sovereign CDS spreads. [Avino and Nneji \(2014\)](#) use some economic and financial factors to investigate whether the iTraxx index spreads are forecastable. Based on the results of the predictive ability of some linear (Structural OLS model and AR(1)) and non-linear (Markov-switching) models, these authors show that the daily changes in the CDS index can be predictable from the yield curve, the equity returns and the changes in the VSTOXX volatility index. Using an error correction model before, during and after the subprime crisis, [Srivastava et al. \(2016\)](#) show that the VIX predicts the future changes in 98% of the studied sovereign CDS markets.

These few studies on the forecastability of CDS spreads rely on the information contained in the theoretical determinants - widely used in the empirical literature - and its ability to predict future fluctuations in the CDS market. Yet, the accuracy of these CDS predictions is assessed through some loss functions criteria that are subject to nonzero mean noise and serial correlation (such as RMSE, MAE. . .). Furthermore, the data studied so far only cover the period of the subprime crisis and end before or right after the outbreak of the Sovereign debt crisis, which is quite a weak point given that all the unexpected changes in the market behavior are not taken into account in their forecasting models. Finally, the most important shortcoming of the aforementioned studies, is that they focus on the first moment order neglect the Variance in forecasting the CDS spreads.

### 3 Data and methodology

This section introduces one of our paper contributions: the sample under study, composed by countries around the world, allowing us to provide international evidences and data time

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<sup>[3]</sup>For a complete theoretical and empirical survey on the use of univariate ARCH processes in financial studies, see [Bollerslev et al. \(1992\)](#).

line covering the both recent two financial and economic crises. Volatility forecasting models are as well presented in this section.

### 3.1 Sample and data description

Our study focuses on a sample composed by 38 worldwide countries belonging to five different geographical areas: Eastern and Western Europe, North and South America and Asia. Besides the developed countries and the emerging countries, the sample under study in this paper includes some Newly Industrialized Countries (such as Brazil, Mexico, Philippines and Thailand...) and some low economic growth countries with the highest credit risk levels (such as Portugal, Ireland, Greece and Spain...). The sample details with the economic and geographical status of each country are given in [Table 1](#).

The dataset used is composed by daily 5-year sovereign CDS spreads, denominated in US dollars and collected from Thomson Reuters®. The extracted series cover a period spanning from January 2006 to March 2017, during which the world financial and credit markets have been affected by two major crises, namely the Global Financial Crisis and the Sovereign Debt Crisis. Thus, modeling, forecasting the CDS volatility and comparing models performances are particularly interesting during this period during which we observed some unexpected fluctuations on the credit market.

### 3.2 Marginal volatility processes: univariate ARCH-type models

The financial markets are generally characterized by periods of volatility clustering, during which the assets' second moment order remains high before regaining its normal levels. [Engle \(1982\)](#) proposes an Autoregressive Conditional Heteroscedasticity (ARCH) model able to capture such financial phenomenon. This volatility persistence is as well observed in the Credit Default Swap market and the use of ARCH-class models to model the variance of the CDS spreads is thus legitimate. As an extension of the ARCH model, [Bollerslev \(1986\)](#) proposes a generalized high-order ARCH process that is more parsimonious and allows for less overparametrization and biasedness in the estimates. This GARCH model is given by:

$$\begin{aligned} x_{i,t} = \mu_{i,t} + u_{i,t} \quad / \quad u_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \quad \varepsilon_{i,t}|\mathcal{P}_{t-1} \rightsquigarrow \mathcal{N}(0, 1); \\ \sigma_{i,t}^2 = V(x_{i,t}|\mathcal{F}_{t-1}) = \alpha_{i,0} + \sum_{k=1}^{q_i} \alpha_{i,k}a_{i,t-1}^2 + \sum_{h=1}^{p_i} \beta_{i,h}\sigma_{i,t-1}^2. \end{aligned} \quad (1)$$

with  $x_{i,t}$  is a financial time series,  $i$  is a given country from the sample and  $\mu_{i,t}$  and  $\sigma_{i,t}$  are respectively conditional mean and conditional volatility. To satisfy the positive-definite condition, some restrictions are imposed:  $p \geq 0$ ,  $q \geq 0$ ,  $\alpha_{i,k} \geq 0$  for  $k = 1, \dots, q_i$ ,  $\beta_{i,h} \geq 0$  and  $\alpha_{i,0} \geq 0$  for  $h = 1, \dots, p_i$ .

For sake of simplicity and suitability, only models with process orders ( $p_i$  and  $q_i$ ) equal to 1 are estimated. In fact, the simplest GARCH(1,1) specification is the most useful and fitted for financial time series ([Bollerslev, 1986](#); [Wei et al., 2010](#)).

The GARCH(1,1) process, as proposed by [Bollerslev \(1986\)](#) is given by the following formula:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1} + \alpha_{i,1}a_{i,t-1}^2 + \beta_{i,1}\sigma_{i,t-1}^2. \quad (2)$$

Table 1: Sample and countries classification into economic categories and geographical positions

Country	Geographical position	Country	Geographical position
<i>Developed countries (20)</i>		<i>Newly industrialized countries (6)</i>	
Austria	Western Europe	Brazil	South America
Belgium	Western Europe	China	Asia
Denmark	Western Europe	Mexico	North America
Finland	Western Europe	Thailand	Asia
France	Western Europe	Turkey	Asia
Germany	Western Europe	<i>Emerging countries (11)</i>	
Ireland	Western Europe	Bulgaria	Eastern Europe
Italy	Western Europe	Croatia	Eastern Europe
Japan	Asia	Czech	Eastern Europe
Latvia	Eastern Europe	Hungary	Western Europe
Lithuania	Eastern Europe	Greece	Western Europe
Netherlands	Western Europe	Indonesia	Asia
Norway	Western Europe	Poland	Eastern Europe
Portugal	Western Europe	Romania	Eastern Europe
Slovakia	Eastern Europe	Russia	Asia
Slovenia	Eastern Europe	Ukraine	Eastern Europe
Spain	Western Europe	Venezuela	South America
Sweden	Western Europe		
UK	Western Europe		
USA	North America		

The countries' economic classification is made according to the NU, the CIA World Factbook, the IMF and the World Bank criteria, in order to have a sample with a sufficient number of countries in each category.

Furthermore to the previous model restrictions,  $\alpha_{i,1}$  and  $\beta_{i,1}$  parameters must satisfy the condition of  $\alpha_{i,1} + \beta_{i,1} < 1$  to comply with the stationarity in the broad sense.

A more restrictive version of the GARCH(1,1) is proposed by Engle and Bollerslev (1986) where the equivalent of the unit root in the mean is included in the variance so we can handle for the stationarity of the variance. The Integrated GARCH(1,1) takes into account the persistence of conditional volatilities<sup>[4]</sup>. The main difference with the GARCH(1,1) is that the IGARCH requires the parameters  $\alpha_1$  and  $\beta_1$  to respect the equality of  $\alpha_1 + \beta_1 = 1$ . Thus, the IGARCH(1,1)<sup>[5]</sup> can be written as follows:

$$\sigma_{i,t}^2 = \alpha_{i,1}a_{t-1}^2 + (1 - \alpha_{i,1})\sigma_{i,t-1}^2. \quad (3)$$

Besides the aforementioned linear models, there exist some nonlinear GARCH-class of models taking into account the other financial markets properties. The Exponential GARCH,

<sup>[4]</sup>Today's shocks on a financial asset (future contracts for example) have a significant impact on the conditional volatility several periods in the future.

<sup>[5]</sup>The IGARCH(1,1) is equivalent to the Exponentially Weighted Moving Average(EWMA) model developed by Morgan (1996).

as proposed by Nelson (1991), is one of these models that accounts for the leverage effect and the asymmetry of the error distribution. While the nonnegativity of linear GARCH model is ensured by several parameters restrictions, the EGARCH model proposes another formulation allowing for a positive volatility without any restrictive constraints. The EGARCH(1,1) is expressed as follows:

$$\ln(\sigma_{i,t}^2) = \alpha_{i,0} + \alpha_{i,1-1}\ln(\sigma_{i,t}^2) + \beta_{i,1}g(\varepsilon_{i,t-1}), \quad (4)$$

where  $g(\varepsilon_{i,t}) = \theta_i\varepsilon_{i,t} + \gamma_i[|\varepsilon_{i,t}| - E(|\varepsilon_{i,t}|)]$ .

The asymmetric relation between assets' fluctuation and volatility changes is depicted by the  $\theta_i$  and  $\gamma_i$  representing respectively the sign and the magnitude of  $\varepsilon_{i,t}$ .

Glosten et al. (1993) propose a model that allows the sign and the amplitude of the innovations ( $\varepsilon_t$ ) to affect the conditional volatility separately. The asymmetric leverage effect<sup>[6]</sup> is represented in the following formulation of the GJR-GARCH(1,1)<sup>[7]</sup> model:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}a_{i,t-1}^2 + \gamma_i I_{i,t-1} a_{i,t-1}^2 + \beta_{i,1}\sigma_{i,t-1}^2, \quad (5)$$

with  $I_t$  is a dummy variable equals to 0 when  $a_t$  is positive and 1 otherwise.

The first model accounting for the long-range persistence of financial assets variance is developed by Ding et al. (1993). This asymmetric power ARCH model allows the volatility to be long-memory<sup>[8]</sup>. The APARCH(1,1) model is:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}(|a_{i,t-1}| - \gamma_i a_{i,t-1})^\delta + \beta_{i,1}\sigma_{i,t-1}^\delta. \quad (6)$$

where  $\delta$  depicts the Box-Cox power transformation of the conditional volatility ( $\sigma_t$ ) and satisfies the condition of  $\delta \geq 0$ .

A more flexible class of GARCH models is proposed by Baillie et al. (1996) who introduce a new feature of the unit root for the variance. In fact, the fractionally integrated GARCH model (FIGARCH) highlights the fact that - unlike stationary processes where the persistence of volatility shocks is finite - in unit root processes, the impact of lagged errors occurs at a slow hyperbolic rate of decay. The FIGARCH model allows, thus, to capture the long memory in financial volatility with a complete flexibility regarding the persistence degree. In fact, the FIGARCH(1,d,1) formulation depends on fractional integration parameter (d) as follows:

$$\sigma_{i,t}^2 = \alpha_{i,0} + [1 - (1 - \beta_i(L))^{-1}(1 - \phi(L))(1 - L)^d]a_{i,t}^2 + \beta_i\sigma_{i,t-1}^2. \quad (7)$$

with  $0 < d < 1$ . When  $d=1$ , the FIGARCH(1,d,1) is equivalent to an IGARCH(1,1) where the persistence of conditional variance is supposed to be complete, while when  $d=0$ , it is rather equivalent to a GARCH(1,1) and no volatility persistence is taken into consideration.  $L$  is the lag operator and  $(1 - L)^d$  is the financial fractional differencing operator.

<sup>[6]</sup>Positive and negative financial shocks revamp asymmetrically the variance. Furthermore, bad news (shocks) generate greater volatility than good news.

<sup>[7]</sup>The volatility's different reactions to signs and sizes of past innovations are also suggested in the Threshold Heteroskedastic model (TGARCH) of Zakoian (1994). The major difference is that in the TGARCH model the conditional standard deviation ( $\sigma_t$ ) is considered rather than the conditional variance ( $\sigma_t^2$ ).

<sup>[8]</sup>The autocorrelation function of time series returns decreases gradually.



Other ARCH formulations are extended to the fractionally integrated GARCH, including asymmetric leverage effect presented in the EGARCH model. [Bollerslev and Mikkelsen \(1996\)](#) propose a new class of model combining characteristics of the FIGARCH and the EGARCH models so-called FIEGARCH(p,d,q). Financial assets' volatilities are, thus, better explained and depicted by a mean-reverting fractionally integrated process. The FIEGARCH(1,d,1) model is written as follows:

$$\ln(\sigma_{i,t}^2) = \alpha_{i,0} + \phi(L)^{-1}(1-L)^{-d}[1 + \psi(L)]g(\varepsilon_{i,t-1}). \quad (8)$$

where  $\phi(L)$  and  $\psi(L)$  are lag polynomials, and - as in the EGARCH(1,1)<sup>[9]</sup> -  $g(\varepsilon_t)$  is a quantization function of information flows such as:

$$g(\varepsilon_{i,t}) = \theta_i \varepsilon_{i,t} + \gamma_i [|\varepsilon_{i,t}| - E(|\varepsilon_{i,t}|)].$$

An extension of the conventional fractionally integrated GARCH model is proposed by [Tse \(1998\)](#) so-called FIAPARCH(1,d,1). The new approach combines the long-range dependencies feature and the asymmetric impact of lagged positive and negative shocks on future volatilities in one fractionally integrated model. The FIAPARCH(1,d,1) is written as follows:

$$\sigma_{i,t}^\delta = \alpha_{i,0}(1 - \beta_i)^{-1} + [1 - (1 - \beta_i(L))^{-1}\phi(L)(1 - L)^d](|a_{i,t}| - \gamma_i a_{i,t})^\delta. \quad (9)$$

More recently, another linear GARCH model, called hyperbolic GARCH (HYGARCH) is proposed by [Davidson \(2004\)](#) who argues that the impact of lagged errors on the conditional variance discloses near-epoch dependence feature. The main contribution of this model is that the fractional integration parameter is negative (-d) instead of positive and that d increases rather when it approaches zero<sup>[10]</sup>. The statistical properties included in the HYGARCH make it the most successful and used approach by financial practitioners in modeling time series volatilities. The HYGARCH(1,d,1) is defined under the following formulation:

$$\sigma_{i,t}^2 = \alpha_{i,0} + [1 - (1 - \beta_i(L))^{-1}(1 - \phi(L))[1 + \alpha_i((1 - L)^d - 1)]]a_{i,t}^2. \quad (10)$$

The volatility estimation of the CDS log returns of the 38 countries is computed for 9 GARCH-class models taking into account, each time, different financial stylized facts such as long-run properties in the conditional mean and volatility clustering and long-memory behavior in the conditional variance. The BFGS-BOUNDS method ([Broyden, 1970](#)) is used to optimize the likelihood function rather than the conventional numerical optimization, in order to respect the parameters constraints, notably the stationary and the nonnegativity constraints.

In addition to the widely used Box-Pierce tests and the LM ARCH effects test, several other diagnostic tests are reported here, namely the Nyblom test, the adjusted Pearson goodness-of-fit test and the Residual-Based Diagnostic (as suggested by [Fantazzini \(2011\)](#)). The Joint Nyblom ([Nyblom, 1989](#)) is a stability test under the null hypothesis of parameters joint constancy over time against the alternative of parameters shift at an undefined

<sup>[9]</sup>When the memory parameter, d=0, the FIEGARCH formulation is equivalent to the conventional EGARCH(1,1) (FIEGARCH(1,0,1)  $\simeq$  EGARCH(1,1)).

<sup>[10]</sup>When d of the HYGARCH is positive, it is considered as a unit root process.

breakpoint. According to [Palm and Vlaar \(1997\)](#), The adjusted Pearson goodness-of-fit test verifies whether the residuals' empirical distribution matches or not the theoretical distribution (namely Gauss, Student or G.E.D depending on the country). The Residuals-Based Diagnostic test ([Tse, 2002](#)) checks for conditional Heteroscedasticity, by complementing and filling the gaps of the Box-Pierce Q statistics.

### 3.3 Loss functions criteria

Following [Wei et al. \(2010\)](#), the forecasting process of the CDS volatility is implemented as follows: the 38 CDS times series' timeline is divided into two subperiods: the in-sample volatility estimation is conducted from January 2<sup>nd</sup>, 2006 to March 31<sup>st</sup>, 2014 (2152 observations), and the out-of-sample model forecasts concerns the last three years, i.e. from April 1<sup>st</sup>, 2014 to March 31<sup>st</sup>, 2017 (783 observations). The twenty-day out-of-sample forecasting are used to assess and compare the predictive performance of the 9 studied models.

The comparison of the volatility models' forecasting ability is not straightforward. Several measures of the predictive ability are suggested in the literature based on some loss functions. According to [Poon \(2005\)](#), [Wei et al. \(2010\)](#) and [Pilbeam and Langeland \(2015\)](#), we can not conclude with certainty the superiority of one model over another in terms of forecasting performance, based solely on the result of a single error statistic since each criterion may be more and less relevant from one case to another<sup>[11]</sup>. That's why the conclusions made in this study are based on the results of rich set composed by the 7 most popular and relevant ones, including:

- The Mean Square Error (MSE):

$$MSE = \frac{1}{N} \sum_{t=1}^N (\hat{\sigma}_t - \sigma_t)^2, \quad (11)$$

- The Mean Absolute Error (MAE):

$$MAE = \frac{1}{N} \sum_{t=1}^N |\hat{\sigma}_t - \sigma_t|, \quad (12)$$

- The Heteroscedasticity-adjusted Mean Square Error (HMSE). As suggested by [Bollerslev and Ghysels \(1996\)](#), the HMSE is calculated as follows:

$$HMSE = \frac{1}{N} \sum_{t=1}^N \left( \frac{\sigma_t}{\hat{\sigma}_t} - 1 \right)^2, \quad (13)$$

- The Heteroscedasticity-adjusted Mean Absolute Error (HMAE). [Andersen et al. \(1999\)](#) proposes a loss function that better accommodates the heteroskedasticity in the forecast bias . The HMAE is calculated as follows:

$$HMAE = \frac{1}{N} \sum_{t=1}^N \left| \frac{\sigma_t}{\hat{\sigma}_t} - 1 \right|, \quad (14)$$

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<sup>[11]</sup>[Diebold and Mariano \(2002\)](#) argue that allowing for forecast errors to be non-Gaussian, nonzero mean and autocorrelated produces better tests' results.

- The QLIKE loss function (QLIKE). This is a test of forecast bias implied by a Gaussian likelihood (see [Wei et al. \(2010\)](#) for a further details.)

$$QLIKE = \frac{1}{N} \sum_{t=1}^N \left( LN(\hat{\sigma}_t) + \frac{\sigma_t}{\hat{\sigma}_t} \right), \quad (15)$$

- The R<sup>2</sup>LOG loss function (R<sup>2</sup>LOG): This loss function assesses the goodness-of-fit of the out-of-sample forecasts based on the regressions of [Mincer and Zarnowitz \(1969\)](#)

$$R^2LOG = \frac{1}{N} \sum_{t=1}^N \left( LN\left(\frac{\sigma_t}{\hat{\sigma}_t}\right) \right)^2, \quad (16)$$

- The Mean Logarithm of Absolute Errors (MLAE): As proposed [Pagan and Schwert \(1990\)](#), the MLAE criterion is written as follows:

$$MLAE = \frac{1}{N} \sum_{t=1}^N LN | \hat{\sigma}_t - \sigma_t |. \quad (17)$$

With  $N$  is the number of predicted data and  $\hat{\sigma}_t$  is the volatility forecasts. The latent daily CDS spreads volatility  $\sigma_t$  is not observed and is thus proxied by the squared daily logarithmic returns<sup>[12]</sup>. Previous studies ([Lopez, 2001](#); [Poon, 2005](#)) report that the use of such a proxy produces unbiased estimates, even though it remains questionable (noisy estimator because of its asymmetric distribution).

## 4 Empirical results

This section presents the summary statistics for the 38 studied time series. The modeling, estimation and testing of the forecasting ability of the 9 GARCH-class models are presented, as well, in this section.

### 4.1 Descriptive statistics

Descriptive statistics, displayed in [Table 2](#), show that the studied countries present dissimilar credit risk levels with CDS spreads ranging from 1 bp to 37081.41 bp. The average daily spreads highlights, as well, this divergence in sovereign financing conditions with the largest value recorded, as expected, in Greece (9508.85 bp) and the smallest value recorded in the USA (24.01 bp). The high levels of standard deviations reveal, on the other side, that the worldwide financial and economic troubles impacted the public finances of the countries under study, doubtlessly with different magnitudes. The least volatile CDS market is Germany (24.50). According to the Augmented Dickey-Fuller test ([Dickey and Fuller, 1981](#)), all the time series present a unit root, implying that the CDS spreads of the 38 countries are non-stationary at 5% statistical level at least.

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<sup>[12]</sup>More methods exist in the literature to proxy the volatility of financial assets, such as, the high-low measure and the realized volatility estimate. For a complete survey of these methods, see [Poon \(2005\)](#).

Table 2: Descriptive statistics and ARCH effect tests for the CDS time series

	Obs.	CDS spreads			Std. Dev	ADF statistics	ARCH-LM (2)	CDS log returns			GPH
		Min	Mean	Max				ARCH-LM (5)	ARCH-LM (10)		
Austria	2936	1.40	36.13	132.77	24.96	-2.45	249.75 ***	127.05 ***	72.58 ***	0.29 ***	
Belgium	2936	2.05	72.39	398.78	74.62	-1.67	508.94 ***	237.99 ***	120.84 ***	0.18 ***	
Brazil	2936	61.50	178.55	606.31	94.86	-2.46	25.01 ***	43.70 ***	37.71 ***	0.11 ***	
Bulgaria	2936	13.22	180.37	692.65	121.88	-2.25	12.71 ***	10.36 ***	6.72 ***	0.08 ***	
China	2936	10.00	82.44	276.30	43.56	-2.82 *	120.85 ***	63.09 ***	39.00 ***	0.22 ***	
Croatia	2936	24.88	244.20	592.50	128.47	-2.15	137.90 ***	58.87 ***	47.62 ***	0.26 ***	
Czech	2936	3.41	66.89	350.00	49.54	-2.62 *	62.52 ***	46.01 ***	29.50 ***	0.14 ***	
Denmark	2936	11.25	36.65	157.46	32.94	-2.17	87.27 ***	41.66 ***	24.36 ***	0.21 ***	
Finland	2936	2.69	26.85	94.00	19.24	-2.33	13.79 ***	7.98 ***	4.43 ***	0.05 ***	
France	2936	1.50	54.30	245.27	50.56	-1.71	276.95 ***	120.56 ***	62.86 ***	0.20 ***	
Germany	2936	1.40	28.77	118.38	24.50	-2.05	252.46 ***	128.31 ***	73.27 ***	0.29 ***	
Greece	2936	5.20	9508.85	37081.41	15351.1	-1.46	5.E-04	4.E-04	6.E-04	-4.E-04	
Hungary	2936	17.34	225.98	729.89	153.05	-2.18	14.48 ***	15.20 ***	8.67 ***	0.10 ***	
Indonesia	2936	118.09	219.29	1240.00	116.83	-2.63 *	139.82 ***	105.31 ***	61.49 ***	0.23 ***	
Ireland	2936	1.75	188.89	1249.30	234.02	-1.36	218.63 ***	103.01 ***	63.33 ***	0.18 ***	
Italy	2936	5.75	151.7504	586.7	127.38	-1.79	127.35 ***	60.46 ***	35.18 ***	0.19 ***	
Japan	2936	2.13	49.26	152.64	33.28	-1.94	71.53 ***	31.30 ***	21.68 ***	0.13 ***	
Latvia	2936	5.50	210.89	1176.30	216.13	-1.62	152.57 ***	68.47 ***	35.36 ***	0.26 ***	
Lithuania	2936	6.00	169.21	850.00	154.01	-1.90	56.75 ***	26.91 ***	13.56 ***	0.15 ***	
Mexico	2936	64.17	141.89	613.11	59.36	-3.03 *	356.35 ***	160.17 ***	127.50 ***	0.39 ***	
Netherlands	2936	7.67	37.13	133.84	29.50	-2.00	10.79 ***	4.33 ***	5.59 ***	0.05 ***	
Norway	2936	10.59	30.95	62.00	17.82	-1.68	3.22 ***	2.46 ***	2.06 ***	0.05 ***	
Philippines	2936	78.30	188.72	840.00	101.70	-1.77	154.83 ***	127.66 ***	90.03 ***	0.23 ***	
Poland	2936	7.67	101.35	421.00	73.12	-2.32	311.98 ***	135.64 ***	75.78 ***	0.21 ***	
Portugal	2936	4.02	289.89	1600.98	323.68	-1.60	53.57 ***	42.23 ***	22.61 ***	0.17 ***	
Qatar	2936	7.8	83.13518	390	53.89	-2.12	37.65 ***	17.33 ***	9.55 ***	0.09 ***	
Romania	2936	17.00	204.20	767.70	144.17	-2.09	57.88 ***	33.74 ***	17.50 ***	0.17 ***	
Russia	2936	36.88	209.09	1106.01	147.84	-2.95 *	258.09 ***	117.58 ***	65.50 ***	0.29 ***	
Slovakia	2936	5.33	77.52	306.01	66.71	-2.03	25.14 ***	24.62 ***	19.31 ***	0.11 ***	
Slovenia	2936	4.25	131.24	488.58	114.88	-1.65	13.23 ***	9.82 ***	34.88 ***	0.11 ***	
Spain	2936	2.55	144.63	634.35	135.01	-1.56	195.02 ***	78.80 ***	39.98 ***	0.19 ***	
Sweden	2936	1.63	27.17	159.00	25.70	-2.64 *	69.49 ***	30.82 ***	20.72 ***	0.16 ***	
Thailand	2936	51.01	120.94	500.00	41.89	-3.64 *	81.52 ***	120.36 ***	96.33 ***	0.17 ***	
Turkey	2936	109.82	217.65	835.01	72.41	-3.72 *	69.04 ***	86.65 ***	46.84 ***	0.21 ***	
UK	2936	16.50	42.89	165.00	28.11	-2.07	27.33 ***	21.12 ***	23.19 ***	0.11 ***	
Ukraine	2936	1.00	2173.76	15028.76	3969.27	-2.15	60.42 ***	32.53 ***	17.13 ***	0.11 ***	
USA	2936	10.02	24.01	90.00	11.11	-3.58 *	94.96 ***	46.67 ***	24.57 ***	0.18 ***	
Venezuela	2936	124.62	1771.08	10995.67	1869.79	-2.00	36.17 ***	38.56 ***	22.73 ***	0.11 ***	

The table reports descriptive statistics for the daily sovereign CDS spreads of 38 countries. Min., Max. and Std. Dev. denote respectively to the minimum, the maximum and the standard deviation. The Augmented-Dickey Fuller (Individual intercept included in the test equation) is a unit root test that informs about the stationarity of time-series with a null hypothesis of the presence of a unit root in the process. The Engle's ARCH-LM test with 2, 5 and 10 lag orders informs about the presence of ARCH effects in the series under the null hypothesis of no autocorrelations in the squared residuals. GPH is the log periodogram test of Geweke and Porter-Hudak (1983) with d-parameter (m=1467). This test is applied to the squared logarithmic returns (as proxy for unconditional volatility) to detect any long-range dependence under the null assumption of no long-memory behavior in the volatility process. \*, \*\* and \*\*\* refer to the statistical significance at respectively 10%, 5% and 1% levels.

Focusing on the evolution of the CDS log returns (computed as  $x_t = \log(\frac{S_t}{S_{t-1}})$ ) over the studied period, as presented in Figure 1, some volatility clustering periods are detected. Results of the ARCH-LM test in Table 2 confirm that the data used clearly exhibit heteroscedastic properties and support the appropriate use of GARCH-class processes to model the conditional volatility. The GPH test (Geweke and Porter-Hudak, 1983) conducted on the squared CDS log returns rejects the null hypothesis of no long-memory behavior in the series' volatility process, suggesting the use of the fractionally-integrated models<sup>[13]</sup>. Figure 2 reports the density estimation and show that the series, composing our international sample, exhibit dissimilar statistical behaviors as to their empirical distribution. The majority of the data returns distribution does not overlay the Gaussian reference, which indicates that the residuals should be allowed to follow a Gaussian, a student and a Generalized Error Distribution (G.E.D)<sup>[14]</sup>.

<sup>[13]</sup>Another commonly used long-range test is the Gaussian semi-parametric (GSP) (Robinson, 1995). Results of the GSP are not reported here because they are similar to those of the GPH.

<sup>[14]</sup>Other statistical distribution should, as well, be taken into account in further studies, such as the Skewed t-student...

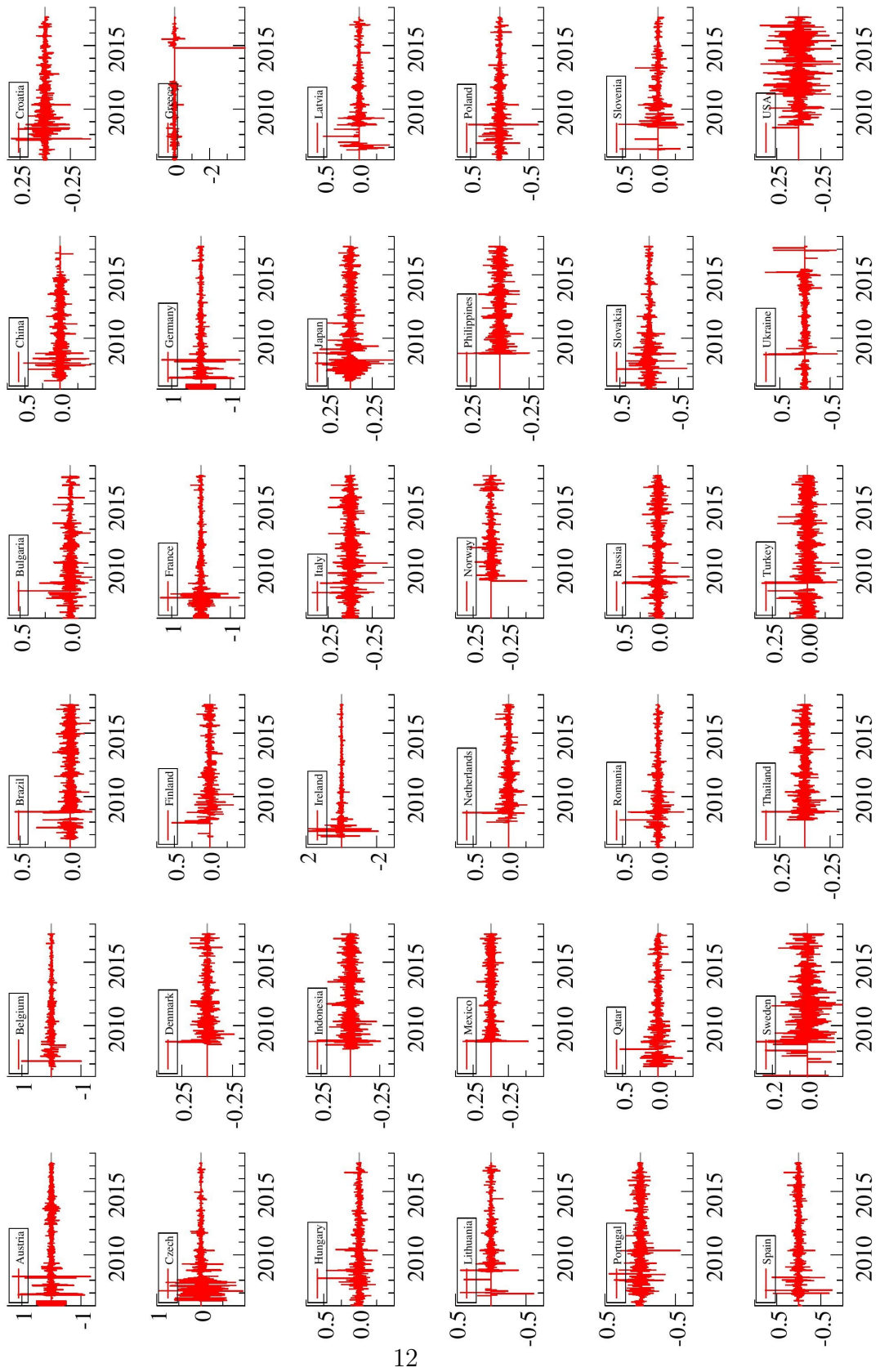


Figure 1: Daily CDS log returns for 38 worldwide countries

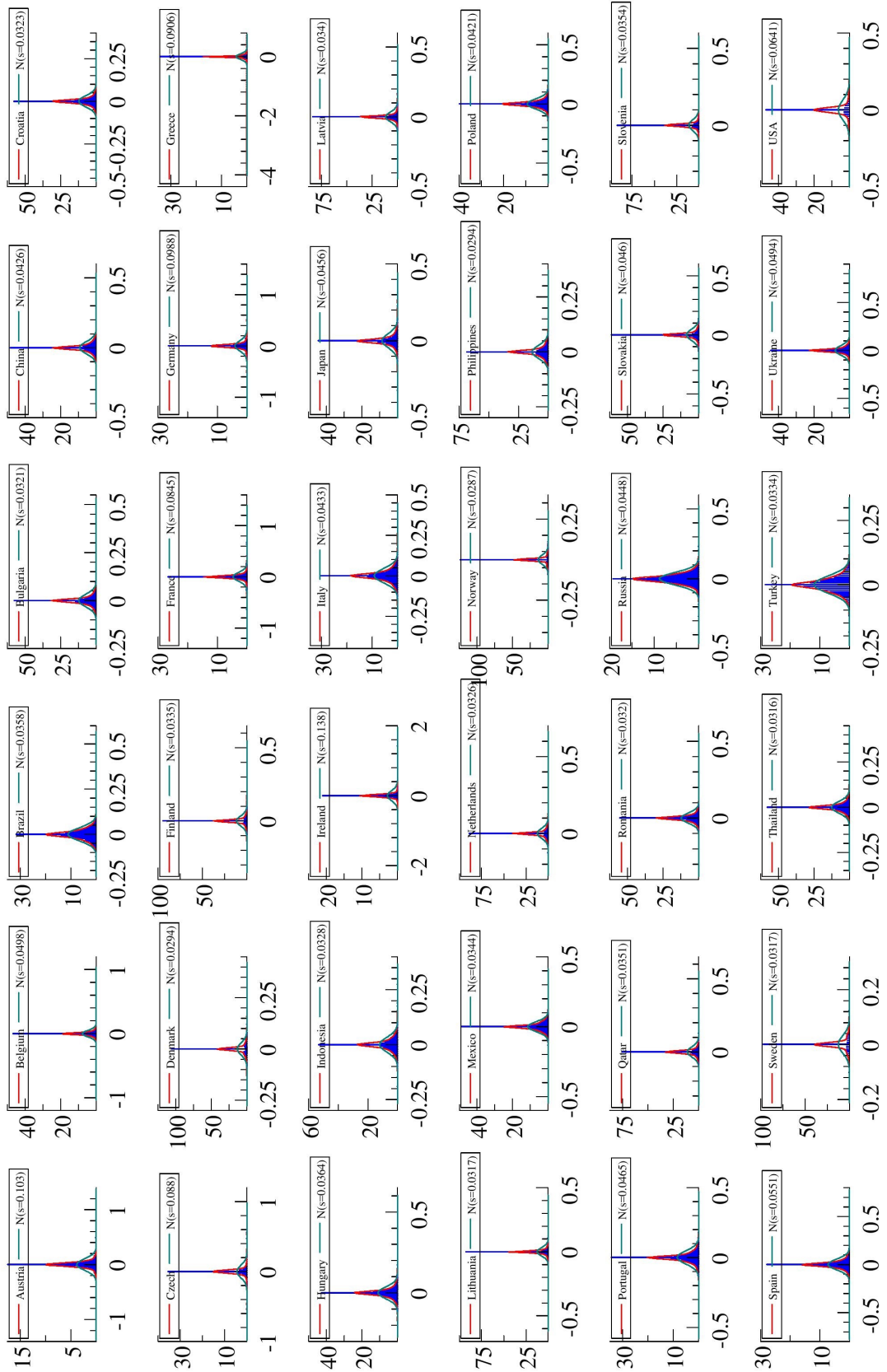


Figure 2: Daily CDS log returns for 38 worldwide countries

## 4.2 Models estimation and diagnostic tests

Results of the 9 GARCH-class model estimates are not reported here but are available upon request. Even though some models are difficult to optimize, no miss-convergences are recorded for any time series. However, at first sight, the major conclusion that could be drawn regarding the models estimation process is that taking into account several financial markets' stylized facts (long memory characteristic, shock persistence and asymmetric leverage effects...) does not necessarily improve the models performances since the more the model is over-parametrized, the more its computation and its convergence are complicated. In fact, different inconsistency and inaccuracy of the estimator parameters in some countries and for some model can result from the complexity of the model's statistical specifications. At the opposite, the models that great perform as to strong numerical convergence and computing-time delay are the GARCH, the IGARCH, FIGARCH and FIEGARCH.

Results of the univariate misspecification tests applied on the standardized residuals are presented in [Table 5 \(Appendix A\)](#). The Q portmanteau empirical statistics with 20 lags of both standardized residuals in levels and squared show that the null hypothesis of no serial correlation is accepted in most cases for all the studied models. The LM-ARCH test up to 10 lag orders shows, as well, that there is no heteroscedasticity in the conditional variance equations of most of time series. The GARCH, IGARCH and FIGARCH models pass this in 100% of cases, whilst the least performant model, in terms of serial correlation, in the FIAPARCH with ARCH effects detected in 6 countries. Moreover, testing for conditional heteroscedasticity through the Residual-Based Diagnostic (RBD) for conditional heteroscedasticity ([Tse, 2002](#)) gives better results, with absolutely no detected serial correlation in all series for the APARCH, IGARCH and FIGARCH. Based on the the Nyblom test, proposed by [Nyblom \(1989\)](#), no possible shifts are detected and the 9 models parameters coefficients are found to be constant over time for all countries. One of the recommended steps in modeling financial data process is to evaluate the goodness of fit ([D'Agostino, 2017](#)). The fitting of our models are thus assessed, in this paper, through the adjusted Pearson goodness-of-fit test. Statistics indicate that mostly there is no difference between the empirical distribution of the residuals and the theoretical one. Interestingly, the basic GARCH model seems to have the highest number (12 over the 38 studied series) of unconformity and discrepancy of the data from the data from the hypothesized probability distribution.

In addition to the diagnostic tests, [Table 5](#) displays the Akaike information criterion (AIC) for each model and country. Result do not allow us to unanimously select only one most appropriate model. AIC results of the studied models are mitigated across the 38 countries of the sample. By minimizing the AIC, the APARCH turns out to be the best fitted model for the CDS data of 34% of the sample, while HYGARCH, IGARCH and FIAPARCH provide the best in-sample fit for respectively 26%, 18% and 11% of the studied countries. However, these results are not in line with the preliminary analysis where all the studied CDS log returns are found to be subject to long-memory feature in the variance. By only focusing in the fractionally integrated subset of models, the HYGARCH is found to majority outperform in 53% of cases, followed by the FIAPARCH in 40% of cases. These results divergence points out the limits of using the 'minimizing loss of information' technique in comparing models appropriateness. Thus, this approach seems to be, in this case, not totally consistent and should only be used tentatively, at least if it is not associated with any other

approaches. Hence the importance of rather rely on the forecasting ability to select the best performant volatility model.

### 4.3 Forecasting performance

Results of the twenty-day out-of-sample volatility forecasts are reported in [Table 3](#) and [Table 4](#). As mentioned before, the forecasting robustness and reliability of the 9 models is studied through 7 error statistics, namely the MSE, MAE, HMSE, HMAE, QLIKE,  $R^2\text{LOG}$  and MLAE. Even though there is no unanimous dominant model in terms of forecasting ability according to all the comparison measure, it is clearly seen that the fractionally-integrated class of model outperforms the basic GARCH models - not taking into account long-memory in volatility process. Ranked in the last position by 5 out of the 7 criteria, the least forecasting performant model for CDS volatility is the EGARCH with the largest recorded errors.

The lowest values of MSE, MAE and  $R^2\text{LOG}$  are recorded for the FIGARCH, whilst the lowest values of HMSE, QLIKE and MLAE are reported for the FIEGARCH, making them preferable, in terms of accurate forecasting abilities, to the other studied models. On another side, and according to the results of the MSE, MAE, HMAE,  $R^2\text{LOG}$  and MLAE criteria, the HYGARCH produce the highest errors, probably due to its computational complexity.

These findings empirically reveal the nonlinear predictability pattern of CDS volatility. In general, our results are in line with the findings of other financial markets: the non-linear GARCH-class models, that allows for leverage effects, nonsymmetrical dependencies and long-range memory in the volatility model provide a more accurate in-sample performance and a more reliable out-of-sample forecasting ability. The improvement of the forecasting power of the studied models depends, thus, on their ability to capture a maximum of financial stylized facts while estimating the CDS volatility of future days.

Table 4: Summary of the number of selected models according to each criterion

	<b>GARCH</b>	<b>EGARCH</b>	<b>GJR</b>	<b>APARCH</b>	<b>IGARCH</b>	<b>FIGARCH</b>	<b>FIEGARCH</b>	<b>FIAPARCH</b>	<b>HYGARCH</b>
<b>MSE</b>	5	1	2	2	7	16	5	13	3
<b>MAE</b>	4	0	1	0	3	14	6	11	4
<b>HMSE</b>	3	3	2	2	2	4	10	7	9
<b>HMAE</b>	4	3	2	4	3	6	7	11	1
<b>QLIKE</b>	6	2	2	3	3	4	10	7	5
<b><math>R^2\text{LOG}</math></b>	5	2	2	1	3	9	6	8	5
<b>MLAE</b>	2	3	0	3	0	6	18	5	4

## 5 Conclusion

This paper aimed to assess the performances of 9 linear and non-linear volatility models. Using daily sovereign CDS data, GARCH, IGARCH, EGARCH, GJR, APARCH, FIGARCH, FIEGARCH, FIAPARCH and HYGARCH are estimated, allowing to take into account different financial markets properties such as the leverage effect, the asymmetric reaction to good and bad news and long-range persistence. The performance comparison being made upon several loss functions criteria and several multivariate diagnostic tests, a certain number of conclusions can be drawn.



Table 3: Results of the loss functions criteria for the twenty-day out-of-sample volatility predictions

Statistic criteria	MAE										MSE									
	GARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIEGARCH	FIAPARCH	HYGARCH	HYGARCH	GARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIEGARCH	FIAPARCH	HYGARCH	HYGARCH
Austria	<b>0.0168</b>	0.2640	0.0181	0.0483	0.0189	0.0189	0.1318	0.0189	0.0194	0.0597	0.1335	0.0603	0.1620	0.0643	0.0643	0.0643	0.1249	<b>0.0527</b>	0.1249	0.0636
Belgium	0.0051	0.1971	<b>0.0050</b>	0.0073	<b>0.0050</b>	0.6694	0.0058	0.0095	0.0095	<b>0.0313</b>	0.2422	0.0323	0.0450	0.0324	0.0324	0.0324	0.2595	0.0332	0.2595	0.0577
Brazil	0.0012	0.1120	0.0014	0.0020	<b>0.0010</b>	0.9925	<b>0.0010</b>	0.0027	0.0027	0.0249	0.1668	0.0276	0.0326	0.0222	0.0222	0.0219	0.5846	<b>0.0216</b>	0.5846	0.0381
Bulgaria	0.0009	0.1813	0.0009	0.0010	<b>0.0008</b>	0.1759	<b>0.0008</b>	0.0022	0.0022	0.0174	0.1050	0.0174	0.0206	0.0105	0.0105	<b>0.0159</b>	<b>0.0159</b>	0.0156	<b>0.0159</b>	0.0277
China	0.0042	0.3349	0.0047	0.0050	0.0055	0.0044	<b>0.0003</b>	0.0044	0.0046	0.0322	0.3501	0.0342	0.0375	0.0410	0.0410	0.0346	<b>0.0270</b>	0.0361	<b>0.0270</b>	0.0342
Croatia	0.0025	0.0046	0.0023	0.0461	0.0008	0.0007	<b>0.0004</b>	0.0008	0.5356	0.0304	0.0418	0.0291	0.1430	0.0161	0.0161	<b>0.0160</b>	0.0409	0.0166	0.0409	0.1537
Czech	0.0082	0.9185	0.0089	0.0092	0.0081	0.0082	0.9067	0.0077	<b>0.0010</b>	0.0380	0.6732	0.0372	0.0504	0.0455	0.0455	<b>0.0369</b>	0.2705	0.0441	0.0441	0.0404
Denmark	0.0070	0.2619	0.0053	0.0076	0.0052	0.0052	0.3797	<b>0.0046</b>	0.0051	<b>0.0456</b>	0.1950	0.0481	0.0621	0.0487	0.0487	0.0476	0.1142	0.0578	0.1142	0.0471
Finland	0.0067	0.7343	0.0065	0.0051	0.0059	0.0165	<b>0.0060</b>	0.0064	0.0064	<b>0.0396</b>	0.4611	0.0408	0.0368	0.0390	0.0390	0.0400	0.0913	0.0410	0.0410	0.0402
France	0.0104	0.0237	0.0114	0.0378	0.0057	0.0055	0.0237	<b>0.0048</b>	0.0290	0.0483	0.0654	0.0504	0.1088	0.0371	0.0371	0.0370	0.0654	<b>0.0358</b>	0.0654	0.0828
Germany	0.0182	<b>0.0165</b>	0.0170	0.0197	0.0171	0.0183	0.1753	0.0205	0.0194	0.0552	0.0603	<b>0.0548</b>	0.0597	0.1203	0.1203	0.0554	<b>0.0548</b>	0.0585	<b>0.0548</b>	0.0666
Greece	0.4862	0.6709	0.4788	0.4704	0.4758	0.4814	<b>0.2765</b>	0.4754	0.4775	0.1550	0.2466	0.1490	0.1483	0.1643	0.1643	0.1511	0.9744	<b>0.1472</b>	0.9744	0.1496
Hungary	0.0032	0.0055	0.0031	0.0012	0.0011	<b>0.0010</b>	0.0049	0.0015	0.0008	0.0367	0.0301	0.0361	0.0200	0.0203	0.0203	0.0190	0.0306	0.0259	0.0306	<b>0.0173</b>
Indonesia	0.4590	0.4590	0.4588	<b>0.4585</b>	0.4587	<b>0.4585</b>	0.6687	0.4586	0.4588	0.1924	0.1925	0.1926	0.1928	0.1925	0.1925	0.1926	<b>0.1539</b>	0.1925	<b>0.1539</b>	0.1930
Ireland	0.0228	0.0889	0.0206	0.0375	0.0197	<b>0.0189</b>	0.7325	0.0175	0.0919	0.0487	0.0625	0.0477	0.0637	0.0477	0.0477	0.0439	0.5391	<b>0.0439</b>	0.5391	0.1077
Italy	0.0043	0.0046	0.0038	0.0015	0.0046	0.0013	<b>0.0012</b>	0.0013	0.0013	0.0496	0.0476	0.0463	0.0286	<b>0.0239</b>	<b>0.0239</b>	0.0256	0.0476	0.0263	0.0256	0.0254
Japan	0.0026	0.6015	0.0023	0.0033	<b>0.0022</b>	0.5609	<b>0.0022</b>	0.0044	0.0044	0.0290	0.8366	0.0290	0.0374	0.0284	0.0284	0.0284	0.1798	<b>0.0283</b>	0.0284	0.0448
Latvia	0.0052	0.1550	0.0051	0.0098	0.0042	0.0044	0.3013	<b>0.0043</b>	0.0046	0.0319	0.2975	0.0330	0.0471	0.0316	0.0316	<b>0.0314</b>	0.1861	0.0327	<b>0.0314</b>	0.0316
Lithuania	0.0073	0.4946	0.0076	0.0074	0.0067	0.4168	<b>0.0063</b>	0.4168	0.0079	0.0389	0.3211	0.0401	0.0405	0.0398	0.0398	<b>0.0381</b>	0.1271	0.0401	<b>0.0381</b>	0.0396
Mexico	0.0024	0.0034	0.0026	0.0026	0.0028	0.0023	0.2575	0.0028	0.0031	0.0397	0.0384	0.0367	0.0381	0.0398	0.0398	<b>0.0365</b>	0.2979	0.0395	<b>0.0365</b>	0.0409
Netherlands	0.0199	0.0184	0.0179	0.0182	0.0177	0.1028	<b>0.0181</b>	0.1028	0.0184	0.0743	0.0758	0.0739	0.0754	0.0732	0.0732	0.0731	<b>0.0000</b>	0.0747	<b>0.0000</b>	0.0765
Norway	0.0675	0.2614	<b>0.0668</b>	0.0693	0.3232	0.0673	0.3232	0.0677	0.0675	0.1514	0.2985	0.1484	0.1575	<b>0.1050</b>	<b>0.1050</b>	0.1480	<b>0.1050</b>	0.1488	<b>0.1050</b>	0.1489
Philippines	0.0008	0.0032	0.0008	0.0007	0.0009	<b>0.0006</b>	0.2773	0.0010	0.0012	0.0226	0.0209	0.0206	0.0199	0.0231	0.0231	<b>0.0185</b>	0.9723	0.0221	<b>0.0185</b>	0.0236
Poland	0.0029	0.0048	0.0029	0.0043	0.0012	0.0088	0.0011	<b>0.0010</b>	<b>0.0010</b>	0.0325	0.0471	0.0321	0.0401	0.0202	0.0202	0.0201	0.0612	0.0197	<b>0.0176</b>	0.0236
Portugal	0.0035	0.0068	0.0038	0.0059	0.0015	0.0113	0.0023	<b>0.0015</b>	<b>0.0015</b>	0.0405	0.0545	0.0422	0.0557	0.0263	0.0263	0.0260	0.0743	0.0334	<b>0.0247</b>	0.0334
Qatar	<b>0.0042</b>	0.0066	0.0043	0.0062	0.0604	<b>0.0042</b>	<b>0.0042</b>	0.0045	0.0044	0.0349	0.0421	0.0353	0.0444	0.0350	0.0350	<b>0.0346</b>	0.0835	0.0362	<b>0.0346</b>	0.0357
Romania	0.0016	0.0198	0.0009	0.0009	0.0008	0.0007	0.7783	<b>0.0011</b>	0.0026	0.0172	0.1187	0.0183	0.0184	0.0156	0.0156	<b>0.0154</b>	0.1630	0.0188	<b>0.0154</b>	0.0292
Russia	0.0012	0.0021	0.0012	0.0012	0.0012	0.0021	0.0021	<b>0.0011</b>	0.0012	0.0239	0.0259	0.0238	0.0237	0.0243	0.0243	0.0243	0.0259	<b>0.0235</b>	0.0259	0.0239
Slovakia	0.0024	0.0339	0.0025	0.0264	<b>0.0018</b>	0.6179	0.0019	0.0067	0.0067	0.0252	0.0633	0.0265	0.0752	<b>0.0218</b>	<b>0.0218</b>	0.0298	0.8524	0.0312	<b>0.0218</b>	0.0420
Slovenia	0.0034	0.0405	0.0034	<b>0.0033</b>	0.0034	0.2103	<b>0.0049</b>	0.0044	0.0044	0.0302	0.0426	0.0300	0.0297	0.0299	0.0299	0.0298	0.8524	0.0312	<b>0.0296</b>	0.0420
Spain	0.0028	0.0279	0.0027	0.0031	0.0025	0.0024	0.0279	<b>0.0023</b>	0.0061	0.0317	0.0827	0.0320	0.0351	0.0295	0.0295	0.0285	0.0827	<b>0.0282</b>	<b>0.0282</b>	0.0299
Sweden	<b>0.0055</b>	0.1411	0.0060	0.0059	0.0056	0.0058	0.2944	0.0060	0.0058	0.0475	0.5913	0.0481	0.0481	0.0485	0.0485	<b>0.0464</b>	0.4491	0.0473	<b>0.0464</b>	0.0465
Thailand	<b>0.0013</b>	0.2656	0.0014	0.0015	<b>0.0013</b>	0.4255	<b>0.0013</b>	0.0016	0.0016	<b>0.0252</b>	0.2061	0.0261	0.0271	0.0256	0.0256	<b>0.0252</b>	0.3849	0.0259	<b>0.0252</b>	0.0299
Turkey	0.0008	0.0117	0.0009	0.0010	0.0007	0.4182	<b>0.0006</b>	0.0013	0.0013	0.0198	0.0583	0.0207	0.0221	0.0186	0.0186	<b>0.0185</b>	0.0773	0.0185	<b>0.0185</b>	0.0270
UK	0.0015	0.0028	0.0017	0.0016	0.0016	0.4541	<b>0.0014</b>	0.0045	0.0045	0.0278	0.0296	0.0303	0.0292	0.0312	0.0312	0.0284	0.3958	<b>0.0281</b>	<b>0.0281</b>	0.0330
Ukraine	<b>0.0042</b>	0.1403	0.0049	0.0052	0.0043	0.1388	0.0045	0.0067	0.0067	0.0274	0.1164	0.0249	0.0290	0.0258	0.0258	0.0250	0.4394	<b>0.0244</b>	<b>0.0244</b>	0.0304
USA	0.0151	0.0177	0.0151	0.0148	<b>0.0146</b>	0.0177	0.0147	0.0163	0.0163	0.0949	0.0965	0.0949	0.0950	0.0929	0.0929	<b>0.0907</b>	0.0965	0.0916	<b>0.0907</b>	0.0990
Venezuela	0.0009	0.0025	0.0009	0.0008	<b>0.0007</b>	0.0662	<b>0.0007</b>	0.0018	0.0018	0.0209	0.0239	0.0207	0.0201	0.0188	0.0188	0.0187	0.0454	<b>0.0176</b>	<b>0.0176</b>	0.0310

Numbers in bold refer to the models minimizing the loss function criterion.

Table 3: Results of the loss functions criteria for the twenty-day out-of-sample volatility predictions (*Continued*)

Statistic criteria	HMSE										HMAE											
	GJR	APARCH	IGARCH	FIGARCH	FIGARCH	FIGARCH	FIAPARCH	FIAPARCH	FIAPARCH	FIAPARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIGARCH	FIGARCH	FIAPARCH	FIAPARCH	FIAPARCH	HYGARCH	HYGARCH
Austria	0.1717	0.8324	0.8449	0.8449	0.5361	0.2306	0.1342	0.4810	0.9513	0.1400	0.1135	0.1160	0.1160	0.1160	0.1160	0.1160	0.1014	0.1504	0.1504	0.1504	0.3334	0.3334
Belgium	<b>0.1511</b>	0.6688	0.1655	0.2009	0.5731	0.4235	0.0048	0.3403	0.3789	0.2287	0.1332	0.9186	<b>0.1058</b>	0.1382	0.1382	0.1382	0.1382	0.6533	0.6533	0.6533	0.5977	0.5977
Brazil	0.3147	0.1508	0.3319	0.8107	<b>0.1001</b>	0.1605	0.1141	0.1071	0.1247	0.4636	0.1299	0.1570	0.8090	0.8774	<b>0.1018</b>	0.8774	0.8774	<b>0.1018</b>	0.9637	0.9637	0.9637	0.9637
Bulgaria	0.2566	0.1750	0.9986	0.2960	0.7895	0.1858	0.3913	0.3834	0.8154	0.4077	0.1066	0.2605	0.1725	<b>0.0815</b>	0.1725	<b>0.0815</b>	0.3563	0.3563	0.3563	0.3590	0.3590	
China	0.4046	0.3621	0.5312	0.3100	0.9334	0.3090	<b>0.1973</b>	0.1656	0.4026	0.4940	0.1455	0.4296	0.1343	0.1503	0.1503	0.1503	<b>0.1302</b>	0.3417	0.3417	0.3417	0.3417	
Croatia	0.7541	0.8237	<b>0.1212</b>	0.9844	0.6604	0.9817	0.9754	0.7310	0.7544	<b>0.7297</b>	0.8821	0.7854	0.7447	0.7559	0.7434	0.7559	0.9875	0.9875	0.9875	0.9875	0.9875	
Czech	0.3804	0.1615	0.1679	0.7687	0.3568	0.1866	<b>0.1210</b>	0.8474	0.9255	<b>0.1102</b>	0.8713	0.8982	0.1697	0.2686	0.2686	0.2686	0.9185	0.9185	0.9185	0.3290	0.3290	
Denmark	0.7105	0.9270	0.5294	0.7015	0.4449	0.1866	<b>0.1334</b>	0.4203	0.3393	0.4420	0.2553	0.5189	0.4186	0.9326	0.9326	0.9326	<b>0.1785</b>	0.5613	0.5613	0.5613	0.5613	
Finland	0.1419	0.7411	0.1322	0.1299	<b>0.1036</b>	0.1107	0.2654	0.2908	0.5239	0.1105	0.4776	0.3471	0.1356	0.2766	0.2766	0.2766	<b>0.0150</b>	0.4860	0.4860	0.4860	0.4860	
France	0.3916	0.3300	0.1807	0.1572	0.9806	<b>0.1188</b>	0.9758	0.5708	0.7225	0.4459	0.7253	0.7944	0.7757	0.7225	0.7225	0.7225	<b>0.1098</b>	0.7805	0.7805	0.7805	0.7805	
Germany	<b>0.2565</b>	0.4401	0.5318	0.8237	0.8339	0.4818	0.5216	0.1277	0.9793	0.7806	0.7225	0.7484	0.1438	<b>0.1015</b>	0.1438	<b>0.1015</b>	0.6760	0.2014	0.2014	0.2014	0.2014	
Greece	0.5632	0.1728	0.3808	0.1166	0.1101	<b>0.1085</b>	0.3914	0.1711	0.2910	0.4335	0.8196	0.4155	0.1881	<b>0.1490</b>	0.1881	<b>0.1490</b>	0.1759	0.3499	0.3499	0.3499	0.3499	
Hungary	0.3482	0.2020	0.1544	0.1241	0.8405	<b>0.1136</b>	0.1327	0.7520	<b>0.7063</b>	0.7510	0.7681	0.7416	0.7162	0.7102	0.7102	0.7102	0.7423	0.7524	0.7524	0.7524	0.7524	
Indonesia	0.6001	0.1478	<b>0.1356</b>	0.9473	0.2363	0.6628	0.4736	0.4057	0.1180	0.1912	0.1402	0.9120	0.1323	<b>0.1130</b>	0.1323	<b>0.1130</b>	0.8306	0.2002	0.2002	0.2002	0.2002	
Ireland	0.2727	0.5478	0.2666	0.8881	<b>0.1456</b>	0.2613	0.4427	0.1308	<b>0.1289</b>	0.1344	0.1949	0.3797	0.2469	0.9929	0.9929	0.9929	0.1316	0.1094	0.1094	0.1094	0.1094	
Italy	0.5332	0.5444	0.8795	0.6657	<b>0.1005</b>	0.7044	0.6773	0.6671	0.6652	0.6584	0.7089	0.6766	<b>0.6411</b>	0.6652	0.6652	0.6652	0.6456	0.6456	0.6456	0.6456	0.6456	
Japan	0.1605	0.2312	0.2424	0.1800	<b>0.1526</b>	0.2020	0.7090	0.1814	0.2176	0.9274	<b>0.1048</b>	0.1202	0.9272	0.2593	0.2593	0.2593	0.8864	0.7186	0.7186	0.7186	0.7186	
Latvia	0.1466	0.4932	0.2723	0.2850	0.3223	0.4290	<b>0.1014</b>	0.1678	0.3185	0.4106	0.6083	<b>0.1436</b>	0.5743	0.1662	0.1662	0.1662	0.5234	0.3183	0.3183	0.3183	0.3183	
Lithuania	0.6748	0.9203	0.7588	0.8314	0.7358	0.0203	0.1169	0.2231	0.3007	0.9843	0.9596	0.3581	0.9213	0.2896	0.2896	0.2896	<b>0.1487</b>	0.3957	0.3957	0.3957	0.3957	
Mexico	0.2091	0.4638	0.3383	0.1779	0.4335	0.3150	<b>0.1786</b>	0.2651	0.4865	0.7087	0.3329	<b>0.1242</b>	0.5829	0.2816	0.2816	0.2816	0.2095	0.2095	0.2095	0.2095	0.2095	
Netherlands	0.0019	0.6496	0.3412	0.2072	0.0637	<b>0.0000</b>	<b>0.0000</b>	0.3186	0.8029	0.0619	0.3484	0.7852	0.6322	0.0961	0.0961	0.0961	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	
Norway	0.8095	0.3640	0.1293	<b>0.1105</b>	0.5344	0.9276	0.9276	0.9625	0.1405	0.4785	0.9825	0.1400	0.1836	<b>0.1005</b>	0.1836	<b>0.1005</b>	0.7092	0.1546	0.1546	0.1546	0.1546	
Philippines	0.1944	0.4249	0.4094	0.5997	<b>0.1045</b>	0.2713	0.1959	0.2934	0.7178	0.2247	0.1017	<b>0.1411</b>	0.4089	0.3749	0.4089	0.3749	0.7089	0.7089	0.7089	0.7089	0.7089	
Poland	0.1288	0.2891	0.5712	<b>0.1017</b>	0.6582	0.1118	0.1405	0.8264	0.7200	0.8248	<b>0.1145</b>	0.8328	0.7086	0.7477	0.7477	0.7477	0.7300	0.7939	0.7939	0.7939	0.7939	
Portugal	0.1711	0.4652	0.9779	0.9757	0.8876	0.9183	0.1444	0.9365	0.6933	0.9749	0.7071	0.7109	0.7116	0.7557	0.7557	0.7557	<b>0.5761</b>	0.7929	0.7929	0.7929	0.7929	
Qatar	0.1555	0.2364	0.5162	0.6170	0.4738	<b>0.1086</b>	0.1277	<b>0.1042</b>	0.1068	0.1197	0.2883	0.1930	0.7598	0.5004	0.5004	0.5004	0.3111	0.3036	0.3036	0.3036	0.3036	
Romania	0.2356	0.2481	0.4849	0.1579	0.2475	<b>0.2162</b>	0.7391	<b>0.1133</b>	0.8750	0.7715	0.7773	0.7664	0.7499	0.7592	0.7592	0.7592	0.8042	0.1228	0.1228	0.1228	0.1228	
Russia	0.5564	0.5567	0.5493	0.5487	<b>0.5422</b>	0.5583	0.5558	0.6024	0.5999	0.6010	0.6019	0.6011	<b>0.6008</b>	0.5999	0.5999	0.5999	0.6019	0.6018	0.6018	0.6018	0.6018	
Slovakia	0.1461	0.3172	0.1572	0.1572	0.2691	0.2237	<b>0.1246</b>	0.2604	0.7703	0.2627	<b>0.1362</b>	0.2946	0.2946	0.2534	0.2534	0.2534	0.3357	0.6022	0.6022	0.6022	0.6022	
Slovenia	0.1043	<b>0.1004</b>	0.9656	0.5711	0.3511	0.5791	0.2105	0.1349	0.1303	0.1343	<b>0.1362</b>	0.1338	<b>0.1214</b>	0.5795	0.5795	0.5795	0.1575	0.1595	0.1595	0.1595	0.1595	
Spain	0.1756	0.3023	0.1479	0.1879	0.8925	<b>0.1131</b>	0.1946	0.5370	0.7852	0.1700	0.2662	0.6153	0.7971	0.7852	0.7852	0.7852	<b>0.1443</b>	0.8480	0.8480	0.8480	0.8480	
Sweden	0.2442	0.6636	0.1900	0.4728	0.7608	0.9317	<b>0.1547</b>	0.2643	0.7898	0.6910	<b>0.1042</b>	0.8460	0.4014	0.8354	0.8354	0.8354	0.9885	0.1769	0.1769	0.1769	0.1769	
Thailand	0.1456	0.2096	0.4946	<b>0.1019</b>	0.2233	0.6441	0.4063	0.8256	0.9634	0.1751	0.2007	0.1378	<b>0.1017</b>	0.1734	0.1734	0.1734	0.3048	0.2604	0.2604	0.2604	0.2604	
Turkey	0.3787	0.3207	0.3749	0.1958	0.2613	0.6156	0.2767	<b>0.1840</b>	0.2208	0.9913	0.2780	0.9056	0.7166	0.8171	0.8171	0.8171	0.6053	0.3471	0.3471	0.3471	0.3471	
UK	0.2678	0.3819	0.3119	<b>0.1041</b>	0.4351	0.8048	0.1135	<b>0.1085</b>	0.1513	0.2461	0.9242	0.4022	0.1572	0.8956	0.8956	0.8956	0.4215	0.1596	0.1596	0.1596	0.1596	
Ukraine	0.1479	0.1547	0.1262	0.4541	<b>0.1003</b>	0.3017	0.3461	0.9458	0.9780	0.1663	0.1696	0.2055	0.9862	0.9370	0.9370	0.9370	<b>0.1562</b>	0.8165	0.8165	0.8165	0.8165	
USA	0.5874	0.5831	0.5456	0.5450	<b>0.5131</b>	0.5807	0.7764	0.4248	<b>0.3882</b>	0.4241	0.4244	0.4127	0.4116	<b>0.3882</b>	0.4116	<b>0.3882</b>	0.4194	0.7035	0.7035	0.7035	0.7035	
Venezuela	0.6039	0.6044	0.6700	0.6665	0.2341	0.1708	<b>0.1064</b>	0.6130	0.6045	0.6066	0.6017	0.6276	0.6244	0.1536	0.1536	0.1536	<b>0.1447</b>	0.2678	0.2678	0.2678	0.2678	

Numbers in bold refer to the models minimizing the loss function criterion.

Table 3: Results of the loss functions criteria for the twenty-day out-of-sample volatility predictions (*Continued*)

Statistic criteria	QLIKE										RZLOG									
	GARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIGARCH	FIAPARCH	HYGARCH	GARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIGARCH	FIAPARCH	HYGARCH		
Austria	0.3750	0.1801	0.3647	0.1122	<b>0.0853</b>	<b>0.0853</b>	0.2368	0.4388	0.2264	0.4459	0.7794	0.3746	<b>0.1370</b>	0.4488	0.4488	0.7341	0.4363	0.4395		
Belgium	0.3249	0.2503	0.2132	0.1330	0.9026	0.9056	<b>0.1228</b>	0.4992	0.4427	0.1054	0.2292	0.1098	0.2206	0.1329	<b>0.1001</b>	0.1417	0.9336	0.1345		
Brazil	0.9257	0.2236	0.4483	0.1299	0.1426	0.6683	0.7384	<b>0.1017</b>	0.8118	0.7850	0.1215	0.1084	0.2500	0.8061	<b>0.0688</b>	0.9786	0.2601	0.9204		
Bulgaria	0.3663	0.9138	0.3905	<b>0.1048</b>	0.2435	0.1555	0.9137	0.3393	0.8627	0.0860	0.1904	0.0860	0.0089	0.8123	<b>0.0809</b>	0.1905	0.8050	0.1035		
China	0.1481	0.2386	0.4922	0.1437	0.4296	0.1168	<b>0.0789</b>	0.1285	0.3236	0.9805	0.1314	0.1498	0.1259	0.5610	<b>0.1031</b>	0.8734	0.1774	<b>0.1013</b>		
Croatia	0.1083	0.1065	0.1081	0.8651	0.9786	0.1022	0.1065	0.1028	<b>0.0301</b>	0.1315	0.1591	0.1290	0.2399	0.9687	<b>0.1007</b>	0.1578	0.1020	0.6233		
Czech	0.6735	0.4635	0.9242	0.7804	0.7404	0.1523	0.7963	0.3167	<b>0.1502</b>	0.7963	0.3167	0.8417	0.1125	0.1092	<b>0.0474</b>	0.6036	0.9240	0.7744		
Denmark	0.2878	0.3238	0.4243	0.2553	0.5189	0.4019	0.9147	<b>0.1785</b>	0.5447	<b>0.1056</b>	0.1821	0.1860	8.9493	0.1191	0.1809	0.3673	0.9240	0.1931		
Finland	0.5708	0.3954	<b>0.1089</b>	0.3253	0.3324	0.1202	0.1426	0.4608	0.4697	0.2240	0.9955	0.1902	0.8405	0.1390	<b>0.1120</b>	0.1508	0.1576	0.1479		
France	<b>0.1680</b>	0.5936	0.3170	0.9615	0.4206	0.4408	0.5936	0.4926	0.6442	0.8680	<b>0.1060</b>	0.8859	0.1270	0.7548	0.7518	<b>0.1060</b>	0.7389	0.1080		
Germany	<b>0.1148</b>	0.2539	0.6543	0.7091	0.7356	0.1309	0.3156	0.6746	0.1885	0.7950	0.8545	0.7606	0.9179	0.8703	0.8189	0.1209	<b>0.1020</b>	0.9904		
Greece	0.1711	0.2741	0.4335	0.6542	0.2582	0.1862	<b>0.1472</b>	0.1756	0.3319	0.9231	0.2916	0.3044	0.1760	0.2234	<b>0.1340</b>	0.1506	0.1627	0.1451		
Hungary	0.8936	0.8885	0.8926	0.8866	0.8261	0.8524	<b>0.7965</b>	0.8634	0.8103	0.1249	0.1249	0.1238	0.9855	<b>0.1046</b>	0.9688	0.1137	0.1132	0.9037		
Indonesia	<b>0.1115</b>	0.1167	0.1895	0.1385	0.8979	0.1307	0.4041	0.8149	0.2002	0.1362	0.3234	0.2156	0.2222	0.1270	0.2141	<b>0.1169</b>	0.1719	0.1636		
Ireland	<b>0.1150</b>	0.1970	0.1187	0.1932	0.3638	0.2309	0.8289	0.1300	0.4509	0.1118	0.9596	<b>0.1110</b>	0.1807	0.1237	0.1165	0.1204	0.1527	0.1330		
Italy	0.7739	0.7709	0.7740	0.6496	0.7049	<b>0.6434</b>	0.7709	0.6969	0.6980	0.8880	0.8694	0.8575	0.6427	<b>0.5658</b>	0.6196	0.8694	0.6496	0.6142		
Japan	0.1677	0.8736	0.9131	0.1186	0.1065	0.9130	<b>0.1034</b>	0.8725	0.5747	0.8854	0.2290	0.1141	0.1226	0.8269	0.1080	0.1235	<b>0.1013</b>	0.9994		
Latvia	0.1488	0.2997	0.3894	0.6083	0.1251	<b>0.0393</b>	0.0413	0.5231	0.3183	<b>0.1304</b>	0.1525	0.1490	0.5939	0.1431	0.1339	0.2170	0.2523	0.1506		
Lithuania	0.2044	0.2832	0.8059	0.9593	0.1943	0.7515	0.2709	<b>0.1485</b>	0.3763	0.1440	0.1894	<b>0.1353</b>	0.3666	0.1400	0.1467	0.2463	0.2881	0.1599		
Mexico	0.2510	0.3569	0.7069	0.3311	0.2391	0.5811	0.2799	<b>0.1242</b>	0.2095	0.1281	0.6893	0.3228	0.2731	0.1787	0.2829	0.4145	<b>0.1144</b>	0.2030		
Netherlands	0.3186	0.7874	0.0602	0.3482	0.6164	0.6164	0.0943	<b>0.0000</b>	0.0000	0.1709	0.1953	0.1873	0.4355	0.1955	0.1852	0.2312	<b>0.0005</b>	0.0205		
Norway	0.9625	0.9873	0.4605	0.9822	<b>0.1382</b>	0.1817	<b>0.1382</b>	0.7092	0.1546	0.1780	0.3609	0.2067	0.6787	0.4132	0.2810	0.4132	<b>0.1752</b>	0.6818		
Philippines	0.2764	0.5689	0.4073	0.9985	0.1411	0.9867	<b>0.2082</b>	0.3731	0.7089	0.1427	0.8120	0.1290	0.2031	0.1566	0.9553	<b>0.1075</b>	0.1789	0.2175		
Poland	0.7821	0.9163	0.7809	<b>0.5306</b>	0.7129	0.8302	0.9038	0.8112	0.7167	0.9412	0.1378	0.9352	0.8913	0.7587	0.8056	<b>0.1298</b>	0.7800	0.7594		
Portugal	0.4618	0.7363	0.4295	0.6806	0.6288	0.6279	<b>0.4300</b>	0.6775	0.5410	0.9387	0.1244	0.9564	0.1147	0.7570	<b>0.1116</b>	0.9194	0.6986			
Qatar	0.8931	0.9255	0.2883	0.2883	0.1782	0.6127	0.3549	0.2950	0.2885	<b>0.1012</b>	0.1193	0.1063	0.5715	0.1140	0.9339	0.1159	0.1160	0.1245		
Romania	0.9652	0.7679	0.8900	0.1095	0.5996	0.8822	0.9351	0.8511	<b>0.1053</b>	0.8422	0.2420	0.8845	0.1900	0.7781	0.8136	0.1131	0.8882	<b>0.1068</b>		
Russia	0.7350	0.7483	0.7483	0.7356	0.7394	0.7396	<b>0.7342</b>	0.7346	0.7355	0.4808	0.5077	0.4789	0.4786	0.4858	0.4856	0.5077	<b>0.4764</b>	0.4806		
Slovakia	0.8763	0.8667	0.8932	0.1168	<b>0.1221</b>	<b>0.1220</b>	0.7453	0.4213	0.4213	0.8999	0.1717	0.9302	0.1374	0.8099	0.8099	0.7402	0.7805	<b>0.1089</b>		
Slovenia	0.3175	0.3700	0.3431	0.3232	0.3232	0.4420	0.3842	0.1575	<b>0.1376</b>	0.2210	0.2209	0.2202	0.2202	0.2200	0.2192	0.1958	0.4102	<b>0.1607</b>		
Spain	0.4066	0.6002	0.1569	0.2526	0.4860	0.6670	0.6002	<b>0.1314</b>	0.7091	0.7214	0.1266	0.7774	0.8834	0.6883	0.7057	<b>0.1041</b>	0.7092	0.9179		
Sweden	0.2641	0.6405	0.6727	<b>0.1040</b>	0.8460	0.4014	0.1769	0.9883	0.1595	0.3345	0.1262	0.1958	0.3440	<b>0.1552</b>	0.1162	0.3369	0.4645	0.1294		
Thailand	0.8093	0.8193	0.1599	0.1990	0.8699	0.1228	0.2893	0.2446	0.1228	0.1665	0.9046	0.1089	0.1216	0.9387	<b>0.1019</b>	0.3224	0.1245	0.1270		
Turkey	<b>0.1695</b>	0.2053	0.8470	0.2632	0.7646	0.5755	0.6912	0.7542	<b>0.0647</b>	<b>0.0647</b>	0.9940	0.6162	0.7366	0.5595	0.5477	0.7946	0.4184	0.7218		
UK	<b>0.1066</b>	0.1342	0.2441	0.9221	0.4022	0.1400	0.8769	0.4213	0.1576	0.2285	0.1329	0.2740	0.3564	0.1378	<b>0.1115</b>	0.3719	0.4688	0.2560		
Ukraine	0.9283	<b>0.0917</b>	0.1489	0.4717	0.8153	0.4308	0.4308	0.1525	<b>0.1233</b>	0.8165	0.5386	0.4148	0.7089	0.7089	0.6997	0.3086	0.7997	0.1493		
USA	0.3082	<b>0.2732</b>	0.3076	0.3077	0.2962	0.2956	<b>0.2732</b>	0.3036	0.5831	0.1617	0.1628	0.1621	0.1624	0.1574	0.1545	0.1628	<b>0.1573</b>	0.1809		
Venezuela	0.8257	0.8405	0.8291	0.8315	0.7997	0.7998	0.0123	<b>0.0011</b>	0.1111	0.4877	0.5290	0.4847	0.4770	0.4530	0.4516	0.7244	0.4906	<b>0.0751</b>		

Numbers in bold refer to the models minimizing the loss function criterion.

Table 3: Results of the loss functions criteria for the twenty-day out-of-sample volatility predictions (*Continued*)

Statistic criteria	MLAE									
	GARCH	EGARCH	GJR	APARCH	IGARCH	FIGARCH	FIGARCH	FIAPARCH	HYGARCH	HYGARCH
Austria	0,1624	0,1566	0,1630	0,9488	0,1566	<b>0,1067</b>	0,1110	0,1589	0,1588	0,1588
Belgium	0,1863	0,1824	0,1836	0,1651	0,1833	<b>0,1339</b>	0,1484	0,1817	0,1469	0,1469
Brazil	0,1796	0,1865	0,1750	0,1672	0,1860	<b>0,1052</b>	0,1930	0,1893	0,1591	0,1591
Bulgaria	0,2087	0,1254	0,2091	0,1990	0,2131	0,2136	<b>0,1253</b>	0,2135	0,1873	0,1873
China	0,1991	0,1922	0,2230	0,2163	0,2025	<b>0,1741</b>	0,1936	0,1756	0,2061	0,2061
Croatia	0,1767	0,1567	0,1789	0,1092	0,2061	0,2034	0,1568	0,2014	<b>0,0012</b>	<b>0,0012</b>
Czech	0,2128	0,6195	0,2197	<b>0,1678</b>	0,1717	0,2149	0,2106	0,1719	0,1999	0,1999
Denmark	0,1664	0,1641	0,1784	0,1610	0,1652	0,1723	0,1604	<b>0,1587</b>	0,1707	0,1707
Finland	0,1771	0,1743	0,1839	0,1809	0,1765	0,1840	<b>0,1251</b>	0,1777	0,1783	0,1783
France	0,1662	0,1409	0,1643	<b>0,1266</b>	0,1754	0,1749	0,1409	0,1757	0,1426	0,1426
Germany	0,1838	0,1603	0,1796	0,1759	0,1804	0,1832	<b>0,1229</b>	0,1821	0,1599	0,1599
Greece	0,2184	0,1930	0,2217	0,1652	0,1427	0,2053	<b>0,1288</b>	0,2246	0,1799	0,1799
Hungary	<b>0,1651</b>	0,1798	0,1659	0,1942	0,1904	0,1941	0,1789	0,1788	0,1987	0,1987
Indonesia	0,1719	0,1723	0,1672	<b>0,1601</b>	0,1665	0,1633	0,1671	0,1641	0,1639	0,1639
Ireland	0,1812	0,1743	0,1814	0,1641	0,1804	0,1860	0,1927	0,1813	<b>0,1378</b>	<b>0,1378</b>
Italy	<b>0,1445</b>	0,1467	0,1481	0,1737	0,1841	0,1767	0,1467	0,1735	0,1773	0,1773
Japan	0,1799	0,6318	0,1818	0,1678	0,1818	0,1826	<b>0,1264</b>	0,1852	0,1569	0,1569
Latvia	0,2083	0,2026	0,2191	0,2075	0,2019	0,2029	<b>0,1468</b>	0,2280	0,2022	0,2022
Lithuania	0,2013	0,1874	0,1961	0,2050	0,2178	0,1868	0,1892	<b>0,1843</b>	0,2069	0,2069
Mexico	0,1763	0,1759	0,1708	0,1667	0,1653	0,1671	0,1679	<b>0,1614</b>	0,1675	0,1675
Netherlands	0,1654	0,1684	0,1700	0,1744	0,1659	<b>0,1615</b>	0,1623	0,1727	0,1670	0,1670
Norway	0,1437	0,1651	0,1523	0,1538	0,1503	<b>0,1381</b>	0,1503	0,1739	0,1646	0,1646
Philippines	0,1795	0,1935	0,1903	0,1895	0,1784	0,1917	<b>0,1560</b>	0,1876	0,1968	0,1968
Poland	0,1784	0,1484	0,1797	0,1742	0,1980	0,1951	<b>0,1391</b>	0,1965	0,2008	0,2008
Portugal	0,1622	0,1432	0,1597	0,1456	0,1789	0,1790	<b>0,1329</b>	0,1632	0,1841	0,1841
Qatar	0,1862	0,1694	0,1848	0,1849	0,1841	0,1904	<b>0,1386</b>	0,1877	0,1859	0,1859
Romania	0,2064	0,9735	0,1975	0,1904	0,2119	0,2063	<b>0,1761</b>	0,1986	0,1834	0,1834
Russia	0,1800	<b>0,1776</b>	0,1804	0,1799	0,1792	0,1794	<b>0,1776</b>	0,1805	0,1801	0,1801
Slovakia	0,2046	0,2085	0,2005	0,1745	0,2107	0,2107	<b>0,1445</b>	0,2113	0,1864	0,1864
Slovenia	0,1792	0,2325	0,1793	0,1805	0,1802	0,1810	0,1997	<b>0,1744</b>	0,2168	0,2168
Spain	0,1740	<b>0,1237</b>	0,1731	0,1682	0,1774	0,1806	<b>0,1237</b>	0,1804	0,1557	0,1557
Sweden	0,1690	0,1603	0,1873	0,1824	0,1660	0,1791	<b>0,0519</b>	0,1810	0,1814	0,1814
Thailand	0,1873	0,1890	0,1884	0,1864	0,1849	0,1848	0,1811	0,1816	<b>0,1763</b>	<b>0,1763</b>
Turkey	0,1901	0,1437	0,1887	0,1847	0,1927	0,1933	<b>0,1416</b>	0,1886	0,1728	0,1728
UK	0,1728	0,1788	0,1775	0,1804	0,1811	0,1772	0,1809	<b>0,1692</b>	0,1701	0,1701
Ukraine	0,1969	0,5542	0,2127	0,2020	0,2016	0,2059	<b>0,0591</b>	0,2100	0,2195	0,2195
USA	0,1222	<b>0,1218</b>	0,1222	0,1221	0,1243	0,1268	<b>0,1218</b>	0,1265	0,1189	0,1189
Venezuela	0,1926	0,1912	0,1943	0,1965	0,1980	0,1992	0,1737	0,1986	<b>0,1716</b>	<b>0,1716</b>

Numbers in bold refer to the models minimizing the loss function criterion.

First, the in-sample estimation shows that all the models almost always pass all diagnostic tests for the most cases, and that the smallest Akaike criterion does not allow us to choose only one best fitted model. Second, none of the volatility models studied in this paper is found to be more relevant than all the others in all situations, in terms of forecasting ability. The chosen model varies from one country to another and from loss function criterion to another. Third, in most cases and according to the majority of the errors statistics criteria, the non-linear GARCH-class models, that capture the long-memory behavior, the leverage effects and the asymmetric dependencies in the volatility process are more relevant in terms of out-of-sample forecasting ability than the others. Fourth, the FIGARCH and FIEGARCH models are found to be the most relevant and robust forecasting models.

Since comparing predictive performance of volatility models is of a paramount in assessing diversifiable risk, in dynamic asset pricing theory and in optimization of portfolio allocation, the economic implication of our findings concerns particularly policymakers, financial practitioners and financial market participants generally. The in-sample performances show that no model clearly outperforms all the others, and since the results are mitigated and differ from one country to another, no volatility model should be selected in an arbitrary way. The model selection should rather be based on the particular features of the data used and the country studied. When it comes to the forecasting performances, some models are preferable and seem to predict accurately and robustly the future volatility of the CDS market. Thus, after taking into account the transaction costs, investors can eventually take advantage of the market's relative inefficiency and generate extra-profits by putting in place a simple trading strategy to exploit the predictability of sovereign CDS volatility. Finally, our study shows that improving the volatility forecasts needs including the maximum of CDS market's stylized facts. However, in practice, the implementation of complex models generates additional costs that are not necessarily reflected in our comparison method, which may controvert the usefulness of using better volatility predictive models.

Our research line can be pursued in several ways. First, a further investigation on the performances of the volatility models can be done by carrying out a comparative study based on the superior predictive ability test rather than on the diagnostic tests and loss functions criteria as in our case. Second, our study can be applied to the corporate CDS market, in order to assess whether the nature of the reference entity impacts the performances of the studied models. Third, since there is a dynamic segmentation in financial markets, it can be interesting to check the robustness of our findings using a different sample from other regions and/or a CDS term structure with different maturity.

## References

- Paolo Agnolucci. Volatility in crude oil futures: a comparison of the predictive ability of garch and implied volatility models. *Energy Economics*, 31(2):316–321, 2009. 3
- Torben G Andersen, Tim Bollerslev, and Steve Lange. Forecasting financial market volatility: Sample frequency vis-a-vis forecast horizon. *Journal of empirical finance*, 6(5):457–477, 1999. 9
- Davide Avino and Ogonna Nneji. Are cds spreads predictable? an analysis of linear and

- non-linear forecasting models. *International Review of Financial Analysis*, 34:262–274, 2014. [2](#), [4](#)
- Richard T Baillie, Tim Bollerslev, and Hans Ole Mikkelsen. Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 74(1):3–30, 1996. [3](#), [7](#)
- Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327, 1986. [3](#), [5](#)
- Tim Bollerslev and Eric Ghysels. Periodic autoregressive conditional heteroscedasticity. *Journal of Business & Economic Statistics*, 14(2):139–151, 1996. [9](#)
- Tim Bollerslev and Hans Ole Mikkelsen. Modeling and pricing long memory in stock market volatility. *Journal of econometrics*, 73(1):151–184, 1996. [3](#), [8](#)
- Tim Bollerslev, Ray Y Chou, and Kenneth F Kroner. Arch modeling in finance: A review of the theory and empirical evidence. *Journal of econometrics*, 52(1-2):5–59, 1992. [4](#)
- Charles G Broyden. The convergence of a class of double-rank minimization algorithms: 2. the new algorithm. *IMA Journal of Applied Mathematics*, 6(3):222–231, 1970. [8](#)
- Amélie Charles and Olivier Darné. Forecasting crude-oil market volatility: Further evidence with jumps. *Energy Economics*, 67:508–519, 2017. [3](#)
- Walid Chkili, Shawkat Hammoudeh, and Duc Khuong Nguyen. Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory. *Energy Economics*, 41:1–18, 2014. [3](#)
- Mauro Costantini, Matteo Fragetta, and Giovanni Melina. Determinants of sovereign bond yield spreads in the emu: An optimal currency area perspective. *European Economic Review*, 70:337–349, 2014. [2](#), [4](#)
- Virginie Coudert and Mathieu Gex. The interactions between the credit default swap and the bond markets in financial turmoil. *Review of International Economics*, 21(3):492–505, 2013. [2](#), [4](#)
- Virginie Coudert, Mathieu Gex, et al. Credit default swap and bond markets: which leads the other. *Financial Stability Review, Banque de France*, 14(2010):161167, 2010. [2](#), [4](#)
- RalphB D’Agostino. *Goodness-of-fit-techniques*. Routledge, 2017. [14](#)
- James Davidson. Moment and memory properties of linear conditional heteroscedasticity models, and a new model. *Journal of Business & Economic Statistics*, 22(1):16–29, 2004. [3](#), [8](#)
- David A Dickey and Wayne A Fuller. Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica: Journal of the Econometric Society*, pages 1057–1072, 1981. [10](#)

- Francis X Diebold and Robert S Mariano. Comparing predictive accuracy. *Journal of Business & economic statistics*, 20(1):134–144, 2002. [9](#)
- Zhuanxin Ding, Clive WJ Granger, and Robert F Engle. A long memory property of stock market returns and a new model. *Journal of empirical finance*, 1(1):83–106, 1993. [3](#), [7](#)
- Robert F Engle. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007, 1982. [3](#), [5](#)
- Robert F Engle and Tim Bollerslev. Modelling the persistence of conditional variances. *Econometric reviews*, 5(1):1–50, 1986. [3](#), [6](#)
- Dean Fantazzini. Fractionally integrated models for volatility: A review-empirical appendix: Some examples with r interfaced with the ox package g@rch. In *Nonlinear Financial Econometrics: Markov Switching Models, Persistence and Nonlinear Cointegration*, pages 104–123. Springer, 2011. [8](#)
- Miguel A Ferreira and Pedro Santa-Clara. Forecasting stock market returns: The sum of the parts is more than the whole. *Journal of Financial Economics*, 100(3):514–537, 2011. [3](#)
- Alessandro Fontana and Martin Scheicher. An analysis of euro area sovereign cds and their relation with government bonds. *Journal of Banking & Finance*, 62:126–140, 2016. [2](#), [4](#)
- John Geweke and Susan Porter-Hudak. The estimation and application of long memory time series models. *Journal of time series analysis*, 4(4):221–238, 1983. [11](#)
- Lawrence R Glosten, Ravi Jagannathan, and David E Runkle. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5):1779–1801, 1993. [3](#), [7](#)
- Massimo Guidolin, Stuart Hyde, David McMillan, and Sadayuki Ono. Non-linear predictability in stock and bond returns: When and where is it exploitable? *International Journal of Forecasting*, 25(2):373–399, 2009. [3](#)
- Donald B Keim and Robert F Stambaugh. Predicting returns in the stock and bond markets. *Journal of financial Economics*, 17(2):357–390, 1986. [3](#)
- CNV Krishnan, Peter H Ritchken, and James B Thomson. Predicting credit spreads. *Journal of Financial Intermediation*, 19(4):529–563, 2010. [2](#), [4](#)
- Francis A Longstaff, Jun Pan, Lasse H Pedersen, and Kenneth J Singleton. How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics*, 3(2):75–103, 2011. [2](#), [4](#)
- Jose Lopez. Evaluating the predictive accuracy of volatility models. *Journal of Forecasting*, 20(2):87–109, 2001. [10](#)

- Jacob A Mincer and Victor Zarnowitz. The evaluation of economic forecasts. In *Economic forecasts and expectations: Analysis of forecasting behavior and performance*, pages 3–46. NBER, 1969. [10](#)
- JP Morgan. Reuters (1996). *RiskMetrics—Technical Document*, 4, 1996. [6](#)
- Daniel B Nelson. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, pages 347–370, 1991. [3](#), [7](#)
- Hongli Niu and Jun Wang. Volatility clustering and long memory of financial time series and financial price model. *Digital Signal Processing*, 23(2):489–498, 2013. [2](#), [3](#)
- Jukka Nyblom. Testing for the constancy of parameters over time. *Journal of the American Statistical Association*, 84(405):223–230, 1989. [8](#), [14](#)
- Luís Oliveira, José Dias Curto, and João Pedro Nunes. The determinants of sovereign credit spread changes in the euro-zone. *Journal of International Financial Markets, Institutions and Money*, 22(2):278–304, 2012. [2](#), [4](#)
- Adrian R Pagan and G William Schwert. Alternative models for conditional stock volatility. *Journal of econometrics*, 45(1-2):267–290, 1990. [10](#)
- Franz C Palm and Peter JG Vlaar. Simple diagnostic procedures for modeling financial time series. Technical report, Maastricht University, 1997. [9](#)
- Keith Pilbeam and Kjell Noralf Langeland. Forecasting exchange rate volatility: Garch models versus implied volatility forecasts. *International Economics and Economic Policy*, 12(1):127–142, 2015. [9](#)
- Ser-Huang Poon. *A practical guide to forecasting financial market volatility*. John Wiley & Sons, 2005. [2](#), [3](#), [9](#), [10](#)
- Peter M Robinson. Log-periodogram regression of time series with long range dependence. *The annals of Statistics*, pages 1048–1072, 1995. [11](#)
- Saker Sabkha, Christian De Peretti, and Dorra Hmaied. International risk spillover in the sovereign credit markets: An empirical analysis. *Working paper*, 2017. [2](#), [4](#)
- Susan Sunila Sharma and Kannan Thuraisamy. Oil price uncertainty and sovereign risk: Evidence from asian economies. *Journal of Asian Economics*, 28:51–57, 2013. [2](#), [4](#)
- Sasha Srivastava, Hai Lin, Inguruwatte M Premachandra, and Helen Roberts. Global risk spillover and the predictability of sovereign cds spread: International evidence. *International Review of Economics & Finance*, 41:371–390, 2016. [2](#), [4](#)
- Yiu Kuen Tse. The conditional heteroscedasticity of the yen-dollar exchange rate. *Journal of Applied Econometrics*, pages 49–55, 1998. [3](#), [8](#)
- Yiu Kuen Tse. Residual-based diagnostics for conditional heteroscedasticity models. *The Econometrics Journal*, 5(2):358–374, 2002. [9](#), [14](#)



Yu Wei, Yudong Wang, and Dengshi Huang. Forecasting crude oil market volatility: Further evidence using garch-class models. *Energy Economics*, 32(6):1477–1484, 2010. [3](#), [5](#), [9](#), [10](#)

Jean-Michel Zakoian. Threshold heteroskedastic models. *Journal of Economic Dynamics and control*, 18(5):931–955, 1994. [7](#)

## A Appendix

Table 5: Results of the diagnostic tests for the 38 countries

Countries	Austria						Belgium						Brazil								
	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)
GARCH(1,1)	-3.347	29.0760	0.2384	0.0117	0.0007	210.80	0.5595	-4.798	0.0826	0.0134	0.0007	210.80	7363.14	3.68E-09	-4.841	0.0657	0.0286	0.0014	350.53	1362.83	1.06E-07
EGARCH(1,1)	-3.415	5.0765	0.0271	0.0043	0.3902	3088.26	0.3902	-5.087	590.777	456.839	68.8840	651.05	4087.95	66.3374	-4.721	1.1139	0.0241	0.0013	336.89		
GJR(1,1)	-0.548	4.8651	0.1230	0.0124	0.0044	3348.04	0.0044	-5.134	0.0271	0.0104	0.0005	274.01	4140.51	6.82E-10	-5.045	0.0115	0.0269	0.0013	303.95	1129.91	3.13E-10
APARCH(1,1)	-3.851	5.7814	0.0140	0.0014	0.0012	1773.18	0.0012	-5.339	0.0252	0.0099	0.0005	299.26	2471.35	3.23E-15	-9.427	0.0069	0.0069	0.0014	344.13		
IGARCH(1,1)	-3.364	15.8233	0.3135	0.0309	0.3850	3206.35	0.3850	-5.177	0.0494	0.0214	0.0011	202.39	3226.50	5.49E-12	-4.970	0.0429	0.0288	0.0014	263.17	965.11	4.00E-08
FIGARCH(1,1)	-3.362	15.8233	0.3135	0.0309	0.3775	3206.35	0.3775	-4.978	0.0494	0.0214	0.0011	185.82	2173.43	6.26E-08	-4.932	0.3063	0.0196	0.0010	236.35	709.56	1.42E-06
FIAPARCH(1,1)	-3.407	23.1804	0.7559	0.0058	38.20	2131.51	0.3927	-4.977	0.0958	0.0127	0.0006	241.35	6964.53	1.02E-06	-1.601	0.0918	0.0140	0.0007	353.85	12945.72	1.86E-10
HYGARCH(1,1)	-3.352	37.6455	4.8655	0.0173	39.73	3427.08	2.2517	-4.776	386.383	266.753	32.6210	306.75	2467.60	1.02E-06	-5.891	0.0197	0.0388	0.0014	284.42	1341.09	1.46E-07
	-3.355	53.4004	29.6438	0.1805	26.14	3427.08	2.2517	-5.258	0.1010	0.0331	0.0016	242.27	6885.60	8.98E-06	-5.087	0.0471	0.0289	0.0014	289.61	1219.48	1.46E-07

Countries	China						Croatia							
	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)
GARCH(1,1)	-5.436	0.0069	0.0627	0.0003	73.69	4489.06	5.34E-16	-5.448	0.5744	0.0443	0.0023	147.78	3162.01	6.71E-07
EGARCH(1,1)	-5.345	25.0836	0.0069	0.0028	77.44	3906.25	1.4874	-5.204	2.8891	0.0950	0.0048	298.22	10.212	0.4670
GJR(1,1)	-5.438	0.0069	0.0069	0.0003	76.87	3546.85	4.70E-16	-6.390	37.8408	2.6997	0.0021	268.22	16.7475	1.6589
APARCH(1,1)	-5.539	0.0069	0.0069	0.0003	39.92	4338.73	7.48E-18	-6.121	4.2886	0.0409	0.0019	233.25	7042.35	0.7049
IGARCH(1,1)	-5.380	69.9080	0.0069	0.0003	30.81	2960.60	2.87E-15	-6.964	0.0069	0.0069	0.0003	263.50	7718.56	1.55E-05
FIGARCH(1,1)	-5.380	69.9080	0.0069	0.0003	31.87	2971.07	3.43E-14	-5.336	0.4017	0.0450	0.0023	86.17	1104.85	7.89E-07
FIAPARCH(1,1)	-5.342	25.0304	0.0627	0.0028	81.79	3886.62	1.4868	-5.333	0.0058	0.0069	0.0003	202.05	10697.77	2.33E-10
HYGARCH(1,1)	-5.384	0.0069	0.0069	0.0003	20.12	2889.18	1.4868	-5.353	0.0085	0.0069	0.0003	204.24	11407.99	3.59E-10
	-5.508	0.0078	0.0069	0.0003	66.07	4352.37	2.41E-10	-6.230	3.8754	0.0485	0.0025	297.64	3817.47	1.08E-07

Countries	Denmark						Finland							
	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)
GARCH(1,1)	-5.460	1.1361	0.3438	0.0186	304.86	5073.43	0.0013	-5.001	22.9743	1.1090	0.0486	4.68	34939.38	0.4752
EGARCH(1,1)	-5.707	178.713	142.618	7.4291	75.39	11394.00	51.2730	-6.320	8.2895	0.0250	0.0013	602.91	3.84E-05	3.84E-05
GJR(1,1)	-5.529	1.1746	0.0491	0.0024	251.86	2065.11	0.0040	-7.209	11.1878	0.9706	0.0737	604.19	14734.60	0.5105
APARCH(1,1)	-4.029	19.3863	29.8426	2.9599	2.48	19300.08	2.6905	-51.583	0.0012	0.0084	0.0000	542.38	4.43E-05	0.1088
IGARCH(1,1)	-5.833	1.0567	0.1636	0.0083	101.94	8931.95	0.0015	-6.632	10.2966	0.4900	0.0178	585.49	30376.46	4.05E-15
FIGARCH(1,1)	-4.076	28.1956	95.3242	9.9514	213.66	8252.67	61.0092	-4.523	2.4864	0.0352	0.0019	401.82	6117.50	5.38E-04
FIAPARCH(1,1)	-4.118	14.9785	34.6246	3.3487	2.84	15533.45	5.5932	-60.873	0.0069	0.0069	0.0000	560.41	30616.72	1.78E-05
HYGARCH(1,1)	-5.660	5.6146	0.2789	0.0195	251.01	2707.45	0.1864	-7.299	3.8123	0.1021	0.0015	708.00	14265.76	6.25E-06

Countries	France						Germany						Greece								
	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RB(10)
GARCH(1,1)	-4.100	0.9096	0.0068	0.0004	96.07	6384.03	4.07E-08	-4.457	12.2807	150.00	0.0115	245.11	5271.93	4.52E-04	-78.634	0.0018	0.0091	890.470	460.24	32006.70	1.02E-08
EGARCH(1,1)	-4.134	0.3972	0.0068	0.0003	101.48	6842.79	5.20E-09	-3.264	64.5231	5.3478	0.1604	3.33	3475.68	2.7357	-4.280	0.0459	0.0069	0.0003	367.73	30265.32	0.0004
GJR(1,1)	-4.038	18.7233	5.7989	0.3162	10.14	4847.63	1.8916	-4.580	0.1858	0.1000	0.0005	277.64	6702.13	0.0125	-7.890	0.0161	0.0074	0.0004	153.05	13822.82	5.01E-08
APARCH(1,1)	-4.041	17.1288	4.2451	0.3667	12.07	4735.22	1.7622	-4.528	38.5063	38.5063	5.7621	60.97	16685.33	9.61E-08	-5.485	11.3689	0.0123	0.0006	535.30	37705.84	1.0815
IGARCH(1,1)	-4.133	23.4623	23.7292	0.7788	68.83	7012.25	21.4607	-4.563	10.3676	0.3631	0.0243	37.88	16535.60	1.8196	-5.535	0.2391	0.0068	0.0003	210.99	10746.67	2.17E-07
FIGARCH(1,1)	-4.032	18.2128	2.3464	0.1837	16.22	4221.79	2.3503	-4.460	0.0030	0.0077	0.0004	78.34	8313.85	7.18E-12	-6.633	0.0048	0.0070	0.0003	209.23	14825.81	0.0233
FIAPARCH(1,1)	-4.184	0.8961	0.0070	0.0004	90.94	5802.09	5.52E-09	-	-	0.0533	0.0005	156.04	8510.04	8.24E-08	-5.188	0.0207	0.0072	0.0004	285.71	9807.51	7.34E-09



Table 5: Results of the diagnostic tests for the 38 countries (Continued)

Countries	Portugal								Qatar								Romania							
	Information Criteria	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	
GARCH(1.1)	-4.213	8.1460	0.00211	0.0007	58.26	2148.92	0.0032	-5.730	0.4905	0.0187	0.0009	390.72	6.144.22	7.14E-06	-5.494	0.0063	0.0069	0.0003	0.0003	65.55	2596.78	1.38E-13		
ERARCH(1.1)	-4.210	18.2045	55.8087***	0.0007	68.31	1148.55	0.0012	-5.497	0.0709	0.0099	0.0005	265.54	17904.04	4.73E-07	-4.804	414.353***	86.8006***	9.1538***	9.1538***	710.67	2766.51	8.2466		
GJR(1.1)	-4.213	5.7994	0.0140	0.0005	58.89	2148.55	0.0012	-5.420	0.8484	0.0182	0.0009	374.56	2428.71	4.36E-06	-5.554	14.2707	0.0307	0.0017	0.0017	710.67	5143.54	0.0003		
APARCH(1.1)	-4.203	1.2002	0.0072	0.0004	87.59	2317.98	8.48E-06	-27.306	0.0069	0.0069	-	309.20	-	-	-5.000	15.9822	0.0377	0.0022	0.0022	58.57	1352.79	0.0006		
IGARCH(1.1)	-4.153	15.9522	70.8632***	0.0004	5.59	2230.79	0.5644	-5.719	0.0357	0.0120	0.0006	334.22	9521.18	7.45E-08	-4.975	12.8627	0.0297	0.0016	0.0016	130.78	1223.21	0.0009		
FIGARCH(1.1)	-4.154	16.0789	74.7565***	0.0004	6.07	2257.34	1.4703	-5.621	0.0953	0.0049	0.0009	132.74	7661.36	0.0005	-5.439	0.0069	0.0069	0.0003	0.0003	130.78	2853.53	1.27E-12		
FIAPARCH(1.1)	-4.154	58.8919***	69.6413***	0.0004	152.73	116.2830***	116.2830***	-5.508	1.1313	0.0154	0.0007	245.86	12152.78	0.0018	-5.430	32.9755**	0.2570	0.0017	0.0017	14.80	2809.52	0.0062		
HYGARCH(1.1)	-4.672	29.5043	73.0205***	4.6828***	385.28	2650.32	10.4886	-5.857	0.0717	0.0137	0.0007	240.42	9689.31	3.12E-07	-5.803	6.1051	0.0244	0.0013	0.0013	140.42	4615.76	0.0020		
	-4.559	36.9416***	49.9189***	1.0429	367.01	3579.22	26.7028	-5.922	0.0250	0.0118	0.0006	362.05	7443.30	1.52E-08	-5.892	29.2195	0.1838	0.0029	0.0029	181.41	1969.51	0.0223		

Countries	Russia								Slovakia								Slovenia							
	Information Criteria	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	
GARCH(1.1)	-3.999	11.5329	20.5284	0.8859	1.61	115.86	126.4820	-5.927	1.2437	0.1159	0.0068	185.57	11600.19	1.21E-05	-4.772	24.7842	8.2290	0.1474	0.1474	2.11	38119.51	1.5993		
ERARCH(1.1)	-3.991	11.9272	17.1360	0.5738	5.15	123.06	3.4639	-4.916	16.9564	0.8987	0.0517	35.08	9322.98	2.2528	-4.269	25.6910	0.7816	0.0436	0.0436	53.05	9279.12	0.6477		
GJR(1.1)	-4.003	11.5791	21.3905	0.8695	1.95	98.86	54.5139	-5.927	1.3406	0.1297	0.0077	186.41	11357.85	1.36E-05	-4.773	24.3940	8.6544	0.1186	0.1186	2.61	19844.39	1.4358		
APARCH(1.1)	-4.005	10.8970	21.0426	0.8779	1.89	117.02	2.0299	-5.965	20.9647	0.3732	0.0133	205.68	4500.97	0.0757	-4.780	23.9669	8.4562	0.1056	0.1056	3.57	35336.01	1.2934		
IGARCH(1.1)	-4.000	11.6009	20.8452	0.9494	1.53	115.63	14.5312	-5.862	2.1215	0.1761	0.0111	9.87	13642.82	7.72E-05	-4.756	32.4648**	9.0434	0.2165	0.2165	1.52	38756.52	2.3373		
FIGARCH(1.1)	-4.099	11.7362	20.1840	0.9201	3.56	112.15	16.1394	-5.861	2.1236	0.1763	0.0111	13.15	13694.25	7.74E-05	-4.768	29.1249	8.5046	0.2446	0.2446	2.30	38813.78	2.7887		
FIAPARCH(1.1)	-4.004	11.9273	17.1361	0.7339	5.53	121.93	3.4049	-4.342	34.7287**	16.5740	0.5374	184.26	7317.98	12.2474	-5.822	4.3473	0.0534	0.0032	0.0032	392.92	7540.61	-		
HYGARCH(1.1)	-3.998	11.6701	19.7961	0.8525	3.71	118.90	16.3329	-5.872	1.3533	0.1081	0.0063	14.90	12610.89	4.94E-05	-5.361	1.0354	0.0343	0.0017	0.0017	334.97	4494.23	4.29E-05		
								-5.991	0.3934	0.1137	0.0060	155.07	7839.59	6.23E-07	-6.688	23.3301	2.1289	0.0592	0.0592	659.86	6649.50	1.3382		

Countries	Spain								Sweden								Thailand							
	Information Criteria	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	
GARCH(1.1)	-4.205	0.1149	0.0072	0.0004	123.66	1230.66	3.90E-07	-6.431	0.0011	0.0739	0.0037	513.78	10848.62	1.86E-13	-6.065	1.7565	0.0071	0.0003	0.0003	581.98	4923.16	0.0003		
ERARCH(1.1)	-4.105	11.8580	0.5806	0.0377	42.62	1542.01	20.6711	-5.793	2.6122	0.0910	0.0044	374.98	7896.91	0.0005	-5.546	30.9012**	0.4201	0.0175	0.0175	459.32	5299.30	0.2655		
GJR(1.1)	-4.208	0.0265	0.0069	0.0003	118.58	3360.48	5.96E-10	-6.658	133.390***	17.560***	17.9000***	542.42	10075.54	292.3860	-5.524	24.6423	0.9679	0.0885	0.0885	548.26	5771.18	7.8470		
APARCH(1.1)	-4.199	0.0269	0.0070	0.0003	224.62	1658.02	3.87E-08	-6.486	14.1611	0.7800	0.0111	535.81	8622.64	-	-6.332	0.3637	0.0078	0.0004	0.0004	553.98	6412.52	4.65E-05		
IGARCH(1.1)	-4.168	0.1050	0.0072	0.0004	74.75	1250.66	3.20E-07	-6.114	0.0069	0.0069	-	434.039	22159.64	1.4E-07	-5.906	32.5837**	1.5782	0.1525	0.1525	570.66	5646.48	1.6670		
FIGARCH(1.1)	-4.154	0.0387	0.0080	0.0004	91.95	1098.28	1.32E-07	-6.114	0.0080	0.0069	0.0003	149.21	18914.88	5.82E-11	-5.952	18.9330	0.1015	0.0091	0.0091	552.50	1333.21	0.1011		
FIAPARCH(1.1)	-4.104	11.8577	0.5806	0.0377	93.64	1541.60	16.4878	-6.059	153.937***	74.9719***	5.8773***	332.10	8389.55	49.8163	-3.326	0.0428	0.0076	0.0004	0.0004	455.17	10646.13	1.48E-08		
HYGARCH(1.1)	-4.199	0.0090	0.0069	0.0003	74.36	1331.34	-	-7.378	0.0750	0.0882	0.0044	608.11	22405.91	2.16E-06	-5.882	212.707***	15.9829	1.6022	1.6022	545.84	2692.58	0.4598		
								-6.300	8.5002	0.6284	0.0327	409.05	3540.58	0.1756	-6.258	2.3372	0.0084	0.0004	0.0004	596.53	8097.17	0.0015		

Countries	Turkey								UK								Ukraine							
	Information Criteria	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q(20)	ARCH(10)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	
GARCH(1.1)	-4.727	0.0167	0.0138	0.0007	92.81	289.45	2.18E-10	-6.575	2.5436	0.0076	0.0004	651.59	6712.97	4.75E-17	-5.957	0.0000	0.0223	0.0011	0.0011	79.77	4790.74	3.20E-14		
ERARCH(1.1)	-4.514	0.0978	0.0135	0.0007	98.71	806.84	1.90E-07	-6.130	167.760	1.9385	616.09	6.6109	7477.76	10.3849	-12.223	884.481***	801.556***	105.8400***	105.8400***	970.74	48665.73	139.62		
GJR(1.1)	-4.739	0.0249	0.0269	0.0013	98.85	254.14	2.63E-09	-6.972	136.972***	12.4021	0.0006	624.47	7192.07	7.22E-05	-5.576	136.035***	151.049***	0.0084	0.0084	188.02	98652.01	0.0002		
APARCH(1.1)	-4.787	0.0064	0.0064	0.0013	151.98	569.65	3.88E-11	-7.785	99.6922***	7.0942	0.0007	569.28	9846.84	4.90E-05	-5.974	0.011	0.061	0.0030	0.0030	183.33	4277.79	8.30E-13		
IGARCH(1.1)	-4.638	0.0386	0.0074	0.0004	68.56	273.77	4.70E-09	-23.832	0.0069	0.0069	-	615.3160	12727.38	1.31E-14	-5.403	2.0310	0.1073	0.0056	0.0056	174.09	4961.98	2.67E-07		
FIGARCH(1.1)	-4.659	0.0852	0.0192	0.0010	89.0917	238.4297	1.95E-08	-6.130	103.137	73.0977	0.0046	663.73	2626.12	0.0006	-5.971	4.1195	0.1000	0.0037	0.0037	83.57	5513.99	7.67E-07		
FIAPARCH(1.1)	-4.420	1.3761	0.0120	0.0007	410.32	981.06	0.0064	-3.981	4.0148	0.0142	0.0002	422.20	6955.95	2.75E-05	-5.095	15.5804	0.3402	0.0318	0.0318	62.30	10353.01	1.0228		
HYGARCH(1.1)	-4.208	17.3871	3.8679	0.1367	1.90	869.68	1.3664	-8.570	2.7105	0.0075	0.0003	701.51	13502.33	1.55E-12	-5.840	0.2173	0.0339	0.0017	0.0017	122.79	9309.59	7.34E-08		
								-7.504	15.8465	0.0864	0.0004	647.42	7236.52	7.66E-08	-7.350	162.703***	187.622***	5.88559***	5.88559***	300.18	11230.89	63.1099***		

Table 5: Results of the diagnostic tests for the 38 countries (*Continued*)

Countries	USA										Venezuela									
	Information Criteria	Q(20)	Q2(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)	Information Criteria	Q(20)	Q2(20)	ARCH(10)	Joint Nyblom	A. Pearson G.o.f (50 cells)	RDB(10)						
GARCH(1,1)	-3.343	22.1954	0.6640	0.0343	2.10	30069.34	0.3515	-4.433	16.8322	7.8329	0.4800	2.83	738.70	4.5171						
ENGARCH(1,1)	-3.219	20.9911	0.6783	0.0249	70.44	10379.22	0.3611	-4.335	22.2545	8.0298	0.2973	69.96	590.98	4.5980						
GJR(1,1)	-3.343	22.6308	0.6768	0.0361	2.90	30020.77	0.3693	-4.433	16.1177	9.7585	0.6520	2.68	905.20	4.3576						
APARCH(1,1)	-3.343	23.2446	0.7894	0.0441	3.33	2994.64	0.4664	-4.450	16.2001	51.5638 ***	4.2005 ***	2.39	1071.96	2.1091						
IGARCH(1,1)	-3.333	25.8119	1.2310	0.0762	1.73	29578.64	0.8526	-4.409	16.9461	23.4663	1.7687	1.5400	1025.57	33.6120						
FIGARCH(1,1)	-3.385	21.7770	1.0326	0.0392	18.30	29371.73	0.2184	-4.410	18.3256	13.0599	0.7184	75.83	720.43	14.4391						
FIEGARCH(1,1)	-3.218	20.9958	0.6784	0.0249	134.90	10202.40	0.3621	-4.518	84.5941 ***	0.1977	0.0138	238.36	200.46	0.1152						
FIAPARCH(1,1)	-3.394	18.9781	0.5677	0.2338	40.45	29411.16	0.2197	-4.771	44.4650 ***	81.4568 ***	8.4904 ***	149.77	340.07	4.0510						
HVGARCH(1,1)	<b>-3.475</b>	21.4223	0.2415	0.0196	15.65	27477.49	0.1690	<b>-4.861</b>	183.988 ***	32.3894 **	3.2795 ***	237.86	403.25	169.0970						

Q(20) and Q2(20) are the Box-Pierce Q statistics under the null hypothesis of no serial correlation in the 20<sup>th</sup> lag order of respectively the standardized residuals and the squared standardized residuals. ARCH(10) is the Engle's ARCH-LM test with 10 lag orders, used to detect ARCH effects in the series under the null hypothesis of no autocorrelations in the squared residuals. The Joint Nyblom is a stability test under the null hypothesis of parameters joint consistency over time against the alternative of parameters shift at an undefined breakpoint. The A. Pearson G.o.f is the adjusted Pearson goodness-of-fit test that verifies whether the residuals' empirical distribution matches or not the theoretical distribution (namely Gauss, Student or G.O.D depending on the country). RDB denotes the Residuals-Based Diagnostic test that checks for conditional Heteroscedasticity, by complementing and filling the gaps of the Box-Pierce Q statistics. \*, \*\*, and \*\*\* denote the statistical significance at respectively 1%, 5% and 10% levels.