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ACTIVITY THEORY IN FRENCH DIDACTIC RESEARCH

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The theoretical and methodological tools provided by the first generation of Activity Theory have been expanded in recent decades by the French community of cognitive ergonomists, followed by a sub-community of researchers working in didactics of mathematics. The main features are first the distinction between tasks and activity and second the dialectic between the subject of the activity and the situation within which this activity takes place. The core of the theory is the twofold regulatory loop which reflect both the codetermination of the activity by the subject and by the situation, and the developmental dimension of the subject's activity. This individual and cognitive understanding of Activity Theory mixes aspects of Piaget and Vygotsky's frameworks. It is first explored in this paper, associated with a methodology for analyzing students' mathematical activities. Then we present findings that help to understand the complexity of student mathematical activities when working with technology.

Keywords: Mathematics, Tasks, Activity, Mediations, Technologies

Introduction

Activity Theory is a cross-disciplinary theory that has been adopted to study various human activities, including teaching and learning in ordinary classrooms, where individual and social levels are interlinked. These activities are seen as developmental processes mediated by various contextual elements – here we consider the teacher, the pair and the artefact (Vandebrouck et al., 2012: 13). Activity is always motivated by an object; a characteristic that distinguishes one activity from another. Transforming the object into an outcome is another key feature of activity. Subject and object form a dialectic unit: the subject transforms the object and at the same time is him/herself transformed. This framework can be adapted to describe the actions and interactions that emerge in the teaching/learning environment, and that relate to the subjects, the objects, the artefacts and the outcomes of the activity (Wertsch, 1981).

Activity Theory was originally developed by, among others, Leontiev (1978). A well-known extension is the systemic model proposed by Engeström and al. (1999), called third generation of Activity Theory. It expresses the complex relationships between the elements that mediate activity in an activity system. In this paper, we take a more cognitive and individual perspective. This school of thought has been expanded over the course of the past four decades by French researchers working in the domain of occupational psychology and cognitive ergonomics, and has since been adapted to the didactics of mathematics. The focus is on the individual as a cognitive subject and an actor in the activity, rather than the overall system – even if individual activity is seen as embedded in a collective system, and cannot be analysed outside the context in which it occurs.

An example of this adaptation is already well-established internationally. Specifically, it refers to the distinction between the artefact and the instrument, which is used to understand the complex integration of technologies into the classroom. The notion of instrumental genesis (or instrumental approach) was first introduced by Rabardel (1995) in the context of cognitive ergonomics, then extended to didactics of mathematics by Artigue (2002) and it is concerned with the subject–artefact dialectic of turning an artefact into an instrument. In this paper, we draw upon and try to encompass this instrumental approach.

First, we describe how Activity Theory has been developed in the French context. These developments are both general and focused on students' mathematical activity. Next, we present a general methodology for analysing students' mathematical activity when working with technology. Then we develop an example of application, and we describe our findings. Finally, we present some conclusions.

1. Activity theory in French context

The first notable feature of Activity Theory in the French context is the distinction between *tasks* and *activity* (Rogalski, 2013). Activity relates to the subject, while tasks relate to the object. Activity refers to what the subject engages in to complete the task: external actions but also inferences, hypotheses, thoughts and actions he/she decides to take or not. It also concerns elements that are specific to the subject, such as time management, workload, fatigue, stress, enjoyment, and interactions with others. As for the task – as described by Leontiev (1978) and extended in cognitive ergonomics – this refers to a goal to be attained under certain conditions (Leplat, 1997).

Activity Theory draws upon two key concepts: the *subject* and the *situation*. The subject refers to an individual person, who has intentions and competencies (potential resources and constraints). The situation provides the task and the context for the task. Together, situation (notably task demands) and the subject codetermine activity. The dynamic of the activity produces feedback in the form of twofold regulatory loop (Figure 1) that reflects the developmental dimension of Activity Theory (Leplat, 1997).

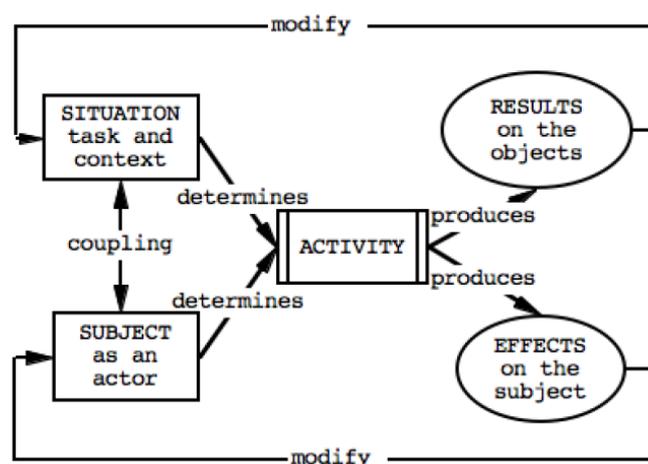


Figure 1: Codetermination of activity and twofold regulatory loop

The concept of twofold regulation reflects the fact that the activity modifies both the situation and the subject. On the one hand (upper loop), the situation is modified, giving rise to new conditions for the activity (e.g. a new task). On the other hand (lower loop), the subject's own knowledge is modified (e.g. by the difference between expectations, acceptable outcomes and the results of actions).

More recently, the dialectic between the upper and lower regulatory loops (shown in Figure 1) has been expanded through a distinction between the productive and constructive dimensions of activity (Pastré, 1999; Samurçay and Rabardel, 2004). Productive activity is object-oriented (motivated by task completion), while constructive activity is subject-oriented (the subject aims to develop his or her knowledge). In teaching/learning situations, especially those that involve technologies, the constructive dimension in the students' activity is key. The teacher aims the students to develop constructive activity. However, especially with computers, students are mostly engaged in producing results and the motivation of their activity can be

only towards the productive dimension. Then the effects of their activity on students' knowledge - as it is stipulated by the dual regulatory loop - are mostly indirect with less or without any constructive aspects.

The last important point to note is the fact that French Activity Theory mixes Piagetian approach of epistemological genetics, together with Vygotsky's socio-historical framework to specify the developmental dimension of activity. As Jaworski (in Vandebrouck, 2013) writes,

“the focus on the individual subject – as a person-subject rather than a didactic subject – is perhaps somewhat more surprising, especially since it leads the authors to consider a Piagetian approach of epistemological genetics alongside Vygotsky's sociohistorical framework”.

Rogalski (op. cit.) responds with

“the Piagetian theory looks from the student's side at epistemological analyses of mathematical objects in play while the Vygotskian theory takes into account the didactic intervention of the teacher, mediating between knowledge and student in support of the students' activity”.

The dual regulation of activity is consistent with the constructivist theories of Piaget and Vygotsky.

The first author (Piaget, 1985) provides tools to identify the links between activities and development, through epistemological analyses. Vergnaud (1982, 1990), expands the Piagetian theoretical framework regarding conceptualisation and conceptual fields by highlighting situation classes relative to a knowledge domain. We therefore define the students' learning – and development - with reference to Vergnaud's conceptualisation.

On the other hand, Vygotsky (1986) stresses the importance of mediation within the student's zone of proximal developmental (ZPD) for learning (scientific concepts). Here, we refine the notion of mediation by adding a distinction between procedural and constructive mediations in the context of the dual regulation of activity. Procedural mediations are object-oriented (oriented towards the resolution of the task), while constructive mediations are more subject-oriented. This distinction can be seen as an extension to what Robert (Robert and Hache, 2008) calls teachers' procedural and constructive teacher's aids. A more detailed exploration of the complementarity Piaget/Vygotski can be found in Cole and Wertsch (1996).

2. General methodology for analysing students' mathematical activities

Following Activity Theory, we postulate that students' learning depends directly on their activity, even though other elements can play a part - and even if activity is partially inaccessible to us and differ from one student to another. Students' activity is developed through the actions that are carried out to complete tasks. Through their actions, subjects aim to achieve goals, and their actions are driven by the motivation for the activity. Here, we draw upon the three levels originally introduced by Leontiev (1978): activity associated with a motive; actions associated with goals; and operations associated with conditions. Activity takes place in a specific situation, such as the classroom, at home, or during a practical session. Actions, involved by the proposed precise tasks, can be external (i.e. spoken, written or performed), or internal (e.g. hypotheses or decisions) and partially converted in operations. As Galperine (1966) and Wells (1993) note, the three levels are relative and, for instance, operations can be considered as actions that have been routinized.

Here, we use the generic term mathematical activities (rather than activity), to refer to students' activity on a specific mathematical task in a given context. Mathematical activities refer to everything that surrounds actions and operations (also non actions for instance). They are a function of a number of factors (including task complexity, but extending to the characteristics of the context and all mediations that occur as tasks are performed) that contribute to regulation and intended development in terms of mathematical knowledge.

Two methodological levels can be adopted from the dynamic of activity within the twofold regulatory loop. First of all, regulations can be considered at a local level as short-term adjustments of activities to previous actions and as procedural learning (also called functional regulations, upper loop in the figure 1). Secondly, at a global level, regulations are mostly constructive ones (also called structural regulations) and they correspond to the long-term development of the subject (in link with conceptualisation).

2.a The local level

At the local level, the analysis focuses on students' activities in the situation, in the form of tasks, their context, and their completion by students with or without direct help from the teacher. The initial step is an a priori analysis of the tasks given to students (by the teacher, the computer...), which is closely linked to the situational context (e.g. the students' academic level and age). We use Robert's (1998) categorisation to characterise these tasks.

First, we identify the mathematical knowledge to be used for a given task: the representation(s) of a concept, theorem(s), definition(s), method(s), formula(s), types of proof, etc. The analysis aims to answer several crucial questions: does the mathematical knowledge to be used already exist for students or is it new? Do students have to find the knowledge to be used by themselves? Do the task only require the direct application of this knowledge without any adjustment (technical task), or does it require adaptations and/or carrying out subtasks? A list of such adaptations can be found in Horoks and Robert (2007): mix of knowledge, the use of intermediaries, change of register (Duval, 1995), change of mathematical domain or setting (Douady, 1986), introduction of steps, choices, use of different points of view, etc. Tasks that require the adaptation of knowledge are referred to as complex tasks and encourage conceptualisation, as students become able to more readily and flexibly access the relevant knowledge, depending however on the implementation in the classroom.

The a priori analysis of tasks leads us to describe what we have called the *intended students' activities* associated with the tasks. Here we draw upon Galperine's (1966) functions of operations, and adapt them to mathematical activities. Galperine distinguishes three functions: orientation, execution and control. Next, we use three "critical" mathematical activities that are characteristic of complex tasks (Robert and Vandebrouck, 2014).

- First, *recognizing activities* refer mainly to orientation and control. They occur when students have to recognize mathematical concepts as objects or tools that can be used to solve the tasks they are given. Students may also be asked to recognize modalities of application or adaptation of these tools.
- Second, *organizing activities* refer mainly to orientation: students have to identify the logical and temporal steps in their mathematical reasoning, together with any intermediaries.
- Third, *treatment activities* refer to all of the mathematical activities associated with execution on mathematical objects. Students may be asked to draw a figure, compute, substitute, transform expressions (with or without giving the steps), change registers, change mathematical domains, etc.

Following Vygotsky, we supplement our local analysis of intended students' activities by developing ways to analyse classroom teaching (a posteriori), and to approach effective students' activities as functions of the different mediations that occur. For this, we use videos and observations in the classroom. We also record students' discussions, teacher's discourses and writings, and capture students' computer screens to identify observable activities. The data that is collected concerns how long students spend working on tasks, the format

of their work (the whole class, in small groups, by pairs of students etc.), its nature (copying, reading, calculation, investigation, written or oral, graded or not, etc.) and all elements of the context that may modify intended activities. This highlights, at least partially, the autonomy given to students, the nature of mediations, and opportunities for students to show initiative, in relation to the adaptation and availability of knowledge. Multiple aspects of mediations are analysed with respect to their assumed influence on student activities. Some relate to their format (interactions with students, between students, with teacher, with computers, etc.), while others concern the specific ways of taking into account the mathematical content (mathematical aids, assessment, reminders, explanations, corrections and evaluations, presentation of knowledge, direct mathematical content, etc.).

Two types of mediations have already been introduced, depending on whether they modify intended activities, or whether they add to activities (effective or at least observed). The first are object-oriented; here we use the term procedural mediations. These mediations modify intended activities and correspond to instructions given by the teacher, the screen or by other students, directly or indirectly, before or during task completion. They are often seen in open-ended questions from the teacher such as ‘What theorem can you use?’ They can be given by the computer giving feedbacks which transform the task to be performed or with some limitations in the provided tools which give indirect indications to students about the way to achieve the task. These procedural mediations may lead to the subdivision of a complex task into subtasks. They usually change knowledge adaptations in complex tasks and simplify the intended activities in such a way that it becomes more like technical tasks (for instance students having to apply a contextualized method).

The second type of mediations are more subject-oriented, here we use the term constructive mediations. They are designed to add something to the students’ activities and the knowledge that can emerge from these activities. They can take the form of a simple summary of what has been developed by students, an explanation of choices, a partial decontextualization or generalisation, assessments and feedbacks, a discussion of results, etc. On some computers, the way a geometrical figure has been achieved by a student can be replayed to recall him the order in which instructions have been given without any wrong ways.

It should be noted here that our framework leads to the hypothesis that there is an internal transformation of the subject in the learning process: constructive mediations aim to contribute to this process. However, the mediations can be constructive for some students and remain procedural for others. On the contrary, some procedural mediations can become constructive for some students, for instance if they extract by themselves a generalisation from a local indication. Moreover some constructive mediation – but also perhaps productive – can belong to some students’ ZPD in the sense of Vygotski or they can remain out of the ZPD. When they belong to the ZPD, they can be identified to appreciate the explicit links between the expression of the general concepts to be learned and their precise applications, in contextualised tasks, according to the necessary dynamic between them. Distinguishing between the kinds of mediations and the way they belong or not to some students’ ZPD can be very difficult.

2.b The global level

The local level can be extended to a global level that takes into account the set of mathematical activities, the link with the intended conceptualisation (long term constructive loops), and teaching practices in the long term. We link mathematical students’ activities to the intended conceptualisation of the relevant mathematical notion, establishing a “relief map” of this mathematical notion. This relief map is developed from an epistemological and mathematical analysis of the notion, the study of the curricula, and didactical analyses

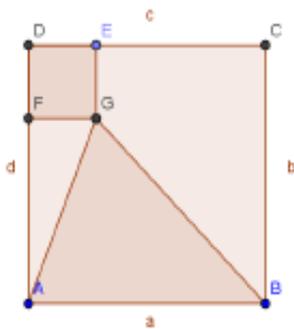
(e.g. students' common difficulties). This global analysis focuses on the similarity between students' activities (intended, observed, effective) and the set of activities that characterise the intended conceptualisation of the relevant notion.

However, the didactical analysis of one teaching session is insufficient. It is necessary to take into account, on a day-to-day basis, all of the tasks students are asked to complete, and teachers' interventions. We use the term scenario to describe a sequence of lessons and exercises on a given topic. The global scenario could be understood as a long term "cognitive road" (Robert and Rogalski, 2005).

3. Example of application: the 'shop sign' situation

To illustrate the utilisation of our Activity Theory, this section presents an example of a situation that aims to contribute to students' conceptualisations of the notion of function. Then we outline some limitations of the methodology at the global level.

The example relates in fact to the GeoGebra 'shop sign' family for learning functions. This family refers to mathematical situations that lie at the interface between two mathematical domains: geometry and functions. There are many examples of shop sign situations, but they share the idea that a coloured area is the lit area of a shop sign (Artigue and al., 2011) which depends on some moving variables in the shop sign.



In Figure 2, ABCD is a square, with A at the origin and $AB=4$. E is a mobile point on the segment [CD]. We consider the sum of the areas of the square (DFGE) and the triangle (ABG). The task is to find the minimum of the sum of the areas as E moves.

Figure 2: Shop sign

The task is set for grade 10 students (15 years old). One solution is to identify DE as an independent variable x . Then $f(x)$, the sum of the two areas, is equal to x^2 (for the square) plus $4(4-x)/2$ (for the triangle): equivalent to $x^2 - 2x + 8$. In the French curriculum at grade 10, the derivative is not known and students must compute and understand the canonical form $(x-1)^2 + 7$ as a way to identify the minimum 7 for the distance $DE=1$ (which is the actual position on the figure).

Students are working in pairs on computers. They have already worked with functions in the traditional pencil and paper context, and they also have manipulated GeoGebra for geometrical tasks that do not refer to functions. In this new situation, GeoGebra helps them to begin the task by making conjectures about the minimum. Students can also trace the graph of the function, as shown in Figure 6. Then, in the algebraic register, they can find the canonical form of the function $f(x)$ and the characteristics of the minimum.

We first identify the relief map on the notion of function and the intended conceptualisation. Then we give the a priori analysis of the task and the intended students' activities. We finish with the observation to two pairs of students to identify observable and effective activities.

3.a The global level: relief map on the notion of function and intended conceptualisation

The function is a central concept in mathematics and links it to other scientific fields and real-life situations. It both formalises and unifies (Robert and Hache, 2008) a diversity of objects and situations that students encounter in secondary school: proportionality, geometrical transformations, linear, polynomial growth, etc. A diversity of systems of representations (numerical, graphical, algebraic, formal, etc.) and a diversity of perspectives (pointwise, local and global) are frequently combined when working with them (Duval, 1995; Maschietto, 2008; Vandebrouck, 2011). As it is summarized by Artigue, Batanero and Kent (2007), the processes of teaching and learning of function entail various intertwining difficulties that reinforce one another in complex ways.

Educational research (Tall, 2006; Gueudet, 2008; Hitt and Gonzalez-Martin, 2016) shows that an efficient conceptualisation of the notion requires a rich experience that illustrates the diversity illustrated above, and the diversity of settings in which functions are used (Douady, 1986). It also means that functions are available as tools for solving tasks, and can be flexibly linked with other concepts. There must be a progression from embodied conceptualisations (where functions are highly dependent on physical experience) to proceptual conceptualisations (where they are considered dialectically and work both as processes and objects), paving the way for more formal conceptualisations (Tall, 2004, 2006).

At grade 10, the intended conceptualisation can be characterized by a set of tasks in which functions are used as tools and objects. They can be combined and used to link different settings (including geometrical and functional), numerical, algebraic and graphical representations, and the dialectic between pointwise and global perspectives. The shop sign task is useful in this respect, as students have to engage in such mathematical activities. A priori, optimisation tasks in geometrical modelling help to build the intended functional experience, and link geometrical and functional settings.

Technology provides a new support for physical experience, as the modelling process provides new systems of representation and helps to identify the dynamic connections between them. It also offers a new way to approach and connect pointwise and global perspectives on functional objects, and supports the building of rich functional experiences. A famous contribution is the one of Arzarello and Robutti (2004), who use sensors to introduce students to the functional domain. The framework is already an activity theoretical framework, together with more semiotic approaches, but it is not in a context of dynamic geometry. Many experiences exist about learning functions through dynamical geometrical situations. For instance, Falcade and al. (2007) study the potential of a didactical engineering with Cabri-Géomètre. The authors take a Vygotskian perspective about semiotic mediations which is more precise than our adaptation of Vygotsky inside Activity Theory, but which is also more restrictive in the sense that they don't consider deep connexions between given tasks and mathematical activities. Moreover, it doesn't concern ordinary classrooms. More recently, Minh and Lagrange (2016) analyze students' activities on functions using Casyopée. This software is directly built for the learning of functions and the authors adopt the model of Mathematic Working Spaces (Kuzniak & al., 2016). They built on three important challenges for students in the learning of functions: to consider functional dependencies, to understand the idea of independent variable, and at last to make sense of functional symbolism. The aims of the 'shop sign' family is consistent with such a progression which is closed to Tall's one introduced above.

3.b The local level: a priori analysis of the task and intended students' activities

The task is to identify the position of E on [DC] in order that the sum of the areas DFGE and AGB are minimal (Figure 2). It requires actual knowledge about geometrical figures and functions. However, it assumes that the notion of function is available, i.e. students have to identify the need for a function by themselves.

In a traditional pencil and paper environment, students first draw a generic figure. They can try to estimate – by geometrical measurements – some values of the areas for different positions of E. They can draw a table of values but this kind of procedure is usually not enough to obtain a good conjecture of the minimum value. Moreover such a procedure can reinforce the pointwise perspective because it doesn't bring the continuous aspects of the function at stake. Usually, the teacher quickly asks students to produce algebraic expressions of the areas. Students try themselves to introduce an algebraic variable ($DE=x$), or the teacher gives them procedural aids.

In the example given here, the teacher provided students with a sheet of paper showing a figure similar to the one given in Figure 2, and the instructions as summarized in Figure 3.

<p>First: construction of the figure (with GeoGebra) Second: conjecture (experimentation and observations with GeoGebra) Third: algebraic proof</p> <p>Troisième partie : Démonstrations</p> <p>1. On pose $DE = x$. Exprimer en fonction de x la somme des aires du carré EDFG et du triangle GAB.</p>

Figure 3: Main instructions given to students

Figure 3 shows that the overall task is divided into three subtasks. Organizing activities are directed by procedural mediations (functional regulation), which is a way to ensure that most students can engage in productive activity.

A priori analysis of the first subtask: the construction of the figure

In the geometrical subtask students have to identify the fixed points (A, B, C, D), the free point (E) on [DC], and the dependent points (F and G). The order of construction is crucial to the robustness of the final figure, but are not important in the paper and pencil environment. Consequently, organizing activities – the order of instructions – are more important in the GeoGebra environment.

The subtask also requires students to make choices. It is possible to draw either G or F first, and the sequence of instructions is not the same. Moreover, there are other choices that have no equivalent in the paper and pencil environment: whether to define the polygons (the square and triangle) with the polygon instruction, or by the length of their sides; whether to use analytic coordinates of fixed points or a geometrical construction; whether to use a cursor to define E; etc. These choices refer not just to mathematical knowledge, but also to instrumental knowledge (following the instrumental genesis approach). This means that treatment activities include instrumental knowledge and are more complex than in the traditional environment. Once the construction is in place, students can verify its robustness – a treatment that is also specific to the dynamic environment.

A priori analysis of the second subtask: the conjecture

There is no task really equivalent to this subtask in the paper and pencil environment. This again leads to specific treatment activities. These are engaged with the feedback provided by the software, which assigns numerical values of the areas DFGE and AGB, according to the position of E. However, students are required

to redefine DFGE and AGB as polygons if they have not already used this instruction to complete subtask 1 (Figure 5). They also have to create in GeoGebra environment a new numerical value that is the sum of the two areas in order to refine their conjecture. It is not clear in what extent these specific treatment activities refer to mathematical knowledge, and we will return to this point later.

A priori analysis of the third subtask: the algebraic proof

This subtask appears similar to its equivalent in the paper and pencil environment. However, as students already know the value of the minimum, the motivation for activity is different and only relates to the proof itself. The most important step is the introduction of x , as a way to pass from the geometrical setting to the functional setting. This step brings recognizing activities (students must recognize that the functional setting is needed), which is triggered by a procedural mediation (the instructions given on the sheet).

Students have to determine the algebraic expression of the function. Existing knowledge about the area of polygons must be available. They also have to recognize a second order polynomial function associated to specific treatments. Treatment activities remain to obtain the canonical form (as students have not been taught about derivatives, they must be helped in this by the teacher). At last, the recognition of the canonical form as a way to obtain the minimum of the area and the position of E which correspond to this minimum is correlate to the importance of the dialectic between pointwise and global perspectives on functions.

3.c A posteriori analysis: observable and effective activities

Students worked in pairs. The teacher only intervened at the beginning of the session (to ensure that all students were working), and at the end (to summarise the session). Students mostly worked autonomously, although the teacher helped individual pairs of students. The following observations are based on two pairs of students: Aurélien and Arnaud, and Lolita and Farah.

Analysis of the first pair of students' activities: Aurélien and Arnaud

This pair took a long time to construct their figure (more than 20 minutes). They began with A,B,C,D, in sequence, using coordinates, and then drawing lines between pairs of points. This approach is closest to the paper and pencil situation, and while it is time-consuming it is not crucial for global reasoning. They then introduced a cursor – a numerical variable j that took a value between 0 and 4 – in order to position E on [D, C]. However, the positioning of F (at 0, 3), was achieved without the cursor, which leads to a wrong square (Figure 4). G was drawn correctly. After they had completed their construction, they moved the cursor in order to verify that their figure was robust; an operation which revealed that it was not (Figure 4).

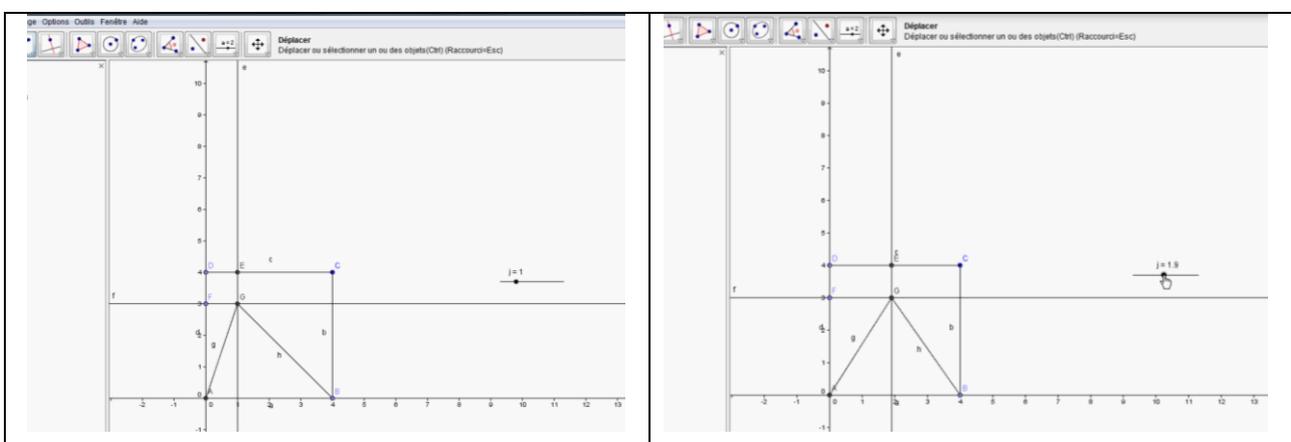


Figure 4: Exploring the robustness of the shop sign

This mediation from the screen is supposed to be a constructive mediation: it does not change the nature of the task and it is supposed to permit a constructive regulation of students' activities (lower loop in reference to Figure 1). However, the mediation doesn't encounter the students' ZPD and it is insufficient for them to regulate their activity by their own. The mediation supposes in fact new recognizing activities, specific to dynamic geometry on computers, that these students are not able to develop.

In this case, the teacher makes a procedural mediation and helps the students to rebuild their figure ("You use the polygon instruction to make DFGE [...] then again to make the polygon ABG"). Once the two polygons have been correctly drawn, the values of their areas appear in the numerical window of GeoGebra (called *poly1* and *poly2*, shown on the left-hand side of the screens presented in Figure 5).

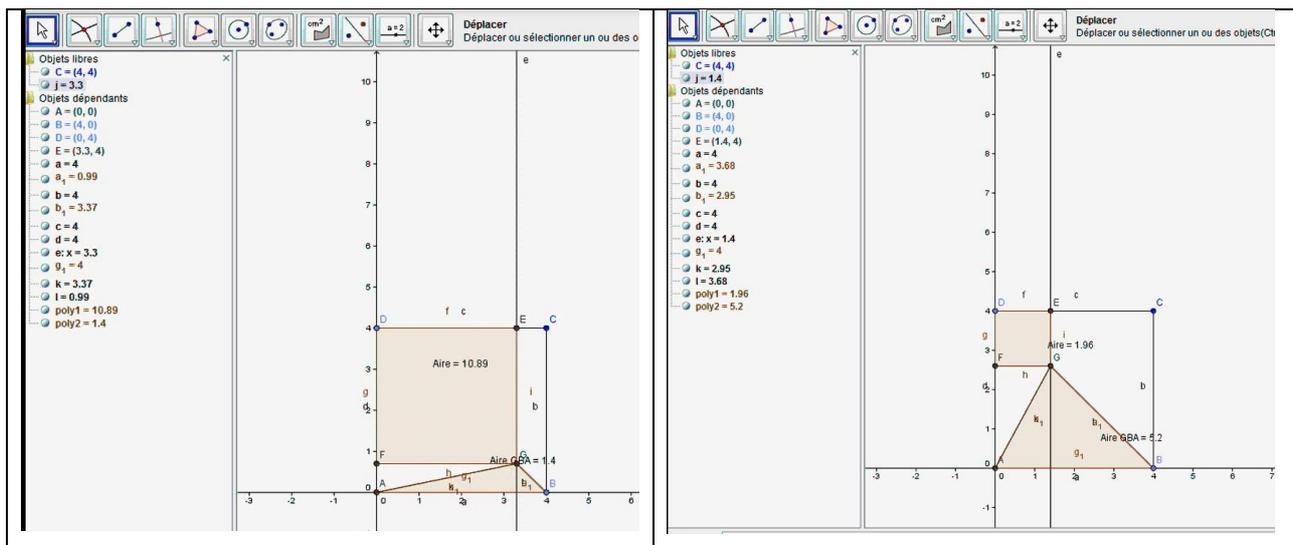


Figure 5: Exploration of varying areas by moving the point E on [DC]

In the conjecture phase (second subtask, 8 minutes), the students made the conjecture that the sum is always 8 ("Look, it's always 8..."), by computing $poly1 + poly2$ in their mind. The numerical window of GeoGebra now shows 18 different pieces of information, including the areas of DFGE (*poly1*) and ABG (*poly2*). Students must introduce another numerical variable (e.g. *poly3*) that is equal to the sum of $poly1 + poly2$. However, this requires new organizing activities that GeoGebra does not help with. In fact, there is already too much information in the numerical window. Here again, the teacher provides direct procedural assistance ("introduce $poly3 = poly1 + poly2$ ").

In the algebraic phase (third subtask, 20 minutes), the students are unable to express the areas DFGE and ABG as functions of x . Analyses reveal that again new recognizing activities are awaited to switch from the computer environment to the paper and pencil environment. These new recognizing activities are not an evidence for the students. They suppose both mathematical knowledge and instrumental knowledge about the potentialities of software and the mathematical way of proving the existence and the values of the minimum. Then, students' attempt to implement $DE = x$ in the input bar leads to feedback from GeoGebra (in the form of a syntax error), which informs them that their procedure is wrong – but does not provide any guidance about what to do instead. It is difficult to know whether to categorise this kind of mediation as procedural or constructive as it does not add any mathematical knowledge.

The teacher asks the students to try to find a solution with pencil and paper (procedural assistance). However, the introduction of x , which is linked to the change of mathematical setting (adaptation of knowledge), seems

very artificial. The students start working on their algebraic formula by looking at their static figure, with E positioned at (1, 4). The base of the triangle measures 4 and its height is 3. One of the pair suggests that “it depends on x ” means that each algebraic expression ends in x , as the following dialogue between the two students shows:

“This is $4x$ / Base times height ... so the base it's 4 / $4x$... it's 4 times x , because it's a function of x / Oh? The height? / Yeah the height... $3x$ / No, it can change / Yeah but in this case / Look, (he moves the cursor) this isn't $3x$ here / Humm... OK, listen ... / Before for the square we found it because x square is always the area... this isn't more complicated than that ... the base is always 4... / No it's not more complicated but... / The base doesn't change / It's $4x$ times... / The base doesn't change / Yes it's sure but we have to find the height...”

At this point, the teacher provides another direct procedural assistance. This once again shows that although the mediation of GeoGebra help students to discuss and progress, it is insufficient for them to correctly regulate their activity. Without procedural assistance from the teacher, they are unable to find the formula for the area of triangle. In the end, the students don't have enough time to finish the task by themselves.

At end of the session, the teacher gives a procedural explanation to whole class of how to find the canonical form (as “ $x^2-2x+8 = (x- _)^2+ _$ ”). Although Aurélien and Arnaud write it down, they do not make the link between it, and their classroom work. Consequently, they do not understand the motivation for the activity and cannot make sense of the explanation of the canonical transformation given by the teacher.

Then the teacher gives constructive explanation about the meaning of the coefficients in the canonical form and the way they give the minimum and the corresponding value of x . But according to Aurélien and Arnaud's activities, it is too early and they do not make the link with their numerical conjecture. In other words, the collective mediation of the teacher seems too far from the students' ZPD and it is not at all constructive for this pair of students.

Analysis of the second pair of students' activities: Lolita and Farah

Lolita and Farah are better students and quickly draw their robust figure. Their numerical conjecture is correct and the teacher gives them another subtask: to find a graphical confirmation of their conjecture. The procedural instruction is to find a new point, M, whose abscissa is the same as E and ordinate is the value $\text{poly1}+\text{poly2}$.

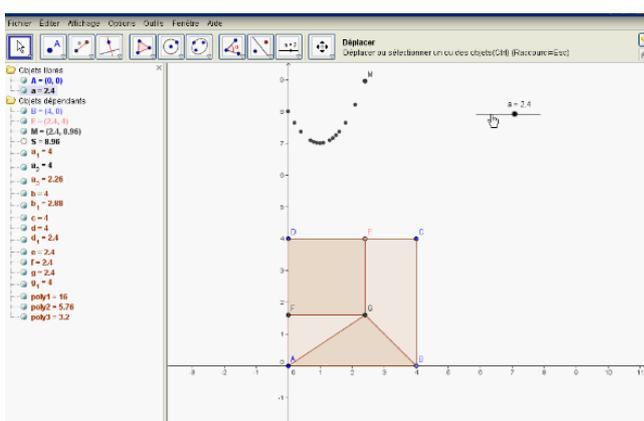


Figure 6: The shop sign task showing part of the graph of the function

When moving the cursor, the trace of M can be interpreted as the graph of the function $f(x)$. However, Lolita and Farah do not recognize this. One says “this is not a curve” and then “the minima, we have seen this for functions but here...”.

They only recognize the trace as a part of a parabola (geometrical setting) and associate its lowest point with the value of the minimum area.

The graphical observation confirms to Lolita and Farah that their numerical conjecture was correct. However, this is a proof for them and they do not understand the motivation of the third subtask which does not make

sense to them. Although they succeed in defining the algebraic expression of the function and they find the canonical expression, they do not make the link with their graphical observation.

Here again, the teacher's summary of how to obtain the canonical form of the function, the value of the minimum and the corresponding value of x is not useful for this pair as it is not the problem they encountered. A constructive intervention about the motivation for the third subtask and how the canonical form was linked to the conjecture would have been a mediation closer to their ZPD.

4. What does this tell us about students' mathematical activities?

The main result concerns complex activity involving technology: here the complexity is introduced by mathematical activities that require either mathematical or instrumental knowledge, particularly knowledge about the real potentialities of technologies in contrast with what is supposed to be solved within the paper and pencil environment. This leads also to new treatment activities (e.g. in the construction and conjecture subtasks) and new recognizing activities. New, onscreen, representations appear, typically dynamic, and students must recognize them as mathematical objects (or not). The example of Aurélien and Arnaud shows how difficult it was for them to recognize a robust figure, and dynamic and numerical representations of variations in areas. Similarly, it was difficult for Lolita and Farah to recognize the trace of M as a special part of the graph of a function.

The second main result concerns the increase in recognizing activities and the new balance between the three types of critical activities. While in a traditional session, the teacher can point out the mathematical objects to use, the screen presents far more information to students, meaning that they have to recognize what is most important in their treatment activities. Organizing activities also increase, both before treatment activities related to construction, and during conjecture. For instance, Aurélien and Arnaud failed in the conjecture task because they were not able to introduce a third numerical variable by themselves. Classroom observation (Robert and Vandebrouck, 2014) has led to the idea that most of effective students' activities are treatment activities as the teacher must make productive interventions before most students can begin the task. Recognizing and organizing activities are mostly activities for best students. These students often have an idea of how to begin the resolution of the task, they are able to adapt quickly their knowledge, and they develop all three types of critical mathematical activities, whereas weaker students find it difficult to engage in the task waiting for any procedurale assistance of the teacher. In classroom sessions that use technology, students are confronted alone with all of these critical activities, which may help to explain the difficulty of weaker students.

A further finding concerns mediations. In such sessions, teacher's mediations are mostly procedural and clearly aim to foster productive activity. Onscreen mediation leads to specific, new recognizing activities (dynamism) but is insufficient for students (not only weaker) to regulate their own activity. It appears that most of the time this mediation is not procedural or constructive enough, leading to more teacher intervention. Moreover, it seems that onscreen mediation is always associated with treatment activities and does not help students in their recognizing or organizing activities.

The last point concerns constructive mediation and the heterogeneity of the students' knowledge (and ZPD). Student activities in classroom sessions that use technology are difficult for the teacher to evaluate. Even if he/she tries to manage the best "average" constructive mediations for all students, our examples show that this is very challenging. This raises the question of what is the real impact of such sessions with respect to the intended conceptualisation. The availability and recognition of functions as tools to complete such tasks was

not really investigated, in the sense that the independent variable x was given to students (on paper), and none of them returned to the geometrical setting as in the traditional modelling cycle - in reference to Kaiser and Blum (Maas, 2006). Moreover, Aurélien and Arnaud did not explore the dynamic numerical-graphical-algebraical flexibility, which was one of the aims of the session; on the other hand Lolita and Farah did, but lacked the constructive mediations needed to complete the cycle.

Conclusion

We have presented Activity Theory in the context of French didactics, notably the dual regulation found in the activity model, which was first developed in ergonomic psychology and then adapted to didactics of mathematics, for studying students' activities. Other works, which we have not discussed here, have looked at teachers' practices in some different ways (Robert 2012, Robert and Rogalski 2005). An important component of this model is the impact of activity on subjects, which represents the developmental dimension of students' activity. This focus highlights the commonalities and complementarities of the constructivist theories of Piaget (extended to Vergnaud's conceptual fields) and Vygotsky. The connection between Activity Theory, the work of Piaget and Vygotsky, and didactics of mathematics, provides a theoretical foundation for a dual approach to students' activity from the viewpoint of mathematics (the didactical approach) and subjects (the cognitive approach).

Our analysis does not provide a model of students' activity (or teachers' practices). However, it leads to the identification of similarities and differences in terms of the relations between subtasks, students' ways of working, mediations, mathematical activities, and compares this complex task with the traditional, paper and pencil environment. One of the specificity of our approach is the deep connection between the students' activities analysis and the a priori tasks' analysis, including mathematical content. But we do not look for the teacher's own intention, unlike what is done in some English research (for instance, Jaworski and Potari, 2009). Moreover, we do not attempt to raise the global dynamic between individual and collective interactions and learning. We should take now a threefold approach to the investigation of students' practices – didactical, cognitive and socio-cultural. As Radford (2016) argues, with respect to Mathematical Working Space (Kuzniak and Tanguay, 2016), the individual- collective dynamic remains difficult to understand in both our Activity Theory and MWS which are discussed together. This represents a new opportunity to better investigate the socio cultural dimension of Activity Theory – especially the one developed by Engestrom - and integrate it into our didactical and cognitive approach.

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