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On-resonance material fast light

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We theoretically revisit the problem of the propagation of coherent light pulses through a linear medium when the carrier frequency of the pulses coincides with the minimum of a narrow dip in the medium transmission. Considering realistic contrasts between the maximum and minimum transmission of the medium and incident pulses of strictly finite duration, we combine temporal and spectral approaches to obtain analytical expressions of the transmitted pulse that reproduce the main features of the exact numerical solutions derived by fast Fourier transform. A special attention is paid to the advance of the pulse maximum over that of a pulse covering the same distance in vacuum and the ratio of this advance to the pulse duration (fractional advance) is optimized.

I. INTRODUCTION

Fast light currently refers to situations in which a smooth pulse of coherent light is transmitted by a linear optical system in such a manner that the maximum of the output pulse anticipates that of a pulse having covered the same distance in vacuum. As extensively discussed in the literature (see, e.g., [1, 2]), the phenomenon is not at odds with the relativistic causality. It appears in particular that the maximum of the output pulse is not as a direct reflection of that of the input pulse but results from the action of the system on the early part of the latter. A direct experimental evidence of this point is reported in [3]. Fast light can be obtained when the system transmission displays a well-marked, narrow dip at the carrier frequency of the input pulse [4]. Note however that this condition is not always sufficient. For optical systems that are not minimum-phase-shift [5], in particular those involving mirrors [6, 7] or polarizers [8–10], it may even occur that a same transmission profile leads to either fast or slow light. In the case of purely propagative systems as considered in the following, fast light simply originates in the dispersive properties of the medium (material fast light) and the Kramers-Kronig relations apply in their common form. Even in this case a dip in the transmission profile does not always entail fast light. For instance when the dip corresponds to a minimum of transmission between two gain lines, there is a range of parameters for which there is no fast light whatever the dip depth is [11].

The simplest way to observe material fast light is to exploit the anomalous dispersion associated with an isolated absorption line of a passive medium. Experiments have been performed on various materials including semiconductor [12], molecular gas [13], atomic vapour at room temperature [14], hot atomic vapour [15] and clouds of cold atoms [16], with propagation distances going from 390 nanometers [15] to 24 meters [13]. In order to control the pulse advance, the transmission dip is artificially

created in other experiments by applying supplementary fields interacting non linearly with the medium and taking advantage of the phenomena of electromagnetically induced absorption [17–23], Raman effect [24, 25], Brillouin effect [26] and four wave mixing [27, 28].

In most experimental reports on material fast light, the emphasis is made on the absolute value of the advance of the pulse maximum over that of a pulse propagating in vacuum and on the corresponding value of the velocity often confused with the group velocity. In fact the true experimental challenge is to obtain advances which are significant compared to the pulse duration with acceptable pulse distortion. On the other hand, in the numerous theoretical papers on the subject, the proof of fast light is generally reduced to that of a superluminal group velocity without simulation evidencing a visible advance of the pulse maximum.

The present article is theoretical but devotes a special attention *to actual or feasible experiments*. Its purpose is to clarify some points about material fast light and to examine what are the practical limitations to the phenomenon, Mother Nature resisting to a violation of her principles even when this violation is only apparent. To do so, we come back to the basic system consisting in a passive medium with an isolated absorption line [12–16]. The simplicity of this system, favourable from an experimental viewpoint, will enable us to obtain analytical results. Insofar as the performances of the medium are essentially determined by the contrast between its maximum and minimum of transmission [4], it may be reasonably expected that these results have some generality. Anticipating a more extensive discussion, we note here that contrast exceeding 40 dB has been actually used in absorbing media [13] whereas it seems difficult to use an equivalent optical gain without generating serious instabilities [29, 30].

The arrangement of our paper is as follows. In Sec.II, we introduce the basic equations of the problem and analyse their physical meaning by privileging the temporal approach. Asymptotic solutions obtained in the short and long pulse limits are given in Sec.III and Sec.IV. Section V is devoted to the optimization of the fractional advance of the pulse maximum, ratio of this advance to

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the pulse duration. The results are discussed in Sec.VI and we conclude in Sec.VII by summarizing the main points.

II. GENERAL ANALYSIS

When coherent light pulses with a slowly varying envelope propagate in a linear medium of thickness ℓ , the envelope $e(\ell, t)$ of the output pulse can be deduced from the envelope $e(0, t)$ of the incident pulse by the general relation:

$$e(\ell, t) = h(\ell, t) \otimes e(0, t) \quad (1)$$

where $h(\ell, t)$ is the impulse response of the medium and \otimes designates a convolution product [5]. We consider here a dilute medium with an isolated Lorentzian absorption-line at a frequency ω_0 coinciding with the carrier frequency ω_c of the incident pulses. We have then [31, 32]

$$h(\ell, t) = \delta(t) - L\gamma k(\ell, t) u_H(t) \quad (2)$$

with

$$k(\ell, t) = \frac{J_1(2\sqrt{L\gamma t})}{\sqrt{L\gamma t}} e^{-\gamma t}. \quad (3)$$

In these expressions, $\delta(t)$, $u_H(t)$ and $J_1(x)$ respectively designate the delta function, the Heaviside unit step and the Bessel function of the first kind of index 1; t is the time retarded by the transit time ℓ/c at the velocity of light in vacuum (retarded time picture), γ is the half-width at half-maximum of the absorption line and $L = \alpha\ell/2$ where α is the resonance absorption coefficient for the intensity. From Eq.(1), we finally get

$$e(\ell, t) = e(0, t) - L\gamma [k(\ell, t) u_H(t)] \otimes [e(0, t)]. \quad (4)$$

From a physical viewpoint, Eq.(4) first shows that, if $e(0, t)$ starts at a given time, it is the same for $e(\ell, t)$ at the corresponding retarded time in agreement with relativistic causality. Second, as noted by Feynman in a more general context [33], the transmitted wave is the sum of the incident wave as if it had propagated in vacuum and of the wave reemitted by the medium. Fast light thus results from the interference between these two waves [34–37].

In the frequency domain, the counterpart of Eq.(1) reads

$$E(\ell, \Omega) = H(\ell, \Omega) E(0, \Omega) \quad (5)$$

where $E(\ell, \Omega)$, $H(\ell, \Omega)$ and $E(0, \Omega)$ are, respectively, the Fourier transforms of $e(\ell, t)$, $h(\ell, t)$ and $e(0, t)$. $H(\ell, \Omega)$ is the transfer function of the medium [5] which in the present case has the simple form [11]

$$H(\ell, \Omega) = \exp\left(-\frac{L}{1 + i\Omega/\gamma}\right) \quad (6)$$

It yields a contrast C between the maximum and the minimum of the intensity transmission and a relative depth D of the transmission dip

$$C = \left| \frac{H(\ell, \infty)}{H(\ell, 0)} \right|^2 = e^{2L} = e^{\alpha\ell} \quad (7)$$

$$D = 1 - e^{-2L} = 1 - e^{-\alpha\ell} \quad (8)$$

We will restrict our analysis to contrasts below or equal to 50 dB [$C_{dB} = 10 \log_{10}(C)$], which seems an upper limit to the contrast that can be actually used in the experiments to avoid parasitic effects. This contrast is attained for $L = 5.756$ and the corresponding dip depth D is very close to 100%. Note that, in most experimental demonstrations of fast light in atomic media, this depth does not exceed the value 86% attained in absorbing media for the moderate optical thickness $\alpha\ell = 2$ ($L = 1$, $C \approx 8.7$ dB).

General properties of the envelope of the transmitted field can be deduced from the transfer function. We note first that $E(\ell, 0)$ and $E(0, 0)$ are, respectively, the areas of $e(\ell, t)$ and $e(0, t)$. The former is thus equal to the latter multiplied by e^{-L} . Second, the mean time of $h(\ell, t)$, i.e. the group delay τ_g , equals the first cumulant κ_1 of $H(\ell, \Omega)$ [38]. Quite generally the cumulants κ_n are given by the following expansion of $\ln[H(\ell, \Omega)]$:

$$\ln[H(\ell, \Omega)] = \ln[H(\ell, 0)] + \sum_{n=1}^{\infty} \frac{\kappa_n}{n!} (-i\Omega)^n. \quad (9)$$

In the present case, they take the simple form $\kappa_n = -n!L/\gamma^n$ and the group advance $a_g = -\tau_g$ reads:

$$a_g = L/\gamma \quad (10)$$

Due the additivity property of the cumulants, a_g is the advance of the centre-of-gravity of $e(\ell, t)$ over that of $e(0, t)$, that is, in our retarded time picture, over that of the envelope of a pulse propagating without distortion at the velocity c . We emphasize that this result is general and holds even when the transmitted pulse is strongly distorted. As shows Eq.(10), the advance a_g is always positive. It should be carefully distinguished from the advance a of the pulse maximum and from that of the centre-of-gravity of the intensity profile $|e(\ell, t)|^2$ which both may take negative values (delay instead of advance) [39–43].

In the following, we consider incident pulses of strictly finite duration, the envelopes of which are good approximations of the Gaussian envelope $e_G(0, t) = \exp(-t^2/2\sigma^2)$ usually considered in the literature. We have retained

$$e_1(0, t) = \cos^2\left(\frac{\Omega_1 t}{2}\right) \Pi\left(\frac{\Omega_1 t}{2\pi}\right) = \left[\frac{1 + \cos(\Omega_1 t)}{2}\right] \Pi\left(\frac{\Omega_1 t}{2\pi}\right) \quad (11)$$

or, if necessary, the more speculative form

$$e_2(0, t) = \cos^4\left(\frac{\Omega_2 t}{2}\right) \Pi\left(\frac{\Omega_2 t}{2\pi}\right) \\ = \left[\frac{1 + \cos(\Omega_2 t)}{2}\right]^2 \Pi\left(\frac{\Omega_2 t}{2\pi}\right). \quad (12)$$

In these expressions $\Pi(x)$ is the rectangle function equal to 1 for $-1/2 < x < 1/2$ and 0 elsewhere. The envelopes $e_1(0, t)$, $e_G(0, t)$ and $e_2(0, t)$ are compared Fig.1 also as the corresponding intensity profiles (nearly undistinguishable). The parameters Ω_1 , σ and Ω_2 are chosen

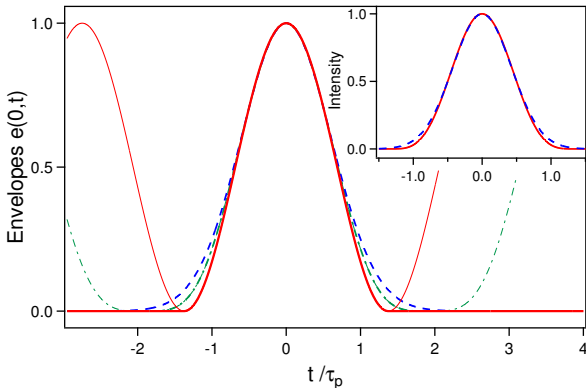


Figure 1: Comparison of the envelopes $e_1(0, t)$ (solid red line), $e_G(0, t)$ (dashed blue line) and $e_2(0, t)$ (dash-dotted green line) of the incident pulses for a same full width at half maximum τ_p of the corresponding intensity profiles. The thin lines are the periodic continuations of $e_1(0, t)$ (solid red line) and $e_2(0, t)$ (dash-dotted green line). Inset: the corresponding intensity profiles $|e_1(0, t)|^2$ (solid red line) and $|e_G(0, t)|^2$ (dashed blue line). The intensity profile $|e_2(0, t)|^2$ (not shown) is quasi confused with the Gaussian profile $|e_G(0, t)|^2$.

such that the corresponding intensity profiles (see inset in Fig.1) have *the same full width at half-maximum* τ_p . This is achieved when $\Omega_1 \tau_p = 4 \arccos(2^{-1/4})$, $\tau_p = 2\sigma\sqrt{\ln 2}$ and $\Omega_2 \tau_p = 4 \arccos(2^{-1/8})$. *The duration τ_p will be taken as universal time unit in all the following.* In this way the profile of the transmitted pulse is entirely determined by the two independent and dimensionless parameters $\gamma\tau_p$ and L (more generally the contrast C). As above mentioned the challenge in fast light experiments is to attain significant advance a of the pulse maximum compared to the pulse duration. From this viewpoint, a merit factor of the experiments is the fractional advance $F = a/\tau_p$. Insofar as the envelopes $e_i(0, t)$ ($i = 1, 2$) start at a finite time ($t = -\pi/\Omega_i$), it is obvious that the fractional advance obtained with these envelopes will reproduce that obtained with Gaussian pulses if and only if its value is significantly less than $\pi/(\Omega_i \tau_p)$, that is 1.37 for $i = 1$ and 1.91 for $i = 2$. As shown later $e_1(0, t)$ constitute a sufficient approximation of $e_G(0, t)$ when $F \leq 50\%$ whereas the better approximation provided by $e_2(0, t)$ is necessary for larger fractional advances. We finally men-

tion that, beyond the advantage of a finite duration, both envelopes offer the possibility of a periodic continuation which will be useful to optimise the fractional advance (Sec. V). Except when contrary specified the envelope $e_1(0, t)$, leading to simpler calculations, is used in the following.

III. SHORT PULSE LIMIT

When the incident pulse is short enough, its spectrum mainly lies in the far wings of the line profile where the medium dispersion is normal [2]. A delay instead of an advance of the pulse maximum might be expected in such conditions. We show in this section that *the opposite occurs*. Simple analytical results are obtained when the duration τ_p of the incident pulse is short compared to the duration of the pulse reemitted by the medium as considered in the discussion of Eq.(4) that is when $\gamma\tau_p(1 + L/2) \ll 1$ [44]. We have then

$$[k(\ell, t) u_H(t)] \otimes [e(0, t)] \approx k(\ell, t) [u_H(t) \otimes e(0, t)]. \quad (13)$$

For an incident pulse of envelope $e_1(0, t)$, it is easily shown that

$$[u_H(t) \otimes e(0, t)] = \frac{1}{2} \left[t + \frac{\sin(\Omega_1 t)}{\Omega_1} + \frac{\pi}{\Omega_1} \right] \Pi\left(\frac{\Omega_1 t}{2\pi}\right) \\ + \frac{\pi}{\Omega_1} u_H\left(t - \frac{\pi}{\Omega_1}\right) \quad (14)$$

and, according to Eq.(4), the envelope of the transmitted field reads

$$e_1(\ell, t) \approx e_1(0, t) - L\gamma [u_H(t) \otimes e(0, t)] \frac{J_1(2\sqrt{L\gamma t})}{\sqrt{L\gamma t}} e^{-\gamma t}. \quad (15)$$

Figures 2 and 3 are respectively obtained for large ($L = 5.756$) and moderate ($L = 1$) optical thickness. They show that the intensity profiles $|e_1(\ell, t)|^2$ determined by Eq.(15) are in perfect agreement with the exact profiles derived by fast Fourier transform (FFT). Quite generally the fractional advance $F = a/\tau_p$ of the pulse maximum has the approximate form

$$F \approx \frac{2L\gamma}{\Omega_1^2 \tau_p} = \frac{2L\gamma\tau_p}{\Omega_1^2 \tau_p^2} \approx 0.38L\gamma\tau_p \quad (16)$$

whereas $a/\tau_p = L/\gamma\tau_p$ and thus $a/a_g = 0.38(\gamma\tau_p)^2$. This means that, in the short pulse limit, the advance of the pulse maximum will be always much smaller than the group advance. This apparent paradox is explained by the fact that the envelope $e_1(\ell, t)$ has a long tail which is essentially negative and artificially increases the advance a_g of its centre of gravity. This tail has the general form

$$e_1(\ell, t) \approx -\frac{\pi L\gamma}{\Omega_1} k(\ell, t) \approx -1.37L\gamma\tau_p k(\ell, t) \quad (17)$$

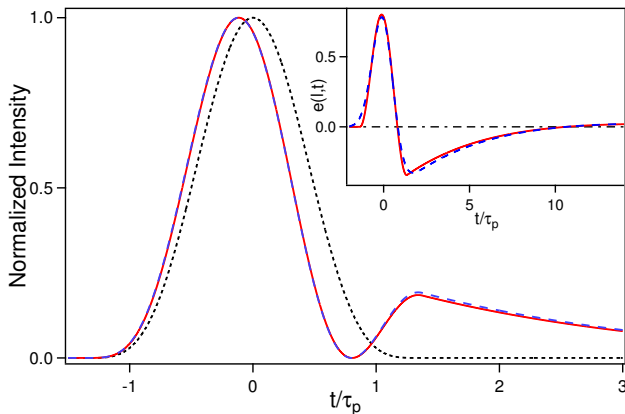


Figure 2: Normalized intensity profiles of the transmitted pulse obtained for $e(0, t) = e_1(0, t)$, $L = 5.756$ (transmission contrast $C = 50$ dB) and $\tau_p = 0.06/\gamma$ ($\Omega_1 = 38\gamma$). The solid red line (the dashed blue line) is the exact solution obtained by FFT (the analytical solution obtained in the short pulse limit). As in all the following figures, the intensity profile of the incident pulse (dotted black line) is given for reference. Its full width at half maximum τ_p is taken as time unit. Inset: comparison of the envelopes of the transmitted pulse obtained for $e(0, t) = e_1(0, t)$ (solid red line) and $e(0, t) = e_G(0, t)$ (dashed blue line). The pulse tail displays a first zero for $t/\tau_p \approx j_{1,1}^2/(4L\gamma\tau_p) \approx 10.6$ (see text).

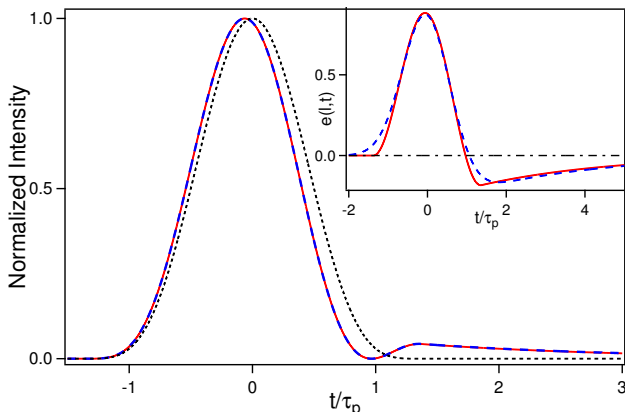


Figure 3: Same as Fig 2 when $L = 1$ ($C \approx 8.7$ dB, dip depth $D \approx 86\%$) and $\tau_p = 0.2/\gamma$ ($\Omega_1 = 11.4\gamma$). The tail of the envelopes (inset) has now an exponential decrease (see text).

For large optical thickness, it oscillates with successive lobes of rapidly decreasing amplitude and zeroes at the times $t_n = j_{1,n}^2/(4L\gamma)$ where $j_{1,n}$ designates the corresponding zeroes of $J_1(x)$. This behaviour is illustrated in the inset of Fig.2. Note that, for the parameters considered, the amplitude of the secondary lobes appearing for $t > t_1$ (not shown) is very weak, below 0.02 for the first and largest one. For moderate optical thickness, the decrease of $k(\ell, t)$ is mainly determined by the term $\exp(-\gamma t)$ and the tail of the envelope is reduced to a simple exponential of the form $-1.37L\gamma\tau_p \exp[-\gamma(1 + L/2)t]$ as shown in the inset of

Fig.3 obtained for $L = 1$. For a five times smaller optical thickness ($L = 0.2$), the intensity of the tail would be 25 times weaker and the fractional advance of the maximum 5 times smaller. The normalized intensity profile of the transmitted pulse is then nearly undistinguishable from that of the incident pulse although the depth of the dip in the transmission profile is significant ($D = 33\%$).

Similar calculations can be performed for Gaussian pulses. We only give below the expression of the envelope $e_G(\ell, t)$ of the transmitted pulse:

$$e_G(\ell, t) \approx \exp\left(-\frac{t^2}{2\sigma^2}\right) - L\gamma\sigma\sqrt{\frac{\pi}{2}} \left[1 + \operatorname{erf}\left(\frac{t}{\sigma\sqrt{2}}\right)\right] k(\ell, t) \quad (18)$$

where $\operatorname{erf}(x)$ designates the error function. There is again a perfect agreement with the exact numerical result obtained by FFT. The fractional advance has now the approximate form

$$F \approx \frac{L\gamma\sigma^2}{\tau_p} = \left(\frac{\sigma}{\tau_p}\right)^2 L\gamma\tau_p \approx 0.36L\gamma\tau_p \quad (19)$$

The envelopes $e_G(\ell, t)$ and $e_1(\ell, t)$ are shown in the insets of Fig.2 and Fig.3. As expected, they are quite comparable.

IV. LONG PULSE OR ADIABATIC LIMIT

We consider in this section the case opposite to the previous one. We assume that the envelope $e(0, t)$ of the incident pulse is *everywhere* slowly varying compared to $k(\ell, t)u_H(t)$. This implies in particular that the pulse duration τ_p is long compared to $1/\gamma$. The function $L\gamma k(\ell, t)u_H(t)$ can then be approximated by $A_k\delta(t - \tau_{kg})$ where A_k and τ_{kg} are, respectively, its area and its centre of gravity. Since $L\gamma k(\ell, t)u_H(t)$ is the inverse Fourier transform of $K(\ell, \Omega) = 1 - H(\ell, \Omega)$, we have $A_k = 1 - H(\ell, 0) = 1 - e^{-L}$ and, by an expansion of $\ln[K(\ell, \Omega)]$ in cumulants [38], $\tau_{kg} = L/[\gamma(e^L - 1)] > 0$. Injecting these results in Eq.(4), we get

$$e(\ell, t) \approx e(0, t) - A_k e(0, t - \tau_{kg}) \quad (20)$$

The envelope of the output field appears equal to that of the input field minus its copy reduced in amplitude and slightly retarded [34, 36, 37]. Assuming that $\tau_{kg} \ll \tau_p$, Eq.(20) takes the form

$$e(\ell, t) \approx e^{-L} [e(0, t) + a_g \dot{e}(0, t)] \quad (21)$$

where the dot designates a time-derivative and where we have taken into account the relations $1 - A_k = e^{-L}$ and $A_k\tau_{kg}/(1 - A_k) = a_g$. In the limit where $a_g/\tau_p \ll 1$, we retrieve the classical result

$$e(\ell, t) \approx e^{-L} e(0, t + a_g) \quad (22)$$

In this limit, the envelope of the output pulse equals that of the input pulse attenuated by the steady state transmission e^{-L} for the field and advanced by a_g (no distortion). It should be however kept in mind that this

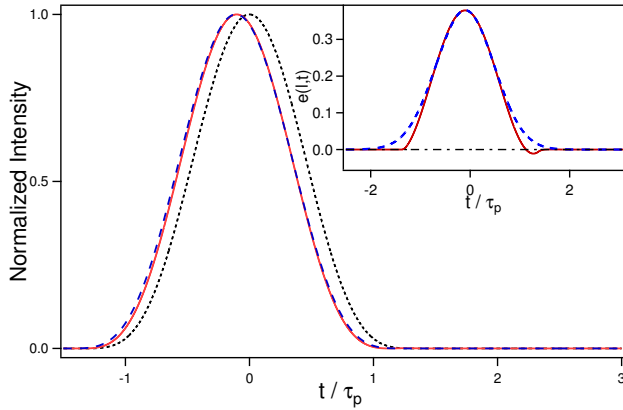


Figure 4: Normalized intensity profiles of the transmitted pulse obtained for $e(0, t) = e_1(0, t)$, $L = 1$ ($C = 8.7$ dB, dip depth $D \approx 86\%$) and $\Omega_1 = \gamma/4$ ($\tau_p \approx 9.15/\gamma$). The solid red line (the dashed blue line) is the exact solution obtained by FFT (the analytical solution obtained in the long pulse or adiabatic limit). Inset: comparison of the envelopes of the transmitted pulse obtained for $e(0, t) = e_1(0, t)$ (solid red line) and $e(0, t) = e_G(0, t)$ (dashed blue line).

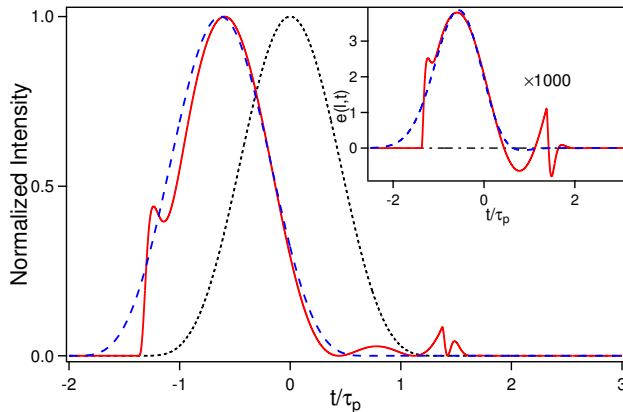


Figure 5: Same as Fig 4 when $L = 5.756$ ($C = 50$ dB) instead of $L = 1$. Note the presence of a precursor and a postcursor which had negligible amplitudes for $L = 1$ and are obviously absent in the case of a Gaussian incident pulse (see inset).

asymptotic result only holds when $a_g/\tau_p \ll 1$ and, consequently, that the corresponding fractional delay will be limited.

Figure 4 shows an example of situation for which Eq.(22) provides a good approximation of the exact result. It is obtained for $e(0, t) = e_1(0, t)$ and the moderate optical thickness already considered in Fig.3 ($L = 1$ leading to $C \approx 8.7$ dB and $D \approx 86\%$) but a much longer pulse duration ($\Omega_1/\gamma = 1/4$ that is $\gamma\tau_p \approx 9.1$). The fractional advance $F = a/\tau_p$ of the pulse maximum is equal to 10.3% whereas $a/a_g = 0.94$, actually close to unity.

The asymptotic solution given by Eq.(22) remains acceptable for long pulses even when the condition $a_g \ll \tau_p$ is poorly satisfied. Figure 5 illustrates this point. It is also obtained for $\gamma\tau_p \approx 9.1$ but for $L = 5.756$ ($C =$

50 dB). We have now a fractional advance $F = a/\tau_p \approx 59\%$ and, despite that the condition of validity of Eq.(22) is far from being fulfilled ($a_g/\tau_p \approx 0.63$), a and a_g remain close. A new feature of Fig.5 compared to Fig.4 is the appearance of transients at the beginning (precursor) and the end (postcursor) of the envelope $e_1(0, t)$, that is for $t/\tau_p = \pm\pi/\Omega_1\tau_p \approx \pm 1.37$. They obviously originate in the discontinuity of the second derivative of $e_1(0, t)$ at the corresponding times, are absent in the case of Gaussian pulses (see inset of Fig.5) and are not taken into account by the adiabatic approximation that assumes that the envelope of the incident pulse is *everywhere* continuous with continuous derivatives. By a cavalier extrapolation of results obtained in the study of the Sommerfeld precursors [45], we have found that the peak amplitude of the precursor roughly scales as

$$A_p = \left(\frac{\Omega_1}{\gamma L} \right)^2 \quad (23)$$

We have numerically checked that this law provides a good estimate of the exact result as soon as the precursor is visible without dominating excessively the main pulse. In the conditions of Fig.5, we get $A_p = 1.9 \times 10^{-3}$ in satisfactory agreement with the amplitude ($\approx 2.5 \times 10^{-3}$) of the precursor shown in the inset of this figure. The order of magnitude given by Eq.(23) also applies to the postcursor but there is now interference of the postcursor with the tail of the main field and it is then difficult to determine what originates in the postcursor alone.

V. OPTIMIZATION OF THE FRACTIONAL PULSE-ADVANCE

The convolution approach used in the previous sections provides both a physical insight on the fast light phenomenon and analytical results in the short and long pulse limits. Unfortunately the corresponding fractional advances $F = a/\tau_p$ may be weak. Larger fractional advances are expected for intermediate pulse durations. The convolution approach can be extended to this case when the optical thickness is moderate, say for $L \leq 1$. The function $k(\ell, t)$ appearing in the impulse response may then be approximated by $\exp[-\gamma t(1 + L/2)]$ and the convolution product can be explicitly calculated. We will not develop this method here. Indeed more general and simpler results on $e(\ell, t)$, can be derived from the envelope $\hat{e}(\ell, t)$ of the transmitted field obtained when the envelope $e(0, t)$ of the incident pulse is replaced by its periodic continuation $\hat{e}(0, t)$ (see the thin lines in Fig.1).

When $e(0, t) = e_1(0, t)$, its periodic continuation $\hat{e}_1(0, t) = [1 + \cos(\Omega_1 t)]/2$ contains only three frequencies ($0, \pm\Omega_1$). Taking into account that $H(\ell, \Omega)$ and $H(\ell, -\Omega)$ are complex conjugates, we get

$$\hat{e}_1(\ell, t) = \frac{\exp(-L)}{2} + \frac{|H(\ell, \Omega_1)|}{2} \cos[\Omega_1 t + \Phi(\ell, \Omega_1)] \quad (24)$$

where

$$\Phi(\ell, \Omega) = \arg[H(\ell, \Omega)] = \frac{L\Omega/\gamma}{1 + \Omega^2/\gamma^2}. \quad (25)$$

The advance \hat{a}_1 of the maximum of $\hat{e}_1(\ell, t)$ on that of $\hat{e}_1(0, t)$ is thus

$$\hat{a}_1 = \frac{\Phi(\ell, \Omega_1)}{\Omega_1} = \frac{L/\gamma}{1 + \Omega_1^2/\gamma^2} = \frac{a_g}{1 + \Omega_1^2/\gamma^2}. \quad (26)$$

So long as the responses to the successive pulses of $\hat{e}_1(0, t)$ do not significantly overlap and as the precursor and postcursor have a negligible amplitude, $\hat{e}_1(\ell, t)$ is expected to be a good approximation of $e_1(\ell, t)$ in their common domain of existence. We have then $a \approx \hat{a}_1$ and the fractional advance reads

$$F = \frac{a}{\tau_p} \approx \frac{\Omega_1 \hat{a}_1}{\Omega_1 \tau_p} \approx 0.44 \Omega_1 \hat{a}_1 = 0.44 \frac{L\Omega_1/\gamma}{1 + \Omega_1^2/\gamma^2}. \quad (27)$$

Equation (26) shows that the advance is the largest and equal to the group advance a_g when $\Omega_1^2/\gamma^2 \ll 1$, that is for long incident pulses. Unfortunately, it results from Eq.(27) that the corresponding fractional advance F is very weak. F attains its maximum value $F_m \approx 0.22L$ when $\Omega_1 = \gamma$ ($\gamma\tau_p \approx 2.29$). The corresponding advance is half of the group advance ($a \approx a_g/2$). Besides the main pulse is followed by a second one, the relative intensity of which reads

$$R = \tanh^2(L/4). \quad (28)$$

Figure 6 obtained for $L = 1$ and $\Omega_1 = \gamma$, shows that the exact intensity profile $|e_1(\ell, t)|^2$ is actually quasi-confused with its periodic approximation $|\hat{e}_1(\ell, t)|^2$. F and R are in perfect agreement with those given by Eq.(27) and Eq.(28), namely $F = F_m = 22\%$ and $R = 6\%$. A simulation (not shown) made for $\Omega_1 = \gamma$ and $L = 2$ ($C \approx 17$ dB) evidences that $|\hat{e}_1(\ell, t)|^2$ is still a very good approximation of $|e_1(\ell, t)|^2$. We get then $F \approx 42\%$ and $R = 22\%$, very close to the values $F \approx 44\%$ and $R = 21\%$ predicted by Eq.(27) and Eq.(28). Note that the relative intensity of the second maximum of intensity is significantly larger than that obtained for $L = 1$.

For large optical thickness, problems arise from the overlapping of the responses to the successive pulses of the periodically continued input envelope and from the precursor and postcursor that can have significant amplitudes. Both problems are solved by replacing $e_1(0, t)$ considered above by $e_2(0, t)$ which is a better approximation of the Gaussian pulse. Indeed the successive pulses of $\hat{e}_2(0, t)$ are better separated than those of $\hat{e}_1(0, t)$ (see Fig.1) and the higher order initial and final discontinuities of $e_2(0, t)$ ensure negligible amplitude of the precursor and postcursor. $\hat{e}(0, t)$ reads

$$\hat{e}_2(0, t) = \cos^4\left(\frac{\Omega_2 t}{2}\right) = \frac{3}{8} + \frac{\cos(\Omega_2 t)}{2} + \frac{\cos(2\Omega_2 t)}{8}. \quad (29)$$

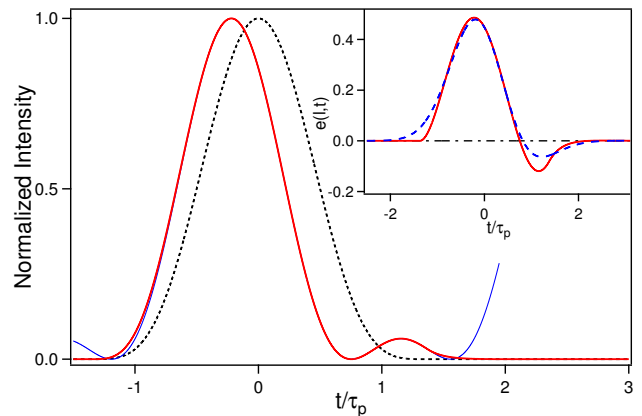


Figure 6: Comparison for $L = 1$ ($C = 8.7$ dB, dip depth $D \approx 86\%$) of the exact intensity profile of the transmitted pulse (solid red line) with the analytical profile obtained by periodically continuing the incident pulse $e(0, t) = e_1(0, t)$ (thin blue line). The pulse duration is that maximizing the fractional advance ($\Omega_1 = \gamma$, $\tau_p \approx 2.29/\gamma$). It leads to $a = a_g/2$ and to $F = L/2\gamma\tau_p \approx 22\%$. Inset: comparison of the envelopes of the transmitted pulse obtained for $e(0, t) = e_1(0, t)$ (solid red line) and $e(0, t) = e_G(0, t)$ (dashed blue line).

It contains five frequencies ($0, \pm\Omega_2$ and $\pm 2\Omega_2$) and leads to

$$\hat{e}_2(\ell, t) = \frac{3}{8} \exp(-L) + \frac{|H(\ell, \Omega_2)|}{2} \cos[\Omega_2 t + \Phi(\ell, \Omega_2)] + \frac{|H(\ell, 2\Omega_2)|}{8} \cos[2\Omega_2 t + \Phi(\ell, 2\Omega_2)]. \quad (30)$$

The advance \hat{a}_2 of the maximum of $\hat{e}_2(\ell, t)$ is obtained by solving the transcendental equation derived from Eq.(30)

$$\frac{\sin[\Phi(\ell, 2\Omega_2) - 2\Omega_2 \hat{a}_2]}{\sin[\Phi(\ell, \Omega_2) - \Omega_2 \hat{a}_2]} = -2 \left| \frac{H(\ell, \Omega_2)}{H(\ell, 2\Omega_2)} \right|. \quad (31)$$

When $L = 3$ ($C \approx 26$ dB), we find that $\hat{F} = \hat{a}_2/\tau_p$ takes its largest value $\hat{F}_m \approx 0.55$ for $\Omega_2 \approx 0.644\gamma$ ($\gamma\tau_p \approx 2.55$). Figure 7, obtained in those conditions, shows that the profile $|\hat{e}_2(\ell, t)|^2$ perfectly fits the exact intensity profile $|e_2(\ell, t)|^2$ with $F \approx \hat{F}_m \approx 55\%$, $a/a_g \approx 0.45$ and a relative intensity of the second maximum $R \approx 33\%$. We have added on Fig.7 the profile $|e_1(\ell, t)|^2$ which would be obtained for the same duration of the incident pulse. For this optical thickness, it does not deviate too strongly from the previous one with $F \approx 61\%$, $a/a_g \approx 0.52$ and $R \approx 35\%$. The situation dramatically changes for the largest transmission-contrast and optical thickness considered in this article ($C = 50$ dB, $L = 5.756$). For $e(0, t) = e_1(0, t)$, the transmitted pulse is then strongly damaged by the precursor and postcursor and the periodic solution $\hat{e}_1(\ell, t)$ fails to reproduce the exact result (even very approximately). On the other hand, the periodic solution given by Eq.(30) remains a very good approximation of the exact result when $e(0, t) = e_2(0, t)$.

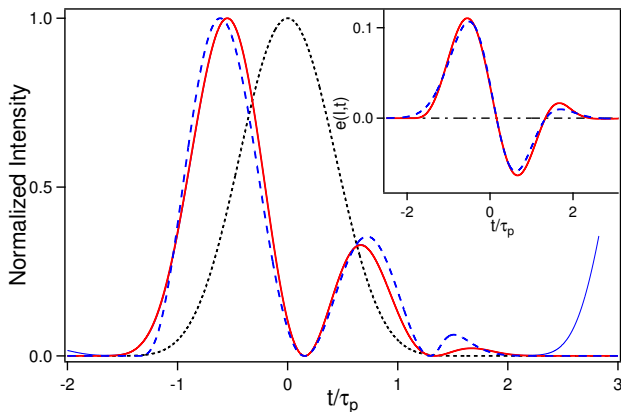


Figure 7: Comparison for $L = 3$ ($C = 26$ dB) of the exact intensity profiles $|e_2(l, t)|^2$ (solid red line) and $|e_1(l, t)|^2$ (dashed blue line) with the analytical periodic profile $|\hat{e}_2(l, t)|^2$ (thin blue line). The pulse duration ($\tau_p \approx 2.55/\gamma$) is that maximizing the fractional advance of $|\hat{e}_2(l, t)|^2$. Inset: comparison of the envelopes of the transmitted pulse obtained for $e(0, t) = e_2(0, t)$ (solid red line) and for $e(0, t) = e_G(0, t)$ (dashed blue line).

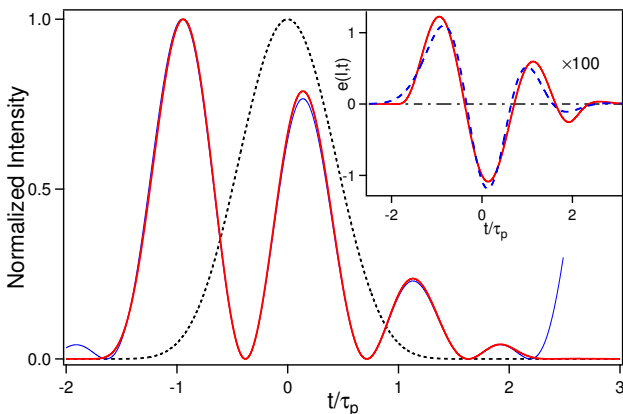


Figure 8: Comparison for $L = 5.756$ ($C = 50$ dB) of the exact intensity profiles $|e_2(l, t)|^2$ (solid red line) with the analytical periodic profile $|\hat{e}_2(l, t)|^2$ (thin blue line). The pulse duration is that maximizing the fractional advance, that is $\tau_p \approx 3.18/\gamma$ leading to $F \approx 94\%$ and $a = 0.52a_g$. Inset: comparison of the envelopes of the transmitted pulse obtained for $e(0, t) = e_2(0, t)$ (solid red line) and for $e(0, t) = e_G(0, t)$ (dashed blue line). Note that the slow light regime is attained for the Gaussian pulse (see text).

Figure 8 shows the intensity profiles obtained for the pulse duration $\gamma\tau_p \approx 3.18$ ($\Omega_2 \approx 0.517\gamma$) leading to the maximum fractional advance $F \approx \hat{F}_m \approx 94\%$ with an absolute advance $a \approx 0.52a_g$. The only difference between the exact intensity profile $|e_2(l, t)|^2$ and its analytical periodic counterpart $|\hat{e}_2(l, t)|^2$ lies in the second lobe, the relative intensity ($R \approx 79\%$) of which is very slightly underestimated in $|\hat{e}_2(l, t)|^2$.

A main result of the above study is that the pulse following the main one becomes invading when the trans-

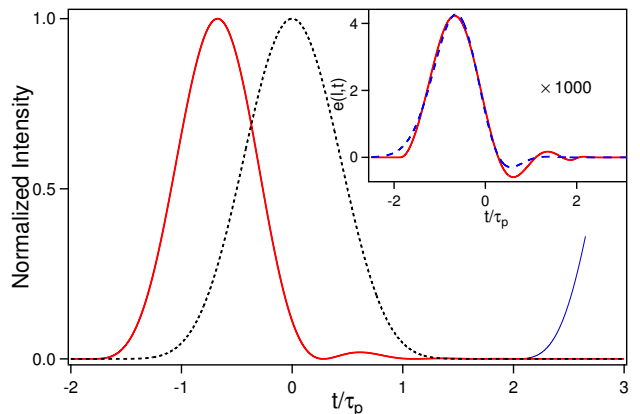


Figure 9: Same as Fig 8 for $\tau_p \approx 7.5/\gamma$ instead of $3.18/\gamma$. The relative intensity of the second maximum is so reduced to $R = 2\%$ whereas the fractional advance falls to $F \approx 67\%$. The inset shows that $e_2(0, t)$ and $e_G(0, t)$ both lead to fast light with very close pulse advances.

mission contrast C increases. For $C \geq 25$ dB ($L \geq 2.9$), there is a range of pulse durations for which the peak intensity of the second pulse exceeds that of the first one. There is then a transition from fast to slow light [40–43]. When $C = 50$ dB and $e(0, t) = e_2(0, t)$, this occurs for $0.185 < \gamma\tau_p < 2.95$. For Gaussian incident pulses, the upper limit increases up to $\gamma\tau_p \approx 3.34$ and the inset of Fig. 8 (obtained for $\gamma\tau_p \approx 3.18$) shows that the slow light regime is actually attained. We however remark that fast light is always obtained when the pulse duration is sufficiently short or long.

As mentioned in our introduction the main challenge of fast light experiments is to obtain a significant fractional advance with *moderate pulse distortion*. The first objective is attained in the conditions of Fig. 8 but not the latter owing to the presence of large secondary lobes in the intensity profile of the transmitted pulse. In order to reduce their relative intensity to a few percents, the incident pulse should be lengthened and this obviously reduces the fractional advance. Figure 9 is obtained in the conditions of Fig. 8 for $\gamma\tau_p = 7.5$ instead of 3.18. The relative intensity of the second maximum (the fractional advance) then falls to $R \approx 2\%$ ($F \approx 67\%$) whereas $a \approx 0.88a_g$. Note that the intensity profile of the transmitted pulse is perfectly fitted by the periodic solution $|\hat{e}_2(l, t)|^2$. This result is general. As long as the periodic solution works for the pulse duration maximizing the fractional delay, it works better for longer pulses. In particular, the approximation $e(l, t) \approx e_1(l, t)$, very good for $L = 1$ and $\Omega_1 = \gamma$ (Fig. 6), is excellent when $L = 1$ and $\Omega_1 = \gamma/4$ (Fig. 4), leading in this latter case to an advance which is exact with four significant figures.

When the intensity of the secondary lobes is weak enough, the distortion of the transmitted pulse mainly consists in a narrowing. This simply results from the negative value of the second cumulant of $H(l, \Omega)$ and the additivity of the cumulants [38]. The narrowing of

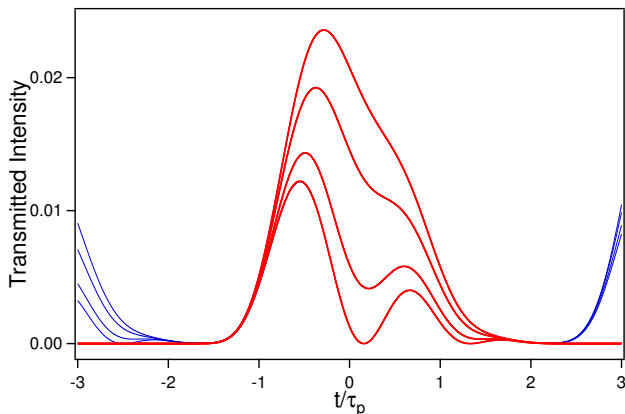


Figure 10: Effect on the transmitted intensity of a frequency detuning of the incident pulse for $e(0, t) = e_2(0, t)$. Parameters $L = 3$, ($C = 26$ dB), $\tau_p \approx 2.55/\gamma$ (as in Fig 7) and, from top to bottom, $\Delta/\gamma = \pm 0.6, \pm 0.5, \pm 0.3$ and 0. The solid red lines (the thin blue lines) are the exact solutions (the analytical periodic solutions).

the pulse originates a difference between the advances a_\uparrow at its rise and a_\downarrow at its fall. The distortion can then conveniently be characterized by the narrowing indicator $\xi = (a_\downarrow - a_\uparrow) / (a_\downarrow + a_\uparrow)$ where the advances a_\uparrow and a_\downarrow are measured at half-maximum intensity [11]. In the conditions of Fig.4 [Fig.9], we get $\xi \approx 8\%$ [$\xi \approx 13\%$] in agreement with the value derived from Eq.(24) [Eq.(30)].

The procedure of periodic continuation used in this section to maximize the fractional pulse advance can also be used to determine the effect of a detuning of the carrier frequency ω_c of the pulses from the resonance frequency ω_0 . In a frame rotating at ω_c , the transfer function reads

$$H(\ell, \Omega) = \exp \left[-\frac{L}{1 + i(\Omega + \Delta)/\gamma} \right] \quad (32)$$

where Δ is the detuning. When $e(0, t) = \hat{e}_2(0, t)$, we get for $\hat{e}_2(\ell, t)$ the analytical expression

$$\begin{aligned} \hat{e}_2(\ell, t) = & \frac{3H(\ell, 0)}{8} + \frac{H(\ell, \Omega_2)}{4} \exp(i\Omega_2 t) \\ & + \frac{H(\ell, -\Omega_2)}{4} \exp(-i\Omega_2 t) + \frac{H(\ell, 2\Omega_2)}{16} \exp(2i\Omega_2 t) \\ & + \frac{H(\ell, -2\Omega_2)}{16} \exp(-2i\Omega_2 t) \quad (33) \end{aligned}$$

which is reduced to Eq.(30) in the resonant case. The envelope $\hat{e}_2(\ell, t)$ is now complex and the corresponding intensity $|\hat{e}_2(\ell, t)|^2$ does not fall to zero in its main part. This behavior is illustrated on Fig.10 obtained in the conditions of Fig.7 for $\Delta/\gamma = \pm 0.6, \pm 0.5, \pm 0.3$ and 0 (from top to bottom). As expected, the advance (the pulse amplitude) decreases (increases) as a function of the detuning, the fractional advance falling from 55% for $\Delta = 0$ to 29% for $\Delta = \pm 0.6\gamma$. Note also the disappearance of the secondary lobes and the *strong asymmetry* of the transmitted pulse when the detuning exceeds 0.4γ .

As expected the analytical periodic solution $|\hat{e}_2(\ell, t)|^2$ perfectly fits the response $|e_2(\ell, t)|^2$ to the pulse of envelope $e_2(0, t)$ in the whole domain where $e_2(0, t) \neq 0$ ($|t| \leq \pi/\Omega_2 \approx 1.9\tau_p$). Similar results are obtained when $C = 50$ dB. Taking $\Delta = \gamma/5$ and $\tau_p = 6/\gamma$, it is in particular possible to reproduce the fractional advance ($F = 68\%$) and the shape (including its asymmetry) of the probe pulse which have been recently evidenced in a four-wave mixing experiment [28]. Again the intensity profile is very well fitted by the analytical periodic solution. We incidentally mention that, for the same contrast and the same detuning, slow light is obtained for shorter durations of the incident pulse. Otherwise said, a small detuning accelerates the transition from fast to slow light. This phenomenon, easily explained by a spectral analysis, is general and an asymmetry of the absorption line has a comparable effect [40, 43].

Results obtained on resonance with a symmetric absorption line can be extended to the case where the absorption is exactly compensated by a gain which is also on resonance at the carrier frequency of the incident pulses. The medium is then *transparent* at this frequency and the transfer function of this twofold resonant medium reads

$$H(\ell, \Omega) = \exp \left(\frac{L}{1 + i\beta\Omega/\gamma} - \frac{L}{1 + i\Omega/\gamma} \right) \quad (34)$$

where L and γ refer to the absorption line as previously and β is the ratio of the widths of the absorption and gain lines ($0 < \beta < 1$). The transmittance $|H(\ell, \Omega)|^2$ actually equals 1 for $\Omega = 0$ (transparent medium). It is maximum for $\Omega = \pm\gamma/\sqrt{\beta}$, yielding an intensity contrast

$$C = \exp \left[2L \left(\frac{1 - \beta}{1 + \beta} \right) \right]. \quad (35)$$

For arbitrary β , the phase [5] and the group advance read

$$\Phi(\ell, \Omega) = \frac{L\Omega/\gamma}{1 + \Omega^2/\gamma^2} - \frac{\beta L\Omega/\gamma}{1 + \beta^2\Omega^2/\gamma^2} \quad (36)$$

$$a_g = (1 - \beta) \frac{L}{\gamma} = (1 + \beta) \left(\frac{\ln C}{2\gamma} \right) \quad (37)$$

In the limit $\beta \rightarrow 0$ (infinitely broad gain-profile), these expressions are obviously identical to those obtained with a single absorption line. The second form of a_g in Eq.(37) might lead to expect that, for a same contrast C , the systems with $\beta \neq 0$ are more efficient than the reference medium. This is true for the absolute group advance but not for the fractional advance. An examination of the phase $\Phi(\ell, \Omega)$ as a function of Ω [5] yields a qualitative explanation of this apparent paradox. Figure 11, obtained for a same value of C when $\beta = 0$ and $\beta = 1/2$, shows that the slope $d\Phi/d\Omega$ at $\Omega = 0$ is actually larger for $\beta \neq 0$. On the other hand the spectral domain where this slope is positive is narrower and, for a same distortion of the transmitted pulse, the incident pulse should

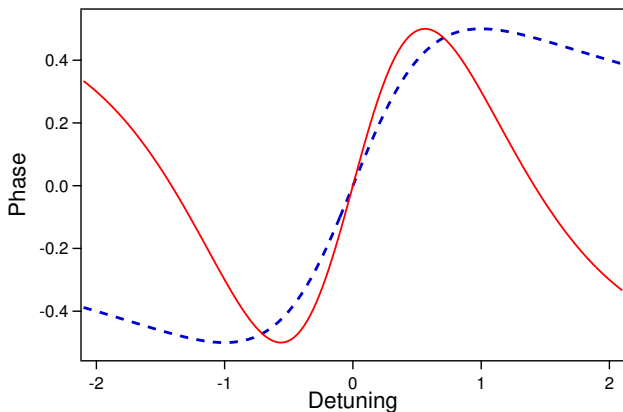


Figure 11: Phase of transparent resonant media as a function of the detuning. The solid red line (the dashed blue line) is obtained for $\beta = 1/2$ ($\beta = 0$). The phase (the detuning) is expressed in units of $\ln C$ (γ).

be lengthened to the prejudice of the fractional advance. In fact the maximum fractional advance does not significantly depend on β and is even independent of it in the range of validity of the approximation $e(\ell, t) \approx \hat{e}_1(\ell, t)$. We then get

$$F \approx \frac{\Phi(\ell, \Omega_1)}{\Omega_1 \tau_p} \approx 0.44L \left(\frac{\Omega_1/\gamma}{1 + \Omega_1^2/\gamma^2} - \frac{\beta\Omega_1/\gamma}{1 + \beta^2\Omega_1^2/\gamma^2} \right). \quad (38)$$

F is maximum for $2\beta^2\Omega_1^2/\gamma^2 = \beta^2 + 4\beta + 1 - (\beta + 1)\sqrt{\beta^2 + 6\beta + 1}$. This maximum reads

$$F_m \approx 0.22L \left(\frac{1 - \beta}{1 + \beta} \right) = 0.11 \ln C \quad (39)$$

and is actually independent of β for a given contrast. Moreover a simulation made for $L = 3$ and $\beta = 1/2$ ($\ln C = 2$) leads to a normalized intensity profile that perfectly fits that obtained with a single absorption line for a same contrast, that is for $L = 1$ (Fig.6).

VI. DISCUSSION

In the numerous theoretical articles on fast light published in the last eighteen years, fast light in absorbing media is generally discarded in favor of gain-assisted fast light (renamed superluminal light). We point out on the contrary that, *for a same fractional advance and a same distortion* of the transmitted pulse, absorbing media are preferable to media with gain. In the most cited experimental report on fast light [24], the authors exploit the anomalous dispersion associated with a minimum of gain between two gain lines created by Raman pumping. As remarked in a premonitory article [29], the use of an amplifying medium raises some difficulties due in particular to the phenomenon of amplified spontaneous emission and a risk of lasing on the gain lines. Cross modulation instability also limits the gain that can be

used [30]. Another point is that, for a given contrast, the gain-doublet arrangement yields weaker pulse advances than those obtained with a single absorption line [4, 11]. The fractional advance evidenced in [24] is actually very small (below 3%) and, as noted in [46], the advances at the rise and at the fall of the pulse are significantly different with a narrowing indicator $\xi = 20\%$. Moreover it should be remarked that, contrary to the title given in a subsequent article [47], the medium is far from being transparent with a transmittance of only 40% and that the experimental results reported in [24, 47] can be reproduced with the single absorption-line arrangement with a transmittance better than 66%. See Fig.1 in [11]. More convincing experiments using the gain-doublet arrangement are reported in [25]. The effects of the cross modulation instability [30] are here overcome by using two separate spatial regions for the two Raman pump beams [48]. The fractional advance is now $F \approx 10\%$ with a narrowing indicator $\xi \approx 13\%$. We however remark that the same fractional advance can be obtained in the single absorption-line arrangement with a fairly good transmittance (14%) and a weaker pulse narrowing ($\xi \approx 8\%$). See Fig.4.

Independently of the abovementioned reasons that restrict the use of gain media and of eventual problems of detection in the case of absorbing media, one may wonder why fractional advances exceeding 40% *with moderate distortion* have not been evidenced in linear media. The response is obviously that the large contrasts C required to attain this purpose entail a dramatic sensitivity of the transmitted pulse to defects of the incident one. As evidenced in the experiment reported in [13] with $C \approx 45$ dB, imperceptible quasi-discontinuities of the envelope of the incident pulse or of its derivatives originate transients (wiggles) in the envelope of the transmitted pulse which become invading when C is enhanced (even slightly). Simulations also show that the effect of a chirping of the incident pulse may become dramatic for large contrast. Although this point is beyond the scope of the present article (it is a subject *per se*), it is also probable that the effect of a partial light-incoherence, moderate for small contrast [49], will be destructive when this contrast increases.

Compared to other arrangements used to evidence fast light, the single absorption-line arrangement has obviously the advantage of its simplicity, from both experimental and theoretical viewpoints. We however mention that the contrast required to attain a given fractional delay can be slightly reduced by using a doublet of absorption lines [11], the experimental implementation of which seems quite feasible. When the lines are separated by $2\gamma/\sqrt{3}$, the lowest order contribution to the pulse distortion cancels and the medium transmittance is flattened around $\Omega = 0$. We then get $\ln C = 3L$ and

$$\Phi(\ell, \Omega) = \frac{2L(\Omega/\gamma)(2/3 + \Omega^2/\gamma^2)}{(4/3 + \Omega^2/\gamma^2)^2 - 4\Omega^2/3\gamma^2}. \quad (40)$$

In the range of validity of the approximation

$e(\ell, t) \approx \hat{e}_1(\ell, t)$, the fractional advance reads $F \approx \Phi(\ell, \Omega_1) / (\Omega_1 \tau_p) \approx 0.44 \Phi(\ell, \Omega_1)$ and attains its maximum $F_m \approx 0.39L$ for $\Omega_1 \approx 1.39\gamma$ ($\tau_p \approx 1.65/\gamma$). We have thus $F_m \approx 0.13 \ln C$ instead of $0.11 \ln C$ with a single absorption-line. This means in particular that the fractional delay obtained for $C \approx 8.7 \text{ dB}$ with a single absorption-line (Fig.6) would be obtained for $C \approx 7.3 \text{ dB}$ with a doublet. A simulation shows that the corresponding intensity profiles are very close, both being perfectly fitted by the analytical periodic solution of Eq.(24). Similar results are obtained for large contrasts. An intensity profile comparable to that obtained Fig.9 for $C \approx 50 \text{ dB}$ with a single absorption-line can be obtained for $C \approx 47 \text{ dB}$, both profiles being now well fitted by the analytical periodic solution of Eq.(30).

VII. CONCLUSION

The observation of significant fast-light effects requires the use of systems with a large contrast C between maximum and minimum transmissions. We have studied in detail the reference case of a dilute medium with a narrow absorption line, the frequency of which coincides with the carrier frequency of the incident pulse. The impulse response $h(\ell, t)$ relating the envelope $e(\ell, t)$ of the transmitted pulse to that $e(0, t)$ of the incident pulse is then real. In a retarded time picture (time delayed by the transit time in vacuum), the group advance a_g is positive no matter the propagation distance and can be identified to the advance of the center-of-gravity of $e(\ell, t)$ over that of $e(0, t)$. This advance generally differs from the advance a of the pulse maximum and, quite generally, *a large group advance is not a sufficient condition to observe significant fast light effects.*

By convoluting $h(\ell, t)$ and $e(0, t)$, fast light appears as resulting from the interference of the incident wave as if it had propagated in vacuum with the wave reemitted by the medium. Explicit analytical expressions of the convolution product are obtained in the short and long pulse limits. In the first case, we get $a \ll a_g$ with moderate attenuation of the peak intensity. The experimental evidence of this behaviour seems to be an open challenge. In the second case, a tends to a_g and the peak intensity is reduced by the factor C . Most experimental results have been obtained in conditions more or less approaching these ones. In both cases the fractional

advance $F = a/\tau_p$ is not optimum.

The observation of significant fractional advances leads to use incident pulses of intermediate duration. Analytical results are then obtained by a method of periodic continuation of the incident pulses. When the contrast C is below 20 dB as in most experiments actually performed in optics, the duration τ_p maximising F has an explicit form and the corresponding advance a is half of the group advance a_g . This latter result remains a good approximation when C is increased but the main transmitted pulse is then followed by large secondary pulses. Their lowering demands a lengthening of the pulses that reduces F below its maximum value.

The range of validity of the method of periodic continuation is very broad. It enables one to reproduce analytically the asymmetry of the transmitted pulses when the carrier frequency of the incident pulses is detuned from resonance. In the case of a resonant transparent medium, it shows that, for a same contrast, the maximum fractional advance is roughly equal to that of our reference medium.

Insofar as the fractional advance is mainly determined by the transmission contrast, the use of amplifying media seems unsuitable to evidence significant fast-light effects. Large gains indeed originate optical instabilities, obviously absent in absorbing media. Anyway, the use of large contrasts requires an exceptional quality of the incident pulse, the smallest imperfections of which can lead to considerable distortion of the transmitted pulse. That is probably the main practical limitation to fast light. Improvement of the quality of the incident pulses combined with the use of a doublet of absorption lines instead of a single line appears to be one way to evidence fast-light effects beyond those observed up to now.

ACKNOWLEDGEMENTS

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- by their conjugates. *The phases are then changed in their opposites* but the final results obviously do not depend on the used convention.
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