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Verônica Gitirana, Takeshi Miyakawa, Maryna Rafalska, Sophie Soury-Lavergne, Luc Trouche

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RE(S)SOURCES 2018

International Conference
ENS de Lyon - May 2018

+ Young Researchers Workshop /
Atelier Jeunes Chercheurs

Editors / Editeurs

Verônica Gitirana - Takeshi Miyakawa
Maryna Rafalska - Sophie Soury-Lavergne
Luc Trouche

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AUF Agence Universitaire de la Francophonie Europe de l'Ouest
EPIC ED 485
Science S2HEP Society
ReVEA Ressources vivantes pour l'enseignement et l'apprentissage
EducTice
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Université de Lyon
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Presentation / Présentation

The theme of the Re(s)sources 2018 international conference is 'teachers interacting with resources' which will be discussed in seven plenary lectures, a plenary panel and four Working Groups. The conference is followed by a young researcher workshop structured under four sessions.

The Re(s)sources 2018 call for contributions concerned participation in the Working Groups, which constitute the essential place for presentation, discussion, and collective work on specific topics related to resources. Each proposal has been peer-reviewed (and subsequently revised), resulting in 70 accepted oral and 7 accepted poster contributions.

The present volume, edited by V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne and L. Trouche before the conference, gives access to the presentations of the main moments of Re(s)sources 2018 conference and of the Young Researchers' Workshop, to the presentations of the lecturers, panelists and WG coordinators, and, essentially, to the whole set of accepted contributions. As such, it constitutes an essential resource for preparing the Conference.

The conference itself will give rise to a book, to be published in October 2018 in the Springer series Advances in Mathematics Education. This book, entitled "Resources in Mathematics Teachers' Professional Activity", will be edited by L. Trouche, G. Gueudet and B. Pepin. It will contain papers written by the plenary lecturers and panelists, and syntheses of the work of each WG, written/authored by their coordinators, in addition to selected and developed contributions. A new resource to come, for keeping alive the results of the Re(s)source 2018 conference!

Le thème de la conférence internationale Re(s)sources 2018 est « l'interaction des professeurs avec leurs ressources », thème qui sera discuté dans sept conférences plénières, un panel et quatre groupes de travail. La conférence est suivie par un atelier jeunes chercheurs, structuré en quatre sessions.

L'appel à contributions Re(s)sources 2018 concernait la participation aux groupes de travail, qui constituent la place essentielle pour la présentation, la discussion et le travail collectif sur les sujets relatifs aux ressources. Chaque proposition a suivi un processus de relecture entre pairs (et révisée en conséquence, débouchant sur 70 contributions orales et 7 posters acceptés.

Le présent volume, édité par V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne et L. Trouche avant la conférence, donne accès à la présentation des principaux moments de la conférence Re(s)sources 2018 et de l'atelier jeunes chercheurs, à la présentation des conférenciers, des panelistes et des coordinateurs des groupes de travail, et à l'ensemble de toutes les contributions acceptées. Il constitue ainsi une ressource essentielle pour préparer la conférence.

La conférence elle-même donnera matière à un ouvrage, qui sera publié par Springer dans la collection «Advances in Mathematics Education ». Cet ouvrage, intitulé « Resources in Mathematics Teachers' Professional Activity », sera édité par L. Trouche, G. Gueudet et B. Pepin. Il contiendra des articles écrits par les conférenciers et les panelistes, et des synthèses de chaque groupe de travail, sous la responsabilité de leurs coordinateurs, qui sélectionneront des contributions qui pourront être développées.

Une nouvelle ressource à venir donc, pour conserver vivants les résultats de la conférence Re(s)sources 2018!

Ten Years after the Emerging of the Documentational Approach to Didactics, Work in Progress, Theoretical Networking, International Enlightening and Programs of Research

Dix ans après l'émergence de l'approche documentaire du didactique, chantiers en cours, croisements théoriques, éclairages internationaux et programmes de recherche

The development of the Internet and the abundance of digital resources, in addition to the emergence of new forms of teacher collective work, lead to new developments and uncertainties in teaching and learning of mathematics. These upheavals have given rise to new theoretical needs: how to analyze teachers' work when they prepare for their teaching? How to conceptualize the relationships between individual and collective work? How to follow the related processes over the long term?

Ten years ago, these theoretical and practical needs led to the proposal of a new frame, the document*ational approach to didactics*, in the field of mathematics education (Gueudet & Trouche, 2009). This approach has developed in relation to other approaches in the field, in France and internationally. It has resulted in approximately twenty theses, crossed other theoretical frames throughout the development of research programs at the national level (ANR ReVEA in France for example) or international (European projects such as MC2, projects in Argentina, Brazil, China, Lebanon or Senegal). The initial field, mathematics at secondary school, has been expanded from Kindergarten to University, and to other fields of application: languages, biology, chemistry, physics.

Alongside these developments the approach has been enriched by new concepts ("daughter resources", "mother resources", meta-resources, disciplinary affinity, documentational incident, documentational expertise or documentational trajectory); the

Le développement de l'Internet, et, comme conséquences, le foisonnement de ressources numériques et l'émergence de nouvelles formes de travail collectif, conduisent à des bouleversements des formes de l'enseigner et de l'apprendre. Ces bouleversements ont suscité de nouveaux besoins théoriques : comment analyser le travail que les professeurs conduisent pour concevoir la matière de leur enseignement ? Comment penser les relations entre le travail individuel et le travail collectif des enseignants ? Comment suivre, dans la durée, les processus en jeu ?

Ces besoins ont conduit, il y a dix ans, dans le domaine de l'enseignement des mathématiques, à la proposition d'une nouvelle approche théorique, *l'approche documentaire du didactique* (Gueudet & Trouche, 2008). Cette approche s'est développée en relation avec d'autres approches du domaine, en France et au niveau international. Elle a nourri une vingtaine de thèses, a rencontré d'autres cadres de recherche à travers le développement de programmes de recherche au niveau national (ANR ReVEA en France par exemple) ou international (projets européens comme MC2 ; projets en Argentine, au Brésil, en Chine, au Liban ou encore au Sénégal). Le domaine d'application initial, l'enseignement des mathématiques au second degré, a été étendu, de la maternelle à l'enseignement supérieur, et pour d'autres disciplines : langues, biologie, chimie, physique.

methodology of reflective investigation has developed in several directions, in particular for the analysis of collective forms of documentation work. At the same time new questions have emerged, highlighting the need for new research programs. It is to take stock of these advances and questions that a three-day conference is organized in Lyon (France) in May 2018.

The conference was followed by a workshop (31th May and 1st June) dedicated to young researchers.

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Dans cette dynamique, l'approche s'est enrichie de nouveaux concepts (ressources filles – ressources mères, méta-ressource, affinité disciplinaire, incident documentaire, expertise documentaire ou encore trajectoire documentaire) ; la méthodologie d'investigation réflexive s'est développée dans plusieurs directions, en particulier pour l'analyse des formes collectives de travail documentaire. Dans cette dynamique aussi, de nouvelles questions sont apparues, mettant en évidence la nécessité de nouveaux programmes de recherche. C'est pour faire le point de ces avancées et de ces questions qu'est organisée cette conférence de trois jours.

La conférence sera suivie par un atelier de deux jours, le 31 mai et le 1 juin, à destination des jeunes chercheurs.

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Sabrina Chatti (French institute of education), Verônica Gitirana (Universidade Federal de Pernambuco, Brazil), Claire Lucarelli (ENS de Lyon), Anita Messaoui (ENS de Lyon), Takeshi Miyakawa (Joetsu University of Education, Japan), Jean-Pierre Rabatel (French Institute of Education), Maryna Rafalska (French Institute of Education) & Luc Trouche (ENS de Lyon)

Working Groups / Groupes de travail

WG 1. Teachers' resource systems, their structure, their evolution, their mapping
GT 1. Les systèmes de ressources des enseignants, leur structure, leur évolution, leur cartographie
Chairs / Coordinateurs : Jana Trgalova (University of Lyon, France) & Moustapha Sokhna (Université Cheikh Anta Diop, Dakar)

WG 2. Analyzing teachers' work with resources, methodological issues
GT 2. Analyser le travail des enseignants avec les ressources, questions méthodologiques
Chairs / Coordinateurs : Catherine Loisy (ENS de Lyon, France) & Hussein Sabra (Université de Reims, France)

WG 3: Instrumentation, competencies, design capacity, expertise
GT 3: Instrumentation, compétences, capacités à concevoir, expertise
Chairs / Coordinateurs : Sebastian Rezat (University of Paderborn, Germany) & Carole Le Henaff (University of Western Brittany, France)

WG 4: Transitions towards digital resources: change, invariance, and orchestration
GT 4: Transitions vers les ressources digitales : évolution, invariants, et orchestration
Chairs / Coordinateurs : Paul Drijvers (Utrecht University, the Netherlands), Verônica Gitirana (Federal University of Pernambuco, Brazil), and John Monaghan (Agder University, Norway & University of Leeds, U.K.)

Young researchers workshop organizing committee / Comité d'organisation de l'atelier jeunes chercheurs

Chair / Coordination : :
Maryna Rafalska (Institut français de l'éducation)

Mohammad Alturkmani (IFÉ), Cibelle Assis (UFPB, Brazil), Verônica Gitirana (UFPE, Brazil), Anita Messaoui (ENS de Lyon), Takeshi Miyakawa (Joetsu University of Education, Japon), Sophie Soury-Lavergne (IFÉ), Luc Trouche (ENS de Lyon)

Lectures and Panel /
Conférences et Panel

Lecture / Conférence

The construct of 'resource system' as an analytic tool in understanding the work of teaching / La construction du "système de ressources" comme un outil analytique pour comprendre le travail d'enseignement

Kenneth Ruthven (University of Cambridge, UK)

Euclid's ingenious organization of mathematical material from different sources to form the *Elements* created a systematic text which served as a base for the study and teaching of mathematics for more than two millennia. To this day, the idea of creating a coherent system of resources following some concerted approach continues to inform the design of textbooks and curriculum schemes. However, when teachers are encouraged to draw on multiple sources to enrich their teaching, they face the challenge not only of comprehending, but often of reconciling, contrasting approaches, so as to create their own system of resources. In an era of flexible curriculum schemes, of reusable learning objects, and of proliferating mediating technologies, this challenge becomes all the greater. This lecture will examine how this aspect of the work of teaching has been analyzed in recent research, notably the lines of investigation initiated by Ruthven (2009 et seq.) and Gueudet & Trouche (2009 et seq.).

L'ingénieuse organisation, par Euclide, du matériel mathématique de différentes sources pour constituer les Éléments a produit un texte systématique qui a servi de base à l'étude et à l'enseignement des mathématiques pendant plus de deux millénaires. À ce jour, l'idée de créer, à partir d'une approche concertée, un système cohérent de ressources continue d'informer la conception des manuels scolaires et des programmes d'études. Cependant, lorsque les enseignants sont encouragés à s'appuyer sur de multiples sources pour enrichir leur enseignement, ils doivent relever le défi non seulement de comprendre, mais souvent de concilier, des approches contrastées, afin de créer leur propre système de ressources. Dans une ère de programmes de programmes flexibles, d'objets d'apprentissage réutilisables et de technologies de médiation proliférantes, ce défi devient d'autant plus important. Cette conférence examinera comment cet aspect du travail d'enseignement a été analysé dans des recherches récentes, notamment les lignes d'investigation initiées par Ruthven (2009 et seq.) et Gueudet & Trouche (2009 et suivants).

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM*, *45*(5), 633-646.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199-218.

Heath, T. L. (1908). The thirteen books of Euclid's Elements. Cambridge: Cambridge University Press.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Éducation et didactique*, *3*(1), 131-159.

Lecture / Conférence

Studying Teachers' documentation work: emergence of a theoretical approach /

L'étude du travail documentaire des professeurs : émergence d'une approche théorique

Ghislaine Gueudet (University of Western Brittany, France)

The documentational approach to didactics is a recent theory –less than ten years old. In this conference I will look back at the process of development of this approach. I will recall the initial context, motivating the elaboration of a specific frame for studying teachers' documentation work. Several kinds of evolutions will be emphasized: theoretical, methodological, but also evolutions of the questions studied in the research works referring to the documentational approach. I will conclude by evidencing the perspectives for future research and further evolutions.

L'élaboration du cadre théorique de l'approche documentaire du didactique a débuté récemment, il y a une dizaine d'années. Dans cette conférence nous reviendrons sur cette émergence. Nous rappellerons le contexte initial qui a motivé le développement d'une théorie spécifique pour l'étude du travail documentaire des professeurs. Nous nous attacherons à mettre en évidence les évolutions des principales propositions conceptuelles et méthodologiques, ainsi que des questions étudiées dans les recherches se référant à cette approche. Ce regard sur les évolutions passées de l'approche documentaire nous permettra de mettre en évidence les perspectives ouvertes actuellement.

Adler, J. (2000). Conceptualising resources as a theme for teacher education, *Journal of Mathematics Teacher Education, 3*, 205–224.

Guin D., Ruthven K., & Trouche, L. (Eds.). (2005). *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument.* New York: Springer.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.). (2012). *From text to* 'lived' *resources: Mathematics curriculum materials and teacher development*. New York: Springer.

Gueudet, G., Pepin, B., & Trouche, L. (2013). Collective work with resources: an essential dimension for teacher documentation. *ZDM, The International Journal on Mathematics Education*, *45*(7), 1003-1016.

Gueudet, G., Trouche, L. (2009). Towards new documentation systems for mathematics teachers? Edu*cational Studies in Mathematics*71, 199-218.

Pédauque, R. T. (coll.) (2006). *Le document à la lumière du numérique*. Caen : C & F éditions.

Rabardel, P. (2002/1995). *People and technology, a cognitive approach to comtemporary instruments /Les hommes et les technologies: Approche cognitive des instruments contemporains.* Paris: Armand Colin, Retreived on http://ergoserv.psy.univ-paris8.fr/Site/default.asp?Act_group=1

Remillard, J.T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. Re*view of Educational* Research, 75(2), 211-246.

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 52-67). Larnaca: CERME-5.

Trouche, L. (2004). Managing complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281-307.

Lecture / Conférence

What is the reality of teacher's work, behind the design of mathematics exercises? Lessons from the scribal schools, 4000 years ago /

Quelle est la réalité du travail des enseignants, au-delà de la conception d'exercices de mathématiques ? Les leçons des écoles de scribes, il y a 4000 ans

Christine Proust (CNRS, Paris)

How did the teachers of mathematics work four thousand years ago? Surprising as it may seem, education in Mesopotamia is remarkably well documented. The reason of this abundance of sources is the nature of the support of writing that was used in the Ancient Near East, namely, the clay, an indestructible material that has survived to the present day. School exercises, including exercises of mathematics, have been found by tens of thousands by archaeologists in Iraq, Syria and Iran. One can detect how teachers worked through a careful analysis of exercises for beginners and sets of problems for advanced students. I will try to answer some quite precise questions: Who exactly were the teachers of the schools of scribes, what was their social function, what were their sources of income? Did teachers design documents specifically for teaching? Did they use reference texts? Was the knowledge to be transmitted fixed in writing, or stored in the memory of the masters? Did teachers have some freedom of initiative ?

Comment les enseignants de mathématiques travaillaient-ils il y a quatre mille ans ? Aussi surprenant que cela puisse paraître, l'enseignement en Mésopotamie est remarquablement bien documenté. La cause de cette abondance de sources est la nature du support de l'écriture qui était utilisé dans le Proche Orient ancien, l'argile, matériau indestructible qui s'est conservé jusqu'à nos jours. Des exercices d'écoliers, entre autres de mathématiques, ont été trouvés par dizaines milliers par les archéologues en Irak, en Syrie et en Iran. C'est de façon indirecte, par l'analyse minutieuse de ces exercices d'écoliers, ainsi que de séries de problèmes destinés à des étudiants avancés, qu'on peut détecter la façon dont les enseignants travaillaient. Je m'efforcerai de répondre à des questions relativement précises : Qui étaient exactement les enseignants des écoles de scribes, quelle était leur fonction sociale, quelles étaient leurs sources de revenu ? Les enseignants fabriquaient-ils du matériel spécifiquement destiné à l'enseignement ? Disposaient-ils de textes de référence ? Le savoir à transmettre était-il fixé par écrit, ou stocké dans la mémoire des maîtres ? Les enseignants avaient-ils une certaine liberté 'initiative ?

Michalowski, P. (2012). Literacy, Schooling and the Transmission of Knowledge in Early Mesopotamian Culture. In W. S. van Egmond & W. H. van Soldt (Eds.), *Theory and Practice of*

Knowledge Transfer. Studies in School Education in the Ancient Near East and Beyond (pp. 39-57). Leiden: PIHANS.

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Proust, C. (2014). Does a Master always Write for his Students? Some Evidence from Old Babylonian Scribal Schools. In A. Bernard and C. Proust (Eds.), *Scientific Sources and Teaching Contexts throughout History: Problems and Perspectives* (Boston Studies in the Philosophy of Science) (pp. 69-94). New York: Springer.

Proust, C. (2015 (issued 2017)). La chanson des mathématiques dans l'Edubba. In M. C. Bustamante (Ed.), *Scientific Writings and Orality (Archives Internationales d'Histoire des Sciences Vol. 65/2*) (pp. 19-49). Brepols.

Veldhuis, N. (1997). *Elementary Education at Nippur, The Lists of Trees and Wooden Objects,* Ph. D. dissertation, University of Groningen.

Lecture / Conférence

Reflecting on a theoretical approach from a networking perspective : the case of the documentational approach to didactics /

Penser une approche théorique dans une perspective de réseautage : le cas de l'approche documentaire du didactique

Michèle Artigue (LDAR, Université Paris Diderot-Paris 7)

One decade ago, the documentational approach to didactics (Gueudet, Pepin, & Trouche, 2012) was just emerging. Since then, it has developed and matured, benefitting from the collaborative work and contributions of researchers with different research background and educational culture. It has also extended its scope beyond the sole field of mathematics education. The diversity of those who have contributed to the development of this approach has expressed through a diversity of theoretical constructions and combinations. In this lecture, I will reflect on this development from a networking perspective. More precisely, relying on the vision of networking as a research practice at stake in (BiknerAhsbahs & Prediger, 2014) and the concept of 'research praxeology' introduced in (Artigue, Bosch & Gascón, 2011), I will study the influence of this diversity on the research praxeologies developed by the researchers working in that area, the Interactions between these praxeologies, and the outcomes of this research.

Il y a une décennie, l'approche documentaire du didactique (Gueudet, Pepin, & Trouche, 2012) émergeait à peine. Depuis lors, elle a développé et mûri, bénéficiant du travail collaboratif et des contributions de chercheurs aux références théoriques et culturelles différentes. Elle a également étendu sa portée au-delà du seul domaine de l'enseignement des mathématiques. La diversité de ceux qui ont contribué au développement de cette approche s'exprime via une diversité de constructions et combinaisons théoriques. Dans cette conférence, je vais réfléchir à ce développement en me situant dans une perspective de réseautage théorique. Plus précisément, en m'appuyant sur la vision du réseautage en tant que pratique de recherche à l'œuvre dans (Bikner-Ahsbahs & Prediger, 2014) et sur le concept de «praxéologie de recherche» introduit dans (Artigue, Bosch & Gascón, 2011), j'étudierai l'influence de cette diversité sur les praxéologies de recherche développées par les chercheurs travaillant dans ce domaine, les interactions entre ces praxéologies et les résultats de ces recherches.

Artigue M., Bosch, M., & Gascón J. (2011). Research praxeologies and networking theories. In, M. Pytlak, T. Rowlad, & E. Swoboda (Eds), *Proceedings of the Seventh Congress of the European Society for* Re*search in Mathematics Education* (pp. 281-2390). Poland: University of Rzeszów.

Bikner-Ahsbahs, A., & Prediger, S. (Eds.) (2014). *Networking of Theories as a Research Practice in* Mathe*matics* Education. New York: Springer.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.). (2012). *From text to* 'lived' *resources: Mathematics curriculum materials and teacher development*. New York: Springer

Lecture / Conférence

Teachers' Use of Mathematics Resources: a look across cultural boundaries /

L'usage par les enseignants de ressources mathématiques. Un regard traversant les frontières culturelles

Janine Remillard (University of Pennsylvania)

Curriculum resources are adopted in school systems around the world, often, to promote curricular coherence or reform (Pepin & Haggarty, 2001; Valverde *et al.*, 2012). The development and design of these resources varies across systems, as do the expectations for and assumptions about their use (Remillard, Van Steenbrugge, & Bergqvist 2016). Ultimately, teachers' interpretations and decisions are at the center of resource use. In this lecture, I present initial findings from a cross-cultural study of elementary teachers' interpretations and use of mathematics curriculum resources in four cultural contexts, Sweden, Belgium (Flanders), Finland, and the United States. We consider differences in how curriculum resources are made available to teachers, and how teachers make use of them to shape instruction. We also examine the extent to which teachers make use of additional resources, particularly those available through the global marketplace on the Internet.

Les ressources curriculaires sont adoptées dans les systèmes scolaires du monde entier, souvent, pour promouvoir la cohérence ou la réforme des programmes scolaires (Pepin & Haggarty, 2001, Valverde et al., 2012). Le développement et la conception de ces ressources varient selon les systèmes, de même que les attentes et les hypothèses concernant leur utilisation (Remillard, Van Steenbrugge & Bergqvist 2016). En fin de compte, les interprétations et les décisions des enseignants sont au centre de l'utilisation des ressources. Dans cette conférence, je présenterai les résultats initiaux d'une étude interculturelle sur les interprétations des enseignants du primaire et l'utilisation des ressources du curriculum mathématique dans quatre contextes culturels (la Suède, la Belgique (Flandre), la Finlande et les États-Unis). Nous considérerons les différences dans la façon dont les ressources du curriculum sont mises à la disposition des enseignants et la façon dont les enseignants font appel à elles pour façonner les instructions. Nous examinons également dans quelle mesure les enseignants utilisent des ressources supplémentaires, en particulier celles disponibles sur le marché mondial d'Internet.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French, and German classrooms: a way to understand teaching and learning cultures. *The International Journal on* Mathe*matics Education (ZDM), 33*(5), 158-175.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht, the Netherlands: Kluwer.

Remillard, J. T., Van Steenbrugge, H., & Bergqvist, T. (2016). *A cross-cultural analysis of the voice of six teacher's guides from three cultural contexts*. Paper presented at the AERA annual meeting, Washington, DC.

Lecture / Conférence

Teachers collective work inside and outside school as an essential spring of mathematics teachers' documentation: Japanese and Chinese experiences /

Le travail collectif des enseignants à l'intérieur et à l'extérieur de l'école comme élément essentiel de la documentation des enseignants en Mathématiques : les expériences japonaises et chinoises

Takeshi Miyakawa (Joetsu University of Education) & Binyan Xu (East China Normal University, Shanghai)

As opposed to the individualistic perspective that has seemed dominant in teacher's work for teaching the lesson, one may find today a wide range of teachers' collective works inside and outside school in different parts of the world, such as collective improvement of teaching practice in Japanese Lesson Study (Fernandez & Yoshida, 2004) and Chinese Teaching Research Group, and collective writing of mathematics textbooks in France (Sésamath series). In this lecture, we try to show how teachers' collective work promotes documentational works on the resources including teaching material, lesson plan, textbooks, etc. for developing, communicating, sharing, and disseminating teaching practice and knowledge, through two East Asian examples which are comparative and complementary each other: the activities of mathematics teachers' local circle outside school in Japan (Miyakawa & Winsløw, to appear) on the one hand, and Teaching Research Group inside school in China (Wang, 2011; Gu & Wang, 2003) on the other hand.

Contrairement à la perspective individualiste qui a semblé dominante dans le travail des professeurs pour l'enseignement d'une leçon, on peut trouver aujourd'hui un large éventail de travaux collectifs d'enseignants à l'intérieur et à l'extérieur de l'école dans différentes parties du monde, comme l'amélioration collective de la pratique de l'enseignement dans les Lesson studies au Japon (Fernandez & Yoshida, 2004) ou les Groupes de recherche sur l'Enseignement en Chine, ou encore l'écriture collective de manuels scolaires mathématiques en France (Sésamath). Dans cette conférence, nous essaierons de montrer comment le travail collectif des enseignants favorise les travaux documentaires sur les ressources, y compris le matériel pédagogique, le plan de leçon, les manuels scolaires, etc. pour développer, communiquer, partager et diffuser les pratiques et les connaissances didactiques, à travers deux exemples de l'Asie de l'Est qui sont illustratifs et complémentaires l'un de l'autre: les activités des cercles locaux d'enseignants de mathématiques en dehors de l'école au Japon (Miyakawa et Winsløw, à paraître) d'une part, et les groupes de recherche sur l'enseignement à l'intérieur des écoles en Chine (Wang, 2011; Gu & Wang, 2003) d'autre part.

Gu, L.Y. & Wang, J. (2003). Teachers' Professional Development in Action Education. *Curriculum-*Text*book-Pedagogy*, *1*(2), 2-10.

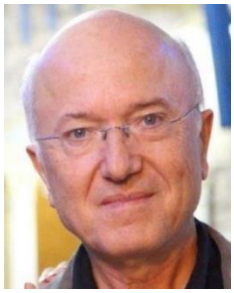
Fernandez, C. & Yoshida, M. (2004). *Lesson study – a Japanese approach to improving mathematics teaching and learning*. Mahwah: Lawrence Erlbaum.

Miyakawa, T. & Winsløw, C. (to appear). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: a case study of "practice research" in Japan.

Wang, J.P. (2012). *Mathematics Education in China: Tradition and Reality*. Singapore: Cengage Learning Asia Pte Ltd.

Lecture / Conférence

Evidencing the missing resources of the documentational approach to didactics, towards new programs of research / La mise en évidence de ressources manquantes de l'approche documentaire du didactique comme fondement de nouveaux programmes de recherche



Luc Trouche (French Institute of Education, Ecole normale supérieure de Lyon, France)

The development of a theoretical approach creates necessarily new needs, conceptual as well as methodological: when meeting new phenomena; when extending the study outside its initial field (in this case mathematics teaching in French secondary schools); when meeting other theoretical frames. Evidencing its missing resources (Chevallard & Cirade, 2010) is then a critical issue for a community of research. We will propose to face this issue, developing a reflective investigation in two directions: asking the recent Ph.D. students having situated their research into the DAD; examining the questions of *denomination* (Rousseau & Morvan, 2000) arising from crossing linguistic, curricular and cultural borders (Libbrecht *et al*., 2008; Pepin *et al.*, 2013), particularly in the Chinese case (Pepin *et* al., 2016). We will draw from this investigation some programs of research for supporting the development of the documentational approach to didactics.

Le développement d'une approche théorique crée nécessairement de nouveaux besoins, tant conceptuels que méthodologiques: lors de la découverte de nouveaux phénomènes; lors de l'extension de l'étude en dehors de son champ initial (ici, l'enseignement des mathématiques dans les écoles secondaires françaises); lors de la rencontre d'autres champs théoriques. L'évidence de ces ressources manquantes (Chevallard & Cirade, 2010) est alors une question cruciale pour une communauté de recherche. Nous proposerons de penser ces questions dans deux directions : en interrogeant des étudiants ayant récemment soutenu un doctorat dans ce domaine ; en examinant les questions de dénomination (Rousseau et Morvan, 2000) découlant de la traversée de frontières linguistiques, scolaires et culturelles (Libbrecht et al., 2008 ; Pepin et al., 2013), en particulier dans le cas chinois (Pepin et al., 2016). Nous tirerons de cette étude la nécessité de programmes de recherche pour soutenir le développement de l'approche documentaire du didactique.

Chevallard, Y., & Cirade, G. (2010). Les ressources manquantes comme problème professionnel. In G. Gueudet, & L. Trouche, L. (dir.), Ressources *vives. Le travail documentaire des professeurs en* mathématiques (pp. 41-55). Rennes: PUR.

Libbrecht, P., Desmoulins, C., Mercat, C., Laborde, C., Dietrich, M., & Hendriks, M. (2008). Cross-Curriculum Search for Intergeo. *Lecture notes in computer science* 5144/2008, 520-535

Pepin, B., Gueudet, G., & Trouche, L. (2013). Investigating textbooks as crucial interfaces between culture, policy and teacher curricular practice: two contrasted case studies in France and Norway, *ZDM, The International Journal on Mathematics* Education, 45(5), 685-698

Pepin, B., Xu, B., Trouche, L., & Wang, C. (2016). Developing a deeper understanding of *mathematics teaching* expertise: Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational studies in Mathematics*, *94*(3), 257–274, http://rdcu.be/koXk.

Rousseau, D., & Morvan, M. (2000). *La dénomination. Le temps des savoirs*, 1. Paris : Odile Jacob

Panel / Panel

Mathematics Teachers as Designers / Enseignants de Mathématiques comme designer

Chair / Coordinatrice :

Birgit Pepin (University of Utrecht - Netherland)

Participants :

Michele Artigue (France); Verônica Gitirana (Brazil); Takeshi Miyakawa (Japan); Kenneth Ruthven (UK), & Binyan Xu (China).

The goal of the panel discussion is to develop deeper understandings of the processes involved when teachers act as designers (Pepin, Gueudet, & Trouche, 2017). In this panel discussion the participants will respond to each of the following questions:

- Why are teacher design activities relevant? (e.g. practical and/or scientific perspective)
- What are the most challenging/interesting design tasks? What would teachers design? (e.g. actions and/or artefacts)

How would teachers design? (e.g. What kinds of design approaches would they use?; Would they design individually or collectively?; Under which conditions would they design, with who m, short-term or long-term?)

L'objectif de la table ronde est d'approfondir la compréhension des processus impliqués lorsque les enseignants agissent en tant que concepteurs (Pepin, Gueudet et Trouche, 2017). Dans cette table ronde, les participants répondront aux questions suivantes:

- Pourquoi les activités de conception des enseignants sont-elles pertinentes? (d'un point de vue pratique et / ou scientifique)
- Quelles sont les tâches de conception les plus difficiles / intéressantes? En quoi les sont-ils des concepteurs? (par exemple, en termes d'actions et / ou d'artefacts)
- Comment les enseignants devraient-ils concevoir ? (Par exemple, quels types d'approches de conception s'agirait-il d'emprunter ? Individuellement ou collectivement ? Dans quelles

After each participant has responded, the audience will be invited to join the discussion.

conditions concevraient-ils, avec qui, à court terme ou à long terme ?)

Après cet échange à l'intérieur de la table ronde, la discussion sera élargie à l'ensemble des participants

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM- Mathematics* Education, 49:799–812

Working groups / Groupes de travail

WG1. Teachers' resource systems, their structure, their evolution, their mapping / GT 1. Les systèmes de ressources des enseignants, leur structure, leur évolution, leur cartographie

Chairs/ *Coordinateurs*:

Jana Trgalova (University of Lyon, France) & Moustapha Sokhna (Université Cheikh Anta Diop, Dakar)

An important facet of teachers' work, done outside the classroom, consists in searching for, selecting and gathering resources for lesson and assessment preparation, for inquiring about institutional requirements and constraints or for professional development. The working group focuses on issues related to this aspect of teachers' work. The following sets of questions can be addressed:

- How do teachers constitute their resource systems? How do they search for resources and where do they get them from? What criteria do they (explicitly or implicitly) use to select new resources? How are these new resources integrated into the existing resource system? What are teachers' representations of "good" resources?
- Do these representations differ according to the subject matter (mathematics, physics…), the school level (primary, secondary, tertiary), the type of activities (lesson preparation, assessment...) or other factors?
- How do the teachers structure their resource systems? Are there central / pivotal resources and what is their role? What is the place of curricular resources (prescribed curriculum) and textbooks in teachers' resource systems?

Une facette importante du travail des enseignants, en dehors de la classe, consiste à rechercher, à sélectionner et à rassembler des ressources pour la préparation des cours et des évaluations, pour s'informer sur les exigences et les contraintes institutionnelles ou pour leur développement professionnel. Le groupe de travail se concentrera sur les questions liées à cet aspect du travail des enseignants. Les questions suivantes seront abordées :

- Comment les enseignants constituent-ils leurs systèmes de ressources ? Comment cherchent-ils des ressources et d'où les obtiennent-ils ? Quels critères utilisent-ils (explicitement ou implicitement) pour sélectionner de nouvelles ressources ? Comment ces nouvelles ressources sont-elles intégrées au système de ressources existants ? Quelles sont les représentations des enseignants des « bonnes » ressources ?
- Ces représentations diffèrent -elles selon le sujet (mathématiques, physique ...), le niveau de l'école (primaire, secondaire, tertiaire), le type d'activités (préparation de la leçon, évaluation ...) ou d'autres facteurs ?
- Comment les enseignants structurent-ils leurs systèmes de ressources ? Existe-t-il

- How do teachers' resource systems evolve? What are the levers of their evolution (experience, cultural environment, curricular changes, teachers' participation at professional development courses….)?
- What are the available resources (for a given subject matter, school level…)? How can these resources be mapped to learning and/or teaching objectives? How do these resources evolve with changes in society (rapid development of digital technology…) or in education (curricular changes and reforms…)?

des ressources centrales et essentielles et quel serait leur rôle ? Quelle est la place des ressources curriculaires (curriculum prescrits) et des manuels dans les systèmes de ressources des enseignants ?

- Comment les systèmes de ressources des enseignants évoluent-ils ? Quels sont les leviers de leur évolution (expérience, environnement culturel, changements scolaires, participation des enseignants à des formations continues ...)
- Quelles sont les ressources disponibles (pour un sujet donné, le niveau scolaire ...) ? Comment ces ressources peuvent-elles être mises en correspondance avec les objectifs d'apprentissage et / ou d'enseignement ? Comment ces ressources évoluent-elles avec les changements dans la société (développement rapide de la technologie numérique ...) ou dans l'éducation (changements et réformes curriculaires ...) ?

WG1. Oral Communications
GT1. Communications Orales

The Metamorphosis of resource systems of prospective teacher: from *studying* to *teaching*[*]

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Within the theme "Teachers' resource systems, their structure, their evolution, their mapping" proposed by the Working Group 1, we are particularly interested in how teachers constitute their resource systems at the beginning of their careers. In order to answer it, we analyze the cases of two prospective teachers, focusing on the events occurring during an initial teacher training course at a Federal University in Brazil. Despite this research is still in development, we will support the hypothesis that their "resource systems for teaching" come from what we name "metamorphosis" of their "resource systems for studying". Our study is comprised in four phases: studying teachers' documentational trajectory; analyzing Geogebra usages in studying vs. teaching situations, and the prospective teachers' interview. Our results allow us to better situate the notion of the metamorphosis of a resource system.

Keywords: Documentational approach to didactics, resources, resource system metamorphosis, teacher education program, mathematics teaching.

Introduction

A preservice teacher education program, according to Ponte and Brunheira (2001), needs to take into account all dimensions of professional knowledge and has to consider the way prospective teachers may develop them. They point out, for example, adequate preparation both in the subject they will teach and in its teaching methods, pupils' learning processes and recognise the influences of sociocultural backgrounds, in addition to knowing the critical features of the mathematics curriculum, and knowing how to prepare and conduct a lesson and evaluate pupils. According to them, preservice teacher education must provide student teachers with the basic knowledge they need to begin their professional activity.

As an important element of this process, we consider the constitution, structure and evolution of teachers' resource system. The understanding of this perspective is, essentially, the understanding of the core of teachers' professional development (Gueudet & Trouche, 2009).

In our study, we propose to answer the following research question: *how do prospective teachers develop their resources for teaching?* We present the hypothesis that they performed a "metamorphosis" of their "resource systems for studying" into "resource systems for teaching".

We structure this proposition in four sections. In the first section, we situate our theoretical proposition in the frame of Documentational Approach to Didactics - DAD (Gueudet &Trouche, 2009) and Documentational Trajectories (Rocha & Trouche, 2017). In the second one, we present

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our associated methodological choices. In the third one, we present our data collection and our first analyses. Finally, we give perspectives of follow-up that would be considered in the next months.

Documentational Approach to Didactics in our research

In this section, we introduce our own propositions according to DAD, referring mainly to the concepts of resources and documents, system of resources, and documentation work introduced by Gueudet and Trouche (2009) and *documentational trajectory* introduced by Rocha and Trouche (2017).We introduce the notions of "resource system for studying", "resource system for teaching" and "metamorphosis" of the resource system, allowing to conceptualize the way *prospective teachers constitute their resource systems*, with respect to the context of the teacher training course.

Based on Besnier (2016), we distinguish two main classes of situations for prospective teachers: *studying situations* and *teaching situations*. The studying situations comprise experiences related to studying any subject during the course, for example: mathematics, specific software or how to prepare a lesson plan. The teaching situations consider experiences at university or at school, with colleagues or students as a target. Corresponding to these two classes, for a given prospective teacher, we have: resource system for studying (RSS) and resource system for teaching (RST). RSS considers everything that support prospective teacher's studying activity and, RST, the resources that support her/his Mathematics teaching.

We consider that both set of resources have their own structure that allows naming them a *resource system*. In addition, there is coherence between them that led us to characterize the changes from RSS to RST as a metamorphosis. We analyze these changes following three dimensions: 1) the *content* which considers material resources; 2) the *structure* that reflects a new relation among the resources and (re)establishment of their roles; 3) the *nature* which comprises the mobilized knowledge from a teaching situation in order to produce resources for teaching.

The associated methodological choices

In this section, we present our subjects, and our focus on Dynamic Geometry – DG. Finally, our contract and the methods of collecting data considering a qualitative study based on two cases.

Cecilia and Marcos were chosen because they developed some projects and subjects during their teacher training offered by a Brazilian federal university in which DG and Geogebra software were studied and applied. Therefore, the DG was previously identified as an element of Cecilia and Marcos's RSS and RST.

Among these experiences that they developed with us, we worked as supervisors in the semester when they produced their *TCC* reports (from April to June in 2016). The *TCC* report - *Trabalho de Conclusão de Curso* - or final essay, is a mandatory report made by each prospective teacher during the last year and, in their case, they described a teaching experience undertaken at school.

We carefully specified what they were expected to do as a *methodological contract* (Sabra 2016): in the TCC period, we worked together in order to develop their own initial choices (the schools, the contents and the grades) and their own initial ideas of teaching with DG. Despite having advised them to use specific resources (the textbook adopted at school, curriculum resources and some

articles about DG) we did not specify what or how to use them. In addition, we did not limit the use of any resource. They have also agreed to entrust us with their recordings, their archives, to film their lessons at school, or to be interviewed by us from the beginning of the research in 2016 until the end, in 2018. The combination between reflective data, from prospective teachers, and our inferences allowed us to map their *documentational trajectories* and their system of resources.

Our research methodology was organized in four phases. Firstly, we asked them to build up their documentational trajectory followed by an interview about each documentational trajectory. It was recorded on audio and video and happened in November 2016. Next, we asked them to send us, by email, some proposals that were previously developed with Geogebra, for studying and for teaching, during their course. This data collection was done in July 2017. We intend to take new interviews considering each presented proposals, including the data about proposal of TCC report.

The data and first analyses

In this section, we present some data and first analyses considering Cecilia's case. First, we consider her documentational trajectory, and then, her proposals developed with Geogebra. As a result, we point out some aspects related to *content*, *structure* and *nature* dimensions.

Concerning the documentational trajectory, Cecilia mentioned events from 2011 until 2016, period of her under graduation. These events and associated resources were mentioned as important for her professional development and gave us a general perspective of classes of situations.

She mentioned events related to experiences of teaching and studying mathematics, studying a subject related to teaching mathematics and studying technologies in order to teach mathematics. Naturally, Geogebra, as a resource, was present in most events. The experiences of teaching were sourced by the work on and with particular resources (lesson plan model, textbooks, curriculum resources) and from experiences developed at school and at university (from stages and projects with students and teachers or with colleagues and educators).

Considering Cecilia's proposals with Geogebra, we received 27 files with .doc, .ppt and .ggb extensions, that comprised tasks on mathematical contents related to fundamental and high school levels. We identified three types of tasks: done by her in order to study mathematics or to study about Geogebra itself, and mathematics tasks proposed by her with Geogebra.

From this set of data, we represented the *content* of her RSS and RST, respectively, from the following intertwined classes of situations: *to accomplish a task with dynamic geometry* and *to create a task with dynamic geometry.* Regarding these classes, we organized the resources according to blocks of activities (Besnier, 2016). For the first class of situation, the blocks are: *studying mathematics*, *following a worksheet*, *using dynamic geometry* and *using the Geogebra tools*. For the second one, the blocks: *conceiving a mathematical task*, *structuring a support for the task*, *creating a worksheet*, *using Geogebra tools* and *using dynamic geometry*.

A first analysis on Cecilia's metamorphosis considers some changes under the *content* dimension. We have identified *professional resources,* new resources for her integrating her SRT: some Brazilian material curriculum resources and some articles about experiences of teaching with DG

and models of lesson plans. These resources have sourced the *creating the tasks with DG* acting in the blocks *conceiving and structuring a support for the task* and *creating a worksheet*.

Considering a change under the *structure* dimension, we have observed the *transitional resources*. It means, resources for *studying* that became resources for *teaching mathematics with DG* through the reestablishment of roles within the SRT. In fact, the textbooks and books, and Geogebra were used for *studying mathematics* and they became resources to *conceive* and *to structure a support for the task*. In addition, the worksheets turned the support for studying into a support for teaching.

Therefore, we have observed that previous tasks (*accomplished* or *proposed with DG*) were resources for *creating a new task*. In these cases, we identified some uses of the DG (drawing and figure) and Geogebra tools (slider, graphic representations and dynamic text). We consider this dispositive of re-producing elements of GD with associated Geogebra tools as a dispositive for creating, once it allowed her to produce tasks differently. We infer that the *nature* dimension emerges from these situations where knowledge is mobilized to produce each new task.

Perspectives

Before considering Marcos's case, we will analyze Cecilia's data related to *creating a task about triangle classification with DG* that happened during the *TCC* event and also her implementation in class. We intend to infer mobilized knowledge to develop this particular set of activities and make associations with previous tasks in order to characterize the *nature* dimension. We will study this dimension considering analyses of *professional schemes* (Goigoux & Vergnaud, 2005).

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Analysing Teacher Candidates' Evolution into Teachers through Documentational Approach

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Abstract: *In this study, we aim to analyse prospective mathematics teachers' evolution through "Documentational Approach of Didactics". We use "reflective investigation" in order to follow the process closely. We study with two prospective mathematics teachers that take "Computer-aided mathematics teaching" course. As the steps of the research, first, we analyse the prospective teachers' lesson plans. Then, we observe their planned lessons. After the lessons, we interview with the prospective teachers to discuss how they realise their teaching, their thoughts and their resource system that shape their lesson. As a part of the interview they draw schematic diagram of their resource system and tell the details of the resource system. After a year, when they graduated and become actual teachers, we ask them to observe a lesson they teach, and we realise a recall session about the factors that affect their teaching. As a result, we find out that there is a difference between before and after the teaching experience about revising the lesson according to students' understanding and ability to re-shape it according to classroom context.*

Keywords*: Prospective mathematics teachers, professional development, reflective investigation.*

Introduction

Teachers as guides to students for finding the way to knowledge, have a very important role in every branches of education. Day by day, the burden of this guidance goes up according to several changes and improvements in the world. Teachers have to keep up with the changes in all categories of life and have to learn how to prepare students to life (MEB, 2017). For this reason, they invent some *document*s for their lessons and conduct their lessons according to these documents. These documents are also indicators for the teachers' professional development (Tanışlı, 2013). In this context, we think that the documents a person built can show his/her evolution professionally.

Purpose of the Study

The documents teachers created give clues about their approaches to mathematics teaching. Some researches show that teacher candidates revise their knowledge about teaching, after they start teaching as a job (Altun, Yazgan and Arslan, 2004). Even if they learn most of the teaching methods at the faculty, they confess that they use just one or two of them while teaching (Baştürk-Şahin, 2014). In this study, we aim to see the scope of the changes that happen while being a teacher. As sub- aims: (1) We aim to see the schemes that teacher candidates have before they become a teacher. (2) The purpose of the study is to analyse teacher candidates' resource systems and their evolution into teachers. (3) In this context, we aim to make a contribution to the theoretical framework on understanding the genesis of the resource system.

Theoretical Framework: Documentational Approach of Didactics

In this study, we utilise "Documentational Approach of Didactics" as a theoretical framework (Gueudet & Trouche, 2009). We aim to comprehend the elements of schemes that the teacher candidates have and the changes that occur in the process of being a teacher. That's why "Documentational Approach of Didactics" is a very appropriate framework for the study.

Methods and Procedures

In order to discern the elements of schemes that teacher candidates have, we utilise qualitative research methods. We designed this study as a case study, because we aim to have a deeper understanding of the process. We studied with two volunteer teacher candidates: Esin and Melodi. They study in the educational faculty and they both take the "Computer-aided mathematics teaching" class. We utilise "reflective investigation" as suggested in theoretical framework (Gueudet & Trouche, 2009). Reflective investigation is a detailed kind of investigation both in and out of the classroom. By using reflective investigation, we aim to get better information about the genesis and evolution of the resource system.

Firstly, we tell the whole class to prepare a lesson plan, about a unit of mathematics that they want to implement in their own class. We ask them to prepare the plan highly detailed and applicable, that means we want their lesson plans to be realistic and appropriate for primary school students. Esin and Melodi are so eager to prepare a lesson that they are the first students to come and make the professor control their lesson plans. After a week, they implement their lesson plans to their friends, that play role as primary school students. We observe and video-record their lessons to inhibit data loss. After the observation, we discuss the interesting parts that we want to get detailed information about their lesson. And we prepare the interview form and audio-record the interviews. With the interview, we get detailed information about the lesson plan and their opinions about implemented lesson. While we interview with them, we asked them to draw their schematic representation of resource system and explain it. The schematic representation has no true version, the representation is just a diagram that shows their resources and how to use them.

After a year, when they become teachers, we asked them to observe their lessons again and we interview with them about their resource system. We record the new observations and interviews, too. This session is recall session and in this session, we want to see their own interpretations about their own teaching experience.

Data Analysis

We transcribe the data of observation and interview, verbatim and coded them as suggested for qualitative researches (Patton, 2014). We analyse lesson plan and schematic representations, by document analysis technique.

Results and Conclusion

The results of cross over study on teacher candidates' resource system and the usage of the resources are presented below in terms of three main categories: (1) Institutional factors, (2) general resources, (3) factors about teaching in the class. We examine these categories and their sub-categories before and after becoming a teacher, shown in Table 1.

Categories	Sub-Categories	Before the Teaching Experience	After the Teaching Experience
Institutional factors	-Private School -Public School	Esin works in private school. Melodi works in public school.	
General resources	-The style of collection	-Melodi had a very full representation diagram of her resource system. -Esin had a very little representation diagram of her resource system.	-Melodi uses less than resources she represented before. -Esin use the resources that shared in the school.
	-Sharing resources with colleagues	-Melodi was willing to share her resources and she was using digital resources to share resources with other teachers or teacher candidates. -Esin was willing to share her resources with other teacher candidates only.	-Melodi is willing to share her resources with her colleagues. -Esin is willing to share resources with her school colleagues, not the other teachers.
	-Criteria of choosing auxiliary resources	-Melodi and Esin were choosing their auxiliary resources according to their visual quality.	-Melodi and Esin are choosing their auxiliary resources according to their appropriateness to exam system.
	-Sharing resources with students	-Melodi and Esin were willing to share her resources with her students.	-Melodi works at a public school which sharing resources are strictly forbidden. -Esin works at a private school which shares all the resources with the students.
	-Teaching arrangement	-Melodi was open to digital resources and digital technologies	-Melodi uses the technology in her classroom.

		using in the teaching. -Esin was anxious about the technology in mathematics class, and were thinking that this can only be visually.	-Esin uses technology in her class.
Teaching in the class	-Ability to revise the plan	-Melodi and Esin were very anxious about students' questions. They could not revise their lesson plan while teaching their own friends.	-Melodi and Esin are much relaxed and they mention that even if they can not revise the plan, they can slow down or fasten the lesson plan according the understanding of the class.
	-Choice of exercise type and number	-Melodi and Esin were thinking the lesson's duration while deciding the number of exercises.	-Melodi and Esin are taking into consideration the students' understanding and they predict student's thinking styles much better than before.

Table 1. The results of the cross-over study on the differences between "before and after" becoming a teacher

As shown in the table, we can mention that the institution type is an effective factor on teachers' sharing their resources both with their colleagues and students. We can mention that the teacher candidates are taking more responsibility on their lesson plans, they are closer to revising their lessons. They can understand students' understanding better and they can predict their students' weaknesses and they can plan their lessons according to them. Exam system of the country, effects their teaching styles even in their first year, which we were expecting the effects of prior knowledge of the faculty classes.

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Analysis of Primary Mathematics Teachers' Lesson Document Preparation Processes

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Abstract: *The question of how teachers built their lessons, as one of the most important elements of learning and teaching process, defines the starting point of this study. In this way, it seems precious to enlighten the documents that teachers built in their entire teaching experience. In our opinion, the document a teacher built is a strong resource because it indicates teacher's teaching and lesson preparing schemas. In this study, it is aimed to reveal the elements of the schemes and factors that affect the lesson document preparation process. "Documentational Approach of Didactics" is utilized as a theoretical framework (Gueudet & Trouche, 2009). Qualitative research methods are used. The study is designed as a case study and the participants are four collaborative and willing primary mathematicsteachers.*

Keywords: Lesson document preparation, documentational approach of didactics.

Introduction

Teachers as guides to students for finding the way to knowledge, have a very important role in every branches of education. Day by day, the burden of this guidance goes up according to several changes and improvements in the world. Teachers have to keep up with the changes in all categories of life and have to learn how to prepare the students to life (MEB, 2017). That's why they invent some documents for their lessons and conducts their lessons according to these documents. Therefore, both the documents and their preparation processes gain importance day byday.

Purpose of the Study

Examining teachers' documents seems important in mathematics education, because the documents teachers' created, demonstrates their approaches to mathematics teaching. Some researches show that teachers not only use public textbooks that are given students for free, but also they use auxiliary books (Altun, Yazgan & Arslan, 2004). However, how teachers decide to use these auxiliary books, how they evaluate them and how they reflect their content to their lessons is shadowy. In this regard, it seems important to examine teachers' document preparing processes. The aim of this research is analysing the elements of the schemes that generated during the preparation process of the lesson documents. In this study, while examining the preparation of documents, the effects of classroom context to documents are taken into account. Because it is thought that teachers can transform their documents according to students'feedback.

Theoretical Framework: Documentational Approach of Didactics

In this study, "Documentational Approach of Didactics" is utilized as a theoretical framework (Gueudet & Trouche, 2009). It is used to comprehend and explain the elements of schemas and factors that affect the document preparationprocess.

Studies that investigates mathematics teachers, generally focuses on teachers' practices, knowledge and beliefs separately (Perrin-Glorian, De Blois ve Robert, 2008). In contrast "Documentational Approach of Didactics" allow for studying on all these concepts together with their evolution and improvement. That's why "Documentational Approach of Didactics" seemed appropriate for the study.

Methods and Procedures

In order to discern the elements of schemes, qualitative research methods are used. The study is designed as a case study. 10-year-experienced, voluntary and collaborative four mathematics teachers' document preparation processes are examined. All participants were working at public schools in Turkey. The participants were known as the consultant teachers for the teacher candidates and they were eager to share their experience with them. So, it is thought that they are appropriate for this study.

Reflective Investigation and Data Collection Tools

While examining the process, reflective investigation is utilized, as suggested in theoretical framework (Gueudet & Trouche, 2009). Reflective investigation is a long term, detailed type of investigation. Main principles of reflective investigation played an important role on choosing it as methodology: Long-term follow-up, in-and-out-of-class follow-up, broad collection and reflective follow-up. Thinking the features of reflective investigation, the method seemed to allow getting better information about the document preparation process.

Correspondingly, several data collection tools are used. First, a *personal statement questionnaire* is applied to participants. Then, in order to follow the teachers both in and out of class, a semi structured diary is given. The format of the diary is taken and translated from the original one from Gueudet and Trouche. Then, a *schematic representation of resource system* is asked to participants. After then, a pre-interview is done to understand teachers' views about the subjects that they will implement in the class. Then, the preparation of the lesson is done together with the teachers separately. After then, the lesson is observed and researcher notes are taken. And, a small interview is done about the interesting points of the class just after than the observation. After one or two months, a recall session is done to make them remember and make them conscious about how they behave and how they implement their documents in the classroom. Participants were asked to make an interpretation about possible changes in their documents and how they woulddo them.

Data Analysis

Observation and interview data are transcribed verbatim and analysed as suggested for qualitative research analysis (Patton, 2014). Personal statement questionnaires and schematic representation of resource systems are analysed by document analysis technique (Yıldırım & Şimşek, 2006). Observation and interview data are analysed by content analysis technique. All the data is coded and categories are offered.

Results and Conclusion

First of all, it is seen that teachers filled the diary just like a lesson notebook in Turkey. It is thought that the "lesson notebook" concept in Turkey may cause this confliction. The display of the diary is similar to lesson notebook that teachers fill in every day in the class. But in the literature, same results are reported that people have a struggle using the diary as a data collection tool (Bolger, Davis and Rafaeli, 2003). Because the diary is kind of a subjective tool, people may match them with other subjective materials they use every day. This result may help further studies to be careful for preparing the diary. That's why this tool is eliminated from the data analysis, in this study.

The results of cross over study on mathematics teachers' lesson document preparation processes and the pre-interviews are presented in terms of five categories: (1) Professional background, (2) General resources, (3) Local resources, (3) Institutional factors, (4) Factors about exam system, (5) Factors about teaching in the class. The categories and sub-categories are shown in Table1.

Categories	Sub- Categories
Professional background	Department
	Experience
General resources	The style of collection
	Sharing resources with colleagues
	Criteria of choosing auxiliary resources
	Sharing resources with students
	Homework routine
	Teaching Arrangement
Local resources	Criteria of choosing resources
	Critical points of the subject
	Methods of the subject
Institutional factors	Community decisions
	Physical conditions
Factors about exam system	Teaching method
	Teaching duration
	Choice of exercise
	Choice of exam question
Factors about teaching in the class	Choice of exercise type and number

Table 1: Categories of the lesson document preparation processes and pre-interviews of teachers

According to data obtained from interviews and observations, some elements that can possibly be schemas of teachers, are found. The elements that teachers take into consideration while preparing the documents are as follows:

- Taking care of students learninglevels
- Taking advantage of prior teachingexperience
- Sharingresources
- Homework from both textbooks and auxiliarybooks
- Choosing resources according to importance of thesubject
- Paying attention to mathematicalnotation
- Taking decisionsindependently
- Directing the teaching according to examsystem
- Making reification according to classroomeffects

All elements of schemas and factors are classified as inner and outer factors. Inner factors are the factors that belong only to the teacher and teachers' character. Inner factors above, are; taking care of students' learning levels, taking advantage of prior teaching experience, taking decisions independently, making reification according to classroom effects. Outer factors are the factors that affect teacher's choices, for example, country's national education strategies, exam system and school instruction methods. Outer factors above are; sharing resources, homework from both textbooks and auxiliary books, choosing resources according to importance of the subject, paying attention to mathematical notation, directing the teaching according to exam system. In this context, it can be said that inner factors are more or less the same as other researches' findings (Ruthven, 2013). However, outer factors are changing because of different educational cultures of countries, for example, exam systems.

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Discussing digital artefacts under design as a process aiming towards the centre of TPaCK

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The study addresses the illumination of TPaCK knowledge of postgraduate mathematics education students and their progression towards the TPaCK centre by means of a threaded forum discussion concerning their on-going designs of digital artefacts for pupil meaning-making. We perceived those digital artefacts as living documents under change and studied both students' constructions and their written exchanges to identify how their initial ideas were placed in the TPaCK model and how these could progress. We studied the exchanges between them and their instructors as a boundary crossing process considering the artefacts as boundary objects. Our preliminary results show that the process was effective in helping us with our understanding of both how the students started and how they were challenged to progress towards TPaCK intersections and centre.

Keywords: Professional development, instrumental approach, boundary crossing, turtle geometry.

Theoretical Framework

This study addresses how to understand and support the process by which mathematics majors, studying mathematics education, jointly design, construct and discuss over half-baked digital artefacts (Kynigos, 2007) aimed at generating constructionist learning in their prospective students Kynigos, 2012). We use the term 'half-baked' digital artefact to discuss an artefact didactically engineered to be faulty or incomplete and mediated as such to the students. Typically the students are required to identify its properties and in particular those requiring change, make those changes and come up with a mathematically sound model. After doing so, the students are encouraged to build a construction of their own using the correct model as a building block. So, our Masters' students were acting as teachers designing such artefacts. We employed TPaCK (Mishra & Koehler, 2006), Documentational Instrumentalization (Guedet & Trouche, 2009, Kynigos & Psycharis, 2013) and Boundary Crossing (Kynigos & Kolovou, in press) in an integrated way in order to focus both on illuminating the 'placement' of our students' thinking on the TPaCK model (Technological, Pedagogical and Content Knowledge of a math teacher as circles having overlaps in a Venn Diagram) and on ways in which they can be supported to progress towards the centre of the model. That is, we identified the students' arguments to be situated mostly at the T and C parts of the Venn diagrams of TPaCK and intervened during their discussion to support the inclusion of P and their progression to the two-way intersections en route to the centre. So far, not so much attention has been paid to the instrumentalization process in the Documentational approach (Pepin,Gueudet&Trouche 2013, Guin,Ruthven&Trouche, 2005)., i.e. how a living document - in the form of digital artefact - involves the reciprocal shaping of the TPaCK and the document (in this case the half-baked artefact), affordances. In our work with our students we wanted to understand

the process by which the changing affordances became the focus over which ideas were expressed and developed towards the centre of TPaCK. The on-going study involved twelve mathematics teachers and two researchers. Most of the teachers had recently graduated from Mathematics Departments in Universities in Greece. Very few of them had experience in teaching mathematics in a classroom, they had mostly acted as private tutors with high-school and junior high-school students preparing candidates for University entry exams. The study took place during the "Pedagogical Utilization of Digital Media in Mathematics" course, which was given as a part of the University's postgraduate studies in Mathematics Education. The two researchers operated in the capacity of giving the course, the senior (academic) researcher in the role of course instructor and the junior PhD candidate and acting teacher in the role of supporting tutor. It is important to say that the latter had previously followed the same course and had hence gained further experience in the design and the use of these media in classroom. As an assignment of the course, teachers in pairs had to jointly design a digital model (artefact) for teaching mathematics, post it in a threaded discussion forum, where only the members of the class had access, and then engage in a critical friend style discussion of each others' artefacts. As a result, there should be the final versions of six artefacts, therefore six parallel threaded discussions. Teachers were also supposed to make comments on each threaded discussion. The two researchers also took the role of critical friend who assist the students with their posts in the thread discussion. During the semester, three such assignments were to take place; each one consisted of another phase of the study. In this paper, we are presenting some early results of the study's first phase. The first assignment for the teachers' pairs was to design a half-baked representation of the capital letter "N" in MaLT2. MaLT2 is a web-based Turtle Geometry environment that affords programmable Logo-based Turtle Geometry in 3D space (hence the avatar is now a 'sparrow' instead of 'a turtle') uniquely integrated with dynamic manipulation of variable values by means of a uni and a bi dimensional variation tool (see Kynigos, 2012). We chose "N", because of the plentiful mathematical concepts that can be integrated in several versions of its construction (Psycharis & Kynigos, 2009).Teachers should design and post the relative Logo program along with a text, as an exercise, asking students what to do with it and explain in a separate text, around what kind of mathematical concepts were their prospective students expected to make meanings during their effort to fix the bug of the specific "N" which each pair half-baked.

Results and discussion

The first phase of the study lasted three weeks. During the first week, most of the groups posted their Logo programs. The discussion started at the end of the first week. By the end of the third week, 77 posts were totally submitted. The most commented object had 24 posts, while the less commented had 8 posts. In this paper we focus on a turn of the discussion under a certain thread.Theextractthat follows is a discussion between a teacher (T1) and one of the researchers (R, the school teacher). The discussion was about the half-baked "N" of T1's group. When running the Logo program, which used a variable "x", the bird forms an ill-constructed N. The bird moved executing sequentially the commands: forward x, right 150 degrees, forward x/cos(150), left 30 and forward x, for a certain value of x. The purpose, according to T1 was to find out that the amount of turn and the argument of cosine must be supplementary angles, in order the N to be well-shaped.

R: If a student fixes the bug, through trial and error, without using the property of supplementary angle or trigonometric numbers, should it be accepted?

T1: But we want to realize it, through the use of Geometry, not trials.I would ask “If the turn changes from 150 to 145 what would be the argument of cosine?”

R: Is it possible that a student, will answer “it will change from 30 to 35, so what?

T1: I think you are right. In this case we should add an extra activity, asking the student to construct “N” by using sine instead of cosine.

The idea of replacing cosine with sine sparked off a succession of posts. A version of the same program were posted by R having“forward (x/sin(30))/a” instead of “forward x/cos(150)”, where “a” is a variable. The second extract is a discussion between a teacher (T2) and the researcher R.

T2: We could ask the students: “If there is a reason that you should not use cosine, but only sine is there a way to change the relationship?” To avoid trial and error, you should use the new variable “a” and ask the students: “Can you find out what the value of “a” depends on, in order to make a proper N?” […]

R: What if a student says: ““a” depends on my mood, since I can manipulate its value, using the slider! And, using sine instead of cosine makes the diagonal of “N” very long and ugly! It’s a mess!”

T2: This reaction of the student made me mad, already! There are most important things than maths, you should explain to this student;mostly about behavior!

R: This might be a little tactless, but the student stresses that, in his/her point of view that “a” is not depended on anything. Is it wrong?

T2: Totallywrong! It must depend on something. You could be driven to formula’s exploration by putting the right questions!

In the first extract, T1 stated that “the student should use Geometry, not trial and error” and he tried to reformulate his/her hypothetical question to the students. In the second extract, T2 described hypothetical student’s attitude, as inappropriate, construing the answer “depends on my mood” more as arude reaction which had no relation with the activity in MaLT2, than related with the context of the activity, which may have no personal meaning for the student. The T1’s and T2’s first reactions may show that they both used CK than PK. In that case R, acting as a critical friend, that assist teachers to develop TPaCK tried to cross the boundary, by using the half-baked construction; he/she made a hypothetical scenario of a student’s reactions that branched off the initial plan of T1 and T2. However, the reaction of T1 and T2 was not the same. T1 reflected on his/her initial design, used the program as a tool, changing its functionalities (sine instead of cosine) and then proposing this change as an alternative activity for the students, a reaction that could be seen as instrumentalization. On the other hand, T2 did not changed his/her position. She insisted that the reaction of the student was wrong. Through this analysis we do not assume that T1 extended his/her development of PK, PCK or TPaCK. Our effort was to find technics and theoretical tools to see in more detail what happened, during a course of TPD. We suggest that the use of threaded discussion as an extra resource during the resource design process could give a potential of a more distinct understanding the TPD’s mechanism. We found the process of critically discussing the affordances digital artefacts under design and the potential student meanings in using them both illuminating and a rich context for student progression to the TPaCK centre. The artefacts themselves as 'documents under change' were particularly useful as boundary objects between the

researchers-instructors and the students playing an important role in the development of a more articulate language to discuss TPaCK issues. The T, P and C knowledge in our students became effectively apparent and a generative starting point to negotiate and progress towards two-way intersections and the centre of TPaCK.

Acknowledgments

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The documentational work in the initial formation of a mathematics undergraduate in training for the teaching of first degree equation

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This article present an analysis of the relationship between the documental work of an mathematics undergraduate in training and a supervisor and a teacher trainer in the process of preparing and implementing first degree equation. The study was based on the documentational approach to didactics and on the methodological principles of the reflective research (Gueudet & Trouche, 2010). The researchers followed the documental work of the participants before, during and after their classes during two months in a school in Recife-PE. The results reveal some challenges and limits of the choice and use of the resources for teaching the subject (exercise card, virtual scale, etc.) that give rise to fruitful investigative paths to initial teacher training in the light of this approach.

Keywords: Documentational approach to didactics. Supervised teacher training. First degree equation. Initial training. Mathematics undergraduate.

Introduction

The desire to improve the dialogue between the actors involved in the supervised teacher training (UFRPE), namely, the supervisor (PS, from the school), the advisor (PO, from university) and a mathematics undergraduate in training (ELM) has been a challenge in the initial teacher training. Teacher training has been a privileged area of questioning and investigation. In this direction, this study is motivated by concerns about the influence of the indications of resources and guidance the supervisors offer the undergraduates in mathematics degree, and the desire to reflect, on the part of the undergraduate in training (EML), on their own documental work, from their lesson plans to their implementation in the classroom. To that end, we take as reference the documentational approach to didactics, which assumes as one of its characteristics that the teachers' daily work takes place inside and outside the classroom, and gives special attention to the use of resources in the process of planning the classes and how these are transformed throughout the teachers' professional development.

Documentational approach of didactics

The documentational approach of didactics (Gueudet & Trouche, 2010) was inspired on the instrumental approach (Rabardel, 1995), which considers that the instruments are subjective, constructed by the subject from a set of artefacts, used as a means for action by combining two processes, the "instrumentation" (the constitution of schemes of use of the artefacts) and instrumentalisation (by which the subject puts the artefacts into hands). Whereas the instrumental approach distinguishes what is available for the activity (the artefacts) and that is developed by the subjects (the instruments), "the documentational approach distinguishes what is available for the activity of teachers, the resources, and what they develop to support

their teaching activity: the documents" (Trouche, 2016, p.10). It takes into account that the construction of a document is made from a set of resources that passes through phases of appropriations, transformations, combinations and adaptations the teacher performs. By identifying a set of resources that the teacher has for a given class of situation of the document that he constructs from this set. That is, in the process of documentational genesis, resources guide the teacher's activity (instrumentation) and, in return, the teacher appropriates, adapts and modifies them throughout its use (instrumentalisation). In this way, we question how a PS document, having a status of resource for the ELM, evolves in the documental work of this ELM. What are the contributions of an analysis of the PS's and ELM's documents, during teacher training, to the professional development of both?

Methodological design

The methodological conception of the documentational approach of the didactics is based on the monitoring of the teacher's activities in a significant period of time, on a continuous basis, and on the reflective principle of information, during the data collection phase, by the researched and the researcher (Gueudet & Trouche, 2010). Based on this conception, we adopted a methodological model to follow the ELM's (UFRPE) training, composed of university and school activities (class observation and teaching practice).

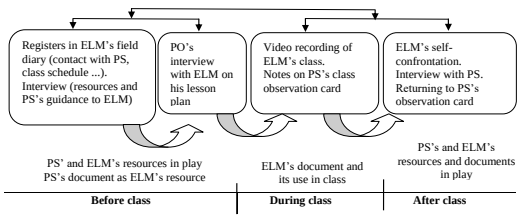


Figure 1: Methodological model

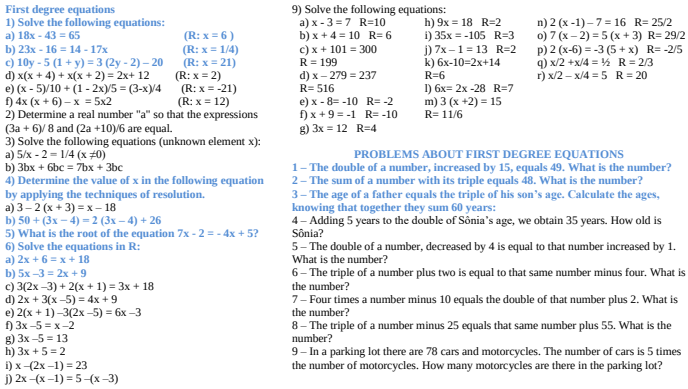
Before the ELM's class at school, we followed the PS's preparation of two classes on the revision of first degree equation and its implementation in the classroom for a 1st year of high school of youth and adults education (Figure 1). At the university, under the PO's guidance, the ELM carried out an analysis of the PS's documental work for those classes and the mathematical organization (OM) of that particular knowledge taught (Araújo, 2009). At the end of this stage, the ELM himself should prepare a lesson on the subject and apply it in the classroom. The challenge here would be neither deviate from what the teacher had already taught, nor from how he had done it. The ELM's class was video recorded and observed by the PS, and after the class, there was a simple self-confrontation (Clot, 2001). Finally, the PO interviewed the ELM and the PS, recovering all material collected during the research.

From the PS's documents to the ELM's mother and daughter resources and uses in class

Before the ELM's class, we identified that the PS prepared his classes using an exercise card obtained in Google[1], which read: "1st-degree equation exercises". From this resource, the PS

[1]Souce: http://www.objetivoboicucanga.com.br/wp-content/uploads/2015/10/Lista-de-VC-Matem%C3%A1tica-8-ano.pdf

selected 40/45 tasks of the type "T1 - solving equations of the first degree", and 9/54 tasks of the type "T2 - solving problems with the equation of the first degree". In his analysis of class observations, the ELM found that the PS proposed to students, on the whiteboard, some of the tasks of his card (Figure 2): one T1.1: ax + b = c (item 1a); four T1.2: a1 x + b1 = a2 x + b2 (items 1b; 5 and 6a; 6b); two T1.3: A1 (x) = A2 (x) (items 1c; 4b); one T2.1: (item 1) and two T2.2: (items 2 and 3). The technique expected by PS was "τNTC - neutralization of terms or coefficients". The PS based such τNTC on the "technology θ-principle of equivalence between equations" (Araújo, 2009), explaining it verbally to the students. For the PS, such tasks would be essential to the revision of the subject, due to the already known difficulties of the students of this class.

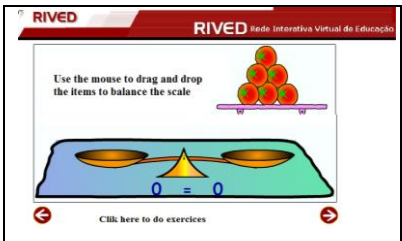
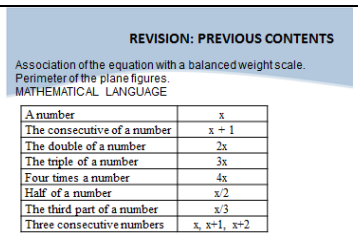
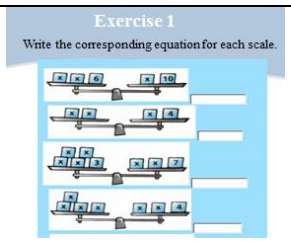

First degree equations
1) Solve the following equations:
a) 18x - 43 = 65 (R: x = 6)
b) 23x - 16 = 14 - 17x (R: x = 1/4)
c) 10y - 5 (1 + y) = 3 (2y - 2) – 20 (R: x = 21)
d) x(x + 4) + x(x + 2) = 2x+ 12 (R: x = 2)
e) (x - 5)/10 + (1 - 2x)/5 = (3-x)/4 (R: x = -21)
f) 4x (x + 6) – x = 5x2 (R: x = 12)
2) Determine a real number "a" so that the expressions (3a + 6)/ 8 and (2a +10)/6 are equal.
3) Solve the following equations (unknown element x):
a) 5/x - 2 = 1/4 (x ≠0)
b) 3bx + 6bc = 7bx + 3bc
4) Determine the value of x in the following equation by applying the techniques of resolution.
a) 3 – 2 (x + 3) = x – 18
b) 50 + (3x – 4) = 2 (3x – 4) + 26
5) What is the root of the equation 7x - 2 = - 4x + 5?
6) Solve the equations in R:
a) 2x + 6 = x + 18
b) 5x –3 = 2x + 9
c) 3(2x –3) + 2(x + 1) = 3x + 18
d) 2x + 3(x –5) = 4x + 9
e) 2(x + 1) –3(2x –5) = 6x –3
f) 3x –5 = x –2
g) 3x –5 = 13
h) 3x + 5 = 2
i) x –(2x –1) = 23
j) 2x –(x –1) = 5 –(x –3)

9) Solve the following equations:
a) x - 3 = 7 R=10
b) x + 4 = 10 R= 6
c) x + 101 = 300 R = 199
d) x – 279 = 237 R= 516
e) x - 8= -10 R= -2
f) x + 9 = -1 R= -10
g) 3x = 12 R=4
h) 9x = 18 R=2
i) 35x = -105 R=3
j) 7x – 1 = 13 R=2
k) 6x-10=2x+14 R=6
l) 6x= 2x -28 R=7
m) 3 (x +2) = 15 R= 11/6
n) 2 (x -1) – 7 = 16 R= 25/2
o) 7 (x – 2) = 5 (x + 3) R= 29/2
p) 2 (x-6) = -3 (5 + x) R= -2/5
q) x/2 +x/4 = ½ R = 2/3
r) x/2 – x/4 = 5 R = 20

PROBLEMS ABOUT FIRST DEGREE EQUATIONS
1 – The double of a number, increased by 15, equals 49. What is the number?
2 – The sum of a number with its triple equals 48. What is the number?
3 – The age of a father equals the triple of his son's age. Calculate the ages, knowing that together they sum 60 years:
4 – Adding 5 years to the double of Sônia's age, we obtain 35 years. How old is Sônia?
5 – The double of a number, decreased by 4 is equal to that number increased by 1. What is the number?
6 – The triple of a number plus two is equal to that same number minus four. What is the number?
7 – Four times a number minus 10 equals the double of that number plus 2. What is the number?
8 – The triple of a number minus 25 equals that same number plus 55. What is the number?
9 – In a parking lot there are 78 cars and motorcycles. The number of cars is 5 times the number of motorcycles. How many motorcycles are there in the parking lot?

Figure 2: From the resource to the PS's documentation

Based on classroom use of the PS's documents and his guidelines (Figure 2), ELM prepared his lesson plan on the subject using five other resources available on the internet, identifiable as mother resources (Hammoud, 2012); the result of which was the daughter resource (a file with six slides), that is, what he prepared from these mother resources to put into practice in class (Hammoud, 2012), can be seen as follows: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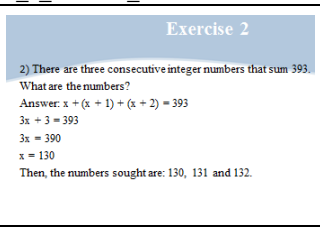	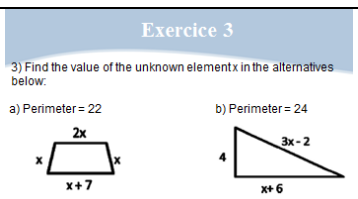	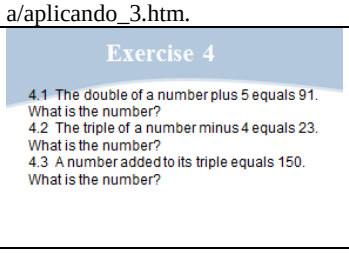
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Figure 3: From PS's re urce to the ELM's daughter resource

In the ELM's daughter resource (Figure 3), we can identify the influence of the OM (τNTC and θ related) of the PS's document (Figure 2) when he chose two activities with virtual scales (slide 1 and 3), aiming at promoting their visualisation and discussion, as well as when he chose tasks the PS had already worked in class (slides 2, 4 and 6). In a way, exercise 3 (slide 5) was different from the other tasks proposed by the PS. According to the ELM, he chose it himself because he thought relevant to articulate the field of algebra with geometry. When he observed his own class (during self-confrontation), ELM explained that as the slides progressed, he tried to give examples on the whiteboard, challenge the students and give them time to answer the assignments. From this specific moment, we selected an example, when he said in class: "Perimeter is the sum of the sides of a polygon". The ELM realized that such definition did not apply to non-polygonal figures, which could be an obstacle to students' understanding. At another moment, the ELM commented that class time (50 min) was not enough for T2 (slide 6), because of the time spent in the activities with the scales. As a consequence of the use of the ELM's document, the PS stated that he would return to the proposition of T2 in another class (Figure 2) and as the activities with the virtual scale (Figure 3) appealed strongly to the students, he would use this resource in other classes.

Final considerations and perspectives

Although the methodological model adopted here still instigates new investigations and evolvements, and although we have focused on the analysis of a single PS resource, we looked carefully into the sources of resources used by the ELM and the PS (how they appropriated, adapted and transformed such resources). The information they exchanged with each other provided, in that specific training, a particular reflection on the documental work as contributing to the professional development of both, for example, the PS's incorporation of the "virtual scales" into his system of resources. Otherwise, it was significant to identify influences of the use of the PS's document in the teacher training process of the teacher in training Just as the kind of technique used to solve 1st-degree equations, due to ELM's engagement in pursuing online activities that deal with the "τNCT", whereas before, he used to use "τTTC: transpose terms or coefficients".

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Collaborative design of digital resources: role of designers' resource systems and professional knowledge

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In this contribution, we report on the analysis of collaborative design of digital resources taking place within a community of interest in a specific socio-technical environment established in the framework of the MC Squared European project. This analysis allowed to bring to the fore how the designers' resource systems and their professional knowledge impact their choices in the design process.

Keywords: digital resource, resource system, socio-technical environment, collaborative design.

Introduction

Collaborative design reported in this paper took place within the MC Squared (MC2) project (http://www.mc2-project.eu/), in a socio-technical environment, called C-book technology, aiming at creating digital resources, called c-books (c for creative). The c-books were produced in order to develop creative mathematical thinking in their users. Through the analysis of the design of one particular c-book, we attempt to enlighten the role of both the resource systems of the designers and their professional knowledge. We start by presenting our theoretical and methodological framework. Then we continue by the presentation and analysis of a case study, before concluding.

Theoretical framework

Activity theory

Collaborative design is a human activity. The activity theory (Engeström, 1987) is a lens for describing and understanding the context of this design. Indeed, this theory incorporates notions of intentionality, history, mediation, collaboration and development in constructing consciousness, i.e. decision making, classification, and remembering, taking into account artefacts. Engeström's model of activity structure (Fig. 1) brings to the fore the interactions occurring among the diverse entities that constitute the activity: subject, object, community, rules, division of labor, and tools and signs.

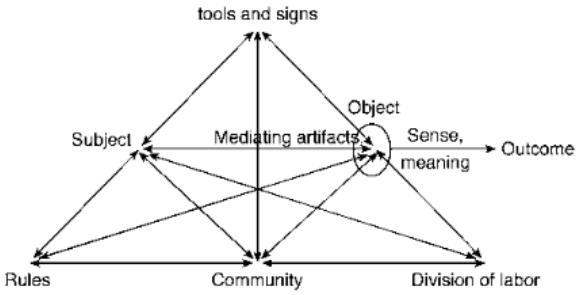

Figure 1. The structure of an activity system (Engeström, 1987, p. 78)

Various communities are involved in the c-book design. We therefore draw on the concepts of community of practice and community of interest in order to better understand the nature of interactions within those communities.

Communities of practice and communities of interest

A *community of practice* (CoP) (Wenger, 1998) gathers people who share a common identity, domain of knowledge and practice, in its core, whereas *communities of interest* (CoIs) "*bring together stakeholders from different CoPs to solve a particular (design) problem of common concern*" (Fischer, 2001). According to Fischer, the diversity of members of a CoI makes the latter having a greater creative potential comparing to a CoP. Thus, in the MC2 project, CoIs were constituted for designing c-books; they gathered people from different professional worlds, such as researchers, teachers, software designers, artists, etc., with different activity systems (Engeström, 1987), different identities, knowledge and practice, to solve a (design) problem.

Collaborative design is studied from the perspective of the designers' resource systems to highlight their impact on the design choices. For this reason, we draw on the documentational approach.

Documentional approach to didactics

The documentational approach to didactics (DAD) (Gueudet and Trouche 2009) introduces a distinction between a *resource*, conceptualized as any "thing" with which the teacher interacts and which re-sources his/her practice, and a *document* resulting from a combination of resources to which a *scheme of utilization* is associated. The process of transformation of (a set of) resource(s) into a document is called *documentational genesis* (DG). The DG consists of processes of *instrumentation* (resources informing a teacher's action) and *instrumentalization* (teacher's transformations of resources). DAD also gives importance to the set of resources gathered and/or developed and organized by a teacher in his/her professional activity, which constitutes a *resource system*. Collaborative design of a c-book can be viewed as a collective DG, starting from a resource or a set of resources contributed to the joint enterprise by the designers and resulting in a c-book resource. Indeed, in the design process, the designers bring resources coming from their own resource systems (*mother resources*), and they combine and transform them into a c-book resource (*daughter resource*) (Hammoud, 2012).

We can now refine our research question: in a collaborative design of digital resources within a CoI, what is the influence of the designers' resource systems and knowledge on the design choices?

Methodology

Our methodology relies on a case study, i.e., a collaborative design of a particular c-book that we analyze through the lens of our theoretical framework to answer our research question. The data were collected mostly via CoICode[1], a communication tool used by the CoI members, but also through minutes of manifold meetings (CoI, CoP, etc.), emails and the designed c-book. The analysis of the design process goes through three stages. The first stage aims to highlight the context of the CoI activity. We describe and analyze entities of the structure of the activity featuring the design of a particular c-book, as well as some *triangles of interactions* to show particularities of the design process. The second stage aims to identify the design choices and to infer relations with

[1] CoICode is a communication environment, part of the C-book technology, offering a workspace within which members of a CoI engaged in a c-book design can communicate.

the designers' systems of resources and professional knowledge. We analyze the designed c-book through its activities, artefacts, and tasks. The third stage aims to locate some episodes of the design process, which bring to the fore learning that occurs during the collaborative design process.

A snippet of the collaborative design of the "Algebra" c-book

In this section, we outline a few elements of the analysis of the design of the "Algebra" c-book aiming to develop algebraic thinking in young secondary school students. Referring to the activity theory, we consider the structure of the activity (Fig. 1) that consists in the design of the c-book (*object*). The CoI (*community*) was composed of thirteen members with a convergent interest in mathematics or mathematics education. The *subject* of the activity was a group of six CoI members engaged in the c-book design (called sub-CoI, Fig. 2), composed of four members at the core with a specific roles assigned from the beginning of the design (designer, moderator…) and two members at the periphery who intervened on request for a precise task. Two CoPs were associated to the sub-CoI, *Aristod* that develops educational software and the *Dynamic elementary algebra* (*DEA*) group whose members reflect on and develop resources for teaching elementary algebra.

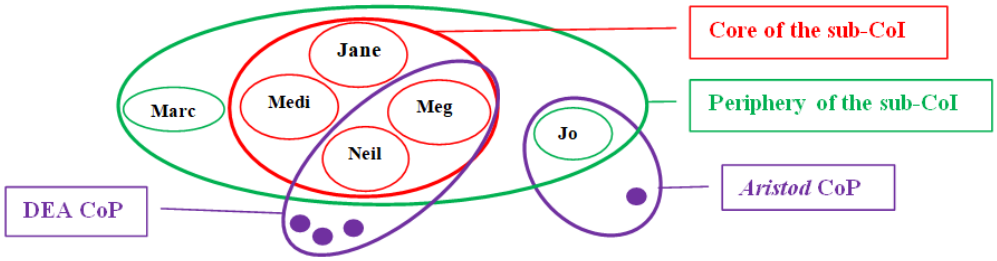

Figure 2. Sub-CoI engaged in the design of the "Algebra" c-book

One *rule* that the CoI adopted for the design of all c-books specified that at least two widgets from two different widget factories available in the C-book technology should be embedded in a c-book. The roles were assigned from the design outset to the "core" members (*division of labor)*: Meg and Medi were the main designers, Jane a moderator and Neil a secretary and an observer. Jo, also a member of the *Aristod* CoP, was asked to collaborate for technical support and Marc intervened to design specific widgets. A *product* of the activity was the designed c-book. From the meeting minutes, it appears that the sub-CoI core has decided to embed in the c-book widgets designed and developed by the *Aristod* CoP, namely *epsilonchat*, a chat tool, and *epsilonwriter*, a dynamic algebra environment (*tools and signs – sub-CoI – object)*, imposing a constraint for the design.

The c-book content is organized around three activities: "Pattern and generalisation", "Calculation programs", and "Equations". From the collected data we note that the choice of starting the c-book with an activity on patterns and generalisation comes from Meg who considers such an activity as a first step in the development of algebraic thinking for Grade 5 and 6 students (didactical knowledge). Via CoICode, she shared a resource from her personal resource system, showing a pattern with bordered squares[2] (*mother resource*). But from her experience, Meg knows that this

[2] It is a well-known activity suggested in the resource for junior high school mathematics teachers written by the Ministry of Education, http://media.eduscol.education.fr/file/Programmes/17/3/du_numerique_au_litteral_109173.pdf

pattern is difficult for young students (knowledge of her students). Meanwhile, in the *DEA* CoP, to which she also belongs, an activity has been developed with another, simpler pattern inspired from Mason (1996, p. 84). The experiment of this activity with Grade 6 students was successful, leading to adopt the latter pattern for the c-book (*daughter resource*). The constraint to use epsilonwriter and epsilonchat led the designers to create tasks calling for the use of these widgets: a questionnaire with epsilonwriter and collaboration and communication tasks with epsilonchat.

Regarding learning within the sub-CoI, we mention the case of Medi. When Meg suggested the pattern activity, Medi showed a resistance as he considered such activity as a way to enhance mathematical creativity in older students. Indeed, his personal resource system contains resources featuring complex patterns Medi uses with students, such as polygonal (triangular, squared…) numbers to enhance their mathematical creativity. Finally, after a long negotiation, Medi, recognized the potential of the activity suggested by Meg to develop early algebraic thinking. It was him who asked Marc to design a widget with this pattern to be embedded in the c-book.

Conclusion

In this paper we attempted to show the influence of the resources from the designers' personal resource systems or those from the associated CoPs on the c-book design choices; for example the idea of a generalization of a pattern (Meg) combined with Mason's article (*DEA* CoP) led to the design of the first c-book activity. Some tasks embedded in the c-book are tool-driven, created around *Aristod* digital tools. On the other hand, the design process made the designers' professional knowledge and representations evolve, as we showed on the case of Medi.

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Analyse des écarts entre les prescriptions institutionnelles et les systèmes de ressources des enseignants : le cas de l'enseignement des limites de fonctions

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Ce texte s'inscrit dans le thème du groupe 1 sur « les systèmes de ressources des enseignants, leur structure, leur évolution, leur cartographie.». Nous abordons ici les questions liées à la façon dont les enseignants constituent leurs systèmes de ressources pour préparer leur cours sur les limites de fonction et les critères qu'ils utilisent pour les sélectionner. Ce travail s'appuie sur la théorie anthropologique du didactique et l'approche documentaire du didactique.

Mots clés : limite de fonctions, la théorie anthropologique du didactique, l'approche documentaire du didactique, la médiation documentaire, systèmes de ressources.

Introduction

L'apprentissage oriente l'enseignement (Maulini 2006) donc les ressources utilisées par les enseignants pour faire leur cours. La réciproque de cette proposition est certainement vérifiée si par ressource, nous comprenons l'ensemble des éléments qui permettent de résoudre un problème d'enseignement (Hitt F., Machietto M., Trgalova J., Sokhna M. 2012). Notons également que certaines parmi ces ressources, certaines proposées par des institutions assujettissent le travail de l'enseignant. Dans les pays où le manuel scolaire officiel est absent, les programmes scolaires constituent les ressources primaires : celles-là qui façonnent les pratiques et officialisent les contenus d'enseignement. Pour mieux comprendre les intentions du programme et les enseigner le professeur sera amené à utiliser les ressources dites secondaires pour préparer son cours. Ces ressources secondaires sont entre autres des manuels, des ressources numériques. Dans certains cas, parce qu'elles sont étrangères au système, elles n'interprètent pas de façon adéquate le programme de ce système et pourtant elles sont souvent utilisables dans les classe. De plus, le « manuel est omniprésent dans la pratique des cours de mathématiques et il façonne notre perception des mathématiques et, en bonne partie, la façon de les enseigner » (Love & Pimm, 1996, p.402). Certains pays ont un système de ressources « fermé » dans le sens où ils ont un programme et des ressources denses dans ces programmes. Ils ont pour chaque niveau un livre au programme agrée ou adapté ou même confectionné par des instituts de références. Dans la plupart des cas, le personnel de ces dites institutions sont associées à la confection des programmes scolaires s'ils ne sont pas les auteurs. Ce qui fait qu'ils n'y a pas souvent de grands décalages entre les ressources secondaires de ces pays et les ressources primaires. Par contre d'autres pays comme le Sénégal, le système de ressources est « ouvert ». Il n'y a pas toujours de ressources officielles qui accompagnent le programme. L'enseignant choisit des ressources qu'il considère plus adaptées pour interpréter et pour enseigner le programme. Avec le développement des ressources sur le web, il est évident que certaines de ces ressources proviennent d'autres cultures et interprètent d'autres programmes. Il serait utile de savoir dans ce cas, quels sont les critères qu'utilise le professeur pour choisir ses ressources. Dans certaines situations ces ressources utilisées par les enseignants peuvent influencer les concepteurs des programmes. La diversité de ces ressources non organisées, leur prolifération, pourrait avoir des conséquences néfastes sur le plan des contenus des cours : il pourrait y avoir une diversité dans les

approches dans l'enseignement d'une notion et une incohérence entre l'enseignement du professeur et le programme. Nous allons illustrer ces propos dans le cas de l'enseignement de la notion de limite de fonction.

Quelques outils théoriques

Les éléments théoriques qui sont au cœur de notre étude font une articulation entre la théorie anthropologique du didactique et l'approche documentaire de la didactique.

Nous nous situons d'abord dans une perspective de complémentarité de la théorie anthropologique du didactique et l'approche documentaire de la didactique.

La théorie anthropologique du didactique (T A D) se distingue des autres théories par le fait qu'elle s'intéresse en premier au savoir alors que les approches classiques s'intéresse en premier au sujet apprenant ou enseignant (Haddad, 2012). Un autre aspect novateur (et de la force) de la théorie anthropologique du didactique repose sur le fait qu'elle considère les objets de savoir comme dépendants des institutions où ils vivent ; en effet, le savoir est le fruit d'une construction humaine (et l'activité mathématique est une activité humaine comme les autres). Pour ces objets de savoir, leur perception, leur rôle, et leur enseignement diffèrent selon le milieu et le temps, mais aussi selon les choix pris par les institutions où ils sont utilisés pour les présenter (Gonzalez-Martin, 2013). Pour Chevallard (2003, p.82). « Une institution *I* est un dispositif social « total », qui peut certes n'avoir qu'une extension très réduite dans l'espace social (il existe des « micro-institutions »), mais qui permet – et impose – à ses *sujets*, c'est-à-dire aux personnes *x* qui viennent y occuper les différentes *positions p* offertes dans *I*, la mise en jeu de *manières de faire et de penser propres*. » On voit là les influences des institutions sur les enseignants et sur les ressources qu'ils utilisent. Cependant pour mieux comprendre et analyser l'activité de l'enseignant sur, avec, et pour les *ressources nous nous sommes appuyés sur* l'approche documentaire du didactique (Gueudet & Trouche 2008). Cette approche peut être décrite autour de trois piliers : la distinction entre ressources et *documents*, le processus de genèse documentaire et *la médiation documentaire*. Cette dernière qui doit constituer le socle de l'œuvre, n'est pas souvent développée dans les différents travaux sur l'approche documentaire. Or si nous voulons éviter cette fiction didactique qui pourrait laisser penser que le système didactique peut fonctionner sans enseignants, nous devons compléter la théorisation de l'approche documentaire par les différentes médiations du système. Dans cette perspective théorique, nous avons introduit la notion de *médiation documentaire,* prolongeant la médiation instrumentale. D'ailleurs pour Rabardel (1999) la médiation instrumentale apparaît comme un concept central pour penser et analyser les modalités par lesquelles les instruments influencent la construction du savoir. Dans le cadre de l'enseignement des mathématiques, les médiations documentaires se font à deux niveaux : d'abord la noosphère qui utilise les ressources principales (les programmes) pour organiser l'activité d'enseignement et implicitement la *collaboration* entre enseignants ; ensuite, chaque enseignant après analyse interprétation du programme utilise des ressources pour « faire son cours ». Nous soustrayions cette disposition des médiations à une organisation qui implique davantage la noosphère dans leur rôle de formateur. La noosphère sera ainsi proactive, c'est-à-dire qu'il doit non seulement répondre aux difficultés des apprenants, mais anticiper sur certaines difficultés afin de réguler les perturbations éventuelles des enseignants. Quatre types de médiations documentaires pourraient apparaitre (épistémique, pragmatique, réflexive et interpersonnelle) (Rabardel, 2002) : la médiation épistémique qui est orientée vers la connaissance de l'objet ; la médiation pragmatique qui, elle, est orientée vers l'action, la médiation *réflexive* qui est orientée vers le sujet lui-même (elle est

présente dans ce que Chevallard (1999) appelle le *moment* de *l'évaluation*) et enfin la médiation interpersonnelle qui se réalise entre les sujets.

Terrain expérimental et outils méthodologiques

L'étude porte sur l'enseignement de l'analyse et particulièrement sur l'enseignement des limites au lycée (18-19 ans).

Si dans le cadre de la formation des enseignants le programme de mathématiques a juste été modifié deux fois, le programme d'enseignement du Sénégal est à sa 5ème modification. La cinquième réforme, celle de 2006 est révélatrice de difficultés à assoir un programme cohérent. On peut considérer que cela n'est pas spécifique au Sénégal. En France, pour la CREM (2003) « il y a nécessairement dans les programmes des notions ou des résultats dont la définition ou la preuve ne peuvent raisonnablement pas être des objectifs de l'enseignement secondaire. Il en est ainsi, par exemple, de la définition des limites». Il faut noter que la difficulté sur les limites est non sur sa définition mais sur l'absence de définition. Actuellement dans le programme de mathématique en France, visiblement la définition privilégiée est : $\forall\varepsilon > 0, \exists\delta > 0, (|x - x_0| < \delta \Longrightarrow |f(x) - l| < \varepsilon)$. En effet dans ce programme si une fonction admet une limite en x_0 alors elle est continue en $x_0$et la fonction f définie ci-dessous n'aurait pas de limite en 0.

Dans le programme sénégalais la conception d'une limite finie en x_0 est $\forall\varepsilon > 0, \exists\delta > 0, (0 < |x - x_0| < \delta \Longrightarrow |f(x) - l| < \varepsilon)$. Ce texte ne revient pas sur un vieux débat relatif à ces deux définitions de limite. Il fait seulement remarquer les difficultés rencontrées par la noosphère et par les enseignants, dans un système ouvert de mettre de la cohérence dans leur système de ressources. Le théorème admis ci-dessous du programme du Sénégal est en contradiction avec la définition de limite du même programme. Les fonctions f et g étant définies de $\mathbb{R}$ vers $\mathbb{R}$.

Théorème admis : Pour tout triplet (a,b,l) de $\mathbb{R}^3$, si $\lim\limits_{x\mapsto a} f(x) = b$ et $\lim\limits_{x\mapsto b} g(x) = l$ alors $\lim\limits_{x\mapsto a} gof(x) = l$	Si $f(x) = \begin{cases} 1 & si\ x = 0 \\ 0 & si\ x \neq 0 \end{cases}$ et $g(x) = \begin{cases} 0 & si\ x = 0 \\ 1 & si\ x \neq 0 \end{cases}$	D'après le théorème admis du programme $\lim\limits_{x\to 0} gof(x) = 1$	Or $\lim\limits_{x\to 0} gof(x) = 0$

Cette incohérence « interne » peut être due à la façon dont la noosphère sélectionne leur système de ressources afin de produire un programme pour son institution. Pour remédier à ce problème, nous proposons d'introduire la notion de médiation documentaire pour outiller les activités des enseignants (noosphériens ou pas) dans la détermination des critères de sélection de ressource qui servent à la conception du programme. Il serait néanmoins possible d'utiliser la notion de médiation documentaire pour expliquer voir justifier les activités liées à la sélection de ressources. Elle permettra de prendre en compte les éléments qui sous-tendent la conception de ressources. Au Sénégal, la notion de médiation prend de plus en plus place dans la conception de programme : au lieu de concevoir un programme, d'expliquer à des enseignants le sens des reformes et de leur demander de les mettre en œuvre en classe, le nouveau programme en cours d'élaboration au Sénégal s'appuie sur un dispositif plus collaboratif avec une obligation pour les concepteurs d'être plus proactif. Ainsi ils pourraient répondre plus efficacement aux difficultés éventuelles des apprenants, mais également anticiper sur celles des enseignants. Celui-ci ne sera pas seulement un critique passif qui dira que le programme est bon ou mauvais mais de par son expérience, sa conception des mathématiques et son métier, il participe à son élaboration et à sa réécriture. La prise en compte des différentes médiations

documentaires articulées produit un système dynamique unifiée comprenant une communauté d'enseignants et les ressources documentaires.

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Participation in a virtual community as a lever for the evolution of chemistry teachers' resource system

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Abstract: The joint emergence of the theme of resources and new collective forms of teachers' work naturally raises questions about the impact of teacher's participation in a community on the evolution of his resources, and thus about the community as a lever and factor of this evolution. In this perspective, this contribution investigates the impact of Lebanese chemistry teachers' participation in a virtual community on their documentation work, and more particularly on the evolution of their resource systems. This community created "by" teachers "for" teachers promotes exchanges, sharing of resources and experience among teachers. Data analysis show that teachers' participation in this community has a double productive / constructive impact on their documentation work, but this impact seems different depending on teacher's profile, whether he is experienced or beginning teacher.

Keywords: Chemistry, community, resource system, documentation work.

Introduction

A central task for teachers is to search, select, design, modify, and recompose the resources they present to their pupils and which support their activities, but also share with their colleagues. Interactions between teachers seem to hold an increasing place; we are indeed in a time of sharing resources and development of collective aspects of teachers' work. This development of a variety of teachers' collectives is due to the growing supply of digital technologies which is also accompanied by the development of new teachers' practices (Gueudet & Trouche, 2012). Digital means and technology tools deeply change conditions of sharing knowledge and convey new forms of communication, networking and association.

Building on these technologies, some Lebanese chemistry teachers have created and animated, for about one year and a half, a professional group named "chemistry teachers of Lebanon". It is a community of a large number of chemistry professors all over Lebanon (more than 900 professors) teaching at middle and high school, that relies on the use of technology, especially a free messaging application called Telegram[1]. This community, created "by" teachers "for" teachers, promotes exchanges, sharing of resources and experience among teachers.

Few studies have focused on the evolution of resources and the development of teachers' practices in such communities created by and for teachers outside the strict institutional circuits. As Quentin (2012) pointed out, shared and created knowledge in teacher networks could be at the origin of a new professional identity and encourage the implementation of changes in the professional practices

[1] https://telegram.org/

of teachers involved in these online networks. Such a perspective considers teachers not as passive users, but as designers, creative “users” and “sharers” of their own resources and sees these resources as “lived” resources (Gueudet & Trouche, 2012).

The joint emergence of the theme of resources and new collective forms of chemistry teachers’ work naturally raises questions about the impact of teacher’s participation in a community on the evolution of his resources, and thus about the community as a lever and factor of this evolution.

Elements of the theoretical framework

We rely on the documentational approach of didactics (Gueudet & Trouche, 2012) for the study of teachers’ professional evolution, where our attention is focused on the resources with a specific interest to teachers ’activity outside the class. We focus more precisely on teachers’ interaction with resources or on what Gueudet & Trouche (2012) named the teachers’ documentation work. Moreover, these authors claimed that teacher’s resources are organized in structured systems.

The study we present here of teachers’ resource systems is intrinsically linked to teacher interactions within a community. Preece & Maloney-Krishmar (2003) point out that in a community, members have a common purpose or need, engage in repeated and active participation, share resources, and create a context of social conventions that determine the framework of interactions. Actually, several terms have been used to describe and designate communities, such as online community, virtual community, community of teachers, community of learning, community of practice, etc. We are referring here to virtual communities (Daele & Charlier, 2006), as the point of articulation between the notion of community and technology tools. However, we will speak of community in general, using the term “virtual” as a descriptive of this community, and not as an element defining a subcategory of community. Daele & Charlier (2006) presented the virtual community of teachers as an interesting way of professional co-training which, whether or not supported by the institution, offers its members significant opportunities to pedagogical innovation, sharing of information and knowledge, self-expression, identity building and professional reflection.

Research questions and methodological elements

This research aims to question the evolution of the resource system of teachers participating in the virtual community of “chemistry teachers of Lebanon”. In what follows, we formulate our research question:

What is the impact of chemistry teachers' participation in this virtual community on their documentation work? How can this participation contribute to the evolution of their resource system and, more deeply, to their professional development?

To provide some answers to this question, we conduct an empirical work through an exploratory approach whose methodology is essentially qualitative. For this first phase of the empirical work and constitution of the corpus, we conducted four semi-directive interviews with a sample of members participating in this community: one leader, one beginning teacher and two experienced teachers (an interview was conducted with each of these members). We privilege the vision of the

teachers themselves and the description of the context of the community. The narrative inquiry (Clandinin & Connelly, 1990), which involves the reconstruction of a person's experience in relationship both to the other and to a social milieu, will be adopted. Indeed, after having clarified our purpose of study to the leaders of this community, we asked them to add us as a member in this community. This allowed us to follow closely this community, to better understand the interactions among members, and thus to better understand this community, its rules, its objectives, the frequency of exchanges and the type of resources exchanged. This step serves to refine later our methodology, especially our data collection.

Findings

Based on the first analysis of the interviews, we first describe the community, then we analyze the impact of teachers' participation in this community on their documentation work, and more particularly on the evolution of their resource systems. Note that our study is still in progress.

Before being gathered through the Telegram application, some Lebanese chemistry teachers were exchanging and sharing resources through WhatsApp groups. The problem with these groups is that they cannot contain more than 250 members in the same group. Thus, several WhatsApp groups were created to integrate as many teachers as possible, but with the multitude of these groups, it became difficult for leaders (who were the same for all of these groups) to share resources in all groups. Therefore, it has been necessary to find an alternative solution that allows all teachers wishing to be part of such a community, and whose number increases more and more, to be able to participate. The Telegram application was the solution, and the community "chemistry teachers of Lebanon" has been created since one year and a half.

Currently, this community contains, until the time of writing this contribution, 941 members and this number is certainly likely to increase. These members are chemistry teachers from different regions of Lebanon, teaching in private and public schools. The exchange and discussion within this community is done in three languages[2]: French, English and Arabic.

According to the interviewed leader, this community is oriented towards beginning teachers in order to accompany them in their profession: from the preparation of lessons and the annual progression to the preparation of assessments. However, any Lebanese chemistry teacher can be involved in this community: indeed, teachers of different profiles (beginning and experienced teachers in public and private schools) take part of this community. In order to participate in this community, a teacher has to contact one of the leaders (there are four leaders for this community), indicate his name and the name of his school, and download the Telegram application on his mobile phone or on his computer. We also noticed that this request for participation is often made by members of this community: they ask the leaders to add colleagues they know, by providing them with the phone number of the teacher who wants to participate. Each member can have access to the names and phone numbers of all the other members. This allows teachers, who wish to discuss a specific topic

[2] Depending on the schools, teaching science in Lebanon is done in French or in English, not in Arabic.

with one of the members, to conduct this discussion privately, and that's happens sometimes as pointed out by the interviewed members.

The follow-up of this virtual community shows that there are daily exchanges between its members. Pedagogical and didactic reflections are conducted within this community and several types of resources related to chemistry, chemistry teaching or education are exchanged: photos, videos, digital textbooks, websites, course summary, lesson models, annual progression, exercises, assessments… When a teacher asks for any resource or asks a question, he will be helped by other members of the community. It is not only resource texts that are exchanged and discussed, but also uses of resources. The interviewed leader, who has 17 years of professional experience in secondary education, appears as one of the most active members of this community. Moreover, he appears as a resource person for members, as stated by the interviewed teachers.

The beginning teacher, with whom we conducted an interview, has been teaching chemistry for grade 12 since last year. Therefore, she had no experience in teaching this class. This first year of teaching was also marked by a critical event: some chemical topics were suspended (they were removed and will not be taught anymore) based on an institutional decision (the Center of Research and Pedagogical Development under the supervision of Lebanese Ministry of Education). This event further disrupted this teacher who did not already know the content that was required before this suspension, and what will be changed as a result of this suspension. As she pointed out, her participation in the community greatly supported her documentation work and have led to the evolution of her *professional knowledge* (Shulman, 1986): knowledge on chemistry, on chemistry teaching and on teaching in general. On the one hand, there has been a lot of discussion and exchange within the community regarding the issue of suspended topics. On the other hand, she stated that she collected the shared resources within this community, especially the exercises, and adapted them in order to implement them in her classroom. Moreover, she mentioned that, at the beginning of her teaching, she had a lot of difficulty in designing assessments for her students, but with the many resources, related to assessments, shared and exchanged within the community, this task became much easier. She explained that her professional development is closely linked to her participation in the community. The resources exchanged within the community seem to be the heart of her resource system. Therefore, her participation to this community enriched her resource system. More than an enrichment, this participation has fostered a *profound* reorganization and restructuration of her resource system, in the sense that she has built and developed her resource system and her professional knowledge from the discussions and the resources exchanged within the community.

In addition, the two experienced teachers (they have 12 years of professional experience), with whom we conducted interviews, stressed the importance of creating such a community, as it helps to reduce the distances between teachers. Sharing experience and different types of resources, exchanging ideas and information within this community are real benefits for them. They stated that the resources shared within this community, in relation to assessment, are those most important to them. The participation of these teachers in this community seems to be contributing to the

reorganization of their resource systems. However, this reorganization appears “local” and concerns more particularly the resources related to the assessments.

Discussion - Conclusion

Data analysis shows that teachers’ participation in this community has a double productive / constructive impact on their documentation work, since they produce new resources (productive impact), based on shared resources within the community, and they develop their professional knowledge (constructive impact). Therefore, this promotes their professional development.

We identified two levels of evolution of the teachers’ resource system as a result of their participation in this community: a “profound” reorganization and restructuration in the resource system of the beginning teacher, and a reorganization and restructuration that we termed “local” in the case of experienced teachers. Therefore, the impact of a teacher's participation in a community seems different depending on teacher's profile: we assume that a beginning teacher needs more support in his documentation work, which could justify the fact that we have identified a deeper reorganization of the resource system of the beginning teacher, compared to the experienced teachers. But in both cases, participation in the community seems to support teachers’ resource system.

As we mentioned, our empirical work is ongoing. We have presented here the first analysis of the data we have collected. We are in a phase of refinement of theoretical and methodological elements of our study which allows us to collect and analyze new data in order to better understand the impact of the participation of a teacher in a community on his individual practices.

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Teachers' resource systems and documentation work: the case of two physics-chemistry teachers in France and two chemistry teachers in Lebanon

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Abstract: This contribution investigates teachers' interactions with resources or, in other words, teachers' documentation work, and teachers' resource systems by crossing two case studies: the first is related to two professors teaching physics-chemistry in France and the second case concerns two Lebanese chemistry professors. First, we present the theoretical framework chosen, then our research questions and some elements of the methodology developed for this research. We also expose some findings and we highlight common points and differences between the documentation work and the resource system of these four professors as well as the impact of the teacher's relationship to the curriculum of his country on his documentation work.

Keywords: Physics, chemistry, curriculum, resource system, documentation work.

Introduction

In their professional activity, teachers interact with a wide range of *resources*, in many ways, in diverse places and moments. An important facet of teachers' work, done outside the classroom, consists in searching for, collecting, selecting and transforming resources for lesson and assessment preparation. Teachers develop a *resource system* (Gueudet & Trouche, 2010) that they constantly modify. Interactions with resources are thus major elements in teachers' work. This paper focuses on issues related to this aspect of teachers' work and develops a variety of perspectives on these interactions, on teachers' resources and their use, and more broadly on teachers' resource systems, their structure, as well as the impact of the teacher's relationship to the curriculum of his country on his work with resources by crossing two case studies: the first is related to two professors teaching physics-chemistry in France. The second case investigates two Lebanese chemistry professors' interactions with resources.

Indeed, in the tradition of French secondary education, physics and chemistry are closely associated: the same professor teaches both physics and chemistry. Nevertheless, the Lebanese educational system does not combine, as in France, physics and chemistry. As such, Barlet (1999) claims that physics and chemistry are not so "sisters": the dialectic modeling-experimentation in chemistry is stronger than in physics while mathematical modeling is less present in chemistry than in physics.

Thus, differences may exist between Lebanese and French teachers in their interactions with resources, in terms of their resource system, since the relationship to discipline, teaching activities and curriculum is not the same in both cases.

Elements of the Documentational Approach of Didactics

We will not develop here the documentational approach to didactics (Gueudet & Trouche, 2010) on which we rely, but we focus on the new concepts that have enriched this approach, namely *mother-resource, daughter- resource, structuring mother-resource*. As Gueudet & Trouche (2010), we give a very broad meaning to the term "resource" (including socio-cultural resources such as interactions with students or discussions with colleagues). The teacher interacts with resources, selects them and works on them within process where design and enacting are intertwined. The expression "*documentation work*" (Gueudet & Trouche, 2010) encompasses all these interactions.

Hammoud (2012) proposed two new concepts, namely "mother-resources" and "daughter-resource" that allow to distinguish between what the teacher uses to prepare his teaching (mother-resource) and what he produces and develops from the initial resources (daughter-resource). Describing a specific type of mother-resource, Alturkmani (2015) proposes the concept of "structuring mother-resource" which is a mother-resource that helps to structure the teacher's documentation work from two points of view: on the one hand, it's around it that is structured the daughter-resource dedicated to this teaching; on the other hand, it induces new equilibria in the disciplinary and didactic affinities[1] of the professor.

Research questions and methodological elements

We investigate here the common points and the differences between the documentation work and the resource system of four professors: two professors of physics and chemistry in France and two chemistry professors in Lebanon. In what follows, we formulate our research questions:

- How do teachers structure their resource systems? Does this structure differ between a physics-chemistry teacher in France and a chemistry teacher in Lebanon? In other words, what is the difference between the resources and, more broadly, the resource system, of a physics-chemistry teacher in France and a chemistry teacher in Lebanon?
- How does the relationship of each teacher to the curriculum in his country influence his work with resources?

To provide some answers to these questions, we have chosen a qualitative approach based on the *reflective investigation* methodology (Gueudet & Trouche, 2010). We chose to focus here only on the data that comes from the interviews. We conduct a semi-directive interview with each teacher aiming at approaching his documentation work and his resource system. The interview thus stimulates the reflexivity of the teacher on the nature and structure of his resources.

Findings

In this section, we present the main results for the four teachers: the place of the textbook and, more broadly, the curricular resources in the teachers' resource system, the impact of the teacher's relationship to the curriculum of his country on his documentation work, the type of resources

1 Disciplinary affinity is defined by disciplinary interest and disciplinary awareness. Didactic affinity is defined by didactic interest and didactic awareness (Alturkmani, 2015).

identified in relation to the type of teachers' activity and the identification of particular resources, especially "*structuring mother-resources*" and "*pivotal*" resources (Gueudet & Trouche, 2010) in the teachers' resource system. We designate the two French teachers by PC1 (he has an affinity for chemistry and 10 years of professional experience in high school) and PC2 (he has an affinity for physics and he has been teaching for 14 years in high school) and the two Lebanese teachers by C1 and C2 (C1 has 12 years of professional experience in middle school and C2 has 11 years of professional experience in middle and high school).

Textbooks, as part of curricular resources, generally respect the curriculum imposed by the educational system of each country. We found that the four teachers used, at the beginning of their career, textbooks as a main mother-resource, but largely deviated from it later. This use is justified by the fact that these curricular resources constitute, for them, a means to follow, appropriate and apply the curriculum that is prescribed for them. Nevertheless, the use of textbooks changes with the years of experience and professional development of the teacher: from a "main" mother-resource to a "secondary" mother-resource.

Furthermore, in France, new programs in high school have been applied since 2010. Changes have affected the teaching of physics-chemistry: this teaching has been structured by themes; as a result, physics and chemistry are mixed within each theme. Moreover, in practical work, experimental approaches as an inquiry approach are recommended, and in the teaching of physics and chemistry, the reduction of mathematical formulas is also advocated. PC1 and PC2 mobilize the disciplinary themes in their teaching and not the themes of the curriculum. They mention that they do not distinguish in their resources the teaching of physics and that of chemistry. The reduction of mathematical formulas in the curriculum allowed PC1 to develop his disciplinary and didactic interest in physics. He puts less and less emphasis on the mathematical aspect in the teaching of physics. Depending on the disciplinary themes, PC1 and PC2 declare that, sometimes, they implement inquiry in their physics-chemistry teaching. Thus, this reform involved a particular documentation work for these teachers, to integrate new resources, and to reorganize resources already produced.

Experimental activities occupy an important place in the French curriculum and allow students to acquire specific skills. Indeed, we found that the place of practical work is much more central in the professional activity of teachers in France than in Lebanon. PC1 and PC2 declare that they begin their teaching with experimental activities, then they institutionalize the knowledge at the end of the session. Therefore, resources related to this type of activity appear also central in the resource system of these teachers. However, the difficult conditions of the laboratories in the Lebanese public schools of C1 and C2 (no laboratory in C1 school, presence of a laboratory with a single bench in C2 school, lack of materials) and the fact that the Lebanese curriculum does not sufficiently emphasize, in its general objectives, the need to implement experimental approaches, are reasons which lead these teachers not to give an important place for the experimental activities in their teaching. Consequently, the documentation work and the resources of C1 and C2 related to this type of professional activity appear *marginal*. More precisely, the use of resources, their importance and their place in the resource system of these teachers appear strongly in relation to the

type of their professional activity: for C1 and C2, the activities related to the "preparation of exercises" and "preparation of assessments" are "*dominant*" compared to those related to "course design" and "development and implementation of an experimental activity". As a result, resources in relation to "dominant" types of activity appear in turn "dominant" and central in the resource system of these teachers.

Beyond the curriculum and textbooks, the interactions between a teacher and his students on the one hand, and the interactions between a teacher and his colleagues, on the other hand, are central resources that nourish the documentation work of the teachers interviewed. We inferred that PC1 discussions with two colleagues from SESAMES[2] group and the resources of his colleagues contribute to the development of his physics knowledge and support the design of his daughter-resource. They thus appear as a "structuring mother-resource" in physics. Moreover, we have noticed that the structure of his daughter-resources is identical to that of SESAMES group's resources (activity and model). Therefore, the latter also appear as pivotal resources in his resource system since he relies essentially on this type of resources to organize his teaching in physics or chemistry. In parallel, the interaction of C1 and C2 with their students (and students' productions), especially with the students to whom they gave private lessons, greatly enriched their resource systems. Indeed, the direct contact with these students and their resources (students' resources related to courses, exercises, and assessments given by their own teacher at school and which C1 and C2 collect during private lessons), and the close follow-up of their difficulties, allowed these teachers to develop their professional knowledge as well as new daughter-resources in chemistry. Therefore, it appears that the interactions of these teachers with these students constitute a "structuring mother-resource" in chemistry. These interactions also appear, beyond a structuring mother-resource, as a *pivotal resource* because they are, on the one hand, involved in several types of teachers' activities: the preparation of lessons, exercises and assessments. On the other hand, more than an enrichment of the resource system, they seem to help restructure and reorganize the resource system of these teachers.

With the technological and internet development and, as a consequence, the proliferation of digital resources (including online resources, simulations, videos), the latter are increasingly occupying a very important place in the resource system of the four teachers. In Lebanon, the two teachers specify a particular role for social networks (Facebook, WhatsApp, Telegram) in the diffusion of digital resources in chemistry: more particularly, C1 and C2 are part of a group of Lebanese chemistry teachers gathering more than 900 chemistry teachers in middle and high school through the Telegram app. This group allows sharing and exchange of all types of resources related to chemistry education and education in general. We found that this group resources constitute a central resource in the resource systems of these teachers, particularly for the preparation of assessments. Similarly, PC1 and PC2 are part of collectives involving professors, with a strong epistemological coherence: PC1 works with SESAMES group, which is distinguished by its point

[2] The SESAMES group brings together researchers in science didactics, and physics-chemistry teachers in middle or high school in order to develop resources for teaching.

of view on science and models, and PC2 is involved in the GFEN group (French Group of New Education) which is distinguished by a strong point of view on socio-constructivism and scientific work. We inferred that these two collectives support the documentation work of the two teachers and nourish their resource systems, stimulating, from the resources conceived, the development of disciplinary and didactic affinity (Alturkmani, 2015).

Discussion - Conclusion

Through this study, we identified differences as well as common elements in the structure of the resource system of Lebanese and French teachers. Textbooks were the main mother-resource for the four teachers at the beginning of their profession, but this resource becomes "secondary" with teachers' professional development and the proliferation of digital resources. The latter are very important and occupy a central position in the resource system of the four teachers. As the experience of these teachers grows, their openness to resources grows as well. We also found that the curriculum in each country influences the documentation work and the resources of the teachers: in the case of French teachers, the resources are used for the teaching of physics-chemistry, with sometimes orientations towards the one or the other discipline, and the resources related to the design and implementation of experimental activities, are central. Nonetheless, these resources seem marginal for Lebanese teachers who use resources for teaching chemistry only. We have also highlighted a close relationship between the use of resources, their importance and their place in the resource system of these teachers, on the one hand, and the type of teachers' professional activity, on the other. Furthermore, structuring mother-resources as well as pivotal resources have been identified in the teachers' resource system.

Resources, their use, institutional conditions and constraints certainly differ in the various contexts we have discussed. It seems to us that all the professors, whatever their specialty, carry out a documentation work which occupies, in all cases, an important part of their activity; but a specificity of this work seems to prevail. Neither the resources nor the uses are generic.

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Resources for Science Teaching in a Foreign Language

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Our article examines how a science teacher, at high school, uses for her CLIL lessons some resources that she downloaded from Internet websites. For our analysis, we refer to two theoretical frameworks: the documentational approach of didactics, the Joint Action Theory in Didactics. We analyse the effects of the teacher's representations of "good" resources according to the language used in class, and of the use of such resources. We eventually produce recommendations for teacher training.

Keywords: Didactics, resources, chemistry, science, English, Content and Language Integrated Learning (CLIL), documentational approach, comparative didactics

Context of the study

Our research examines the use of resources within the context of "Content and Language Integrated Learning" (CLIL), which consists in the teaching of a particular subject matter in a foreign language. We present a case study on the teaching of chemistry in English at high school in France. The data that we study have been collected as part of a national research project in France ("ReVEA"), funded by the French National Agency for Research (ANR). We look more specifically at what is a "good resource" for a science teacher in this case and what are the consequences of its use on language and science learning. Several studies have shown that, during CLIL lessons, the scientific knowledge is frequently withdrawn, relatively to the linguistic contents (for instance, Maître, 2017). It has also been showed that the vocabulary used during the interactions is more related to everyday speech than to specific domain knowledge contents (Auer, 1988; Caffi & Janney, 1994). Some authors (for instance, Roussel, Tricot, Joulia & Sweller, 2017) have also demonstrated that the cognitive load endured during CLIL lessons provides no advantage to content learning. Our research questions are as follows: what are the criteria that guide the teacher in choosing a "good resource"? What are the consequences of using such resources on language and science learning? For instance, a video resource, associated with the use of a fill-in-the gaps text?

Theoretical framework

In this paper, we study on one hand, the preparation of the teacher's class in order to characterize her documentary work and, on the other hand, how Laureen uses her resources (Adler, 2010) during a lesson on atoms which is entirely in English. We are using the perspective of the documentational approach of didactics (Gueudet, Pepin, & Trouche, 2012). We also refer to some concepts that have been developed within the framework of the Joint Action Theory in Didactics (JATD), in order to understand didactic activities (Sensevy & Mercier, 2007; Sensevy, 2011). In the JATD, the concepts of didactic contract and milieu help define a strategic system, which is the strategic system of teaching and learning. We refer to the concepts of topogenesis and chronogenesis (Sensevy, 2011).

Topogenesis describes the partitioning of knowledge between the teacher and the students, and chronogenesis describes the temporal progression of the knowledge at stake.

Methods

Our methodology is based on the "reflective investigation" (Trouche, 2014), which provides methodological tools enabling us to track how the studied teacher, both inside and outside her educational institution, uses and produces her teaching resources. We followed several principles. The teacher's work has been observed by the researchers in and out of class; a long term follow-up has been organized, during several school years. The teacher and the researchers have worked together, and the roles occupied by each other had been formerly defined. The teacher has had a reflective stance on her own work: for instance, she has provided comments on videos of her activity filmed by the researchers, during an auto-analysis interview (Jameau, 2015). Video data have been collected during the lessons and our initial results are based on the transcriptions of these lessons, reconstructed from video recordings.

Main results

Firstly, we analysed interviews with the teacher, about her system of resources. While she was looking for resources specific to CLIL lessons on websites dedicated to this teaching, we noticed that she was rather looking for English language activities and videos, explaining that the resources selected for her lesson should allow oral interaction in the classroom and learning of a specific vocabulary. The criteria that guided her in choosing "good resources" for science in English were therefore very much oriented towards the practice of English, notably spoken English.

Secondly, we analysed the effects of using such resources on language and science learning. During the lesson, after playing a video, the teacher distributed a fill-in-the-gaps exercise. We noticed that, during this activity, Laureen either gave the answers, or explained the meaning of the words related to the experiment, or came back to the experiment so as to help the students answer the exercise. She also translated the words in order to help them. By doing that, we could say that she integrated new elements into the milieu, and thus re-shaped her resources by adding translations and explanations, so as to make didactic time, the chronogenesis, progress more rapidly. In order to continue a form of communication between her and her students, based on a "question and answer" activity, the learning process is more oriented towards the didactic contract, characterised by the continuity in the communication, instead of the comprehension of the milieu (the text). The communication logic takes over the epistemic logic (Sensevy, 2011).

A little later in the same session, while the teacher worked, on the blackboard, on a sketch symbolizing Rutherford's experiment, the teacher's insistence upon the use, and the pronunciation, of a phrase ("straight through") shows that she uses several resources (the fill-in-the-gaps text, the sketch) so as to shape them strategically as "good" resources to speak in English. This insistence upon the pronunciation is a reminder of the didactic contract: during CLIL lessons, it is important to pronounce well. Even if it would have been interesting to examine and to compare other CLIL teachers' practice, we can say that this extract is emblematic of what is at stake during CLIL lessons. Indeed, teachers have to search for a balance between a focus on specific scientific knowledge and on the language practice.

Discussion - Conclusion

What are the consequences of using such resources on language and science learning? For instance, a video resource, associated with the use of a fill-in-the gaps text? We saw, thanks to our analysis, that the criteria guiding the teacher in choosing "good" resources for CLIL teaching are mainly oriented towards the will to make her students practice spoken English, with respects to the contract regarding continuity in communication. This is partly in line with official expectations as regards CLIL teaching, which is often described as an opportunity to develop students' fluency in a foreign language. Yet, in order to make students experience both science and the language of science, Laureen has to make her students deal with linguistic and scientific knowledge. Our study has demonstrated that, thanks to a didactic analysis based on effective actions, analysed through JATD concepts, it is not so much that there is a question of withdrawal of one content relatively to the other, but that CLIL lessons require, for the teacher, an ongoing search for a balance between epistemic density and specificity of the linguistic and scientific resources. Here, the teacher has to adjust the epistemic density of scientific and linguistic knowledge to each other. All the delicacy of her work consists in trying to alter as little or as abruptly as possible the epistemic density of one content relatively to the other. We could say that CLIL lessons are characterized by the fact that the grounding of the epistemic contents regularly moves from one content to another.

As for the consequences of using the resources in question, on language and science learning, we can say that the resource selected by the teacher contains many occurrences of idiomatic and specific linguistic knowledge, which is too remote from what the students already know. The resource also contains scientific knowledge that is very specific to the scientific language relative to atoms. Indeed, as, among others, Roussel, Tricot, Joulia and Sweller (2017, p. 4) have shown, it can be difficult for learners "to acquire large amounts of secondary knowledge simultaneously". Furthermore, we have seen that the students experience difficulties in engaging in the practice of oral English, notably because they had never heard about the scientific experiment before the lesson. For the training of teachers, as Gruson and Marlot put it (2016), "teachers must be adequately trained to seriously investigate the knowledge involved, mainly before implementing the situation". The knowledge involved, here, is Rutherford's experiment on atoms, but also the English language associated with this experiment. Teachers, thus, should be aware of the necessity for them to be precisely informed about their students' mastering of the scientific contents at stake in CLIL lessons. Adler (2015, as cited in Gruson & Marlot, 2016, p. 12) also insisted on the importance "to create time for teachers out of school where they themselves have opportunity to participate in various practices, practices that deepen their [..] knowledge for and in teaching □..□, providing opportunity for teachers to appreciate that their knowledge-in-use is a key resource in their practice".

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The complex process of classifying resources, an essential component of documentation expertise

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In a time of abundance of resources and curriculum reform in France, we propose to analyze the evolution of teachers' practices through their interactions with resources. This contribution focus on an unexplored stage: the process of classifying a new resource selected by the teacher in one's resource system. First, we situate our work in the thread of the documentational approach to didactics, and develop the concept of documentation expertise rely on Personal Information Management studies. The ability to manage and organize one's resource system is a part of the documentation expertise we want to characterize in this paper. To reach this aim, we examine a case study of three teachers who prepared a new lesson. Lastly, we analyze data through the concept of scheme, and highlight a component of documentation expertise: transposition of teaching activities in a classification system.

Keywords: Teachers practices, Resource system, Documentation expertise, Scheme, Personal Information management

Introduction

The available resources constantly increase and a lot of diversified resources are used by teachers (e.g. textbooks, Internet). They spread a lot of time to search and collect curriculum materials. This is especially true for French teachers in secondary school since the 2016 curriculum reform have been in force. This reform introduced big changes to two specific points. Firstly, the organization of the curriculum now follows a 3-year cycle. In each school, subject teams have to decide which content is taught in each grade. Secondly, new topics were introduced as algorithmic in mathematics and technology. French teachers must design new lessons and deal with resources, particularly digital ones, more than ever. For contemporary teachers, the challenge is the efficiency to find resources when they need it without wasting time. So, the ability to organize and manage information and resource system is a critical issue. In this context, we proposed to analyze the evolution of teachers' practices through their interactions with resources.

Our contribution focuses on an unexplored stage: the process of classifying a new resource selected by the teachers in their preexisting and already structured resource system. First, we present our theoretical framework, which built around the documentational approach to didactics and the personal information management studies. Then we explain our methodological choices of a case study of three teachers who prepared a new lesson. And lastly, we propose some findings.

Theoretical construct

The theoretical framework mobilized is at the crossroad of different fields: education sciences and information sciences. First we present the concepts of documentation work and resource system, then we focus on the concept of scheme. Lastly, we introduce a new concept, the documentation expertise (DE).

Documentation work and resource system

We situate our work in the thread of the Documentational Approach to Didactics (DAD) (Gueudet & Trouche, 2012). Our starting point is the documentation work, a complex process which takes place anywhere, whenever and encompasses a broad diversity of activity (finding, selecting, classifying, using, modifying…resources). The documentation genesis results in a document, a hybrid entity composed of recombined resources and of a scheme. According to these authors, documentation work is linked with teachers' professional development.

Our PhD work[1] focus on the professional knowledge associated to information management. Personal Information Management (PIM) encompasses three activities (Jones, 2007): (1) finding / refinding information (2) keeping information (i.e. select and store) and (3) organize and structure the personal information system (PSI). "A PSI contains a person's books and paper documents, e-mail messages (on various accounts), e-documents, and other files (on various computers)" (Jones, 2007 p. 462). This definition is close to the definition of the resource system given in the DAD, the concept of resource system clusters all the material resources, paper or digital, which are stored by the teacher. But, for Gueudet and Trouche "the resource system of the teacher constitutes the 'resource' part of her documentation system (i.e. without the scheme part of the documents)" (2012, p. 27). Following this definition, identify and characterize the scheme of classifying one's resources that we propose to do, is a way to clarify scheme part of the documentation system.

From skill to scheme

Definitions of skill are diversified and, for us, don't appear operational to analyze documentation work. We refer to professional didactics (Pastré, Mayen & Vergnaud, 2006) for choosing the concept of scheme than skill, for work analysis. According to Vergnaud (2009), a scheme is an invariant organization of the activity for a given class of situations, comprising goals, rules of action, inferences and operational invariants (i.e. knowledge in action). We retain the distinction of Rabardel (2002) between two kinds of schemes: the first is *instrument-mediated action schemes* (e.g. prepare a new lesson); the second one is *usage schemes*, related to secondary tasks (e.g. select or modify a resource). The usage schemes could be entailed in different instrument-mediated action schemes: "usage schemes constitute specialized modules, which, in coordination with one another and also with other schemes, assimilate and mutually adapt in order to constitute instrument-mediated action schemes" (Rabardel, 2002, p. 83). Usage schemes, like skills, are part of finalized action, they support our analysis to identify the components of documentation expertise.

Documentation expertise

To conclude this part, we introduce the concept of documentation expertise (DE), a work-in-progress concept (Messaoui, 2017; Wang, to be published). Teachers' documentation expertise combines two types of schemes, media-instrumented scheme for the global task (e.g. preparing a new lesson) and usage schemes for the secondary tasks (e.g. classify resources). We supposed that

[1] PhD supervised by Luc Trouche and Béatrice Drot-Delange.

the operational invariants which guide teacher action contain knowledge linked with PIM and knowledge linked with the teaching content. One of the goals of our research is to define the components of the documentation expertise and this paper is also an opportunity to deepen this concept.

Methodological design: a case study of three teachers

To observe and analyze the documentation work, we based our methodology on the four principles of the reflective investigation (Gueudet & Trouche, 2012). We also built new methodological tools.

Reflective investigation

The first principle of the reflective investigation is a reflective follow-up: a lot of processes involved in the documentation work is unconscious, the data collect system leads the teacher to explain and awareness these processes (e.g. resource system maps). The second and third principle are a long-term and anywhere follow-up: a long period is necessary to catch the evolution of the organization of the resource system, that's why our data collect spread over 3 years (the follow-up began just before the curriculum reform and it's still continuing); moreover, the documentation work takes place both at home and at school, even in the virtual spaces (e.g. cloud storage, mail). The last principle is to store a broad collection of data.

Methodological design

For observing the documentation work and its deep links with the professional development, we chose to follow up teachers from different subjects: one in mathematics, Laurie, because of the introduction of algorithmic in the curriculum, and two in English, Amy and Mary, because of their needs for authentic resources. A lot of French teachers manage their resources on a digital form[2], so, we observed the organization of resource system through the computer classification system adopted by the teacher, with the use of software to record the activities on the computer. Nevertheless, we also consider paper resources in a complementary point of view. To observe the whole stage of documentation work, we investigated a new lesson plan designed by the teacher and follow up its life cycle for 2 years. We collected data in action (i.e. self-capture screen video when the teacher was preparing a lesson) and explanation of teachers about their activities (i.e. resources-guided tour interview or reflective map). Through our analysis, we also produced inferred map (e.g. figure 1). All the data may be used in further interview to increase the reflective dimension.

First results

In the same situation of preparing a lesson, we identified common features for the usage scheme *classify one's resources* between the three teachers. Firstly, the main goal is always to enable access quickly and easily to the resources. Secondly, rules of action depend on teachers but the classification is organized by three criteria: chronology, teaching level, and types of activities (figure 1), which are a reflection of teaching activities.

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[2] According to the Profetic 2016 survey (http://eduscol.education.fr/cid107958/profetic-2016.html), 80% of French teachers are using digital to set up classroom activity sequences

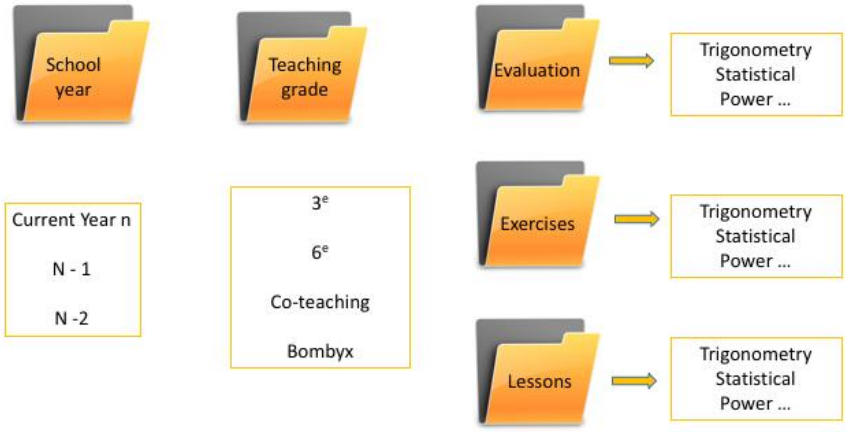

Figure 1: Inferred map of the classification system of Laurie's digital resources

We inferred operational invariants that we cluster in two categories: teaching content and information management. Classify resources by types of activities suppose didactic knowledge to attribute a type of activity to a resource. The teaching level refers to knowledge of the organization both of school and of the curriculum. Chronology reveals knowledge of the yearly progression of the courses and the annual routines. These operational invariant are based on teaching content and school organization. We also identified operational invariants linked to information and digital literacy. First, knowledge about the proprieties of digital objects (e.g. create a file, drag and drop a folder), then building a hierarchical relation between classification criteria, and lastly knowledge of representations synthesized in the file names.

To conclude, the expertise in classifying resources depends on information management; however, our results show that this activity is interwoven with the teaching activity. Even if the resource system is more than the digital resources organization, through our analysis of the scheme classify one's resources, we can highlight a component of documentation expertise: transposition of teaching activities in a classification system.

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Technology integration into mathematics classrooms: case study of a high-school teacher's use of GeoGebra

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This paper centers on the use and integration of resources in mathematics classrooms. It presents an experienced teacher's use of GeoGebra and of worksheets in a class with 11th grade high-school students, and contrasts it with his reflection during a semi-structured interview that took place after the class. We aim to show how this teacher structures and integrates his resource system (including a technological tool) into the educational model (and the curricular characteristics and analyse the role that his experience plays in such integration). The data analysis shows the difficulties of this teacher in integrating the technological tool into his classes, and how his vast teaching experience is insufficient for him to efficiently achieve that resource integration.

Keywords: Teacher's practice, digital resources integration, high-school, GeoGebra.

Introduction.

Using Pepin, Xu, Trouche, and Wang's (2017) research related to mathematics teachers' expertise, the relevance of collective work in professional teaching development, and the theoretical approach on the use of resources (Gueudet & Trouche, 2009), this article analyses data from an interview of an experienced high-school teacher and compares it against the teacher's real practice. Our aim was to determine the characteristic elements of this teacher's documentational work related to how he integrates technology in his lessons. We contrast the teacher's practice, based on what we observed in one of his classes, with comments he made in the interview at the end of his lesson. Then, we attempt to answer the following research questions: How does the teacher articulate and integrate technology, as a resource, in his lessons? To what degree does the teacher's experience allow him to integrate -in some way- the use of technology as a resource for his classes?

Theoretical framework.

Our research uses the documentational approach of didactics (Gueudet & Trouche, 2009), which focuses on studying the teacher's activity, including the factors involved in the evolution of the teacher's professional development (Gueudet & Trouche, 2012). Gueudet and Trouche (2012) consider the teacher's activity a social activity, and thus it is necessary to pay attention to the context in which it takes place, specifically, those of the institution and the different social groups. The theory is centered on the resources, taking into account the mediating role of the artifacts. The conceptualization and scope of a resource goes beyond the notion of an artifact and represents the result of the human activity, with a defined purpose. The notion of artifact is defined by Wartofsky (1979), who states that artifacts not only have a use, but also represent the mode of action in which they are used or have been produced. In consequence, in the documentational approach of didactics, resources play a key role in the teacher's work, from their selection, to their transformation and implementation. In this approach, the interaction and ways of social organization are also regarded as resources, and constitute a key factor. Then, considering that it is not possible to organize a

teaching without resources supporting this teaching; the work, then, is to identify these resources and its uses. Therefore, because the documentational approach to didactics has been developed with the objective of identifying teachers' resources, we use this approach to analyze how a teacher integrates the technology into his teaching practice.

Methodology.

The work presented here is a qualitative case study of a mathematics teacher who was observed conducting a Grade 11 GeoGebra-supported mathematics class in a public high-school in Mexico City. The teacher is a chemical engineer with 40 years of teaching experience, and who also had a master in mathematics education (from 20 years ago). The researchers attended one of the teacher's two-hour lessons on trigonometric functions to observe the use of resources during class, especially that of GeoGebra. At the end of the class, a 50-minute semi structured interview of the teacher was conducted and video recorded. For this paper we rely on the teacher's responses concerning the use of technology: in his teaching experience, in his lessons and in the class that we observed prior to the interview.

Results analysis and discussion.

Integration of technology as a resource during class.

As stated above, we observed the teacher in a lesson on trigonometric functions where he used the GeoGebra software. During the lesson, the teacher used the software to show the students the behaviour of the function $y = sen\ x$, when varying some parameters. It must be considered that the curriculum suggests students use the software to observe how graphs behave (see Figure 1). However, first we observed a difference between what the curriculum states and what the teacher implemented. The syllabus suggests students to engage actively in dynamic constructions (see Figure 1). Furthermore, at this particular school, even the classrooms were initially configured to promote a greater student involvement and collaboration (see transcript below), and prevent the teachers from having the central role. In the observed lesson, however, the students were given no opportunity to interact with the software.

Aprendizajes	Temática	Estrategias sugeridas
Analiza e identifica los parámetros que aparecen en las funciones: $f(x) = D + Asen\ (Bx + C)$ $f(x) = D + Acos\ (Bx + C)$ D desplazamiento vertical, A amplitud, B frecuencia, y desfasamiento.	Gráfica de las funciones: $f(x) = D + Asen\ (Bx + C)$ $f(x) = D + Acos\ (Bx + C)$ Análisis del comportamiento de la gráfica respecto de los parámetros: A, B, C y D	• El alumno, en equipos, realice gráficas de las funciones seno y coseno con cada uno de los parámetros observando el cambio, con dos parámetros, con tres y con todos los parámetros. • El alumno por medio de un *software* dinámico realice diferentes gráficas variando los parámetros A, B, C, D y comprueba las gráficas realizadas con lápiz y papel.
Utiliza las funciones trigonométricas para representar fenómenos de variación periódica	Problemas de aplicación.	• El profesor propone problemas de variación periódica para que los alumnos los resuelvan en equipos. • Los alumnos se auxilien con un *software* dinámico para comprobar la solución de los problemas.

Figure 1: Image of the curriculum on the topic of trigonometric functions.

Teacher: The original CCH [*high school*] model proposed the shape of the classrooms […]; we used the two boards [*there were two whiteboards in the room where the lesson and interview took place*] and the [*student*] desks were arranged [*in cooperative*

clusters]; there was an [*empty*] space in the middle and [*the teacher*] could work on both boards and that made a difference when working with the kids.

Even though there were two boards in the classroom, the teacher used only one and his class was a traditional one: the teacher was standing in front of the board, and was the only one who used the GeoGebra software. Moreover, he used GeoGebra completely in a static way to simply project graphs of functions: i.e. he did not use at all the dynamic characteristics of the software (for instance, he didn't use a slider tool to change the parameters of the graph). During the interview, the teacher also said:

Teacher: The technology: I do use it more or less, as you saw [*in the class that we observed*]. […] [*The students*] don't use graphic calculators […] I do it all there [*on his computer*], only so they can see. […] To me, the calculator is not so necessary. When [*the students*] know how to graph, when they have a clear notion, then I let them use the graphic calculators. But not while they're learning; […] they shouldn't use them [*graphic calculators*] because they lose much information.

Later he said:

Teacher: My resources are still the board and the eraser, the chalk [*or marker*].

We observed that although the teacher has 40 years of experience (and even a Masters degree in mathematics education), from how he conducted his lesson and from his statements ("I do use it more or less, as you saw" and "I do it all there, only so they can see") it is clear that he has not managed to integrate technology in a meaningful way into his practice, and avoids the use of technology among the students.

The teacher's expertise.

On his evolution as a teacher, the teacher pointed out the following during the interview:

Teacher: I have a diploma on the use of tablets and I learned many things. […] Here, I took GeoGebra courses. […] And every year we [*the teachers*] take refreshing courses. Here [*at the school*] some teachers of the school [*teach the courses*] […]: the teachers of the school teach refreshing courses to the rest of the faculty.

The teacher did not say whether the courses are a factor that made a difference in the way he teaches. He focused on the mathematics knowledge he has acquired through the years. The professional development and refreshing courses he has taken seem to be relevant only in terms of his own knowledge ("I have a diploma on the use of tablets and I learned many things"), but do not seem to have much of an impact for him to integrate the use of technology to his lessons ("My resources are still the board and the eraser"). Furthermore, he even prevents his students from using technological resources, such as graphic calculators ("not while they're learning […] they shouldn't use them") or GeoGebra on their tablets: During our lesson observation, we saw a couple of students using, covertly, GeoGebra in a personal tablet, to reproduce and even go beyond the use that the teacher did of GeoGebra during his lesson; but in an informal chat, they told us that the teacher did not like them using GeoGebra, especially in class.

Conclusions.

In this paper, we briefly analysed a case study of a lesson and interview of teacher with 40 years of experience, concerning his use of technology. With respect to the research questions, the analysis reveals the teacher's struggle to integrate the use of technology to his resource system, particularly the use of a dynamic software. His resource system revolves around traditional methods and resources: traditional lessons, individual work, and a dominant use of the board. He seems, therefore, to be stuck in that traditional way of teaching –as is shown by the analysis of the lesson and the interview– even though he, himself, considers that his teaching practice has evolved through the years**,** Thus –despite a Master's degree in Mathematics Education (albeit from a time before digital resources were commonplace), his years of teaching practice and regularly taking courses in the use of technology– he is unable: (i) to take advantage of the affordances of the technological resource (in this case of the dynamism of the GeoGebra dynamic geometry environment) to go beyond that traditional way of teaching and promote new mathematical knowledge; and (ii) of implementing a key component of the curriculum: an active use, by students, of technology. This shows a lack of expertise in terms of this teacher's appropriation of technological resources for mathematical teaching and for promoting his students' learning. Pepin, Xu, Trouche and Wang (2017) consider that teachers' expertise lies, among other factors, in their involvement with other teachers' training. In this case, the teacher, despite attending courses from, and with, colleagues at his institution, by his own admission, does not engage much with the rest of the faculty in terms of collaboration (during interview he said: "[*Interaction*] is not common, really. We are many teachers and there is little academic interaction").

These research results emphasise the need for finding ways to assist teachers in integrating, meaningfully, new elements to their resource system. A possibility to consider is the interaction between teachers, expert teachers and mathematics education researchers, engaging in lesson analysis, discussions, reflections, sharing of experiences –from the lesson planning stages to the implementation and lesson analysis– in order to strengthen the teachers' evolution.

Acknowledgment

We thank the participant teacher for allowing us to conduct the interview.

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Reflecting on the paths for selecting digital resources for (geometry) teaching: the case of a first grade teacher

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WG 1. Teachers' resource systems, their structure, their evolution, their mapping

We present some results from a research project that studies how elementary teachers select digital resources to teach geometry and the reflections they make about this process. To analyse the selection of resources, we use the idea of "paths", that is, documentary traces in the practice of teachers. We present the case of a primary-school first grade teacher from Colombia. Our results show how the selection of digital resources implies a mobilisation of the teacher's documentational system (and her corresponding professional knowledge) and describes a path of reflective selection that involves professional knowledge about resources, how to use them in specific situations and the teacher's opportunities to reflect (and learn) about her own practice.

Keywords: Selection of digital resources, case study, teaching of geometry, reflective investigation

Introduction

Our contribution focuses on a key process in the practice of teachers (in our case, for primary school): the selection of resources for the class. The use of resources (e.g. manipulative materials) in math classes in primary school is a commonplace practice in the early grades. We have paradigmatic examples such as the use of Dienes blocks (Dienes, 1969) for the initial study of shapes and counting; recently, however, elementary teachers are increasingly inclined to include digital resources in their classes. This increasing use of digital resources raises interesting research questions regarding teachers' practice; for example: What kind of digital resources tend to be selected by primary teachers? With what criteria? And how the use (and reflection on the use) of digital resources produces feedback for the practice and for their selection?

These are the driving questions of our research project, where we are studying a group of primary teachers who work in three public schools in Colombia. These teachers have access to digital technologies in their classrooms (e.g. internet, computers, tablets, etc.), and have been exposed to various teacher training programs for the use of digital resources in their classes (e.g. internet, computers, tablets, etc.). In this paper, we report part of our research results, by presenting the case of a first grade teacher who decided to use digital resources in her geometry classes and gave herself the opportunity to reflect (and learn) about her own selection process.

Theoretical framework

Our research is based on the documentational approach (Gueudet & Trouche, 2012), which studies the relationship between resources, documents and practice. As explained by Gueudet and Trouche (2012), resources are artefacts with a specific aim (that can support the teacher's practice) and they

form resource systems which comprise “material elements [and] other elements that are more difficult to collect, like conversations between teachers” (p. 27).

When the teacher interacts with resources (by using and developing them), a document emerges, through documentational genesis. This documentational genesis involves two interrelated processes: the instrumentalisation that allows the teacher to transform the resources s/he uses into documents, and the instrumentation that allows resources to influence the knowledge and practice of the teacher (Gueudet & Trouche, 2012). Thus, a document involves the resources along with schemes of use of those resources. For Vergnaud (1996), a scheme results in an invariant organisation of activity for a class of situations and represents “knowledge-in-act” of the subject. Schemes constitute a way to understand how the professional knowledge of teachers is structured (Gueudet & Trouche, 2012) – a knowledge that allows the teacher to develop professionally and that includes processes as complex as the selection of resources for the class.

The professional knowledge of a teacher, which develops as part of the documental genesis, is thus structured and organised around a documentational system (that includes resources and their schemes of use – in terms of contexts, times, situations and specific objectives). However, this professional knowledge is capable of qualifying itself by generating increasingly refined documents through the teacher’s retrospective reflection, which we consider leaves traces or “footprints”. During the selection of resources, teachers also go through a process of reflective selection of resources for their class, which thus also leave a traceable path (which includes knowledge-in-act about resources, how to select them and how to use them). For example, as already explained in Sacristán and Santacruz (2016), this selection of resources implies a mobilisation of the teacher's documentational system and of their professional knowledge for the definition of criteria (e.g. ergonomic, mathematic, curricular and didactic) that sustain that selection.

Methodological design

Our research is qualitative and relies on some elements of the methodology of reflective investigation, also defined in Gueudet and Trouche (2012), which gives importance to the schematic representation of the teacher's resource system. We take as data, lesson videotapes, questionnaires, interviews to the teachers, as well as the documentation valise of the teacher (Trouche, 2014) (that includes teachers’ resources, their notes, etc.).

We use the metaphor of path as a tool to visualise the evolution of documents produced by teachers from their retrospective reflection. We present the case of Laura, a Colombian public primary school teacher. Laura has been a first grade teacher for several years and has attended several professional development training programs on the use of digital technologies for teaching. She considers herself a successful teacher. She is currently participating in a professional development program (in which we are not involved), where teachers show video recordings of their lessons in order to discuss their classroom work. We took advantage of that program in order to work with the teacher. We met Laura when we interviewed her about her selection process of digital resources. A little over a year later the main author of this paper engaged with her in a reflective investigation that was induced through questions, being asked to analyse the resources she selected and used, and being asked to graphically represent how she selected resources and how she used them.

During the interview, and the mediated reflective investigation, Laura's left traces (a path) of her selection process and use of resources for her classes, during three phases. Phase 1 was an *exploration*, where Laura became aware of the resources that she commonly uses in her classes and her main selection criteria. Phase 2 took place during the collaborative meeting, where Laura described a specific class and the selection path she followed (see Fig. 1), in order to analyse the resource selection process prior to a class. Finally, in Phase 3, she carried out a retrospective reflection of her selection process, in which she constructed and revised three versions of her path. Her descriptions evolved in a way that represented the evolution of her documentational genesis, in terms of her selection of resources.

Analysis and discussion of Laura's reflection and path

Our analysis is divided into the reflective moments that Laura went through as she created descriptions of her path when she selected a resource for a lesson: in this case, the resource was a digital Tangram, that Laura uses for the children to explore some characteristics of flat figures (e.g., for them to recognise that there are figures with rectilinear and curved edges).

"Path version 1.0"

In the exploration phase, which Laura at the end called "Path version 1.0", she highlighted the importance of teachers having a broad and specific resource universe and specific knowledge about them. Laura considers that one of her qualities is her interest in learning from her practice. Her reflective process during the first phase, focused on what she did and not why she did it. At first she only explained some of the structuring elements of her selection path. As the reflective investigation progressed, it allowed for her to reconstruct her actions and think about other important elements. During the reflective work, the main author and Laura engaged in a discussion of how primary-school teachers do not give enough importance to the geometrical properties of figures (focusing more on the form, rather than properties) and that they usually propose situations where there is little to explore in a construction. As a result of that discussion, Laura realised that many of the resources that she (and her first-grade colleagues) use regularly can either limit or potentiate students' learning.

The path as an object of reflection

In Phase 2, Laura noticed interesting things: e.g. that she tends to use resources that she has used before or that resemble something she has already used. In the case of the tangram, its selection was a product of a long school tradition of using it in lessons with aims similar to those of Laura. As Laura advanced in her reflection of why she selected the tangram, she identified three "reflective pauses": What resource do I use? How do I select it? And what did I learn from that use? From then on, the reflection and representation of her path involved always those pauses. We consider these as important learning opportunities for her, that gave her feedback on her practice.

The reflective selection path of Laura: version 3, "still under construction"

As her reflection progressed, Laura strengthened her opportunities to learn about her practice. She represented her selection path as shown in Figure 1, although she considered it a preliminary version. In this figure, not only did she graphically represent her three "reflective pauses", but she

also added transversal elements to her knowledge of the resources she selects and how she uses them (these gave insights of her resource usage schemes).

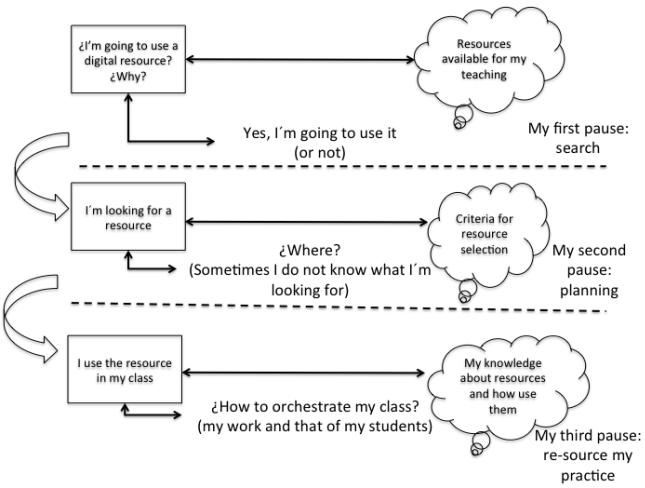

Figure 1: Laura's third version of her resource selection path

Conclusions

Laura expressed explicitly that the teacher's knowledge determines his or her practices. This knowledge (often called usage schemes) is usually invisible. Reflecting and tracing the selection (and usage) path that teachers follow, can be a strategy for re-constructing, visualising and making more evident the underlying criteria and knowledge involved in the selection process of a resource. It is not easy, however, for a teacher to carry out such reflective investigation on his/her own. In the case of Laura, it took several reflective sessions for her to become aware of the knowledge she used unconsciously in her resource selection. This study encourages us to look for further ways in which reflective investigations can be used to promote teachers' professional development.

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RESOURCES2018: What resources and assessment logic to design tests in mathematics at primary school in France?

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The assessment practices of primary school teachers in mathematics are complex and still barely studied in the reality of their realization in the classrooms. To develop materials for assessing pupil learning, teachers use a multitude of different resources (digital, professional, institutional, personal, etc.), which they assemble, transform and recompose to adapt it to their didactic scheduling. Institutional, social, personal and professional components, particularly visible at the time of assessment, give specificity to the assessment documents drawn up by the teacher. In the French educational institution, these assessment documents can take different forms more or less used by teachers, more or less familiar to teachers, according to their level of education, their training, their preferences, etc. I propose to study the assessment documents drawn up by primary school teachers in mathematics from two surveys (qualitative and quantitative).

Keywords*: assessment document, assessment practices, primary school, mathematics*

Assessment practices of primary school teachers

The assessment practices of primary school teachers in mathematics are complex and still barely studied in the reality of their realization in the classrooms. The didactic frame of the assessment that I developed in my "Habilitation à Diriger des Recherches" (Sayac, 2017), aim to analyze these practices to get a better understanding, a better knowledge of it, and to make it evolve. To study assessment within a didactic approach, I developed the concept of *assessment episode*. This concept enable us to figure out assessment in mathematics whatever its shape and beyond its functions. An assessment episode is a specific moment chosen by a teacher or an institution to which pupils are confronted to assess[1] a state of knowledge relating to a required or taught know or know-how. In order to study an assessment episode, it is necessary to take into account the moment when it took place, its constitutive tasks, its stakes, and the way to manage it. It is also necessary to take into account the contract that binds teacher and pupils with regard to knowledge whatever the assessment episode, which I will specify later.

The frame I proposed could be split in two parts: one is linked to the assessment episodes designed and proposed by the teacher and the other one is linked to the assessment logic that guides these episodes. I have chosen indicators to apprehend the assessment logic. These indicators enable us to describe the personal logic and make it evolve. These indicators include the design of various assessment episodes that mark out the study of a mathematical knowledge, in other words the way teachers design it.

To develop materials for assessing pupil learning, teachers use a multitude of different resources (digital, professional, institutional, personal, etc.), which they assemble, transform and recompose

[1] Whatever the meaning of this term (negotiator, regulator, certification, diagnosis, verification, etc.).

to adapt it to their didactic scheduling. The documentational work of teachers has been the subject of many works (Gueudet & Trouche, 2008, 2010; Wozniak & Margolinas, 2009, 2010), but only a few has chosen the assement of pupil learning as a subject, which is nevertheless identified as "one of the main moments of a lesson" (Gueudet & Trouche, 2010). During the documentational production process (Gueudet and Trouche, 2010), teachers must develop documents based on their professional beliefs and their usual practices (Gueudet, 2013). These documents will be used as a basis for the assessment episodes they plan to do with their pupils. Concerning the assessment episodes, I proposed to name assessment document the document developed by a teacher during a documentational production process (Sayac, 2017). The pedagogical freedom granted to teachers in France to assess their pupils' learning makes even more useful the use of this specific denomination, which both incorporates a wide variety of resources and patterns of use which are specific to the nature of each assessment episode. Institutional, social, personal and professional components, particularly visible at the time of assessment, give specificity to the assessment documents drawn up by the teacher. They are intimately linked to the teacher's representations of assessment and learning, but also linked to their disciplinary, didactic and professional knowledge. They are hybrid entities, composed of reorganized resources and patterns of use structured by operative invariants (Gueudet & Trouche, 2010, p.2). The mathematical coherence (Margolinas & Wozniack, 2010) of these assessment documents is all the more relevant to assess pupils' learning, that teachers of mathematics have an abundant supply of online resources (Artigue & Gueudet 2008), which also affects primary school (Bueno Ravel & Gueudet, 2014).

Assessment documents

These assessment documents are of various kinds. They are adapted to the assessment episode, which they are intended for, and to the purposes expected by the teacher. The teacher has more or less designed and finalized it before the assessment episode, but even if he did not, it will be the result of previous documentational production processes. In the French educational institution, these assessment documents can take different forms more or less used by teachers, more or less familiar to teachers, according to their level of education, their training, their preferences, etc. As mentioned above, they can also be more or less worked out. An assessment episode may have been programmed with a specific objective without the teacher explicitly writing on his preparation sheet the different assessment tasks that constitute it, or, a teacher may improvise an informal (oral or written) assessment episode during a session if he or she feels the need at some point. The diversity of the assessment documents corresponding to the various assessment episodes can be very large.

The contribution I am proposing is based on the assessment documents made by primary school teachers of mathematics in France. It is part of a perspective that I want to develop in the continuity of my work to get my HDR, but for the moment, it has not been the object of a specific research. As part of this seminar, I wish to enrich the debates around this particular type of document, which is intrinsically linked to the assessment practices of teachers in mathematics, based on results from qualitative and quantitative surveys in progress or past.

<u>Quantitative survey:</u>

In June 2017, an on-line questionnaire for primary school teachers was delivered in the Creteil academy. The purpose of this questionnaire was to get a better understanding of the assessment practices of primary school teachers.
Six hundred teachers answered to this online questionnaire including 35 questions.

Vous construisez vos évaluations en mathématiques à partir : *

Plusieurs réponses possibles.

- [ ] du manuel de la classe
- [ ] du livre du maître
- [ ] de ressources disponibles sur Internet
- [ ] d'exercices du même type que ceux déjà traités en classe
- [ ] des compétences visées dans les programmes
- [ ] de votre progression personnelle
- [ ] Autre : _____

Figure 1: question from questionnaire

Of the 600 teachers who answered to the online questionnaire, the following distribution is found:

Schoolbook: 30, 5%
Master's book: 26.7%
Internet Resources: 45.1%
Exercises of the same type as those treated in class: 71.6%
Skills targeted by official programs: 73%
Personal way to program learning in his class: 47.7%
Other: 2.8%Other: 2.8%

Généralement, vous concevez vos évaluations : *

Une seule réponse possible.

- ( ) Seul(e)
- ( ) Avec des collègues
- ( ) Autre : _____

Figure 2: other question from questionnaire

Concerning the second question, 76.8% of the teachers indicated that they themselves designed their assessments, 21% did it with colleagues, some teachers indicated their choice depends on parameters (for example, if two classes of the same level collaborate with each other).

Thus, it can be seen from the responses to this survey that the resources used by teachers to design their assessment documents are both numerous and varied, although the questionnaire does not indicate how these resources are used and especially how they are exploited and which schemes are mobilized to design assessment documents corresponding to expectations and aims of teachers.
The fact that more than three-quarters of the teachers design alone these documents is also an important result because they are not the product of an exchange or discussion. Considering that assessment training are barely provided, it can be assumed that the design of mathematics assessment documents in primary school is very personal and contingently professionalized.

Qualitative survey

In a collaborative research within a LéA (EvalNumC2), a specific work has been done with a group of 6 teachers teaching in CE1 (grade 2) around their assessment practice in mathematics. This

research aims to have a better understanding of the way teachers design and manage the assessment episodes that they provide to their pupils. The adopted methodology incorporates interviews with each teacher at the beginning and at end of the collaboration year. These semi-structured interviews are made to question the teacher assessment logic and to study its evolution during the collaboration year. Among the asked questions, three questions are specifically linked to the way teachers design their assessment documents and the resources they use for this purpose. The teacher answers are currently being processed, but they seem to confirm the results of a previous survey (Sayac, 2016), which highlighted that assessment practices of teachers are varied and very personal regarding the designing of assessment documents.

Acknowledgment

I would like to acknowledge Olivia Martinelli for its translation assistance.

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Analyse de la structure d'un système de ressources, articulation entre aspect dynamique et aspect statique : cas d'une enseignante de mathématique

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Résumé –

Une intégration des ressources de Sésamath a eu lieu dans l'atelier mathématique d'un collège algérien par une ex-inspectrice (coordinatrice de mathématique). Nous approchons le système de ressources de l'enseignante en s'appuyant sur des concepts déjà présents dans l'approche documentaire (Gueudet et Trouche, 2009). Nous mettons en évidence des aspects statiques et des aspects dynamiques de ce système. Nous proposons un modèle d'articulation entre le système de ressources et les composantes des schèmes (Vergnaud, 1991), notamment les règles d'actions et les invariants opératoires.

Mots-clés: Approche documentaire, système de ressource, schème.

Abstract –

An integration of Sésamath's resources took place in the mathematical workshop of an Algerian college by an ancient inspector (coordinator of mathematics). We are approaching the teacher's resource system based on concepts already present in the documentary approach (Gueudet and Trouche, 2009). We consider static and dynamic aspects of this system. We propose a model of articulation between the system of resources and the components of the schemes (Vergnaud, 1991), in particular the rules of actions and the operating invariants.

Keywords: Documentary approach, resource system, scheme.

1. Introduction.

Notre communication vise à aborder la structure du système de ressources de l'enseignant de mathématique, en s'appuyant sur des concepts déjà présents dans l'approche documentaire, nous analysons ce système sous l'aspect statique (taxonomie de ressource) et sous l'aspect dynamique (transformation, évolution) justifié par le concept de schème se rapportant à une situation donnée.

2. Terrain expérimental

Le collège IMTIYAZ est un collège privé arabophone, selon les règles institutionnelles, tout enseignement doit se faire en langue arabe. L'enseignement des mathématiques n'en fait pas exception, toutefois, l'écriture et le symbolisme mathématique se font de gauche à droite en lettre française. Dans ce collège active sous la responsabilité de la coordinatrice de la matière, l'atelier mathématique (classe non institutionnelle) qui a servi pour les enseignants du collège, comme milieu de découverte des ressources de Sésamath[1].

[1] Sésamath : association à but non lucratif, spécialisée dans la vulgarisation de l'usage des Tice pour l'enseignement des mathématiques. http://www.sesamath.net/

Au sens de Wenger(1998), nos collectifs suivis appartiennent à deux communautés de pratiques : Cop(Tice), communauté de pratique autour de l'usage des ressources Tice de Sésamath dans les classes institutionnelle et Cop(Sésamath) autour de l'usage manuels des ressources de Sésamath. où l'enseignante Meriem[2] en fait partie.

3. Le positionnement théorique et questions de recherches : L'approche documentaire et travail collectif des enseignants de mathématiques.

Nous assistons ces dernières années à un foisonnement de ressources pour l'enseignement des mathématiques. Cette évolution entraîne des mutations profondes, non seulement des ressources, mais du travail même de l'enseignant de mathématique, et de son développement professionnel (Gueudet & Trouche, 2009). Ce constat a été à l'origine de l'élaboration d'une approche spécifique: l'approche documentaire du didactique (Gueudet & Trouche, 2010). Pour assurer sa tache didactique, l'enseignant des mathématiques utilise, modifie, et adapte un ensemble de ressources matérielles et immatérielles dans un contexte individuel et/ou collectif. Au terme ressource, nous acceptons la définition donnée par Adler (2010). Nous utiliserons les termes ressource primaire, ressource intermédiaire et stabilisée dans le sens où l'enseignant s'approprie de ressources initiales (primaires), pour créer ses documents, on leur attribue le statut de ressources intermédiaires, après usage, elles seront révisées, réajustées, pour l'enseignant cette ressource est toujours en évolution dans un cycle de vie récursif déterminé, au bout duquel elle passe au statut de ressource stabilisée, qui fera partie de son système de ressource, et qui pourra contribuer à la création de nouvelle ressource. Le document issu de cette transformation doit répondre à une intention didactique, aux besoins de l'enseignant, à des règles organisatrices de son usage. Gueudet, & Trouche, (2009) définissent un document par l'équation : Document = Ressources recombinées + schème d'utilisation (Gueudet et Trouche 2010, p. 59). Nous nous basons sur les travaux de Vergnaud (1991) pour décrire les éléments du schème pour une classe de situation, «c'est une organisation invariante de l'activité pour une classe de situation donnée» (Vergnaud, 1991) décrit par son but et ses sous buts, des anticipations, des règles d'actions de prise d'information et de contrôles, et des invariants opératoires ce qu'il généralise par l'organisation de l'activité.

4. Questionnement et méthodologie.

Nous proposons, dans cet article de nous intéresser à l'évolution des pratiques collectives de l'enseignante Meriem en utilisant les ressources de Sésamath, nous étudierons en quoi le collectif d'enseignants organisé autour de ses ressources joue un rôle dans l'évolution de son système de ressources et de sa pratique d'enseignement ?

Pour son suivi, notre méthodologie s'inspire de la méthodologie d'investigation réflexive (Gueudet & Trouche, 2009) autour, d'entretiens, des représentations schématiques de son système de ressources, des observation de la Cop(Sésamath[3]) dans une séance de travail pour la sélection d'une ressource de Sésamath sur la géométrie dans l'espace pour sa classe de sixième , et enfin, observation de mise en oeuvre de cette même ressource traduite en arabe en classe institutionnelle.

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[2] Meriem est ingénieur en informatique (Bac+5), toutes ses études universitaires sont en langue française, elle a trois (3) années d'expériences en enseignement au moment du démarrage de notre expérimentation.

[3] Notre choix pour les ressources de Sésamath est lié à la proximité de programmes de mathématique algériens et français

5. Analyse du système de ressources de Meriem

Pour l'analyse de l'évolution de son système de ressource, nous avons procédé à une analyse de sa représentation schématique de son système de ressources initial (voir Figure 1) et (voir Figure 2), puis à l'analyse de ses interactions dans la communauté Cop(Sésamath) pour la sélection de sa ressource intermédiaire issue du manuels de Sésamath se rapportant aux cours de géométrie dans l'espace pour sa classe de sixième. Cette analyse est étroitement liée avec les données recueillies par les autres outils (les différents entretiens). L'analyse des interactions de Meriem dans la communauté Cop(Sésamath) est décrite par le schème «sélection d'une ressource de Sésamath pour la classe de 6è», la description des constituants de ce schème nous a permis d'approfondir l'évolution de son système de ressources (comparativement à sa description initiale (voir Tableau, 1) .

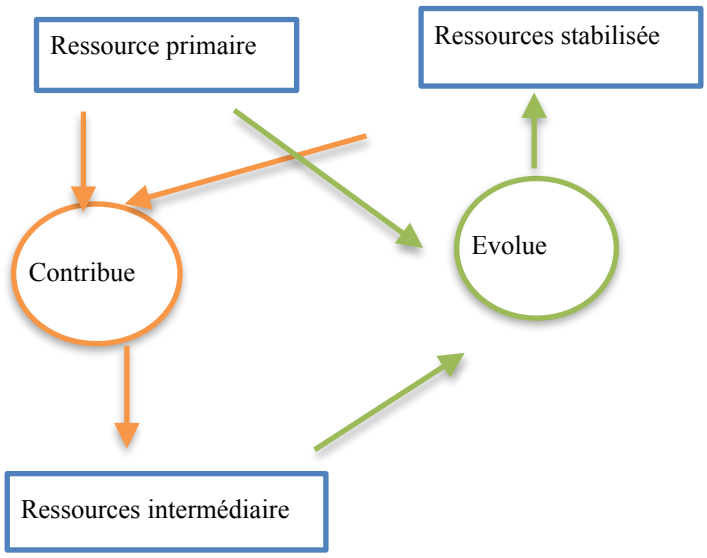

Figure 2- Modélisation du système de ressources de Meriem (RSSR initiale)

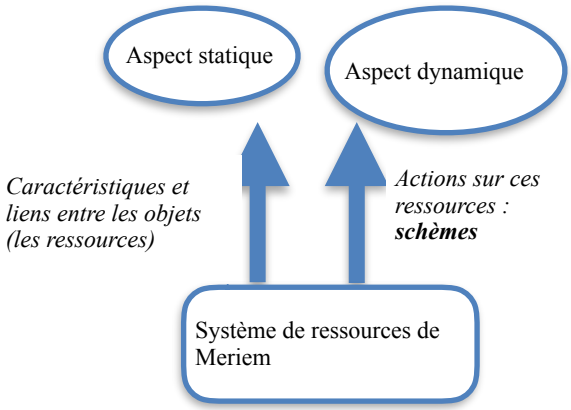

Figure 1- Approche d'analyse du système de ressource de Meriem.

Schème : sélection d'une ressource de Sésamath QCM sur l'espace pour la classe de 6è	
But :intégrer et s'approprier les ressources de Sésamath dans le programme institutionnel	Sous but 1 : elle veut faire une évaluation pour ses élèves dans une courte durée. Sous but 2 : elle veut que l'évaluation soit englobante (regroupe tout le cours sur la géométrie dans l'espace) Sous but 3 : elle veut axer l'évaluation sur les situations problèmes.
Anticipation (valable pour les 3 sous buts)	• elle veut appuyer son enseignement par des ressources diversifiées • elle veut introduire l'évaluation par les QCMs basés sur des situations problèmes.
Invariants opératoires (sous buts 1)	• je sais que mettre les élèves en situation problème est plus significatif pour leur apprentissage ; • je sais que les élèves sont plus motivés face à un travail sur QCM ; • je pense que les ressources de Sésamath diversifient le type d'activités de mes élèves ; • je sais que les contenus de notre programme de mathématiques et de celui contenu dans les ressources de Sésamath sont identiques ou se rapprochent fortement ; • je sais que les activités sélectionnées poussent mes élèves à réfléchir pour décider de la ou les réponses jugées justes.

Schème : sélection d'une ressource de Sésamath QCM sur l'espace pour la classe de 6è	
Inférences et règles d'actions	• elle projette que ses élèves seront capable de résoudre toutes les situations proposées dans le QCM ; • elle avance que certaine notions ont été traitées dans des cours précédents et ça sera un bon rappel lors de cette évaluation ; • elle voit que certaines situations peuvent avoir plus d'une réponse juste ce qui va pousser ses élèves à une meilleures réflexion autour de la ou les réponses juste.

Tabeau 1- Elèments du schème «sélection d'une ressource de Sésamath QCM sur l'espace dans la classe de 6è

4- Conclusion :

Notre méthodologie a tenté de ramener une réflexion sur l'analyse de la structure du système de ressources de l'enseignant de mathématique en faisant abstraction entre ressources et interactions avec ces ressources. Le concept de schème, croisé avec les autres outils de la méthodologie d'investigation réflexive, nous a permis de rapprocher le système de ressources de l'enseignante, pour répondre à notre question : en quoi le travail collectif autour des ressources de Sésamath, leurs sélections, adaptation et traduction, contribue à l'évolution du système de ressources des enseignants dans un contexte arabophone. Nous précisons que nous avons analysé le schème pour la classe de situation «sélection d'une ressource de Sésamath» où nous avons essayer de définir ses constituants à travers plusieurs moments (lors des entretiens, lors de la visite guidée du système de ressources de l'enseignant).

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Systèmes de ressources des enseignants et des élèves : quelles évolutions et quelle cartographie ?

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Cette étude s'inscrit dans une recherche en cours sur l'évolution des systèmes de ressources des élèves et des enseignants. Elle fait la cartographie des systèmes de ressources en explicitant leur nature et leur rôle. Elle s'inscrit ainsi dans le thème du groupe 1 sur « les systèmes de ressources des enseignants, leur structure, leur évolution, leur cartographie.». L'étude propose une caractérisation des systèmes de ressources des enseignants.

Mots-clés : L'approche documentaire, caractérisation de systèmes de ressources, TICE,

Introduction

Le développement des Technologies de l'Information et de la Communication pour l'Enseignement (TICE) ont renforcé dans l'esprit populaire, ce que l'on voit plus facilement dans l'éducation : l'aspect informationnel. En effet ce que l'on voit plus facilement dans les TICE c'est l'aspect informationnel, et ces côtés se renforcent mutuellement (Papert & Jaillet, 2003). Pour les chercheurs par contre ont compris que leurs études ne peuvent porter sur des éléments isolés mais elles doivent prendre en compte l'unité du système d'enseignement. Cette étude est en cours. Elle repose principalement sur ce que le Sénégal appelle la promotion des Sciences et de la technologie. Une politique qui est mise en œuvre par le Programme d'amélioration de la qualité de l'équité et de la transparence (PAQUET). La loi d'orientation N° 91-22 du 16 février 1991 et la lettre de politique générale pour le secteur de l'éducation et de la formation (LPGEF) ont mentionné l'intérêt et le rôle très important des Technologies de l'information et de la communication (TIC) dans l'amélioration de l'éducation et les performances des élèves. (REPUBLIQUE DU SENEGAL, 1991). En effet les TICE, permettraient d'accéder avec plus de facilité à des ressources de qualités en quantité suffisantes. La diversité des ressources et leur gratuité semble être un des critères de choix des autorités. Le renforcement de la qualité, composante essentielle du PAQUET, passe nécessairement par l'introduction et le développement des (TIC) (PAQUET, 2013). Mais quelles usages les élèves et les enseignants font-ils des ressources ?

Nous envisageons dans cette contribution de décrire la situation des TIC et des ressources pédagogiques dans l'éducation au Sénégal et d'étudier l'utilisation qu'en fond les élèves et leur professeur.

Outils théoriques

Les outils théoriques qui fondent notre étude et qui la soutiennent sont surtout inspirés de l'approche documentaire du didactique (Gueudet & Trouche 2008) qui vise à analyser l'activité de l'enseignant sur, avec, et pour les *ressources*. Pour eux, un ensemble de ressources donne naissance, pour une classe de situations, au cours d'une genèse documentaire, à un *document*. Le document se construit ainsi, progressivement, par et pour le

professeur, à travers deux processus duaux : *l'instrumentation* (le processus qui fait émerger les fonctions constituantes des ressources) et *l'instrumentalisation* qui est liée au développement des fonctions constituées des ressources (Hitt & al., 2012).

Le travail documentaire du professeur est le moteur d'une genèse documentaire, qui développe conjointement une nouvelle ressource (composée d'un ensemble de ressources sélectionnées, modifiées, recombinées) et un schème d'utilisation de cette ressource (SOKHNA & TROUCHE, 2015). Un schème d'utilisation comporte en particulier des règles d'action, et des invariants opératoires qui se forgent au cours de l'activité de l'enseignant. L'ensemble des ressources d'un professeur est un ensemble vivant, en permanent renouvellement, et en interaction avec l'activité qu'il déploie. Nous complétons le processus de genèse documentaires par la prise en compte des différentes médiations documentaire articulées produit un système dynamique unifiée comprenant une communauté de travail (subject) ; les outils conceptuels et matériels (instruments) ; les objets de l'action (object) comme dans les travaux de Engeström (1991).

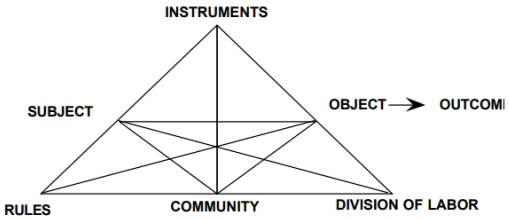


Figure 1 : Le modèle de Engeström (1991)

Terrain expérimental et outils méthodologiques

Nous décrivons la situation des ressources dans l'éducation au Sénégal et leur utilisation par les élèves dans quatre collèges d'enseignement moyens de Dakar. Nous avons interrogé 160 élèves avec un questionnaire qui a déjà été utilisé en 2013 pour le même nombre d'élèves des mêmes collèges de Dakar. Nous avons un échantillon pas significatif par rapport au nombre total d'élèves des collèges d'enseignement moyen du Sénégal qui était de 656 459 élèves soit 0,024%. Pour la collecte des données nous avons eu recours à des entretiens, des questionnaires et une documentation. L'outil d'analyse prend en compte le niveau d'instrumentation, d'instrumentalisation et le concept de système dynamique unifié.

Résultats

De cette étude, nous avons obtenu deux catégories de résultats :

Résultats de l'analyse stratégique des TICE au Sénégal

Le tableau ci-dessus présente le panorama des TIC dans l'éducation. Il laisse apparaitre au sein d'un contexte plus général de l'éducation les caractéristiques des structures (les institutions), leurs évolutions dans leur contexte.

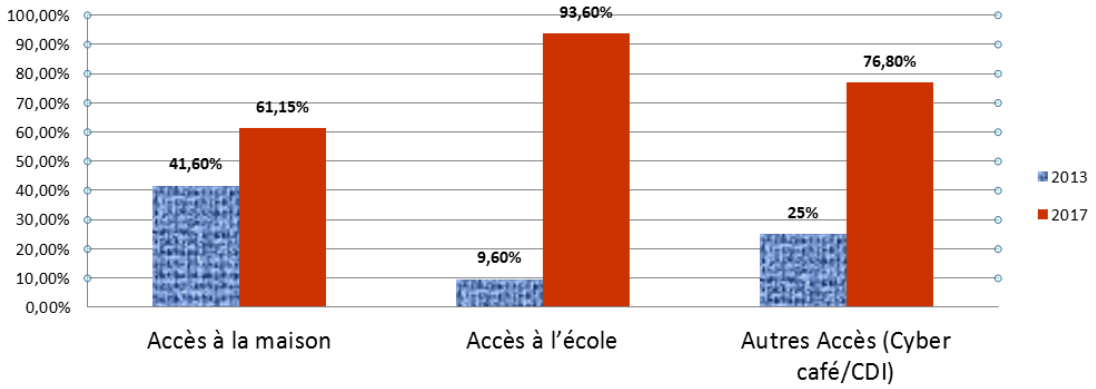
Acteurs	Intérêts/buts	Stratégies	Quels instruments?
Ministère de l'éducation	Promotion TIC pour la gestion et les pratiques pédagogiques.	Education au numérique.	- Lettres de politique - PDEF, PAQEET-EF 2013-2025 - Conventions de partenariat
Les inspections d'académies et les inspections de l'Education et de la Formation	Promouvoir les disciplines, séries et filières scientifiques et techniques à travers les cellules d'animation pédagogique	- Renforcent les capacités des enseignants. - Appui à l'équipement, en salles spécialisées et ressources matérielles et numériques adaptées	-Projets : (amélioration de la qualité et de l'équité dans l'éducation de base (PAQEEB) ; ADEM-Dakar ; amélioration de l'éducation de base en Casamance (PAEBCA) et Ressources numériques du MEN
Les établissements scolaires **(CEM et Lycées).**	Promouvoir les projets d'établissement pour réaliser les plans d'action.	Les établissements mettent en place des dispositifs de mise en œuvre projets d'établissement.	Projets d'établissement ; contrats de performance ; contrats d'objectifs chefs d'établissement-enseignants ; Fiche suivi activités.

Tableau 1 : Analyse stratégiques des TICE au Sénégal

Analyse du questionnaire

Le même questionnaire administré en 2013 a été ré-administré en 2017. Sur un même nombre de 160 questionnaires administrés, 128 sont retournés. A la question :

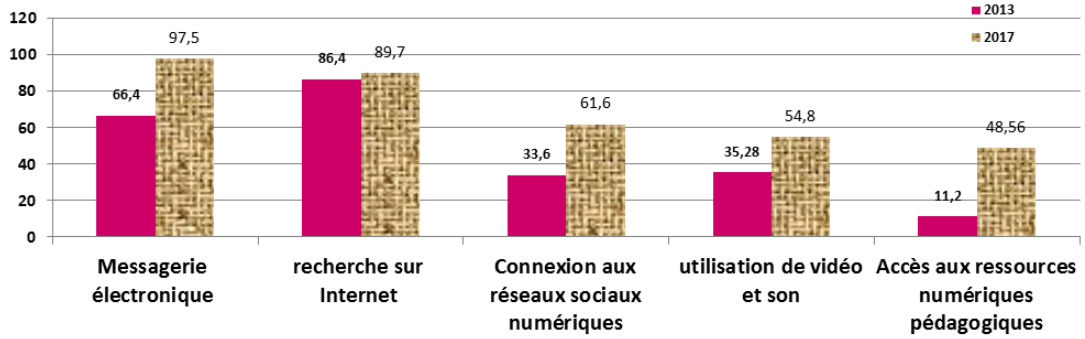
Quelles possibilités avez-vous de vous connecter à internet ?



(CUSE/Enquête : Salimata Sène, 2013,2017)

Le graphique montre une nette différence dans l'accès aux outils technologiques aussi bien à la maison qu'à l'école et au Cyber café. Les pourcentages de l'année 2017 sont de loin plus élevés que ceux de l'année 2013 et il montre une disponibilité dans les écoles de l'accès comparés à 2013. En quatre années on constate une nette évolution de l'accès. L'écart entre 2013 et 2017 est de plus de 20% pour l'accès à la maison, plus de 80% pour celui de l'école et d'environ 50% pour l'accès dans des CDI et de Cyber Café. Cela explique la volonté politique dans laquelle s'est inscrit le gouvernement pour les TICE et l'engagement des institutions.

Quels types d'utilisation en faites-vous pour vos apprentissages ?



(CUSE/Enquête: Salimata Sène, 2013, 2017)

On constate globalement une utilisation croissante pour toutes les modalités par rapport à l'année 2013. Même si le taux d'utilisation des ressources numériques pédagogiques reste faible, il a augmenté de manière significative de 2013 à 2017. Les élèves interrogés s'intéressent de plus en plus aux TIC. Toutefois l'utilisation des ressources numériques pédagogiques reste très faible par rapport à la tendance constatée.

Conclusion

La politique de l'éducation et de la formation au Sénégal est orientée vers la promotion des disciplines scientifiques et technologiques en vue d'améliorer les performances des élèves. Les différentes stratégies utilisées ont conduit à un certain nombre de changement au niveau de l'accès et l'utilisation par les élèves. L'analyse stratégique a montré un engagement des plus hautes autorités pour les TIC à l'école. Les efforts fournis ont entrainé une nette évolution de l'intérêt des élèves pour les TIC. Les différents types d'utilisations révèlent le besoin urgent de consolider le processus d'utilisation des TIC pour améliorer les enseignements apprentissages. Les questions adressées aux élèves à l'utilisation des ressources numériques pédagogiques sont les principales priorités pour une mise en œuvre effective des politiques TICE. L'utilisation et le développement des pratiques des élèves supposent un accompagnement des enseignants. Nous poursuivons cette étude avec les enseignants pour étudier leurs pratiques et les corrélations éventuelles avec celles des élèves.

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Systèmes de ressources et intégration des TICE : quels critères de sélection ?

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Cette étude s'inscrit dans une recherche en cours sur l'évolution des systèmes de ressources des enseignants. Elle fait la cartographie des systèmes de ressources en explicitant les critères de sélection. Elle s'inscrit ainsi dans le thème du groupe 1 sur « les systèmes de ressources des enseignants, leur structure, leur évolution, leur cartographie.». L'étude propose des critères de sélection des ressources qui soient indépendants des disciplines et des niveaux d'étude.

Mots clés: L'approche documentaire, médiation documentaire, systèmes de ressources, paramètres de qualité, critères de sélection.

Introduction

Le développement des didactiques a pris ces dernières années un tournant important lié, en grande partie, par le développement du numérique. Ce développement secoue le cœur de l'activité didactique. Il n'étudie pas seulement des éléments isolés du système didactique (la notion de variable didactique, de milieu, topos etc.), mais il tente de faire de l'unité du système un objet d'étude. Vygotski (1934) attirait notre attention sur la méthode qui consiste à décomposer en éléments les totalités psychologiques complexes. Pour lui, « On peut se comparer en faisant l'analyse chimique qui décompose l'eau en hydrogène et en oxygène, aucun des deux éléments ne comprenant les propriétés du tout et, chacun, possédant des propriétés qui ne sont pas présentes dans la totalité. Ceux qui appliquent cette méthode pour comprendre par exemple la propriété de l'eau d'éteindre le feu, découvriront avec surprise que l'hydrogène l'allume et l'oxygène le maintien. Ces découvertes ne nous aideraient pas beaucoup dans la solution du problème». La pertinence de cette réflexion s'est plus illustrée dans l'étude des ressources pédagogiques et du large consensus sur sa définition englobante. En effet pour Hitt & al (2012) les ressources sont l'ensemble des éléments qui permettent de résoudre un problème d'enseignement. Mais, devant une telle définition de ressources quels critères de sélection utilisés ? Quels critères de qualité utilisés ?

Outils théoriques

Les outils théoriques qui fondent notre étude et qui la soutiennent sont surtout inspirés de l'approche documentaire du didactique (Gueudet & Trouche 2008) qui vise à analyser l'activité de l'enseignant sur, avec, et pour les ressources. Elle peut être décrite autour de trois piliers : la distinction entre ressources et documents, le processus de *genèse documentaire* et la médiation documentaire.

Le premier élément de cette théorisation est la distinction ressources document. Cette distinction est à rechercher dans ce que Rabardel (1995) appelle une conceptualisation généralisée de la notion d'instrument. Pour lui l'instrument est à la fois un artefact matériel ou symbolique et un ou des schèmes d'utilisation associés résultant d'une construction propre ou de l'appropriation de schèmes sociaux préexistants. Gueudet & Trouche (2008) font résulter les schèmes d'utilisation d'une

construction- redéfinition et reproduction par le sujet des schèmes sociaux et culturelles préexistants dans les ressources.

Un autre intérêt de l'approche en termes de genèse instrumentale est qu'elle permet de fonder théoriquement l'articulation et la continuité entre les processus institutionnels de conception des artefacts et la poursuite de la conception au sein des activités d'usage. Les processus d'instrumentation participent au processus global de conception en s'inscrivant dans un cycle : modes opératoires (prévus par les concepteurs), schèmes d'utilisation (élaborés par les utilisateurs), nouveaux modes opératoires prévus par les concepteurs à partir des schèmes d'utilisation. Les processus d'instrumentalisation s'inscrivent dans un cycle parallèle au précédent : fonctions constituantes de l'artefact (définies par les concepteurs), fonctions constituées (par les utilisateurs), inscription de ces fonctions constituées dans une nouvelle génération d'artefacts (par les concepteurs). (Rabardel 95, P 3). Se fondant sur des principes similaires un ensemble de ressources donne naissance, pour une classe de situations, au cours d'une genèse documentaire, à un document. Le document se construit ainsi, progressivement, par et pour le professeur, à travers l'instrumentation et l'instrumentalisation des ressources.

Le troisième élément la notion de *médiation documentaire*, prolongeant la médiation instrumentale. D'ailleurs,

> « Vygotski développe le concept d'instrument psychologique comme médiateur entre sujet et lui-même et les autres sujets. L'instrument psychologique constitue, à ce moment du développement de sa pensée un type d'instrument particulier correspondant à des artefacts spécifiques (plans, langages, signes, schémas…). Il nous semble qu'il est nécessaire de dépasser ce caractère trop limité de cette conceptualisation en distinguant plusieurs directions de la médiation : le rapport médié à l'objet d'activité externe (qui chez Vygotski caractérise l'instrument technique ou matériel), le rapport médié à soi-même et aux autres, caractéristiques pour l'auteur, de l'instrument psychologique. » (Rabardel 2002).

Lorsqu'un enseignant met en œuvre une situation pour que ses élèves q*uatre types de médiations instrumentales* apparaissent : la médiation épistémique qui est orientée vers la connaissance de l'objet ; la médiation pragmatique qui, elle, est orientée vers l'action, la médiation *réflexive* qui est orientée vers le sujet lui-même. Et enfin la médiation interpersonnelle qui se réalise entre les sujets.

Terrain expérimental et outils méthodologiques

Aujourd'hui la détermination de la qualité des ressources pédagogiques est marginalisée dans les études. Or la non détermination des critères de qualité a rendu difficile les choix qui sont faits par les enseignants et institutions pour document les formations. Devant cette difficulté nous avons interrogé des recherches pour répondre à la question : comment et avec quels critères les ressources devraient-elles être sélectionnées par les usagers ?

Pour se faire, nous avons essayé de spécifier les paramètres qui déterminent les « qualités internes » et les « qualités externes » d'une ressource pédagogique. Ces paramètres constitueront les instruments de mesure à partir desquels seront étudiées des ressources pendant les différents moments de leur utilisation. Ils permettent ainsi, à un temps t, à partir d'une coupe d'un processus

d'évolution de la genèse documentaire, de mesurer l'état d'évolution de la ressource et de faire des pronostics sur son utilisation éventuelle.

Ces paramètres seront rangés en deux catégories :

- la première catégorie que nous appellerons paramètres internes permet d'étudier les fonctions de ces ressources. Les critères théoriques qui sont en train d'être éprouvés permettront de suivre l'évolution d'une ressource donnée ;
- la seconde catégorie que nous appellerons paramètres externes permet de faire une étude prospective de « modèle » de ressources.

Paramètres internes de qualité

Les paramètres internes de qualité de ressources pédagogiques s'appuient sur les travaux de Choppin (2005). Nous retenons les quatre fonctions essentielles qu'ils ont décrites pour les manuels :

- la fonction référentielle, curriculaire ou programmatique, qui vient du fait que la ressources est en quelque sorte l'« interprétation pratique » de programme officiel ;
- la fonction instrumentale correspond au rôle pédagogique de la ressource. Les ressources vont exposer des méthodes d'apprentissage, propose des activités et structure des progressions.
- la fonction idéologique et culturel correspond au fait que les ressources sont des véhicules idéologiques et culturels, de manière plus ou moins marquée selon les époques, les pays et les disciplines.
- la quatrième fonction est la fonction documentaire, qui fait des ressources un lieu de stockage et de proposition de divers types de documents.

Paramètres externes de qualité

Cette seconde catégorie que nous appellerons paramètres externes permet de faire une étude prospective de modèle de ressources.

La qualité est la prise en compte de certaines caractéristiques qui ont guidé les travaux sur les NORMETIC. Il s'agit de la réutilisabilité, de l'adaptabilité, de la collaboration, de la reconnaissance, de la pertinence pédagogique, de l'accessibilité, de la durabilité et de l'interopérabilité.

Nous avons d'abord choisi les notions de ***réutilisabilité*** et de l'***adaptabilité*** parce que, dans une approche instrumentale, elles marquent la vie d'une ressource et présupposent la prise en compte des usages. D'ailleurs tout laisse penser que ces deux notions, à elles seules, pourraient suffire pour résumer toutes les caractéristiques des ressources, les autres n'en seraient que des conséquences. Précisons que, pour CREPUQ - Novasys (2003, p. 31) :

> « La réutilisabilité et l'adaptabilité est « le caractère de ce qui permet la réutilisation de ressources d'enseignement et d'apprentissage à différentes fins, dans différentes applications, dans différents produits, dans différents contextes et par différents modes d'accès ».

La ***collaboration*** sera retenue parce qu'elle répond aux objectifs de partage au sein d'une communauté de pratique et donc aux médiations interpersonnelles.

Nous avons retenu la ***reconnaissance de la propriété intellectuelle*** des ressources car elle permet d'identifier les initiateurs et permet d'étudier la vitalité des secteurs producteurs.

L'***accessibilité*** et la ***durabilité***, par rapport au public cible et au besoin d'enrayer leur isolement, apparaissent comme un objectif fondamental pour les ressources. Ces ressources doivent être disponibles dans l'espace et dans le temps (sur papier, sur cédérom, dans une plate-forme dédiée, etc.) sans que le mode d'accès n'altère le contenu.

S'agissant de la ***pertinence pédagogique***, étant donné que chaque ressource doit prendre en compte les la formation, l'enseignement et l'apprentissage », des efforts considérables doivent être faits pour rendre lisibles les situations dans lesquelles elle sera mise en œuvre. La pertinence pédagogique est, pour CREPUQ - Novasys (2003, p. 18), « *le caractère de ce qui rend compréhensible les contextes pédagogiques des ressources d'enseignement et d'apprentissage* ».

L'***interopérabilité*** est importante si on pense à l'enrichissement des ressources par d'autres ressources de la toile et la conservation pour les générations à venir. Précisons que :

> « L'interopérabilité est le caractère de ce qui permet l'utilisation des ressources d'enseignement et d'apprentissage développées par une organisation dans un environnement technologique donné par d'autres organisations dans d'autres environnements technologiques » (CREPUQ - Novasys 2003, p. 10).

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The concept of resources and documents as means to understand mathematics teachers use of digital platforms in the classroom

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Abstract

Currently, digital learning platforms are being implemented in Danish elementary schools. These platforms are developed with a dual aim of both supporting teachers' planning and classroom teaching. This paper investigates and discusses the opportunities of using the documentational approach to study Danish mathematics teachers' use of these platforms for classroom teaching and preliminary findings here of in the context of an ongoing PhD project.

Keywords: Learning Management Systems, resources, documentational genesis

Introduction

The current amount of available digital resources such as e-textbooks and online teaching materials have made it a complex matter for mathematics teachers to choose, combine and redesign curriculum materials that meets the learning goals for at specific group of students (Abar & Barbosa, 2011). Digital resources for teaching are often found at various websites, and in different platforms, portals and fora, requiring teachers to navigate between many digital sites when planning a lesson (Nokelainen, 2006). In Europe, we currently see efforts from educational ministries, municipalities, school leaders and other stakeholders in education to reduce this complexity by purchasing and implementing digital learning platforms (DLP) (see for example KL, 2016; Johnson, Becker & Hall, 2015). The aspirations associated with DLP's is often that a single-portal-solution, where teachers can access all resources at one site, will make teachers' work easier (KL, 2016). Due an increase in 'bring your own device' policies, it has also become possible to use DLP's for classroom teaching (Johnson, Becker & Hall, 2015). The DLP's currently implemented in Denmark is characterized by exactly such a dual purpose in that they both are designed to support teachers' planning *and* classroom teaching (KL, 2016). The Danish DLP's has a student interface allowing teachers to use the platform to distribute lesson plans, tasks and activities to students in the classroom. This feature of is associated with hopes of freeing up teachers' time in the classroom (KL, 2016). DLP's allow students to manipulate on-screen objects, to log their academic progress and to make a range of mathematical inputs during the class. Such new available actions however often entail an increase in the complexity of real-time input for teachers to make sense of (Clark-Wilson & Noss, 2015). There is therefore good reasons to investigate *how* and *with what implications* teachers use DLP's in their teaching.

Studies of mathematics teachers use of DLP is certainly not new. Particularly the documentational approach (Gueudet & Trouche, 2009) have inspired research about mathematics teachers' individual and collective work with various kinds of platforms (Gueudet, Pepin, Sabra & Trouche, 2016). There however seems to be a tendency in many of these studies to focus on teachers' work

with platforms *outside* the classroom (Gueudet & Trouche, 2009; Gueudet, Pepin, Sabra & Trouche, 2015) or what Remillard (2005) refers to as the design arena. This aspect of mathematics teachers' work with DLP's is no doubt of high relevance. The current Danish DLP however also brings new conditions and opportunities for classroom teaching that are critical to understand. Similar studies have been done previously (for example Ruthven, 2012) but these have often focused on content-specific technologies. The Danish DLP is different in that the it is a general-purpose technology that, nonetheless, most likely have implications for *mathematics* education.

The purpose of this paper to initiate a discussion about the opportunities of using the documentational approach (Gueudet & Trouche, 2009) as a framework to study Danish mathematics teachers use of DLP for teaching and to provide preliminary findings of this. The paper address this purpose tentatively in the context of my ongoing PhD project by suggesting a research design based on the documentational approach. In the paper, I seek to give an address the following three questions:

- How can the concept of resources and documents (Gueudet & Trouche, 2009) help understand the opportunities in and implications of using a DLP for classroom teaching?
- Which research questions are we able to answer by building on this framework, and which methods are appropriate to use?
- What role does DLP's play in mathematics teachers' classroom teaching?

Theoretical approach: Documents and resources

The documentational approach (Gueudet & Trouche, 2009) is designed to study teachers' selection, design and appropriation of resources. It draws on inspiration from the instrumental approach (Guin, Ruthven & Trouche, 2005) and Adler's (2000) concept of resources (Gueudet & Trouche, 2009). Inspired by the instrumental approach, teachers work with resources in considered a dialectic process, where teachers' usages and knowledge and the resources mutually affect each other (Gueudet & Trouche, 2009). Based on Adler's (2000), resources are defined as "*a range of other human and material resources, as well as mathematical, cultural, and social resources*" (Adler, 2000, 210). A *document* is thus considered as the product of combined resources, usages and knowledge (Gueudet, Soury-Lavergne & Trouche, 2012)

The advantage of the documentational compared to other available frameworks (for example TPACK (Misra & Köhler, 2006) and instrumental orchestrations (Drijvers et. al., 2010) is that it allow us to think of teachers choices of using and not using DLP's in classroom teaching in various situations as resource and document 'management'; as efforts in creating or maintaining situations in which certain resources are available. It thus enable us to study how the DLP's enters teachers' resource systems and documents and whether/how, DLP's create new infrastructures for documentational work. The framework also enable us to understand why teachers often uses DLP's partly and differently depending on the specific situation. Finally, if offers a sympathetic way of considering the reasons teachers might have *not* to use the DLP in certain situations.

Towards a research question

Drawing on the documentational approach, the third research question in this paper can be refined as follows:

How, when and with whom do mathematics teachers use and DLP in classroom teaching? Which resources and combinations of resources are enabled and disabled as a consequence of the above, and how do mathematics teachers manage and appropriate these resources?

Methodology and data and preliminary findings

To answer the research question posed above, information about teachers use, partial use and non-use of DLP are needed. Information about teachers' reasons and thoughts of why they use the DLP of parts of it in particular ways and when and why they choose not to use it is also needed. To collect data about this, I am conduction classroom observations and individual interviews with 4 individual mathematics teachers in grade 5-7 (age 12-14). The observations intend to generate insight in the following: *When are the platform used, and who and what are involved in the use of the platform? What resources and combinations or resources are enabled from the above? Which and how are these resources appropriated and which documents are thereof?*

Currently, I have completed observations and interviews with two teachers and I will complete observations and interviews of two additional teachers during November/December 2017. During the fieldwork done so far, I have both conducted formal and informal interviews. The formal interviews aimed at gaining an insight in the teachers conceptions and experiences of using (or reasons of not using) the LP's in the classroom. Further, the interviews investigated what resources the teachers was sensitive towards, and how they would draw on such resources (in the form of student expressions, classroom atmosphere, materials or other) in their teaching. The interviews included questions such as: *What do you consider important or relevant information when you are teaching? Which factors or information can make you adjust your plan for the lesson? In what situations are such information provided, and is it related to or influenced by the DLP?*

As I am currently processing the data, I will only briefly introduce preliminary findings here and elaborated the finding in the oral presentation at the conference. The interviews of the four teachers showed diverse perspectives on using DLP's and its implications. A teacher uttered the perspective that DLP's reflects managers' need to *"control what the students are doing in the classroom"* and that a DLP have no positive impact for student learning. He felt that his relational contact with the students were compromised when using a DLP, as students work on computers made it difficult for him to engage in their work. His resource management consisted in balancing between living up to the requirement from the management of using the DLP and maintaining a good relation to the students. Another teacher emphasized that using a DLP contributes in "*including students in the lesson on a much higher level. I use it to distribute the content of the lesson"*. He also thought the DLP as a resource helping him managing teaching resources in a single digital site.

Conclusion

This paper have discussed the opportunities of using the documentational approach to study mathematics teachers' use of learning platforms for classroom teaching in the context of my

ongoing PhD project. By using this framework in empirical studies, the paper suggested that in this context, the documentational genesis allow us to study how teachers work with DLP's for classroom teaching as *resource management*. Thereby, we are able to understand the reasons why teachers in some cases choose to use parts of DLP's partly, and different parts in different situations. This can provide important insights into how technologies not only bring new resources, but also may other resources previously available, and how teachers navigate professionally under such conditions. The preliminary findings also suggest that the potentials of DLP's must be studied in the light on the individual teachers' beliefs and perceptions of good mathematics teaching.

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Understanding teachers' use of instructional resources from a cross-cultural perspective: the cases of Sweden and Flanders

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Acknowledging the expanding field of instructional resources available to teachers and students, this contribution aims to advance our understanding of what instructional resources teachers draw on, how they use these resources, and reasons for use of resources. Twenty teachers from Sweden and Flanders, ten per region, were interviewed twice. An insider – outsider approach was applied to advance understanding of their use of instructional resources. Preliminary findings suggest a range of factors influencing teachers' use of instructional resources, including factors related to the official curriculum, operational curriculum and instructional resources.

Keywords: cross-cultural study, use of instructional resources, insider-outsider approach, elementary school.

Introduction

Characteristic for the 21st century is the availability of online and other digital resources in addition to printed curriculum resources such as student textbooks and a teacher's guide. This evolution provides a fertile ground to study how teachers navigate the expanding landscape of instructional resources and select and use these newly available resources in addition to print curriculum resources (Gueudet & Trouche, 2009). This contribution addresses this challenge from a cross-cultural perspective by means of focusing on the two distinct educative contexts of Sweden and Flanders (Belgium). In particular, we aim to better understand what instructional resources teachers use when they plan for and enact lessons, how they use them, and the reasons behind their use.

Guiding framework

We use the term instructional resources to refer to the resources designed to support curriculum enactment. These resources include curriculum resources that sequence a particular content such as student textbooks and teacher's guides, but also other resources such as digital (online) applications. Because we understand teachers' use of resources to be situated in a broader school system, we draw on the curriculum enactment process as conceptualized in Remillard and Heck (2014). Remillard and Heck differentiate between an official and operational curriculum. The official curriculum, authorized by governing agencies includes curricular aims and objectives; assessments; and the designated curriculum – a set of instructional plans specified by a governing agency. The

operational curriculum captures the enactment process. It acknowledges the central role that teachers have in interpreting and mobilizing curriculum resources and differentiates between a teacher-intended and enacted curriculum and student outcomes. The location of instructional resources outside of the official and operational curriculum allows to fit both (centralized) systems in which the instructional resources are part of the official curriculum, and other systems in which they are not. We found Remillard and Heck's conceptualization helpful because it allows to situate the participating teachers' use of instructional resources in their particular Swedish or Flanders school systems. It also allows to relate to a range of factors, including factors that influence the official curriculum, teacher-intended and enacted curriculum, factors that influence instructional materials, and factors that influence student outcomes.

Mathematics education and curriculum resources in Sweden and Flanders

To better situate the findings in relation to each region, we describe for both regions the objectives and aims of elementary mathematics education, common classroom practices, curricular reform, the design of printed curriculum resources, and teacher support in common printed curriculum resources.

Sweden and Flanders have similar aims and objectives for mathematics education. Mathematics education in these two regions favours goals such as problem-solving skills, mathematical reasoning, communicative skills, and conceptual understanding.

Classroom observations in Sweden (Boesen et al., 2014) and studies of discourses about effective teaching revealed that teachers are responsive to students' needs and encourage their motivation and personal responsibility for learning (Hemmi & Ryve, 2015a, 2015b). Students typically work individually in their student textbook for the major part of the lesson and the teachers' major role is to facilitate this student – textbook interaction. In Flanders on the other hand, teachers have a proactive role in that they are supposed to organize students' encounters with mathematics. Students are typically supposed to learn mathematics by means of a whole-class introduction to the content followed by individual practice in their student textbook (Van Steenbrugge, Remillard, Verschaffel, Valcke, & Desoete, 2015).

Aside from the development of new curriculum resources, the mathematics education landscape in Flanders has been stable since the launch of attainment targets in 1998. The situation in Sweden is different in that a new curriculum has been issued in 2011. Also, characteristic for mathematics education in Sweden is a changing discourse that stresses the teacher's central role in facilitating student learning. Indeed, the discourse stresses the importance of introducing, explaining and discussing mathematical content as central aspects of teachers' classroom practices (Ryve, Hemmi, & Kornhall, 2016). Supportive toward this discourse is the enrolment of a nation-wide professional development, to which foremost of the teachers subscribed during one complete school year.

In Sweden and Flanders, as in many other regions, printed curriculum resources are (still) extensively being used by teachers and students during mathematics lessons (e.g., Boesen et al., 2014; Hemmi, Krzywacki, & Koljonen, 2017; Van Steenbrugge et al., 2015). The design of these curriculum resources in the two regions is a commercial enterprise dominated by a handful of

publishers without interference from the government (Remillard, Van Steenbrugge, & Bergqvist, 2016).

As to teacher support in printed curriculum resources, in Sweden, lesson guidelines in the teacher's guide are short and relate to one to two pages of descriptions of which half the page includes a representation of the corresponding student textbook page. Teachers are provided with descriptive information about the exercises in the student book, but with little guidance as to how to enact it (Hoelgaard, 2015; Remillard et al., 2016). In Flanders, lesson guidelines in the teacher's guide are extensive and detailed to be in line with the official curriculum. They contain clear directions for the teacher as to what to do and say (Remillard et al., 2016). Curriculum resources are typically highly structured as well. Chapters contain of numbered lessons, lessons are sequenced into lesson phases and there is little support for teachers to alter the suggested lesson structure.

Method

Our methodological approach builds on having insiders and outsiders to both contexts (c.f. Clarke, 2013; Hemmi & Ryve, 2015a; Pepin & Haggarty, 2001). We draw on interviews with ten teachers per region. These twenty teachers were interviewed twice. The first is a general interview in which we focused on teacher background and school characteristics, the instructional resources being used by the teacher, teacher's view about these resources, and teacher beliefs about teaching and learning mathematics. The second interview zooms in on one particular lesson by having the teacher to walk us through her/his planning for and enactment of that particular lesson to reveal what resources the teacher is relying on and better understand her/his motives. Interviewed teachers were using one of the two selected printed curriculum resources in each context. These resources are representative for the two contexts and express the variation of printed resources within a context. As to the Swedish context, we included one curriculum resource that provides teacher support in line with the descriptions in the previous section. We also included another curriculum resource that is gaining rapidly popularity and is more directive and structured than many of the available resources on the Swedish market. As to the Flanders context, we also included one curriculum resource that is in line with the abovementioned descriptions and a second one that differs in that the curriculum resource distinguishes between different types of lessons (teacher-steered, guided practice, and individual practice) and lessons typically address more than one content domain.

Preliminary findings

Ongoing analysis of interview data allows to center on three issues: use of a central curriculum resource, adaptations made to curriculum resources, and use of digital instructional resources.

Both Swedish as well as Flanders teachers rely on their printed curriculum resource for sequencing of the content and to ensure coverage of the curricular aims and objectives as stated by the governing agencies. Teachers do so by relying on specific features of the curriculum resource. In Flanders, for instance, teachers rely on the sequence in the student workbook, the chapter overview and/or the year overview in the student book or teacher's guide.

From the interviews, it also became apparent that Swedish and Flanders teachers adapt, sometimes replace, and complement their main printed curriculum resource. Whereas this comes with no

surprise, the reasons they do so seem to differ based on a number of factors. In Flanders, for instance, teachers working with the curriculum program in which lessons typically address more than one content domain consistently critiqued how lessons were organized and modified the sequence within chapters, and structure of lessons and even tests so that lessons and tests relate to one content domain. Teachers relate this to their experiences as to how students best learn mathematics, their personal preferences (e.g., it is easy to work with a resource that has structured lessons per content domain), and stance to learning (e.g., structure is very important for students' learning). Adaptations in Sweden both reflect the prevailing practice as well as the changing discourse (see "Mathematics education and curriculum resources in Sweden and Flanders"). Resonating with the changing discourse is that teachers working with the traditional curriculum resource extend or add a lesson's instructional phase. Reflecting the prevailing teaching practice is that teachers working with the new curriculum resource have a tendency not to skip the tasks and exercises that students can work with on their own.

Whereas foremost of the Swedish teachers make use digital resources, use of digital resources by Flanders teachers expresses much more variation. In Sweden, use of digital resources is supported in the nationwide professional development project; this might be one reason for teachers' use of digital resources. This is not the case in Flanders, and use of digital resources seem to depend in part on teachers' stance toward digital resources. When used, these resources are brought in the common lesson structure, which differentiates an instructional phase from a practice phase.

Contribution to working group

By means of describing the instructional resources used by teachers, how they use them, and relating this to factors tied to the official and operational curriculum, and the instructional resources, this contribution aims to advance discussions in the working group concerning teachers' perceiving and mobilization of instructional resources during planning for and enactment of lessons.

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WG1. Posters
GT1. Posters

The importance of documentary and digital resources in the definition of quality of higher education by its actors

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This study takes place in higher education institutions in Ivory Coast. We present interviews where the participants talk about the state of documentary resource system and its importance for quality of higher education. Questionnaires, interviews and press review were used to collect data. Results show the importance of documentary resources in higher education quality for the respondents.

Keywords: Quality, Higher education, Documentation.

Introduction

Digital and documentary resources are important sections in delivery of courses. This importance is emphasized in higher education where appropriation of courses is really done after a work with bibliographic references.

The context for this study is higher education. Many issues are decisive for higher education quality. Among these, academics training, students level, materials and equipment laboratories. In Africa, there is a lack of library and documentation resources. It seems that the use of technologies reinforces the gap between developed and developing countries in Africa. When we ask to higher education actors, generally the section of documentation center is described very poorly. This includes lack of digital connection and materials (computers, devices, tablets…). This impacts negatively the quality of higher education in Africa.

In our research, subjects evoke others issues for quality but we focus on documentation and digital resources to emphasize their importance in the definition of higher education quality by its actors. Those detail the route of documentary resources in the improvement of teaching and therefore quality of higher education. These actors giving their opinion about the quality of higher education say that an explanation for the crisis in higher education resides in the lack of documentary resources and digital devices. Digital resources going hand in hand with documentation are evoked.

Methodology

We used questionnaires about the definition of higher education quality. We wanted to know different domains in which people evoke higher education quality. They are also invited to define evaluation and its methods concerning higher education. Interviews show the same domains in the definition of quality and evaluation. The interviewees talk about different subjects and more freely, always about quality of higher education. For interviews and questionnaires, public institutions of higher education are our ground study with subjects like academics, students, administrative and technical staff.

The last method is press review where we accumulate articles in electronic press talking about higher education quality in Africa. Generally they describe the meaning of higher education quality for the persons.

Our research emphasizes the importance of documentary resources in the definition of higher education quality. The data extracted (questionnaires, interviews, articles) have a link with the place of library, documentation center and resources in higher education.

Results

Results concern especially interviews and questionnaires where documentary resources largely contribute to development and improvement of courses. Our study focuses on documentary center and resources according to students, academics and administrative staff. The results show that quality in this domain is dropping, with insufficient resources, scarcity and non-existence of digital materials in higher schools. Press review confirms this trend.

Academics related the fact that it isn't easy to prepare their courses because of a lack of resources, and to do some courses because the working environment (Ruthven, 2008) doesn't fit with the use of digital technology and material. Teachers' practices are various. It's very important to have availability of digital resources for a metamorphosis of curriculum material (Remillard, 2005; Monaghan, 2004). This isn't the case in higher education in Africa. Teachers can't appropriate themselves the documentation (paper and digital) to adapt it for the course. It stays at the stage of artefact, and can't be seen as instrument (Rabardel, 1995).

Conclusion

Documentary resources are a decisive issue in higher education. This notion is evoked by the actors of higher education who relieve its importance in quality of higher education. Others tendencies show that, according to participants of the study, teacher's resource systems evolve very slowly because libraries are empty and there isn't enough digital resources for the new society of knowledge, which require students and teachers well equipped.

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Resources and didactic decisions of a teacher in the teaching of combinatorial analysis

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This study aims to identify the resource system of a mathematics teacher at the levels of their activity and their relation with the didactic decision making on the teaching of combinatorial analysis. The theoretical framework is based on Trouche & Gueudet (2010), Margolinas (2002) and Bessot et al. (2013). The reflexive methodology is used in the analysis of the documental work of a teacher in Brazil. We demonstrate how decisions in n +1 are influenced by epistemic factors and by the history of teaching and relate to the documental trajectory and to the system of teacher's resources in n+3 and n+2 of their activity.

Keywords: documentational approach of the didactics. Levels of teacher activity. Didatic decisions. Combinatorial analysis.

Introduction

What resources are mobilized at different levels of teacher's activity? In what way does the choice of a resource in the preparation of a lesson is inscribed in the system of resources of the other levels of the teacher's activity? What factors influence the didactic decision-making from the choice of a given resource? These questions guided the present study.

Theoretical framework

The documentational approach of the didactics (ADD) was inspired by the instrumental approach - AI (Rabardel, 1995). Because AI distinguishes between what is available for the activity (the artefacts) and what is developed by the subjects (the instruments), the ADD distinguishes what is available for the teachers' activity, the resources, and what they develop to support their teaching activity: the documents (Gueudet & Trouche, 2010). This work is restricted to the first three levels of the teacher's activity (Margolinas, 2002): n+3. Values and conceptions about teaching/learning; n+2. Construction of the theme and n +1. Lesson plan. Given only macro-decisions, those that take place outside the classroom. Based on two categories of factors that influence didactic decisions (Bessot et al., 2013): *epistemic*, which nourishes the teacher's personal relationship with institutional, epistemological, didactic and pedagogical knowledge, and of *didactic history* (between teacher and a particular student, students of a class, or in general).

Methodology

For two months, we followed the documental work of a mathematics teacher of the2nd year of high school for the teaching of combinatorial analysis in a public school in Pernambuco - Brazil, in a continous way and based on the reflexive principle of the information, during data collection process (Gueudet & Trouche, 2010). Using the following instruments: in n+3 (documental trajectory, schematic representation of the resource system and interviews); n+2 (class diary, RSSR, interview) and n+1 (idem n+2).

Results

In n+3, we highlight the teacher's documental trajectory, resources related to interdisciplinary projects developed at the school and presented at the event “Feira Brasileira de Ciências e Engenharia”. In the RSSC, we highlighted: the curriculum, the textbook and specific websites. These resources were taken up by the teacher in n+2 with better visibility regarding the preparation of sequences on "combinatorial analysis" (ex: multimedia mathematics site, games, videos). On n+1, in the analysis of the choice of the resource "Taxi and Combinatorics (multimedia mathematics site)[1], we identified didactic decisions related to epistemic factors such as: didactic (eg., experimental activities), epistemological (eg., relation between mathematics and other disciplines and their application in everyday life). And factors related to the educational history of students in general (eg., students think it is possible to find the correct solution of a problem by a single technique) and the students in the class (eg., difficulties in the concept of "combination"). These results show elements of n+3 and n+2, where the "interdisciplinarity and applications of mathematics" were relevant in the teacher's work.

Final considerations

This study is in progress. However, we believe that early results on a view of the teacher's resource system at different levels of their activity may lead us to better understand some factors that influence their teaching decisions.

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The textbook for field public schools in Brazil - PNLD field

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Many teachers priviledge the textbook as a resource. Among the reasons for this protagonism are the lists of exercises that they provide and their feature of a handbook to assist the teacher. In Brazil, the strengthening of the National Textbook Program (PNLD), which includes the evaluation and distribution of teaching materials to help teachers from public schools, is also one of those reasons. At the center of this theme, we present some characteristics of the PNLD Field aims especially at students and teachers of the initial years of the elementary school in the Brazilian rural area schools.

Keywords: Textbook, PNLD Field, Field Schools, Mathematics Teaching.

The Book as Didactic Resource

Several researchers of mathematics didactics have studied the resources used by the teachers, as well as the processes of choice and construction, whether individual or collective, using the didactic documentary approach as theoretical framework (Gueudet & Trouche, 2009; and Trouche, 2015). In this context, a resource is defined, in the broadest sense, as an artifact or a set of artifacts that can assist the teachers in their professional development. The textbook is one of such artifacts. Researchers such as Pitombeira (2002) and Nogueira et al. (2017) show that the presence of the textbook is remarkable in Brazilian schools and, consequently, influences teachers' activity, what is revealed through the conceptions and knowledge mobilized by the teachers at various moments of their activity.

This scenario emphasizes the relevance of the debate about both the role the textbook plays in the teaching and learning processes of the contents and concepts worked in the different areas of knowledge, including mathematics, and the teacher who uses it. In fact, Skovsmose's (2014) studies highlight how relevant it is that teachers recognize the plurality of realities of their students, with a view to relate the teaching of mathematics to the construction of full student citizenship.

Materializing teaching in this perspective necessarily passes through the didactic choices of the teacher. This raises a complex question, because the possible solutions depend on several factors such as, for example, the types of didactic resources constructed or chosen. In the case of textbooks, we can question both the knowledge mobilized by the authors in the elaboration of works and their epistemological, didactic and methodological choices.

The Textbook in PNLD Field

Moving forward in this debate, we present some characteristics of the Programa Nacional do Livro Didático - PNLD Campo (National Textbook Program - PNLD Field) (Brasil, 2011,

2012)[1], a Brazilian public policy that aims to assist the work of public school teachers, including the education of young people and adults, through the evaluation and distribution of didactic materials, mainly textbooks, for the teaching of knowledge areas. The works are included in the program through public notices and are submitted to an evaluation process that includes both the graphic and editorial aspects regarding the adequacy of the language used, methodology, activities proposed and conceptual correction, among other aspects.

In 2011, the first public notice was published for the evaluation of didactic works destined to the teaching of the initial years of elementary school in schools located predominantly in rural areas, called *escolas do campo* (field schools), and, in 2013, the second and final public notice was published (Brasil, 2011, 2013). This public policy resulted from the demands of social movements that, through field education, strive for a socially referenced quality education for rural area people (Caldart et al., 2012).

The evaluative process gives rise to the TextBook Guides[2], which are intended to assist the teacher in the choice of books. In the first edition of the PNLD Field, only two collections were approved out of the eighteen enrolled, and two in the second edition, out of the ten enrolled in the Program. It is worth mentioning that only one of the collections compose the two editions of the Guide. The significant number of excluded works gives important clues that there is still a long way to go in building textbooks for field schools.

Final considerations

The almost continental extension of Brazilian geographic territory, the ethnic-racial and socio-cultural diversity, as well as the plurality of means of production and productive cycles make the task of constructing didactic materials to assist public school teachers in the rural areas a great challenge. This scenario requires that teaching resources be produced by people (researchers, teachers, rural area people...) who know and recognize the realities of the Brazilian rural area with its difficulties and potentialities, aiming to contribute effectively to the teaching and learning processes and, above all, with social transformation.

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A comparative study on the content and use of mathematics teaching resources between China and France: from the exercise perspective

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This poster aims to present the mathematics resources on exercises that teachers use in China and France. The structure of the poster is situated within the framework of the Documentational Approach to Didactics. The poster mainly contains three parts: 1) try to present the exercise resources that teachers use; 2) figure out how the teachers design and organize exercises; 3) discover the similarities and differences of Chinese and French exercise training system.

Keywords: Mathematics teachers' resources; mathematics textbooks; exercises; variation problems; documentational approach to didactics.

INTRODUCTION

The generalized availability of digital resources for mathematics teachers entails a complete metamorphosis of curriculum material (Remillard 2005). It also yields a deep change in teachers' professional knowledge and development (Gueudet & Trouche, 2009). Besides textbooks, teachers use various resources to design and organize their teaching. Therefore, it is essential to study mathematics teaching resources at present.

Exercise is a critical resource for mathematics teaching and learning. Most of mathematical concepts, principles and methods are abstract, exercise becomes a concrete approach to make students comprehend the concepts, principles and mathematical methods (Wang & Bao, 2014). Accordingly, mathematics teaching is always accompanied by the training of exercises. Focused on the value of mathematical education and the demand of reducing unnecessary academic burden, it is necessary to improve the quality of exercise.

The mathematics teaching in China and France are both in a time of transition nowadays. The main purpose of this poster is to present a general view of teachers' exercise resources and figure out how the teachers design and organize the training of exercises. At the same time, the author illustrates one typical training form in China (exercise variant training). The main research questions of this poster are as follows: 1) What are the teachers' main curricular exercise resources? 2) How do the teachers design and organize the training of exercises? 3) Comparing Chinese mathematical teaching with that of French, what are the similarities and differences of their exercise training system?

METHODOLOGY

The structure of the poster is situated within the framework of the Documentational Approach to Didactics (Gueudet, G., & Trouche, L., 2009), focusing on the documentation work of the teacher.

[1] Paper written in the frame of a PhD co-supervised by Luc Trouche at ENS de Lyon and Jiansheng Bao at East China Normal University, began in September 2017

Teachers' main curricular exercise resources

Questionnaires study are carried out here to find out mathematics teachers' main curricular exercise resources. The questionnaire should propose open questions. By combining this result with the relevant literature, we can have a general picture of teachers' main curricular exercise resources. Author chooses the research objects for the following questions from these resources.

Teachers' design and organization of the training of exercises

The author chooses textbooks (three editions for each country), teachers' guide books, exercise books, e-books, associated exercises online to study. The study in this part mainly contains two parts: the structure of the resources and illustrations of variation problems cases. The author firstly classifies different parts and gives a definition of each part, then illustrates what activities involved in this part, the hierarchy of exercises (examples and exercises on the same section or chapter, or one main question with several sub-problems whose difficulty increases progressively), and at last chooses specific items (variation problems cases) to analyze. Text analysis will be used here.

Comparison between Chinese mathematical teaching and French

The comparative analysis between China and France is conducted under the framework of the paper (Pepin, Gueudet, & Trouche, 2013). By doing this, the differences and similarities of Chinese teachers and France teachers can be explored by comparative analysis, and some feasible suggestions are hoped to be put forward to benefit teachers' documentation work and professional development.

FURTHER STUDIES

To make the study more comprehensive and credible, we should conduct interviews and observations of teachers for further studies. Based on the questionnaires results above and objective conditions, some typical teachers will be interviewed specifically and observed about their documentation work on exercises. After that, the result of text analysis can be collated with that of observation, to see if they support each other.

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WG2. Analyzing teachers' work with resources, methodological issues / GT2. Analyser le travail des enseignants avec les ressources, questions méthodologiques

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Analyzing teachers' work with resources requires considering the work of teachers 'as a whole'. Actually, teachers' interaction with resources is deployed in space and time. This involves taking into account activities in and out of the classroom, at home, in the lab rooms, computer rooms, etc. In addition, a teacher interacts with his/her resources from previous years, for different teaching objectives, derived from his/her involvement in different collectives over time. All of these interactions with resources determine his/her teaching experience and his teaching beliefs. Furthermore, redoubtable methodological issues would have to be taken into account, with consideration on the teaching content knowledge and on the level of teaching. The "versatility" of primary teachers determines their interactions with resources in a different way, than for secondary teachers, or university professors who have generally a dual role of teacher and of researcher.

- How do we analyze teacher interaction with resources 'as a whole'?
- How do we analyze the structure of the resource system?
- How does this interaction take shape according to teaching contents and to levels of teaching?

Analyser le travail des enseignants avec des ressources suppose d'appréhender leur travail comme une totalité. En fait, l'interaction des enseignants avec les ressources de développe dans l'espace et le temps. Cela implique de prendre en compte les activités à l'intérieur et à l'extérieur de la salle de classe, à la maison, dans des laboratoired, des salles d'informatique, etc. En outre, un enseignant interagit avec ses ressources des années précédentes, pour différents objectifs pédagogiques, à travers son implication dans des collectifs différents au fil du temps. Toutes ces interactions avec les ressources déterminent leur expérience d'enseignement., et leurs convictions personnelles. Des questions méthodologiques complexes sont alors à prendre en compte, dépendant eu contenu et du niveau d'enseignement. La «polyvalence» des enseignants du primaire détermine leurs interactions avec les ressources d'une manière différente de celle des enseignants du secondaire et les enseignants de l'enseignement supérieur combinent un rôle d'enseignant et un rôle de Chercheur.

- Comment analyser l'interaction des enseignants avec les ressources « dans leur ensemble » ?
- Comment analyser la structure des systèmes de ressources ?

- How do we study in the long term the evolution of the interactions of teachers with resources?

- Comment cette interaction prend-elle forme en fonction des contenus pédagogiques et des niveaux d'enseignement ?

- Comment étudier à long terme l'évolution des interactions des enseignants avec les ressources ?

WG2 Oral Communications / GT2 Communications orales

The Documentary Approach to Didactics in a Flipped Classroom Proposal

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This paper presents a study derived from a proposal that suggests a connection between the Documentary Approach to Didactics (DAD) and the Flipped Classroom (FC), goal of a research project approved and developed in a Virtual Learning Environment (VLE) at the Pontifical Catholic University of Sao Paulo. The study analyzed the choice, preparation and development of resources and the documentary work performed by teachers involved in this Hybrid Teaching proposal, i.e., the Flipped Classroom. We found that the didactic decisions made to produce and transform the resources into documents were strongly influenced by the teachers' prior knowledge from a mathematical, technological and didactic point of view.

Keywords: Hybrid teaching, Documentary approach to didactics, Flipped classroom, Virtual learning environment.

Introduction

The proposal of this study is part of the Working Group 2, of the Re(s)ources 2018 Conference as it aims to analyze the work of a group of teachers and the respective connections with some resources, based on the Documentary Approach to Didactic (DAD) and a Flipped Classroom (FC) proposal. This study is part of a research project developed by two teachers and the author of this article in a Virtual Learning Environment (VLE) at the Pontifical Catholic University of São Paulo (PUC-SP)[1].

The main goal of the project was to analyze Hybrid Teaching models that may be applied to contents that are Math prerequisites in Higher Education using GeoGebra, a free software program, as technological support. The specific goals were first to investigate an FC model that involves both classroom and online teaching using the Moodle Environment and second to identify if the strategies applied created favorable conditions to teach and learn certain Math contents.

The study presented in this article, elaborated by the author, highlights didactic decisions taken during the preparation of classes and the resources developed by the team for a Hybrid Teaching proposal, i.e., the Flipped Classroom. The following questions guided this study: what resources are available to teachers who take part in an FC proposal that includes the use of a VLE? What resources are actually taken into account? What documents are created based on the choices made? What modifications are required to be able to use them in a VLE?

[1] This project was set up to answer a call made by the university to assist them in performing a Research Applied to Teaching (PIPAD).

Hybrid Teaching and Flipped Classroom

The study presented in this article started as the teachers became involved with the Hybrid Teaching methodology. By studying this topic, different types of Hybrid Teaching, which according to Christensen et al (2013, p. 2) are *formal education programs that combine online teaching with traditional schools*, were investigated and one of them, the Flipped Classroom, was chosen and used in the project developed. It is also the object of study of the present article.

Christensen et al. (2013) state that teachers interested in adopting hybrid teaching are given two options: the *sustained* version that allows to profit from "the best of both worlds", the advantages of online teaching and the benefits of the traditional classroom, and the *disruptive* version, which adopts online teaching based on new models that move away from the traditional classroom. The authors state that, in any case, the hybrid education solution may certainly become a future educational model.

Among Hybrid Teaching models, we selected a rotation model of learning called *Flipped Classroom*. It allows students to use their own time to read texts and to study at home, i.e., to get familiar with topics that will be treated on the occasion of a face-to-face meeting with their teacher, who then discusses and processes contents previously tackled by students, probably in a VLE, thus playing a more critical role rather than just transmitting content. Teachers are also given the opportunity to evaluate students before they meet, as the students who study topics in a VLE also perform a series of activities that can be checked by the teacher, thus guiding his future teaching actions.

That methodology was applied to our project as follows: students accessed certain Math contents in a VLE before meeting their teacher in person, developed assimilation activities and were asked to solve some given tasks. After that, they discussed their questions and difficulties with their teacher face-to-face. Their meeting, in this case, was not a lecture, but rather a moment of reflection, discussion and student learning.

Creating Documents

Documentary work for using an FC involves resources that may be either available or that need to be created, developed, or transformed into documents that match both the teacher's standards of use and his experience.

Creating documents is a continuous process and occurs when resources become documents according to both the teacher's standards of use and his experience which, regarding the FC, involves prior knowledge in terms of Math, didactics, face-to-face teaching and VLE and last, but not least, material that supports developing the proposal.

A wide range of resources has to be selected by the teacher, both individually and collectively, which involves textbooks, videos from reliable sources, software programs, tutorials etc., in addition to a VLE, made available in an institution with its specifics. Hence, mathematical, didactic and technological knowledge is fundamental to develop and transform these resources into documents.

Trouche & Gueudet (2015, p. 14) claim that much of a teacher's documentary work takes place outside the classroom and is often created in a private space, the "hidden part of the teacher's work," but always in interaction with others and keeping in mind the final purpose of these documents. All teachers involved in the project developed their work individually and after that, the proposals were discussed and improved with the team. The teachers' interaction with the resources is determined by their teaching experiences and the knowledge acquired over time. This context was present in the situations described below.

Choosing Math contents was an object of study one of the teachers selected, taking into account the topics that would be studied by eleven volunteer students of a Computer Science course in the following semesters. In this regard, she needed to analyze the course schedule and the respective subjects that would be attended by the students and use that as a data source that could help her create her activities, select the texts and use GeoGebra.

This experience was shared with two other teachers who, as a team, started to choose the resources they would make available in the VLE. A first challenge involved finding reliable digital resources that could be used and as a result, a free virtual library was put at their disposal by the National Publishing Group (GEN)[2] in response to their request. The team selected videos that were related to the defined contents and three e-books that were available at a digital library and could be accessed by students using a login and an individual password, which would be necessary to develop the project.

According to Rabardel (1995), these actions reveal instrumentation, as they guided the activities of the teachers in choosing their resources. In configuring GeoGebra's required tools aimed at student activities, we identified that instrumentalization.

The documentary work of the teachers also shows in the reading of the texts and the watching of the videos. The videos had to "dialogue" with the texts so that the students could safely learn the contents and be able to solve the tasks.

The FC proposal was developed in two different spaces: in the Moodle environment and in face-to-face meetings. Moodle was used to provide students with the documents that guided their activities, the texts with their Math contents, the GeoGebra resource, the nine videos and the steps to be followed by means of a "learning path", a detailed script of all activities that had to be developed before the face-to-face meeting and according to the Flipped Classroom methodology.

Documents were created for five different topics and nine videos were included. Videos could be accessed via Moodle and were rather long in duration compared to the standards found in the literature on didactic videos. However, students were advised that they could pause them or watch them in stages according to their availability. To look up e-books, students were redirected to the respective pages containing texts and activities. According to the FC model, after students study on their own, personal meetings with their teacher take place. Figure 1 shows the learning path of Topic 3 of the five topics developed.

[2] Grupo Editorial Nacional GEN [National Publishing Group] - https://www.grupogen.com.br/

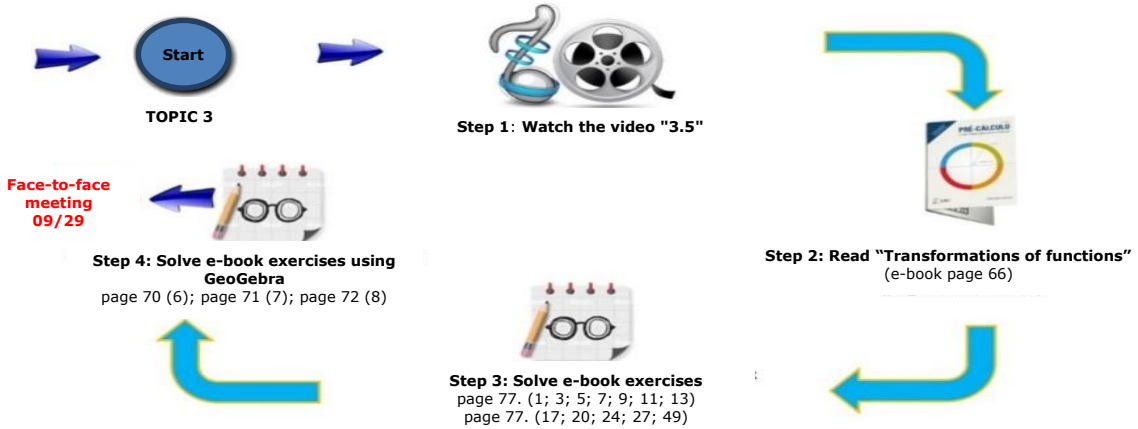

Figure 1: One of the learning path available on Moodle

Moodle configuration required additional resources to create new ones, such as guidelines in the form of "learning paths", tutorial videos on how to use GeoGebra, setting up question and presentation forums, documents containing important information about the project schedule, the final evaluation questionnaire, and feedback regarding the project proposal.

Final considerations

We conclude that to create new teaching proposals, one needs to organize, to be skilled in finding and collecting information, and to identify the most appropriate ways to process resources, taking into account the documentary approach to didactics so as to favor learning.

The resources used were specifically aimed at the FC proposal using a VLE containing videos, e-books and activities. Based on these choices, we may infer that the work plans developed for each class are documents that may be adapted to guide new proposals.

This study made us aware of the fact that the prior knowledge of teachers was crucial to develop and carry out the project, as well as to select the right resources. New knowledge was needed to create documents that met the FC proposal, that were innovative, and that supported the proposed contents.

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Between designing and use, how to address the question of resource's coherence ?

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Instrumented by a resource, a teacher develops his own project for the class. As designers of resources, we wanted to present in this paper a framework for the analysis of the convergence between projects' authors and those of teachers using these resources. We use the framework of the documentational genesis and structuring milieux in a didactical situation to define an indicator of convergence that will allow us to define the concept of resource's consistency; we propose a didactical engineering in order to study the differences between the intentions of the authors of a resource and the teacher's achievement in the classroom. Then we give an account of the test of this method in the context of an experiment.

Keywords: resource's coherence, documentational genesis, structuring milieux, experimental approach, research situations in the classroom, didactical situation

Introduction

Reflection on resources, their creation, their uses and their quality is not new, but is becoming more and more important when the diversity of the offer and the modification of the supports imposes choices. In this proposition, we are interested in the enrichment of a teacher documentary system when he/she integrates a resource in his/her documentational system and the relationships between the intentions of the resource authors and the way a teacher uses it. The concept of coherence, as often defined (Newmann & al., 2001 ; Schmidt & al., 2005 ; Boning, 2007, Mangiante-Orsola, 2012), concerns a global logic within a curriculum or in teachers' practices. It does not allow to conceptualize the relationships existing between a resource, necessarily constructed by authors with particular intentions, and its actual uses. From this perspective we propose a definition of the coherence of a resource in situation in order to identify fundamental elements for the analysis of the relationships of the resource use and the resource conception. To model this coherence, we will rely on the articulation of two theoretical frameworks: the documentary genesis (Gueudet & Trouche, 2008) and the structuring of *milieux* in the theory of didactic situations (Brousseau, 1986).

Theoretical frameworks and methodology

The structure of the milieu which has been proposed by Brousseau (1986) and extended and clarified by Margolinas (2004) and Bloch & Gibel (2011) gives a framework for analyzing both the teacher and the students' work. The teacher plays a fundamental role in different positions: designer, when he/she elaborates her didactic project, observer, when students are working in a a-didactic situation or teacher, when organizing the devolution of the situation and, later, the institutionalization of knowledge. In our work we consider the point of view of the teacher and we

use the structure of the *milieux* within a "decreasing analysis", that is to say an analysis starting from the noospherian teacher confronted to the designing milieu in a situation not yet finalized. Margolinas (2004) proposes the concept of didactic bifurcation: when a teacher designs a situation for the pupils, he/she projects his/her intentions of teaching, that is, his/her willingness to change the pupils' knowledge system. He/she then constructs a didactic situation based on a milieu. In their position, the pupils are unaware of the teacher's intentions, but they can imagine them and in turn project their own a-didactic situation by relying on the physical environment that the teacher has build in an objective situation. There is a bifurcation when, confronted with this material environment, the pupils invest a situation different from that envisaged in the intentions of the teacher. The model of the structure of the milieux was not built to take into account the authors of resources. Yet, in the process of constructing resources, the authors are part of the set of potential didactic situations that can be built on this resource. We propose to add to the model of the structure of the milieux an additional point of view, that of the Author in interaction with the structure of the milieux of didactic situations. We then define the concept of coherence of a resource in situation: the teacher, in the situation of construction, confronts himself/herself to the resource and instrumentalises it to build his/her own document, support of a didactic situation. He/she then interprets the resource, as a musician interprets a musical work, translating the intentions of the resource for his/her own project. In a noospherian situation, the teacher interacts with the design milieu including his set of resources. In particular, the teacher interprets the intentions of the Author of a resource through the interactions with the resource. Following a process similar to that of the didactic bifurcations explained in the previous paragraph, the construction situation may carry intentions distinct from those of the resource to lead to a different didactic situation of the didactic situations potentially carried by the resource. In this case, we will speak of a bifurcation of construction. The coherence of a resource in a situation is then judged by the non-bifurcation of construction. We use the concept of didactic engineering as a method based on didactic realization, validated by a confrontation of an *a priori* analysis and an *a posteriori* analysis. The *a priori* analysis is built around the analysis of the resource, and a collection of the Author's intentions. The *a posteriori* analysis is built on the observation and analysis of the implemented situation. The confrontation between these two analyzes allows to conclude at a bifurcation or not, that is to say to the non-coherence or the coherence of the resource in the considered situation. After having defined the concept of coherence of a resource in situation and proposed the method, we'll apply to a particular case study taking the resource "Exprime" (Aldon & al., 2010) as an example, allowing both to study the coherence of this resource and the consistence of the concept.

Case study

In order to consider the coherence of the resource in situation, we begin to analyze the intentions of the Author before considering how a teacher includes it in his/her set of resource and how he/she uses it in the design of a didactic situation.

Author's intentions

We based our analysis of the Author's intentions on the text published within this resource as well as on a direct contact with authors. In the frame of problem solving and open problems studied in the IREM of Lyon (Arsac & Mante, 1988, 2007), it appears as a necessity to consider the

relationship to the mathematical objects at stake in a mathematical situation and the construction of knowledge. The Author of the resource takes up the socioconstructivist framework. To assert that every student constructs his knowledge has led him to produce problem-solving situations in which the objective milieu is such that the student can engage himself or herself in a process potentially leading to constructions of knowledge that is to say a structured integration of new elements into a dynamic system of thought. The author considers, on the one hand, that mathematical activity relies on a field of experiences, consisting of a set of objects sufficiently familiar to enable prospecting and, on the other hand, that the relevance of the objects products in the activity judges at one time or another, to the quality of the feedbacks they produce on the "reality". Thus, for him, essays, formulations of conjectures, attempts of proofs can only appear meaningful when the student, acting on mathematical objects or some of its representations, material or not, produces new, relevant knowledge in the sense that they allow a new understanding of the mathematical situation. Thus, for the Author, the study of the relationship to mathematics is directly linked to the consideration of actions on objects, to the result of these actions and to the analysis of the actual and felt relevance of the activity, in terms of effectiveness in understanding the studied phenomena. This focus on mathematical activity strongly linked to objects leads the Author to put forward situations that should favor the return journeys between experiments on mathematical objects, formulations of conjectures, elaborations of reasoning, demonstration, algorithms.

Analysis of the case study

In this paragraph, we confront the intentions expressed by the Author with the actual construction of a didactic situation by JFC, the teacher who has agreed to implement a problem-solving situation by relying on the resource EXPRIME. The constraints which he undergoes or which he takes in the institution prevents him from setting up a particular pedagogical organization:

> I did things, ah, some problems that were a bit open, I will say, with a class organization that is not a group work, but a classical organization... (teacher's interview)

JFC clearly states his difficulty in considering the progression of the prescribed curriculum under this teaching practice. In the situation of project, the teacher uses the resource to plan the didactic organization of the session, based on the problem chosen and on the sessions already proposed:

> I showed them an open problem that of the two straight lines that cut off the sheet and I told them, well this is an example of an open problem, they'd love to, they have searched a bit, we talked about it again, I proposed a solution and it was fine because we had just learnt the homothety with complex numbers so it was dead on time. (teacher's interview)

The extract also shows JFC aiming at a specific objective, beyond the plurality of possible approaches. In this situation of project, JFC puts himself in the position of the students to see what they could do, the resource appearing as an help to complete the analysis of the mathematical situation. JFC uses the resource to refine his skills to organize a research situation in the classroom. In the didactic situation, the teacher presents his situation to the students by following a scenario proposed in the resource. He adds, in accordance with his particular noospherian position, a reflection on meta-cognition. JFC in the didactic situation quickly places his pupils in an "over-didactic situation" by anticipating *a priori* a reflexive return on their "cognitive adventure". During

the phase of research in the classroom, the teacher, as an observer of the situation, measures the distance between his intentions in terms of the progress of didactic time and the actual work of the pupils but maintains a willingness to start from the students' proposals; all the dialogues and observations show the tension between the desire to propose to students a parametrization of the problem and the desire to let the students explore the tracks they wish to explore:

> I was a bit surprised that it was so long to start and I was also surprised, [...] that they do not set the problem faster. Uh well it's a good awareness... (teacher's interview)

However, one can think that the teacher enlarges his repertoire of didactic answers in such a situation, and completes his documentary system even if the resource did not prevent this possible difficulty. In the implementation of the situation, JFC resolutely places himself in a potential situation proposed by the resource and orchestrates a research situation that joins in many aspects the expectations of the Author.

Conclusion

This study allows us to implement the concept of coherence of a resource in situation and makes it possible to envisage its use in wider fields. Beyond the specific didactic engineering, the concept of coherence will have to be tested in other contexts and with other resources. The first field of experimentation will have to deal with contexts of ordinary class, that is to say outside the construction of a specific engineering. It is in this context that we can, through the study of the coherence of a resource in situation, consider expanding the subject to the study of the dissemination of resources and their impact on education of mathematics.

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Institutional resources at university admission: emerging documentational identity for the analysis of the professor´s work

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Professional identity is an evolving construction in which various elements converge. How does it relate to the work of the professor around institutional resources? From the case study of a professor of mathematics at university admission, we will analyze this relation focusing on his identity traits. A construct of the research, which we will call documentational identity, *will emerge as a first result, and is our contribution to the discussion of the WG2 organized on the occasion of the 10 years of the Documentational Approach to Didactics.*

Keywords: Documentational Identity, Documentational Approach, Professional Identity, Institutional Resources.

1. Introduction.

This article, part of a doctoral thesis in progress, is our contribution to the reflection on the documentation work of the professor. Our central question is "What is the relation between document genesis and identity genesis?" In response, we will present the notion of documentational identity. We will begin by stating some aspects of the theoretical trends that we integrate, and the notion of resource that we consider a bridge in bringing them together. We will then introduce the most salient features of the methodological design in our case study: the choice of professors and work at university, the tools used for the intentional data collection, and essential aspects of the negotiation of the methodological agreement between researcher and actors. Next, we will focus our analysis on the self-confrontation interview to a professor to analyze the use of the institutional resources proposed for university admission in Argentina. This way, we will present some features that characterize what we call documentational identity.

2. Theoretical framework.

We use the term *resource* in the sense of "re-source": "return to the source" as an element where life arises from. In agreement with Adler (2000), we assume that it feeds the work of the professor, that it is a source for his work and at the same time re-nourishes his activity; and this is not exclusively a material thing: it can be exchanges with colleagues, reflections on a class event, etc.

We analyze the integration of resources as a human activity organized around two poles: the triad instrument-scheme-situation and that of experience. The first pole was problematized by the instrumental approach, and at the same time integrated to didactics by the *documentational approach to didactics* –hence ADD-. The document genesis allows us to explain the process of transforming a resource into a document. Synthetically, professors interact with resources: they select, reorganize, modify them and produce new ones for teaching. Both this process and its result are called *documentation work* (Gueudet & Trouche 2008).

The pole of experience involves two stages: the accumulation of the past and the appropriation that the person makes of it. From the productive aspect, the person seeks to transform reality and with this action obtains tangible products; from the constructive aspect, it implies the transformation of the person him/herself; an intangible process, not necessarily intentional. The activity of the professor is developed around resources and can be seen in its productive phase - the generation of concrete products for teaching - and in its construction phase - how this task also shapes the work of the professor, his own identity as a teacher. As the documents geneses explain how the resources are integrated, the *identity geneses* (Pastré 2005) explain the personal process of developing a professional identity.

3. Methodological framework.

As a methodological model, we adopt *reflective investigation*, rooted in the ADD (Gueudet & Trouche 2008). This model is based on the principles of long-term follow-up, inside and outside the classroom, wide range data collection and reflection on documentation work. We will briefly present some elements of the experimental field where the research is carried out; some decisions that justify the selection of the studied cases and a brief description of the profile of the case study discussed here.

3.1. Experimental field.

The UNAJ is a newly created university located on the outskirts of Buenos Aires, Argentina. We will consider the mathematics professors in charge of the initial courses for admission at higher education level. It is a group of about 50 teachers with diverse profiles: engineers, architects, biotechnologists, mathematicians as well as mathematics teachers. The work with the students is developed on the basis of a textbook as an institutional resource which guides teaching, other materials - called guidelines- that contribute to the development of teacher practice, and collective workspaces. To encourage participation and actively support the teachers, we invited them to join the research and some of them have voluntarily taken on the commitment of being part.

3.2. Tools for data collection.

Based on the type of methodological agreement established between researcher and teacher (Sabra 2011), we have used various tools for the intentional collection of data: semi-structured in-depth interviews - initial, class preparation, self-confrontation and reflection-triggering strategies on the professor's work (as the proposal of drawing a schematic representation of resource system -SRRS for its acronym in French). In this communication, we will focus on the analysis of some excerpts from the first self-confrontation interview to begin to characterize the traits that constitute professional identity. Previously, we will account for the profile of the case study presented here, taking elements of the initial interview and class preparation.

3.3. Claudio´s case.

Claudio is 58 years old. He is a professor of mathematics, physics and cosmography. He began his teaching career some 35 years ago as a laboratory assistant in physics. He had initially started studies in electrical engineering, but then decided to train as a professor. For several years now, he has been teaching mathematics in different university institutions. At UNAJ he participates in a

collective around the production of resources for virtual classrooms. During the class preparation interview, he showed the main resources he uses for planning. In Figure 1, we can see the centrality of the institutional resources - we mean the textbook and the guidelines -.

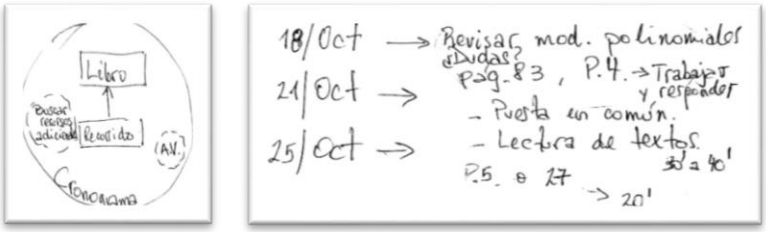

Figure 1: On the left, Claudio SRRS. To the right, his notes on the organization of the topic.

Also, how he "peripherally" indicates additional resources to which he turns to, as well as the space of the virtual classroom - in the SRRS identified as A.V. - on which he relies to extend the work done during the classes. Finally, appears the reference to the schedule of the course that, according to his notes, would seem to be a key element in the organization of the topics and scope of each class.

4. Data analysis and discussion.

Next, we will present some excerpts from the self-confrontation interview with the purpose of providing an account of the elements that make up the professional identity. This was done after Claudio had seen the film recording of his class on the revision of polynomial functions and the introduction to the exponential models, and he had e-mailed us some previous reflections that will later on be recovered during the dynamics of the interview. Here we will focus on the first part about the polynomial models.

According to the SRRS, the textbook seems to be of prime importance in the preparation of the class. However, he begins with a problem of his authorship. He thus justifies this decision:

Investigator: (...) you started off with a skateboard ramp problem. Did you get it from somewhere else?

Claudio: No, no. I invented it.

Investigator: Ah. Okay. In the e-mail you say: "My expectation was that they could relate the questions or the values of the variables to the graphic peculiarities of the model. For example, how to calculate the distance to the edge of the ramp (...)"

And he later expands on the sense of this work:

Claudio: We work on two types of problems with quadratic models. One departs from the description of the situation to arrive at the expression, as in the case of the box that is in the textbook, but in the case of the ramp it is not so easy to reach (...) I provide them with the simulated model straightaway. What I want, what I ask is that they draw the consequences of the expression, for example the roots. (...) I do not ask them for the root, I ask them about the distance to the floor (...)

These expressions illustrate the teacher's autonomy to interweave the problems of the textbook (institutional resource), with others of his “authorship” that allow him to assemble different elements necessary for mathematical work. They show a certain “sensitivity” to what he perceives from the students' work and their needs:

Claudio: (...) In order to make an analysis of the model itself, of the consequences, of the model as a variation of a quantity relative to the other, basic capacities are required otherwise the student cannot make any analysis. I'm worried. I see it in the development of the class, they (the students) do not know how to solve a linear equation of the simplest kind ... well, it seems simple to me and I am asking them to perform an analysis of the variation of the function from the graph (...)

These actions reveal elements that would allow beginning to delineate the components of a professional identity. The accumulated experience and the way in which Claudio explains it, allows considering a strong component of autonomy in his identity that enables him to behave with freedom and to make decisions in his own way. His openness to take into account the students’ demands shows a feature of flexibility to integrate them into the development of the class and thus modify its original plan. These reflections allow us to think of the intertwining of the documents geneses with the identity geneses, their mutual influences, and how this characterization of traits shapes a way of being. The set of these traits is what we call *documentational identity*.

5. Perspectives.

The continuity of the research promises the emergence of new elements to integrate into this characterization. For example, those elements which shape the collective work dynamics within the university - not present in this text, but in the research - would allow us to analyze the sense of belonging and the extent to which an individual relates to an institutional proposal. They will also require a re-negotiation of the methodological contract to incorporate some tool for the intentional collection of data, which would account for these traits. At the same time, in order to continue analyzing the place of institutional resources, a new interview is planned in which Claudio will be asked to consider the possible substitution of a textbook. This strategy seeks to reveal how a situation of destabilization affects the professor's resource system requiring its reorganization, and allows highlighting the substantive features of his documentational identity to face this situation.

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Preparing to teach mathematics: results from a survey on mathematics teacher resources in the united states

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Planning for a lesson has long been recognized as a primary factor impacting the efficacy of instruction. Significant to such planning are the resources used which shape the mathematics content and processes teachers intend for students to engage in and develop. The author presents results from 154 Grades 6-12 (student ages 11-18 years) purposively sampled U.S. mathematics teachers surveyed on lesson preparation and lesson plans. Results indicate teachers utilized an average of 2.6 resources with course textbooks, the internet, and math colleagues the most common resources. Furthermore, pairs of resources tended to be utilized together, and the number and types of resources used differed with teachers' years of experience and grade level taught.

Keywords: Instructional planning, mathematics instruction, resource materials

Planning for a lesson has long been recognized as a primary factor impacting the efficacy of instruction (e.g., Schoenfeld, 1998; Smith, Bill, & Hughes, 2008). In mathematics, focused lesson planning has been shown (e.g., Boaler & Staples, 2008; Smith et al., 2008) to support teacher enactment of cognitively demanding tasks, help teachers anticipate students' cognitive challenges, and support the generation of questions teachers can ask that promote and elicit student thinking. As described by Schoenfeld (1998, p. 17), a teacher's lesson image is "the teacher's envisioning of the possibilities and contingencies related to a lesson" and includes the teacher's expectations for how their students will engage with certain tasks or activities, what students might find straightforward or challenging, potential student responses to the lesson's tasks and activities and how the teacher anticipates dealing with such responses. Significant to teachers' lesson images are the resources used "which shape the mathematical content presented to, and used by, pupils in their mathematics learning" (Pepin, Gueudet, & Trouche, 2013, p. 929). Pepin et al. (2013, p. 929) define mathematics teaching resources as "all the resources which are developed and used by teachers (and pupils) in their interaction with mathematics in/for teaching and learning, inside and outside the classroom". Existing research (e.g., Chao, Chen, Star, & Dede, 2016; Tarr et al., 2008) has examined the use of digital resources as a means to increase students' motivation and engagement, teachers' conceptions of resource needs and use, and the impact of various types of district-adopted textbooks and curricular materials on student achievement. In this report, the author presents results from a study designed to examine how teacher characteristics, and school and district requirements impact the resources teachers utilize when planning instruction. Specifically, the study addressed the following research questions:

- What resources do mathematics teachers use when planning instruction?
- How do/does (a) school and district requirements regarding lesson plans, (b) grade level (ages and experience of students), and (c) teachers' years of classroom experience impact the resources mathematics teachers utilize when planning instruction?

Methods.

Results were generated from a survey on lesson preparation and lesson plans and focus on responses to the question: "What resources do you use when preparing lessons or writing formal (or informal) lesson plans? Please explain". Although prior research has identified teachers' formal qualifications, textbooks, time, teachers' guides, manipulatives, family support, and technologies as resources (e.g., Adler, 2000; Cohen, Raudenbush, & Ball, 2003; Gueudet, 2017), the study allowed respondents to report resources they actually utilize in their own words using their own conceptions of the term 'resource'. Study participants (n = 154) were comprised of Grades 6-12 (student ages 11-18 years) mathematics teachers, purposively sampled by state and district. Potential respondents were chosen throughout the U.S., rather than a specific district, set of districts, or state, in order to attempt to minimize redundancy in teachers' use of specific (state) standards, textbooks, and curricula. For each state, 2–3 public school districts were identified that met combinations of high-, medium-, or low-level criteria related to: (1) percent minority students, (2) percent poverty, (3) enrollment, (4) student achievement, and (5) type of developed environment (e.g., urban, rural). A compliance-approved correspondence with accompanying survey link was emailed to those 6th-12th grade math teachers whose email could be located from this list of districts. Over 6,800 emails were sent to public school math teachers (an average of 136 per state) with an additional 520 emails across 18 states sent to teachers from private schools. The survey response rate was 2.1%, with a 95.5% completion rate.

Analysis.

Analyses involved descriptive statistics, Pearson's chi-squared, and Fisher's exact tests. Particular data examined included teachers' lesson plan requirements, grade levels taught, and years of mathematics teaching experience. Descriptive statistics are illustrated in Table 1 below.

Grade Band	Years of Experience	Number of Resources (Most Common Resources)	Lesson Plan Expectations
6th–8th (n = 49)	Mode: [1, 5] years; Mean: [6, 10] years	Mode: 3; Mean: 2.6 (Internet, Textbook, District Materials)	Mode: None; Mean: For Evaluations Only
9th–12th (n = 105)	Mode: Over 20 years; Mean: [6, 10] years	Mode: 2; Mean: 2.6 (Textbook, Internet, Colleagues)	Mode: None; Mean: For Evaluations Only
6th–12th (n = 154)	Mode: [1, 5] years; Mean: [6, 10] years	Mode: 2; Mean: 2.6 (Textbook, Internet, Colleagues)	Mode: None; Mean: For Evaluations Only

Table 1: Descriptive statistics

As illustrated in Table 1, the mean years of classroom experience for respondents was between six and 10 years, inclusive. The mean number of resources utilized when planning instruction was 2.6, with course textbook (70.1% of teachers), the internet (64.9%), and math colleagues (33.1%) being the most common resources identified. Finally, expectations regarding lesson plans were that they be submitted only at the time of teacher evaluations (which included formal classroom observations). Although the mean Lesson Plan Expectations for Grades 6-8 math teachers was 'For Evaluations Only', 55.1% of such teachers worked at schools with 'Weekly' or 'Daily-Limited or Detailed', expectations. Pearson's chi-squared tests were performed to determine the independence of Number of Resources with (1) Grade Band, (2) Years of Experience, and (3) Lesson Plan

Expectations. Results indicated only differences in Number of Resources between Years of Experience were significant, $\chi^2(5, N = 154) = 11.87, p = 0.037$. In particular, see Table 2 below for percent of teachers that indicated they utilized three resources.

Teacher Years of Experience (n)	Number of Resources (Percent of Teachers)
n ≤ 10 years	1(13.5%); 2(33.8%); 3(41.9%); 4(10.8%); 5(0.0%); 6(0.0%)
n > 10 years	1(18.8%); 2(32.5%); 3(23.8%); 4(16.3%); 5(6.3%); 6(2.5%)

Table 2: Number of resources by teacher years of experience

Fisher's exact tests were performed to determine the independence of Grade Band, Years of Experience, and Lesson Plan Expectations with each of the following: Textbook, Internet, Prior Experience, Colleagues, Supplemental Texts, District Materials, and Standards. Results indicated only differences between Grade Band and Internet (p = 0.030) and Grade Band and Colleagues (p = 0.001) were significant. Table 3 illustrates the percent of teachers identifying these resources.

Grade Band	Internet – Yes (Percent of Teachers)	Internet – No (Percent of Teachers)	Colleagues – Yes (Percent of Teachers)	Colleagues – No (Percent of Teachers)
Grades 6–8	77.6%	22.4%	14.3%	85.7%
Grades 9–12	59.0%	41.0%	41.9%	58.1%

Table 3: Grade band and resource data with significant differences

As illustrated in Table 3, a significantly larger proportion of 6th–8th grade math teachers indicated the internet as a resource than 9th–12th grade math teachers. Conversely, a significantly larger proportion of 9th–12th grade math teachers indicated colleagues as a resource than their 6th–8th grade counterparts. Fisher's exact tests were performed to determine the independence among pairs of the following resources: Textbook, Internet, Prior Experience, Colleagues, Supplemental Texts, District Materials, and Standards. Results indicated all differences were not significant except: Textbook and Internet (p = 0.027), Internet and Prior Experience (p = 0.029), and Internet and Supplemental Texts (p = 0.002). Table 4 illustrates the percent of teachers for corresponding pairs.

Resource	Textbook – Yes (Percent of Teachers)	Textbook – No (Percent of Teachers)	Prior Experience – Yes (Percent of Teachers)	Prior Experience – No (Percent of Teachers)	Supplemental Texts – Yes (Percent of Teachers)	Supplemental Texts – No (Percent of Teachers)
Internet – Yes	59.3%	78.3%	52.1%	70.8%	89.3%	59.5%
Internet – No	40.7%	21.7%	47.9%	29.2%	10.7%	40.5%

Table 4: Resource data pairs with significant differences

As illustrated in Table 4, if a teacher did not indicate the course textbook as a resource, then a significantly larger proportion of these teachers identified the internet as a resource than did not. If a teacher identified texts alternative to the course text (i.e., supplemental texts) as a resource, then a significantly larger proportion of these teachers identified the internet as a resource than did not. Finally, if a teacher did not indicate their use of personal collections of tasks and activities (through

prior teaching experiences) as a resource, then a significantly larger proportion of these teachers identified the internet as a resource than did not.

Conclusion

A sample of 154 6th-12th grade mathematics teachers in the U.S. reported course textbooks, internet, and math colleagues as the three most common resources utilized when planning instruction ($\bar{x} = 2.6$ resources). In addition, a significantly larger proportion of teachers with 10 or less years of experience utilized three resources than teachers with more than 10 years of experience. A significantly larger proportion of 6th–8th grade math teachers indicated the internet was utilized as a resource than 9th–12th grade math teachers. Utilization of the internet as a planning resource played a role in teachers' use of other resources. In general, when either the course textbook or the use of personal collections of tasks and activities (through prior teaching experiences) were not indicated as a resource, then a significantly larger proportion of teachers identified the internet as a planning resource than did not identify the internet as a resource. Conversely, when texts alternative to the course text were indicated as a resource, then a significantly larger proportion of teachers also identified the internet as a planning resource than not. Further research is needed to examine why teachers make the resource choices they do.

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A primary documentation system embodied in a system of training activities for trainee teachers of mathematics

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The experimental context of this paper takes place in France where the practice of research and proof problems amongst pupils are rare at the primary level. We present our theoretical and methodological approach for fostering trainee teachers to enact this kind of activity. This approach is theoretically based on intentional communities of practice (Wenger, 1998) and an ergonomic approach of teachers' resources (Georget, 2009, 2010). Methodologically, it consists of designing a primary documentation system embodied in a system of training activities likely to trigger dynamic processes of professional development at short, mid, or long-term.

Keywords: Communities of practice, ergonomic resources, mathematics teacher education.

Introduction

This contribution derives of our research project pertaining to intentional communities of practice of mathematics teachers and resources for teachers (Georget, 2007, 2009). The project is based on the theory of communities of practice (Wenger, 1998; Wenger, McDermott, & Snyder, 2002), on an ergonomic approach of resources (Georget, 2010), and began with a 3-year experiment of an intentional community of practice of mathematics teachers at the primary level enacting research and proof problems amongst pupils (Georget, 2007, 2009).

We define what we call a *primary documentation system* as a set of thematic documents given to teachers to practice a kind of activities with their pupils and that is design to be intertwined with their own documentation system. We hypothesize that a primary documentation system designed with tools of the theory of communities of practice, with ergonomic characteristics (e.g. utility, usability, accessibility), and supported by a specific system of training activities, can help teachers to enact richer mathematical activities in their classrooms and trigger their interest in resources, internal or external to this primary documentation system, more adapted for the actual curriculum.

In this research project, we operationalized several concepts central to the theory of communities of practice including *reification* and *participation*. These both processes are dialectically linked and participation and reification can also be considered as products of these processes. From the Wenger's perspective, reifications are seen as means to facilitate the activity of a community and the activity of each of its members with different level of participation. Wenger, McDermott, and Snyder claim that "planning a community is more a matter of finding the triggers to catalyze evolution than creating a full design" (2012, p. 73). In our research, the primary documentation system and its documents are thus considered as triggers to facilitate a variety of professional developments and documentational genesis (Gueudet & Trouche, 2012) at short, mid, or long-term, as reifications to foster participation. From the dimensions of the design for learning (Wenger, 1998) and the principles laid down by Wenger, McDermott, and Snyder (2002), we built a frame to "rethink" the design for emerging communities and especially for designing a primary documentation system (Georget, 2009).

Let us illustrate one of the elements that can be taken into account: the *identification/negotiability* dimension. The primary documentation system must contain elements of identification for teachers (e.g. extracts from the national curriculum, links with traditional activities daily enacted in ordinary classrooms) and also elements of negotiation (e.g. ways for enacting only parts of a given activity, options for enacting a given activity). This is relevant to facilitate the adoption of a given part of this documentation system. Our previous work showed that the resources for teaching often give only one way to proceed and don't describe opportunities to not exploit these resources at an optimal level (Georget, 2009). Such resources also generally pay little attention to different teaching contexts and to different levels of expertise of the teachers. In ergonomic terms, the level of usability of resources is very poor (Idem) and this point is important to take into account for professional development.

Context of experimentation

Our research project mainly concerns research and proof problem between pupils at the primary level (6-11 years old) in France. The goal of these activities is to put pupils in a situation similar to that of mathematicians solving a problem. As in other countries, this kind of complex classrooms activities is required by the national curriculum but also does not often exist in the ordinary practice of teachers (Artigue & Houdement, 2007). Many factors contribute to explain this fact. In particular, teachers have a few helpful resources for enacting practices that could be more efficient in classrooms (Kieran, Tangay, & Solares, 2012; Georget, 2009).

This contribution focuses on the issue of resources and of support for trainee teachers in the context of our university. These trainees are undergraduate students – rarely in scientific domains – that have validated the first year of their Master of Education program. During the year, they alternate between training sessions at the university and a traineeship in one classroom of a primary school. At the end of the year, trainees can obtain a permanent status of in-service teacher and a graduate degree. We re-designed the training sessions in 2016 with another teacher educator in mathematics. Sessions are now based on a *primary documentation system* that is distributed to each trainee teacher at the beginning of the year, mainly distributed under electronic format.

A primary documentation system embodied in a system of training activities

The primary documentation system is design by educators. It is written to be easily understood and be practical for the trainees. It contains internal references and can be thus use without a particular order. It contains also useful external references (e.g. to national curriculum, books, websites) and insights to explore these external resources. The primary documentation system is thus as ergonomic as possible and built for fostering participation of the trainees to enact rich activities with their pupils but, due to what we called the *incompleteness paradox of resources* concerning the quantity of information usable and acceptable by a teacher (Georget, 2009), the primary documentation system must also be accompanied by a specific system of training activities.

The main aim of the system of training activities is to make the trainees explore the documentation system and use it concretely for preparing for their classes. Within these activities, some focused on general considerations of mathematics teaching or on specific mathematical topics, some were dedicated to the preparation of sequence of teaching, others were dedicated to the consultation of

the documentation system and some external references. A first example of these training activities is a 2-hour session to explore freely the documentation system or its external references, and then report about this exploration amongst a small group of trainees, then amongst the whole group. A second example is the two assessments during the year which consist in writing an *a priori* and an *a posteriori* analysis of a course or a sequence of courses chosen by the trainees during their traineeship, supported by references to the primary documentation system or external references.

In summary, the trainee teachers are here considered as a community coordinated by a teacher educator and this community is considered as a complex system about which we know “few” things. Therefore we don’t know exactly if and how each resource or each training activity could trigger a “positive” reaction from students, but we think that designed resources and activities have a potential to trigger particular dynamic processes at short, mid, or long-term for each teacher. Facing the complexity of the community, we answer by a complex primary documentation system of resources embodied in a system of training activities that give both some room for individual choices linked to individual experience, confidence, or context; and some constraints as the fact that both the *a priori* and *a posteriori* analysis must contain at least one research and proof problem.

Methodology

To evaluate the influence of our global design, the following data were collected: answers to a questionnaire after a 2 month training session (n=35 trainees), observed and videorecorded lessons enacting one research of proof problem in their classroom in the middle of the year (n=4 trainees), and post-lesson audio-recorded interviews (the same 4 trainees), evaluation of the training sessions by the trainees at the end of the year. The observed trainees were not known to have specific difficulties in their classrooms as far as the administration was concerned. The role of the questionnaire was to know more about the use of the primary documentation system by the trainees and about the practice of problems of research and proof with their classroom. The observed-videorecorded lessons and the audio-recorded post-lesson interviews were a way to investigate the practice of the trainees and the links with the documentation system and the training activities.

Only preliminary analysis are made to evaluate globally the relevance of our approach.

Results and perspectives

The designed training session contains important constraints on the practice of the trainee teachers. At the end, these novices accepted all of this and furthermore they ask for increased “pressure” at the very beginning of the year for the next trainees. As one of them said during his interview: “I'll give it a try, it's the beginning of the year so it doesn't matter" (Jim). By these words, enlightened by the complete interview, he meant that he felt a lot of pressure from the university at the beginning of the year but he would have the excuse of being a beginner if something was to go wrong. He also thought that it was a “one-shot” activity in his practice to satisfy the university. During the interview, he declared that he finally enacted other similar activities in his classroom and felt more comfortable with them, for his own benefit and the benefit of his pupils. The other interviewed trainees declared more or less the same things.

What about the analysed practices? We observed enactments of research and proof problems but we saw that these could be enhanced in several ways by linking with specific resources from the

primary documentation system. For example, mathematical debates are more or less “sacrificed” like in other ordinary classrooms and lessons lacked a real conclusion. Finally, the influence of our design on the “quality” of the lessons is not prominent, the influence seems greater in terms of confidence, enthusiasm and desire of the trainees for their professional development. We consider that these preliminary results confirm the relevance of our general approach.

Today our research questions are oriented towards the study of the practice of the observed teachers that are now in-service teachers and the enhancement of the primary documentation system. The study of the practice of teachers is dedicated to knowing more about the mid-term effect of the training sessions. The enhancement of the primary documentation system consists of facilitating access and of enhancing specific resources and training activities to study if we can observe a more prominent effect during the training sessions.

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The dynamic interactions between teacher and resources in the use of the textbook and teacher guide

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Textbooks and the accompanying teacher guides are recognized as important resources in mathematics education. They can be used in different ways by teachers in the planning phase: teachers may closely follow the textbook or guide content, they may adapt it or improvise. This study focuses on investigating the interrelationship between teachers and the mobilization of textbooks and teacher guides. The two participants used the textbook as the basis for lesson planning, consulted the teacher guide as an additional resource for activities and ideas, and then went back to the textbook to finalize their lesson plans. The results suggest that the process of teachers' lesson planning cannot be described as linear, but rather as a dynamic and complex process.

Keywords: Curriculum resources, mathematics education, teacher guide, textbook.

Introduction

The curriculum resources have always had an important role in the planning of mathematics lessons because they support the teacher's pedagogical content knowledge, help them in the lesson design, serve as a source of tasks, and help organize individualized teaching (Davis & Krajcik, 2005). Teachers as instruction designers actively interpret and transform curriculum materials in the planning phase (Remillard, 2005). Textbooks are important resources in mathematics education and are widely used in the planning and enacting of a lesson (e.g. Pepin & Haggarty, 2001). Teacher guides are also found to be important in terms of teachers' planning for practice (Remillard, Harris & Agodini, 2014). The results of the previous studies (Glasnović Gracin & Domović, 2009) show that the textbook is the basic resource for teachers' preparation in lower secondary education in Croatia, and that the teacher guide is the next most frequently used resource. These findings motivated us to conduct a qualitative investigation into the interaction of these two resources in the process of teachers' lesson planning.

Theoretical framework

Taking a stance of teaching as design, Brown (2009) built a Design Capacity for Enactment Framework (DCE) which represents the idea that both the curriculum and a teacher's personal resources affect the designing and enacting of the instruction. The DCE framework includes three basic aspects of curriculum: physical objects, representations of tasks, and representations of concepts. Together, the curriculum and teacher factors provide a basis for understanding teachers' offloading, adapting, or improvising with curriculum resources. *Offloading* refers to teachers' literal use of the curriculum materials, placing the responsibility for guiding instruction directly onto the material. *Improvising* refers to teachers' use of their own instructional strategies to reach the instructional goal. *Adapting* indicates an equally-shared responsibility for the delivery of the lesson between teacher and curriculum resources. These concepts are called different types of mobilization of curriculum resources (Brown, 2009).

The aim of the study presented in this paper is to investigate the dynamic interactions between teachers and the two curriculum resources. For this purpose, we formulated the following research question: How do the textbook and the teacher guide affect the designing and enacting of the instruction? The question is formulated in accordance with Brown's types of mobilization of curriculum resources.

Method

The case study involved two mathematics teachers from lower secondary education in Croatia (grades five to eight). They are named Beth and Debbie and they both used the same textbook in the classroom. These participants were chosen because of their teaching experience (about 20 years), they both have participated in previous studies and because they have differing opinions about the textbook used: Beth was satisfied with the textbook, while Debbie was not satisfied with its content and pedagogical objectives. The participants also had the accompanying teacher guides at their disposal for lesson preparation.

The study included content analysis, on-site observations of participants' teaching and semi-structured interviews. We observed four lessons in Beth's classroom and four in Debbie's classroom. Content analysis included the lesson plans for the observed lessons and activities provided in teacher guides; they were compared to textbook content and to the observed actions in the classroom. The analysis of the transcribed interviews, observation notes and textual material focused on different mobilizations of resources (offloading, adopting, improvising). The interviews also provided an opportunity for the teachers to explain why a particular type of resource mobilization was used.

Results

The analysis of the structure and content of the teacher guides shows that they are primarily made up of lesson plans for each lesson. At the back of each guide there are an additional 20-30 activities which are not directly referred to in the lesson plans, but are provided as ways to adapt or supplement the given lesson plans.

Teachers' mobilization of curriculum resources

Both participants offloaded, adapted and improvised with the materials, but not in the same way and not to the same extent in each lesson. In terms of *offloading*, Debbie, the participant who is not satisfied with the textbook, only uses the exercises from the textbook, giving them as homework or for practicing, and she does not consult the teacher guide for this. Beth, however, consults the teacher guide when she decides to offload an entire lesson from the textbook:

> Beth: When I decide to use the textbook for almost the entire lesson, I consult the lesson plans in the teacher guide. I believe they [the authors] suggest the best way of using the textbook. I mean [...] they [the textbook and teacher guide] are both written by the same people.

In Beth's case, comparison of the observed lessons with the lesson plans from the teacher guide shows that the lessons follow the structure and suggestions from the teacher guides.

Concerning *adapting*, the observed lessons included several instances where Debbie and Beth adapted an activity from the textbook. They used worked examples similar to the ones provided in the textbook rather than using them from the textbook directly. Beth explained that she cannot use every activity exactly as it is in the textbook or is suggested in the teacher guide; she has to adapt it to her students' needs where necessary. Also, in the observed lessons both participants used the

same activity from the teacher guide, but they adapted it differently (Grade 6, Fractions). Debbie explained that she liked this activity but that in her opinion the original activity contained too many tasks for students to investigate, so she "had to modify it and adapt it to her students". She did this by decreasing the amount of tasks in the activity. Beth used the same activity from the teacher guide but she added an extra task as an additional step in the learning process:

Debbie: I added the last column to help them [the students] to come to valid conclusions.

The *improvising* observed in the lessons was mostly used by Debbie. She found that the suggestions in the teacher guides did not suit her goals. And she did not rely on the textbook for the motivation phase and introduction of new content; she introduced rational numbers, for example, with an approach that was not set out in the textbook or suggested in the teacher guide. In the acquisition phase, she added some content which is not contained in the curricular outlines. Her reason for this is that that particular content will be important for the students later on:

Debbie: So… based on experience, I know what will be important to students in the next grade, and there are some things, which I would consider important, which are not in the textbook.

So, her improvisation included consulting the planned curriculum for the upper grades as another resource. Beth rarely improvised, but, for example, she made her own worksheet with guided discovery learning to introduce rational numbers. In the interview, she explained that she was prompted to do this by the guided discovery worksheets found in the teacher guide, as she considers the technique to be very beneficial to students:

Beth: I designed the worksheet myself. I wanted my students to use discovery learning. In the teacher guide I saw there are activities involving guided discovery, so I used my knowledge to achieve that in this worksheet.

Sequence of the interaction with resources

The data obtained in interviews show that both participants do use the teacher guide in lesson planning, but not as the primary resource. The resource they consult as the starting point is the textbook:

Debbie: First, I look at the textbook to see what I can use, what suits me. And then I look at other resources like the teacher guide.

After consulting the teacher guide (and other resources), the participants either return to the textbook to finalize their lesson plan with textbook exercises, or they proceed directly to lesson implementation. These steps are presented in Figure 1. The findings reveal a non-linear, dynamic and complex nature of the lesson planning process in which various resources interact and influence teachers' choices.

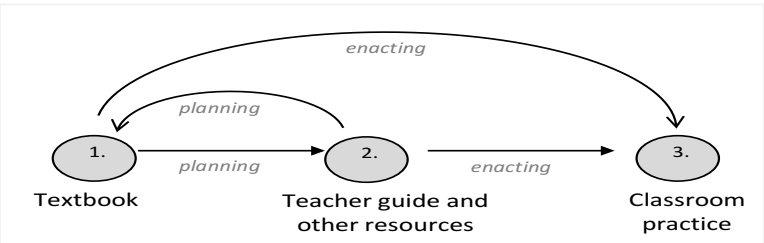

Figure 1: Steps of using resources in participants' lesson planning

Discussion and Conclusions

The study presented in this paper showed that the textbook and the teacher guide affected the design and enactment of the instruction in different ways in the cases studied, depending on the teachers' beliefs, experience and use of other personal resources. The differences in the enactment could be seen in the different types of mobilization of resources. Beth, the teacher who liked the textbook, often used offloading with the textbook and the teacher guide because she believed the authors knew how best to use the given resources. However, Debbie improvised a lot because she was neither satisfied with the didactical intentions in the textbook and the teacher guide, nor was she completely satisfied with the intended curriculum. Therefore, she improvised by adding content in order to better prepare the students for subsequent grades. She explained that her experience influenced this decision. Both participants adapted the same activity from the teacher guide, but in different ways.

The study confirmed that teachers' lesson planning is a complex and dynamic process, where various resources interact and have an influence on teachers' choices. The participants actively and in different ways modified the curriculum resources in planning instruction (Remillard, 2005). Their actions were influenced by the curriculum and their personal resources and beliefs about mathematics teaching (Ball & Cohen, 1996; Davis & Krajcik, 2005), showing that the process of teaching is indeed a design process (Brown, 2009).

The study results showed that the teachers' personal resources are the key element in deciding on the type of mobilization of curriculum resources, including their deconstruction and reconstruction. For that reason, teachers' mathematical and pedagogical knowledge should be examined in more depth through further research in the area of teachers' lesson planning.

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Documentational trajectory as a means for understanding teachers' interactions with resources over time: the case of a French teacher

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Our work aims to contribute WG 2, discussing the issue: How to study, in long term, the evolution of the interactions between teachers and resources? We propose two notions: documentational trajectory and documentational experience, grounded in and in return contribute to documentational approach to didactics. For studying the relevance of these notions, we followed up one middle school teacher since 2015. We adapted reflective mappings of the teacher's resource system, and documentational trajectory which was developed according to the reflective investigation proposed by documentational approach to didactics. Our work in this study mainly are consisted with: using reflective mappings of her resource system designed by the teacher at different moments of her career, crossing them with her description of her documentational experience and of her documentational trajectory, evidenced the sensibility of her interactions with resources to some events occurring in her professional life.

Keywords: Teacher professional development, Documentational approach to didactics, Documentational experience, Documentational trajectory, Reflective investigation.

Background of the study

Deep evolution in Education, such as new curriculum purposes, students' new needs, new pedagogical practices and new digital resources, stimulates the professional development of teachers. In their daily work, teachers face many professional problems that lead them to develop their knowledge and create their own strategies to improve their teaching. In particular, Adler (2000) highlights the importance of the resource design process in a teacher's professional development. This complex and dynamic process is core in teachers' professional activity (Gueudet & Trouche, 2008). In this context, we hold special interests in understanding how teachers' interaction with resources benefits their professional development over time. This is also one of the interests of a national French research project, ReVEA (Living Resources for Teaching and Learning, 2014–2018, http://anr-revea.fr), which works for studies in the process of design, sharing and integration of resources by teachers.

In this paper, we discuss the issue: *how to study, in the long term, the evolution of the interactions between teachers and resources*. We propose two concepts that help us to analyze the evolution: *documentational trajectory and documentational experience*, and we situated our study in a case of a middle school mathematics teacher. In order to achieve what we propose, we organize this paper in four sections. Firstly, we introduce our theoretical framework. Secondly, we discuss some methodological choices inspired by reflective investigation. Thirdly, we present the preliminary results of the analysis. Finally, we present some considerations based on our partial analysis.

Theoretical Framework

This research is based on and in return aims to contribute the development of the *documentational approach to didactics* proposed by Gueudet and Trouche (2008). This approach considers that teachers are responsible for designing their resources. They defined *documentation work* as all works done by teachers in collecting, organizing, selecting, adapting and designing resources. *Resources* could be everything (can be explained) that teachers use to *re-source* their work (Adler, 2000). They are part of a complex and structured set of resources named *resource system*. To prepare their lesson, teachers develop their knowledge interacting with resources. The result of this process is called a *document*, which is one hybrid entity that comprises teachers' resources (explicit dimension) and the knowledge (implicit dimension).

In professional didactics, Pastré (2005) presents one framework to analyze how adults learn in their professional life. He defines *experience* as the accumulation and appropriation by the subject of his past. In our work, we are interested in teacher's experience acquired during his interactions with resources, and we name it *documentational experience*. Over time, many events take place feeding this experience. We name *documentational trajectory* the set of events, both of individuals and collectives, along with teachers' documentational experiences. To understand teachers' documentational trajectory and documentational experience, we present in the following section some methodological choices and tools.

Methodological choices and tools

Our methodological choices are guided by four principles of reflective investigation proposed by Gueudet and Trouche (2012, P.27–28): (1) "long-term follow-up" allowing to identify stabilities and changes in teachers' documentation work; (2) "in- and out-of-class follow-up" covering many moments when teachers documentation work happens; (3) "reflective follow-up," leading teachers to explain and reflect about their past experiences; and (4) "broad collection of the material resources" keeping the vestiges of materials designed and used by teachers.

Our methodological tools are classified into three categories: (1) face-to-face follow-up to capture the teachers preparing or implementing their lesson (2) distance follow-up for capturing the moments that we are not with them; and (3) reflective follow-up interviewing the teachers. In this paper, we analyze some data about the tool of *reflective mapping* that belongs to the third tool category. The reflective mapping is an adapted methodological tool from *Schematic Representation of Resource System (SRSS)* proposed by Gueudet and Trouche (2012, see more details in Rocha, 2018). We propose two ways to investigate a teacher's documentation work:

—Teachers' mapping of their documentational trajectory. In Rocha (2018) there is an example of teachers' *Reflective Mapping of Documentational Trajectory (RMDT)* in which teachers present a continuous timeline with events that were remarkable to their documentation work.

—Teachers' mapping of their resource system in several moments (starting career, currently, etc.). In this article, we present two *Reflective Mappings of Resource System (RMRS)*. In one of them, the teacher represented the resources used during the first five years of her career. In the other one, she presented the resources were currently used, in 2017. We use these mappings to understand her

documentation work evolution in these two periods of her documentational trajectory. In this article, we explore the data of the middle school teacher, named Anna, whom had been followed since 2015. We chose this teacher because of her long teaching experience (26 years). In next section, we will present some preliminary results.

Preliminary results

We analyze Anna's documentational trajectory combining with her interview and of her reflective mappings of her documentational trajectory and her resource system (Figures 1 and 2).

In Figure 1, we highlight four points. Firstly, textbooks' position is core in her lesson preparations (courses and exercises), especially the official textbook in her school. Secondly, resources from formal professional collectives in France are very important ones for her. Thirdly, working in collectives provides her a chance to reflect about teaching. Fourthly, digital resources start to be integrated into her resource system.

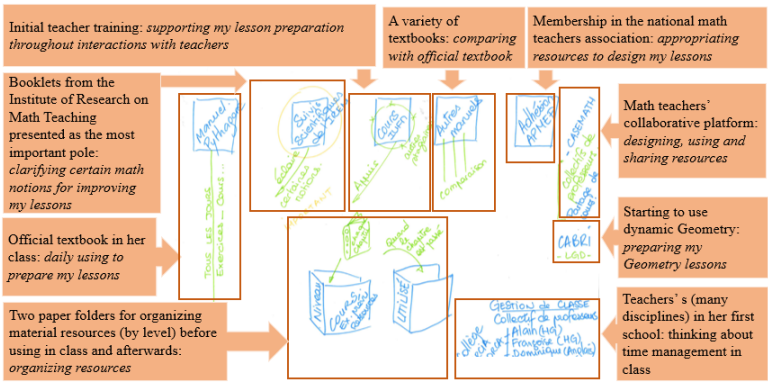

Figure 1-Anna's RMRS of her first five years in her career

A set of resources, important to her during this period, was booklets from Institute of Research on Mathematics Teaching (IRMT). These resources remained a long time in her documentation work. She explained that these resources "took up some notions, explained to them and gave examples of how they could be taught". Then, these resources helped her to learn about teaching mathematics, giving her mathematical didactic advices. She presented in her mapping of documentational trajectory that she found these resources during her university studies.

In Figure 2, we observe some changes in the same four pointed as in Figure 1. Firstly, textbooks change; they are used only to offer exercises to the students. Moreover, this change is very clear for Anna "I need to understand why we chose this exercise over another, and the principle of textbooks as a catalog of exercises without any analysis does not suit me."

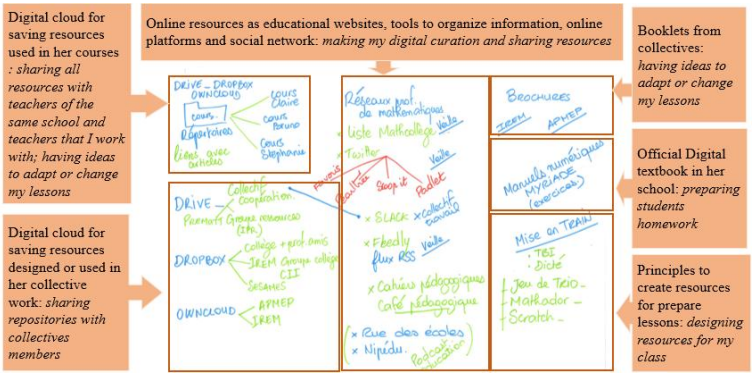

Figure 2-Anna's RMRS represent resources used in 2017

Secondly, resources from formal professional collectives in France were still being used to get new ideas to teach. However, she is more than a user, now she proposes resources in these booklets. This a result of one important event in her trajectory, her participation in Sésames group. Sésames is a collective that reflects about resources to teach algebra. There, she learned more about teaching algebra in middle school and elaborated new resources and new pedagogical practices (Rocha, 2018).

Regarding the third point, we observed that she is engaged in many collectives inside and outside of her school. These collectives are now one source for her resources. For example, she always prepares her lessons with her colleague, Cindy. Anna participates in IREM and Mathematics Teachers' Professional Association. Collectives work is very important to her documentation work and she said, "I cannot work alone, […] because I cannot be sure of myself, if nobody gives me any advices. I need the opinions of others to do something." And the last point, digital resources now had one structuring status in her documentation work. They have (at least) two functions: (1) One is to have a shared space with the colleagues that she works with, in which she shared her resources and meanwhile she could access to resources from others. Such shared space of resources is important to her documentation work because she can turn to others' resources for new ideas when she did not feel satisfied with her lesson. (2) The second one is that she created one system to monitor new digital resources related to mathematics teaching. For example, she follows many twitters (https://twitter.com/) and once she finds any interesting resources she saves them as her favorites. These resources give her ideas that she could try to implement in her class.

Final considerations

Finally, we come back to our initial issue: *How to study, in long term, the evolution of the interactions between teachers and resources?* We presented some elements to start to discuss this question. Firstly, thinking about teachers' documentational trajectory helps us to consider teachers' whole professional life in which they interacted with various resources, and helps us to understand which events change and nourish their documentational experience. Secondly, reflective mappings turn to be one powerful tool to analyze their evolution, because they can show how some resources change states and new functions appeared. In the future, we will propose more links between different tools employed in our work to analyze teachers' documentational trajectory. Our analysis of teacher professional development is hoped to help a better understanding on teachers' craft knowledge, which in a further step, could offer supports in deciding teachers training and new government policies.

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MOOC as a resource for teachers' collaboration in educational program

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MOOCs can be understood as digital resources for teachers, because they are a repository of many other digital powerful resources for teaching mathematics, with which teachers can interact by themselves or each other inside the MOOC environment. The aim of our paper is to answer the following specific research question: how to analyze the interactive nature of a MOOC and its influence on teachers' learning? The answer is linked to the elaboration of a suitable theoretical framework. It was born thanks to our involvement in different teaching experiments with some Italian MOOCs, designed and delivered for mathematics teachers' professional development.

Keywords: MOOCs, teacher professional development, MOOC-MDT, meta-didactical transposition, teacher collaboration.

Introduction

A Massive Open Online Course (MOOC) is an online course aimed at unlimited participation and open access via the web. MOOCs are a recent and widely researched development in distance education which were first introduced in 2008 and emerged as a popular mode of learning in 2012. Early MOOCs often emphasized open-access features, such as open licensing of content, structure and learning goals, to promote the reuse and remixing of resources. Some later MOOCs use closed licenses for their course materials while maintaining free access for students. In addition to traditional course materials such as filmed lectures, readings, and problem sets, many MOOCs provide interactive user forums to support community interactions among students, professors, and teaching assistants. Therefore, a MOOC can be considered as a digital resource with many other digital resources inside. Despite their big success, the emergence and use of MOOCs for professional teacher development is still uncommon, especially in mathematics, but on the verge of gaining a foothold. Moreover, the specific intersection of MOOCs and professional teacher development is poorly researched.

Mathematics MOOCs for teachers' professional development

In our Department at University of Turin, we are involved in a project called Math MOOC UniTO. The project aims to design and deliver MOOCs for mathematics teachers, mainly from secondary schools, with the aim of increasing their professional competencies and improving their classroom practices. The project started in the spring of 2015 and four MOOCs were designed, one for each of the main topics in the official Italian programs for secondary school: Arithmetic and Algebra, Geometry, Change and Relations, Uncertainty and Data. So far, the first two have been delivered and the third one will be delivered in January 2018. These MOOCs are open, free, and available online for teachers on a Moodle platform (http://difima.i-learn.unito.it/). It is important to underline that each MOOC is weekly based and from 6 to 8 weeks long in total. Every week the trainees become

familiar and interact with resources focused on specific mathematical topic (i.e. height, angle, sense of the number, …) related to the specific mathematical area considered in that MOOC. Therefore, in each week trainees watch videos where an expert introduces the topic of the week; read the mathematics activities based on a laboratory methodology (and, optionally, experiment these in their classroom). Trainees are invited to share thoughts and comments about the activities and their contextualization within their personal experience, using specific communication message boards (Forum, Padlet: https://it.padlet.com/, and Tricider: https://www.tricider.com/). From our experiences, there is a need for designing and implementing MOOCs for teachers' PD in mathematics education with a focus on the development of communities of practice (Wenger, 1998) and the collaborative work among teachers as a basis for their PD. Indeed, when people co-work (work together collaboratively) they can also co-learn (learn together collaboratively): teachers can learn through discussion, conversation and reflection on their own teaching, on student learning and on the teaching of others (Robutti et al., 2016). The methodology of our MOOCs aims to create collaborative contexts for teachers' work, where they can learn from these kinds of practices. Taking into account the necessity for teachers to be supported in exploiting technology affordances, the objectives are: accompanying teachers in the design of teaching resources, by examples of activities and reflection on their work in progress; fostering a reasoned use of technology, with the use of appropriate digital tools for classroom. A research question comes out: *How to analyze the interactive nature of a MOOC (understood as a digital resource with many other digital resources inside) and its influence on teachers' learning?*

The MOOC-MDT

Despite the fact that a rich literature exists about how teachers can develop their PD in traditional face-to-face courses, on MOOCs specifically there is not the same richness: hence our proposal is to present a theoretical framework aimed to interpret the interactive nature of a MOOC environment and its influence on teachers' learning. We revised the *Meta-Didactical Transposition*[1] model (MDT: Arzarello et al., 2014) and re-elaborated it from a framework apt to describe face-to-face teachers' PD to a new one, suitable for describing the PD dynamics within a MOOC environment. To do that, we integrated MDT through an *hybridization process* (Arzarello, 2016) with *Connectivism* (Siemens, 2004) and the *Instrumental Approach* (IA: Verillon & Rabardel, 1995), obtaining what we call MOOC-MDT. With the hybridization, you consider a particular component of a theory. This is "implanted" in another theory that, for this reason, will be hybridized: the old theoretical framework is so enriched and the language as well. In Connectivism, the notions of *personal knowledge* and *learning* are closely connected to the network theory. *Personal knowledge* is a particular kind of network, whose nodes are any entity that can be connected to another node (information, data, images, ideas...), while an arc is a connection, a relationship, a link between two nodes (Siemens, 2004). *Learning* is a continuous process of building, developing, self-organizing knowledge – understood as

[1] MDT is a model that describes the process of teachers' PD, aimed at describing teachers' and researchers' activities over time, in terms of meta-didactical *praxeologies* (Arzarello et al., 2014, pp. 353-355): tasks in which they are engaged in the educational program, along with the techniques used to accomplish them and their theoretical justification.

a network (Siemens, 2004). Therefore, learning not only adds new nodes, but also connects existing nodes and makes sense to these connections. The personal knowledge can be understood as an evolving network: the knowledge at a given moment corresponds to a timely conformation of the network, while the act of learning, of increasing knowledge, corresponds to the process by which the structure and complexity of the network expands. Taranto considered these two key concepts of Connectivism and made a hybridization with the IA (Verillon and Rabardel, 1995) taking into account also successive refinements (Trouche, 2004; Drijvers & Trouche, 2008).

In this frame, a MOOC can be considered as an *artifact*, that is a static set of materials. Connectivism considers the *MOOC-artifact* with its own network-based knowledge: its nodes are the used content, ideas, images and videos; the connections are the links between their node pairs. When a MOOC module is activated, it dynamically generates a complex structure that we call ***ecosystem***: "all the relations (exchange of materials, experiences and personal ideas/points of view) put in place by participants of an online community thanks to the technological tools through which they interact with each other, establishing connections within the given context" (Taranto et al., 2017). The network-knowledge of the *MOOC-ecosystem* is dynamic: it evolves as the MOOC-artifact's network, thanks to the participants' contribution. Also, the network-knowledge of individuals evolves as a personal self-organization (Siemens, *ibid*, p. 4) of the ecosystem. The process of transformation from artifact to instrument (Verillon & Rabardel, *ibid*) is here replaced by the evolution artifact-ecosystem/instrument. In particular, what the IA indicated as instrumental genesis, here is replaced by an hybridization between IA and Connectivism, triggering what we call "***double learning process***": from the one side, the MOOC-ecosystem is a specific learning tool for the individual; from the other side, the use of the MOOC-instrument by the individual generates learning for the whole ecosystem. The dynamic process has the following components, intertwined and self-feeding each other: (i) **Instrumentation/Self-organization** (from the ecosystem to the individual): process by which the network of MOOC-ecosystem expands the individual's network-knowledge. In particular, the **instrumentation** is the process by which the chaos (Siemens, *ibid*) of the ecosystem network reaches the individual. The many novelties of views and experiences make sure that the individual compares himself with new usage schemes. A phase of **self-organization** of the MOOC's information follows this process: namely, when the individual selects which usage schemes proposed by the MOOC are valuable and which are not. (ii) **Instrumentalization/Sharing** (from the individual to the ecosystem): process by which the individual's network-knowledge expands the network of MOOC-ecosystem. The **instrumentalization** is the process by which the individual, with her/his renewed network-knowledge independently builds new connections. The individual is stimulated by a task requested by MOOC and (s)he caters to the ecosystem to turn it according to her own (new) usage schemes. (S)He wants to integrate it with her/his own cognitive structures. **Sharing** is the process by which the MOOC welcomes the contribution of the individual and makes it available to all: information goes towards all members.

Discussion and Conclusion

The hybridization takes place considering two components of the Connectivism: personal knowledge and learning. These are implanted in the IA, which is so adapted to MOOC's own dynamics, where the community of participants becomes subject and object of a new, more complex kind of

instrumental genesis: the double learning process. In fact, it maintains the structure of the instrumental genesis, with directions from the subject to the object and vice versa, but it is also enriched with the Connectivism standpoint. If in the MDT, the trainers shape their proposals according to the practices they think appropriate, and so they can estimate how much the trainees learn such proposals, in the MOOC-MDT the process of training appears to be more difficult to control. The trainers do not know "what" the user has really looked at among the presented materials, nor they can know how (s)he interpreted them. Equally, the trainees benefit from material provided by trainers and other trainees sharing their materials and ideas using the communication boards. The process evolves stochastically: a determining role is played by each trainee feeling part of an interacting community. MOOC-MDT facilitates the study of the specific dynamics of the interactions among trainees and between trainees and trainers, which occur online in virtual environments. We will show the MOOC-MDT efficacy through examples of collected data emerging from the experiences of our MOOCs.

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A mathematics teacher's use of digital resources for formative assessment

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This paper explores the formative assessment practices of a mathematics teacher in an English High School. It uses data from ongoing PhD research on mathematics teachers' appropriation of digital resources and the impact on classroom practices. This paper shows that the teacher uses a set of digital resources which allow the teacher to teach, collate and e-analyze students' work and feedback in real time and thus inform the teacher's 'emergent lesson planning'.

Keywords: Digital resources, Mathematics, Teacher, Formative Assessment

Introduction

This paper explores the formative assessment (FA) practices of a mathematics teacher Joe[1] in a state school in England. The paper is structured as follows. I begin with a brief review of literature on formative assessment and digital resources, and the theoretical framework which guides the PhD research. I then present the context of the research and outline the methodology of the research. I then present the findings and a discussion on the practices observed during the lesson cycle.

Formative assessment, theoretical framework and research question

My approach to research is informed by the 'documentational approach' (Gueudet & Trouche, 2009) – and the affordances of digital resources. In the course of observing lessons, I repeatedly witnessed FA in classrooms. The milestone publication on FA is Black & Wiliam (1998). In a later paper (Black & Wiliam, 2009, p.7) they define formative assessment as:

> evidence about students' achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited

For them, the feedback is interpreted and used by three players (teachers, learners and peers) towards making a decision that could impact on the instructional practices. It would, of course, be naive to assume that all kinds of feedback to students about their work are equally effective. Digital technologies are central to recent efforts to enhance FA decision making and strategies in mathematics (Cusi, Morselli, & Sabena, 2017). This digital focus on FA resonates with my lesson observations and leads to the research question for this paper:

What digital resources does a mathematics teacher employ in classroom FA?

[1] This is a pseudonym used for the purpose of this research.

Research Context and Method

Joe is 29 years old and has been teaching for 5 years. He works in a school for students aged 11-18 years. The school hosts one of the Maths hubs[2] where the use of digital resources are encouraged and supported. I focus attention on one of Joe's lessons to a class of 11-12-year-old students with the mathematical focus on representing numbers as fractions, decimals and percentages. Data informing this paper (semi-structured interview, classroom observation, collated documents and field notes) were gathered in the context of ongoing PhD research. My analysis focuses on whole-class episodes in which digital resources were used in the process of FA. The units of analyses are 'events of interests' where the teacher organizes the digital resources as tools for enhancing the FA practices during the lessons. In the findings below I focus on: identifying the resources used by Joe; identifying connections between these resources; and identifying student actions in the FA process.

Findings and Discussion

The findings are essentially a description of observed and reported FA practices during a lesson. I begin by describing how the lesson unfolds.

- Students get an iPad each and scan in designated sets of tasks from select banks of questions[3] via a QR[4] code (Quick Response Code) this provides students with links to the tasks material that instantly pops open on their iPad devices.
- Students work on tasks (Individually for 5mins, then teacher's whole-class intervention with constructive feedback).
- Teacher using Plickers[5] app on iPad scan the students' responses from customized cards[6], collates the results in real time, e-analysed, and colour-coded on a spreadsheet.

C	D	E	F	G	H	I	J	K	L
Last name of student	Question 1	Question 3	Question 2	Question 4	Question 5	Question 6	Question 7	Question 8	Total Mark
[redacted]	2	3	1	1	2	0	2	3	14
	4	3	1	1	3	0	3	3	18
	1	1	1	1	2	1	2	3	12
	2	2	1	0	3	1	2	3	14
	2	2	1	1	3	1	2	2	14
	3	3	1	1	2	1	3	0	14
	2	2	1	1	3	2	0	0	11
	3	2	1	1	1	1	2	1	12
	3	3	1	0	2	2	3	0	14
	2	0	0	0	1	1	1	0	5
	2	0	1	1	3	1	3	3	14
	1	1	1	1	2	0	0	3	9
	3	2	1	1	3	0	1	0	11
	4	2	1	1	2	1	3	0	14
	1	2	1	1	0	0	2	3	10
	1	1	1	0	0	1	1	3	8
	1	3	0	1	1	1	1	1	9
	2	1	1	1	1	0	3	0	9
ed	6	5	16	14	6	2	6	8	
ed	33	28	89	78	33	11	33	44	

Screenshot 1: Joe's e-analyzed FA data

The screenshot 1, above shows the list of students, number of tasks given, number of questions mastered and the percentages of students that mastered specific questions.

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[2] http://www.mathshubs.org.uk/A UK collaborative national networks of schools.

[3] https://www.10ticks.co.uk/ and https://www.socrative.com/

[4] A machine-readable optical label, - consisting of black squares arranged in a square grid on a white background, - that can store up numeric and alpha-numeric characters – URLs, text, math problems, etc. It contains information about items to which it is attached and this can be read and interpreted using an imaging device like a camera, iPad and cell phones.

[5] See Umameh, M., & Monaghan, J. (2017). A classification of resources used by mathematics teachers in an English high school. *International Conference on Technology in Mathematics Teaching* 13 Lyon, 3-6 July 2017.

[6] There are Plickers generated and customized cards for each individual student.

- Then the e-analysed and colour-coded whole-class performance is displayed on IWB, this triggers class discussion, formative intervention, extension tasks[7] are given to students in pairs and tasks done on mini-whiteboard while teacher move around the class. This lasts for 10minutes.
- Nominated/volunteer students share solution and peer-checking takes place.
- Students' e-analysed feedback informs teacher's 'emergent lesson planning.

Joe's use of sets of digital/non-digital resources (Visualization tools: IWB and iPads; Data-capture tools: QR codes, Plickers; data-handling and display: Spreadsheets; and resource banks) provide opportunities for task differentiations and real-time feedbacks during the lesson. The e-analysed feedbacks and follow-up tasks on mini-whiteboards enable the teacher to modify the instructional practices and adjust the level of cognitive demand of the tasks as experienced by students. The evidences from the data supports the finding by Cusi, Morselli, & Sabena (2017) that the integration of digital resources into mathematics teaching extends and enhances FA strategies.

The immediate effects of displaying a colour-coded e-analyzed feedback on the IWB is that it makes where the students are in their learning visible to all, it motivates and acts as stimulus for whole-class discussion. The IWB becomes a collective space to trigger off interactions amongst students, with Joe facilitating the shared ownership of the learning. Several instances, students volunteered/nominated to share their strategies in solving a task on the adjacent chalk board, while the rest of the students were encouraged to work on the extension tasks in pairs. Then randomly selecting the mini-whiteboard from a pair, displays it, the teacher then asks them to explain and justify their solution. Another pair of student were unable to make the connection between fractions and decimals, and between fractions and percentages. Joe's stated intention in the pre-lesson interview was; "I don't want students to compartmentalize ideas. I want them to link everything together, because then they can understand it.[8]"

From the post-lesson interview the teacher had anticipated a variety of solutions and possible misconceptions by students, hence built in the remediation. But one thing he did not anticipate was that the cognitive demand of the task was beyond the students' present ability, hence a need to adjust the tasks and develop an 'emergent lesson'. Joe reacted to this thus:

> "I can tell from the questions they have answered correctly on there, who has made fundamental error, how that should feedback into my next lesson, so if there is any glaring error that many of them made.[9]"

At the end of the whole-class discussion he made useful correction, validated a variety of solution (re-voice) and went on to reteach some aspects of the task and then gave a moment for peer-checking each other's solutions. This deepens students' sense of ownership of their learning, confidence that their inputs count and a feeling of belonging to a community of mathematics learners.

The whole-class performance (screenshot 1) and mini-whiteboard tasks show that they struggle with basic mathematics operations. This gave rise to an 'emergent (in lesson) task design' afforded by

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[7] Extension tasks drawn from https://www.mangahigh.com/en-gb/
[8] Pre-lesson interview 01/02/2016
[9] Post-lesson interview 01/02/2016

access to multiple digital resources aligning with Joe's earlier remarks *feedback into my next lesson* this led to a new task design mid-stream. He introduced a new resource *Mangahigh*[10]. This enabled the teacher reteach and refresh the previous learning on the 'sense of fractions and decimals' using bar modelling.

In a previous interview on the impact of digital resources on his assessment practices, Joe stated the changes this affords:

> "It can really impact on planning time. Within the lesson I can assess them, I can find out what their weaknesses are, I can help them with those weaknesses[11]"

Conclusion

The premise underlying this exploration is that the mathematics teacher's integration of digital resources into mathematics teaching could extend FA practices. We identified a variety of resources that have potentials to enhance FA. The affordances of these resources enable the teacher to provide immediate feedback to students and facilitates the teacher's informed choices and decision. In addition, for an effective FA practice with digital resources, a cycle of carefully planned and enacted lessons and well-regulated whole-class discussion is indispensable tools for learning.

In spite of the growing evidence of the potential of digital resources enhanced FA practices, the challenges of the 'complexity' (Monaghan, 2004), the 'hiccups' (Alison-Clarks, 2015) of integration of digital resources could be the catalysts for further teacher professional development focused on the development of teachers' assessment literacy with digital resources.

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[10] https://edshelf.com/tool/mangahigh/ a game-based-learning sites with adaptive contents that changes the difficulty levels of tasks as learner progresses.

[11] Interview 03/02/2016

WG3. Instrumentation, competencies, design capacity, expertise

GT3. Instrumentation, compétences, capacités à concevoir, expertise

Chairs / Coordinateurs : Sebastian Rezat (University of Paderborn, Germany) & Carole Le Hénaff (University of Western Brittany, France)

Teachers use resources in order to support their teaching, to support students' learning, and to advance their own pedagogical and content knowledge. Using resources requires particular knowledge and skills. These are conceptualized within different theoretical frames as competencies, aspects of design capacity, teacher expertise, professional knowledge, or utilization schemes within the instrumentation process. The working group focuses on empirical studies, theoretical advancements and methodological contributions related to these aspects of using resources within the documentational approach or other frameworks related to the following questions

- How are these aspects of teachers' use of resources approached theoretically and methodologically?
- How do these aspects develop over time in teachers' practices?
- How do these aspects relate to teachers' previous experiences and knowledge?
- How do these aspects relate to features of the resources and the subject matter?

Les enseignants utilisent des ressources afin de soutenir leur enseignement, de soutenir l'apprentissage des élèves et de faire progresser leurs propres connaissances disciplinaires et pédagogiques. L'utilisation de ressources nécessite des connaissances et des compétences particulières. Celles-ci sont conceptualisées dans différents cadres théoriques en tant que compétences, aspects de la capacité de conception, expertise, connaissances professionnelles ou schèmes d'utilisation dans le processus d'instrumentation. Le groupe de travail se concentrera sur les études empiriques, les avancées théoriques et les contributions méthodologiques liées à ces aspects de l'utilisation des ressources dans l'approche documentaire ou d'autres cadres liés aux questions suivantes :

- Comment ces aspects de l'utilisation des ressources par les enseignants sont-ils abordés théoriquement et méthodologiquement ?
- Comment ces aspects se développent-ils au fil du temps dans les pratiques des enseignants ?
- Comment ces aspects concernent-ils les expériences et les connaissances antérieures des enseignants ?

- How do these aspects relate to teachers' capacity to re-design them during their teaching action?

- Comment ces aspects se rapportent-ils aux caractéristiques des ressources et à la matière ?

- Comment ces aspects sont-ils liés à la capacité des enseignants à les redéfinir lors de leur action didactique ?

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WG3. Oral Communications
GT3. Communications orales

The evolution of a resource stemming from research

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Abstract: The resources produced over many years by the researches of the ERMEL team (Ifé) were becoming less and less accessible to teachers because of the evolution of the training courses. Therefore, we had to produce new resources based on those researches which focus on most important situations for the student's mathematical learning, and also accurately describe the teacher's professional gestures. This communication presents the analyses and the choice that build the evolution of a resource as well as several questions that arise from it.

Keywords: Production of resources, Teachers' practices, didactic

I. Presentation.

1. The ERMEL team's researches and the production of resources.

The ERMEL team has conducted researches on mathematical learning in school, building on problem solving first in the numerical field then geometric field. These researches were meant from the very beginning to be broadcasted in the form of books to teachers and trainers in primary education.

The research methodology includes:

- An identification of the teachers' needs, for example about the implementation of didactic situations.
- An analysis of mathematical knowledge (problems, properties, symbolic representations…).
- The organization of the study of all numerical (or geometric) notions of a cycle (in the French definition, a cycle spreads over several years).
- The construction of didactic situations and the experimentation in many classes of different social background in several academies.
- The identification of pupil potentiality is also coming from the experiments, in interacting with previous component.
- The writing of books including:
    - Further elaboration of the stakes of learnings and problems of the teacher
    - And the teaching devices experimented upon; their description allows the teacher to anticipate the mathematic activities of the pupils.

These books are often considered references on teaching and are particularly analyzed in conference proceedings on teacher training (Bautier et al., 2004) or researchers (Gueudet, Aldon, Douaire, Trgalová, & Winslow, 2010), (Houdement, 2013) (Butlen, Masselot, Pézard, Amigues, & Kherroubi, 2003). The situations produced are often qualified as "strong", an assessment based on the experimentation conducted in many classes.

2. A new inquiry on the use of resources

The use of those books as resources for teachers presumes of the understanding of the stakes of learnings exposed in the theoretical chapters at the beginning of each book even though the teachers

are not experts on the subject. Furthermore, the implementation of activities, for example devolution, requires professional gestures as defined by Paquay et al. (1996). However, these last few years, feedback from teachers and trainers has shown that the understanding of the learning stakes or the characteristics of didactic situations has become more difficult for teachers who discover our resources, regardless whether they are beginners or experienced teachers. We may hypothesize that the evolutions of the training courses, initial or continuous training, and in particular the necessary mediation given by the trainers for the appropriation of those resources have become more uncertain, also taking into account the general evolution of the profession.

Therefore, in order to ensure the broadcasting of the research results and to allow teachers to use them in current situations, we have decided to rethink their use and produce a new resource. It is that process and its hypothesis and questioning that we wish to explore here.

II. process analysis

Several observations are formulated by teachers and trainers on these ancient resources.

1. An analysis of potential appropriation difficulties

First of all, the reading of the theoretical parts can be difficult for a teacher who discovers the resource if it is not articulated with a training course.

The most important situations – the ones which allow for a significant and qualitative jump in the pupils' knowledge – are not always clearly identified among the numerous activities.

In those situations, decisions about the organization of the work sessions (i.e. individual work, group work,…) are sometimes left up to the teacher and the criteria are not often explained. Other decisions or interventions made by the teachers are also not always detailed.

This analysis leads us to offer new strategies for the appropriation of the content.

2. Leading choices

The new resource we are constructing completely rethinks the benefits of researches in the following ways:

- An entry based on the most important situations which leads us to offer a clearer and year-long progression centered a fortnightly situation. Those 15 situations are the ones which provide a qualitative jump allowing for an increase of learning over the year.
- A precise description of these situations and their course while removing the options that were not relevant in the mathematical activities of the pupils.
- A further explaining of the many decisions that the teachers can make in regards to the characteristics of their class by giving the choices with their criteria.
- Answers to questions that the teacher may ask themselves while implementing these situations, whether is about the mathematical activity of the pupils or the conduction of those sessions (i.e. giving the instructions, managing the sharing session or how to deal with a multi-level classroom). The reflexions on those professional gestures rely, for example, on chronicles illustrating the actions of the teachers.
- The stakes of learning or the progression choices are further explored later.

The underlying hypothesis is that teachers can change their own practice through the analysis of a personnel experience and practice where uncertainties are limited rather than through the prior understanding of theoretical frameworks.

To encourage this analysis, we offer three "tools":

- A very precise description of the actions of the teacher
- A list of technics to implement those tasks, commented throughout the description
- Gain some distance, thirdly, through a speech/discourse that shows that these techniques are common to many situations.

Furthermore, the potential use of those resources by beginner teachers leads us to offer a progressivity in the implementation of the professional gestures required in the situations during the year, for example identify the pupils' knowledge and give them a problem at the beginning of the year and then accompany a research or differentiate the tasks or conduct a sharing session subsequently.

During our communication we will offer further elaboration on those questions by comparing the situation in the old and new resource.

III. conclusion: drafts of answers to the study group questions

We will approach the questions asked by this study group by trying to elaborate on this "empirical study", on the "theoretical progresses" (or rather the theoretical questions) and the "methodological contributions" linked to these aspects of the use of the resources.

1. How are these aspects of teachers' use of resources approached theoretically and methodologically?

The identification of the needs of the teachers is at the very root of our researches and their very purpose is to produce resources for teachers. The knowledge of the pupils are analyzed through several didactic works and the proposed situations rely on the theory of didactic situations (Brousseau, 2011) (notion of didactic contract, variable, validation…) and also the works of Vergnaud, Douady, Chevallard… Their appropriation works through the comments about the implemented activity.

2. How do these aspects develop over time in teachers' practices? How do these aspects relate to features of the resources and the subject matter?

We currently do not have research results on this point as our resource is still fairly new, but only some opinion of teachers or trainers. We have elaborated on the progressivity of the professional gestures necessary to the implementation of the situation while considering that the resource must be accessible to a beginner teacher who probably doesn't have sufficient markers in their practice to anticipate, for example, the consequences of a premature intervention during the research phases. To produce a resource, we must take the teachers' skills into account. Even without the beginner's case, there's such a large variety of experiences that it is difficult to list them. For example, during a previous research[1], we noticed that several teachers offer problems but quickly "take charge" again

[1] Reference (« Faire des maths en classe »)

by handing out judgements on the productions during the validation phase which in turn become the grading phase. The stakes of certain validation phases are described and accompanied, when required, with chronicles illustrating the possible actions for the teacher.

3. How do these aspects relate to features of the resources and the subject matter?

Many gestures that the teachers make are not specific to teaching mathematics. For example, the management of language interactions: sometimes in a didactic situation's description, we see a lot of the interaction between the situation and the pupil but the "teacher is outside"; we therefore try to explain this part of silence… or this part of language activation.

4. How do these aspects relate to teachers' capacity to re-design them during their teaching action?

We could analyze the appropriation of this new resource by teachers, whether they are alone or accompanied through a training course or its use in a training session as an analysis tool for professional gestures – these both seem to be extremely interesting leads.

However, the processes through which these professional gestures are acquired would be a research focus about primary school teachers pratices. In what conditions are those generalizable to other subjects[2]? And therefore, how can we enable their acquisition through the use of a resource? We can ask the following theoretical question what is the relationship between didactic knowledge and pedagogic skills?

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[2] référence possible à n° 75

Math Trails: a resource for teaching and learning

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This paper presents a study about the potential of math trails as a resource for the teaching and learning of mathematics in a non-formal context. This researched focused on future teachers of basic education and is of qualitative nature. Results showed that the participants evidenced a more positive attitude towards mathematics, broadening their perspective about the connections between mathematics and everyday life. It was also possible to conclude that, despite the design of the tasks through the trail not being diversified or easy, the motivation increased along the experience.

Keywords: Tasks, math trails, problem solving and problem posing, teacher training.

Introduction

Much of the mathematical failures presented by students are originated in the affective environment fostered in the classroom that can compromise their expectations and motivations. To reverse this situation, and since teachers play a key role in what happens in the classroom, teacher training must promote a new insight into the nature of mathematics and its teaching, allowing future teachers to experience new approaches to later use with their own students. We believe that mathematics is accessible to all and present in everything that surrounds us. Only actions that show these features may lead to a greater awareness, and will enable us to reverse the demotivation and the negative image of mathematics. The contact with a contextualized mathematics, inspired by a non-formal context like the outdoors, can be a powerful approach to the teaching and learning of mathematics. The main purpose of this study was to construct mathematical tasks associated to a trail outside of the classroom, to foster a new attitude towards mathematics, through the observation and exploration of the urban environment in the context of elementary teacher training. Simultaneously it is an opportunity for future teachers to formulate problems, applying and mobilizing mathematical knowledge, in a realistic situation. Grounded on these ideas, the following research questions were posed: 1) Which mathematical contents can emerge from tasks formulated from the surrounding environment?; 2) What kind of difficulties are expressed by future teachers when formulating these tasks?; 3) How do future teachers relate to non-formal learning environments?

The importance of the tasks in mathematical activity

The learning of mathematics is heavily dependent on the teacher and the tasks proposed to students (e.g. Stein & Smith, 1998). The tasks are an important mediator between knowledge and students in the teaching and learning process. In this sense, it is important that teachers develop certain skills, based on a deep mathematical and didactical knowledge of mathematics, allowing them to build, adapt, and explore good mathematical tasks. Students' learning should include more than routine tasks, it should be enriched with challenging tasks such as problem solving. Nevertheless, a prescriptive perspective on problem solving, reducing it to the teaching of strategies, has proved to be insufficient. Students should be involved in processes like discovery and invention, refinement

of methods and forms of representation, looking for different ways to use mathematical knowledge, and be persistent in solving problems. The tasks focused on problem solving and posing have great potential for learning, contributing to the acquisition of mathematical knowledge but also to the development of important skills. Contexts where students have the opportunity to solve mathematical problems using a variety of strategies, and formulate their own problems, make them more involved, increase their motivation and encourage them to investigate, make decisions, look for patterns and connections, generalize, communicate and identify alternatives. Furthermore, problem solving and problem posing give teachers important information about how students understand and use the concepts and mathematical processes, allowing also the identification of their attitudes towards mathematics. In particular, it helps students reduce the anxiety levels concerning their learning and at the same time promotes a higher level of creativity (Brown & Walter, 2005). The majority of mathematical failures come from the affective environment (attitudes, emotions, conceptions, feelings), and this can influence the teaching and learning process of mathematics, inside and outside the classroom (Hannula, 2006).

Math trails as a means to approach problem solving and problem posing

The process of acquiring information and promoting the development of knowledge occurs in many ways and many places (Kenderov et al., 2009). In this sense, the outdoors is a possibility. The use of the surroundings as a classroom environment can foster positive attitudes and an additional motivation for the study of mathematics, allowing students to understand its applicability. A mathematical trail consists of a "sequence of stops along a pre-planned route, in which students study math in the surrounding environment" (Cross, 1997, p. 38) and offers concrete learning experiences for any of the mathematical concepts taught in the school mathematics curriculum. This type of activity facilitates the creation of a non-formal meeting space centred on the learning of mathematics, simultaneously addressing problem solving, the establishment of connections, communication and other mathematical skills in a meaningful context. Since it occurs outside of the classroom, a math trail creates an atmosphere of adventure and exploration and, at the same time, gives students and teachers the opportunity to solve and pose problems. If we believe that learning mathematics is heavily dependent on the teacher and the tasks, it is necessary to provide future teachers diverse experiences, to develop their abilities in this area. Teacher education must promote a new vision of mathematical knowledge and the teaching practice, enabling future teachers to experience the same tasks that they are expected to use with their own students, and a math trail is an example of a great resource to engage students in a meaningful activity (Barbosa & Vale, 2016).

Method

We adopted a qualitative methodology. The participants were 70 students of a teacher training undergraduate program for basic education (3-12 years old children). During the classes of a Didactics of Mathematics course, students were exposed to teaching modules focused on problem solving and posing, as well as on processes such as communication, reasoning and mathematical connections. After these modules, they developed a group project called “Mathematical Trail", which consisted in defining a route in an artery of the city of Viana do Castelo (Portugal), which integrated multiple tasks, appropriate to basic education students, based on characteristic elements

of the city (e.g. monuments, windows, gardens, maps, tiles, crafts). They started by choosing the artery and then they walked through it several times in order to take photographs of elements of the local environment that would allow a mathematical exploration. During the classes the students shared their ideas and got feedback to improve the construction of the trail, in terms of the tasks and the description of the route. Data was collected in a holistic, descriptive and interpretative way, including classroom observations, a questionnaire focused on the students opinions about the experience (implemented at the end of the semester), and the written work they produced. This evidence was analysed according to criteria such as: diversity of the tasks/contents, rigor of the mathematical content, reactions of the students. The classroom observations originated field notes, to which was added the analysis of the written work associated with each mathematical trail. Through this data we perceived the contents privileged by the students, the difficulties they felt, the typology of the tasks proposed, among other aspects. The questionnaire was a personal reflection about students' attitudes and allowed to complement the already collected data.

Some results

Considering that, in this work, students started from a static situation, the photographs, from which they posed questions or formulated problems without changing what was presented, they mainly used *Accepting the data* as a problem posing strategy (Brown & Walter, 2005). In Figure 1 we present some examples of the tasks created by the students.

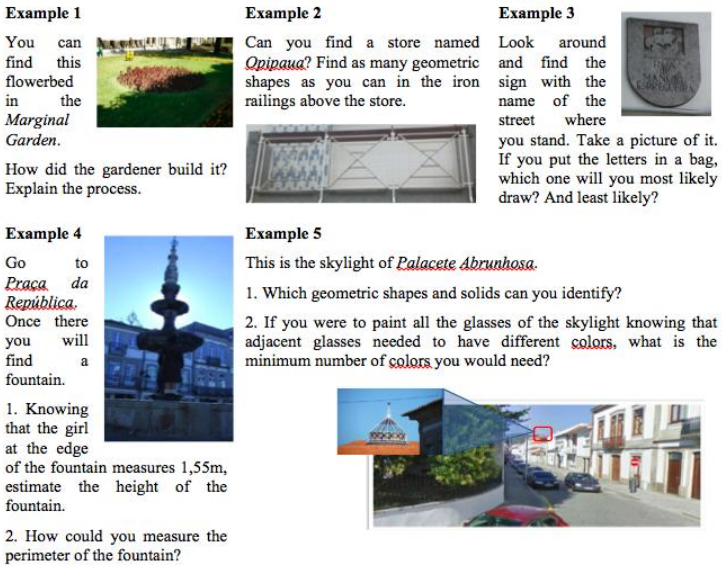

Figure 1: Examples of tasks formulated by the students

Most of the groups created tasks that were familiar to them or had expression on textbooks, like problems and exercises. Designing the tasks was one of the main difficulties for these future teachers, due to their lack of experience in problem posing. The contents involved in the tasks were elementary mathematical concepts and generally could be applied in a formal context like the classroom. Overall these future teachers showed a tendency to mobilize geometry concepts, choice that can be explained by the fact that the elements involved in the trail were of visual nature. Nevertheless, as can be seen in the examples presented in Figure 1, some groups were able to diversify the underlying contents, approaching themes like probability, algebra, measurement or

numbers and operations. After selecting the photos and formulating the tasks, future teachers organized them in the form of a trail, considering the route, the stops along the way and the sequence of tasks to be solved. They came up with different structures to present the trail, but the majority used maps, flyers and traditional structures similar to a worksheet. Some groups proved to be more original, presenting the trail in the form of a treasure map or books with the description of the route and the tasks. At the end of this project the participants were given a questionnaire to access their opinions about difficulties, positive aspects of this work and the impact on their perspective about mathematics teaching and learning. They recognized the difficulty of organizing a trail, assuming the role of the teacher, mentioning aspects like: formulating the tasks (correctness, clear language, diversity of the type and the contents); sequencing a balanced trail (e.g. distance, number of stops, time of exploration); there are more natural themes than others (e.g. figures, area, perimeter, patterns). However, the majority also valued the potential of this type of work to promote a positive image of mathematics, highlighting the opportunity to experience its applicability.

Conclusions

This work contributed to provide future teachers with a more positive attitude towards mathematics and to acquire a broader view of the possible connections that can be established between mathematics and the world around us. Non-formal learning contexts, like the math trails, have the potential to trigger meaningful learning, involving students in a direct way, discovering the invisible mathematics in the elements of the local environment. The interaction with this resource (the tasks originated along a trail) was a crucial experience for these future teachers, conceiving the math trails as an opportunity to support learning. The design of the tasks was not an easy process, particularly from the point of view of the mathematical knowledge involved, whether in the degree of challenge but also in the diversity of the nature of the tasks, focusing on the most obvious and simple concepts associated to geometry. However, the students became more aware of the applications of mathematics, showing willingness to overcome the difficulties they faced, valuing the advantages of the math trail as a teaching and learning resource.

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Two case studies on the interaction between historical investigation and the documentary work of teachers

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The paper is based on two cases questions how research on sources studied by historians of sciences car interact with the design of teaching projects. One case concerns the use of excerpts of Diophantus's Arithmetica*, an antique text written through arithmetical problems, in the framework of an interdisciplinary project associating French and Mathematics; the second concerns the work done by a trainee teacher to gather problems that enhance critical thinking in the student's apprenticeship in mathematics. Commenting on these studies, we question to what extent, and in which way, the features of the historical documentation influence the design of pedagogical projects, and conversely how this design can trigger historical inquiries. We highlight in particular the crucial importance of the epistemological and didactical questions of the teachers involved, pre-existing any interaction with the historical matter investigated by researchers.*

Keywords: History of science, series of problems, teacher's documentary work, probability, critical thinking.

1. Context and research questions

The proposed contribution is part of a research project begun in Académie de Créteil in 2016. Part of this research is focused on clarifying the way in which the kind of historical sources and studies, investigated in the project, might become resources for teaching and professional development; and conversely, how the documentary work of the participants of the project, can trigger historical inquiries on the corpus we are concentrating on.

The concrete basis of this research is the existence of a group (belonging to *IREM de Paris Nord*) in which historians, mathematicians and teachers (of history and mathematics) can interact in complex ways. The group has organized since 2013 several in service teacher training sessions, in relation to a research project in history and anthropology of science on historical sources written as collection of problems (Bernard 2015). When the above mentioned project about "laws of randomness" was launched in 2016, this IREM group was enlarged to welcome new members that would contribute to his new project. In all cases, we are speaking with very narrow groups: no more than ten persons each time, including researchers.

The special effect produced, in the mentioned IREM group that serve as an "experimental area" for our inquiries, by the interaction between researchers in history and teachers, has already been analysed (Bernard & Gosztonyi, 2014, 2017). These first studies have emphasized the crucial importance of one working principle: namely that teachers usually came to this group not just because there were interested in the historical material discussed within it, but also *because the proposed sources echoed their preexisting experience and questions*. Working on the basis of this working principle amounts to postulate there is an *encounter* between historical reading and

research on the one hand, and the professional experience of the participants, on the other. This hypothesis implies that the work these teachers do with the historical sources they find in this kind of group is not a function of the sources themselves, but primarily of their usual work and the whole set of resources they usually imply and elaborate in their work. This pre-existing work and reflections gives the historical sources they come to meet, its proper meaning.

Our first research question, then, focuses on the *fact* of this encounter, and the second on its *nature*. As for *fact*, our first purpose is to show that this hypothesis is an adequate one, because it reflects the reality and complexity of the internal exchanges within this kind of group, in which heterogeneous perspectives can interact with each other. The two case studies chosen here are meant to illustrate this fact, following our first attempt in (Bernard & Gosztonyi, 2017). As for the *nature* of the encounter, the issue is to show that it is not unique but can take several forms. In the first case study, we illustrate how the historical interpretation of an ancient text could meet the long term reflection of two teachers working on an interdisciplinary project: the historical interpretation thus contributes to enrich a preexisting didactic thinking. In the second case study, we show that the reflection of teachers on their proper objects - namely small problems meant to trigger critical thinking by the students - influenced the historians' investigation on similar problems found in their corpus. Here the historical investigation partly becomes a function of the didactic reflection of other members of the group.

In the two case studies chosen here, the work of the associated teachers was done in interaction with researchers in history of mathematics. The data, on which we rely and that we discuss are mainly the resources produced by teachers and exposed to the group and/or for the associated training session, and interviews of the involved teachers in the first case; in the 2nd case, we rely on the same kind of material, as also on a master memoir written by a teacher trainee.

2. First case study: historical material incorporated into the design of an interdisciplinary teaching

This first case is based on the interdisciplinary cooperation of two teachers at the level of middle school, who have come to organize in 2016-17 for their students an interdisciplinary workshop (Classical litterature and Mathematics) on the general topic of *Humanistic figures from Antiquity and the Renaissance period.* Among the cultural figures of knowledge and science retained by the colleagues, there was Diophantus of Alexandria, an antique mathematician and the author of the *Arithmetica*. This important treatise was originally constituted of arithmetical problems arranged in an order of progressive difficulty and that are solved by a seemingly algebraic method. It is introduced by a sophisticated preface that borrows several key commonplaces of ancient rhetorical education, thus lending the treatise a flare of classical and philosophical prestige (Bernard & Christianidis, 2012; Bernard, 2015). These features also superficially explain why the treatise appeared as a perfect fit for the above mentioned teachers, who were naturally looking for a text that could be exploited both in mathematics and in *Lettres classiques*.

At the surface, the work of the colleagues in question could be understood as stemming from a first encounter (within a teacher training session) with historical material that later triggered an interdisciplinary workshop. Closer inquiry (through interviews and mutual clarification) shows that

the didactic reflection of the teachers was developed much before the training session and was already turning around key questions like the role of language in mathematics, the importance of translation between various semiotic registers, or questions related to pupils' perception of their own apprenticeship. These reflections echoed the leading research questions behind the historical interpretation of Diophantus: namely, that the technique of ancient *paraphrasis* is a key feature of Diophantus' method of exposition.

The effect of this interaction can be followed against the evidence furnished by the concrete way, in which the text of the translation of Diophantus was changed in a pedagogical document through the addition of specific tasks, altogether obeying a set of didactic objectives made explicit by the two teachers.

2. Second case study: the conception of resources triggering historical inquiries

The second case study is based on the work of some participants to the IREM group working on themes related to the relationship between statistics, probability and citizenship. One of them was in 2016-17 both a trainee teacher (*stagiaire*) in mathematics, working in a high-school for 10-th grade students (*secondes*), and a student in a master curriculum. As such, the theme of her master thesis was related to the topics of the research project. She thus attended the first workshop of the project held in Jan 2016. On this occasion, several possible resources were presented and discussed, some coming from the historical corpus under investigation, others coming from the published work of the former "mathematics and citizenship" IREM group (Dutarte et al. 2007).

In this work, she concentrated on the various ways in which critical thinking could be cultivated in mathematics (in the framework of her own teaching) and how these various levels of activity could coordinate with each other (Chabot-Déjà 2017). Among other possibilities, she constituted and categorized a collection of small problems that are meant to trigger the students' curiosity and to call to their critical attitude, on the model of preexisting material found either in school manuals or in the kind of IREM booklet mentioned above. At a further meeting of the group gathered around the research project mentioned in introduction, it appeared that this work and the underlying questions were shared by other members.

The attention paid to daily situations, that can awake the students' consciousness of the need of a critical attitude toward "biased" information, met some questions raised by the historians implied in the group about the existence of *recreative problems* on the topics of probability and statistics, developed at the turn between the 19th and 20th cent. Some of these "historical" problems could indeed ultimately find place, among the proposed collection mentioned above, and could give the didactical questions raised some historical depth.

This case therefore illustrate a didactic reflection leading to new resources met the *historians'* inquiry and triggered new questions, on a corpus on which they would perhaps have put less emphasis without this experience and interaction with pedagogical concerns.

3. Theoretical underpinning

The *encounter* illustrated by the two case studies then represent the meeting between two intellectual logics of interpretation that, for deep reasons that are discussed in particular in (Fried

2001), are not entirely compatible with each other. But the fact that they pursue different ends does not mean that the one can interact with the other, as in a system of echoes. The one logic, then, (that of historical interpretation) met the other (the didactic one) and produced a kind of deviation and enhancing of the latter.

For the study of these questions, furthermore, the focus on teacher's resources and documentary work (Gueudet, Pepin, et Trouche 2011) is an adequate framework of understanding, against which the impact of historical studies on teachers's documentary work (and conversely) can be studied. Others have appeared useful: the notion of transitional objects or boundary objects (Bernard and Gosztonyi 2017).

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Reflecting on didactic metadata of learning sequences

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With the emergence of e-textbooks, together with expectations to integrate technology in instruction, teachers are coming to take an active role in designing the curriculum they teach. In order to maintain instructional coherence, teachers need to develop sensitivity to various didactical aspects of the curriculum. We suggest the use of novel tools for tagging didactic metadata of learning resources, and for representing learning sequences from the perspective of this metadata. We report on an ongoing experiment with practicing teachers. Over a period of several weeks, participating teachers tag all of the learning objects that they use in class, construct various representations of the taught curriculum, highlighting different didactic aspects of the sequence, and reflect on the instructional coherence of the sequence in light of these representations. We ask what effect this intervention may have on teachers' pedagogical design capacity.

Keywords: Tagging, instructional coherence, pedagogical design capacity.

Literature review and theoretical framework

The linear arrangement of conventional textbooks implies a "teaching sequence" of the intended curriculum – a collection of learning activities to be undertaken in a particular order. Yet, in enacting the curriculum, teachers may change the sequence, adding and removing tasks, or ordering tasks differently from the author's intention. In so doing, teachers are exercising *pedagogical design capacity* – "their ability to perceive and mobilize existing resources in order to craft instructional contexts" (Brown, 2009, p. 24). Such changes can influence *instructional coherence* of the intended curriculum in significant ways, for example, Yerushalmy and Chazan (2008) have shown that sequencing can influence ways in which inherent discontinuities in the content are addressed. As e-textbooks become more common, teachers are taking a more active role in designing sequences of activities. This is especially true in the case of the interactive model of e-textbooks (Pepin et al., 2015), where tasks and interactives can be linked and combined in different ways. This emerging practice is blurring Pepin et al.'s distinction (2015) between the *coherence of design* of a textbook (in its intended curriculum) and its *coherence-in-use* (as enacted).

Whatever model of textbook is in use, the instructional coherence of the intended and the enacted curricula is liable to remain opaque to teachers. To address this, we are developing a coupled pair of tools to support teachers as co-designers of curriculum: a tagging tool for associating didactical metadata with learning resources, and a tool for representing and navigating a collection of tagged resources. The overarching question that guides our research is: How do these tools, along with the categories of metadata that they support, influence teachers' curricular discourse, and in particular, their pedagogical design capacity? In addressing this question, we follow Drijvers & Trouche (2008) in focusing on two complementary aspects of *instrumental genesis* of these tools; *instrumentalization*

is the process by which subjects shape the instrument and its use, and *instrumentation* is the process by which the artifact influences subjects' activity and thinking. Both these aspects influence and are influenced by teachers' pedagogical design capacity.

Our broader research project can be described along two orthogonal dimensions; A. We are investigating both intended curricula (textbooks, or teachers' curricular planning), and curricula as enacted by teachers; B. We view these curricula in terms of *balance* – relative prominence of various types of learning resources – and of *sequence* – the order in which these learning resources are presented. Our research to date has focused mainly on textbook balance (see Cooper, 2017 for an overview). In the present report we turn our attention to the balance and sequencing of enacted curricula, and ask: What are the affordances of tools for tagging and representing didactical aspects of learning sequences as a means for eliciting and promoting teachers' pedagogical design capacity?

Methodology

The study described herein is taking place in the 2017-2018 school year, currently with the participation of 2 secondary-school teachers in Israel over approximately 8 weeks of teaching.

- For each learning resource used (e.g. tasks), teachers tag didactic metadata (see the following section for some categories of metadata), pertaining to both *design* (e.g. nature of operations on mathematical objects, type of student interaction with technology, available representations) – and *use* (e.g. class arrangement, duration, class-work or homework).
- Researchers independently tag the learning objects in the first two weeks, as validation for the teachers' tagging. Discrepancies are discussed and resolved with the teachers.
- At the end of the 8-week teaching period we prepare "representations" of learning the sequence from two perspectives – balance and sequence (see appendix).
- We are conducting semi-structured artifact-based interviews with teachers, through which we elicit the meaning that they ascribe to categories of metadata, and their significance in reflecting on didactic aspects of the enacted sequence.

Analysis and findings

Our analysis begins with representations of the tagged sequences, in which we search for patterns. Findings will later be discussed with teachers, to elicit aspects of their design capacity. We expect patterns of three types: 1. Intentional results of teachers' thoughtful sequencing; 2. Unintentional yet reasonable upon reflection; 3. Unexpected or surprising. Patterns of the 2nd and 3rd type will serve as a springboard for discussing alternatives to the sequence that was enacted.

We have analyzed one teacher's tagged sequence and present preliminary findings, noting that their significance will emerge through interviews which will be conducted and analyzed at a later date.

Balance: Tasks tagged as *modeling* tend to have a prominent *numeric representation*; tasks tagged as *transforming* have a prominent *symbolic representation*, longer tasks (15-45 minutes) require *drawing conclusions*, are arranged for *individual* work, and do not include *numeric representation* (see Figure 1 in appendix).

Sequencing: The object *circle and points* is sequenced spirally – in consecutive tasks at the beginning of the sequence, and then intermittently (Figure 2A); similarly, *graphic representations* is minimal at

first, is high in a number of consecutive tasks, and then returns intermittently (Figure 2B); *numeric representation* is prominent at the beginning of the sequence, and later fades (Figure 2C); *whole-class* activities appear once every lesson (Figure 2D).

Interviews with teachers will be analyzed to characterize teachers' curricular discourse, focusing on their considerations in selecting and sequencing tasks.

Acknowledgments

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Appendix: Representations of a learning sequence

Figure 1 is a screenshot of the balance representation tool, developed on an open platform for data analysis (Keshif LLC). Locking on a value of metadata (in this case the value *medium* in the category *duration*) highlights the relative prominence of these tasks within categories of the collection.

Figure 2 includes screenshots of the sequence representation tool, developed on the same open platform. The tool shows the progression over time of each category of metadata.

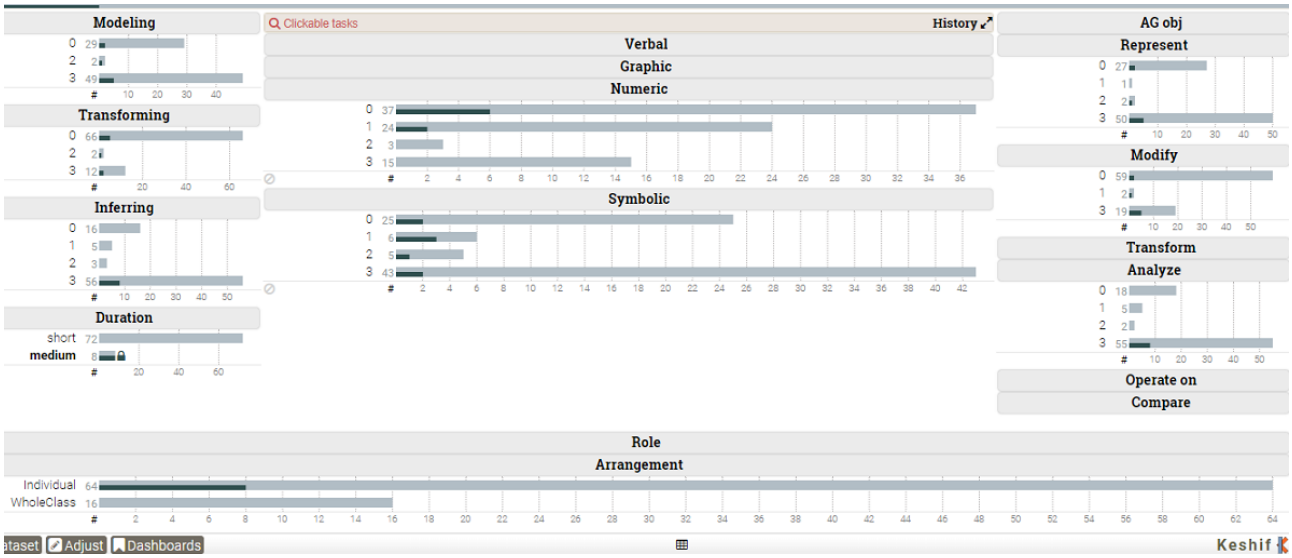

Figure 1. ***Dashboard representing balance of the sequence.*** **Locking on** ***medium duration*****, we see that all 8 of these tasks require** ***inferring and drawing conclusions*** **(at level 3), were arranged as** ***individual*** **activities, and do not include** ***numeric representation*****.**

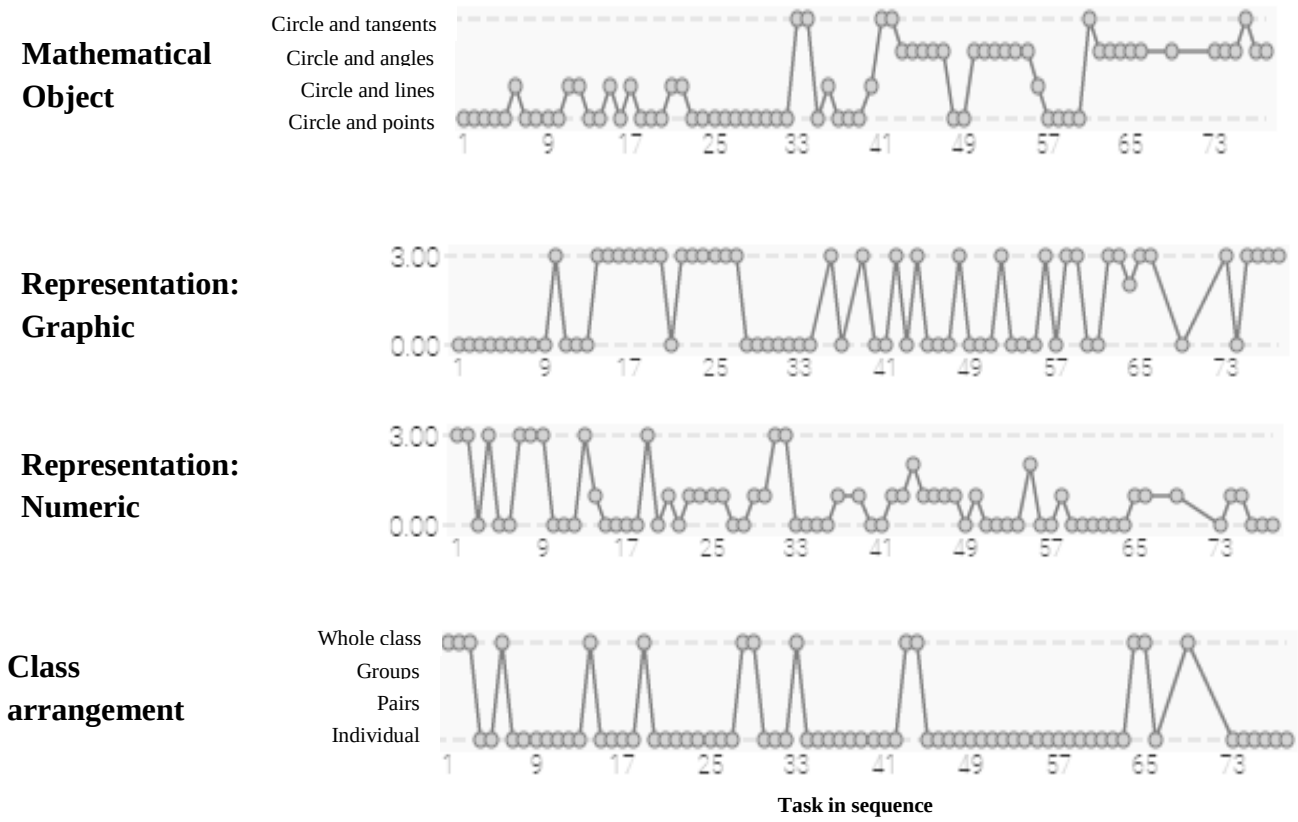

Figure 2. Representing the sequence of tasks from 4 perspectives, corresponding to 4 categories of tagged metadata.

Teachers documentary work with series of problems

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This paper discusses the role of series of problems in the Hungarian "guided discovery" tradition of mathematics education and some questions related to the use and the construction of these series by teachers. First we analyze how series of problems are present in the mentioned guided discovery approach in Hungary; then we discuss potential difficulties of teachers to use existing resources or create their own ones; finally we present a new project aiming to better understand teachers' work with and on series of problems and to create new resources in order to help teachers' work.

Keywords: Series of problems, guided discovery, conception of teaching resources, teacher competencies

Introduction

The research presented in this paper is part of an ongoing Hungarian research project[1], focused on the Hungarian "guided discovery" tradition of mathematics education based on problem solving and mathematical discovery. This tradition exist first of all in the teaching of gifted students, but several efforts were made to use similar methods in ordinary mathematics education: the main example is the reform led by Tamás Varga in the 1960s and '70s which is recognized in Hungary as a relevant and inspiring model of mathematics education until today. A narrow and relatively well defined circle of teachers follow today the approach developed in these frames, while its transmission to a broader circle of teachers appears problematic. Our research aims on one hand to reconstruct the theoretical bases of this approach which are only implicitly present in the documentation of the Hungarian tradition, and on the other hand, to contribute to its more efficient transmission amongst nowadays teachers.

In this paper we focus on one of the main characteristics of the approach in question, namely on the crucial role of ordered series of problems in the conception of teaching processes (Gosztonyi, 2015a, 2017). In the first part we analyze the role played by series of problems in the Hungarian guided discovery approach and the available resources. Then we discuss potential difficulties of teachers to use existing resources or create their own ones. Finally we present a new project aiming to better understand nowadays teachers' work with and on series of problems and to conceive new resources in order to help teachers overcome the identified difficulties.

A Hungarian tradition of using series of problems in mathematics education

We identified in precedent studies (Gosztonyi, 2015b) two main characteristics of the Hungarian "guided discovery approach" represented by Varga and other authors: organizing short and long

[1] MTA-ELTE Komplex Mathematics Education in the 21th Century project supported by the Hungarian Academy of Sciences (ID number 471028)

term teaching processes on the basis of series of problems, and guiding classroom dialogues around problems. Our analysis was principally based on written sources, inspired on one hand from historical research (Bernard, 2015), considering written documents as traces of existing teaching practices, and on the other hand, from didactical research considering these documents as resources for teacher's work in a sense similar to (Gueudet & Trouche, 2010).

One of the first written appearances of series of problems in the Hungarian context can be found in a book popularizing mathematics, *Playing with infinity* of Rózsa Péter, published in 1944: this book aims to present the way mathematicians construct mathematics by guiding the non-mathematician reader through problems, attempts of solutions and new problems emerging from them. The author contributed later to a collection of textbooks (Gallai & Péter, 1949) in which tasks are systematically organized in series depending one to the other. We can also refer to the works of G. Pólya (1945) who proposes (and sometimes analyses) series of problems in several of his books and emphasize on the references to already solved problems while solving new ones.

All these mentioned sources (as well as the authors themselves) exerted important influence on Varga's reform. The primary school teacher's handbooks related to the reform contain a number of reflections on the organization of problems into series. However, they don't always propose completely developed series: in some cases they insist on a given order, in other cases they leave the work of organization to teachers or they give some advices to the development and organization of a series with only some suggestion of problems. In this way, the authors leave important autonomy and responsibility to teachers in the elaboration of series of problems: this autonomy being considered as necessary to the convenient adaptation of teachers work to the special needs of their classes. It is important to underline that the series suggested in these books are conceived not only for the short but also for the long term (which might mean even several years[2]), and can cover several domains of mathematics (which should have as much intersection with each other as possible in Varga's conception). This means that these series of problems might be quite complex and are closely related to the long-term progression of teaching – in fact, the handbooks encourage teachers also to elaborate their own teaching progression, even if they give some examples.

The middle-school textbooks of Varga's reform present also numerous organized series, as well as fictive examples of classroom dialogues around a short series of problems; but much less commentaries explicate the organizing principles for teachers.

Analyses of the reform's resources helped to understand several possible roles and organizing principles of these series of problems in Varga's conception of mathematics education. One of the main ideas seems to be the construction of mathematical knowledge through progressive generalization (Gosztonyi, 2017): a big diversity of problems with repetitions and variations in diverse aspects give a large basis of experiences, while the discussions led on these problems help to recognize analogies between them, and generalize on a more and more abstract way. But we can also find other kind of uses of series of problems, from the exploration of heuristic methods to questioning a primitive version of definitions in order to develop a more general or more stable one

[2] Hungarian teachers teach the same class during several (in most of the cases during 4) years.

(Gosztonyi, 2015b). We plan to continue the theoretical analysis of series of problems issued from the Hungarian tradition and from other contexts to better understand their potential role in mathematics education.

In the contemporary Hungarian mathematics education we can also see several examples of the conception of series of problems: they appear for example in the textbook collection for middle and high schools, *Sokszínű matematika*, which was the most generally used during the 2000s.

We mention finally the work of the mathematician Lajos Pósa with gifted students which represent an influential example for many Hungarian teachers nowadays. He works on the basis of sophistically constructed "networks of problems" with long series of problems crossing each-others in several nodes.

Understanding teachers' work with and on series of problems

According to Ervin Deák, who was present at some forums in 1949 presenting the new textbook written by Rózsa Péter and her colleagues, teachers reacted quite negatively to the idea of series of problems presented in the textbooks, saying that they eliminate their pedagogical liberty. In fact, as the tasks were build one on the other, they couldn't choose freely the ones they wanted to discuss with students but they had to treat the whole series. This is a real difficulty while completely constructed series of problems are presented to teachers: if they don't understand the aim and the logic of the construction or they don't feel enough creativity to modify it, they might have difficulties to adapt them to their real teaching context. The same difficulty can be observed while nowadays teachers try to find inspiration in Lajos Pósa's mentioned work: the complexity of the construction of his "network of problems" makes it quite rigid. This is a construction, elaborated during several decades, of a high level mathematician who is a particularly creative teacher in the same time: transforming its construction (for example to adapt it to less gifted students) or taking only some parts of it suppose a profound understanding of the organizing principles and also a dose of didactical creativity.

Varga's primary school teachers' handbooks seem to take into account this problem: this might be one reason while they don't always develop whole series but offer suggestions for constructions. However, these suggestions are presented only on a very concrete level and dispersed in small remarks in the books which made them quite difficult to use.

In order to answer to these problems, in a recent project led in the frame of the Hungarian Academy of Sciences[3], we first aim to better understand teachers' actual work with series of problems. We work in a mixed group of researchers and teachers who are recognized followers of the guided discovery tradition (direct disciples of Varga and his colleagues in most of the cases) who accord an important role to series of problems in their work. They usually construct their series themselves and use textbooks or other published resources quite rarely in their classroom. We collect examples of series of problems from them, if possible with several variations developed during the years. We

[3] MTA-ELTE Komplex Mathematics Education in the 21th Century project. https://sites.google.com/view/mtakomplexmat

classify these examples according to their aim, there didactical role in the teaching process, and we analyze the structure of the series. The elaboration of this analysis' methods is in progress: we try to represent links between problems by graphs, interview participating teachers and/or ask them to write narrative text about the process the series is intended to guide. We also aim to interview them about the variations and transformations of the series and videotape some classes where they use these series during teaching.

Toward new resources to help teachers work with series of problems

In a next step of this project we aim to prepare new resources to help teachers work with series of problems. As we saw above, the only presentation of concrete series might be insufficient. On the other hand, giving only general principles of organization don't seem convenient either, as the organization of series of problems can be much related to the content and the characteristics of problems. For these reasons, we choose the idea of a commented collection of examples were the presented series of problems would play the role of generic examples, while the commentaries help them to find inspiration to transform the examples or to elaborate their own ones.

This collection of examples will contain extracts from earlier published resources, with their analysis in the commentary, but also examples issued from teachers, with explanations of their actual use and with several variations. Once a first version is prepared, we aim to experiment this resource involving teachers who aren't familiar with the guided discovery approach.

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Teacher Capacity for Using Existing Resources Productively

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Based on analyses of the data from elementary teachers using a range of mathematics curriculum programs in the United States, this paper elaborates teacher capacity needed for using existing resources productively. The capacity elaborated in this paper includes (1) articulating mathematical points of the lesson and steering instruction toward the mathematical points, (2) recognizing affordances and constraints of the resources, (3) using the affordances of the resources, and (4) filling in the holes and gaps in the resources. Each of these aspects is explained along with examples from data and related literature. This paper also discusses the need of nurturing proper operational invariants in teacher education (teacher preparation and professional development), and the role of resources in increasing teacher capacity.

Keywords: Resource use, teacher capacity, teacher education, role of resources.

Introduction

In this paper, I describe teacher capacity needed for using existing resources productively, based on analyses of the data gathered in the *Curriculum Use for Better Teaching* (ICUBiT) project, whose goal was to identify components of the capacity that Brown (2009) calls *Pedagogical Design Capacity* (PDC, i.e., "a teacher's ability to perceive and mobilize existing curricular resources" in order to design instruction) and to develop tools to measure PDC. The data were drawn from 25 elementary teachers in grades 3–5 in the United States. These teachers were using five different curriculum programs (each program includes resources for students and teachers for daily lessons), ranging from commercially-developed to reform-oriented. Analyzing the content and pedagogical support of the five curriculum programs, and analyzing how each teacher using their curriculum program to teach everyday lessons from various perspectives shed light on teacher capacity needed for effective use of existing resources. Specific aspects of the teacher capacity are summarized along with examples and related literature in this paper. Eventually, this paper attempts to answer to the following question: What are the components of teacher capacity for productive resource use?

Theoretical Background

Although I draw on Brown's (2009) notion of PDC, exploring teacher capacity for productive resource use is in a broad research context. First, this capacity is critical in teachers' documentation work and documentation system (Gueudet and Trouche 2009). Also, this capacity is fundamental in teacher decision making around using existing resources, which is elaborated in *pedagogical reasoning and action* by Shulman (1987) and aspects of using knowledge in teaching practice as elaborated in Rowland, Huckstep, and Thwaites' (2005) notion of *knowledge quartet*. Finally, exploring teacher capacity for productive resource use is based on the participatory relationship between teachers and resources (Remillard, 2005). Teacher capacity needed for using resources productively is grounded in bilateral influences of teachers and recourses that shape both parties. This relationship generates the research context that examines not only the components of the

teacher capacity needed for using resources productively, but also the role of the resources in supporting teachers to develop such a capacity.

Data Sources

In order to explore the capacity needed for productive use of existing resources, I drew on data gathered from the *Improving Curriculum Use for Better Teaching* (ICUBiT) project. In the project, 25 teachers using five different curriculum programs (5 teachers per curriculum program) were (1) asked to keep a Curriculum Reading Log (i.e., indicating parts they read, parts they planned for instruction, and parts influenced their planning on a copy of written lessons), (2) observed in three consecutive lessons in each of two rounds, and (3) interviewed after each round of observations. All observations were video-taped and all interviews were audio-taped. Then, both video- and audio-taped data were transcribed for analysis. This paper drew on data analyses on various aspects of resource use by the teachers (e.g., Kim, 2018, under review).

Teacher Capacity for Productive Resource Use

Four specific aspects of teacher capacity for productive resource use are described along with brief examples from the data in the ICUBiT project. More details will be in the final paper.

Articulating mathematical points and steering lessons toward mathematical points

Using existing resources to teach mathematics, teachers first read and make sense of the written lessons. In doing so, they need to identify the mathematical points of the lessons and evaluate how well the lesson activities, tasks, and problems support students' learning of the mathematical points (Remillard & Kim, 2017; Sleep, 2012). Then, they need to organize lesson activities toward the mathematical points in instruction (Brown, Pitvorec, Ditto, & Kelso, 2009). Failing to articulate the mathematical points of the lessons, teachers orchestrate lessons activities away from the mathematical points (Kim, 2018, under review). For example, a teacher in the ICUBiT project considered identifying and using keywords as the goal of the lessons on creating and solving multiplication and division story problems, and emphasized keywords instead of the meaning of multiplication and division (Kim, under review). Moreover, teaching the lessons, the teacher omitted important lesson components that had a great potential to support students' understanding of multiplication and division.

Recognizing affordances and constraints of the resource they use

As teachers read and make sense of the resources and identify the mathematical points of the lessons/activities/tasks, they can also recognize aspects/components of the lessons/activities/tasks that support or do not support students' learning of the mathematical points. In order to use existing resources well, teachers need to recognize affordances and constraints of the resources they use (Atanga, 2014; Kim, under review, 2018). Teachers who were not able to recognize the affordances may not use them in instruction. Also, teachers who do not recognize the constraints hardly try to make up the limitations. Depending on their evaluation of the affordances and constraints along with their students' need, teachers can decide whether they use, change, or omit components of lessons/activities/tasks, or add new elements to enact lessons (Kim, under review). Therefore, recognizing affordances and constraints is critical to use the existing resources productively. Not

seeing the usefulness of representations for subtracting a fraction from a whole number, which were provided in the resources, a teacher in the ICUBiT project totally dismissed the need of the representations in supporting students' conceptual understanding of the procedure for subtracting a fraction from a whole number (Kim, 2018).

Using affordances

Recognizing the affordances of existing resources is important, so is using those affordance in instruction. Brown's definition of PDC includes both "perceive and mobilize" the existing resources. In particular, using those resources together as a coherent set seems critical in using the existing resources well (Atanga, 2014). Different components of the resources are designed to support students' learning of mathematical points. Resources as a set rather than separate elements indicate the synergy that they can generate in supporting teachers to steer instruction toward the mathematical points. In the ICUBiT project, when using resources productively to teach lessons, teachers were using a range of elements provided in the resources toward the mathematical points of the lessons. Otherwise, as seen in the earlier example of the teacher focusing on keywords, teachers altered or omitted useful, important resources (e.g., representations and tasks), sometimes to include additional elements they chose to do instead.

Filling in holes and gaps

Recognizing constraints of the existing resources does not necessarily lead to productive ways of overcoming them, which is another important aspect of the capacity needed for effective use of existing resources. In the ICUBiT project, teachers tended to add new elements to the written lessons to enact them (Kim, under review). Some were effective; others were not. For example, noticing that her students needed a review on symmetry before starting a task of finding the area of a coat, one teacher asked students questions about symmetry, which effectively supported students' work on the task. In contrast, one teacher asked students to use cubes to determine the mean of four different numbers but this was not productive because using cubes were not supporting students to understand what mean really means. There are no perfect curriculum resources that fit in any classroom situation; proper change, omission, or addition is needed. Yet, the way teachers fill in the holes and gaps in the existing resources should be determined toward students' engagement with the mathematical points of the lessons they teach.

Supporting Teacher Capacity Development

Different approaches can be taken to support teachers to develop their capacity for productive use of existing resources. Two approaches are highlighted in the paper.

The data used for this paper revealed that the teachers in the ICUBiT project had certain operational invariants (i.e., cognitive structure guiding resource use) that Gueudet and Trouche (2009) explained as part of teacher documentation system. Unproductive use of existing resources is often rooted in operational invariants that are not appropriate (Kim, under review). For example, the teacher emphasizing keywords in multiplication and division story problems believed keywords helped students' learning of the operations. Therefore, in order to support teachers to develop the capacity needed for productive use of existing resources, teacher education (teacher preparation and

professional development) need to support teachers to examine their own operational invariants and generate such opportunities in individual teachers' documentation system.

In addition, the role of curriculum resources in increasing teacher capacity is fundamental. Educative features in the resources support teacher learning (Davis & Krajcik, 2005). Above all, it is recommended that curriculum resources be designed to support teachers to clearly see the mathematical points of lessons, activities, and tasks, and provide guides to make proper decisions.

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The capacity to think of transmission of knowledge from learning supports: a proposition of a conceptual model

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Teachers have to get ready for teaching: it means that everything must be thought of beforehand and they have teaching resources at their disposal and can use them whenever needed. In this « behind-the-scenes» work, the teacher must think about the supports that will support/assist the pupil learning activity. We propose a model linking teaching postures and learning supports. The learning supports selected or designed by the teacher and the uses that are envisaged crystallize and materialize these postures and the tensions that cross them. These postures refer to the factors that condition the teacher preparation activity in which interactions between teacher and teaching resources take place.

Keywords: Teaching resources, teaching postures, learning supports.

Teachers in order to design/formulate their teaching practice, especially activities in which they engage their pupils, have resources at their disposal. In these, we find resources "explicitly designed for teaching". The “specific intentions of their designers and institutions involved in their design or their uses "are in these resources (Gueudet & Trouche, 2010, p. 357). The internet teaching resources increase this offer. Now, teachers access more easily and more quickly teaching resources designed by school publishers, school institutions, but also by others teachers and teachers collectives (Gueudet, 2013, p.89). During this exercise, they're interacting with these resources

Interactions between teacher and teaching resources is conditioned by various factors

In Brown’s model “The Design Capacity for Enactment Framework” (2009), the characteristics of curriculum resources (“physical object and representations of physical objects, representations of tasks – procedures – and representations of concepts – domain representations”) and the characteristics of teacher resources (“subject matter knowledge, pedagogical content knowledge, and goals and beliefs”) influence the use of curriculum materials. For Remillard (2010), these characteristics determine “modes and forms of engagements”. In our thesis, we have studied the relationship to teaching materials[1] of the primary school teachers during the preparation work in mathematics (Leroyer, 2013). This relationship to teaching materials is expressed in specific uses.

[1] We defined the relationship to teachnig materials (pupil worksheets, teacher’s guides, etc.) as the following: “The set of relationships that a teacher, as an individual, has with the constraints and resources of the professional context in wich they evolve, as well as to those linked to the common forms of his professional life”

We have established a typology of those uses, specifying those defined by Brown (2009). And, we have highlighted factors influencing those uses: teachers' opinions on teaching materials, context (class level), relationship to discipline, but also teachers' knowledge level.

In our work, the teacher's ideas about: knowledge, learners, learning and teaching still have to be examined. During his preparation work, the teacher is led to ask himself several questions. These questions are about: knowledge to be studied and transmitted, logic of their programme, pupils place and pupils activities. These ideas influence the teacher preparation work and more particularly the thought about the pupil activity supports that we named "learning supports". We consider those learning supports as artefacts (Rabardel, 1995) made up of a medium (paper, digital, video ...) and a content in which can be included pupils instructions. We must also consider the schemes of utilization associated with this support. In reference of the "documentational approach of didactics" (Gueudet & Trouche, 2008, 2012), the learning support is "recombined resources" which, associated with schemes of utilisation corresponds to a "document".

Toward a model linking teaching postures and learning supports

Our work on the resources in the training design work of teacher trainers (Leroyer, in press) led us to develop a model in which trainer postures and training supports are linked. The trainer's postures allowed us to understand the trainer / training support interactions in the complexity of the training design activity. The theoretical model of "the teacher multi-agenda" (Bucheton & Soulé, 2009) defines a posture as a "pre-constructed scheme of" think-say-do" in response to a given school situation or task ". This posture is "constructed in the social, personal and scholarly history" of the individual. And, this one "may change posture in the course of the task according to the new meaning they attribute to it". The posture is both on the side of the individual "in a given context, but also on the side of the object and the situation" (Bucheton & Soulé, 2009, p. 38). In our model, based on Bailleul and Thémines work (2013), four trainers postures (guide, engineer, didactician and epistemologist) are differentiated. This differentiation is according to relationships with: others, time and organization, tools and science. The training supports designed and their anticipated use crystallize these trainers postures and the tensions that cross them (Leroyer & Georget, 2017).

We wanted to base our work on this model to relate teaching postures and learning supports. So, learning materials selected or designed by the teacher and their anticipated uses crystallize and materialize these postures and the related tensions. These postures also refer to the set of factors that condition the teacher preparation activity in which interactions between teacher and teaching materials take place.

We suggest presenting this model and the results of its testing. The research corpus consists of transcripts from two primary school teachers interviews. We also have, for each teacher, the teaching scenario on a mathematical concept and the learning support. Semi-directional interviews are based on the presentation and explanation of their preparation work. To facilitate explanation, they rely on their preparation and on the learning supports chosen, adapted or designed. In order to grasp "the reality of the activity" (Clot, 1999), teachers are also asked to express what they would have liked to do and what they could not to do. During the interview we also ask questions about

the context: material, relational and organizational environment (Chatigny & Vézina, 2008), and questions related to the individual: teacher professional and academic career, place of mathematics in this career, sense of mathematical competence in disciplinary and didactic knowledge, personal and institutional training in mathematics, ideas of knowledge, learners, learning and teaching.

A conceptual model to apprehend and develop the capacity to think transmission of knowledge by using learning supports

This model aims to contribute to a better understanding of teacher interactions with teaching resources in the complexity of the teacher preparation activity. During this activity, the choice of learning supports is questioned. Faced with a diversity of resources, teachers have to estimate the quality of these resources. In a research on the professionalisation of teachers in initial training, we noticed that students's interest for having a critical eye on learning supports deteriorated (Leroyer & Bailleul, 2017). The use of this model in training could constitute a tool for the analysis of learning supports in order to build teachers' reflection. This tool could allow them to question the effects of learning supports they are going to use, but also their teacher's posture.

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What do teachers attend to in curriculum materials?

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In this paper, we describe an emerging methodology using eye tracking to explore teachers' curricular attending as they interact with curriculum materials to design a lesson in order to learn what teachers pay attention to and how this attention shifts during planning. We propose affordances of this new method, remark on some of its limitations, and propose future directions.

Keywords: Curriculum research, Instructional design, Instructional materials and practices, Teacher education, Research methodology.

We are convinced that the curricular work of mathematics teaching is challenging and not well understood. Throughout many professional experiences in teaching future teachers and participating in conferences, we have observed mathematics teacher educators in the United States face challenges navigating and making sense of unfamiliar textbook materials. We have also witnessed prospective mathematics teachers (PSTs) completely overlook what we view as key aspects of curriculum materials. This raises our general research question: How can we study the process by which teachers participate with curriculum materials? Answering this question may enable us to identify potential barriers present when teachers interact with curriculum materials and to propose strategies for overcoming these barriers. Studying the teacher-curriculum interaction may also help us describe the design capacity (Brown, 2009) required of teachers as they interact with curriculum materials.

Thus, rather than conceptualize this curricular work as a simple process of selecting and using parts of curriculum materials, we instead draw from Remillard (2005) and Gueudet and Trouche (2009) to recognize a complex interaction between teachers and textbooks in which the teacher's influence (the *instrumentalization*) and the influence of the materials (the *instrumentation*) on the interaction are tangled and messy. To describe this interaction, we draw from professional noticing theory (e.g., Mason, 2002; Jacobs, Lamb, & Philipp, 2010) to recognize three interrelated phases of curricular interactions, which we conceptualize as *curricular attending* (what teachers look for and at as they look at the materials), *curricular interpreting* (what sense teachers make of what they see as they look at the materials), and *curricular responding* (what teachers decide to do with the materials based on their interpretations) (Males, Earnest, Dietiker, & Amador, 2015). This enables us to place specific focus on the challenges presented in each of these phases and foregrounds the importance of a teacher's attention to materials, something that has remained elusive in the literature.

In this paper, we describe an emerging methodology to explore teachers' curricular attending as they interact with unfamiliar curriculum materials to design a lesson in order to learn what teachers pay attention to and how this attention shifts throughout planning. We propose affordances of this new method and remark on some of its limitations. To close, we describe potential future directions.

The Context of this Research

The methodology described in this paper was developed as part of a larger study to gain insight into the curricular noticing process of PSTs. At the time of interviews, the PSTs had little experience planning lessons or using materials as a teacher. We collected data by asking PSTs to plan a hypothetical lesson while wearing Tobii Pro Glasses 2, a wearable eye tracking device. The lesson, "2.1.2 How can I measure steepness?" from the *Core Connections Algebra* (CPM Educational Program) was selected because it focuses on slope, a topic that PSTs are familiar with and will likely encounter as teachers and both, the approach to developing slope using a geometric growth pattern, and the structure of the materials varied from materials with which PSTs were most familiar.

Analytical Views of the Teacher-Curriculum Materials Interaction

One way to study the attention of a PST to materials as well as the shifting between teacher and student materials is by comparing their *Planning Timelines*. Figure 1 shows a timeline of two PST's planning sessions, where yellow indicates attention to teacher materials, blue attention to student materials, and black no attention to the materials (e.g., focusing on the written lesson plan).

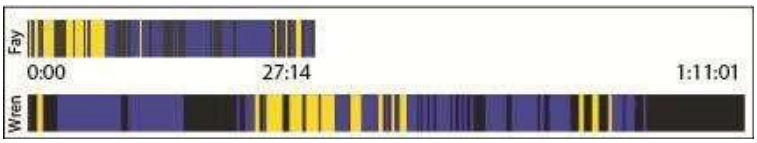

Figure 1: Planning Timeline of Fay and Wren's Entire Planning Sessions

In these examples, we can see that the PSTs attended to student and teacher materials quite differently. First, the different lengths of the timeline indicate that Wren spent more than double the length (in minutes) planning for the lesson than Fay. Second, although both PSTs spend more time attending to student materials, as evidenced by the larger sections of blue, we see that Fay moves more quickly between the teacher and student pages, as evidenced by the smaller widths of many of the sections of her timeline. In addition, both timelines have periods of not attending which usually proceeds attention to the student materials. Since much of the non-attention to the materials occurred when PSTs were writing their plan, this may indicate that what they wrote was triggered more by the student materials. With this representation, we can study how the teacher and student materials are viewed in relation to each other throughout planning. If we wish to analyze a specific switch between materials, however, we must incorporate the use of another visual.

In contrast, a *Planning Map* shows how particular aspects of the teacher or student materials may trigger attention to other aspects of the materials. For example, Figure 2 represents 30 seconds of a PST's planning session. The circles indicate locations of attention while the numbers within the circles indicate the order to which they were attended. This mapping reveals how certain parts of text led to a shift in attention between the teacher materials (on the left) and those of the student (on the right). Circle 2 portrays Wren's focus on a question concerning steepness on the student page, which is followed by a shift back to the objective on the teacher page (circle 3). Although maps offer insight into how some text might influence attention to other portions, these work for only small durations of time, since lengthy time results in complex, overlapping paths.

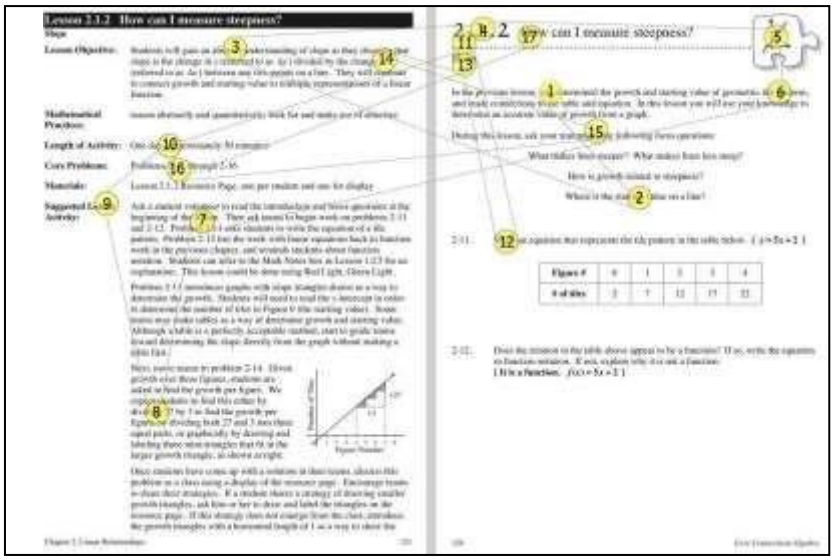

Figure 2: Planning Map of 30 Seconds of Wren's Planning Session

While *planning timelines* and *planning maps* only represent an individual teacher's attention, a *Heat Map* represents the attention of a group of teachers. In Figure 3, we depict heat maps that represent visit duration (i.e., how long PSTs attended to portions of the text) on the left and visit count (i.e., the number of times PSTs attended to portions of the text) on the right. In both maps, green indicates portions with durations or visits that were shorter/fewer than the red which represents longer durations or more visits. Taken together, heat maps can reveal what teachers find important, useful, difficult to comprehend, or requiring more attention at a later point.

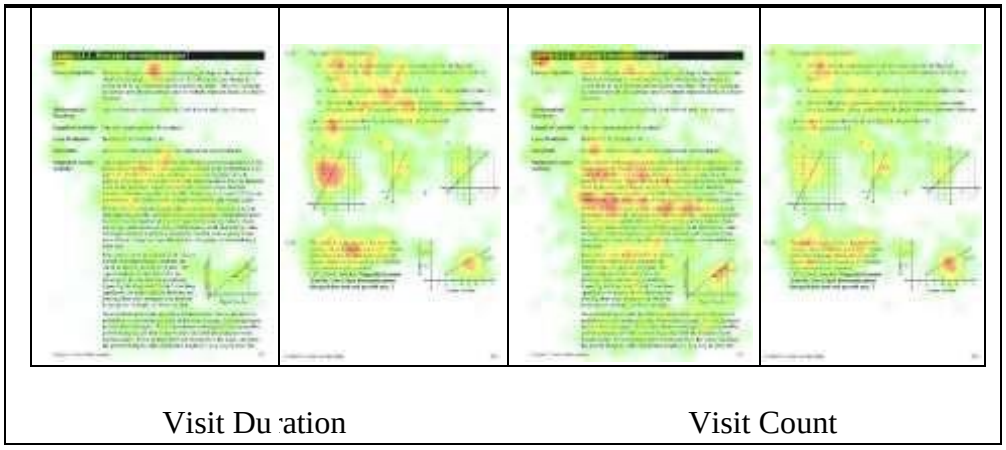

Figure 3: Aggregate View of Visit Duration and Number of Visits

Discussion

The analytical snapshots presented in this paper illustrate the methodological power of eye tracking to support researchers in describing the critical process of attending to curriculum materials and provide insight into how we might use this technology to further explore *instrumentalization* and *instrumentation* throughout an interaction (Gueudet & Trouche, 2009). We contend that the decisions that teachers make with regard to using curriculum materials rely on the interpretations

that they make when interacting with their materials and that teachers can only interpret what they attend to.

As teachers use materials, they develop a sense of the design, content, and philosophy of the materials that enable new ways of interaction (Choppin, 2011; Drake & Sherin, 2009). The eye tracking technology, coupled with interviews and observations, can allow us to learn how these factors influence what teachers do and do not attend to in their materials and how curricular attending may change over a teacher's career. By including a wider range of teachers, we can tease apart how years of experience, beliefs, orientations, and vision influence the curricular attention of teachers. In addition, we view eye tracking as an exciting methodological advancement, offering insight into instrumentation. This is encouraging as there is currently little knowledge on how the structure, format, and the features of textbooks impact teacher attention. We are interested in potential patterns in how teachers attend to different textbooks features, such as those identified by Rezat (2006).

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The mathematical teacher: a case study of instrumental genesis in the UAEM

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This paper presents a case of instrumental genesis at the Autonomous University of the State of Mexico, in particular at the Valle de Chalco University Center, where university teachers who teach subjects in the area of mathematics have access to institutional resources that are part of a formal program, as well as encouraging the creation and use of alternative didactic materials that include digital technology, with the intention of giving feedback to their teaching practice in order to move from instrumentation to instrumentalization and orchestration processes in the teacher. Although the documentary trajectory (Monaghan & Trouche, 2016) is incipient, the tendency of teachers moves towards this methodological framework.

Keywords: Teacher instrumental genesis, integration technology, mathematics teacher.

Introduction

The Autonomous University of the State of Mexico (UAEM) is the main public university of the state, with headquarters in the capital Toluca and a population of 75,000 students. Before a growing demand, a desconcentration process began in 1984, creating the Valle de Chalco University Center located east of the State of Mexico to provide higher education to a population of 2,600 students. At 21, the institution has 120 teachers to cover this educational offer in undergraduate: Computer Engineering, Administrative Computing, Law, Accounting, Nursing and Industrial Design; and three postgraduate programs: Master's and Doctorate in Computing and Master's in Nursing. The 90% of teachers are hired per hour to teach, so they have several jobs in the education sector at different levels or in the business sector, in the same area.

The Mathematics teacher that teach classes in the different careers face the problematic at the national level of high rates of reproved, low operational level of the students and lack of conceptual dominio (INEE, 2017; Martinez, Soberanes & Castillo, 2016), reason why the UAEM created a system of resources of institutional support which puts at their disposal, with the intention of bringing educational resources and means of feedback of their teaching to attack this problem. Despite the good intentions of the institution, the teacher faces different dilemmas: how to select "good" materials for his class of mathematics? How to incorporate the resources selected in his teaching practice? For this paper, we point of the question: what is the role of the teacher, the student and the technology?

These concerns, which are not new (Cuevas, Pluvinage & Martinez, 2005; Monaghan, 2016), require a framework of analysis that allows teachers to give a response beyond a description to make decisions, also considering the conditions of the locality. Therefore, this work is framed in the theory of instrumental genesis and the processes of instrumentation and instrumentalization of teachers, to establish the importance of the resources as support of the activity and mediators of

knowledge, the design of the didactic activities to generate instrumentation processes, and the moments in which they intervene: teacher, technology and student. That is, the different configurations of instrumental orchestration.

In the UAEM the importance of the planning, choice of material and instructional design of the teacher is clear. There are official instruments (semester planning, tutorial report, student performance indicators in official statistics), which are modified per year. The didactical resources that use: multimedia, official software according to their matter, laboratories with computers, projector, etc., they are complemented each semester. The teachers make weekly programs where indicate contents, materials, objectives and ways of decentralizing contents, have a defined methodology.

Metodology

The teacher interacts with the resources, selects them under some criteria and works them (adapting, revising, reorganizing, etc.); in this process, the planning of the class session, the distribution and management of the resources and the documentation of the activity are vital processes. Hence, there is a first problem regarding the selection and use of resources with a defined didactic intention. Becoming aware as a teacher of this decision making is complex if you do not have a theoretical framework. For this reason, within the instrumental genesis, the processes of design and representation are intertwined (Gueudet & Trouche, 2012): instrumentation and instrumentalization. This first phase will be described in this paper evidencing the "old" resources and the "new" resources to which the mathematics teachers have access in the UAEM. On the other hand, the teacher works for a class under certain situations, selects a set of resources of different nature and defines a way to use them, giving rise to a scheme of use or an incipient orchestration, in the sense of Monaghan & Trouche (2016). The documentation of this activity is essential; giving rise to an incipient work of documentary trajectory (ibid.) that has been modified in the university over the past 10 years, although in its beginnings without a method for it.

The instrumental orchestration as a complex process contemplates identifying the characteristics of the artifacts, their potentialities; to the subjects of the activity, their knowledge; and forms a working method: the goals and the anticipations, the rules of action, gathering of information, control-taking and the operative invariants. The instrumentation (subject) and instrumentalization (artifact) processes are designed, and an orchestration instrumental defined by didactic configurations is emphasized (Trouche & Drijvers, 2014). This process will be described immediately. So, how far is an adequate scheme for the teacher in the role he plays?

Description of resources

The UAEM makes official resources available to mathematics teachers, such as:

a) Curriculum for each subject with didactic indications and suggestions for the use of resources, Indicators of students: results of the entrance exam at a higher level, performance of the students through academic tutoring and results of student appreciation. The objective of these resources is that they are a source of support to develop a general planning of the course (distribution of activities and resources), based on an initial quantitative diagnosis of the students.

b) Repository of didactic materials (pedagogical, didactic and apprenticeship evaluation guides, anthologies, notes, audiovisual presentations, exercise or problem books, monographs and software), access to online textbooks, to repositories of objects of learning, theses and scientific articles.

c) Infrastructure conditions: computer room with 25 machines, internet access and software with license, projector and interactive whiteboard.

Mathematics teachers organize these resources: diagnosis by subject (pre-test), selection based on previous experience in operational and conceptual problems of the students, incorporate results of student appreciation on organization and dynamics in the classroom. This initial orchestration responds to an institutional scheme, a political-administrative issue. This changed as the teacher changes his paradigm incorporating instrumental orchestration and confirming his experience, it presents in academic and research events. In addition, it ventures into the development of didactic material that is included in the repository and in updating courses in mathematics, didactics and technology. The documentation of this change follows an incipient documentary trajectory through some monitoring tools and logs. Some alternative resources that the mathematics teachers incorporated in the last five years:

a) Technological developments on the Internet to administer courses (Edmodo, Google Drive, Dropbox, etc.), non-institutional repositories of teaching materials (LAReference, National Repository Conacyt, Merlot, Univermath, etc.) and some virtual communities (Teaching Seminar of the Calculation, Geogebra Community, etc.). These resources have modified their teaching practice, they have incorporated them gradually.

b) Resources derived from teacher training: materials that include Virtual Reality, Learning Objects, Games, Teaching guides, Exercise Notebooks, etc., prepared by Mathematics teachers.

c) Feedback from academic events showing the use of educational software or apps: CalcVisual, Lirec, Virtual environment (Pulleys and Ballon), etc. These materials have been incorporated since 2007 into the classroom, modifying the organization of the resources, moving towards instrumental processes.

Organization of resources and results

It is important to understand the teacher's work in its continuity in and out of the classroom. For teachers hired at the UAEM the majority must prepare their material and their class outside working hours, since the institution only pays class hours in front of a group. The activities in academy are not necessarily within their working day, their participation in research projects and preparation of papers are extras. Thus, the teacher works most of the planning and follow-up outside the institution in interactive platforms, which hinders feedback activities with other teachers and requires an effective organization of work.

The need of teachers of mathematics in relation to instrumental orchestrations, in the sense of Trouche & Drijvers (2014), is due to: an intention to incorporate technology to encourage conceptual processes and means of operational practice in the classes; that is, a didactic configuration alternative to the traditional; an adjustment the way to carry out the dynamics in the

classroom, which It obeys the variety of their knowledge and skills. It is a way of answering the question about the roles played by the teacher, the technology and the student.

The UAEM teacher went from the refusal to incorporate new resources due to ignorance of their handle to the gradual integration of the resource, going from superficial schemes to more elaborated schemes, since integrated better results. Analyzing the professional development of teachers as an indicator in the incorporation of their teaching resources is still precarious, but progress is made towards it within the framework of the documentary trajectory. This is an example of slow maturation, which depends on a climate of trust in resources and their implementation, as well as on the documentation of their use and the evolution of their practices, as mentioned by Guin & Trouche (2008).

For example, in the use of the pulleys virtual environment that has been used since 2007 (http://mattec.matedu.cinvestav.mx/univermath/escenarios.didacticos/), the factors for the appropriation of the resource by the teacher can be analyzed and its evolution in the design of this resource. The change of paradigm and the methodological monitoring will allow the teacher to be aware of the process and move towards the modification of the resource. Also, the collectivization of the activity with other teachers allows generating a creative appropriation, thus demonstrating the way in which an instrumental genesis with emphasis one conceptual genesis. The follow-up within the framework of the documentary trajectory maintains the use of logs, questionnaires and interviews.

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Collective designing of Open Educational Resources: what effects on teachers' design capacity ?

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Abstract : In this paper the case of a language resource production group is introduced. Due to the centrality of the OERs in the members' activity I show that the documents they produce are essential to their work. The theoretical framework of the pedagogical design capacity is used in order to analyse the design of the group's resources and the resources of one of its members. The comparison between the two will allow the identification of the effects of collective work on the members' professional development.

Keywords : OER, collective teacher work, professional development.

Introduction

The availability of Open Education Resources (OERs) has introduced drastic changes in education (European Commission 2013), regarding both the nature of resources and teachers' interactions with such resources, as *users* as well as *designers*. Designing and using an OER, in particular OERs like lesson plans incorporating the use of software, require new competences and more specifically with digital resources. In aiding the development of teachers' practices it is essential to involve them in the designing of tools and resources (National Digital Council, 2012). In collective work the interactions of teachers with their resources have an impact on professional development (Gueudet & Trouche, 2010). In this paper, the effects of collective designing of OERs on individual work are analysed in order to highlight their impacts on teachers' professional development.

Theoretical frame

In the documentational approach Gueudet and Trouche (2009) consider not only artifacts, but all kinds of resources. They argue that resources influence the teacher's activity (instrumentation) and that the teacher acts on his/her resources in return by adapting them (instrumentalization) during documentational genesis, which produces a document, encompassing resources and a scheme of use of these resources (Vergnaud 1998). Teachers develop structured resources and documents systems during their work. Their professional development is directly linked with the development of these systems.

In recent evolutions of the documentational approach, Pepin, Gueudet & Trouche (2017) associate their theory with Brown's work on curriculum materials (2009). The authors underline that the characteristics of curriculum materials described by Brown can be connected to the documentational approach and more particularly to the concepts of instrumentation / instrumentalization. Indeed, they explain that the characteristics of the curriculum materials in Brown's theory "represent an interface between the knowledge, goals, and values of the author and the user" (Pepin and al., 2017, p.801). They link this point to the instrumentation's concept. The authors show too that Brown presents the curriculum materials as "[..] inert objects that come alive

only through interpretation and use by a practitioner" (Pepin and al., 2017, p.801) which can be connected to the concept of instrumentalization.
Brown presents teachers' work on their curriculum materials as a design activity. To describe it he introduces the two concepts of *design work* and *pedagogical design concept*. *Design work* can be defined as the creation of a new resource by teachers. The resource can rely on something existent or can be created with completely new elements. The second concept, the *pedagogical design capacity (PDC),* is described as the knowledge / abilities of the teachers to make pedagogical choices to prepare his/her resources in order to reach his/her education goal.
In their work on this theory Pepin, Gueudet & Trouche (2017) describe a method adapted to a case study. They present ten questions (see Pepin and al., 2017) that can be used to analyse teachers' *design work* and can promote the comparison between the collective design work and the individual design work. Teachers' *design capacity* can be analysed with three elements : the *goal*, a *set of design principles* and *reflection-in-action*. The *goal* helps to understand where the new resource will be situated in the teachers' work, (in the progression of the lesson for example). The second element is a *set of design principles.* They are the robust design rules teachers can apply/follow to each design. The principles include a little flexibility. It means the design rules can be slightly adapted to the situation: level, context, for example. The third element is *reflection-in-action.* The design work might evolve during the use of the resource. For example, teachers can adapt a part of the resource according to students' reaction. To conduct the analysis the two concepts of *design work* and *teacher design capacity* are used as tools to describe a case of a collective and an individual design work.
In this paper the following question will be treated: Does a participation in a collective designing resource contribute to the development of teachers' design capacity, and how?

Data collection and analysis: presentation of the group and methods

The data is collected from a language resource production group created in 2013. The group followed is named: group of pedagogical integration of the uses of digital technology (GIPUN). It is composed of 7 language teachers (4 English teachers, 2 Spanish teachers and 1 German teacher). The collective concentrates a part of its activity on designing resources for a national platform named Cartoun. The latter offers a collaborative geopositioning service listing pedagogical activities designed and uploaded by teachers, or pedagogical experts, for teachers. Its stated objectives are to allow the consultation of educational activities, to encourage the exchange of practices or to publish didactic activities based on the use of digital technology.

The qualitative methodology is built as follows: regular follow-ups of meetings, audio recorded, and all shared resources used in the online collaborative space used by the group are collected. To process the data, synopsis (synthesis of the meetings with their main episodes) and transcriptions are made. The first elements collected from the meeting are analysed to highlight collective design work. Secondly, an interview with a member of the group, Dunbar, is conducted to better understand her *design activity.* She has been an English teacher for twenty four years at high school. She joined the GIPUN four years ago as a designer with one objective to « respond to a need for stimulation and sharing of knowledge ».

Design work collective and individual

Based on the series of ten questions presented by Pepin, Gueudet & Trouche, a collective and an individual *design work* is described.

Collective design work : GIPUN
For the collective design work, the members have one meeting a year under the supervision of a regional inspector. They use a mailing list during the rest of the year for their exchanges. During this meeting, they mainly discuss the new orientation(s) of the group's work. Members present and discuss the new resources. The resources are produced in a digital version and they are made available to teachers for free.

Individual design work : Dunbar
The example introduced describes an activity that Dunbar designed earlier in the year. The teacher was not observed in her classroom. An interview was conducted in which she describes the cycle of the resource design from the idea to the enactment in class. The focus is on her resource named Book in Project[1] (students select a few books in a list, other students do the same in another school then they share their reading impression on Twitter). Dunbar started working on her resource in December for an implementation around March in her class. Then she designed a document for the Cartoun platform based on this activity.

Comparison of collective and individual design work
The analysis emphasizes the importance of digital tools in the design work of group's members and of Dunbar. In their design work, using Cartoun has a lot of impact on the members' practices. Indeed the members put the resources they design on the platform but are themselves using the same media to develop their design work. In the case of Dunbar, she explains the evolution of her practices. She uses both Cartoun and Twitter to find resources. Previously, she conducted her research using a search engine. She declares that this change is directly linked to her participation in the group.

Dunbar's development of design capacity

Dunbar explains that she finds different resources on line (Twitter, Cartoun, eteach.net) and then categorizes them using lists on her computer to be worked on later. With her work on this new resource, Dunbar wants to show that it is possible to use a social network, like Twitter, for sharing interests and hobbies in a safe manner. Some elements interpreted as keys to understand the development of her *design capacity* are identified in her speeches.
Dunbar 's *goal* is to design an activity with a digital resource she is familiar with and uses daily. She starts her design work from the resource itself, from Twitter but not from the lesson. Concerning her *robust design principles,* she explains she needs at least two hours to learn how to use a new tool but sometimes more when the project is more complex. She also states that a new resource has to be tested. In the case of her Twitter's resource she tested it with her husband and children at home before using it in class. Dunbar has *flexible design principles*. For example, before using a resource in class, she might work with other teachers when she is less knowledgeable about the topic. In the case of her Twitter's resource, she collaborates with the school documentalist to design her activity.

1 https://sites.google.com/site/defibabelio/book-in-project

In order to realize the activity in class, Dunbar, the documentalist and the Canadian assistant were there. Dunbar has a *reflection-in-action*. She had not anticipated the lack of her students' knowledge of Twitter. When she implemented her activity, she noticed their difficulties to follow Twitter's rules such as the 140 characters limit. She declares she has to be sure now how much the students know about a digital tool before using it.

Conclusion

In this paper, the link between collective and individual *design work* is shown. Dunbar's participation in the collective influenced her way of looking for resources. In return, the group give her new elements to look at and a massive interest in sharing. Some elements can be interpreted as the development of her *design capacity.* Her set of design principles can evolve with the situation when she is working with a resource. Using the two concepts of *design work* / *design capacity,* the interactions between teachers and their resources are highlighted as a vector of professional development. These first observations will be refined in the coming months. These concepts will be used to analyse a collective designing work and the implementation of the ressource designed in class in order to uncover the effects of this work on the development of the teachers' design capacity over a long period.

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Documentational genesis during teacher collaborative development of tasks incorporating digital technology

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A way to improve conceptual use of digital technology (DT) in secondary mathematics teaching may be to increase teacher involvement in task development. This research considered the documentational genesis of four groups of teachers designing and implementing DT tasks. We analysed the tasks they produced and considered factors involved in their documentational genesis. Results suggest the value of task development in supportive groups that encourage sharing of ideas.

Keywords: documentational genesis, task design, teacher collaboration, digital technology

Background and Theoretical Framework

This paper considers a potential role for teacher-developed tasks incorporating digital technology in secondary classrooms. Often researcher-developed tasks have been employed in studies with teachers because they have been carefully designed to embody key theoretical principles. However, the teacher has to adapt these resources to fit their own pedagogical and epistemological perspectives and the needs of their students (Leung & Bolite-Frant, 2015), shaping and transforming them (Cobb, Zhao, & Visnovska, 2008). This process of shaping and transforming resources into documents has been described in the documentational genesis (DG) framework (Gueudet & Trouche, 2009), which presents two dimensions of genesis, instrumentalization and instrumentation. Here the former dimension conceptualizes the appropriation and reshaping or transformation processes, while the latter conceptualizes the influence on the teacher's activity of the resources drawn on. In the words of and Gueudet and Trouche (2009, p. 214) "Document = Resources + Usages + Operational Invariants". In their theory resources, which always belong to a set, may include a variety of artifacts, such as: a textbook; a piece of software; a student's sheet; a discussion with a colleague; etc. The study reported on here considered groups of teachers working together to produce a document incorporating digital technology (DT) resources. The teachers worked in groups of three, intended to form a small, supportive community of inquiry (Jaworski, 2003) into DT task design. The researchers provided professional development (PD) for the teachers designed to assist them to develop DT tasks for their students while improving their confidence and attitude to teaching with DT. We propose extending the concept of resources in DG to embrace aspects of Schoenfeld's (2010) decision making theory of resources, orientations and goals, which includes teacher knowledge as a primary resource. This has been recognised by Gueudet and Trouche (2009), who refer a document being 'saturated with the teachers' experience', which implies their knowledge base. Taking teacher knowledge as a resource enables a link between DG and the theoretical model, of Mathematical Pedagogical Technology Knowledge (MPTK) (Thomas & Hong, 2005; Clark-Wilson & Hoyles, 2017) also employed in this research. This model, which has at its core a focus on the enhancement of mathematical thinking through DT use, incorporates the principles, resources, conventions and techniques required to teach mathematics through the technology. Thus while a teacher needs to be a proficient user of the DT, it

is vital that they understand the principles and techniques required to build didactical situations incorporating DT, in order to enable mathematical learning. Intrinsic factors involved in MPTK include a teacher's resources, orientations and goals (Schoenfeld, 2010), their instrumental genesis (IG) of the DT tools and their mathematical knowledge for teaching. To assist with a student-centred approach a three-point FOCUS framework for lesson planning, delivery and review (Choy, 2016) was introduced. It focused on identifying, for each lesson, key concepts, possible points of student confusion, and proposed teacher courses of action to remediate these. We hypothesise that to use DT to design documents including rich tasks with epistemic value, teachers need to align their mathematical knowledge for teaching and personal and professional IG with positive orientations, such as confidence in using DT. Our research question was: What factors influence secondary mathematics teachers' development and implementation of digital technology algebra tasks, and how?

Method

The participants comprised eight female and four male Sri Lankan Advanced ('A') Level secondary school teachers. These formed four groups (variables such as age, gender and experience were not used in construction of the groups) to work together as small communities of inquiry with a common goal to design a task, implement it and reflect critically on the classroom teaching. These reflections were taken into account for further task modification. None of the teachers had ever taught with, or produced, tasks incorporating DT. The research design involved three stages. The first included a Likert-style attitude questionnaire with five scales and semi-structured individual interviews, after which each group produced an initial design for a DT task using the software GeoGebra. The second stage comprised a two hour PD intervention led by the first author during which theoretical principles and criteria for rich DT task design were presented. They were also assisted with how to plan a lesson using DT, and the FOCUS framework was discussed. In the third stage, prior to classroom implementation, the teachers were able to modify their task. They also took part in a group interview focusing on how they had worked on their task, planning for the implementation, modifications they made and factors that influenced the process. The teachers then completed the attitude questionnaire a second time. The teacher groups chose either *graphs of quadratic functions* or *domain and range of functions* as their topic and GeoGebra as their DT.

Analysis and Results

Analysing the task design process using the DG framework we identified a number of items in the set of resources employed by the teacher communities of inquiry. These include artifacts, such as the criteria for rich task design, the three-point FOCUS framework and an exemplar task, all from the professional development, as well as the software GeoGebra, the student worksheet and the 'A' level syllabus. The tasks produced before and after the intervention were measured against the Rich Task Framework comprising twelve factors: Focuses on mathematical ideas e.g., epistemological obstacles; Considers the role of language & discourse; Students' written interpretations and reflections; Goes beyond routine methods; Encourages student investigation; Has multi-representational aspects; Appropriate for student instrumental genesis; Provides opportunities for instrumental feedback; Integration of DT and by-hand techniques; Aims for generalisation; Students

think about proof; and Develops mathematical theory. The tasks were rated against these factors, each of which had a score of 3, 4 or 5, giving a maximum total of 46. The results of the *t*-test analysis presented in Table 1 show that the intervention significantly improved the quality of the tasks produced. The reasons for this improvement are multi-factorial. For one, it was clear that the teachers valued collaborating together in a supportive community. They spoke about how "…solutions came from the discussions we had in the group" (A1), "I think working in this group helped us a lot in this activity. If we work individually, even though we write three tasks none of them will be [as good as] this task will it?...I know that this is much better than I would do by myself." (A3), and "It's better than doing it alone because it helps to minimise mistakes and we end up with a rich one, a perfect one." (A2).

Group	Pre-Intervention (Max 46)	Post-Intervention (Max 46)	*t*	*p*
A	18	29	3.11	<0.005
B	12	25	4.03	<0.001
C	12	32	4.44	<0.0005
D	21	27	1.80	<0.05

Table 1: Analysis of pre- and post-intervention rich task framework scores for the tasks

This supportive collaboration seems to have had a positive influence on teacher confidence and attitude to teaching with DT. For example, Group C's confidence in teaching mathematics increased significantly based on a Wilcoxon signed-rank test (N = 7, $S_{pre} = 23.5, S_{post} = 28.1$, W = 0, p<0.05), as did their attitude to teaching with DT (N = 8, $S_{pre} = 28.1, S_{post} = 31.5$, W = 0, p<0.05). In addition, Group B significantly increased their confidence in using DT ($N = 10, S_{pre} = 22.2, S_{post} = 26.4, W = 4.5, p < 0.05$). One positive aspect of working collaboratively and valuing each other's contributions was that it enabled these teachers to discuss the factors of a rich task from the PD, leading to a stronger task. They commented: "Aiming for generalization. Like in this [shows the list of factors of a rich DT task]. In our case there's no generalization. Is there any? I don't know. I have a doubt there." (B3), and

C3 Shall we check whether the suggested features of a rich task are there in our task?

C2 This is beyond the routine. So that's there. Address a mathematical concept or an idea. Yes. Ask students to write how they interpret their work. They do that as well.

The community discussions also resulted in development of their personal IG. For example, when considering a problem with the values from the quadratic function $f(x) = ax^2 + bx + c$ Group C worked out together that to input a function $f(x) = ax^2$ they had to use $a * x$^2. One member (C3) said "Yes that is the problem here I think. Here the computer takes it in another way. a, multiplication, and then x^2…Let's put the multiplication sign. x^2 is alright. In this place [between a and x^2] we have to put the multiplication sign." They were assisted to consider what students would think of the tasks and any potential difficulties, as presented in the three-point FOCUS framework. This helped to develop their professional IG; how to teach mathematics through the DT.

C3 Yes it's a bit difficult to get that point. That's the problem. …Because when the graph changes from positive to negative there should be a place it becomes zero. Students might get that idea.

C2 Yes, it is bit difficult to get that. What if we change this [scale] to 0.5s? …

C1 Yes now you can see. Look it cuts the axis. If a student asks with the graph in this situation we can help them. We can show them it has not yet touched.

In terms of the collaborative task development process we found that groups where teachers made equal contributions benefited more and that the composition of the groups was a key factor. Two who did well included Group A which comprised three male, relatively experienced teachers, all aged 31-40 and Group C, which was also homogeneous in age, and experience but heterogeneous in gender. In these two groups each member felt more confident to share his/her ideas with the others, while in other groups less experienced teachers sometimes lacked the confidence to express themselves in front of experienced teachers with strong ideas. Considering the instrumentalisation dimension of DG, working together to produce a task lessened the need for teacher appropriation and re-shaping of an existing task, although they still needed to make the content and focus their own. It seems that this was easier for the groups more homogeneous in age and experience. Being better able to discuss the three points from the student-centred FOCUS framework influenced the instrumentation dimension of their DG as their attention was focused on how the implementation would best proceed. Further, the groups freely sharing ideas were more flexible in their DT use than the others, searching for and incorporating appropriate DT techniques into the task to help students understand the mathematical concepts. This enabled them to improve their personal IG, learning new techniques and accompanying schemes. In addition their professional IG was enhanced. Overall, this research suggests there is merit in encouraging teachers to design DT tasks by working collaboratively in small groups, provided specific PD support is given to assist them. In turn, there may be beneficial effects on wider documentational and instrumental genesis.

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Analysis of a Task in three Environments: paper and pencils, manipulative materials and Apprenti Géomètre 2*

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This paper discusses how students (from 10 to 12 years old*) of 6th grade of elementary school solves a task of unit change in different environments: paper and pencils, manipulative materials and Geometry Software Apprenti Géomètre 2. Conceptual Field Theory (TCC) and didactic model for conceptualization of area as a magnitude comprise our theoretical framework. As a research methodology we use elements of didactic engineering. The results indicate that the environments manipulative materials and Apprenti Géomètre 2 provided a greater diversity of possibilities of action than paper and pencil, rising true theorems in action as: area is invariant by isometry, area is the number of tiles needed to cover certain figures.*

Keywords: Area, Geometric magnitude, Conceptual Field Theory.

Introduction

Research on mathematical education over the years has used a diversity of resources to intervene, and sometimes to diagnose, difficulties in learning area of flat figures in basic education, such as the use of tangram (Ferreira, 2010), squared paper (Pessoa, 2010), geometry software (Silva, 2016), among others.

Silva's (2016) master dissertation investigated the treatment given by students of the 6th year of elementary school to situations that give meaning to the area as magnitude (area comparison, area measurement, unit change and surface) in different environments: paper and pencils, manipulative materials and the Software Apprenti Géomètre 2. In this paper, we discuss how these students deal with the task of unit change, that were similar in the three environments mentioned above. Therefore, we will present here our theoretical framework, the method of research and analysis (a priori and a posteriori analysis); and finally, our final remarks and references.

Theoretical Foundations

Approach area as magnitude

The proposal to address area as geometric magnitude in this research is based on Douady and Perrin-Glorian (1989). These researchers observed several mistakes made by French students regarding the concept of area in the different school grades, such as: the area is attached to the figure and it does not dissociate from other characteristics of that figure; if the perimeter of a figure changes; their area as well (and reciprocally); if two figures have the same perimeter, they have the same area and the use of formulas in situations where they are not valid. These difficulties in area learning according to Douady and Perrin-Glorian (1989) occur because of the treatment given to area problems, based on: a "shape conception", in which, area is linked to the geometrical picture (confusing it with the figure); or a "Number design", in which, area is linked to the numerical framework (confuse the area with the area measure); or both simultaneously, without establishing relationships between them.

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Faced with these difficulties, French researchers understand that in the area learning, three frames must be considered: the geometric frame, the framework of the magnitudes and the numerical frame. The geometric frame refers to the flat surfaces; the numerical frame refers to the measurements of the surface area, interpreted as non-negative real numbers; and the picture of magnitudes refers to the establishment of equivalence classes formed by figures from the same area. Douady and Perrin-Glorian (1989) still argue that the approach of the area as a magnitude favours the pertinent articulations between geometric and numerical frameworks, which contributes to students' overcoming of learning difficulties and errors like those mentioned above.

Theory of conceptual fields

Vergnaud (1996) consider that a concept consists of a triad C = (S, I, R), in which S is the set of situations that gives meaning to the concept (the reference), I is the sets of invariant on which the operability of schemes (meaning) is based, and R is the set of forms belonging and not belonging to language that symbolically represent the concept, its properties, situations and procedures of treatment (the signifier). In our text, we will focus on operative invariants, more specifically, on theorems-in-action that are propositions, which may be true or false. These propositions remain, for the most part, implicit in the actions of the subject.

In her study of area and perimeter as magnitude, Ferreira (2010) used as a theoretical framework the CBT and a set of four large classes of situations that give meaning to the area as autonomous magnitude: area purchase, area measure, change of unit and surface production. The situation of unit change, which we approach in this text, refers to the procedure of representing the area of the same figure with different units of area measure (in this text we use also unit of area and unit of measure with the same sense), that is, to the same surface can correspond different numbers according to a chosen unit of measure, but the area does not change (Ferreira, 2010). The numerical framework and the passage from magnitude to number are highlighted in this situation by choosing a unit of area.

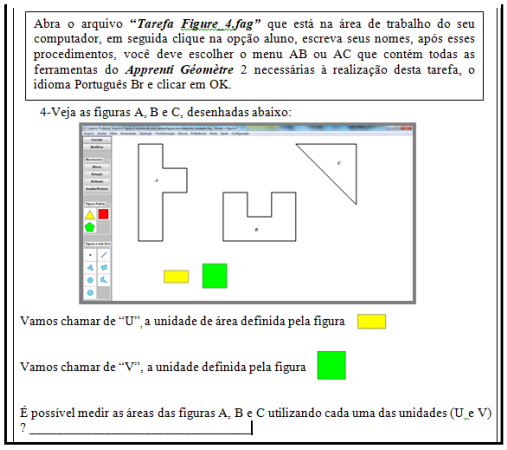
Methodology

Our research methodology is based on elements of Didactic Engineering (Artigue, 1996), more specifically, the phases of a priori and a posteriori analysis. The subjects of the research were 12 students who attended the 6th grade of elementary school (10 to 12 years) of a state school, located in the northern forest area of Pernambuco (Brazil). The school grade was chosen because it is the first grade of a new stage of Basic Education in Brazil, in which pupils are expected to systematize basic skills, in relation to the study of area of flat figures in previous school grades. We organized them in pairs, two per environment and delivered a set of five similar tasks (one for area comparison, two for area measurement, one for unit change and one for surface production). As a resource to answer the tasks, the students of: "Paper and Pencil" environment, received only blank paper and pencil; "Manipulative Materials", had available baking paper, a paper with different tiles to be cut and used, squared papers, scissors, glue and adhesive tape; and "Apprenti Géométre 2", received only the software in a computer, and they could use all tools of Apprenti Géométre. We organized in this way because we would like to investigate students' strategies emerged within the different environments.

A priori analysis

The task, chosen to be discussed in this text, was to investigate how the students deal with the possibility of effective tiling the same figure, by using different units to tile (change of units) and with the impossibility of this tiling a figure with the units of tiling given. We present in figure 1 below task 4 of Silva's dissertation (2016), classified as unit change.

Figura 1: Task 4 (Silva, 2016, p. 316)


Abra o arquivo "*Tarefa Figure_4.fag*" que está na área de trabalho do seu computador, em seguida clique na opção aluno, escreva seus nomes, após esses procedimentos, você deve escolher o menu AB ou AC que contêm todas as ferramentas do *Apprenti Géomètre* 2 necessárias à realização desta tarefa, o idioma Português Br e clicar em OK.

4-Veja as figuras A, B e C, desenhadas abaixo:

Vamos chamar de "U", a unidade de área definida pela figura

Vamos chamar de "V", a unidade definida pela figura

É possível medir as áreas das figuras A, B e C utilizando cada uma das unidades (U e V) ? _____

The way we constructed the task, it aims to led students to distingue area and the number that represents it, by controlling the correspondence between figure and numbers. In the designed task, the area of one figure corresponds to different numbers according to the unit of area chosen., without changing the area. This kind of activity is important to students' overcoming numeric conceptions. That is why we opted to discuss here the task of unit change. We highlight here that *unit of area* and *unitary of tiling* are different elements. When we refer to unit of area, we have different figures that can have the same area measure and be used as the same unit, whereas unitary surface, as the case of *unit of tiling*, has its shape as an important feature that differentiate it from others. In this sense, two different tiles with the same area can be considered as different units of tilling but the same unit of area. In the case of the task, we have two different units of area.

As a correct answer, they can state that it will be possible to measure all the figures including the triangle with units U and V. To assume that the students must accept that measuring does not depend on the way a set of the unit of tiling fit exactly the figure. With respect to the measurements of the areas of A, B and C, we have A = 10 U and A = 5 V, B = 10 U and B = 5 V, $C = 6\frac{1}{4}U$ and $C = 3\frac{1}{8}V$ (decomposed). As an incorrect answer, it may be stated that it is only possible to measure the areas of Figures A and B with units of measure U and V, and that C will not be possible given the impossibility of effectively brick this figure with such units of tiling, or even admit that it is possible measure the area of Figure C using for the measurement only the quantity of tiles that fit entirely within the triangle or the amount of tiles that partially cover the triangle C.

A posteriori analysis

In order to measure the area of Figures A, B and C within Apprenti Géomètre 2, the two pairs of Students used the same strategy. They reproduced the figures that represent the unit of area (U and V) in sufficient quantity to cover A and B. So, they paved the figures and stablished their areas. Regarding Figure C, after verifying that it is impossible to pave it completely with the tiles, they decomposed and recomposed figures U and V to pave Figure C. Finally, they counted the used units to obtain their areas. These procedures were based on correct theorem-in-action, according to which: decomposition and (re)composition without loss or overlap preserves the area, and the area is the number of units necessary to cover a figure.

Regarding the Manipulative Materials environment, one of the pairs deduced the areas measures by counting the squares of the grid, and verifying how many units of area would fit in figures A, B and

C. They counted by compensating, two half parts of the square of the grid as a square. Nonetheless, they forgot to compensate one of the half part of the square of the grid for figure C. In this environment, the figures were given to them designed in a squared grid paper. The other pair reproduced the sufficient amount of tile sufficient to cover the figures. Nonetheless, they also fail to reach the measure of area of the triangle.

As for the Paper and Pencil environment, one of the pairs attempted to sketch on Figures A and B, the units of tilling given in the task. So, they estimated how many tiles would be needed to pave these figures and then established the correct area. To sketch the tilling on the figures they transferred the length of the rectangle (square) sides using the pencil. Regarding Figure C, they get confused to reproduce the tiles on the figure. As for the other pair, they were not able to express the areas, they kept asking to the tutors for measuring instruments, such as graduated ruler.

Final considerations

The availability of different resources in the manipulative material environments and in the Apprenti Géomètre 2, led the pairs of students to extend their strategies of resolution. It also promotes that true theorem-in-act arise such as: *area is invariant by isometry;* and *the area of a figure is the number of tiles needed to pave the figure.* In the case of the paper and pencil environment, even if one of the pairs correctly set the task for figure A and B, we observed they were more stacked to previous knowledge they had, connect to area as number, for example, one pair always requested measure instruments, with conventional units of length, to help them in the resolution.

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Relation between teaching and research in mathematics and physics at university: the resources point of view

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We question the relation between research and teaching in the practices of university professors. We examine this issue from the documentational approach point of view. We extend this approach to take into account the interaction with resources in research activity. The collecting of the data consists in audio recorded interviews with 6 university professors (3 in Mathematics and 3 in Physics). The analysis shows different forms of relation between research resources and teaching resources. It seems that the perception of research resources determine the relation that university professors maintained between research activity and teaching practices

Keywords: teaching practices at university, research-teaching interaction, resources, mathematics, physics.

Introduction

The professional activity of a professor at the university usually involves teaching activity and research activity. The study presented here constitutes an exploratory study. We aim to contribute to the understanding of teaching practices at university and the factors governing these practices. We consider particularly the factors related to the research activity of university professors. We study it particularly by the prism of interactions with resources.

Some research in science education have attempted to find evidence of positive or negative correlations between research and teaching (Neumann, 1992) without taking into account a specific discipline. Madsen and Winslow (2009) emphasize that the relationship between research and teaching differ significantly from one discipline to another. They consider the case of mathematics and physical geography. Therefore, our research aims at understanding the interaction between teaching and research within the mathematics and physics disciplines.

We will discuss the potential of the *documentational approach to didactics* (Gueudet, Pepin & Trouche, 2012) to study this issue. It would be also an opportunity to discuss the scope of the documentational approach to address several disciplines issues. Gueudet (2017) analyses the interactions between university teachers and teaching *resources*. She did not investigate the interactions between university teachers and research resources. Our study is an extension of the (Gueudet, 2017) study on the theoretical level as on the methodological level.

Documentational work in research and teaching institutions

The documentational approach to didactics (DAD) (Gueudet & Trouche, 2012) proposes a holistic point of view on teachers' work, considering the activity of the teacher as a continuous process. In

the DAD, there is a distinction between *resources* and *documents*. We consider in this contribution the materials resources defined like all things that could *re-source* university professor activity (research and teaching). The interaction with the resources generates a document, which is the association of resources and a *scheme of use* of these resources.

A scheme is used here as defined by Vergnaud (1998) as the invariant organization of conduct for a set of situations having the same aim. According to Vergnaud (1998), a scheme is a dynamic that has four interacting components: *aim, rules of actions, operational invariants*, and *possibilities of inferences*. A *class of situations* includes all situations having the same aim.

A university professor develops a professional experience by interacting with the *teaching institution* and the *research institution* simultaneously (Madsen and Winslow, 2009). The interactions with resources in each of institutions are related on the one hand on the specific classes of situations (research classes of situations, teaching classes of situations) and on the other hand, the specificities of the discipline. The relation between research and teaching could take place like the migration and adaptation of resources between institutions, or also like a dissemination by a university professor of the professional knowledge and mode of teaching (the "operational invariants" component of scheme of use resources, Gueudet & Trouche, 2012).

The research question studied here is: when and how do the resources coming from research activity enrich the professor capacity to re-design them for his teaching work?

Methodology

Context and data collection

We have conducted 6 interviews with 3 mathematicians and 3 physicists. To keep the anonymity of the university professors interviewed, we will call them M1, M2 and M3 for the mathematicians; P1, P2 and P3 for the physicists. M1 and M2 are specialist in the mathematical modeling of scientific phenomena. M3 is a theorist mathematician. P1 and P2 are nuclear physicists. P3 is specialist on "electronic microscopy".

We constitute the interviews guidelines from two parts: research activity part and teaching activity part. We did not ask direct questions about resources; the interviewed expresses himself freely about his research and teaching activities. Choice that will allow us to identify the resources quoted in their answers, which would be main resources. We focus on scientist approaches, which allow us to make comparisons between the two disciplines, according their epistemological specificities.

Analysing the data: documents table of teaching and documents table of research

We consider two different tables (cf. figure 1): teaching documents tables (Gueudet, 2017) and research documents tables (which is our contribution in this paper).

Research documents tables			
Research aims	Resources	Rules of action	Operational Invariants

Teaching documents tables			
Teaching aims	Resources	Rules of action	Operational Invariants

Figure 1. The documents tables that permit us to consider a document in research institution and a document in the teaching institution

We will study the correlations that could exist at resources level and operational invariants between the two tables (teaching document, research document). Firstly, we tracked in each interview the information to complete the tables (figure 1). Then, we repair the resources in the research document table that exist in several classes of situations and used in teaching (it figures in teaching documents table). Finally, we discuss the resource mobilization process from research institutions to teaching institutions, by considering a horizontal analysis of the document table corresponding to this resource.

Some findings and discussions

Relation between teaching and research: the case of mathematics

According to the research documents tables of M1 & M2, the numerical analysis softwares are the key resources in their research activity. According to them, using these softwares with students is "complicated", particularly to find adequate problems with basic mathematical tools. M1 uses the softwares with the master degree students for promoting the appropriation of "mathematical modeling tools". He uses it according the same *epistemic mediation* (Gueudet et al., 2014) in both research and teaching institutions. M2 uses the numerical analysis softwares in two teaching classes of situations: 1) "conjecture mathematical properties in Master degree" where he uses it according the same epistemic mediation in research activity; 2) "Designing lesson for programming (including algorithm development and verification)" where he shapes and recontextualises the use of the softwares differently for teaching purposes.

In the case of M3, we have not identified resources used in both research and teaching tables. Therefore, there is no explicit relation between the two activities from resources point of view. However, M3 teaches the proof following the same process lived in his research: "it should make hypotheses and determine the properties to be mobilized".

Regardless of resources, M1 and M3, despite the resource gap between teaching and research, try disseminating their scientific approach research in teaching (repaired in the operational invariants that justify some rules of action).

Relation between teaching and research: the case of physics

For P1, the "bibliographical references" are crucial resources in her research activity. Whereas, "nuclear materials" is a resource that P2 mainly uses in his research activity. For P3, the "electronical microscope" is a key resource in his research activity.

P1 encourages students to use the bibliographical references she suggests in tutorial classes (where students must develop hypotheses and carry out some experiments). She disseminates partly her scientific in learning situations. P2 selects some materials that has similar characteristics to nuclear one in the aim to teach the associated physical model. To preserve the same epistemic value of the resources in both research and teaching institutions. He makes an adaptation of a general model; a movement of instantiation of a general model. Otherwise, P3 perceives the electronical microscope as "a very complex tools". Therefore, students could not use it in the similar context of his research.

P3 manages strictly observation and demonstrations courses in Bac+3 around electronical microscope without any link to his research work.

The three interviewed physicists highlight think that students from different undergraduate degrees (whether Bachelor degree or Master degree) need to have a "solid" conceptual knowledge in physics. For them, the lack of solid basic knowledge brakes the possibility to make explicit correlation between research and teaching.

Conclusion and perspectives

As results of the current study, we can conclude that the documentational approach is relevant to determine some aspects of relation between research activity and teaching practices at university. Through the analysis of the data, we observed different kind of correlations between teaching and research (case of M1, M2, P1 & P2). The relation maintained between research activity and teaching practices depends closely on the university professors perceptions of research resources.

The documentational approach offers a possibility to characterize *tangible nexus* (Neumann 1992) between research and teaching (via the kind of interaction with resources), but also *intangible nexus* (Neumann, 1992) linked to the interactions links to the specific professional knowledge of the university professors; the operational invariants resulting from the research activity determine partly teaching practices).

The collective design of resources in both teaching and research institutions constitutes an important direction to future research. It can elucidate the complex forms of relation between research and teaching in the practices of university professors.

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A case study of a Chinese teacher's lesson: problems, conceptions and instrumental orchestration

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Using the theory of instrumental orchestration and the quadruplet model of conception as the main interpretative frameworks, this study attempts to provide an insight to an experienced Chinese teacher's lesson, focusing on the conceptions she might intend to challenge using some problems, her way to propose these problems and organize a technology-rich classroom.

Keywords: problems, quadruplet model of conception, instrumental orchestration

Theoretical framework

Balacheff (2013) once defined a conception as the state of dynamical equilibrium of an action/feedback loop between a learner and a milieu under proscriptive constraints of viability, modeled by a quadruplet including "problems, operators, representations and control structure" where the problems "corresponds to the class of the disequilibria the considered conception is able to recover. In mathematical terms, this is the problems it allows to be solved. In pragmatic terms, we will speak of the sphere of practice of the related conception" (Balacheff & Gaudin, 2014, p.190). Problems could destabilize the learner/milieu system by modifications of constraints on the interaction between the learner and the milieu or modifications of the characteristics of the milieu, like the problems questioning the control structure and/or the representation system. This is how the learning proceeds—student's conceptions being challenged, transformed and reinforced—with the evolution of conceptions as the outcome. This paper would not figure out how to find meaningful problems destabilizing the learner/milieu system thus motivating the learning process, which proves to be a very difficult topic. Instead, we give an investigation on what could these problems be like, making use of the examples of an experienced Chinese teacher's lesson.

Another theoretical perspective, which informs our description of teacher's behavior in a technology-rich environment, is the notion of instrumental orchestration introduced by Trouche (2004) and developed by Drijvers et al (2010). An instrumental orchestration is defined as the teacher's intentional and systematic organization and use of the various artefacts available in a (computerized) learning environment in a given mathematical task situation. Drijvers (2010) once distinguished three elements framing this organization (a didactic configuration, an exploitation mode and a didactical performance), and described briefly six types of orchestration identified from theory and data in terms by introducing the first two elements of theses orchestrations.

Research questions and method

Under the framework of quadruplet of conception and instrumental orchestration, the research questions are formed as follows:

1) According to the mathematics problems proposed by a Chinese teacher on her lesson, which aspect of mathematics conception (from the perspective of the quadruplet model) might she intend to challenge using the problems?

2) Which types of instrumental orchestration emerge in technology-rich classroom when the teacher tries to encounter students with these mathematical problems?

All data in this paper are from the recording and observation of only one mathematics lesson given by an experienced teacher (Female, with 23 years) in a technology-rich classroom (in a famous private school). We answer the first question by analyzing the quadruplet of the conceptions addressed in this lesson and grouping the problems by the quadruplet elements of the conceptions that contribute to solve the problems. As for the second question, we observed that Ms. Jia's instruction is significantly characterized by the "spot-and-show" orchestration, which would be illustrated later in this paper.

Results

One main conception addressed in this lesson is the position relationship between point and circle on the same plane. Among the problems proposed centered on this conception, three categories were identified: problems challenging the representation system, challenging the control structure and those challenging the operators.

The quadruplet of the conception "position relationship between point and circle" and the problems challenging the conception

The operators in the quadruplet of a conception correspond to the set of operators needed either to perform "concrete" actions on the milieu, or to transform and manipulate linguistic, symbolic or graphical representations. In this case they might be "*judging the position relationship between any point and the given circle on the plane, based on the measurement of distance and radius or just perception", "drawing a circle that satisfy particular conditions, with the use of compass",* etc. The control structure refers to the components which support the monitoring of the equilibrium of the [Student - Milieu] system (Balacheff & Gaudin, 2014). It often goes with the operator as the condition to be required, like *"drawing a circle with trial passing the given point"*; or serves as criteria to make judgment such as "*only the center and the radius of a circle are fixed can the circle be fixed*". Other examples include "*a circle's center will fall on the perpendicular bisectors of any chord formed by any two points on this circle", "only the three points are not aligned can we find a circle that passes them*" etc.

During the instruction practice, the homework problems that Ms. Jia had proposed to students before this lesson are categorized as questioning the operators of the conception

1. How many circles can you draw with all these circle trials passing one point A/ two points A, B/ three points A, B, C?

Then she collected students' answers through the computer system available in this school, chose the (in)appropriate answers as a point of departure to initiate the whole class discussions. In the class-teaching phase Ms. Jia laid much emphasis on the problems challenging the "control structure".

2. (two kinds of answers was presented, figure 2 just simulate that due to lack of real photos) Which of the two drawings satisfies the condition "all circle trials passing one point" and why ?

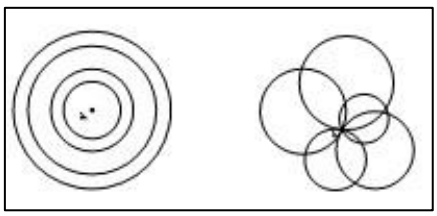

Figure 2

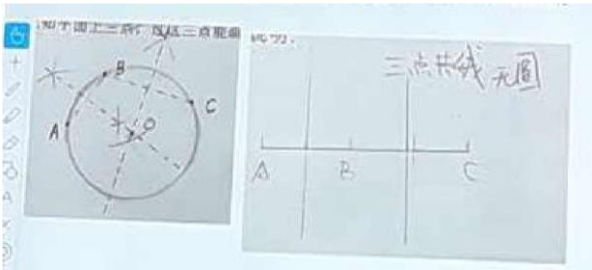

Figure 3

3. (two kinds of student answers are presented on the screen, as in figure 3)Can we always find a circle that passes 3 points on the plane?

In Balacheff and Gaudin's (2014) definition, the representation system of a conception describes the linguistic, graphical or symbolic means which support the interaction between the subject and the milieu, either actions or feedback, as well as their outcomes. The representation system in this case include the symbol *"⊙A",* letters *"A,B,C…"*for the representation of poin*t, "R, d"* for the representation of length, linguistic words like *"circle, segment, chord, perpendicular bisector, on, in and outside", and the graphic pictures of circles,* etc.

In an activity of predicting the influence range of typhoon "Lan En" using the photo taken by meteorological satellite, Ms. Jia paid special attention to students' expressions of position relationship.

4. (when a student was invited to give a prediction in the class: "If the edge of typhoon touches the city…" The teacher instantly added) what do you mean by TOUCH?

The orchestration types related to the problems

The main orchestration type introduced by Ms. Jia to encounter students with these mathematical problems is Spot-and-show, in which 'Spot' means that the teacher, while preparing the lesson, spotted the (in)appropriate answer in the students' work, and 'show' referring to the teacher's decision to display these students' results as a starting point for whole-class discussion. This is especially true in dealing with the problem 1, 2, 3. As Drijvers (2010) described, the didactical configuration in this orchestration type includes access to the computer system regulating student information during lesson preparation. As exploitation modes, teachers may have the students whose work is shown explain their reasoning, and ask other students for reactions, or themselves provide feedback on the student work. We would illustrate the two elements embodied in Ms. Jia's lesson, while the third element that involves the ad hoc decisions taken on how to perform while teaching—the didactical performance—is unable to be described for the time due to the limitation of data collection.

Didactical configuration (for the preparatory phase): the computer regulating system's option for students to upload the images of his/her homework and option for Ms. Jia to look at the students' work at any time. This permits Ms. Jia to choose the (in)appropriate answers, screenshotted them into her iPad and then display these results to guide students compare, differentiate and discuss. In addition, students need some apparatus to take photos. The monitoring of students' work is instrumented by the computer system's facilities that are not available to students.

Didactical configurations and exploitation modes for class-teaching phase: an applet in the iPad and a system connecting the iPad to the flat panel television, which allows of invoking and projecting photos in the iPad to the television at any time.

The arrangements include putting the flat panel television in the front of classroom, there is a whiteboard beside the television and the teacher can write some main points or clarify the knowledge structure of this class. The configuration enables the television screen and whiteboard coexist in students' visions, so they can freely transfer attention from the television screen to the whiteboard and reciprocally. Students were divided into groups and each group sit around a table for the convenience of discussion. Ms. Jia could walk freely in the classroom with iPad in the hand, taking photos of students' productions during the discussion.

It is worth noting that the Spot-and-show orchestration in Ms. Jia's lesson is not so student-centered. Students were designated randomly to answer the teacher's question; the technology has nothing to do with the student discussion, neither with the mathematics content. However, all the images she projected in the spot-and-show orchestration were from student work. As this orchestration depends on the facilities offered to the teacher for browsing through the student work, it is technology-specific, instrumented by the computer system and could hardly be achieved in in non-technological teaching settings.

Conclusion and discussion

Most of these problems/tasks are proposed to students before class as well as in class. However, in class Ms. Jia obviously allocated more time on the discussion of the problems concerning representations and control structure, while the problems questioning about operators are mainly for the students to solve before the class. This allocation doubtlessly ensured the class more effective and well-directed, at the same time placing greater demands about the work outside classroom on students. It is absolutely not an ordinary school, and of course not all students are appropriate for this kind of lessons. It is necessary to clarify that this paper just makes an assumption on the aspect of mathematics conception that the teacher might intend to challenge using the problems. Whether Ms. Jia indeed wanted to challenge these conceptions, and if students have actually experienced a conception evolution, remains to be studied empirically.

The computer system in this school has ensured the Spot-and-show orchestration by Ms. Jia: where teachers have the option to look at the students' work at any time, to spot the inappropriate answer in the students' paper-pencil work, and to show these students' results on the screen as a starting point for whole-class discussion.

The frequent use of Spot-and-show orchestration, well-organized problems and the good exploitation of student work constitute the prominent characteristics of this teacher's teaching behavior in a technology-rich environment, which enables her to better understanding student difficulty and give instant feedbacks. Nevertheless, other types of instrumental orchestration are few in this class, and the potential of technology for doing math and improving instrumental genesis (Gueudet & Trouche, 2009) remains to be developed.

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Visual solutions as a resource to promote mathematical learning

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Seeing is an important component in mathematical understanding, that can be developed, learned and taught. It is important to teach students how to see what is necessary, proposing challenging and adequate tasks. The use of visual solutions can be an alternative to the use of concepts that the student may not have learnt, and an easier way to solve a given situation. Therefore, the mathematics class should include practices and resources that lead students to think visually from experiences that imply this type of thinking. We intend to discuss some of these ideas and present examples of tasks, with multiple solutions, developed in the scope of a teacher training course.

Keywords: Tasks, visual strategies, problem solving, teacher training.

Introduction

Tasks have a great influence on students' learning, especially if they lead to the understanding of mathematical concepts and structures. The purpose of this study is to analyze the role of tasks, that allow multiple solutions, as a resource to enhance students' mathematical knowledge and their repertoire of problem solving strategies, highlighting the potential of visual solutions. In this sense, we used some tasks, in different contexts (visual and non-visual), in order to encourage students to present different solutions, especially visual strategies, from which two examples are described.

The tasks as a resource to trigger mathematical activity

The role of the teachers in the classroom is quite demanding. Among many aspects, they have the responsibility of choosing the resources that are most adequate to the students and the educational context. In particular, the tasks proposed deserve special attention, because they trigger the mathematical activity. Furthermore, different tasks, with different cognitive demands, induce different modes of learning. Therefore, it is important that teachers select tasks that promote students' interest and involvement, leading them to the pleasure of discovery, an inescapable condition to enjoy mathematics. The learning of mathematics should go beyond the experience with routine tasks, it should be enriched with problem solving, which provides the understanding of concepts and mathematical structure, and leads students to progressively acquire a set of strategies that may be useful in other approaches. Our experience with problem solving, both in terms of teaching and teacher training, supports the belief that teaching strategies has potential. We don't advocate a prescriptive teaching of strategies, but their analysis in a natural way in the classroom. Through a diversity of tasks students should be able to acquire a set of strategies that will be added to their personal repertoire, making them more competent in the approach to problem solving (Vale & Pimentel, 2017). Going further, teachers should provide tasks that enable multiple solutions, that allow several approaches, hence the application of different strategies, which favours fluency and flexibility of thought. Through the analysis and discussion of the feasible strategies, students' naturally increase their repertoire (Vale & Pimentel, 2017).

Visualization and problem solving

Visual representations can diminish the cognitive load in problem solving, allowing students to work on a part of a model without having to mentally analyse the model in its whole, also supporting analytical reasoning (e.g. Arcavi, 2003). Such representations may help highlight relevant information that might otherwise be more difficult to identify, and even create a scaffold for algebraic reasoning. In summary, visualization is being recognized as a key component of reasoning, problem solving, and even demonstration, which means that its status may need to be revised, in order to be considered more than just a useful tool to support learning (e.g. Arcavi, 2003). There is a set of problems that have great potential for visual solutions. To approach those problems we propose the use of a specific strategy that we call *seeing*. This strategy involves an activity that may be associated with the more traditional range of strategies (e.g. draw a picture, look for a pattern) and consists of a cognitive perception of the mathematical objects, combined with previous knowledge and experiences. The *seeing* strategy implies the use of visual representations (e.g. drawing, diagrams, figures, graphs) as the main approach to reach a solution. However we recognize that visual strategies are seldom used in the mathematics classroom (e.g. Barbosa & Vale, 2014). Several authors (e.g. Presmeg, 2014) advocate the use of visual means in problem solving, including not only records in the form of images, but also more abstract spatial representations, involving graphics and patterns. In this scope, Fujita and Jones (2002) argue the importance of developing a *geometrical eye* - the power of seeing geometric properties detach themselves from a figure - an essential tool for the construction of geometric intuition, that can also be used with topics other than geometry. In the field of mathematics, it has been argued that the use of visual images is an important support for all sorts of problems, including problems in which the visual component is not evident (e.g. Presmeg, 2014; Vale & Barbosa, 2017). Literature states that the activity of “seeing" is not a clear and innate process, but something you can develop, learn and teach, hence the importance of creating the opportunities for students to contact with this form of reasoning.

Methodology

Considering the purpose of the study we adopted a qualitative methodology. The participants were 14 pre-service teachers, in the context of a Didactics of Mathematics course, that included a module on visualization. Giving the purpose of the study, and in the framework of tasks with multiple solutions, our aims were: (1) to identify and understand the strategies used by the students when solving those problems; (2) to analyse the nature of the strategies (visual, non-visual) used. During the classes, these students had the opportunity to experience the power of visualization, in particular visual solutions, in problem solving and in the promotion of conceptual understanding. They solved a sequence of ill structured tasks that allowed the use of multiple approaches. Data was collected through observations and students’ written work in the several tasks used, being analysed inductively, from which emerged several categories concerning the presented solutions (e.g. style of thinking – analytical, integrated, visual; type of strategy).

Some results

We present two examples of the tasks used and some of the solutions obtained.

Task 1: What is the area of the rhombus, if M1, M2, M3, M4 are middle points of each side of the square and the square as 1 unit of area? Find out more than one process of getting the solution

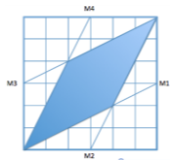


Figure 1: Task 1 – The Vasarely Rhombus

In Figure 2 we can observe some solutions of this task. This problem has multiple solutions that we grouped into four categories, in accordance with the written productions presented by the students.

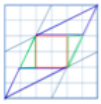
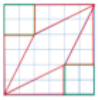
			
Area Formula - PythagoreanTheorem $A_{\Delta} = \frac{\frac{\sqrt{2}}{3} \times \frac{\sqrt{2}}{2}}{2} = \frac{1}{6}$ $A = \frac{1}{3}$	Decomposition A = 4+3+1+3+1 = 12 $A = \frac{12}{36} = \frac{1}{3}$ $A = \frac{1}{3}$	Framing A = 36 – 2 x (4+8) = 12 $A = \frac{12}{36} = \frac{1}{3}$ $A = \frac{1}{3}$	Decompostion-composition $A = \frac{1}{3}$

Figure 2: Some solutions of Task 1

The students were able to present more than two solutions, although not always in different categories of reasoning. We can say that they used the tools they've learned. Yet, only one solution was unique because it broke with the expected approach to this type of task - the last one on Figure 2. It is clearly a visual solution, contrary to the other three categories that show an analytical and integrated reasoning. This solution proved to be simpler, because the mere observation of the figure allowed to conclude that the area of the rhombus is 1/3 of the area of the square.

Task 2: In a school, there are 18 male teachers less than the female teachers. If 45% of the teachers are male, how many teachers are there in the school?

Figure 3: Task 2 – Number of teachers

This is a traditional word problem involving rational numbers. Figure 4 shows some of the students' solutions.

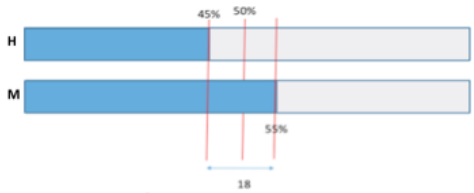
$\frac{55}{m} = \frac{45}{h} \Leftrightarrow \frac{55}{m} = \frac{45}{m-18}$ $\Leftrightarrow 55(m-18) = 45m$ $\Leftrightarrow 55m - 990 = 45m$ $\Leftrightarrow 55m - 45m = 990$ $\Leftrightarrow 10m = 990$ $\Leftrightarrow m = 99$ $h = m - 18 = 99 - 18 = 81$ $T = 99 + 81 = 180$ 180 teachers	$H = M - 18$ $T = M + M - 18$ $T = 2M - 18$ $M = 0,55T$ $T = 2x0,55T - 18$ $T = 1,1T - 18$ $0,1T = 18$ $T = 180$ $\quad 0,55x180 = 99$ $0,45x180 = 81$ T - 180 teachers: 99 women e 81 men	 55%-45%= 10% → 18 teachers, hence 100% → 10x18 =180. There are 180 teachers

Figure 4: Some solutions of Task 2

The first and second solutions use manipulation of numbers, equations and proportionality. These students chose to solve the problem in an analytic way, privileging the traditional approach to a problem involving rational numbers. These were the most common solutions. Other strategies emerged, based on the use of a diagram, that did not lead to the intended result. However, the last

example on Figure 4 shows a solution that we identify as visual. The bar model represents all the information stated on the problem and facilitates a direct reasoning with percentages.

These examples evidence that students may have different ways of thinking and of representing data when solving problems, but, frequently, when we recur to the visual aspects of a situation it can be easier to solve it, as well as making less mistakes in calculus. There is a strong relationship between visualization and problem solving, since visualization often provides students additional strategies, having therefore more to draw upon within their repertoire.

Concluding remarks

The majority of the students were not visualizers, possibly because of their previous experiences, fact that was noticed in the choice of strategies. However, due to the nature of the tasks, and the discussion of the different approaches, students became gradually more sensitized to the use of visual strategies. The examples presented show the importance of having *geometrical eye* (Fujita & Jones, 2002) to achieve more efficient solutions in problem solving. But they also show us that it is very important to have good tasks to develop such a competence, serving as a trigger to highlight these features and develop mathematical knowledge. A final discussion with the students is always necessary to identify the best strategies to approach the given tasks, making it evident that some of them are clearly better. In fact, if the students are alerted to the advantages of a different strategy based on *seeing*, it is more likely they use that process in subsequent problems. Throughout this paper we value the *seeing* strategy, since visualization can be a powerful alternative approach that increases the window of possibilities regarding problem solving, providing different, non-traditional solutions and frequently imbedded with meaning. But, it is also important to point out the importance of the teacher's role: when these strategies do not appear naturally, the teacher must necessarily demonstrate that way of seeing, in order to develop the visual skills of students and to increase their strategies repertoire (Arcavi, 2003).

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Designing to support teachers' reconstruction of instructional design rationales

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This paper is an initial report on an ongoing research and development study conducted with small groups of teachers in Australia. While we do not, at this point, report on research findings, we share some of the conceptualisations that informed the design of the study and the key features of this design. In particular, we aim to clarify why the goals we pursue for teachers' learning are formulated in terms of teachers' reconstruction of the instructional design rationale, what we mean by this, and why these goals shape profoundly how we design resources for teachers.

Keywords: Mathematics teachers' resources, teacher learning, instructional design, instructional sequences.

Resources and teacher learning

This paper is an initial report on an ongoing research and development study that is being conducted with small groups of teachers in Australia. The overarching aim is to contribute to understanding of teachers' learning with resources, in particular in situations, where resources are designed and used to support teachers' transition to instructional practices that aim at ambitious and equitable students' mathematical learning (Jackson, Gibbons, & Sharpe, 2017).

We are conceptualising teacher learning with an instructional resource as a process of documentational genesis (Gueudet & Trouche, 2009; Trouche, Gueudet, & Pepin, in press), an expression of the productive and constructive dialectical relationship between teachers' interpretations of the resource (document) on the one hand, and forms of their participation with this resource (instructional practices) on the other hand. In this context, we aim to clarify why the goals we pursue for teachers' learning are formulated in terms of teachers' reconstruction of the instructional design rationales, what we mean by this, and how these goals shape profoundly how we design resources for teachers. We will foreshadow our anticipated contribution to the questions proposed within the Working Group 3 (instrumentation, competencies, design capacity, expertise).

Developing ambitious teaching practices in mathematics

Teaching inherently involves adapting given (prescribed, shared, or previously used) instructional materials and ideas to specific contingencies of the classroom in which these materials are to be used. It is in this sense that teachers are being viewed as designers of classroom mathematical activities (Pepin, Gueudet, & Trouche, 2017). When the curriculum or the expectations for the nature of classroom mathematical activities change, guidance is needed on which resources to select from and how. Importantly, guidance is also needed on how to make specific adaptations to new resources when addressing the unanticipated challenges that emerge in different classrooms.

In our view as designers of instructional resources, the latter type of guidance is provided by the instructional *rationale* we develop and refine during the process of designing the teachers' resource materials. It primarily consists of conjectures about the collective mathematical development of the classroom community, and how this development can be supported. Rationales of this kind outline a progression of clearly formulated and specific long- and short-term learning goals, as well as key adaptations that made these goals achievable in different classrooms. They also include notions that typically fall within one's pedagogical repertoire, such as how classroom activities can be organized to be most productive and why, and what types of classroom discourse made it possible for students to progress in classrooms where the activities were previously used. This long list of sources of guidance on which we draw when adapting the existing resource to the contingencies of a new classroom should help to clarify that our history of use of the resources and deliberate analysis of this use were essential in making the rationales accessible to us.

While teachers' access to a design rationale is key to their effective use of a new resource, it cannot, in principle, be readily available to them from the outset. Nevertheless, teachers we worked with in past came to reconstruct the rationale in the course of sustained professional development interactions that productively built on both their current instructional practices, and on the designed resources they were learning to use (Visnovska & Cobb, 2013; Visnovska, Cobb, & Dean, 2012). This is why we find it productive to conceptualise teacher learning with novel resources as a process of their increasingly central participation in practices of a community, within which the reconstruction of the rationale is proactively supported (Wenger, 1998).

Collaborating to learn

The first author is developing professional development (PD) collaboration(s) with teachers within a single state primary school (year levels 0 through 6). The design of specific PD interactions was co-developed between the participating teachers, their school-based mathematics couch, and the first author. Up to this point, the first author collaborated with a pair of year 3 teachers over a period of 2 months, followed by collaborating with one year 4 teacher, over another 2 months. The collaborations started in the second half of the 2017 school year and will continue in 2018, when the teachers will start working with new classrooms.

The mathematics couch was part of PD meetings whenever she was available. Her part-time role within the school involves supporting teachers in their mathematics teaching with a particular focus on problem solving. A part-time research assistant, a secondary mathematics teacher on a 2-year study leave from her teaching job, who provided data collection support was also an active participant in PD interactions. It is anticipated that she will continue collaborating with the first author in 2018 when she returns to her teaching of Year 7 students in a high school.

The work meetings with the teachers all took place at the school site, during (or immediately prior to) the teaching hours. The main organizing feature of PD collaborations was a series of classroom teaching events, in which the first author, the classroom teacher, or both, taught lessons informed by the instructional sequence of *Fractions as Measures* (Cortina, Visnovska, & Zuniga, 2014). Mathematical goals for the lessons targeted initial fraction learning, specifically supporting all students to *reason soundly* about the inverse order relation of unit fractions, and, in year 4, about

proper and improper fraction comparisons, including equivalences. The material resources included brochure-like teacher notes on key lessons in the sequence with advise on organization of classroom activities, types of likely student solutions, and possible follow up questions, and student-made rods for measuring unit-fraction lengths relative to a given length—the wooden *stick*

Each classroom session was complemented by 15-60 minute planning and/or debriefing meetings between the first author and the respective teachers. In year 3 collaboration, the second teacher observed the lessons and took an active part in planning and debriefing sessions. Simple pre- and post- data on students' understanding were also collected to provide teachers with additional means of evaluating the impact of the learning experiences on individual students.

Following most of the teaching sessions, the two authors discussed the student learning progress, teachers' perceptions of how their students are learning, and what pedagogical and mathematical issues teachers brought up as they recounted their classroom observations and justified planning decisions. They also planned for types of PD experiences that could be beneficial to teachers and how those could be framed within the existing collaboration structures and routines.

It became abundantly clear within these interactions that the teachers were primarily interested in learning what were the components of the instructional sequence (tasks, organisation of classroom activities). During planning sessions, the teachers' expertise on their students' learning circumstance was essential to making adaptations. At the same time, the researcher contributed the explanations of the design rationale and illustrative examples. In this way, both parties contributed to planning where opportunities emerged for discussions of aspects of the design rationale.

To this point, teachers' interactions with the resource appear to satisfy their need for forming initial interpretations of the resource components, and what they as a teacher 'need to do' in the classroom. We do not suggest that teachers' participation in the co-planning process equipped them to plan sequence activities independently. Instead, we frame these experiences as constitutive of their initial document (cf. Gueudet & Trouche, 2009) for *Fractions as Measures* sequence, one that will be further revised and refined. During the second iteration of the teachers' work with the sequence, in 2018, we will pursue following research questions: (1) Which aspects of the design rationale do the teachers reconstruct independently/collaboratively? (in planning and/or justifying enactment of the lessons) and (2) Which additional aspects of the sequence rationale can be reasonably addressed (or become of interest to teachers) in conversations with the researcher? Data from planning and debriefing sessions will be analysed against similar data from the first iteration with focus on similarities and differences in teachers' justifications of their decisions. We will report on the learning of the research team, and draw implications for designing teachers' resources during the Re(s)sources conference.

Concluding remarks

When it comes to designing for students' mathematical learning, designing for students' guided reinvention of mathematical ideas is a long accepted practice (Gravemeijer, 1999). Designers formulate (and test for viability in classrooms) a progression of mathematical ideas and design successive instructional activities to support the emergence of these ideas in the classroom.

We argue that when it comes to designing for *teacher* learning, the resources cannot *give* teachers the instructional rationales that underpin them. They could, however, (a) identify (and test for viability with teachers) a progression of mathematical and pedagogical ideas that are central to teachers' instructional decision-making and to their reconstruction of the design rationale, and (b) design successive professional experiences to support the emergence of these ideas as teachers interact with the resource (or, more specifically, with the successive *documents* they are creating in the process). Our goal in the ongoing study is to contribute to understanding of teachers' learning with resources in terms of teachers' progressive reconstruction of the rationales that underpin the resource design. While we adopt design research perspective, this focus is directly related to efforts at understanding teachers' design capacity, competencies, expertise, and what is discussed as utilization schemes within the instrumentation process.

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An exploration of Documentation Expertise through a collective lesson preparation: A Chinese case of two mathematics teachers

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This contribution aims to explore the expertise aspect of teachers' documentation work in the frame of WG3. Thus, a notion of Documentational Expertise (DE) is proposed and developed to represent the expertise in teachers' documentation work. Designed as a case study, two Chinese mathematics teachers, a mentor, Gao and her apprentice Yao, were chosen, and a series of intense collective lesson preparation (named MOKE "磨课") between them was filmed and will be analyzed. The subject of this lesson was about "the properties of in-equality", content in grade 6, which was assigned as an open lesson task to Yao, with the instructions of her mentor Gao. Based on Documentational Approach to Didactics (DAD), the results will be presented in two parts: firstly the evolution of the different versions of lesson plans (the aspect of documentational genesis), and secondly the schemes in interacting with resources showed in Gao, during her instructions to Yao (aspect of component of a document). It is hoped to refine the definition of DE from a deeper dimension of scheme, as well as how its development could benefit from collective work in the end.

Keywords: Mathematics teachers' resources; teachers' collective work; teaching research group; teacher expertise; documentation expertise; scheme

Introduction

With an aim of contributing WG3, in particular the issues in of "instrumentation, competencies, design capacity, expertise" and "how do these aspect develop over time in teachers' practice", this study takes teachers' expertise in resource work as a main concern.

In one of our pilot studies in China (Pepin, Xu, Trouche, & Wang, 2016), three mathematics expert teachers from a same high school were interviewed, and the results showed that (1) mathematics teacher (knowledge and) expertise has many facets, and one of them relates to their design activity (e.g. lesson preparation); (2) teachers' expertise in interacting with resources and their design activities seem to be enhanced by working in collectives; and (3) the 'institution' of Teaching Research Group (TRG), a school-based teachers' collective working unit consisted by teachers who teach same or similar disciplines, is a special form of mathematics teachers' collective work. In fact, several studies had verified the positive influences of collective work on Chinese teachers (Wang, to be published). Thus, in this case study, where a collective resource design work (MOKE, which was a colloquial expression of "collective lesson preparation" used among teachers (Chen, 2006)) happened in TRG, an extra aspect of "collective" is also need to be cared.

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In the following sessions, the theoretical framework and research question will be presented firstly; secondly a detailed explanation about the methodology and tools; subsequently, some elements of DE, and its development in collective work will be discussed.

Theoretical framework

This study is based on Documentational Approach to Didactics (DAD) (Gueudet, Pepin, & Trouche, 2012). Hence, some notions from DAD were kept: the broad meaning of resource as "everything can re-source an activity"; the lens of documentatational genesis to see through teachers' professional development; the definition of documentation work; and the formula of "a document = a resource + the scheme of usage".

As an important component in document, scheme was defined by Vergnaud (2009) as "*the invariant organization of activity for a certain class of situations"* with four components: *goals* that guide the activity, *rules* that retain "*sequences of actions, information gathering, and controls*", *operational invariants* that the "knowledge in action", and *inferences* that allow to take into account the singularities of the situation.

Based on these notions, Documentational expertise (DE) was proposed to represent the expertise in teachers' documentation work. As Berliner (1988) stated, expertise is "specific to a domain and developed over hundreds and thousand of hours". When focusing on a design phase (e.g. a lesson design), Brown (2009) used to proposed Pedagogy Design Capacity (PDC) as "a teacher's skill in perceiving affordances, making decisions, and following through plans". Further studies were made to deepen this notion. Pepin *et al.* (2017) defined and refined, the notion of mathematics "teacher design capacity", distinguishing the following three components: (1) the goal/s of the design activity, (2) a set of principles, and (3) reflection-in-action. These components kept a clear resemblance with Vergnaud's conceptualization of operational knowledge and his notion of schemes. The notion of DE is similar to the teacher design capacity of Pepin *et al.* (2017), enlarging it by evolving not only the phases of lesson design and implementation, but the whole process of documentation work: a life circle of producing a document through resource retrieving, selecting, organizing, modifying, adapting, implementing and sharing off, of an individual teacher, and also of her interactions in collectives.

To summarize, DE was defined preliminarily as the operational knowledge (knowledge in action), situated in schemes of teachers' interactions (retrieving, selecting, organizing, modifying, adapting, implementing and sharing off) with resources. The research questions are clear: what is DE and how DE gets developed through collective work? The specific methodology and research design for achieving the answers will be presented in the following section.

Methodology and tools

Methodology as well as the research design will be explained here.

Reflective investigation

The reflective investigation involves teachers as part of the study throughout the data collection, which means teacher's reflections on their previous answers could become a continuous data. Four principles are emphasized: long-term follow-up; in- and out-of-class follow-up; broad collection of material resources used and produced throughout the follow-up; and reflective follow-up of the documentation work (Gueudet *et al.,* 2012, p. 27).

Documentation-working Mate (DWM)

DWM was proposed in order to better understand and studying the influences from collective interactions on teachers' documentation expertise. DWM in this study was defined as a person who works closely with the targeted teacher, with mutual influences on their documentation work and DE development. *Mate* (in the Oxford Dictionary) refers to "a fellow member of joint occupant of a specific thing, like table-mate". This indicates four characteristics of DWM: (1) s/he is chosen by the targeted teacher, not the researcher; (2) each teacher could have several DWMs, such as mentor/apprentice, trainer/trainee; (3) the most important indicator for choosing DWM is "interacting most frequently"; (4) there is no boundary or constrains of age or education background or expertise levels for being DWMs.

Research Design

Gao and Yao are two mathematics teachers who worked closely in a same mathematics TRG within a same middle school of Shanghai, China. According to the five-stage model of pedagogical expertise of Berliner (1988), Gao belonged to the group of expert teachers (with 24 years' teaching experiences and ability to deal with teaching problems effectively and effortlessly), while Yao was a novice teacher (with 1 year's teaching experience). Gao was the ex-leader of Mathematics TRG, and she worked as the leader of Lesson Preparation Group in grade 8 during our data collection. Gao chose Yao as one of her DWMs, who worked as her apprentice since 2015, when Yao graduated and started to work in her middle school. In return, Yao also chose Gao as her DWM. Although working in different grades with Gao, she always turned for Gao's instruction when she met some teaching problems.

At the beginning of the follow-up, a three-week full-day observation of Gao's schoolwork was conducted. In the end of the follow-up, interviews with Gao and Yao about their resource work and mutual beneficiation on each other were conducted. During the follow up, Yao was asked to prepare an open lesson entitled "the properties of in-equality" (contents in Grade 6) through MOKE, with the instructions of her mentor Gao, which was a quite regular work for novice teachers in their school. They conducted a total of three MOKEs (a MOKE consisted of lesson preparation-implementation-collective discussion). We interviewed Yao for each version of lesson plans and filmed her collective discussions with Yao after each lesson.

First results from the data analysis

Four versions of lesson plans were generated along with the three rounds of MOKE activities. The changes of each appeared as: (1) The introduction activities of the lesson; (2) The quantity and quality of the exercises prepared for students; (3) the coherence between the current teaching contents and students' previous knowledge. Meanwhile, although decisions of changing were made by Yao, the influences from Gao's suggestions can be evidenced.

In Gao's propositions to Yao, there are several DEs can be seen: (1) searching for the available resources from learning-aid books, which were a main source for her own resource system; (2) adapting and adjusting the resources according the students' reactions; (3) adding some innovative resources for a the interesting and rhythm of the lesson. As an experienced teacher, Gao holds some deeper conceptions of "student-centered" towards lesson preparation: "lesson is not isolated", so when teaching "properties of in-equality", it is needed to build the links with students' previous knowledge ("properties of equality"), which was used to be considered as "off-topic" by another teacher, and life experiences (making comparisons).

With the instructions of her DWM, Yao experienced a complete process of preparing a topic through several MOKE activities. Although such collective work is not a traditional cooperation with labor divisions, Yao obtained the way to deal with the detailed problems appeared in her lesson implementations, while Gao also experienced a process of trying the ideas in her mind. Further results and detailed data will be presented in May 2018.

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WG3. Poster
GT3. Poster

The sequence of mathematics tasks in the computer environment

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Abstract

Dynamic geometry software affects teachers' choice and decision of mathematics tasks and their organization. But there is the deep gap between real status in teaching and the results of didactical research. Two examples from China and France were chosen to contrast the tasks and possible orchestration of different teachers. In Chinese example, software is used to generate new tasks and suggests a teacher-centered way to use the software. The role software played in French tasks is more like an amplifier of paper-pencil activities, supporting a more student-centered organization of the class.
Key words: Dynamic geometry software, Instrumental orchestration, Lesson preparation

With the broad use of ICT in all the area of our life, mathematics education paid more emphasis on the effects computer technology brought into mathematics learning process. In this paper, I want to solve the following questions: What is the difference in the mathematics tasks teachers use based on dynamic geometry software?

Theoretical framework

The role of dynamic geometry software (DG) played in mathematics tasks (Laborde, 2001; Soury-Lavergne, 2017): 1, DG is an amplifier of the paper and pencil, like facilitating material aspects of the task while not changing it conceptually. 2, DG is the genesis of constraints, affordances and new task, like "Boite-noire-sourire".

My analysis of technology integration into teaching is based on the instrumental orchestration (Trouche, 2004; Drijver, 2010). This paper focus on the first two elements: didactic configuration and exploitation mode, the sequence of the task teachers showed to the students and the role mathematical software played.

Methodology

In order to answer the question mentioned above, two examples from China and France were chosen to contrast the tasks and possible orchestrations of different teachers.

The use of computer technology is still limited in mathematics

There is still a gap between the research and status of teachers' use of software. In Chinese example, software is used to generate new tasks and suggests a teacher-centered way to use the software. In French tasks, it is more like an amplifier of paper-pencil activities, supporting a more student-centered organization of the class.

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WG4. Transitions towards digital resources: change, invariance, and orchestration

GT4. Transitions vers les ressources digitales : évolution, invariants, et orchestration

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Digital resources have become an important part of teachers' and students' resource systems. The integration of digital resources into teaching and learning practices, however, raises many questions to teachers and educators.

- How to choose appropriate resources from the myriad of available options?
- How to adapt these resources to the specific learning goals at stake?
- How to orchestrate the students' use of the digital resources?
- What do student resource systems look like? How to prepare pre- and in-service teachers for these challenging tasks?
- Which role can digital resources play in assessment?

Les ressources numériques sont devenues une partie importante des systèmes de ressources des enseignants et des étudiants. L'intégration des ressources numériques dans les pratiques d'enseignement et d'apprentissage pose de nombreuses questions aux enseignants et aux éducateurs.

- Comment choisir les ressources appropriées à partir de la myriade d'options disponibles ?
- Comment adapter ces ressources aux objectifs d'apprentissage spécifiques en jeu ?
- Comment soutenir l'utilisation des ressources numériques par les élèves ?
- À quoi ressemblent les systèmes de ressources des étudiants ? Comment préparer les enseignants, en formation ou en exercice, pour ces tâches difficiles ?

- Which opportunities do they offer for new learning formats, such as blended learning and flipped classrooms? How do classroom experiences inform the (re)design of a digital resource?
- What are the options for personalized learning in adaptive environments?

In this Working Group, these issues will be addressed from theoretical perspectives, including instrumental genesis, instrumental orchestration and documentational genesis.

- Quel rôle les ressources numériques peuvent-elles jouer dans l'évaluation ?
- Quelles possibilités offrent-elles pour les nouvelles formes d'apprentissage, tels que l'apprentissage mixte et les classes inversées ? Comment l'expérience de la classe aide-t-elle à (re) concevoir une ressource Numérique ?
- Quelles sont les options pour l'apprentissage personnalisé dans des environnements adaptatifs ?

Dans ce groupe de travail, ces questions seront abordées à partir de points de vue théoriques, comme la génèse instrumentale, l'orchestration instrumentale et la genèse documentaire.

WG4. Oral Communications
GT4. Communications orales

LEMATEC Studium: a support resource for teaching mathematics*

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This work presents the educational platform "LEMATEC STUDIUM". This paper shows the conceptual elements that support its construction and use. LEMATEC Studium is a support for teaching practice, based on the idea of teaching documentation, in which the created document is on the basis of a webdocument comprised of a system of resources chosen and integrated into the webdocument by the teacher.

Keywords: Resources, Mathematics, Teaching Resources, Lematec studium.

Introduction

Teaching practice has always been dependent of resources that supports it. Nowadays, digital resources are increasingly present in mathematics classes and teachers need to orchestrate class in as such way that the resources used are articulated among themselves. Ruthven (2010) discusses the "resource system" as the complex teaching work in articulating a set of resources.

As regards the complexity of the resources articulations to help mathematics teaching, the platform *LEMATEC Studium* was created to support teachers to model their digital and computational documents. In such platform, teachers can use a set of features and schemas of utilization to create a document to be used in a specific class. Since we are speaking of digital resources, the produced document is a webdocument. So, LEMATEC Studium can be used to help a teacher to build his own webdocument. The platform conception and development aims:

- To systematize and conception process contributing to the advances of educational software engineering (Tchounikine, 2011);
- To identify theoretical-methodological principles of instrumental orchestration and documentary approach to specify Lematec Studium requirements. This process of identification as an engineering process to produce the environment contributes in these principles evolution;
- To produce a pragmatic validation of the engineering process as well as an experimental environment for validation of the didactic engineering theoretical approaches that guides its conception.

As regards the validation of the platform, a workshop was held with a teacher, wich is presented in the methodology of this work.

Theoretical Foundation

Resources

In the past, educational resources for mathematics teaching in most academic papers were focused on didactic material, such as textbook. This led Adler (2000) to present the notion of resources as

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everything that helps teaching activity, everything that "re-source" teachers' action. The author understands the term "resource" as a noun and as a verb. Resource as a noun is related to the object that supports teaching activity, and as a verb is related to the actions made by a teacher in using it.

Trouche and Bellemain (2016) following the notion of resources presented by Adler, embody the activity as a teaching resource. For them, the activity resource is interpreted as a webdocument activity, since teachers organize both, the activity that should be done by students and the students actions in the activity, generating a new document.

Resource and Document

While resource is everything that support teaching practice, document is the product generated by a teacher in a process of documentation when they clusters and models an existing set of features and schemas of utilization. Based on the instrumental genesis, Verillon and Rabardel (1995), which are the schemes of utilization created by teachers during their interaction with a resource, arise the dialectic resource-document.

According to Gueudet and Trouche (2008), document is a result of a teacher action after modeling a set of resources and schemes of utilization aiming to teach a specific content. The document can be considered as a living entity and can be modified to create a new document to new teaching situations. In this way, LEMATEC Studium is like a resource to support a webdocumentation, as development and result.

Webdocument

A Webdocument can be considered as a document on the web. In our perspective, embodied in LEMATEC studium perspective, this document is a teacher construction to support his/her own activity. The idea of a document is based on the documentary approach (Gueudet and Trouche, 2008), which a teacher selects and combines a set of resources and usage schemas for a specific teaching situation. There are three types of webdocument classification, it can be:

- Static: in case of no interaction is allowed within the webdocument. For example, when a teacher inserts a file in pdf. format, it is unchanged, there is no way to modify it.
- Dynamic: in case of the webdocument can be set up according to content specificities. A teacher can choose resources and modify it within the webdocument.
- Active: in case of webdocument developed as a software developed by the teachers combining resources, webdocument-software which will be the locus of the student actions. When a new webdocument is inserted or modified, a new version of this software is created.

In this way, webdocument is a software composed by others software programs. Therefore, a webdocument is not only a product, but it is also considered as the support for the elaboration of a product.

The platform LEMATEC Studium

The LEMATEC Studium is a platform developed in our research group (LEMATEC) and currently tested by the member of the group for teaching mathematics contents. The platform is a support for construction and use of webdocuments in face-to-face, e-learning and blended education classes.

LEMATEC Studium is based on the theoretical-methodological principles of instrumental orchestration (Trouche, 2005) and the documentary approach (Gueudet and Trouche, 2008). In that platform the registered teachers can edit a discipline and insert webdocuments already created by them or create a new one.

The platform is structured into three interfaces:

- Resources: where teachers select or insert a resource. There are many formats of resources that can be inserted, such as: video, image, pdf file and dynamic geometry software file.

Figure 1. Lematec Studium

- Webdocument: interface that supports the construction of teachers' lessons. They can select a previously created webdocument or create a new one, serving as a microworld for teachers documentation. There are two types of Layout: text or list. In the first one, it is possible to insert a resource in the middle of a text, in the other one the resources are placed in sequence.
- Studium: interface that supports orchestration of the environment that will be used in class. In that interface, teacher selects the webdocuments he will use in each meeting and decides if an inserted webdocument will be an activity (keeping track of student activity) or just a material. When an activity is applied, it can help teachers to evaluate his students. In the case of a webdocument that has a GeoGebra file as a resource, for example, a student can have and save a GeoGebra activity, without having to access the software outside the platform. After completing the activity, the student can select the option to send for evaluation, enabling his/her teacher to access to his/her construction.

Classes are presented in a calendar available in the platform. In this way, students select the date and to access to the class orchestrated by the teacher.

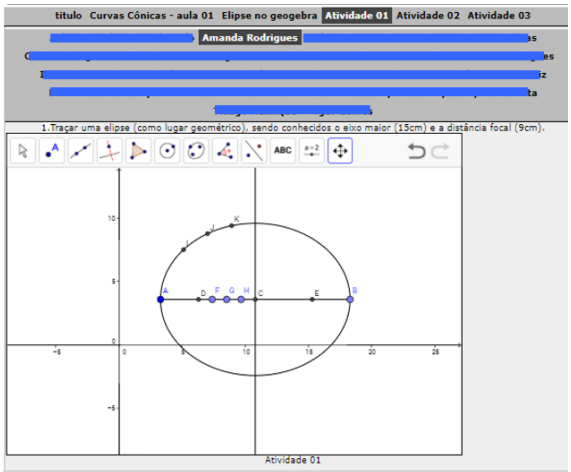

Figure 4. webdocument activity

Methodology

Recently tests conducted by a teacher during the teaching of conic curves, the platform demonstrated potentiality to assist action. In order to present the potentialities of the platform LEMATEC Studium o support construction of teaching situations of mathematical contents, a workshop was held with a teacher. The workshop was performed with a teacher of higher education and focused on the teaching of conic curves for a class from a first period of a bachelor's degree.

At the first moment the conceptual points that were the basis for the construction of LEMATEC Studium were presented. In the second moment the teacher had acess to the environment and can interact and know the tools to develop a webdocumento.

With the end of the workshop, one of the researchers who observed the subject's interaction with the environment, followed a elaboration of a lecture aiming at the teaching of conic curves in the platform made by the teacher. Subsequently an interview was conducted to find out about the potentialities of this platform.

Final considerations

Regarding the students, the platform favored the visualization of the geometric elements due to the implementation of dynamic geometry software in the environment. Besides the practicality the fact that it doesn't need to use externally the software of dynamic geometry:

- reduce the time destined to send each geometrical construction separately to the teacher,
- made the discipline more organized, since it is enough to acess the tab of the activity,
- provide an environment to combine and articulate resources in a webdocument-software for supporting multirepresentational mathematic activities.

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Orchestrations at kindergarten: articulation between manipulatives and digital resources

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In this paper, we study the uses of digital resources for teaching mathematics at kindergarten. We focus on articulation between tangible and digital resources. At kindergarten manipulatives take an important place in teacher's system of resources. In this context, integrating digital resources implies for teacher designing new situations. How teacher design these situations? Which professional knowledge are involved? To study these aspects, we refer to the theoretical framework of documentational approach and to the concept of orchestration. We identify a new type of orchestration, "manipulatives and software duo".

Keywords: software, manipulatives, mathematics, orchestrations, kindergarten

Introduction

Our work deals with teaching numbers at kindergarten and using digital resources for this aim. Studies have highlighted the fact that technologies can make the teaching situation more complex: implementing technologies in its practices requires to design new situations (Trouche, 2009). Designing these situations involves changes in his/her resources, practices and professional knowledge. At kindergarten, teachers seem to consider manipulatives as crucial resources for teaching numbers. Concerning manipulation in mathematics, according to Briand (2014) "it is necessary to question the often widespread notion that mathematics is constructed by simple manipulation of objects […] This means […] creating devices where the student will anticipate the effects of his or her action on a material environment, and learn from the results different from those expected, to modify his actions." (p.2). Create and manage these sort of situations raises professional issues. In this context, we explore articulation between manipulatives and digital resources. How kindergarten's teachers associate manipulatives and digital resources? Our research takes place in a research group working in France and involving researchers and teachers, the MARENE[1] group. This group develops sets of resources (software, manipulatives, scenarios) for teaching numbers. In the following part of this text we present one of this set of resources called Cars & Garages (CG).

Cars & garages: use number as a memory of the quantitity

The situation is as follows: pupils have at their disposal a lot of garages. They have to take in a single journey, in a remote place, just what is needed of cars so that each garage has a car and that remains no cars without garage. For pupils, the challenge is to realize that they can use number-based procedures (eg counting garages, then cars) to keep the memory of the quantity. The material used in the group is made from matchboxes (Figure 1). The group developed software based on this situation

[1] MARENE: Schools 'resource kit for learning the concept of numbers; http://python.espe-bretagne.fr/blog-gri-recherche/wp-content/uploads/marene_main.pdf

in a tangible environment (Figure 2). The teacher has also scenarios suggesting ways to implement this situation and articulate tangible and digital (software) resources.

Figure 1: manipulatives, matchboxes

Figure 2: software

Theoretical frame work and research questions

We retain the theoretical framework of the documentational approach (Gueudet & Trouche, 2009) associated to the concept of orchestration (Trouche, 2009; Drijvers, 2012). We consider that the teacher interacts in his teaching with a set of resources. The teacher seeks out resources, chooses them, modifies them, associates them. He implements them in class, modifies them again. We call this activity the documentation work. In the documentational approach, we consider a subject, the teacher, engaged in a finalized activity the aim of which may be, for example, "teaching numbers for 4- 5 years old pupils". During this activity, we consider that the teacher develops a document, composed of recombined, modified resources, and a scheme of use of these resources. First this scheme comprises the aim of the activity; the rules of actions which are the ways of proceeding of the teacher (the stable elements of these ways of proceeding). Secondly this scheme is composed of and the operationals invariants which we consider as professional knowledge for teacher. We also refer to the concept of orchestration. Instrumental orchestration is defined as a systematic arrangement of available artefacts in a given environment, for the implementation of a given activity (Trouche, 2009). At the secondary school level, Drijvers (2012) identified different types of orchestrations. In the context of kindergarten, Carlsen, Erfjord, Hundeland and Monaghan (2016), Gueudet, Poisard and Bueno-Ravel (2014) have completed Drijvers's typology with new types of orchestrations concerning primary school. Our work led us to identify yet other types of orchestrations at kindergarten. We consider orchestrations as an emerging part of the scheme. We observe new orchestrations as elements to consider teacher's professional development and teacher's knowledge. We explore here the following question: how teacher adapt resource to specific learning goal, teaching numbers at kindergarten? Which orchestrations are implemented by teacher for teaching numbers?

Methodology

In this paper we focus on the case of Mia, an experienced teacher and MARENE's group member. We followed her documentation work during two years (year 1 and year 2), while she taught number with 4-5-year olds pupils using the set of resources Cars and Garages (CG). We conducted an initial interview with Mia about her resources for teaching mathematics and her documentational work. In year 1 and 2, Mia implemented two sequences from the CG resource designed in the group. We filmed

sessions from these two sequences. The sessions are preceded by exchanges pre-sessions and followed by exchanges post-session. At the end of the implementation of each sequence, we organized an interview. Professional writings (session preparation sheets, etc.) were collected over the two years.

Results

We focus on one moment in teacher activity and learning situation. We study the case of the implementation of a phase of discussion about students' procedures. This phase takes place in year one after several working sessions with the software in year 1. In year 2, this phase takes place after several working sessions with manipulatives (here matchboxes) and the software. We identified during two years the same orchestration associated to a moment of discussion about procedures. Mia displayed a system of labels containing the visuals of the software. Pupils were invited to play the game of garages and cars, not on the software but on the board, instruct each pupil to make a try in front of the group and propose his solution to win at this game, as well as shown for example in Figures 3 and 4.

Figure 1: System of labels, year 1

Figure 2: System of labels, year 2

This material was not originally present in the resources designed in MARENE; it was created by Mia during the sequence in year 1. It is linked to a type of resource we can often find in kindergarten classes, and that Mia often uses in other subjects, (labels on the classrom's board). Here the system of labels "CV" is used in what we call "a tangible and software duo" orchestration, defined as follows: it involves manipulatives displayed on the board and operated collectively in connection with pupils' experience with the software. The use of this resource was observed over two years, associated with the conduct of discussions on procedures moments. The "manipulatives and software duo" orchestration is new compared to the typologies developed in other research. We consider that the development of this new orchestration is linked to the development of a scheme, and in particular of a new operational invariant, developed by Mia along the work with the software. This new orchestration is linked to Mia's emphasis on verbalization in mathematics and the idea that peer-to-peer exchanges promote learning, especially in kindergarten. This point is illustrated by the fact that Mia is developing this new resource: it is important for students to be able to collectively see the problem and discuss it (which is more difficult when working individually or in pairs on a single computer workstation). In year 2, Mia insists on comparing the system of labels with the students' experience with the software. To reinforce this link, the system of labels is modified by Mia. She reconstructs as accurately as possible the interface of the software to the board (adding a gray strip). We perceive in the design of this resource by Mia the influence of the software, which is organized in three distinct zones (the garages' zone, the cars' zone and the zone of the cars to be parked in the

garages by a gray paper strip). This transformation is justified by Mia during the balance sheet interview:

Mia: I wanted to get closer to the software... I wanted more ... to make situations identical.

We note a professional knowledge identified in year 1, which is reinforced by the implementation carried out in year 2: pupils may have difficulty recognizing the same situation when the material varies, when the context changes.

Conclusion

In this text we studied following questions: how teachers adapt resources to specific learning goals, teaching numbers at kindergarten? Which orchestrations are implemented by teachers for teaching numbers? Regarding these aspects, we identified a new orchestration compared to the typologies defined in previous research: “the manipulatives and software duo” orchestration”. This orchestration and its implementation are linked in the case of Mia to professional knowledge related to importance of verbalization and peer exchanges. Mia does not have a video projector. To allow students to expose the procedures they have experimented mainly in Year 1 on the software and discuss them with each other, it is necessary for Mia to create a new resource, the label system. She implements here a new orchestration: “a tangible and software duo" orchestration. We see a link between one of Drijvers’ orchestrations: “link screen board”, but in the case of “tangible and software duo”, software’s screen isn’t really there on the board. In addition, knowledge about the way connecting pupils’ experience on tangible and digital’s environment has emerged.

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From digital "bricolage" to the start of collective work – What influences do secondary teachers non-formal digital practices have on their documentation work?

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Our communication aims at studying teachers' non-formal digital practices influences on their documentation work, especially documentational genesis, either through collective work around digital resources or within digital common spaces.

Keywords: Documentational genesis, non-formal digital pratices, collective work.

The present educational context in France is characterized by a series of injunctions conveyed by official texts, projects or artefacts urging teachers to share their personal resources and to integrate collaborative practices and digital resources within their ordinary schemes and resource systems: the Digital Plan for Education ("Plan numérique pour l'éducation"), launched in may 2015, as well as a new curriculum for secondary education (applied since september 2016), or institutional platforms designed to foster collaborative work (e.g the Banque de Ressources Numériques pour l'École, BRNE, Canoprof or the social media Viaeduc). This institutional context led us to wonder if teachers' non-formal digital practices and their use (consultation, participation) of non-formal digital « common » spaces could have an effect on their documentation work, especially on documentational geneses.

The theoretical framework we used for our communication is based on the definition Gueudet and Trouche (2008, p.1) give of teachers' documentation work: "[…] to collect resources, select them, transform them, recompose them, put them into practice, share them, review them… Documentation encapsulates both this work and its results; it lies at the core of teachers' professional activity[1]". In the same article, they insist upon the social dimensions of documentary work, which according to them is performed by teachers not only for themselves but also for their students, through a net of "professional liabilities[2]" (p.17), while meeting other teachers in different contexts, either constrained or chosen. We would like to investigate further this social dimension by trying to understand how teachers use non-institutional digital spaces or apps in order to enrich their documentation work, and what are their goals in doing so.

We also use the adjective "non-formal" referring to Anne Cordier's work on teenagers

[1] [Our translation] "[…] collecter des ressources, les sélectionner, les transformer, les recomposer, les mettre en œuvre, les partager, les réviser… La documentation désigne simultanément ce travail et ce qu'il produit ; elle est au cœur de l'activité professionnelle des enseignants.".

[2] [Our translation] "assujetissements professionnels".

informational practices (2015), where she describes such practices as: "[…] practical purposes [which] are achieved through interactions with the environment and others. Many conscious or unconscious acquisitions linked to non-formal learnings are built on observing, 'impregnating' oneself from, self-training or imitating either relatives or society" (p. 40). We believe that this concept encapsulates a wide range of teachers' digital practices that are part of teachers' professional routines whereas remaining understated either by institutional agents or industrial designers.

Therefore, we will focus our analysis on digital "making do" (*manières de faire*) (de Certeau, 1990), occurring at different moments of teachers' work. Indeed, we formulate the hypothesis that sharing and discussing digital resources whether with students, peers or academical agents in non-formal spaces, is a crucial, dynamic element of documentational genesis.

This proposal echoes our ongoing PhD research, during which we used a "comprehensive methodology" (Paillé & Mucchielli, 2016, p. 42) in order to perform a "qualitative analysis" (Paillé & Mucchielli, 2016) of the collected data. We led 35 recorded "comprehensive" (Kaufmann, 2016) interviews with teachers regarding their documentation work: 4 from the primary level and 31 from the secondary level, teaching different subject matters (biology, litterature, technology, physics, mathematics, history, italian, physical education, arts, german and english). The range of their professional experience was quite diverse, from 2 to 20 years. We transcribed each interview (lasting between 45 minutes and one hour and a half) and analysed them owing to an "analytic questioning strategy" (Paillé & Mucchielli, 2016.) based on the building of an "investigative canvas" (Paillé & Mucchielli, 2016, p 214), which will be used to answer progressively to the different sets of questions drawn from data itself.

Main results

The analysis of the interviews we led showed that "bricolage" (referring to Claude-Lévi Strauss' work *The Savage Mind*), which could be defined as "the art of using what is at hand" (Plantard, 2016), was one of the most common teachers digital practices, and also one of the most disregarded. Each teacher we interviewed admitted they consulted, inspired from, copied, printed or used colleagues resources available on the internet, through personal spaces like blogs, websites, social media like Facebook pages or subjet-matter dedicated groups, most of the time "whithout saying thank you". This pedagogical monitoring activity seemed to trigger a documentational genesis, starting with a selection of the initial resources owing to a long time follow-up of the colleagues work, even if there was no communication between the teachers.

Nevertheless, as the teachers we interviewed understated it themselves, "sharing" resources on the internet is an ambiguous practice. The following "recurrent contradiction" (Kaufmann, 2016, p. 96) was often expressed during interviews: "To see everything while showing nothing", and the feeling that this was the most common behaviour among teachers could repel some of them. Considering these contradictions, we tried to understand if "bricolage" could evolve and give birth to processes of collective design within digital spaces, especially those considered non institutional, and under which conditions.

Gueudet, Pepin and Trouche (2013) reflected upon conditions which were necessary for collective work to take place and listed the following criteria: a common working room, 'official' working hours, and possibly the intervention of institutions linked to school. We believe that common digital spaces can play the same part as common rooms to initiate collective work processes, under specific conditions.

Indeed, the analysis we carried out showed that networks and platforms that aren't originally dedicated to education or linked to school institutions could offer favourable settings for collective work, for instance small groups of teachers connected through apps (Google Drive, Dropbox, Whatsapp) e-mail correspondence or social media (Facebook groups or pages which are not institutional but linked to subject matters or groups dealing with teachers professional identity and experiences.) These groups appeared to be either defined by precise circumstances (teachers who met during their "internship year" during their teacher education and wanted to stay in touch), or made by teachers who already knew each other personally or professionally or who already met or built an online relation because they shared affinities or a similar status.

We also found out that teachers valued these non-formal groups because they had the feeling they could express themselves or ask questions about didactical practices or resources without fear of judgment or assessment. We could therefore say that expression of shared trust and goodwill are two essential criteria to reach the first step of collective work: not only getting teachers to upload their resources but also getting them to express their own "voice[3]" (Remillard, 2010, p 206) about their resources. Other connections between participants of these non-formal groups were their desire to develop their resource system alongside their didactical practices in order to avoid routines and to adapt to their students; a feeling of loneliness regarding these interrogations or due to their interest in digital resources among their school team.

Interestingly, we found similarities between the dynamic process described by Gueudet and Trouche (2010) [our translation] as documentational genesis: "teachers documentation work triggers documentational genesis, which develops both a new resource (composed of a set of selected, modified, recombined resources) and a utilization scheme for this resource […] (p 59.)" and teachers expectations regarding what true digital collective work should be on dedicated platforms. According to the teachers we interrogated, they wished to nourish their documentational work by uploading their resources into common digital spaces, in order to gain reflexive insights on their resources, or to solve '"shared professional difficulty" (which required a shared approach for solving it)" (Pepin, Gueudet, Trouche, 2013).

Finally, we would like to show that teachers' collective work is also shaped by and for students. The interviews we led even showed that professional cultures associated with certain subject-matters, especially scientific ones, led teachers to co-design their resources not only with peers but also with students. For instance, a biology teacher who had been teaching for 23 years when we met

[3] Defined by Janine Remillard as "the way in which the authors/designers' words are represented, and their ways of adressing the teacher. In most of curricular resources that [the author] examined, authors are invisible and we're given little information about themselves or their own experience." [our translation]

him explained to us that he shared a Google Drive with his colleagues, which served as a collective experiment collector used to improve inquiry-based activities with students.

To conclude, we would like to insist upon an element which should be considered as a part of collective work: several teachers underlined that the fact that their students could express feedbacks on their resources (digital or not) and their teaching owing to social media or common digital spaces (on platforms such as WebPedago.com, websites or blogs with "guestbook" or personal Facebook pages) offered them a precious symbolical recognition that they didn't receive from school institution.

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Exploring teachers' design processes with different curriculum programs

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In this paper I explore how teachers perceive and use curriculum resources. Teachers were interviewed planning lessons from two curriculum programs that differed according to their primary communicative function (monologic or dialogic). Using a framework that conceptualizes teaching as a design process in which there is a participatory relationship between users and materials, I analyzed teachers' processes according to their curriculum practices, their descriptions of mathematics in the materials, and how they viewed the pedagogical affordances of the materials. I found that teachers who used innovative programs were habituated to the epistemological stance in the materials, while teachers using conventional programs were agnostic with respect to the epistemological stance. I draw implications for the design of digital curriculum materials.

Keywords: Curriculum design, teaching experience, instructional design

Recent research has theorized curriculum materials as tools that mediate teachers' instructional design processes (Brown, 2009; Remillard, 2005). While the literature has amply demonstrated the impact of teacher characteristics on the ways teachers recognize and take up resources in curriculum materials (Lloyd, 1999; Remillard & Bryans, 2004), there is considerably less research that focuses on how the characteristics of curriculum materials influence the ways teachers perceive and use those materials. Given the emerging reliance on digital curricula, and teachers' role in selecting from a plethora of materials, it is imperative to better understand how teachers recognize and take up resources in curriculum materials.

Theoretical framework

Curriculum materials contain messages about the nature of mathematics and the learning of mathematics and provide structure for how mathematics and mathematical activity is presented to students. Brown (2009) refers to the *teacher-tool* relationship, where the use of a particular curriculum (tool) constrains and extends how teachers design and implement lessons (Wertsch, 1998). Remillard (2005) terms this relationship *participation with the text* in which "teachers and curriculum materials are engaged in a dynamic interrelationship that involves participation on the parts of both the teacher and the text" (p. 221). Thus, while curriculum materials do not determine what teachers do, they provide a structure in which teachers operate and thus influence their instructional decisions. As such, we think of features of curriculum materials in terms of messages and structures that influence the kinds of representations of mathematics and pedagogical activities ultimately enacted with students.

Messages in and structure of the designated curriculum

I conceptualize materials in terms of their primary communicative functions. Wertsch and Toma (1995) discuss the dual functions of text as monologic and dialogic. The monologic function follows the delivery or conduit metaphor (Reddy, 1979), in which the main functions of text are encoding, transmission, and decoding. The enactment of texts that emphasize monologic function are often conceived of in terms of fidelity, in that it is assumed that the sender's intent is unambiguous and that misunderstandings result from inaccurate decoding of the sender's intended content. Wertsch and Toma state that "communication models based on the unidirectional

transmission of messages cannot be amended in any simple way to deal with the issue of texts as thinking devices" (p. 166), suggesting that a priori decisions made by authors influence which of these functions is most prominent in texts.

Lotman describes the dialogic function of text in terms of text as *thinking device* in which the "main structural attribute of a [dialogic] text is its internal heterogeneity" (Lotman, p. 37, as cited in Wertsch and Toma, 1995) and second is that of a *generator of meaning*. Internal heterogeneity refers to the extent to which different approaches or interpretations are afforded. These different approaches or interpretations are generators of meaning when their differences and underlying similarities are explicitly discussed, in the process generating new understanding or meaning in that community. The dialogic function of curriculum materials can thus be viewed as promoting *interactions that generate understanding*. Thus, curriculum can be thought of as text that primarily serves as a conduit to transmit information or as a thinking device, with curricula encompassing both functions, though one is typically dominant (i.e., the primary communicative function is monologic or dialogic).

In terms of the structure of the curriculum, we refer to the ways mathematical activity is organized in the student text, including: the nature, sequencing, and organization of tasks; the timing of formalization of mathematical language and procedures; and the delegation of autonomy (cf. Munter, Stein & Smith, 2015; Stein, Remillard & Smith, 2007). The structure is connected to the epistemological stance conveyed by the message; together, structure and message work in tandem. Particular structural features, such as the early formalization of language and the modeling of specific algorithms before students encounter problems, are associated with a monologic approach: the intent is to deliver information to students, whose role is to master the terminology and procedures provided to them. Other structural features, such as problems that involve multiple approaches or that are designed to help students make key mathematical connections before they are explained to them, are associated with a dialogic approach: the intent is to provoke student reasoning and to get students to interact with each other around mathematical ideas.

Research Questions

I posed the following research questions:

1. How do teachers perceive and mobilize curriculum resources (e.g., messages and structure) from curriculum programs that have substantively different characteristics?
2. How are teachers' interpretations of features in curriculum materials evident in their lesson planning?
3. How are teachers' interpretations of pedagogical resources in curriculum materials related to the materials and their experience with the materials?

Methods

In the larger research project, we conducted 50 interviews with teachers as they planned lessons with their regular curriculum materials. We also asked these same teachers to plan lessons with curriculum materials that we classified as being substantively differences in terms of the messages and structure compared to their regular materials (e.g., aligned with monologic or dialogic communicative functions).

There were two types of interviews, one of which involved curriculum based on a monologic communicative function and the other involved a curriculum based on a dialogic communicative function. The first type of interview involved the teachers planning with their regular materials and the second type, which we termed Staged Lesson Planning, involved different materials. The Staged Lesson Planning interviews were designed to capture teachers' thinking and intentions when

working with curriculum materials in planning. We provided a set of published curriculum materials (e.g., student textbook, teacher's edition, and assessment materials) and asked them to plan a hypothetical lesson. In the interviews, we asked teachers to describe their goals, the sequencing of activities in the lessons, the rationale behind that sequencing, and how the activities are intended to help students learn the specified topic. In addition, we asked what curriculum resources they used to plan the lesson and why they used those resources.

The analysis involved a multi-stage process in which the data were divided into the following categories: (1) Curriculum practices (what resources teachers read and for what purposes); (2) Description of mathematics content (what was the primary mathematical focus of the lesson); and (3) Description of pedagogical affordances (how do the materials – and the activities in the materials – help students develop an understanding of mathematics). See Table 1 for a sample of these analytic categories.

Curriculum Practices	Mathematics	Pedagogical affordances
The teacher explained that the TE provided few insights beyond what she gained by doing the problems. She found no guiding questions, and there was little support for helping explain how to help students make connections, especially struggling students.	Teacher described how visual models were used to demonstrate scaling to determine equivalence of fractions and percents	Teacher stated that the problems were narrow and scaffolded and that the materials forced students to make certain connections. Teacher explained that the problems introduced new representations or concepts without providing adequate explanations for how those new representations or concepts were connected to what preceded them in the student text.

Table 1 Example of Sample Analysis within each Data Category

A second analytic activity involves constructing a lesson plan from the interview in which we re-created the lesson, listing the steps and justifications for those steps for both lessons.

Results

The results indicated that the variation with which teachers responded to messages and structures in curriculum programs was related to their prior curriculum experiences. Most notably, the teachers that were highly experienced with the two most innovative programs – and the two programs that varied most from conventional US programs – tended to exhibit practices that aligned with the epistemological resources of those programs. That is, the use of these programs had habituated them to seeking a connected set of student experiences in the student materials and guidance in the teacher resources in terms of organizing lessons and anticipating student responses (beyond just errors). When these teachers used conventional programs in the Staged Lesson Planning interviews, they struggled with the lack of richness and connectedness of the tasks in the student materials and with the lack of guidance in the teacher resources with respect to student exploration and anticipating student thinking.

By contrast, teachers who were experienced with conventional programs tended to use curriculum materials (either type) mainly as sources of problems to pose to students (were agnostic with respect to the epistemological stance of the resources), and remarked on the lack of examples and explicit procedures in the two innovative programs. When they looked at the teacher resources of the innovative materials, they typically stated that they wished such detailed information about student thinking were present in their usual (conventional) materials.

The design processes of a third group of teachers who had experience in both types of programs were contingent; that is, they conformed their design processes to the perceived intent of the

designers. The designs of lessons for these teachers were dependent on the type of materials being used.

Implications

As teachers exercise greater roles in curriculum design processes as a result of the increasing use of digital resources, it is important to understand how prior curriculum experiences influence how they take up new materials, especially materials that have different epistemological characteristics and different structures than their current materials. Conversely, it is important to consider how the design of curriculum materials can influence the ways teachers design lessons. This potential to influence use was most evident in the contingent group but also evident in the group who had deep experience with innovative materials. Understanding how design can influence teachers' design processes will be valuable for curriculum designers whose goal is to challenge teachers to enact rich forms of instruction.

Acknowledgment

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Digital resources: origami folding instructions as lever to mobilize geometric concepts to solve problems

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Engage future teachers of basic education in tasks where geometric knowledge is mobilized to create digital resources could be a challenge. Using origami as lever, we defined two tasks in a qualitative study to understand how manipulative and virtual artefacts promotes the representation of geometric concepts and cognitive processes. Results suggest a development in the students' cognitive processes (visualization, construction and reasoning) when performing this type of tasks.

Keywords: digital resources, geometric concepts, artefacts, origami, dynamic geometry software.

Introduction

Critical thinking, problem solving, creativity, persistency and curiosity are examples of a broad range of so called XXI century skills (WEF, 2015). Preparing future teachers demands they must develop not only the skills required to assess and select digital resources but the expertise needed to be producers, to challenge their creativity, to promote their abilities to communicate and to solve and pose problems. In this sense, dynamic geometry environments play an important role when teaching and learning geometry, allowing the creation of learning environments that favour the development of these competences. On other hand origami can provide a rich environment for the posing problems. The contribution of these artefacts, manipulative and virtual may be at a practical but also on a cognitive level, challenging us to perceive how they can interact with the mind and how we can take advantage of them in the design of tasks in an educational context.

The main purpose of this study is understanding how the creation of synergies between manipulative (origami) and virtual (software) artefacts promote the representation of geometric concepts when solving problems, and how they intertwine the processes of visualization, construction, and reasoning.

Artefacts

The construction and use of artefacts is intrinsically linked to human activity. According to Norman (1993), cognitive artefacts are tools of thought, cognitive tools "that complement abilities and strengthen mental powers" (p. 1), but the interaction with the mind and the results produced depend on how they are used. The nature of artefacts can be experiential or reflective. The "experiential artifacts provide ways to experience and act upon the world, whereas reflective artifacts provide ways to modify and act upon representations" (Norman, 1993, p. 5). This leads us to the idea that "any representation is supported by an artifact" (Bussi & Mariotti, 2008, p. 747) focusing on the importance of artefacts to promote the representations and consequently the powers of cognition, laying the foundations where new abstractions can emerge.

Teaching and learning Geometry

According to Duval, the learning of geometry involves three types of cognitive processes with specific epistemological functions (Jones & Bills, 1998): a) *visualization processes*: related to the

visual representation of a geometrical statement; b) *construction processes*: the use of artefacts like dynamic geometry software, ruler, compass, etc., to construct figures that can be used as a model; and c) *reasoning processes*: discursive processes that arises in the context of a formal proof, in the deepening of knowledge or in the explanation of a certain content. According to Jones and Bills (1998), Duval points out that the development of these processes can be done independently. However, he refers that "these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry" (p. 124).

In order to obtain this proficiency in geometry, and at the same time to engage students in tasks where geometric knowledge is mobilized for a situation that will be part of his reality as future teacher, we intend to design tasks that promote the creation of digital resources mediated through two kinds of artefacts: manipulative (origami) and digital (software). The challenge is to understand how the creation of synergies between origami and dynamic geometry environments promote the intertwine processes of visualization, construction, and reasoning in this type of task.

Method

The adopted methodology is qualitative. This study took place on the curricular unit Technologies in Mathematical Education of the course of Basic Education and had the participation of 33 students. The students studied two dynamic geometry software, namely Geometer Sketchpad (GSP) and Geogebra (studied in this order). Teacher and students made simultaneous constructions, generally involving the formulation of conjectures, allowing the demonstration of several software tools.

To understand if the students were able to use the previously explored software (GSP) and were simultaneously capable of understanding which concepts were being mobilized by these to solve geometric problems, we selected two simple but challenging origami constructions.

In the first task, the students had to reproduce the educational resource presented in Figure 1. In the second task, we provided the folding instructions of the several steps required to make a cup, as shown on Figure 2, in order for them to create an educational resource. In this task, we wanted to assess the student's autonomy when creating resources.

Data was collected through**:** classroom observation, digital resources submitted by the students**,** video recording and photos**.** The analysis focused on the following aspects: mobilized geometrical concepts; strategies and autonomy.

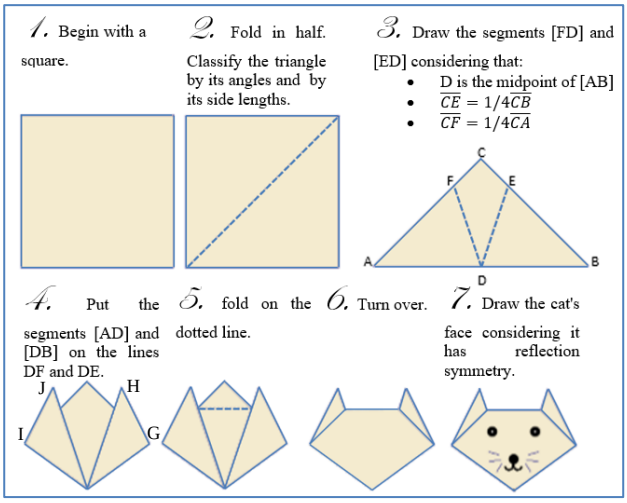

Figure 1: Origami folding instructions for a cat.

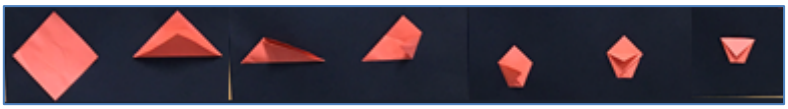

Figure 2: Origami cup.

Results

The students demonstrated a progressive domain over the software. The main difficulties arose when using untested tools or with those which are not very intuitive (for example, the selection of the rotation centre or the axis of symmetry which are achieved by double clicking).

In task 1, the students autonomously constructed the figures shown on steps 1 and 2 without difficulties (Figure 1). Using *visualization processes* and *construction processes*, the *representation* was made using the virtual artefact (GSP) and the information given in the task. The figures presented on steps 3 and 4 were more challenging. To reproduce the figure shown on step 3, most students copied one of the triangles formed on step 2 and request assistance to place it in the same position (*visualization processes)*. Several had trouble selecting the rotation centre and struggled to determine the correct angle at which to rotate and if the rotation sense was clockwise or counter clockwise. When questioned about the chosen angle amplitude, many students were unable to respond with certainty until the teacher aided with more directed questioning.

The next step was to determine points E, F and D. The students easily determined point D but requested assistance to mark points E and F. They realized they had to find the midpoint in the [AC] segment and in the [BC] segment (the concepts were correctly mobilized to draw the points - *visualization processes*) but needed assistance to overcome the technical difficulties (they had to draw the segments first in order to be able to mark the midpoint) in order to make a representation.

The construction of the figure shown on step 4 was the most difficult for the students. This was the first time all students needed to build the origami (manipulative artefact). They started from the triangle drawn on step 3, and resorted to folding to aid them determine points G, H, I and J, as shown on Figure 1. Although many students assumed points I and G were midpoints for segments [CB] and [CA], there were some who autonomously concluded otherwise, stating that "we need to [find] points I and G which are not midpoints" and when questioned if they had tested that statement they responded "of course. We folded it here and the midpoint is way behind" (they did not rely on the *visualization processes*, proceeding to artefact manipulation). All students requested the teacher's assistance on determining these points. When she asked them to analyse the relation between angles HDG and GDB, they immediately concluded they were congruent but only a few answered correctly on the geometric location of the points which are at the same distance on the sides of the angle. To determine points H and J, the teacher questioned the class for them to conclude that $\overline{DB} = \overline{DE}$. Some students immediately suggested building a circumference with centre on D going through B, but many needed further questioning to reach that conclusion. Finally, they concluded they needed to intersect the circumference with line DE. All remaining figures were executed autonomously.

Task 2 was not executed by all students. Those who had time revealed more autonomy and dexterity in the mobilization of geometric concepts when defining a construction strategy. The manipulative artefact (origami) promoted *reasoning* based on *visualization or constructions processes.*

Student 1:	We did the bisector to know this (point marked on Figure 4A). We will find the other point doing the same thing on the other side. What we see is that this point (Figure 4B) is half of this (folds the paper to prove the statement as shown on Figure 4C).
Student 2:	And we feel this (dashed segment on Figure 4 D) is perpendicular [to the bisector].
Teacher:	Why is it perpendicular? How does the folding guaranties it really is a perpendicular?
Student 1:	You can see it by folding it this way (Figure 4 E).

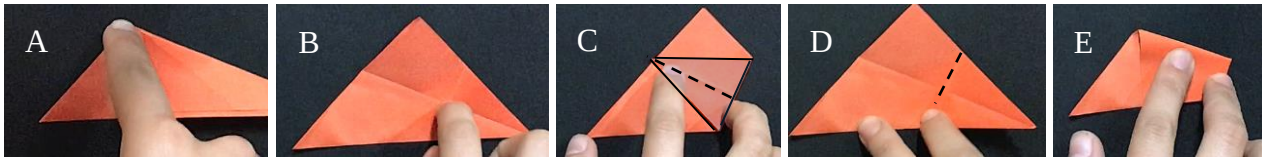

Figure 3: Explaining the argument

Besides the strategy shown on the transcription above, resulting from mobilizing concepts such as bisector, midpoint and perpendicular, other students used different strategies like drawing the bisector, tracing a line parallel to the base through the point found on Figure 4 A and, after concluding the triangle shown on Figure 4 C was isosceles, determined the unknown point through drawing a circumference.

The students stated the challenge was hard because it demanded already forgotten concepts and highlighted steps 3 and 4 of Task 1 (Figure 1) were the hardest.

Conclusions

This study allowed for future teachers to create digital resources through interaction of manipulative (origami) and virtual (software) artefacts, promoting the representation of geometric concepts, while developing three types of cognitive processes (visualization, construction and reasoning). During the fulfilment of the first task, the students showed greater difficulties when constructing the figures shown on steps 3 and 4 (Figure 1). In this case, the concepts mobilized by most students to define a strategy came from questioning the teacher, although some could not properly name them. Some students relied on visualization (visualization processes) and formulated incorrect conjectures which were proven wrong when they started folding (construction processes). Regarding the software, the difficulties were related to using less intuitive or unknown tools. When solving the second task, some students developed some reasoning processes based on *visualization or constructions processes*, showing greater trust and autonomy when making a digital resource.

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Orchestrating the use of student-produced videos in mathematics teaching

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This study aims to identify and characterise different orchestrations used by a teacher in a mathematical discussion with regards to student-produced videos. Open coding was used to propose a framework adopted from Brown's (2009) degrees of artifact appropriation. The findings show that the teacher orchestrated the use of videos in distinctly different ways, capitalising on the affordances and working around the constraints of the medium.

Keywords: Resource orchestration, student-produced videos, inquiry-based learning.

Introduction and context

Digital Interactive Mathematics is a joint venture project of teachers and researchers developing innovative ways of using technology in inquiry-based mathematics teaching at lower secondary school in Norway. In the project, students have been using the videos to present solutions and strategies to inquiry-based combinatorics tasks. Videos produced by students have been investigated by, among others, Kearney and Schuck (2004, 2005, 2006) with emphasis on the students' production process. I intend to investigate student-produced videos from the teacher's perspective.

The student-produced videos were used as a resource for planning a full class discussion about the strategies and solutions provided. The videos were produced at the behest of the teacher regarding combinatorics tasks towards the end of a two-hour group work and lecture session. Student-produced videos had never been used by the teacher before this lesson, and it is therefore a new resource for the teacher. The area of research for this oral presentation is: "How does the teacher orchestrate the use the student-produced videos in a mathematics discussion?".

Using the documentational approach (Gueudet & Trouche, 2009) and Brown's (2009) degrees of artefact appropriation, I have analysed the teacher's use of videos in a full class discussion. These orchestrations were in some way or another based on presentations in the student-produced videos. The concept of orchestration is to be understood as the interaction between the teacher and the presentation medium (i.e. the student-produced video itself, the presenting group or a single student).

Recognising how the teacher incorporates the resource in different parts of the lesson, I classify the orchestrations within a one-dimensional model adopted from Brown (2009) (see Figure 1). As the model itself does not incorporate the "why"-question, I further develop the orchestrations according to the teacher's apparent schemes of utilisation.

Methodology and scheme for analysis

The method for data collection was observation, I collected video and audio recordings of a 45-minute full class discussion. Also, I collected the teacher's notes for the discussion and the student-produced videos used for both planning and use in the full class discussion.

I aimed at identifying key aspects of the teacher's orchestration and construct a framework for categorising, describing and recognising the various aspects of the teacher's orchestrations. I utilised data reduction and open coding to frame a rough sketch of the categories. After recognising key aspects of the categories, I refined the framework applied in this paper.

My scheme for analysis was based on Brown's (2009) degrees of appropriation; i.e. offloading (use as is), adapting the resource, and improvising (disregard the resource and enact without specific guidance from the presentation).

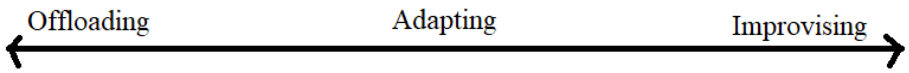

Figure 1: Brown's (2009) degrees of appropriation as a one-dimensional model

While coding the observations, I first looked at the interaction between the teacher and the presentation (i.e. the group itself, or the video), and developed the categories presented below. Field notes were used to substantiate the analysis of the observations. The interactions helped me explore the apparent schemes of utilisation at work within a given orchestration. This is not to say that the schemes are identical within a given orchestration, but some general ideas have been observed within a given category.

Findings

The teacher applied what appeared as a quite fixed framework for every mathematical problem presented in the discussion. First with a presentation of the problem, following up with an elaboration through a back and forth discussion, and ending with a conclusion and connecting the current problem with the next. This fixed framework was evident throughout the lesson; a new problem was never presented without a conclusion to the former. Within this framework, a number of orchestrations related to the student-produced videos emerged.

Offloading orchestrations

The most notable examples of offloading were when the teacher used the videos as an introduction to a problem, or as the conclusion to the problem. Videos as an introduction were more prominent in the discussion, often more than one video for each problem. As the students had worked on the same inquiry-based tasks, there were many different views on the solutions. The teacher then used the videos as a presentation of the problem and possible solutions and follows up with questions to elaborate and lead the discussion into the conclusion. This can be regarded as offloading because the teacher used the videos as they were. He neither stopped the videos and elaborated, nor asked the students questions about the presentation. He used the information from the videos to segue into a full class discussion.

A student-produced video segment was used to conclude only once during the discussion. In this section, three students were conveying the idea of the multiplication principle followed by a video explaining the same principle. The teacher did not add to the video's statements but stated that he thought the video explained the idea thoroughly. The reason for not using more videos as conclusion seemed to be that the problems elaborated in the discussion generally arose from the videos. As the conclusion of each problem was connected to an instructional goal, it would be fair

to assume that the videos did not explicitly state a conclusion to the issues at hand in the full class discussion.

Adaptation orchestrations

The teacher chose to adapt most of the student-produced videos in some way or another. Within this category, there were two main orchestrations; video with commentary and student presentation with commentary. In contrast to offloading (use as is), the adaptation of the videos with commentary was an interaction between the teacher and the combination of the video and the producers of said video. The teacher would start the video, pause it, and direct a question to the presenters in the video. This type of orchestration appeared to be used to clarify or to elaborate statements made by the students.

The teacher seemed to use student presentation with commentary when the video suffered from more issues than what the teacher could clarify with questions, this being a lack of "showing the mathematics" in addition to statements the teacher wanted to clarify. The need for a more thorough investigation led the students to redo the presentation with teacher support, mostly in the form of asking questions regarding their statements in the live presentation. Although the video itself was not used in this orchestration, the teacher still adapted the students' video presentation, knowing what the students would say having watched the video in preparation for the discussion.

Hybrid orchestrations

A hybrid form of offloading-adapting observed was student presentations without commentary. The teacher asked a group of students to present something from their video. While the presenters used the presentation from the video to describe the problem at hand, the teacher requested them to reproduce it without the video. This form of adaptation was mostly used when the video did not show the mathematics the students were presenting. The teacher asked them to "redo" the presentation from the video live with more resources for instructions. With this orchestration, the teacher adapted the material before the presentation begins, and it is therefore not a clear-cut adaptation. Both student presentation without comments and offloading orchestrations were mostly used in the presentation part. In my analysis (see Figure 2), I have placed this orchestration between offloading and adaptation, as the emphasis is on the students' presentation without teacher interference.

The other form of hybrid orchestration observed was adapting-improvising. While the offloading-adapting orchestration was used in the presentation of a problem, the adapting-improvising orchestration was used to elaborate the problem. As the teacher wanted the students to elaborate the issue themselves, and not provide answers, he asked specific questions to specific students. As one student's ideas might contrast with another, this prompted comments and questions from the other students. In most cases, the teacher's notes stated the name of the student to be challenged. Whether the teacher selected based on statements in the videos or his's knowledge of the student's mathematical competence is hard to determine with the data collected.

Improvising orchestration

While the categories above prompted most of the discussion, the teacher did present and prompt some discussions without using videos or its presentation in the orchestration (as offloading or

adapting). However, the issues presented were still directly connected to the resource. There is an important distinction to be made between a resource for planning, and a resource for instruction. The improvising orchestrations were all prompted by the presentations in the videos, even though they were not used to present or elaborate the questions. It appears that if the teacher found flawed statements in the videos, he then used this as a prompt for discussion. In some cases, this orchestration led into the hybrid of adapting-improvising, as the teacher asked specific students to present their thoughts on the issue.

Below (Figure 2) I have proposed a model for the orchestrations within the degrees adopted from Brown (2009).

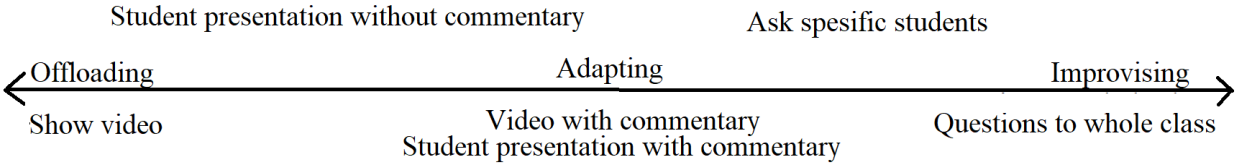

Figure 2: The different orchestrations within the one-dimensional model (Brown, 2009)

Final remarks

This paper has presented a variety of orchestrations identified in a lesson using student-produced videos in teaching combinatorics. The different use of the resource implies perceived affordances the teacher identified while planning the lesson, culminating in schemes of utilisation for the set of resources used. To fully understand the process of the documentation genesis in the teacher's work, a long-term study is needed. As the resource was new to the teacher, the orchestrations reported in this paper is without reference to earlier experiences. The document is dynamic, and as the resource is becoming more familiar to the teacher, its affordances for mathematics teaching will be further understood.

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Instructors' decision making when designing resources: the case of online assessments

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This article will show the progress of an ongoing thesis project that analyzes the impact of instructors' participation in designing an online assessment system; particularly, we analyze the assignments on affine and quadratic functions for a math levelling course at a Chilean university. In addition, we will show the progress regarding the impact on the resources that were created, analyze epistemic value using the Mathematical Working Space (MWS) and how closely related these are to a suitable MWS for instructors. In this contribution, we will only explain two of the observed phenomena: the almost exclusive use of whole numbers both in the coefficients of the functions and in the numbers of the formulation, as well as in the response that is expected from the students and some characteristics from the graphs and their use, along with the limitation in their variability, limited configuration in the Cartesian plane and the accurate reading of the graphs used for feedback

Keywords: instructor design, digital curriculum resources, mathematical working space, online assessment, and functions.

Introduction

According to multiple research, instructors are the key players in the process of integration of technological resources (Abboud-Blanchard, 2014; Drijvers, 2015; Ruthven, 2007). These studies coincide in that this integration depends on several factors such as a) their beliefs about the technology, b) the cost/benefit ratio between the time spent on planning, implementation, and actual student learning, and c) how closely related the technological resources are to the actual curriculum.

The learning benefits can be understood trough the pragmatic and epistemic value of the tasks in digital environments proposed to students (Artigue, 2002). An important part of the cost that planning and implementation entail is the time that takes to become familiarized with the resources in the instrumental dimension and when solving the proposed mathematical assignments (Abboud-Blanchard, 2014). Finally, how closely related the technological resources and the curriculum are is paramount because, as pointed out by Ruthven (2007), an instructor's lesson is a system of thought-out and automated routines adapted to specific circumstances, and the integration of technology usually implies modifying this system.

Combined with the above-mentioned factors, the integration of resources can also be complex when the instructor is seen only as an implementer of what was created by other professionals. One way of addressing this difficulty is by giving more importance to the instructor as a designer. This offers other perspectives for adopting the resources, which can help with integration and serve as professional development (Pepin, Gueudet, & Trouche, 2017). Nevertheless, this participation must be promoted and supported institutionally for it to occur.

In this contribution, we would like to show the progress of our thesis work, in which we are studying the impact that the participation of instructors as designers has on the epistemic value of the resources they create. We are using regular task that instructors assign to their students as a reference of comparison. To study this, we are using the case of a project whose objective is the development and implementation of an on-line assessment system for a mathematics levelling course for freshmen students at a university in Chile. The distinctive feature of this project is that the instructors employed at same university have been invited to participate as designers. We will specifically study the assignments designed for affine and quadratic functions.

Context, Methodology, and Theoretical Framework

This research is part of a project for the development and implementation of an online assessment system for a levelling mathematics course at a Chilean university. Since 2015, this project has steadily grown from 1000 students in its first version. It is projected to expand to 8 campuses and 13,000 students (out of a total of 26 campuses) in 2018. In the design of assignments stage, teams of 3 or 4 instructors have been put together. So far, they have created 306 assignments for 13 different topics. From those assignments, we analyzed the 29 that were related to affine and quadratic functions.

These assignments were designed on the Moodle institutional platform integrated with the mathematics software Wiris, which allows the creation of questions with random parameters of formulation and with a step-by-step feedback system based on these parameters (which are set up by the instructor through an algorithm). Students can also use a mathematics editor for their responses, which allows them to write specific mathematical objects. Additionally, a CAS (computer algebra system) is used, which can recognize mathematically equivalent responses.

The data used for the analyses are the formulations of the assignments on the platform, as well as their programming algorithms, the lessons on affine and quadratic functions of 3 out of the 6 instructor designers were recorded (in May and June of 2017). Additionally, based on the analyses of the tasks on the platform and the class recordings, we carried out interviews that were focused on the design of the assignments on the platform and the use of the latter in the classroom.

The Mathematical Working Space (Kuzniak, Tanguay, & Elia, 2016) is used to study the epistemic value of the assignments designed by instructors, to characterize the instructors' lessons, and to study how closely related the assignments are to the curriculum. Furthermore, as we are studying the design process in a digital environment, we will use the notion of instrumental genesis linked to design (Rabardel, 1995).

The aforementioned MWS is built around two planes: cognitive and epistemological. These plans, are linked through three geneses: semiotic , instrumental and discursive, and vertical planes, which correspond to processes that cannot be divided into only one genesis. It is important to note that we differentiate between the instrumental genesis linked to design in the platform from the one linked to the mathematical work done by students.

Three MWSs can be noted in an educational institution: reference, suitable, and personal. We will mainly work with the notion of a suitable MWS, which is defined as the working space that is designed by the instructors for the students' coursework.

In terms of the theoretical framework, in this article we propose the following research question: What is the potential MWS of the assignments on the platform compared to the suitable MWS of the instructor designers?

Preliminary Results

To date, we have analyzed the assignments on the platform and we have partially made progress in the characterization of a suitable MWS of the instructors whose lessons were recorded. The analyses of the assignments have shown certain phenomena that are common among instructors. Some of these phenomena are related to the use of contexts, such as the use of "artificial" contexts or odd elements in real contexts. Other phenomena are related to the types of assignments, such as focusing on specific kinds of assignments or related to mathematical objects (their representations and characteristics) that appear in the formulations and feedback. In this contribution, we will only explain the phenomena that are related, specifically, to the last aspect: the almost exclusive use of whole numbers both in the coefficients of the functions and in the numbers of the formulation, as well as in the response that is expected from the students and some characteristics from the graphs

and their use, along with the limitation in their variability, limited configuration in the Cartesian plane and the accurate reading of the graphs used for feedback.

As an example, we will examine an assignment that elicits the speed of a person whose movement is represented by an affine function that is graphically represented (see Figure 1). The assignment activates de semiotic-discursive dimension because it asks for the speed, and it establishes a relationship between that and the slope of the function. Then, to calculate the slope of the function, the student must visualize two points of the straight line, which activates de semiotic genesis; however, the digital environment imposes certain restrictions such as not being able to draw lines on the screen.

Once the points to calculate the slope have been identified and obtained, the students could follow at least two strategies: 1) use the formula of the slope of a straight line or 2) use the same graph to draw a triangle from the identification of two points to later calculate the ratio between the cathetus. In the feedback designed by the instructor, we can see how the first strategy is preferred because the accurate reading of the graphs is used for feedback (this was not included in this summary because of lack of space).

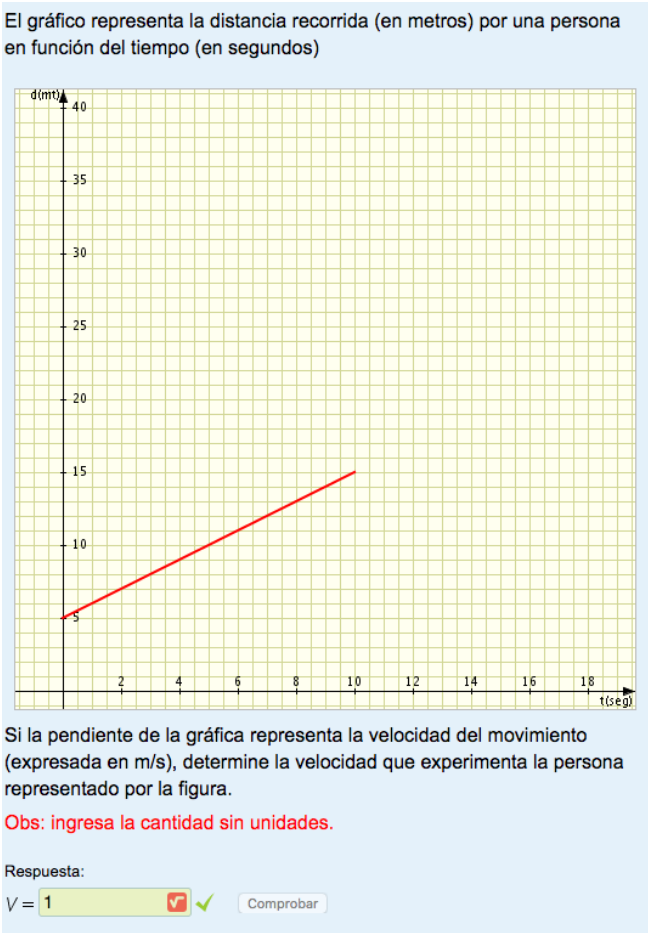

Figure 1: Example of assignment

We can observe that the slope, the images, and pre-images are whole numbers. When analyzing the algorithm, we can see that this occurs independently from the iteration as the function was defined as $f(x)=ax+b$, where a and b are whole random values between 1 and 3 and between 0 and 5 respectively. As we can see, the variability of the parameters is reduced. The reason for this may be that the instructor set the width, length, and the center of the Cartesian plane on which the function is displayed as a graph instead of adjusted according to the coefficients of the function. This leaves a space in the graph, which may its reading difficult as there is a very small area assigned for the function.

One of the questions that emerged from these analyzes was whether the decisions, in this case for example, the decision to use whole numbers, was influenced by instrumental factors such as software and programming knowledge limitations or by the suitable MWS of the instructor.

Up to now, based on the analyses of the lessons, it has been observed that one of the characteristics of suitable MWSs of instructors is the almost exclusive use of whole numbers. When we complement this analysis with the interviews, the instructors said they were aware of the limitations this poses in the potential work of the students. Nevertheless, they justify it based on the low capacity of students to work with a different type of numbers, for example, working with fractions.

In the specific case of this instructor and the questions she asked about the graphs, the use of whole numbers is also due to the way she understands the graphs:

Instructor: I could ask those students who always get an F, but they won't provide a correct response. They could estimate it .

Interviewer: And that troubles you, apparently.

Instructor: Yes, because the concept is not something that is close to a number or around it. It is ten meters or it is not ten meters away.

As it can be observed, the instructor regards the graphic register as a representation from which information can be obtained accurately. This influences the decisions that she makes as an assignment designer, and therefore, the potential work of the students.

Preliminary Conclusions

Tentatively, we can observe how the instructor participation in the design has a direct impact on the epistemic value of the task created in a digital environment. This influence, in turn, depends on several factors where the suitable MWS appears to be more prevailing than the instrumental factor linked to design (i.e., what the instructor can do) or the software limitations (i.e., what can be done).

The influence of the suitable MWS has a positive impact. As a matter of fact, the proposed assignments are more similar to those that instructors usually use, which may ease their integration. A downside is that this similarity implies the reproduction of the elements that characterize their suitable MWSs, which limits or makes the potential work of the students more difficult. Some examples are the exclusive use of whole numbers and the accurate reading of graphs, among others.

The analyzed example, the interview, and the analyses of the lessons show that a better use of the tool will not necessarily enrich the potential mathematical work of the students, as the design decisions that the instructor makes are more influenced by how she understands the mathematical objects and student learning.

Nevertheless, the fact that the assignments have parameters allows us to more clearly observe some phenomena that may not be noticed in fixed assignments; this will facilitate working with instructors in the future to enrich the suitable MWSs and to improve the resources. To finish, we can conclude it is necessary to continue with an instrumental assistance work, but more importantly a didactic assistance to enrich the resources and the teachers' suitable MWS.

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Using an app to collect data on students' use of resources for learning mathematics

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In the last few years, the number of functions that apps serve has increased. Now, one can even use an app to manage one's bank accounts. Young people spend more of their time online on their phone than on the computer (Wong et al.). Given this shift in preference, it is worth considering whether apps could be used more in research. Specifically, I will look at the potential for using a questionnaire app in research on mathematics for undergraduate engineering students, since such an app has been developed for my current project.

Keywords: Undergraduate engineering studies, questionnaire app, documentational approach, resources

Introduction

In my ongoing PhD project I look at how engineering students use resources for learning mathematics during their first year at university. Data collection takes place during the fall semester of 2017, and the spring semester of 2018. This paper focuses on one of the tools for data collection, an app named *Studert* (Norwegian for 'studied') which was created for the project for participants (engineering students) to fill in information regarding their use of resources after each study section. There does not seem to be literature on the use of an app for this kind of data collection. Given the widespread diffusion of mobile technology today, it is worthwhile to examine its potential for use in research. Therefore, this paper will focus on my implementation of the app, the quality of the data collected and the implications for how and whether to implement questionnaire apps meant for regular use into research.

Focus and research questions

The project is exploratory, aiming to analyze students' use of resources in a descriptive manner. The intent is to find a balance between looking at the students' resource use in depth, through collecting and analyzing qualitative data, and more quantitatively through collecting data using the app. The main purpose is not to construct theories from observed trends in the data but to describe those trends, potentially doing groundwork for future research in the area. This paper focuses on the use of the *Studert* app and addresses the following questions: (i) How was the app *Studert* implemented (ii) How can one improve upon the implementation? (iii) Does using a questionnaire app for regular use like *Studert* produce high-quality data? (iv) Can such questionnaire apps be useful in future research?

Theoretical framework

The project adopts the 'documentational approach' (Gueudet & Trouche, 2009) which proposes that interaction with resources is a significant part of a teacher's professional development and practice. In the approach, the combination of a set of resources, strategies for using them and rationales for how strategies vary depending on the situation is called a 'document'. The set of a teacher's documents coupled with their strategies for when and how to implement them is referred to as their 'document system'. Teachers' work to develop their document system is called 'documentation work'. My work focuses on students' document systems and

documentational work. My rationale is that students also have a multitude of resources available (textbooks, calculators, web sites, multi-representational software, etc.) and need to develop strategies for how and when they use them. However, given a combination of practical concerns, my project's focus and the difference between examining students and teachers, I have decided to diverge slightly from the guidelines of the documentational approach. For instance, I use a working definition of resources that is not quite as broad as advocated by the documentational approach, but closer to students' perception of what is meant by resourcers (when I tell them I use a wide definition). Gueudet and Trouche note that: "resources exceed artefacts: the reaction of a student, […] can also constitute resources, for a teacher who draws on them in her activity." (Gueudet & Trouche, 2011, p.3). My research will not look at social interactions related to learning in any depth, but simply at what interactions students engage in.

Methodology

The project is a multiple case study (Yin, 2009), following students from three different Norwegian universities through one semester. Each of the three cases is a cohort of students using resources whilst studying for mathematics modules. I use mixed methods in data collection (Johnson & Onwuegbuzie, 2004). The qualitative data includes hierarchical focus interviews (Tomlinson, 1989) as well as students' schematic representation of resource systems, as used by Pepin, Xu, Trouche and Wang (2016). Participants construct mind maps immediately prior to interviews; the Norwegian term for mind map is familiar to Norwegian students. The quantitative data is described below. The initial goal was for 5 interviewees from each site and at least 30 participants per site using the app (but open to all volunteers).

An Android and an iOs version of *Studert* (each available in Norwegian or English) was developed. The intent was for students to enter information after each study session. I think the Norwegian term for study session is intuitive to the students. Study sessions are either separated by time or by way of working. If one attends a lecture and then does exercises afterwards, those are two separate sessions, as one works differently. I retrieve data via a web portal set up by the developer. Participants can opt to get reminder e-mail if they have not filled in anything on a given day.

From the main page in the app, students press a plus icon to add a session. On the new session screen, they are presented with seven questions. For the first question, participants fill in what kind of session it was; for the second, where they studied; and for the third, what mathematics topic they studied. The main question is the fourth, where participants fill in the resources they used. Multiple answers are possible. The app will remember resources filled in for previous sessions to suggest them when you start writing the word. For the fifth question, participants use a drop-down menu to answer whether they studied alone, with others or in an organized setting with or without group work. If they studied with others or in groups, they are asked how many they studied with. For the sixth question they use a drop-down menu to answer how long they studied for, and lastly, they give the date the study session took place. It defaults to current date.

Results and discussion

Implementation and students' comments

In general, recruitment for using the app in the first semester was disappointing. Factoring in some attrition between recruitment and the first round of interviews, 13 started using the app,

of which 9 were interviewed. Across all sites the goal had been 90 and 15 respectively. I think the main issues concerned general principles of recruitment, and I will be able to recruit more participants and attain big data in the second semester.

Some of my plans for the app did not make it into the final product. While I had commincated my plans to the developer, I had not supervised development, and the developer did not seem to have realized its importance when making executive decisions during development. For instance, because I expected the prospect of filling in something on a near-daily basis would seem daunting to potential participants, I wanted to be able to demonstrate that filling in information was quick and simple. The main difference between my plans and the final project was that the first three questions were supposed to memorize previous answers just as the fourth question does.

At the last interview, I asked the interviewees how they had experienced using the app. First I asked an open question, and then I asked specifically about how their thoughts on the amount of work, the clarity of questions and information given and what could be done differently if one were to update the app to use it again. Out of eight people responding, five expressed a positive overall opinion on using the app, two gave neutral answers and one negative. Four respondents suggested the app should remember their responses to the first three questions, and one suggested the ability to save a study session as a template for entering future study sessions. Even so, only two considered themselves unhappy with the workload. Six respondents reported having difficulty remembering to fill in study sessions. Related to this, two specifically mentioned liking the e-mail notifications, while three suggested using push-notifications instead, and one requested greater ability to customize which days to receive reminder e-mails on. Related to clarity, two respondents thought it was clear enough. The rest were unsure of the difference between the first and third question, but all reported the information document clarifying the issue.

Quality of data from the app

Some issues I have observed when personally filling in digital or physical questionnaires are: 1) When they ask me for how often I use X, I have not been very conscious of it, so thinking back, I have trouble estimating. 2) When trying to estimate, I think I subconsciously make myself seem better by describing how much I use X during a good week, rather than an average week. I justify this to myself, because I am uncertain, so I merely give myself the benefit of doubt. 3) For simplicity of the questionnaire, the researchers often make rather large intervals, and placing myself within one of those intervals do feel like I am misrepresenting my actual habits, especially if I have problems estimating and my use is somewhere around the border between two intervals. I assume the idea is that with enough responses, the number of people that fit into each interval will still be quite precise, but for smaller studies, this lack of precision in individual answers would be a problem.

Using the *Studert* app, participants were able to enter information that represents each individual study session with high accuracy. This enables me to calculate how much each participant used each resource, how much they worked overall, and how much they worked alone and with others. Because the data is retrieved digitally in the form of a spreadsheet, the data is quite flexible. If I come up with a new research question I want to investigate, I can write a program to investigate it. If for instance, I want to investigate whether the amount of time students spent studying math differed from month to month, or how a particular participant's

use of resources varied depending on whether they studied alone or with others, I would be able to write programs to investigate those questions.

However, while participants were able to enter information accurately, thethey did not always do so. Only two people consistently answered the first two question for every study session. It is not a detriment to my project as they are the least important questions to my analysis and everyone consistently filled in information on all the other questions, but it is worth taking into account. Also, only two participants filled in any sessions at all during their exam preparation period. It is hard to say whether the rest stopped because they were busy, or because I had already interviewed them for the last time and they perceived their participation in the project as a whole to be finished.

Utility of questionnaire apps for future research and potential challenges

Based on my work so far, I conclude that questionnaire apps aimed at regular use can provide high quality data, but from my limited experience, it seems that if students do not perceive a question as important, they may not answer it productively. Another potential issue is recruiting participants, as many may find the idea of regularly filling in a questionnaire over a long time unappealing. This might mean that such an app is not well suited for large, quantitative studies, and better suited for being one of the types of data in a case study.

On a practical level, one of the largest obstacles for creating a questionnaire app for a research project is how much time and money goes into it. If questionnaire apps were to be put into widespread use, the most realistic option would be to develop a meta-app, in which researchers can design their own studies. Such a meta-app sound similar to existing apps such as Survey Monkey, but there are several features that Survey Monkey does not have, that could make it more convenient for researchers specifically. For instance, based on the feedback from my participants, a meta app should have the ability to send notifications to participants who have not filled in anything on a day when they are expected to. It should also remember and suggest previous answers, and possibly have the ability to save a session as a template for future entrys. The researcher should be able to retrieve the data as a spreadsheet. Also, for several research projects, an option to insert information relevant to the project into the app itself, would be quite beneficial.

Conclusion

If you look at the potential of incorporating questionnaire apps for regular use into research from a perspective of best case scenario analysis, there are several aspects that make the prospect intriguing. The researchers get a good amount of quantitative data on each participant, that is based on something the participants can easily remember and do not have to estimate based on educated guesses. The researchers do not need to engage in busywork to get this data on a flexible, digital form. However, there are many factors that must be taken into account before deciding if and how to use such a questionnaire app. It is worth questioning how many one will realistically be able to recruit, and if it is a satisfactory number. If one is to use it, one should minimize the workload for participants as well as only including the most important questions, and communicate the importance of said questions to the participants. Questionnaire apps for regular use are unlikely to become widespread unless a meta-app in which researchers can design their questionnaires is developed.

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Un support numérique pour le travail de documentation des enseignants de mathématiques de l'EFII (Collège, en France)

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La conception et l'utilisation de ressources en classe de mathématiques est un thème actuel et important dans le domaine de l'enseignement des mathématiques et il est traité de diverses façons dans la littérature. L'article présent s'insère dans ce thème, résultant d'un projet de recherche dont l'objectif principal est de construire une communauté d'utilisateurs visant le partage de ressources pour l'enseignement des mathématiques de l'Ensino Fundamental II (EFII) au Brésil (niveau Collège en France). Cette communauté a pour but de rassembler des enseignants et des chercheurs afin de discuter, d'analyser et d'élargir l'ensemble des ressources pour ce niveau d'éducation, ayant évidemment pour objectif principal d'améliorer les conditions de l'enseignement des mathématiques. Ce partage s'opère à l'aide d'un support numérique, en l'occurrence l'environnement Wordpress. On espère pouvoir inclure des enseignants français au niveau Collège dans cette communauté afin de recenser les différences et les similitudes entre les attentes des enseignant à ce niveau d'éducation des deux pays.

***Mots-clés**: Travail de l'enseignant, Communauté, Enseignement des mathématiques, Ressources*

Introduction

Ce projet porte sur l'enseignement des mathématiques et analyse plus particulièrement les questions liées au travail de documentation des enseignants de cette matière lors de la préparation de leurs cours. Il vise à promouvoir l'échange d'informations entre les chercheurs de l'enseignement des mathématiques et les enseignants de mathématiques de ce niveau scolaire. Il analyse ensuite le développement et l'utilisation des ressources par les participants, ainsi que l'application des théories de l'enseignement des mathématiques et des domaines connexes[1]. Ce projet porte donc sur les ressources numériques, en accord avec Gueudet & Trouche (2010, p. 43) qui affirment que « ces ressources sont à la fois révélatrices de phénomènes profonds et de facteurs d'évolutions importantes dans les dimensions collectives de l'activité des professeurs ». Pour pouvoir développer le projet, un environnement informatique (www.ensinodematematica.com.br) a été créé sur WordPress et utilisé comme support de communication et d'échange entre les chercheurs et les enseignants. Cependant, l'organisation de cet environnement ainsi que la structuration du dispositif humain du projet restent à développer. Les références qui guident ce projet comprennent la notion de la Genèse Documentaire de Gueudet et Trouche, les PCN (Brésil, 1998), le Programme d'Enseignement de São Paulo (2008), ainsi que des études sur l'enseignement des mathématiques au niveau EFII. Ces dernières, qui portent sur le développement de ressources par les enseignants de mathématiques, ont été développées au Brésil grâce à la diffusion des théories de Rabardel et de la Genèse Documentaire de Gueudet et Trouche. Les auteurs de ce projet ont d'ailleurs utilisé ces références pour leur thèse de doctorat (Almeida, 2017). Les références qui concernent le dialogue avec les enseignants incluent l'expérience

[1] Cet article fait partie d'un projet au niveau postdoctoral de l'auteur principal, Capes/ENS LYON, sous la supervision de Luc Trouche.

de la formation des enseignants, à la fois initiale et continue, ainsi que le suivi de recherche au niveau d'études supérieures. De fait, on observe que les enseignants, de façon générale, font preuve d'une forte disposition à appliquer les « nouveautés technologiques » en classe. Pour conclure, le livre paradidactique de Abar & Igliori (2010) est également une des références pour l'élaboration de ce projet. Les utilisateurs ont jugé les activités intéressantes, mais ils ont noté qu'il était nécessaire de discuter et d'analyser leur utilisation en classe.

Présentation

Initialement, des tâches considérées comme motivantes ont été mises à disposition dans l'environnement informatique. Elles ont pour fonction de façonner le projet et portent sur des thèmes de mathématiques de l'Enseignement Fondamental II. Les contenus que l'on retrouve sur www.ensinodematemática.com.br. ne sont pas uniquement liés à la géométrie, mais en raison de la taille de cet article (quatre pages), nous avons choisi au hasard des problèmes de géométrie spatiale que l'on retrouve d'ailleurs dans le livre de Abar & Igliori (2010). Ces activités étaient motivées par des composantes du développement cognitif, telles que l'attribution de significations (activité adaptée de Pohl [1994, p. 178]), la diversité des représentations plates d'objets spatiaux (Parzysz, 1989, p. 54-55) et l'identification des solides en fonction de leur planification (Brasil, 2011).

Cadre théorique et questions de recherche

Cette recherche est basée sur la notion de la Genèse Documentaire de Gueudet et Trouche (2009, 2010), qui tient pour acquis que les ressources développées par les enseignants pour préparer leurs cours sont au cœur non seulement de leurs activités, mais aussi de leur développement professionnel et impliquent la recherche de ressources inédites, la sélection et la création de tâches de mathématiques pour la planification et le développement de séquences, la gestion du temps et des objets disponibles. Nous faisons également référence à Rabardel (1995) qui affirme que l'utilisation de la technologie invoque la nécessité d'un dispositif technologique quelconque qui devrait se transformer en un instrument lorsque ce dispositif devient porteur de sens pour le sujet. Selon Gueudet et Trouche (2009), ce processus se produit lorsqu'un « instrument est le produit d'un processus, nommé Genèse Instrumentale, par laquelle le sujet construit un schéma d'utilisation de l'objet pour une certaine classe de situations » (p. 204).

À partir de ce cadre théorique, nous assumons plus spécifiquement « le travail documentaire, la dialectique ressource/document et la notion de genèse documentaire, les systèmes documentaires » (Gueudet & Trouche, 2010, p. 43).

D'ailleurs, « l'étude de la documentation des enseignants fournit une nouvelle matière à un croisement de cadres théoriques : ceux-ci doivent ainsi mettre à l'épreuve leurs outils et méthodes au feu de nouvelles questions et se situer par rapport à d'autres approches. » (Gueudet & Trouche, 2010, p. 5). Nos problèmes de recherche portent sur les connaissances préalables des enseignants participants, leurs croyances sur l'enseignement et sur les mathématiques, ainsi que les difficultés du rapport, en principe hiérarchique, entre les chercheurs et les enseignants. Ainsi, nous posons le problème de recherche suivant : Comment développer l'action conjointe des chercheurs et des enseignants ?

Méthodologie

L'apport de réponses au problème de recherche peut s'opérer de trois façons : sur la base d'un développement collaboratif de ressources, en examinant les ressources qui répondent aux besoins des enseignants ou encore en examinant les ressources existantes et en les complétant.

En ce qui concerne le dispositif humain, le projet dispose de deux groupes. Le premier est composé de chercheurs et d'étudiants de l'université brésilienne, PUC-SP, le second comprend 15 enseignants de mathématiques EFII de différentes écoles, choisis de façon pragmatique, en tenant compte de leur volonté de faire un premier pas. Les procédures méthodologiques permettent d'établir un environnement relationnel ouvert et permanent entre les participants afin de pouvoir transmettre les ressources à la classe de mathématiques. Les scénarios d'apprentissage développés permettent créer un laboratoire en ligne de problèmes de mathématiques qui sert d'espace pour la recherche permanente et l'interaction entre les chercheurs et les enseignants. Afin d'atteindre ces buts, nous avons suivi les recommandations de la méthodologie du *design - research* (Cobb et al., 2003) pour soutenir l'équipe responsable de la conception et de la production des scénarios d'apprentissage dans le domaine des mathématiques. Selon ces auteurs, les expériences de conception visent à contribuer au développement et à la compréhension des « écologies d'apprentissage », c'est-à-dire, de systèmes complexes impliquant de multiples éléments de natures différentes.

Activités

Les activités présentées ont été insérées dans l'espace à titre de mise en forme initiale et elles font partie de son répertoire. L´espace destiné à cet article ne permet que la présentation d´une activité.

La Figure 1 montre un solide spécifique (prisme de base triangulaire), un curseur et quatre boîtes qui montrent/cachent des objets. Lors de cette activité, l'utilisateur déplace le curseur pour planifier le solide. Il est prévu que l'utilisateur observe le « démontage » du solide dans la Fenêtre de Vue 3D et les figures géométriques, soit des polygones, qui composent les faces du solide, le polyèdre, dans la Fenêtre de Vue (2D). Les trois autres boîtes contiennent d'autres plans du même solide, ce qui permet à l'utilisateur de se rendre compte qu'une planification peut être réalisée de différentes façons.

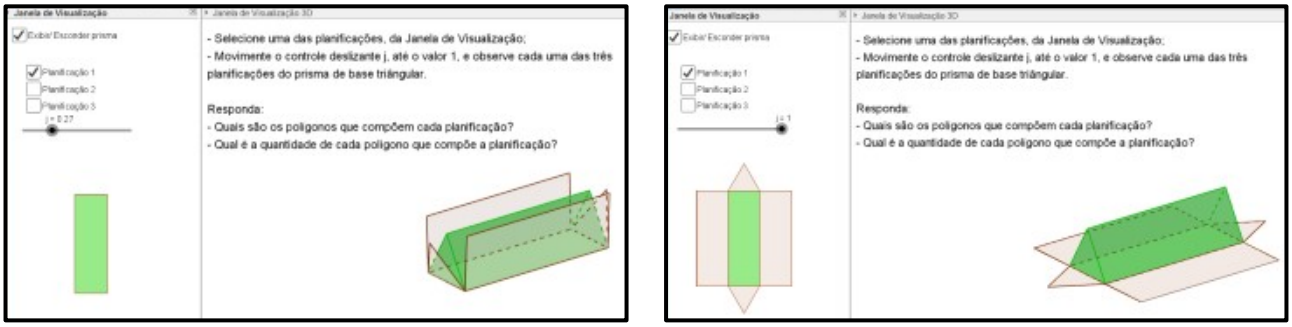

Figure 1 – Planification de prisme de base triangulaire

Considérations finales

L'environnement crée dans WordPress est en cours de développement. La version préliminaire de l'activité présentée est disponible sur www.ensinomatematica.com.br/, offrant d'ailleurs un espace d'activités en langue française et des outils d'inte on entre enseignants et chercheurs.

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The birth of the documentary system of mathematics pre-service teachers in a supervised internship with the creation of a digital textbook chapter*

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This presentation, part of a doctoral research, aims at identifying interconnections between professional development of mathematics pre-service teacher during a supervised internship and the resources used to build and use a chapter of digital mathematics textbook (about the introduction of algebraic functions) in a classroom with 30 pupils from 12 to 13 years-old. Based on the Documental Approach to didactics (Guedet and Trouche, 2009), we bring into discussion partial analysis of one of the two data collection comprised of registers of planning process, prototyping of the chapter and its use in classrooms, regarding the first version of the resource. The partial results suggest that the pre-service teacher took control of part of his own formative path while decided what he needs to produce his own resource, using as reference his interaction with pupils since his first experience of using the digital chapter.

Keywords: Digital Textbook preparation, documentation, pre-service teacher education, mathematics teachers, pedagogical content knowledge.

Introduction

This research study encompasses great themes related to initial formation of Mathematics teachers (IFMT): teaching practice as a curricular component; acquisition of professional teaching knowledge; production and use of didactic resources, specifically textbook in its digital format (DT). The involvement of such broad themes was necessary to develop a study that aims at filling a gap that we find in academic research on the Mathematics Supervised Curriculum Stage (MSCS).

Lopes et al. (2017) indicates the increasing number of papers addressing several aspects of MSCS. However, we do not find reference to research on the development of professional teaching knowledge during Mathematic teaching practice as curricular component within didactic resources, to deal (select, organize, modify, adapt, create, use, reuse ...) more specifically with Mathematics e-textbooks.

Although the focus of the research is the stage curricular supervised of the Mathematic we analyze part of the trainee's academic background prior to the beginning of the research, since in Brazil the IFMT is based on the legal principle that practice should become a curricular component of all course disciplines and not just in some specific disciplines. It is important to emphasize that in the Brazilian context, the IFMT is developed in a single course that includes all the forming time for the development of the necessary knowledge to the teaching practice.

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Study Design and Theoretical Foundations.

The objective of this research was to analyze the process of development of professional teaching knowledge of a Mathematics teacher trainee, related to an experience of authoring an Mathematics e-textbook chapter using the Geogebra-Books / Geogebra-Groups platform as a project for his MSCS. To this end, we base our procedures on the documentational approach to didactics - DAD (Gueudet and Trouche, 2009). Our hypothesis are that such aF project, assumed with a central axis of the supervised stage, can become a means for the trainee to structure his documentation system for, developing knowledge oriented to teaching practice through documentational genesis process. In this article we will show some aspects of integration of resources developed before the internship with others developed during the stages of creation and use of the e-textbook chapter. This is due to the flexible nature of the project that allows the development of a reflective process of future teacher about his actions towards the professional practice mediated by the resources with which he deals.

For Gueudet and Trouche (2009), the term documentational genesis in DAD refers to a process whereby the resources that the teacher uses are integrated by the cognitive structures that the teacher develops / mobilizes to act on classes of situations. The authors understand that the resources (in a broad sense of everything that can nourish the teaching practice) to be adapted by the teacher for their activities, assume a specific role in the intermediation of their actions. It is called the document, the duet formed by the resources and the cognitive structures developed by a teacher to use them in their teaching practice. We have had in a document, therefore, a composition of elements re-sourcing the teaching task (the resources) with those who organize such activities into practical action (the *schemes*) aiming at reaching their goals.

A project for creating a Mathematical e-textbook chapter, by a pre-service teacher, mobilizes a varied range of resources such as printed textbooks, software programs, the curriculum organization of school mathematics, the interventions of MSCS supervising teacher, pupil's interventions during lessons, etc. If, on the one hand, the resources are generally explicable, a greater research effort must be made to infer the cognitive structures that pupils may have developed and / or mobilized. Shedding light on this, in DAD, documentation Genesis concept adopts the schema definition as "the invariant organization of behavior for a given class of situations. It is in the schemes that one must research the subject's knowledge-in-action, that is, the cognitive elements that make the action of the subject operative" (Vergnaud, 1993, p.2, our translation).

Methodological Procedures and Partial Data Analysis

We undertake the data collection of the research with a pair of licensees enrolled in the second (of four) MSCS discipline from a University in the State of Pernambuco / Brazil. The data collection lasted from the second academic semester of 2016 until the end of the first academic semester of 2017 when only one of the individuals, whom we will name Luis, was enrolled. Participation was given voluntarily after the researcher gave (with the consent of the traineeship teacher) an exploratory workshop for Mathematical e-textbook. The school, the field of practical activities, had already been selected by the trainees, but they had not yet presented themselves to a supervisor. The choice of content (introduction to functions) was partly due to the prediction of remaining with the theme for later stages of the MSCS and part due to the availability of contents not yet taught in the final stage of the school year.

We record on video all the planning and development meetings of the Book as well as the regencies. Another part of the data corresponded to access to material exchanges through a shared

folder on the Internet and, eventually, through emails. The regency activities were performed in a laboratory with fifteen computers connected to the Internet, which is why the trainee chose to predict the organization of the students in pairs. We had access to all production of the pairs, including interactions with the trainee, who were registered in the platform online.

Some preliminary, emerging elements can be highlighted from the video-recording data we have made from two report presentations made by the intern at seminars outside the internship field. We were interested in identifying the origin of knowledge mobilized for the achievement of the project stage, identifying the actions that may be associated with them and that were present in the subject's discourse during their presentation. This analysis is also supported by consulting other elements of our data. Using an analysis tool that is being developed by Rocha (2016), we associated these actions with the resources that were selected and those that were eventually produced. We thus obtained a visual representation of the documentary trajectory of the trainee, understood by the Rocha (2016) as the joint development of the resource system of a teacher and his professional knowledge.

Initially we identified twelve moments of the academic trajectory (Figure 1) which the student considered relevant for the development of his internship. In all of them it was possible to associate some feature that he selected to promote his actions.

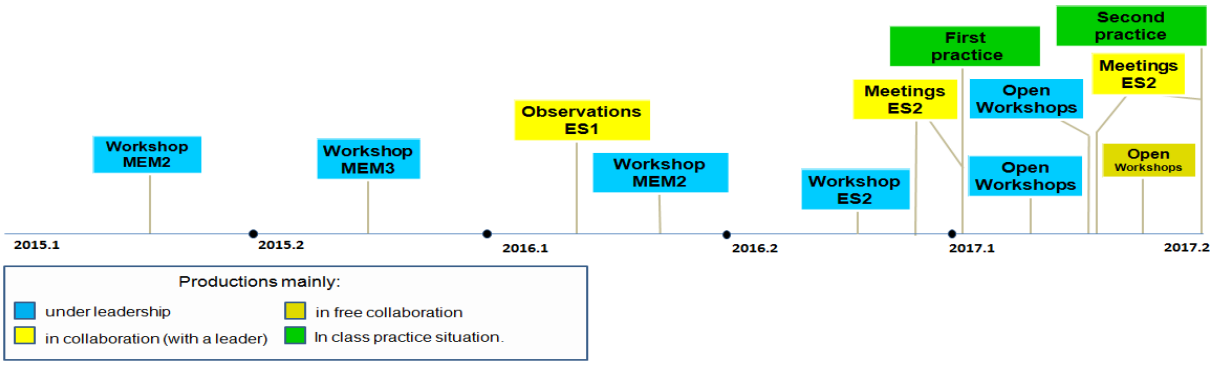

Figure 1: schematic representation of Luis's documentary trajectory

In reference to one of the events in which his production of resources occurred under orientation (the ones in blue), Luis, present one of his resources used to make the chapter of the Mathematical e-textbook recognizing the limits of his decisions at the time states that:

- "And this is what I'm showing you, it was there in methodology 3. We were not even thinking about internship yet. It was one of the activities we had". Luís

However, after the start of his project, the resources related to the workshops and lectures he sought had an affinity with his e-textbook development activities and, therefore, we assumed that the pre-service teacher began to take part of his formative trajectory when deciding what to study, considering the resources it seeks to incorporate into its documentary system. On this point, a possible inference about his knowledge is that prior to the project they were mobilized and developed to meet the goals of their teachers and passed on to the knowledge that drives them to deal with resources in order to respond to their own demands generated by their engagement in the development of his Mathematical e-textbook.

In events marked in yellow, the pre-service teacher's level of autonomy regarding goals and proposals was shared with experienced teachers. Initially his speech to propose guidelines were based on academic readings and activities that he considered disconnected from a possible practical experience, as it demonstrates the following testimony about what he thought at the time:

- What will that do? I do not know, I just know that we did and got the knowledge, had several discussions but we think. Will I use this in practice, day in and day out? What will that do? I do not know, I just know that we did and got the knowledge. (Luís)

However, in his presentation, the pre-service teacher uses pupils' responses as a resource to argue about the relevance of their overall proposal to the chapter, using the recordings on the platform to present strategies used.

In the events in green, when its level of autonomy over the actions was the highest, having total freedom to act in classroom, the pupils' interventions occurred both within the platform (by records that the system makes and that allows an asynchronous dialogue) and in loco, pointing out small inconsistencies in the elaboration of the activity or presenting difficulties of understanding that were unexpected. Always relying on the material produced, the pre-service teacher demonstrated flexibility of actions both to respond to pupils' demands and to adapt the tools to those demands. We postulate that the knowledge that guided the trainee to act in the classroom was possibly originated in ideas developed during his experiences of creating and using resources in previous disciplines. His strategies try to follow the guidelines he obtained in the discipline of MEM2, when he studied on Theory of Didactic Situations by Guy Brousseau.

Preliminary Conclusions

Although the pre-service teacher's academic trajectory already contemplated the involvement in production and experimentation of resources for the study of learning theories, his actions were limited to correspond to the strategies of his teachers. Far from being used as a repository of materials, the chapter of e-textbook served as a resource on which the pre-service teacher guided his actions. In this way, he became selective about the resources he included in his material so that he could identify with the proposal he had for the pupils.

The possibility to follow student's solutions both in class time and through the records left in Mathematical e-textbook has become a didactic resource that the trainee mobilized to improve his chapter of e-textbook and through it to promote modifications in their predictions of future practice of teaching. It is therefore a learning process of the trainee in relation to the knowledge about the nature of the teaching activity and mastery of the tools that he needs to develop his e-textbook.

In the next phase of the research we are verifying to what extent this integration between the knowledge (mobilized and produced) and the resource (Mathematical e-textbook) that it developed presents characteristics of a document.

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Teaching computational thinking in classroom environments: a case for unplugged scenario

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This article aims to share the experience in introducing computational and mathematical thinking to teachers and students, from different range of ages, in a workshop context, through modeling a mathemagical trick. The activity hereby described constitutes in itself a resource usable in the classroom. We argue that the introduction of technology, is not mandatory to enhance computational thinking, but it is a valuable tool on the verification step of solving a problem. That is to say, using unplugged approaches to teach algorithms, it is possible to achieve the goals of the curricula as well.

Keywords: Teaching algorithms, Algorithms, Mathematics, Computational Thinking, Mathematical Thinking.

Introduction

Since the beginning of the scholar year of 2016, the new mathematics curriculum became effective, its main difference being the introduction of Computer Science into the Mathematics program. Its goals are developing the following competences in students:

- Decompose a problem;
- Recognize schemas;
- Generalize and Abstract;
- Design and translate algorithms into a programming language.

Computational thinking is a phrase that has received considerable attention over the past few years. Considering, Computational Thinking as knowing how to program may be too limiting. Shute and All (2017) highlight the National Research Council (2010), which says that everyone should acquire CT, not only programmers. CT skills include managing information effectively and efficiently with technologies in our data-driven era (Burke, O'Byrne, & Kafai, 2016; Kim, Kwon, & Lee, 2014; Lu & Fletcher, 2009; Sanford & Naidu, 2016; Wing, 2010).

Definitions of Algorithms and Computational Thinking

An algorithm can be defined as the process of solving a problem, using a finite amount of steps, in a way which is effective, non-ambiguous and organized, translated into instructions which are replicable to solve problems of the same instance, by an individual or a computer.

Observing the notion of obvious to the kids like, "to spread butter on the bread": using or not a knife is very interesting. It demonstrates that an algorithm must return the same result, independently of who performs the steps.

Following the article from Shute and All (2017), Computational thinking (CT) stems back to the constructionist work of Seymour Papert (Papert, 1980, 1991) and was first coined as a term in a seminal article by Wing (2006). The most of-cited definition of CT comes from Cuny, Snyder, and Wing (2010), defining CT as a process where "… solutions are represented in a form that can be effectively carried out by an information-processing agent".

Weintrop et al (2016) proposed a definition of CT for mathematics and science in the form of a taxonomy consisting of four main categories: data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices, illustrated on the. Fig 1.

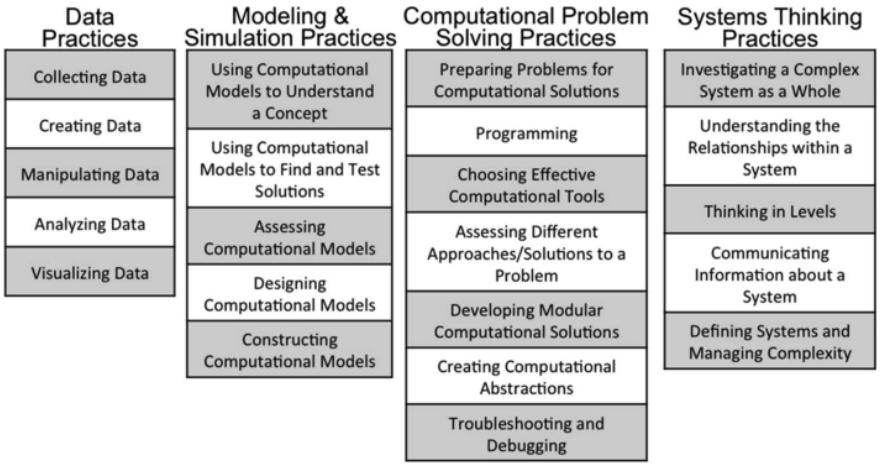

Figure 1: Computational thinking in mathematics and science taxonomy

Defining resources in mathematics education isn't an evident task. However, Adler (2000) suggests that mathematics teacher education programs need to work with teachers to extend common-sense notions of resources beyond material objects and include human and cultural resources such as language and time as pivotal in school mathematical practice. We understand the resource as a transparent tool which is used to achieve a goal. In this paper, we intend to demonstrate the use of the taxonomy proposed by Weintrop et al. (2016), in modeling the mathemagical trick: Binary Code, understood as a CS-unplugged resource, which will aid in the development of didactical strategies to teach computational thinking.

Method

The Workshop, MathMagic: Computational and Mathematical Thinking, was offered to students, teachers, graduate students and researchers in various contexts, totaling more than a thousand people. The aim of the workshop is to introduce the concept of Computational and Mathematical Thinking by modeling the following trick:

The binary magic trick is a set of 5 cards with numbers on them. The mathemagician asks someone to choose a secret number between 1 and 31 (for example her birthday in the month), show her each card one after the other in order to select whether the card contains the secret number. Using his magic powers, the mathemagician figures out which number she has chosen.

3	7	23	19
17	9	11	31
5	27	21	13
1	25	29	15

14	26	10	27
3	30	15	18
31	6	19	23
2	7	11	22

6	5	20	14
28	23	15	13
31	29	21	22
4	30	7	12

14	10	15	31
11	27	29	13
26	28	9	12
8	30	25	24

17	23	19	25
27	30	29	18
22	21	26	24
16	28	31	20

Table 1: Cards from Binary Code Trick

For instance, let's say that one chooses the cards, from left to right: [1st , 3rd , 4th]. Then, I can guess that the date of the birthday is 13, which is the only number in common to these three cards and not the other ones. After performing the trick, almost all the participants react on the same way saying: "Why, How, Can you do it again"? Building on this frustration fuels the motivation to understand the trick and deconstruct the algorithm.

Then, the following tasks were given to participants:

- Understand the trick and write an algorithm which performs it. It means, given a set of five yes or no answers, one for each card, the algorithm returns the correct number chosen by someone.
- Write an algorithm which, given a card rank between 1 and 5, return all the numbers on the corresponding card.
- Write an algorithm which, given a number between 1 and 31, returns the set of cards in which it is featured.

Understanding the problem: The magician has five cards, each of them filled with sixteen numbers. The range goes from 1 to 31. The magician asks if the number is or isn't on the card. The possible answers are yes or no. On this step we don't need any digital tool.

Devise a plan: Before devising a plan, it is important to understand all the data we have to model the phenomena, then choosing a strategy to approach the problem. Examples of strategies described by Pólya are (Guess and check, Make an orderly list, Eliminate possibilities, Use symmetry, Consider special cases, Use direct reasoning, Solve an equation, Look for a pattern, Draw a picture, Solve a simpler problem, Use a model, Work backwards, Use a formula). We can use the taxonomy of CT performing the Data Practices to identify *a priori* strategies:

a. Collecting Data: The numbers in the cards, the number of cards, the answer given to perform the trick.
b. Manipulating Data: Data must be organized by students in order to easy describe the phenomena.
c. Analyzing Data: After organizing the data it is possible to *look for a pattern.*

Eventually the student perceives, after organizing the data, that the relationship between the smallest number of each card and the gap on the sequences has a pattern of increasing by powers of 2. It means that each time one says ***yes*** to the magician, he/she takes the power of 2 corresponding to that card, which is the smallest number of that card, and makes the sum of them. For instance, my own birthday is 23rd. And indeed, 23 is on the 1st, 2nd , 3rd and 5th cards: 1 + 2 + 4 + 16=23.

Carry out the plan: To write the algorithm, individuals can design it in natural language, or pseudocode. Giving the instructions to another person to perform the trick. Identifying the data structure is essential, understanding the set of information available to carry out the plan. For instance, one of the simplest ways to link the elements of a set is creating a list. It's a structure in which the operations insert, withdraw and locate are defined. In the case, a table could be designed to clarify the data structure present in the problem.

Conclusion

This article shows the possibility of working with computational thinking competences even without the use of computers, using CS-unplugged as resource. A primordial step in the convergent thinking (Mercat et al., 2017) in designing algorithms is the translation of thinking, an abstract and subjective construct, into a concrete, objective and meaningful method which makes any information-processing agent return the expected result. Solving the task of writing an algorithm to perform the trick and to solve this particular problem, didn't need the use of any digital tool. Implementing it on a computer requires further work in understanding, collecting, visualizing, structuring and analyzing the data in order to translate the phenomena into a programming language.

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Meta-Orchestration Instrumental for Teacher Education*

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This paper discusses the theoretical model Instrumental Meta-orchestration, which has been developed focusing on the formation of future teachers to use digital technologies integrated with mathematics teaching. A meta-situation of formation emerges from the experience of an instrumental orchestration with a mathematical situation, an experience that points out the need to create a formation model that favors a discussion about Instrumental Orchestration and the learning of its main elements and theoretical assumptions. It was also created a micro-genetic analysis structure to identify events that demonstrate the participants' understanding, doubts or non-comprehension about the theoretical model.

Keywords: Instrumental Meta-orchestration, instrumental orchestration, schemes, teacher's knowledge, teachers' formation.

Introduction

One lesson, designed and undertaken in a masters' class of Mathematics and Technological Education course of a Brazilian university, based on the theoretical model of Instrumental Orchestration (Trouche, 2005), pointed to the need for a theoretic-practical formation based on the use of digital technologies. The development of a theoretical model that aims at such formation, which we named Instrumental Meta-orchestration, is the object of a doctoral research of the first author, guided by the co-authors.

The model of instrumental meta-orchestration is the focus of this paper.

> A meta-orchestration instrumental is a systematic and intentional design of artefacts and human beings, in an environment of formation by an agent, to execute a meta-situation of formation which aims to guide teachers in their instrumental genesis instrumentals about the theoretical model of instrumental orchestration (Lucena, to appear).

In addition to discussing such structure, we are also interested in discussing the structure of data analysis generated from experimentation within a meta-situation of an instrumental meta-orchestration designed for initial teacher training. A meta-situation is a complex situation that can be analyzed as a combination of situations articulated within each other and that has its own nature and difficulty. The meta-situation of formation about instrumental orchestration consists of discussing the theoretical model of Instrumental Orchestration (Trouche, 2005; Drijvers et al, 2010) based on an experience and analysis of this experiment built in the light of this model. We take the notion of situation according to Vergnaud (1996).

Theoretical Framework

An orchestra, in general, can be recognized as an instrumental grouping which comprises one maestro and instrumentalists, their instruments and partitures, all well arranged in a space for

* Partially financed by CAPES

the purpose of performing a music. The concept of instrumental orchestration aims to model teacher's practice to sustain the instrumental genesis of students in a rich mathematical learning perspective (Trouche, 2005).

According to Rabardel (1995), the instrumental genesis is a transformation of an artefact by the action of someone, transforming it into an instrument meanwhile the subject goes through the process of instrumentation by integrating it into their practice. The transformation of the artifact into an instrument is not characteristic of the structure of the tool, but of the schemes that the subject develops to integrate it. Within the perspective of the instrumental genesis students, two concepts are fundamentals to teacher that orchestrate: first, the concept of scheme.

> A scheme is the invariant organization of behaviour for a certain class of situations. A scheme is a dynamic and functional totality; the organization of behaviour must be considered as a whole (totality); it takes place over a certain period of time (dynamic); and it is aimed at achieving something (functional). Although it is a totality, a scheme is a combination of different kinds of elements: goals and expectations; rules to generate actions (...), operational invariants (...) concepts-in-action and (...) theorems-in-action; inference possibilities. (Vergnaud, 1997, p. 12-13)

Second, the concept of situation

> does not assume here the meaning of didactic situation, but the meaning of task; the idea is that any complex situation may be analysed as a combination of tasks, which their own nature and difficulties are important to know. (Vergnaud, 1991, p. 167).

Instrumental genesis is not something simple and takes time and mutual effort, students and teachers, to happen (Drijvers & Trouche, 2008). Designing an instrumental orchestration is the same as creating a fertile environment and providing it with adequate support structure to the actions that it will be developed, to make it emerge. In this direction, three components constitute an instrumental orchestration: the didactic configuration, the mode of execution, both characterized by Trouche (2005) and the didactic performance, by Drijvers et al. (2010).

The analysis of an orchestrated class provided us with crucial elements for the development of a training design based on the notion of Design Experiments (Brown, 1992) and on the theoretical model of Instrumental Orchestration (Trouche, 2005). The Instrumental Meta-orchestration designed was undertaken with a group of pre-service mathematics teachers of a state university.

From the definition of meta-situation, that consists of situations organized to discuss the theoretical model of instrumental orchestration based on the experience and analysis of an experiment built on the basis of this model, we structure the design of the meta-orchestration instrumental. This design consists of five levels, four of which present a training situation, all started within an instrumental orchestration (IO_p) whose proposed situation is mathematical. The IO_p is crucial to the design of meta-orchestration not only because it gives rise to the other orchestrations, but, especially, because its execution highlights the theoretical elements of the theoretical model, either through observation of the experiment or experimentation. The last

level presents situation to elaborate an Instrumental Orchestration. Each level has its own purpose for the formation and are analyzed with specific purpose, as shown in Figure 1.

Regarding the data analysis, we adopted the Video Micro-Genetic Analysis. According to Meira (1994), this technique proposes a detailed analysis of the identified and selected events of videos produced and watched repeatedly, so that such selected events are closely related to the research questions that one wishes to answer. A similar structure of analysis was also used in this work to analyze data not from videos, but from observation forums and protocols.

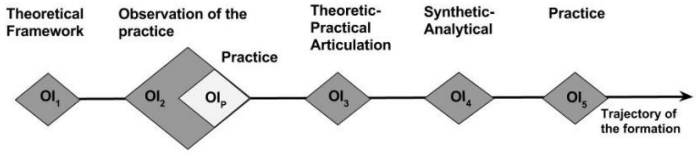

Figure 1: Student's formation trajectory

Analysis of the Results

By recording the students' experience within the meta-situation, with the support of the micro-genetic analysis, we have mapped events of different natures that reveal the students' knowledge about theoretical elements of instrumental orchestration, such as: elements of didactic configuration and mode of execution, genesis instrumental, the difference between artefact and instrument, the sense of integrate a technology into a lesson. As regards the notion of didactic performance and the schemes, the students showed more difficulties.

An analysis of this trajectory also reveals how, and at which point of the meta-orchestration, the design instrumental orchestration ceases to be an artifact to be an instrument for students, in what concerns the use of analysis of an experiment to reflect and develop a better understand the theoretical model.

At theoretical background level, we have mapped the students' posts about the theory, regarding their connection with the elements of the theory. At the observation of practice level, we have identified, in the observation protocols, the criteria developed by the students, based on theory, to observe the execution of the instrumental orchestration (IO_p). We also analyze IO_p which identifies the student's' schemes and reveals theoretical elements in practice such as the importance of the choice of artefacts available, the role of each students, the instrumental genesis, etc. At the theory/practice articulation level, we mapped the events (forum posts) of the students that established a relationship between the theory studied and the observed practice (IO_p execution). At the level of synthesis and analysis, we will try to identify events extracted from the synthesis and analysis of IO_p carried out by the students. At the creation level, we will analyze the instrumental orchestration of the students, which aspects of the model is better carried out by the students.

Final Remarks

Some data already analyzed reveal that the design of the instrumental meta-orchestration has become an instrument for the students, helping them to understand both, aspects of theoretical model and of the theories underlying the creation of the theoretical model of instrumental

orchestration, for example: both the notion of genesis instruments such as didactic configuration showed to be understood. The same did not occur with the notion of scheme.

The didactic configuration of IO_p, presented to the students even before its execution, enable students' understanding of organization of a didactic scenario, decisions regarding availability of resources, definition of roles for each participant, proposition of a mathematics situation integrated with one or more artifacts, were the most respected points for them.

During the development of the theoretical model of instrumental meta-orchestration, the flexible methodology of design experiment has gradually allowed us to define and refine the elements of the model, such as meta-situation, didactic meta-configuration, while allowing us to validate and / or enhance such constructs. The first experiment with a master's class led us to define the model itself; the redesign led us to need the notions of didactic meta-configuration and meta-mode of execution. However, only after analyzing the data of the experiment, we did arrive at a definition of meta-situation.

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Prospective teachers' interactions with interactive diagrams: semiotic tools, challenges and new paths

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The goal of our research is to look closely at prospective teachers' (PTs) interactions with curriculum interactive materials and to provide practical understanding of possible ways for teachers' and PTs' supporting the development of new practices for teaching-learning processes with the new kind of materials. The PTs in the study were involved in developing new practices and experiences during their interactions with the interactive materials and the students' outcomes. The three stages procedure of the interactions served as an intermediate stage between what normally may happen in the classroom according to the initial PTs' orientations and the implementation of the innovative practices in their future teaching with the new kind of materials. The semiotic framework for pedagogical functionality of the materials and the three stages procedure enabled facilitating the PTs' design processes.

Keywords: *Mathematic teachers' resources, interactive textbooks; student-textbook-teacher interactions; semiotics; prospective teachers.*

Introduction

One of the challenges that educators of prospective teachers (PTs) face today is to support PTs in using interactive textbooks for teaching-learning processes. New technologies allow the production of interactive textbooks that are envisioned as allowing the learner and teacher to approach texts in an exploratory mode, rather than simply receiving it in a fixed, prepackaged form. The mathematical activities in interactive textbooks are presented by software applications, which we call interactive diagrams (IDs). IDs can be used for different purposes: an exposition, a task, an exercise, etc. The IDs' include various interactive representations, tools that allow linking between the representations in the text, as well as tools that allow constructing, manipulating and activating examples. To address the pedagogical issues concerning the components of interactive textbooks, Naftaliev and Yerushalmy (e.g. 2017) have developed and elaborated a semiotic framework for analyzing the pedagogical functionality of IDs. The framework for pedagogical functionality is a set of categories that help educators (teachers, designers, researchers, etc.) to decide how to use, design, or choose an ID to support their educational goals. The semiotic framework proposed three dimensions (Table 1) for defining the pedagogical functionality of IDs that address a variety of learning and teaching settings: presentational (refers to a type of example in the ID), orientational (refers to a mode of representations in the ID), and organizational (refers to the connection between all the components of the ID).

For teachers, adopting interactive textbooks, which are tailored to deliver the curriculum and are designed to accommodate the inquiry situation, poses new challenges. To allow students' engagement with interactive texts, teachers must figure out how to promote and guide the

exploration while bridging the tensions between standards required by the curriculum, opportunities for students' active personal learning, and teachers' orientations (beliefs, values and preferences) (Naftaliev, 2017).

Presentational function	Orientational function	Organizational function
Specific Random Generic	Schematic Metric Schematic and/or Metric	Illustrating Elaborating Guiding

Table 1: The semiotic framework (Naftaliev and Yerushalmy, 2017)

Studies of the last years indicate that integration of technological interactive curriculum resources, and changes in mathematics teaching associated with these resources are challenging processes for a teacher (e.g. Trouche et al. 2012). Reasons for the teachers' reluctance to use the new approach include (a) teachers' proficiencies in mathematics and their perceptions of the nature of mathematical knowledge and how it should be learned with technology; (b) their understandings of the pedagogical principles required to teach mathematics with interactive resources (e.g. Pepin et al. 2013; Monaghan and Trouche 2016). Studies based on extensive research populations have accumulated vast knowledge on mathematics teachers' education (see e.g. Rösken - Winter et al. 2015), though little of this knowledge is related to the design of teachers education programs involving usage of learning technologies , and little is known on the impact of such programs on teachers' professional education.

The goal of our research is to look closely at PTs' interactions with curriculum interactive materials (IDs) and to provide practical understanding of possible ways for teachers' and PTs' supporting the development of new practices for teaching- learning processes with the new kind of materials.

The first study in the research (Naftaliev, 2017)

Twenty-five PTs participated in the study. The PTs were involved in the next three steps of the research process: (a) developing teaching plans based on similar IDs distinguished by the designed semiotic functions (developing Intended Curriculum); (b) analyzing classroom scenarios demonstrating interactions with the IDs (analyzing Enacted and Experienced Curriculum); and (c) developing representations of scenarios about classes engaged with the IDs (imagination of Enacted and Experienced Curriculum).

This study implemented a media-rich environment LessonSketch. LessonSketch allows creating experiences around classroom scenarios performed with cartoon characters in the form of a slide show. "The range of controls available to comics makes them especially useful for representing scenarios that seldom exist" (Herbst et al. 2011, p. 94). These were particularly important for our study where we have dealt with innovation materials in student-curriculum-teacher interactions. Depicting the interactions by using the comic allows us to present it for the PTs' interpretation and allows the PTs "to project their own circumstances onto an interaction, to share their own perspectives" (Chazan & Herbst, 2011, p.12). The environment played two roles in our research: as a professional development tool and as a research tool that allowed us to examine the PTs' engagement with the interactive curriculum materials.

Our findings include evidence that the PTs were aware of the different IDs' functions in the three stages of the interactions with the materials. They developed teaching plans that included each of the similar IDs in different steps of the plans and defined different teaching-learning goals for each ID. The PTs used the Illustrating ID only for the presentation of a new activity around the specific given example. They suggested open inquiry with the Elaborating ID to develop a generic example for the familiar form of linear function expression. The Guiding ID was used for a systematic inquiry of the linear function expression' new form. The PTs were also aware of different learning outcomes of the students with the different IDs in the classroom scenario. So the semiotic framework for pedagogical functionality of IDs was productive as a guide in the PTs interactions with the IDs and may serve as a tool for teachers' professional development. However at the same time the PTs did not always take an advantage of the wide variety of options for teaching-learning processes available with the IDs and some of them preferred the well-trodden paths in teaching.

The PTs faced a number of challenges while interacting with the materials. One of the challenges was correspondence of the IDs' orientations to those of the PTs. The PTs were aware of various pedagogical possibilities of the IDs but some of them continue to use conventional instructional practices in their interactions with the materials. Another challenge the PTs dealt with involved discussing a balance between what the students learned by engaging in the variety of interactions that the IDs offer and what the students should learn according to the teachers' learning goals. The third challenge was that the PTs had difficulties dealing with students' knowledge developed while engaging with the IDs and designing teaching-learning processes to help them progress.

The second study in the research

Following the first study, the second study was managed and now it is the stage of data analyses. We added two new stages to the procedure. The first stage was the three-step procedure from the preliminary study. In the second stage the PTs had experience in learning math units with IDs. The stage finished with reflective analyses of their own processes of learning, the teacher's pedagogical decisions and what was the influence of the design decision on the processes of learning. For the PTs it was the first experience of such kind of learning. In the last stage the PTs designed their own unit for math teaching-learning with IDs and presented an episode of classroom scenario in which the class was engaged with the units.

Data analyses of the study is on its preliminary stages and will be completed and presented at the conference. We may say that from the preliminary data analyses we see changes in the PTs' interactions with the materials as teachers. We can conclude that in order that the PTs become able of guiding their students towards productive and meaningful processes of development the mathematical knowledge, they should first of all experience such processes on their own.

The literature describes that teachers have a mode of interactions with textbooks, which have developed along their use of various textbooks, and conduct their using of a new textbook, independently of the features of this textbook (e.g. Pepin et al. 2013). Moreover, an important condition for adoption of curriculum materials is its potential integration into the teacher's 'normal' practice. Teaching with an interactive textbook should be considered more than a technological change; indeed, it is an attempt to create new paths for the construction of mathematical meaning.

Developing the new practices and experiences in teaching-learning with interactive textbooks is a necessary condition for PTs' implementation of these sources in their future teaching. The PTs in the study were involved in developing new practices and experiences during their interactions with the IDs and the students' outcomes. The three stages procedure of the interactions served as an intermediate stage between what normally may happen in the classroom according to the initial PTs' orientations (beliefs, values and preferences) and the implementation of the innovative practices in their future teaching with the new kind of materials. The semiotic framework for pedagogical functionality of IDs and the three stages procedure enabled facilitating the PTs' design processes, to share, to discuss and to modify their decisions.

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Creation of innovative teaching situation through instrumental genesis to maximize teaching specific content: acid-base chemical balance

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Abstract

This paper reports the creation of an innovative teaching situation for acid-base balance content. This "situation" occurred through instrumental genesis process. Here the computer is the artefact that becomes an instrument when a new element is introduced. The situation reported herein was observed during classes given to graduating student-teachers of Licentiate Course of Chemistry of Feira de Santana State University, Brazil. The theoretical framework is based on several concepts integrated: documental and instrumental genesis, instrumentation and instrumentalization, orchestration and anthropological theory of didactics.

Keywords: Teaching situation, innovation, instrumental genesis.

Introduction

Due to difficulties to teach notion of acid-base chemical balance in class, always reported by chemistry teachers community, there's a constant need to make methodologies more effective. Teachers and students' usual difficulties are related to resolution of problems caused by lack of a better control of variables, use of terminology, students' misconceptions, etc. (karpudewan, *et al*., 2015). I order to address such difficulties is imperative to create innovative teaching situations. And this is the main issue of this paper: report and analyze a "teaching situation" specially created to maximize teaching acid-base chemical balance content among graduating student-teachers in Bahia, Brazil, in the course of 1st academic semester of 2017.

The theoretical framework of this study is based on several integrated concepts which are implicit but splitted in the body of this work: documental and instrumental genesis, instrumentation and instrumentalization, orchestration (Artigue, 2002; Trouche & Drijvers, 2014; Rabardel, 1995), anthropological theory of didactics (Chevallard, 2009).

Praxeology

Conception of "teaching situation" mentioned above is taken from didactic engineering of study and research path (SRP) which is unfolded from first generation engineering to develop teaching devices (Chevallard, 2009).

Apparently the object of study is content "the chemical balance", which is a central subject for enabling appropriation of chemistry concepts. The selection of this subject wasn't by chance. The teaching situation occurs during the classes given to student-teachers of Licentiate Course of Chemistry of Feira de Santana State University. The data was collected in a universe of 14 students, seven 3-hour sessions, total of 21 hours/class, period of May and August 2017. The model of study

adopted was questions and answers, and the students worked individually. The narrative is divided into six moments of study following the Chevallard model.

1st moment - lasted a total of 6 hours (two sessions). The point of departure is when the teacher proposes this type of task (T): "how to determine concentration of acid solution". In the sequence, all students started moving to find the answer for the question, Q_O, by using all resources available on the occasion: their own previous knowledge of chemistry, consulting handouts, books online, Google search, asking teacher and classmates. In the end of efforts, the results were: 14,4% achieved answer expected, 28,5% got to incomplete answer and 57,1% didn't find the answer and had difficulty to deal with the task. Such results served as initial data collected for the analysis and also helped to measure the degree of previous knowledge of the group, and provide "insights" to develop didactic organization.

Next step was reviewing in class a historical and epistemological approach of chemical balance notion: its genesis, evolution, and main researchers on the subject. The approach was necessary to create conditions to propose next task (t_1): "determine concentration of a vinegar sample [solution of acetic acid, CH_3COOH]".

Then, the group starts building a technique (τ_1), based on volumetric method (experimental titration), needed to perform the task (t_1). Actually, students are familiar with the technique already. With this, the group was able to conclude the task after a certain time, achieving reasonable positive results.

Here is the point which requires to stop and reflect: to meet the need "to make methodologies more effective", the main issue of this paper, "reasonable positive results" are not sufficient "to create innovative teaching situations". They could be better, in many aspects: in terms of time consumption to find solutions and range of students capable to consistently assimilate and appropriate concepts. In short, we needed something new and different to potentiate learning, specially content herein.

The solution emerged by introducing a new element, software titrAB, a free web applicative designed to simulate acid-base titration whose some basic functions are: allows users to simulate acid-base titrations, under various conditions by controlling variables as acid strength, concentrations of samples, colored indicators, etc. The results are provided in tabular and graphical forms. Concentration values of sample to be determined are shown in the page. Like a game, it presents 4 levels of difficulty and number of "wrong" attempts is shown on screen.

For both students and teachers, the advantages of software are quantitative and qualitative. For example:

In a 50 minute-class, when applying traditional method (experimental) students are capable to perform only one experiment. The software allows performing 3 or more tasks on the average within same timeframe. While running experiment is possible to view/control alterations occurring with variables. It drives students to better develop and describe their experiments in reports/protocols with clarity and orderly. Consequently it potentiates their capabilities and

communicability. As a game, ludicrous aspect really contributes for a better environment in class, holding students' attention.

For teachers and researchers specific advantages are: allows them to measure degrees of students' difficulty with more accuracy; and acts as scaffold in mediation of appropriation of concepts. Experience of students with such practices [task/technique (t_1/τ_1)] is to enable them to perform more complex tasks and describe them in protocols, in a way they can be followed/reproduced by their peers (4th moment reflects it).

2nd moment – here is when creation of a "new teaching situation" effectively occurs, by introducing the innovation named software titrAB through process of instrumental genesis. The artifact is the computer which, hence, became an instrument of chemistry with the mediation of software titrAB. Actually some students' cell phones platforms too. Both computer and cell phones became chemistry instruments in the environment. Next, the teacher gives instructions how to apply parameters defined at the top of the software page.

When teacher assumes the students are already familiar with the instrument and simulation technique (t_1/τ_1), it's time to propose task again "how to determine concentration of vinegar sample". Evolution of results can be compared as it follows:

Students performance	**Experimental Method/τ_1**	**1st simulation software/τ_2**	**2nd simulation software/τ_2**
Hit upon fully	14,4/%	21,4%	71,4%
Hit upon partially	28,5%	42,8%	21,4 %
Failed, partially or fully	57,1%	56,9%	7,1%

In addition to results, the more students get familiar with the technique and the instrument less time is needed to perform same task. The benefit of reduction is maximizing learning in class. The teacher and students have more time to perform more tasks and discuss theoretical aspects in depth.

3rd moment: here is where documental genesis strongly occurs and it is reflected by manipulation of different didactic materials available (papers, books, handouts, protocols), and also by development of other ones in different platforms (web). Construction of a theoretical – technological environment that has been occurring since first meeting with group enabled this moment. After 2 first moments the acid-base theories of Arrhenius, Brönsted-Lowry, Lewis had already been appropriated by student-teachers.

4th moment: here, the simulation technique with software τ_2 is applied to perform task t_2: develop a protocol describing experience with titrAB, by using the volumetric method again, which had been required in task $\mathbf{t_1}$, in the beginning of SRP. From discourse of techniques is possible to evaluate theoretical/technological block [θ/Θ] which justifies both techniques [τ_1/τ_2] employed to perform $\mathbf{t_2}$. Now the students carry out the task by following protocols written by group, as it is told in 2nd moment (see comparative table above).

5th moment: here is possible to note in depth quality of student-teachers communicability. It is expressed in their individual reports and protocols which validate learning. Now is the time of reading all material in class with their participation. It is a very important practice which strongly

contributes to develop their critical thinking and analytical view. That's what is expected from them institutionally: appropriation of concepts and terminology, and didactic organization.

6th moment – here is the time for an overall evaluation and check extent of introduction of all elements and application of concepts in which this study is based.

Conclusions

How was the appropriation of the concept "acid-base chemical balance" By using the software?

The more students got familiar with the technique and the instrument less time was needed to perform same task. This time reduction has maximized learning and consequent appropriation of the concept. The teacher and students had more time to perform more tasks and to discuss theoretical aspects in depth.

Besides, experience of students with task/technique block enabled them to perform more complex tasks and describe them in protocols. While writing the protocols the students faced difficulties which pushed them to learning how to "mathematize" acid-bale balance notion. This process led them to deeper reflections of Chemistry in human activity.

Here it is opportune to remind that this paper is just to report "the creation of an innovative teaching situation for acid-base balance" which occurred through instrumental genesis process. However, this episode is much more than that. There are two levels of reading. In fact, the first reading is the narrative of the instrumental genesis, itself, and the students' performance with tasks. The second reading is described in the following additional conclusions.

Among the diverse resources available for teachers today, it was also included in this training course develop students' competences to choose their appropriate resources to conduct their work. In other words, teach them to teach in a given situation.

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Planning of the teaching of the standard deviation using digital documentary resources

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This research concerns initial formation (DeBlois, 2012; DeBlois & Squalli, 2002) for the context of the documentary genesis (Gueudet & Trouche, 2010) of the teaching project of Christelle, a preservice teacher of mathematic. She was invited to explore a directory of digital documents in order to improve statistical contents of the Cameroonian textbooks, in particular the standard deviation. We observed how this preservice teacher moved from documentary resources to documents containing to involve documentary genesis. Thus, this article presents the variables of arrangement, particularly the way in which she introduces her planning of the teaching of the standard deviation, the manner in which she recognizes students' interest, the modifications she does in the contents of the documents she exploited.

Key words: initial formation, teaching statistics, documentary genesis, planning, standard deviation.

Problematic

In many developing countries and in some developed countries, textbook is at the same time the learner's book and the teacher's guide. It is thus a very important resource for teachers since their planning tasks are organized based on it (Lepik, Grevholm & Viholainen, 2015). It is therefore "the multifaceted mirror that reflects programs, classroom activities, but in which actors project their questions" (Bruillard, 2010, p. 230, free translation). Concerning these questions, Belinga (2009) criticizes the structuring of some Cameroonian school textbooks, whose contents are presented without taking into account the Cameroonian context.

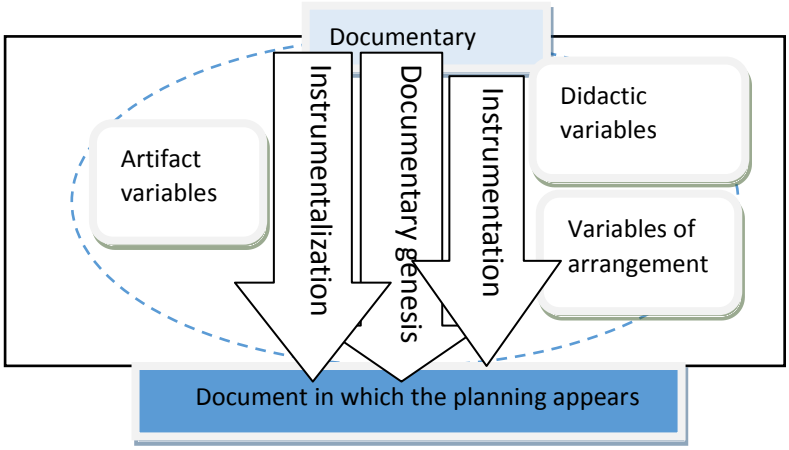
With regard to these educational contents, we are focusing, in this research, on the statistical contents to be taught in the 6th year of secondary (pupils aged 15 to 17 years), especially the standard deviation. So, Cyr and DeBlois (2007) noticed in their researches that several prospective teachers have difficulties in talking about statistical concepts and their interpretations. We hypothesize that planification could help preservice teachers to improve their understanding of the notion to teach. In fact, anticipate the difficulties encountered by the pupils could favor a teaching centered on the development of reasoning (DeBlois & Squalli, 2002). We thus propose to study the documentary genesis (Gueudet & Trouche, 2010) when a preservice teacher is exploiting documentary resources on measures of dispersion.

Theoretical Framework

During a teaching project, the preservice teachers can adopt three epistemological postures: the ancient pupil, the university student and the teacher (DeBlois, 2012). It can be characterized by the preservice teacher's role as "organizer" of his teaching and annalist of "problems or didactic materials". Thereby, Gueudet and Trouche (2011) identify a variety of variables that may be involved in the orchestration of planning: the variables of arrangement, artifact and didactic

variables. The variables of arrangement constitute a system of scenarios of didactic exploitation linked to knowledge to be constructed with the aid of artifacts (Trouche, 2007). For him, artifacts are designed by the teacher for the class, and integrate the usage scenarios, the modes of integration in the implementation of a teaching device which represents in our research the document in which the planning appears. We characterize the decision taken by the preservice teachers when they are realizing the document in which their planning appears as it is illustrated in the following figure.

Figure 1: orchestration of a planning



In this article, we will observe the variables of arrangement of the documentary genesis (Gueudet & Trouche, 2010) when Christelle realized her planning. We will then answer the following research question: How does the preservice teacher Christelle take variables of arrangement into account when she is realizing the document in which her planning appear?

Research Method

This document presents the case-study of a preservice (Christelle) of the fifth academic year in Higher Teacher Training College (HTTC) of the University of Yaounde I. The data collected were consisted of 2 seminars, 1 planning of the teaching of the standard deviation and a semi-structured interview. Christelle participated in a first seminar with these 5 other peers. During the seminar, she discussed with these peers on the different components that can be considered when preparing the planning of the teaching of statistics. At the end of this seminar, we have put at her disposal a Cameroonian school textbook (L'Excellence en Mathématiques [6th year of secondary]) and a website[1] containing the links for access to several digital documents which can help in the teaching of statistics. This website contains lesson plans, scientific and professional articles and worksheets. Thus, the preservice teacher has to carry out a planning of the teaching of the standard deviation at home by exploring the documents of this website put at her disposal to enrich the contents of the textbook. After she completed her planning of the teaching of the standard deviation, she carried out a second seminar with these 5 other peers. During this seminar, she discusses with these peers about the differences arrangement to obtain the document in which the planning appears. Finally, she answered a semi-structured interview to compare the results obtained during the seminars and in her planning. The seminars and the semi-structured interview were recorded on audiotapes and transcribed word for word. After the transcription of the data, we analyzed all the data collected (planning, transcription verbatim) according to the characteristics of the variables of arrangement.

[1] https://statistiqueeduc.fse.ulaval.ca/

Analysis Results

Nongni and DeBlois (in press) showed variables activated by Christelle. They observed that for her the use of documents open mind about teaching a concept, to be attentive to a diversity of examples, activities and topics to offer students but requires discernment to select the content in connection with the official program, and to avoid presenting erroneous content. In addition, she offered to know what it is happening when we add 2 to all data for one distribution or when we multiply by 3 all the values of one distribution. At the end, it seems that arrangement variables could be a springboard to develop an anticipation of the didactic variable. It is why we study this variable.

During the seminar 1, she suggests that teaching be oriented towards contexts close to the students' reality, such as their marks and ages. After her planning, she participated to a seminar 2. At that moment, she affirms that the students will be interested in her planning because it is geared towards a comical context (number of love letters written by the students). For Christelle, it is a context that will make students laugh. In addition, her analysis of problems and didactic materials helped to deepen her mathematical knowledge by situating her teaching project from her epistemological posture as a university student (DeBlois, 2012). Thereby, she mentions how to bring the students to build the notion of a standard deviation in three points.

First, concerning the introduction of concepts in her planning, we observed that she introduces her teaching of the standard deviation with a problem situation inspired by Cyr and DeBlois (2007):

Activity: For two groups A and B of 7 boys of the 6th year of secondary, here are the different number of love letters written by each boy to a girl during the month of March: Group A)1-1-1-2-10-10-10 Group B)0-0-6-6- 7 - 7 - 9 1) For each group, what is the number of love letters written in March? 2) What is the most balanced group (the group in which there is the least deviation between the numbers of love letters written by the pupils)? (Extract from the planning of Christelle)	Let us take the following three groups which represent the goal scored by each player of three different teams during a football season Team A) 1-1-1-2-10-10-10 the range is 9 Team B) 4-4-5-5- 5 - 6 - 6 the range is 2 Team C) 0-0-6-6 -7 - 7- 9 the range is 9 The goal is to identify the team's best-balanced attack (the team in which there is the least deviation between the numbers of goals scored per player… Thus, the deviation of each of the data from the average becomes the central element around which we can introduce the idea of the standard deviation. [extract from the article of Cyr and DeBlois (2007, p. 56)]

Given that these distributions have the same average, her objective is to allow to observe the limits of the average in order to bring the attention of the pupils on how the data are distributed around the average. We observe that to construct her activity, she uses the distributions of data having the same average and range, and she changes the context.

Second, her planning contains several aspects inspired by the documents that don't appear in the textbook. Thereby, her planning informs that unlike the standard deviation, the average deviation doesn't emphasize the influence of extreme data. We also observe some properties like those used to recalculate the standard deviation when one adds or multiply the modalities with a same number. For example, "if we multiply all the values of a statistical series by the same number k, the variance will be mutilated by k^2 and the standard deviation by $|k|$" (extract from the Christelle planning). These properties seem to be the springboards that allow the use of didactic variables.

Third, regarding the interpretation, her planning of standard deviation teaching contains questions centered on the interpretation of results obtained through by comparing the values of the standard

deviation of several data distributions. Apart from interpretation, her planning does not reveal other criteria related to the development of statistical reasoning. For example, a picking or regrouping of data by pupils, the use of real data to enable pupils to test the conjecture, and class discussion in order to develop the critical thinking of the students (Garfield & Ben-Zvi, 2009).

Discussion

The variables of arrangement of the documentary genesis of Christelle are characterized her way of recognizing the interest of students, by a problem situation in order to bring the attention of the pupils on how the data are distributed around the average and by the difference between the average and the standard deviation. These last two properties seem to develop an anticipation who could explain why variables of arrangement contribute to the understanding of didactic variables. Then, it seems that her preoccupations forms in the posture of the university student who analyzes the documentaries resources and who discusses with her peers could shift in her project of teaching towards the epistemological posture of the teacher (DeBlois, 2012).

Conclusion

The documentary genesis of Christelle by using documents is manifested by going beyond the content of textbook to improve her planning. Her improvement is observed by the properties (previous enumerate). However, we observe that Christelle might need support, not only to know how to determine the subjects that may interest the pupils, but also to arrange a teaching of the standard deviation by orienting it towards the development of statistical reasoning.

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Pre-service mathematics teachers' investigation of the constraints of mathematical tools

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This study examines pre-service mathematics teachers' investigation of the constraints of mathematical tools while they were working out the Triangle Inequality theorem. We use constraints of epistemological, didactical and ontogenic origins as our theoretical lens. Nine student teachers used hands-on manipulatives and GeoGebra. Constraints on how tools signify mathematical objects (e.g., points and sides) and subtle representations of mathematical objects are discussed.

Keywords: instrumentation, tools, constraints, the Triangle Inequality theorem

Documental genesis demonstrates a process in which teachers use a set of resources (e.g., tools, worksheets, activities, etc.) influenced by their scheme of utilization (and techniques). Affordances and constraints of a tool (instrumentation) and methods that teachers employ to use the tool (instrumentalization) give rise to this influence (Gueudet, & Trouche, 2009). The process in which a set of curriculum resources becomes a teacher-created document is subject to modifications in the scheme of utilization with successive planning, revisions and refinements in the resources (Gueudet, & Trouche, 2009). Monaghan (2016) bisects an angle using a book as a construction tool and demonstrates that tools may not have a pre-assigned function. Similarly, teachers may alter tools when they teach a mathematical topic and play an important role in identifying the affordances and constraints of tools (Leung & Bolite-Frant, 2015). However, research studies most often focus on the affordances of mathematical tools. There is little research that indicates how student teachers identify the constraints of tools. The current study demonstrates pre-service teachers' (PSTs) investigation of constraints of mathematical tools while they were working out the Triangle Inequality theorem. Our research question is: "In what ways are student teacherreported constraints of mathematical tools related to the didactical constraints of tools?"

Theoretical Framework: Didactical Constraints of Tools

When students are given a mathematical task, they may encounter *didactical obstacles*. Brousseau (1997) identifies different origins of didactical obstacles. Obstacles of *epistemological* origin are helpful in constructing new knowledge as students link their prior mathematical knowledge to accomplish the task. Obstacles of *didactical* origin "are the result of narrow or faulty instruction" (Harel & Sowder, 2005, p.34). Harel and Sowder (2005) emphasize that students' thinking may include epistemological and didactical obstacles such as "multiplication makes bigger" belief. *Ontogenic obstacles* are the result of developmental skills (e.g., neurophysiological development) or lack of prior knowledge.

Obstacles serve as important constructs for a tool-based learning environment (Joubert, 2013; Leung & Bolite-Frant, 2015). For example, a tool may give rise to a faulty instruction if it does not stay true in mathematics–that is also known as *mathematical fidelity* (see Dick & Hollebrands, 2011). Lack of mathematical fidelity in a tool is a didactical constraint. On the

other hand, tools with epistemological constraints may push students to think harder and link their prior mathematical knowledge to come up with new ideas, refine their thoughts, etc. An ontogenic constraint of a tool may include (subtle) mathematics that students are "not ready for the mathematics required to complete the task" (Joubert, 2013, p.71). For example, the *Tangents* tool in GeoGebra constructs two tangent lines to a circle through a point outside the circle. This tool hides the complexity of the construction and the mathematics used.

Methods

A case study (Stake, 1995) was designed to characterize pre-service teachers' investigation of the constraints of mathematical tools while they were working out the Triangle Inequality theorem. The participants of the current study were nine PSTs (4 female, 5 male) who were enrolled in an elective methods course, Teaching Geometry. The class met three hours per week and the first author was the instructor. The data were collected during in-class activity. PSTs worked in groups of three, and identified the affordances and constraints of mathematical tools in three rounds. The researchers provided a set of pre-determined mathematical tools and each group used one tool per round. A worksheet guided PSTs to investigate the Triangle Inequality theorem and each group created a poster. Afterwards, they shared their findings during the whole-class discussion.

In the first round, PSTs used rods and spheres, geo-strips with split-pins (henceforth geostrips) and geo-strips without split-pins (henceforth poly-strips). Each sphere (vertex ball) is 1.7 cm in diameter and has small holes provided on its spherical surface. Small holes on the geo-strips are provided to build different triangles with split-pins. The poly-strips can be joined together on the corners only. In the second round, PSTs used string and scissors, bars made of construction paper, and a GeoGebra sketch. Students adjusted the given lengths in the worksheet using sliders in GeoGebra and swung the endpoints of the line segments to check if it made a triangle. In the third round, PSTs used rulers and straightedges, fraction bars, and a GeoGebra sketch in which the endpoints of the line segments were preset to be joined together.

Each group of PSTs was videotaped. Triples of students using a specific tool in this task is our unit of analysis (Stake, 1995). Each PST contributed to identify the constraints of mathematical tools during in-group and whole-class discussions. Data were transcribed and PSTs' designations for the affordances and constraints of the tools were identified. However, the current study is limited to the constraints of the tools. Due to the limited space, we present results concerning the constraints of tools in mathematical representations of objects. In other words, we characterized student teacher-reported constraints that were associated with mathematical representations, and categorized the constraints using the theoretical lens of didactical constraints.

Results

According to the participants, mathematical representations of line segments and vertex points are important as they are figural units of a triangle that are in a lower dimension (zero- and one-dimensions). However, they stressed that tools, except compass and straightedge, had limitations in signifying the vertex points and sides of the triangles. For example, line segments had thickness and were in three-dimensions in most of the tools. Also, one of the students demonstrated that the sides she cut out of string did not stay straight. Some participants raised questions about the way that line segments were joined together. For example, poly-strips had

oval vertices (Figure 1). They believed this tool could be confusing for students because they may focus on the inner or outer triangular regions of the tools (Figure 1). Therefore, understanding what the tools signify could be an epistemological obstacle for students.

Figure 1: Poly-strips

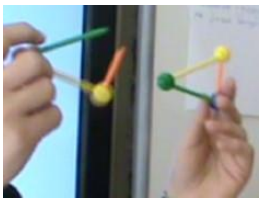

Figure 2: Rods joined different holes

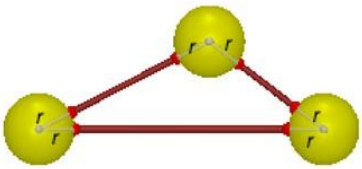

Figure 3: Subtle representation together using of a triangle

Some PSTs had difficulty assembling the sides of the triangles. For example, Murat used two pairs of color-coded side lengths and illustrated two ways of assembling the rods (Figure 2). He said: "…there are a number of holes on the spheres. We both can make a triangle and cannot make a triangle (Figure 2) using the same pair of color rods, which could be confusing for students…" He emphasized that finding the optimal holes on the spherical surface was difficult, in particular when they tried out the side lengths whose sum of two side lengths was close to the length of the third side. Also, the endpoints of the line segments did not meet on the spherical surfaces of the vertex balls. The tool provided a subtle representation, in that a length of *2r* was added to side lengths of the rods where *r* is the radius of the spheres (Figure 3). This subtle representation would be an ontogenic obstacle for students who have not learned the properties of spheres.

The participants most often tried out if the tools made a triangle when the sum of the two side lengths was equal to the length of the third side. In the first GeoGebra sketch, students noticed that it made a triangle because the endpoints of line segment had a size (Figure 4). The representation deviated from the intended mathematics. Some students appreciated the second GeoGebra sketch did not make a triangle if the sum of the two side lengths was equal to the length of the third side. However, in that sketch, two of the sides went missing (Figure 5). One of the students, Dilek, raised questions about the representation of the missing two side lengths as shown in Figure 5. She emphasized that the reason why it did not make a triangle was because the three side lengths lay on the same line segment. She modeled the 3 cm, 3 cm and 7 cm side lengths using the Whiteboard (Figure 6) and pointed out the subtle representation in GeoGebra. Accordingly, the design of the task gave rise to an epistemological constraint.

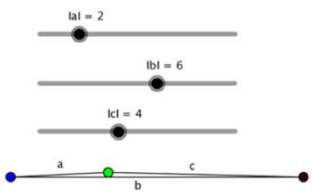

Figure 4: An unexpected triangle representation in GeoGebra

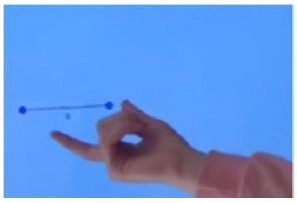

Figure 5: Subtle representation of triangle

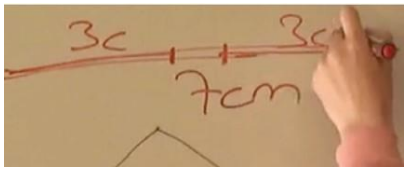

Figure 6: Dilek unveils the subtle representation in Figure 5

Concluding remarks

In this brief report, we presented PSTs' evaluation on the representations of mathematical objects. PSTs identified the constraints of the tools considering from the student perspective

(e.g., difficulties they may encounter when they use the tools). The results suggest that tools may not signify mathematical objects sufficiently and may give rise to didactical or epistemological obstacles. Also, subtle representations in tools may cause epistemological or ontogenic obstacles, which may require further investigation of the representations deviating from intended mathematical objects.

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A proposal of instrumental orchestration to introduce eigenvalues and eigenvectors in a first course of linear algebra for engineering students

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We present an advance of our doctoral research on the complex process of instrumentation that students develop when using various artifacts in a teaching situation to introduce the concepts of eigenvalues and eigenvectors in a traditional course of linear algebra in engineering. The activity to be proposed involves a lot of numerical calculation, so the need to configure the different instruments is evident, to propose a certain instrumental orchestration. The objective of this work is to integrate digital artifacts in the teaching of mathematics, for once transformed into cognitive instruments, an orchestration can be configured. As part of the pilot phase of the study we observed two students. We present results of this case study that allowed to identify from interviews and classroom observations of the use of resources.

Keywords: Instrumentation process, linear algebra, instrumental genesis, student behavior, digital technology integration.

Introduction.

In spite of the recent appearance in the school curriculum of linear algebra (Cowen, 1997), the current digital age positions it as one of the most important mathematics subjects in the study of engineering and sciences, due to its possibilities of application to the solution of very diverse problems in different disciplines (Salgado & Trigueros, 2015; Sierpinska, 2000), including mathematics itself. On the other hand, the reports of problems in their teaching are frequent, where research indicates it as a difficult subject for both teachers and students and a great obstacle for the engineer; due to multiple factors such as the enormous burden of formalistic rigor, causing frustration, bewilderment and dropping out of school (Salgado & Trigueros, 2015; Hillel, 2000; Dorier et al., 2000; Carlson et al., 1993; among others). The concepts of eigenvalues and eigenvectors become part of the school curriculum in the latest topics to develop.

In this sense, we present an advance of our doctoral research on the complex process of instrumentation that students develop by using various artifacts in a teaching situation to introduce the concepts of eigenvalues and eigenvectors in a traditional course of linear algebra in engineering.

Background

In this section, we provide a brief description of the investigation. We conclude with a brief introduction to eigentheory in the hierarchical analysis process.

There is little research that has been done on the teaching and learning of the eigenvalues and the eigenvectors. In Thomas & Stewart (2011) they report that students struggle to connect the two

mathematical processes captured in $A\vec{x} = \lambda x$ (on the left side is the process of multiplying a matrix by a vector; on the right side is the process of multiplying a scalar by a vector however, on each side the final object is a vector), where A is an $n \times n$ matrix, $\vec{x}$ is a vector $\mathbb{R}^n$, and λ It is a scalar, to give sense of equality. The authors emphasize that the difficulties of the students are due to the fact that many teachers and texts do not introduce the geometry of eigenvalues and eigenvectors. They explained that, although students know the algebraic procedures that participate in the search for values and eigenvectors, when the geometric representation is left aside and the relationship of algebraic and geometric representation is not discussed, many students learn these concepts mechanically, while that when both representations are considered in the teaching of these concepts, students understand the concepts better.

Salgado & Trigueros (2015) report the design of a situation in an economic context by taking an economic model as a reference, in parallel, they designed a preliminary genetic decomposition for the concepts of own value, own vector and own space. Likewise, a questionnaire was designed in terms of genetic decomposition. The results of the interviews showed that some students presented some barriers that seem difficult to overcome. For example, the relationship between the number of eigenvectors corresponding to a given eigenvalue and their coordination with the dimension of the space generated by the associated eigenvectors to each proper value. The construction of the concept of own space also proved to be difficult for most students.

Brief introduction to eigentheory in Analytic Hierarchy Process

The Hierarchical Analysis Process (AHP) is a method for making decisions that involves multiple criteria, developed by Saaty (1977). Based on the pairwise comparison matrix, AHP estimates a priority vector of the factors involved through the eigentheory.

Theoretical framework

The research will be developed from the educational model Cuevas & Pluvinage (2003), a didactic that initially proposes the need for a practical action project that seeks to achieve a better understanding and, consequently, internalization of mathematical concepts. That is, propose a problem that is of interest to students and in this sense the development of digital technology comes to offer multiple examples of application of linear algebra. But the problem that arises is that a simple example referred to some application of digital technology offers a complex numerical calculation, from where the need to include within our configuration several software's that allow the student to overcome these obstacles.

Instrumental Orchestration

The integration and use of new technologies in mathematics education has had an impact and even in many cases anarchic. The digital era in which we have entered induces dramatic changes in the way we access information and build knowledge, in the way we communicate, interact and work (Artigue, 2016). At present there is indeed a variety of mathematical graphical, symbolic and numerical software (Matlab, Mathematica, Derive, Octave, Geogebra), Computer Algebra Systems (CAS). Professional mathematicians and engineers know that these sophisticated new tools do not become immediately efficient instruments teaching-learning for the student: "its complexity does not make it

easy to master, and fully benefit from, their potential" (Artigue, 2002, p. 245). The instrumental approach (Guin & Trouche 1999) is a framework that allows taking into account the role of technology in learning and teaching mathematics. The role of the teacher within this framework is fundamental, for example, recognizing that the instrumental geneses of students should be guided by the teacher through orchestrations (Drijvers et al., 2010). The teacher, in this process, has to analyze the characteristics of the technology, taking into consideration previous research in the field of computer-based learning environments, so as to anticipate the changes it may introduce to the learning environment (Guin &Trouche, 1999). Also teachers teaching in a computer room spend a lot of time to technical scaffolding while technology would help their students to work alone and that they could act as a catalyst for mathematical thinking (Lagrange & Monaghan, 2009). Guin & Trouche (2002) emphasize the importance of choice and the way in which activities are promoted by the teacher to make an effective learning tool. A way to configure all the activities to optimize the results is instrumental orchestration. However, this task is not simple Guin & Trouche (2002) defines instrumental orchestration through four components: a set of individuals; a set of objectives; a didactical configuration and a set of exploitation modes of this configuration.

Methodology

The methodology followed in the doctoral research is the one proposed by the action-practice project (Cuevas & Pluvinage, 2003). The data for this preliminary report will be obtained from students in the 5th semester (3rd year) at a public university in Mexico, from a linear algebra course with 4 and a half hours of class per week (2 weeks), in a Computer lab (25 PC's); equipped with Computer Algebra System: Octave, Dynamic Geometry Software: Geogebra, paper/pencil, calculator CASIO fx-991MS. A pilot phase 1 was carried out, in the computer room with three sessions of an hour and a half per week, the students worked in pairs. The activity, integrated by a didactic sequence, was carried out with an adaptation of the hierarchical analysis process. It was proposed to 35 students, but only one pair is analyzed (to reinforce the weak points of each one). Next pilot phase 2: it will be developed in a group of about 40 students and its purpose will be to complement the data obtained in the pilot phase 1, in order to design and implement the appropriate mathematical activities.

The results obtained and analyzed will improve the design and the sequence will be applied again, without the arithmetic-algebraic calculations being a complex obstacle in the reflection and acquisition of such concepts.

Conclusion

It was thought that the integration of technology was something simple, it was enough to give the students the technology and explain how to use it. The action-practice project does not contemplate the behavior of the student, the process of appropriation of an artifact and construction of an instrument (instrumental genesis), how they instrumentalize the computer and the mathematical effects of this transformation process, the individual and collective control of a tool.

For our purposes the action-practice project (Cuevas & Pluvinage, 2003) can be combined with the theory of instrumental genesis (Guin & Trouche, 2002) thus forming a didactic engineering with more elements related to our requirements. Our study will allow us to identify these levels of resource orchestration in the classroom.

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Teaching and learning of function transformations in a GeoGebra-focused learning environment

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Using the Theory of Instrumental Genesis (TIG) as a theoretical framework, this exploratory study investigates which instrumentation processes are dominant in a GeoGebra-enhanced mathematics-learning environment to support students to develop an understanding of function transformations. Students in a bridging program at a South African university participated in this qualitative study. The results showed that the instrumentation of GeoGebra afforded an opportunity to better understand function transformations. Qualitative discourse and content analysis showed students' thinking moved to higher levels of abstraction. The results affirm the potential benefits of using GeoGebra to teach function transformations that lend themselves to technology integration. The results also showed that the researchers' instrumental orchestration of GeoGebra as a teaching resource afforded the students instrumentalise the artefact to construct physical and logico-mathematical knowledge.

Keywords: Instrumental genesis, instrumentation, affords, gestures, knowledge theory.

Introduction

Through interaction with students in a Science and Mathematics at the University of Stellenbosch (SciMathUS) bridging program, the researchers noticed that their beliefs were that Mathematics should be memorized. They wanted to be given notes on procedures and a few examples so that they could memorize them. The researchers, however, observed that the students could not explain why they were executing certain algorithms in a specific way. Some of the students' responses when explaining their methodology were: "My teacher told me to do it in this way" or "I did it like that last year". Results from a pre-questionnaire at the beginning of the program showed that students did not view technology to be useful in the teaching and learning of Mathematics.

Lagrange and Monaghan (2009) argue that the availability of technology challenges the stability of teaching practices: techniques that are used in 'traditional' settings can no longer be applied in a routine-like manner when technology is available. In order to help teachers to benefit from technological resources in everyday mathematics teaching, therefore, it is important to have more knowledge about the new teaching techniques that emerge in the technology-rich classroom and how these relate to teachers' views on mathematics education and the role of technology as a teaching resource therein (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010). Drijvers, Godino, Font and Trouche (2013) also contend that a deep understanding of students' learning processes is one of the core challenges of research in mathematics education. The study is therefore building on the above views and illustrates how a teaching approach with GeoGebra was utilized to teach function transformations so that students can move to higher levels of abstraction. The lived experiences of students from this study can give teachers and researchers some insights on how to utilize GeoGebra in the teaching of this topic specifically and others generally.

Theoretical frameworks

This study considers the following theories to underpin the knowledge base of the phenomenon to be researched: Theory of instrumental genesis (TIG) and Piaget's types of knowledge theory

Theory of instrumental genesis (TIG)

The main theoretical framework that informs the investigation of the students' progression to higher levels of abstraction in dynamic geometry environments (DGE) is the theory of instrumental genesis (TIG). The TIG ascribes a major role to artefacts that mediate the human's activity for carrying out a task (Drijvers et al, 2013). Trouche (2004) describes instrumental genesis as a combination of two processes: an instrumentation process and an instrumentalisation process.

According to Fahlgren (2015) instrumentation is how the user shapes the artefact by his/her knowledge and previous working approaches. Fahlgren (2015) adds that instrumentalisation, by contrast, is how the artefact shapes the user by its constraints and potentialities. Ndlovu, Wessels and De Villiers (2011) also view instrumentation as the process by which the artefact influences the user by allowing him or her to develop activity schemes within some boundaries. Such limits include constraints which oblige the user in one way and impede in another; enablements which effectively make the user able to do something, and potentialities which virtually open up possibilities and affordances that favor particular gestures or movement sequences (Trouche, 2004). The instrumentation processes also guide the students' thinking towards a solution (Drijvers et al., 2013). Ndlovu et al. (2011) view instrumentalisation as the user's personalization (discovery and selection) of relevant features of the artefact to accomplish tasks in innovative ways, including those unforeseen by the designer.

The instrumental genesis focuses therefore on how the learners can use the affordances, potentialities and enablements of the GeoGebra resource to engage them in usage schemes that enable them to construct and represent function transformation concepts. TIG is also complemented by die notion of instrumental orchestration, which refers to the teacher's intentional and systematic organisation and use (exploitation modes) of the GeoGebra artefact in a given mathematical task situation, to guide students' instrumental genesis (Drijvers et al, 2013).

The TIG framework in this study explains the role of the teacher-researcher's instrumental orchestration of GeoGebra to create a learning environment to teach function transformations.

Piaget's types of knowledge theory

Piaget (1952) differentiates between three types of knowledge and how students learn each of them: social, physical and logico-mathematical knowledge.

Social knowledge refers to conventions that created by people over time (Kamii, 2014) and can only be gained through interaction with others (Lutz & Huitt, 2004). Physical knowledge is obtained through hands-on interaction with the environment (e.g. DGE). Exploring concepts with GeoGebra can help students acquire physical knowledge, which forms the basis for logico-mathematical thinking (Piaget, 1952). Logico-mathematical knowledge is an abstract reasoning that is applicable beyond physical interaction with a concrete stimulus (Lutz & Huitt, 2004). The manipulation of objects in different patterns and contexts enables generalizations and abstractions to be formed

(Lutz & Huitt, 2004). Students utilized GeoGebra therefore to make conjectures in order to make generalizations and abstractions. This study therefore also investigated how GeoGebra manipulation might enable students to construct physical and then logico-mathematical knowledge.

Research questions

The main research question which guided this study are:

How did GeoGebra afford students an opportunity to understand function transformations and move to higher levels of abstraction?

The main question will be answered with the following two sub-questions:

1. What instrumentation processes of GeoGebra enhance the learning of function transformations?
2. How did the students negotiate meaning whilst working with GeoGebra?

Methods

The study was done with a convenient sample of 48 Science and Mathematics bridging program students at Stellenbosch University (SciMathUS). The first author was also the teacher-researcher of the study. The hypothetical learning trajectory (HLT) for the teaching of function transformations was done with GeoGebra. The students started with the exploration of the properties of transformation geometry. The rationale behind this was that the researchers viewed the properties of rigid transformation and dilation as a prerequisite to understanding function transformations. The students were given a worksheet and had to create their own activities to explore the properties of transformation geometry in GeoGebra. This activity was followed the exploration of function transformations with given GeoGebra applets.

The lessons were video recorded and analysed to report on gestures that students utilized to supplement the visual affordances of GeoGebra and help with the process of negotiating meaning. The different types of instrumental orchestrations utilized by the teacher-researcher were also observed. Both sets of activities were done in six sessions of 50 minutes each. The responses from in-depth and focus group interviews and, content analysis will focus what the students attribute to which instrumentation processes of how GeoGebra made an impression on them to understand function transformations better.

Findings and discussion

The analysis of students' interactions with GeoGebra is based on the information that students produced while they were exploring transformation of functions with GeoGebra activities in the computer lab and also from in-depth and focus group interviews. Students attributed their understanding of the concept of function transformations to the instrumentation processes of GeoGebra, i.e. *visual affordance*, *potentialities* and *enablements*. The analysis of the observations showed the teacher-researcher's different types of instrumental orchestrations utilized during the sessions in the computer lab.

The students explained to their group members, or teacher-researcher (first author), their instrumental genesis processes and usage schemes for constructing and representing the intended function transformations. In this way, they first engaged in informal reasoning and then moved to formal

reasoning. Students' responses from in-depth and focus group interviews also confirmed that most of them were better able to represent and explain function transformations. The responses, in a way, showed how instrumental genesis with GeoGebra helped them acquire physical or informal mathematical knowledge and more easily construct formal/logico-mathematical knowledge of the representation and meaning of $-f(x)$, $f(-x)$, $af(x)$, $f(x+a)$ and $f^{-1}(x)$ of exponential function. The modest learning gains in the post-test on transformation questions like, showed how the GeoGebra-enhanced instruction in this study fostered a firmer understanding. Some students used a technique, or heuristic, to solve this problem, such as drawing sketches to answer a question.

Challenges observed whilst the students instrumentalised GeoGebra in the computer lab and based on students' responses in the interviews, were constraints posed by syntax and menu commands.

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TPACK addressed by trainee teacher educators' documentation work

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In this paper we study the forms of technological pedagogical content knowledge (TPACK) addressed by trainee teacher educators' documentation work for teachers. The study took place in the context of an in-service program during the trainees' engagement in their practicum fieldwork activities. The documentational approach of didactics and the TPACK framework were combined to identify categories of documents and operational invariants.

Keywords: Documentational approach, TPACK, teacher educators' education, digital resources.

Introduction

In this paper we combine the documentational approach of didactics (Gueudet & Trouche, 2009) and the technological pedagogical content knowledge (TPACK) framework (Mishra & Koehler, 2006) to study the TPACK forms of knowledge addressed by trainee teacher educators' (TEs) documentation work (DW) in the context of their practicum. Practicum was provided shortly before the end of an in-service, reform-oriented professional development (PD) program within the Greek educational system aiming to educate TEs into the pedagogical use of digital tools in the teaching of mathematics. Recent literature reviews on TPACK (e.g., Voogt et al., 2013) identified the need for further research on the interrelation between TPACK, teacher practical knowledge, and teacher beliefs as well as the effects of PD courses in the development of TPACK. In this study, we analyse TEs' DW so as to identify possible interrelations between these elements.

Theoretical framework

Teachers' DW involves "looking for different kinds of resources, selecting/designing mathematical tasks, planning their succession, managing available artifacts, etc." (Gueudet & Trouche, 2009, p. 199). In this process, curriculum materials are not conceived as static bodies of resources that guide instruction, but rather as objects amenable to changes and modifications depending on the teacher's didactical design. Through a class of professional situations and teachers' experiences, the existing resources can be modified as documents according to the formula: Document = Resources + Usages + Operational Invariants. The term 'document' describes not only the material component of resources and the observable parts of teachers' stable behavior for a given class of situations (called usages), but also the implicit knowledge 'piloting' their usages, i.e. the operational invariants.

In order to describe the knowledge needed for integrating technology in teaching, Mishra and Koehler (2006) developed the TPACK framework taking into account the interplay between content, pedagogy, and technology: (a) Technology knowledge (TK) concerns knowledge about technical aspects of hardware and software; (b) Technology content knowledge (TCK) refers to the ways that technology can be interrelated to subject knowledge; (c) Technological pedagogical knowledge (TPK) describes a broader knowledge of technology in relation to pedagogical strategies concentrating on the knowledge of tools and its functionalities, as well as on the interrelation

between specific tools and tasks; (d) Technological pedagogical content knowledge (TPCK) describes an emergent form of knowledge that requires understanding of concepts, pedagogical techniques for communicating mathematical content in constructive ways, knowledge of students' difficulties in learning particular topics, knowledge of students' prior knowledge, and how all the above can be readdressed through the use of technology.

By analyzing the trainees' DW, the research question we aim to address is: Which TPACK forms of knowledge are targeted by the trainee TEs in their documents and which operational invariants are related to these forms of knowledge?

Methodology

The PD program we refer to was implemented in University Centers (UCs) all over Greece for 350 hours and the plan was to employ the newly trained TEs in wide-scale courses to educate groups of teachers in specific Centers for Teacher Education Support (CTES). Practicum was part of UC official structure. Its duration was 30 hours and involved: (a) observation of other teacher educators' teaching in CTES, (b) design of a 3-hour lesson for teachers in CTES under the supervision of a mentor and implementation in CTES, (c) presentation of design and implementation in UC special reflective sessions. The present study took place in the context of a UC course at the University of Athens where we had the role of academic trainers and mentors of trainees. The 16 trainee TEs were experienced, qualified secondary mathematics teachers. Our data consisted of: (1) the researchers' notes from the UC reflective sessions; (2) the trainees' designs for their lessons in CTES (scenarios, worksheets, etc.); and (3) the trainees' activity reports (i.e. templates in which trainees described aspects of their designs and their experiences from the implementation). In the first step of the analysis, under a grounded theory approach (Charmaz, 2006) we categorized the TEs' documents in relation to the TPACK forms of knowledge targeted for teachers in CTES. In the second step, in order to shed light on the interconnections between these forms of knowledge and the underlying operational invariants we triangulated the analysis of trainees' DW, our notes from the reflective sessions and the trainees' activity reports.

Results

The 16 trainee TEs' designs were categorized in two broad categories of documents concerning the targeted TPACK forms of knowledge: (1) documents prioritizing TK and TCK and (2) documents combining TPK and TPCK. Below, we provide a short description of each one of them.

Prioritizing TK and TCK: Instructive documents The documents of this category emphasize the correct use of the available technologies and how they can be interrelated to subject knowledge. This is usually evident in specific worksheets that are structured in the form of step-by-step instructions ensuring the correct use of the tools for the requested activities. For instance, one trainee TE working on an official scenario introducing linear functions through transformations of y=ax with Function Probe (FP) (Confrey, 1991-2002), designed a worksheet including instructions such as: "Table window: Fill in the 3rd column with the ratio y/x and press enter"; "Write down the resulting values and explain what they show"; "Table window: Send points to the graph"; "Graph window: Graph → graph choices → Tick 'Show transformations' → Click on the icon 'y=…' to create the graph of y=x." Common TEs' justifications for this kind of DW provided during the

reflective sessions concerned their observations in CTES that focused on: the different levels of teachers' familiarization with digital technologies; difficulties in linking the different digital representations; the official trainers' teaching model and agenda. Also, documents belonging to this category, seem to prioritize a view of teachers in CTES mainly "as students" who need to be familiarized with tool functionalities rather than a view of them as "teachers of students" who need to be engaged in reflection on how to transform the scenario for their students in school. The implementation of these documents in CTES was characterized by instructive teaching practices targeting skills' mastery and correct performance through a strict following of the worksheet. The operational invariants underlying DW of this category are related to: (a) TEs' emphasis on constraints and opportunities afforded by the context (e.g., time restrictions, technological environment); (b) TEs' observation in CTES concerning teachers' difficulties (with tools or more general) and the previous teaching model adopted by the official trainer in CTES; (c) TEs' epistemology for the teaching and learning of mathematics with technology considering the correct use of tools as a prerequisite for linking tools and content.

Combining TPK and TPCK: explanatory and facilitative documents A distinct feature of the documents included in this category is the emphasis on the P aspect of TPACK that is expressed either through 'explanatory' documents or 'facilitative' ones. Both of them are primarily based on materials designed by the TEs aiming not only to familiarize teachers in CTES with technological tools, but also to support them in conceiving pedagogical uses of these tools for their classroom teaching. However, the P aspect is targeted in different ways in each one of the above subcategories. The explanatory documents consist mainly of tasks involving the use of specific digital tools in parallel with explanations for their pedagogical value while the facilitative ones include mainly open tasks favoring exploration and teamwork.

More specifically, the explanatory documents were created by the TEs mostly through transformation of existing UC materials with the aim to explain to the teachers in CTES – from the TEs' own perspectives – issues related to the efficient integration of technology in the teaching and learning of mathematics. The design of these documents involves new didactic tools aiming to bridge the distance between the language of official resources and the language of practitioners. This kind of documents echoes a consideration of teachers "as students" who need familiarization with technology. At the same time, these documents target teachers' transition to "teachers of students" by explaining the link between tools and pedagogic aims. For instance, one TE prepared a worksheet for the teachers corresponding to an official scenario related to the study of sinusoidal function with FP. The worksheet had the form of a two-column table: in the left column, he inserted the worksheet questions, and in the second, the rationale. He engaged students in exploring the role of ρ in y=ρsinx kinesthetically by stretching the graph of y=sinx (through the "stretch tool") until coinciding with the graph of y=2sinx and y=3sinx which had been constructed. On the right column of the worksheet he explained the added value of the dynamic manipulation tools: "Study of function y=ρ*sinx," "What is the role of ρ." "Multiple representations of the relation between y=sinx and y=2sinx, y=3sinx, Table: visually, Graph: visually and kinesthetically."

The facilitative documents favor teachers' exploration of mathematical ideas, connections between different representations and collaborative work in the form of open tasks allowing TEs to confront

potential teachers' difficulties with digital tools on the spot. The design of these documents does not focus on the technological skills per se but rather how these can be incorporated pedagogically by the teachers for their classroom students. For instance, one of the facilitative documents included open tasks without any kind of instructions (even for aspects of the software). The teachers were asked to work in groups of two and provide both an algebraic and a geometrical solution for a given problem and then to discuss in groups about the potential students' approaches. Then, they were engaged in class discussion of the above findings and the added value of the software. Finally, they were asked to collaboratively design indicative questions for their students and present it in the CTES classroom. The implementation of these documents in CTES was usually characterized by teaching practices facilitating problem posing and solving, and favoring the development of meaningful material by the teachers themselves.

The operational invariants underlying DW of this category include: (a) TEs' epistemological conceptions of the ways mathematical knowledge should be approached though the use of technology; (b) TEs' observation in CTES concerning teachers' difficulties (e.g., use of tools, design of didactical materials); (c) TEs' conceptions of trainees "as students" who need particular supporting structures or pedagogical strategies (e.g., groupwork) to reinforce their TPK and TPCK.

Conclusion

The analysis showed two categories of documents concerning the targeted TPACK forms of knowledge. In each category, three kinds of documents have been identified, addressing differently the integration of technology in the teaching and learning of mathematics by the teachers. Operational invariants underlying trainee TEs' DW are directly linked to the teaching practice in teacher education contexts as well as to their epistemologies for the role of technology in the teaching and learning of mathematics and the ways they conceive trainee teachers ("as students"/"of students"). DW emphasizing the T aspect of TPACK is related to TEs' emphasis on the correct use of tools, conception of teachers "as students" and instructive teaching practices. On the contrary, DW emphasizing the P aspect of TPACK is related to trainees' conception of teachers' learning through connecting tools and pedagogic aims as well as through challenging teachers' engagement in open tasks favoring exploration and teamwork.

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Teachers' interventions to foster inquiry-based learning in a dynamic technological environment

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This study aims at identifying types of teachers' interventions that can foster or hinder the processes of inquiry in the classroom as a result of the teacher's behavior. The Method of Variation Inquiry developed by Arzarello guided this study, which took place in a learning setting where students learnt pre-calculus concepts by exploring real-world phenomena using dynamic technological tools. We present two extreme cases: one illustrates interventions that boost inquiry processes; the other breaks them down. Insights from these cases are discussed.

Keywords: dynamic tools, inquiry-based learning, teachers' interventions, variation theory

Introduction

The introduction of technological tools into educational settings has challenged teachers' roles in managing their students' learning practices. To take full advantage of technology in teaching practices, the role the teacher plays in classrooms is critical and must be taken into account when designing learning environments. To shed light on the ways teachers use technological tools in their classrooms, Drijvers, Doorman, Boon, Reed and Gravmeijer (2010) investigated which types of orchestrations teachers develop when they use technology. However, the technological tools themselves are not the only components that challenge the teacher's role in the classroom. For example, a teacher who believes that learning is based on a process of inquiry can pursue it also by promoting the evolution of productive argumentation processes in the classroom through inquiry processes. This research strand has drawn the attention of scholars in the past (for example, Arzarello and Paola, 2008; Mariotti, 2009; Schwarz et al., under review).

Within this research strand, our study is derived from the Deweyian tradition according to which learning is an inquiry process that "develops as interplay between known and unknown in situations where some individual or group of individuals is faced with a challenge" (Artigue & Blomhøj, 2013, pp. 798-799). In such a setting, the teacher's role is crucial and includes, among other things, preparing challenging tasks, stimulating inquiry processes as students are working in small groups, and leading collective discussions in order to foster the inquiry process. In this paper, we focus on the teachers' role in stimulating the inquiry process among the students within the groups. Our aim is to identify types of intervention that could foster or hinder the inquiry process. The study took place in a learning setting where 9-10th-grade students in scientifically-oriented schools learn pre-calculus concepts by exploring real-world phenomena using dynamic technological tools.

Theoretical framework

With the goal of developing a stable model for inquiry-based learning, Arzarello (2016) developed the Method of Variation Inquiry (MVI). The MVI model results from combining two theoretical perspectives: the logic of inquiry (Hintikka, 1999) and the variation theory (Marton et al., 2004).

The main idea behind the logic of inquiry approach involves seeking rational knowledge by questioning (Hintikka, 1999). Hintikka conceived the process of seeking new knowledge as an interrogative process between two players. Th irst player (the inquirer) has the role of asking

questions, and the second player has the role of answering and is called the verifier (or oracle or nature). The former is the seeker of knowledge who tries to prove a conclusion to be reached from prior experiences or even from theoretical premises. The latter is considered the source of the knowledge. By interrogating the verifier, the inquirer may reach the truth he/she is seeking. In this approach, strategies of questioning play a central role in interrogative processes: they involve choosing different questions and the order of these questions.

In order to design educational situations that may promote inquiry processes, Arzarello (2016) referred to the variation theory (Marton & Morris, 2002; Marton et al., 2004), which defines learning as a change in the way something is discerned, i.e., seen, experienced or understood. According to this theory, meanings emerge as the learner focuses his awareness on the object of learning. When the learner focuses his awareness on an object, some aspects of the object appear at the forefront of his attention. Yet, not all aspects are discerned at the same time or in the same way. In order to understand an object of learning in a certain way, various specific critical aspects must be discerned by the learner.

In the MVI model, Arzarello (2016) proposes that drawing students' attention to critical aspects, asking them to vary them and observe their effects on the phenomena may foster students' inquiry processes. MVI is a multi-layered spiral approach that guides students to become engaged in inquiry-based learning, namely, asking questions, raising conjectures, carrying out experimentations and providing explanations. The main idea of the MVI is creating challenging situations by varying some aspects of the phenomena (real-world or mathematical) while keeping the others invariant. In each layer, the teacher creates a new challenging learning situation by varying new aspects of the phenomena while keeping the others invariant. For example, a teacher may request her students to explore the mathematical model of a ball rolling on an inclined plane (as in the well-known experiment of Galilei). In this situation, the time and velocity of the ball vary, and the angle of the inclined plane is invariant. In the second layer of the exploration, the teacher (or the students) are requested to vary the angles of the plane and explore how this new situation affects the mathematical model. Exploring various aspects of the same phenomena may lead the students to grasp the intended object of learning, for example, quadratic function and its properties.

Study design

The study took place in a high school in Turin (Italy) in which two experienced teachers used the MVI. We followed four lessons of two different teachers as they taught for 1.5 hours. Initially, the students were required to work in small groups, sharing a worksheet (containing a task) and a computer. When the students were working in small groups, the teachers walked around and interacted with them according to his/her choice. The teachers were senior, and a short introduction to the MVI methodology was given to them. The students' actions with the computers and the teachers' interventions were video-recorded. Regarding data analysis, segments of the recorded clips including the teachers' interventions were identified. Finally, the effects of the teachers' interventions on the students' inquiry process were considered.

Preliminary results

In this section, we illustrate two extreme types of intervention. The first illustrates interventions hindering the students' inquiry process, while the second illustrates how teachers' interventions prompted the inquiry process.

An intervention hindering the inquiry process

In the first episode, a group of students is engaged in learning the linear function by exploring Hooke's law with a dynamic applet. Their task is to explore how the change of the mass affects the spring's stretch.

Before the teacher comes to the group, the students discuss whether the relationship between the mass and the spring's stretch is directly proportional or not. Following this discussion, they find an algebraic formula that expresses the relationship between the mass and the spring's stretch. They first check the validity of the equation, then go on to a new inquiry situation, which is interrupted by the teacher:

1	S	Let's see if it works the same way with easier values… let's try!
2	M	5 times 1.5 + 3... wow, guys, it works!
3	P	But how did we arrive at this?
4	Teacher 1	Look at the numbers...
5	Teacher 1	Are you answering question 2?
6	S	Yes, before we thought they were directly proportional, but it should have started from 0!
7	Teacher 1	OK, but here we're talking about the spring's stretch... what is it?
8	G	The difference between the two lengths...
9	Teacher 1	Ok, and here it's true that it starts from 0! Do you need the last column? The difference between the lengths… Do you know how to use it? What do I have to do to calculate the difference between the lengths?
10	G	The B coordinates... I mean, these coordinates less the others!

In [1], 'S' demands to validate her hypothesis (the equation 1.5x+3 represents the relationship between the mass and the spring's stretch). In [2], 'M' checks the correctness of the equation and verifies it. In [3], 'P' starts a new inquiry process by asking the question "how did we arrive at this?" This inquiry process is broken down by the teacher's intervention when she asks them a clarification question [14]. Her question refers to an item in the written task to which the students had not yet arrived. Her intervention is indeed directed to the concept's meaning, i.e., the spring's stretch. After 'G' answers her question, the teacher asks them another clarifying question, but this time she refers to technical aspects regarding the applet's design and its affordances.

An intervention fostering the inquiry process

The episode that follows illustrates a teacher's intervention that boosts the inquiry process. This episode is taken from a learning session aimed at exploring the mathematical model of a ball rolling on an inclined plane within an applet. The applet design allows the user to vary the inclined plane's angles. The default angle was set at 24°.

The teacher walked around the small groups of students. When she came to one of them, she stood beside them and observed what they were doing. After a while, she asked them about their work:

1	Teacher 2	What are you doing? Tell me.
2	Rico	We are considering the acceleration, which is constant and equal to 2.
3	Teacher 2	The acceleration is equal to 2? Why?
4	CR	It is constant.
5	Teacher 2	In all of the cases, will it be constant? Or for a specific value of the angles? What would happen if you changed the angles?

6	Rico	Perhaps it is because we used the 24° angle. Let us change the angles to see what happens.

The teacher initiates her intervention by asking the students about what they are doing [1]. Rico's answer refers to the invariant aspect of the ball rolling on the inclined plane, namely, the acceleration of the ball's movement. In [3], the teacher prompts the students to explain why the acceleration is equal to 2. After this attempt, the teacher changes her question. This time [5], she requests the students to generalize their conclusion. She lets them choose between two options "in all of the cases or for a specific value." Moreover, she suggests that they vary the angle and observe what would change and what would remain invariant. As a consequence, Rico makes a hypothesis [6], i.e., that the acceleration is constant and equal to 2 is a specific answer for $\alpha = 24°$. His statement, "Let us change the angles to see what happens" shows his intention to do an experiment with the applet to check his hypothesis.

Final remarks

Considering the overall data analysis (of which we could provide only short examples here), we found that standing aside without making active interventions for a while is a necessary but insufficient action by the teacher to foster inquiry processes. It seems that this action ensures that the teacher's intervention will be in accordance with the students' deeds – as it happened with the second teacher – and not to divert their minds from what they are doing – as it happened with the first teacher. After a short passive intervention – observing what the students were doing, asking the students about their exploration processes and requiring them to provide a short summary – it was found that this was a useful strategy for focusing the teacher's attention on the objects of learning that the students were considering, e.g., the acceleration of the rolling ball. This attention may help the teachers to recognize the variant and invariant objects the students were considering through their exploration process. Asking the students to vary the invariant objects they were focusing on and observe the effect of this variation was found to be an effective strategy for stimulating inquiry processes among the students.

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Analysis of the Use of Resources on Internet by Pre-service Mathematics Teachers

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The aim of this research is to analyse the use of the meta-resource –Internet– by pre-service mathematics teachers on the one hand in their daily life and on the other hand in their professional life for the preparation phase of the teaching sequences. This study has therefore attempted to determine, via documentational approach, some factors influencing the choice of resources on the internet in a situation of preparation of teaching sequences by mathematics' preservice teachers. As a result, it has been identified the external and internal factors influencing this choice. It has also been highlighted the importance of the presence of a training module linked to the use of this meta-resource, namely the Internet, to prepare teaching sequences in the training curriculum for future teachers.

Keywords: Internet use, pre-service mathematics teachers, resource, documentational approach

Introduction

In teaching and learning mathematics, the teacher play a key role. The constructivist approach defines the teacher as an organizer of the learning environment, as a guide in the construction of knowledge by the students. A teacher who sets up a teaching module does not just use textbooks. He/she also interferes with other paper or computer resources, his/her representations, his/her teaching experiences, his/her social character, his/her ways of class or time management etc. It is in this perspective that we find important to refer to the documentational approach to analyse the organization between the different resources that a teacher uses to set up a teaching sequence. Documentational approach is at the core of research on the teacher, in the sense that it provides the means to understand how a teacher organizes, implements, changes, and adjusts his/her teaching.

Among the multitude of resources that a teacher has at his/her disposal (textbooks, materials, etc.) this research focuses on those related to the use of Internet. According to Gueudet (2012), "very diverse digital resources are now widely available to teachers. This evolution results in profound changes not only in the resources but also in the teacher's own work and in his/her professional development." Hence, Internet is a significant resource in the documentation process. On the one hand, the Internet is omnipresent because of its mobile nature (smartphone, tablet) and on the other hand it is a resource that poses as a mediator of the knowledge, at once accessible and rich (Gueudet and Trouche, 2011). It is therefore important to characterize the use of the Internet by pre-service mathematics teachers for didactic purposes. This meta-resource requires that future teachers have the capacity to develop complex schemes of utilisation in order to make from it a relevant usage, adapted to the learning objectives and intended audience. In this perspective, this research aims to analyse the factors influencing the choice of resources on the internet in a situation of preparation of teaching sequences by mathematics' preservice teachers.

Methodology

The research was carried out at the end of the 2016-2017 school year with 50 preservice mathematics teachers, at the end of their third year at an education faculty in Turkey. Participants took the following courses concerning mathematics education: ICT in Mathematics Education, Dynamic Geometry in Mathematics Education, Methods Specific to Teaching (Teaching Mathematics), and Mathematics Teaching and Organization of Materials. The participants in the research were selected as part of the course "Teaching Technologies and Materials Organization". It was demanded to them to prepare a teaching sequence for a desired subject, as a homework within the course. Data collection tool consisted of semi-structured interviews conducted with each of the 50 participants. The interviews included eight “yes or no” questions and the justification of the responses; their duration was on average 5 to 10 minutes. The interviews were recorded and the recordings were transcribed to make a written document. The data were analysed using the descriptive analysis method.

Findings and Results

This section presents data and results of semi-structured interviews to analyse which elements influence the choice of resources on the internet in a situation of teaching sequence preparations.

To develop the teaching sequence,	**Yes (%)**	**No (%)**
Did you use the internet	94	6
The following percentages are based on the 94% of preservice teachers using the Internet: n = 47		
Did you consult a forum or a discussion group	63,8	36,2
Did you post on a forum or a discussion group	0	100
Did you visit a web hosting website (YouTube, Daily motion ...)	55,3	44,7
Did you use a search engine (google, yandex ...)	100	0
Did you use the site of a publisher	2,1	97,9
Did you use a resource pooling site (smart exchange, cabri exchange)	95,7	4,3
Did you use the email	2,1	97,9

Table 1: Answers of the pre-service teachers to the closed questions

Table 1 shows that a very large majority (94%) of preservice teachers used the Internet to prepare the teaching sequence and used a resource pooling site. It is noted that most of them consulted a forum or a discussion group but no one posted there. Half of the preservice teachers used a video hosting website. Practically no student has used the email communication tool or a publisher site. To understand in depth the student yes or no responses, the analyses of their answers’ justification are given. It is noted from the analyses of the preservice teachers’ speeches that the Internet is a meta-resource indispensable for almost all the preservice teachers preparing their teaching sequence. The three preservice teachers who did not use the internet expressed preference for using

paper. We note then a phenomenon of instrumentalisation since the trainee teacher among all the resources he/she has at disposal, rejects some that do not suit his/her usual habits and schemes.

E-mails have been almost non-used. Indeed, this mean of communication is asynchronous or it can be assumed that the teachers expect immediate feedback or that they had no expert knowledge to ask for a notice by e-mail. We observe here a rejection of a device as a resource by reasons on the one hand external due to the existence of relevant recipients and on the other hand internal linked to the preference to use synchronous communication means such as the telephone which are more immediate.

All the preservice teachers used a search engine which was an expected result because in the majority of web browsers the default home page is a search engine and the browser address bar is also used as a search engine. The students' justification regarding this point can be summarized by the following example:

PST 7: Why I used google ... why ... how can I look for something on the internet otherwise?

It can be supposed that this is a scheme that can be shown through the following operative invariant: Whenever you have to look for information on the internet, you have to go through the use of a research engine.

Moreover, even if all the preservice teachers used the google search engine, the problem of the "wandering" phenomenon during the research was mentioned in a recurring way. An example of students' phrases illustrating this phenomenon is:

PST 35: ….indeed you said that it must be constructivist, there is not really any constructivist example on the internet ... anyway I could not find but yet I searched for a week ... I tried to do with what I could.

It is noticed by the speech of PST 35, on the one hand the weight of the institution which is the professor who has asked for a homework; therefore it is necessary that the homework is in accordance with the expectations of the professor: the preservice teacher knows that the professor expects a constructivist homework. On the other hand, internal factors are evident: the trainee teacher does not have sufficient knowledge to do research with keywords and to sort the information appropriately. Other preservice teachers have also pointed out the difficulty of finding resources, that are both professional and constructivist, on online video sharing sites.

To justify their non-participation in forums or discussion groups, the preservice teachers unanimously mentioned, to a greater or lesser degree, a lack of motivation linked to the procedure of registration to a forum considered to be tedious. They also cited fear of being tried. Indeed, according to the speeches, most of them have the perception that the discussion groups are reserved solely for experienced interveners. The following excerpt of conversation illustrates this phenomenon. In these cases, it is a phenomenon of rejection of an internet resource for internal reasons, at the sentimental level, namely the motivation and the fear that they have a-priori or induced by the experience.

PST 6: Ah yes... the page of teachers of mathematics in Facebook... I feel like there are only teachers who write, I thought it would have been stupid... indeed I do not know... once I had read on a page, a student had written a post... the teachers on the page had answered him by saying why you do not do your homework yourself, you wait for the answer from us without doing any research, things like that... since that... indeed it is, they would have written same for me also, not? ...

Researcher: Do you think they are right?

PST 6: Yes indeed they may be right... in the end this homework is given to us to do research ourselves.

Some students expressed the fact that videos on YouTube even in low quality (480p) consumed too much package via the mobile network. Here, the existence of an external economic factor that influences the choice of the resource is to notice.

Conclusion

One of the conclusions of this research concerns the search engine resource which has proved to be almost ineffective in the context of the teaching sequences preparation by the future teachers. This conclusion may be caused by the fact the pre-service teachers did not have sufficient knowledge to do research with keywords and to sort the information appropriately. The future teachers'knowledge and shortcomings constitute a factor of influence on the instrumentalisation process.

It may also be noted that the institutional context plays a key role in the construction of schemes related to the use of the Internet to prepare teaching sequences. Indeed, future teachers also have the status of student in a faculty of education whose technological training modules take an important place. The results of the semi-structured interviews suggest that the demands of the institution - faculty/professor- strongly influence the characteristics of the preparations of their teaching sequence, in particular the use of ICTs and constructivist approaches.

Another conclusion is that a trainee teacher accepts or rejects a resource in his/her documentational genesis for external and internal reasons. The constructing schemes process of utilisation related to the use of different resources embedded in the meta-resource Internet is not obvious. The training of the teachers on this point seems to be essential.

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An Examination of Teacher-Generated Definitions of Digital Instructional Materials in Mathematics

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In this study, eleven U.S. elementary and middle school teachers of mathematics provided written definitions of digital instructional materials (DIMs). Text mining methods were applied to analyze the terms teachers used most frequently in their written definitions. Teachers' most frequently used definitional terms not only suggested interpretation of DIMs as resources, but also consideration of how, where, and by whom those resources could be used. We relate teachers' definitions to research-based definitions of related terms and implications for research and practice.

Keywords: Mathematics education, Elementary school teachers, Technology uses in education

Background and Purpose of Study.

The purpose of this research is to explore how elementary/middle school teachers define digital mathematics materials. In previous work, the authors defined *digital instructional materials (DIMs)* as "instructional materials that are conveyed via digital materials. Examples include digital textbooks, applets and applications, and online supplementary resources" (Edson & Thomas, 2016, p. 240). In this study, we broadly referred to the intersection of curriculum materials and technology use as *digital instructional materials*, but tasked teachers with generating their own specific definitions. That is, we were interested in how practitioners interpreted the intersection of curriculum materials and technology. Our decision to focus on teachers' interpretations aligns with evidence of teachers' increasingly active roles driving the design, selection, and implementation of digital resources in the U.S. and multiple European countries (Pepin, Gueudet, and Trouche, 2017, p. 799; Gueudet et al., 2016). As we attempt to understand how digital resources impact teachers' work, it is important to not only consider how research defines those resources and materials, but also to understand "what counts" as digital instructional materials from the perspectives of teachers who are designing, selecting, and implementing resources for mathematics classrooms.

Our selection of the term *digital instructional materials* draws from the definition of *instructional materials* offered by Remillard and Heck (2014), who "use the term *instructional materials* to refer to resources designed to support or supplement instruction, including textbooks, curriculum guides, descriptions of mathematical tasks, and instructional software" (p. 707). This particular definition focuses on resources that support or supplement instruction, which might include technologies that are specific to mathematics or those that are more general in nature, but used in a way that supports mathematics instruction. Pepin, Choppin, Ruthven, and Sinclair (2017) use the more specific term *digital curriculum resources (DCR)* to refer to "sequencing—of grade-, or age-level learning topics, or of content associated with a particular course of study (e.g., algebra)—so as to cover (all or part of) a curriculum specification, which differentiates DCR from other types of digital instructional tools or educational software programmes" (p. 647). Digital

instructional materials, then, might include DCR, but extend beyond sequenced topics of content associated with a particular course of study.

More broadly, the notion of documentational genesis, introduced by Gueudet and Trouche (2009) distinguishes between *resources* and *documents*. In this sense, the term *resources* is defined as "the variety of artifacts we consider: a textbook, a piece of software, a student's sheet, a discussion with a colleague, etc. A resource is never isolated; it belongs to a set of resources" (Gueudet and Trouche, 2009, p. 205). Through a process of *genesis*, involving a scheme of utilization, resources produce *documents* (p. 205). Considered in the context of *documentational genesis*, DIMs might be viewed as a specific type of *resource* that, through a scheme of utilization, become a *document*.

Niss (2016) defines *materials* as "the instructional materials and resources, including textbooks, artifacts, manipulatives, and IT-systems employed in teaching and learning," (p. 241) which serve as one of six components of *curriculum* (goals, content, materials, forms of teaching, student activities, assessment). Remillard and Heck (2014) also identify instructional materials, as well as tools and resources, as components within the larger notion of mathematics curriculum. In this sense, DIMs might be interpreted as a specific type of *materials* or *resources* that help to comprise mathematics curriculum. To understand how teachers perceived of digital instructional materials, we investigated the following research question: *What terms do elementary and middle school teachers of mathematics use to define digital instructional materials?*

Methods

Research Setting, Participants, and Data Sources.

We draw from a larger study that examined eleven elementary and middle school U.S. teachers who enrolled in a summer graduate course focused on technology for teaching elementary mathematics. All participants were female and taught in the local suburban and nearby rural schools with between 2 and 12 years of classroom experience. Nine participants taught in elementary grades, one participant taught in middle grades, and one participant was transitioning from teaching high school mathematics to middle grades in a different school. Throughout the duration of the summer course, participants explored a variety of technology apps and tools, read and discussed research and problems of practice relating to technology use in mathematics, and worked in one of three randomly assigned groups to develop indicators in a framework for evaluating DIMs for mathematics. As part of a final project for the course, participants individually generated written definitions for *digital instructional materials.* These definitions serve as data sources in this study. Teachers' evaluation of digital instructional materials is reported elsewhere (e.g., Thomas & Edson, 2017) and lesson planning with and enactment of digital instructional materials in mathematics classrooms is the focus of ongoing work.

Data Analysis.

To identify the terms that the teachers used to define DIMs, we conducted a text analysis that would enable us to visually summarize the teachers' responses. Text mining involved three key phases: text pre-processing, text processing/analysis, and actionable information (He, 2013). During

the pre-processing phase, the eleven individual responses were combined to formulate a *corpus of textual information* (Moreno & Redondo, 2016) containing 1,107 words. We used the online application TagCrowd (Steinbock, 2006), a text processing/analysis tool that generates "word clouds" from provided text. The clouds make visually prominent the words that appear more frequently in the text corpus and can provide word counts. Using this particular online tool, we could automatically omit common terms (e.g., *I, you, am*) and eliminate a list of redundant terms we opted out of the analysis (*DIM, DIMs, digital, instructional, material, materials)*. In this study, we report on the actionable information from the text mining results that summarize the teachers' definitions.

Findings

The terms that teachers most frequently used to define DIMs included: *technology, used, students, tool, learning, classroom,* and *teaching*. Table 1 lists frequencies of the most frequently used terms (those appearing 5 or more times in the corpus) in teacher-generated definitions for *digital instructional materials*.

Technology (18)	Classroom (10)	Framework (7)	Curriculum (5)	Resources (5)
Students (15)	Teaching (8)	Include (6)	Enhance (6)	Support (5)
Used (15)	Teacher (7)	Standards (6)	Purpose (5)	Transform (5)
Tool (12)	Think (7)	Action (5)	Replace (5)	Types (5)
Learning (11)				

Table 1: Terms appearing 5 or more times in teacher-generated definitions of DIMs

Discussion

Frequently used terms such as *technology, tool, standards, curriculum, and resources* suggest an interpretation of DIMs as *resources* (Gueudet & Trouche, 2009), or *materials* (Remillard & Heck, 2014; Niss, 2016). Two of these five terms, *curriculum* and *standards*, may relate to the more specific notion of *digital curriculum resources* as sequenced content that addresses curricular specifications (Pepin et al., 2017). The five most frequently used terms represent a small overall proportion of teachers' most frequently used definitional terms. A larger number of frequent terms such as *students, used, learning, classroom, teaching, teacher, think, action, enhance, purpose, replace, support* and *transform*, indicate that, in defining what constitutes a *digital instructional material*, teachers considered not only the resource itself, but also how, where, and by whom it might be used. That is, teachers' definitions tended not to distinguish between the *resource* and the *genesis* through which it becomes a *document*. Instead, the summary of teachers' terms identified both *resources* and elements of a *scheme of utilization,* which are indicative of *documents* (Gueudet & Trouche, 2009).

Unlike instructional materials such as textbooks which can be defined and evaluated based on static content, the interactive nature of DIMs may make the content of the materials seem inseparable from the their use. Conceiving of instructional materials as synonymous with their use

raises questions about teachers' practices for selecting and evaluating digital materials for classroom use, such as: *How do teachers evaluate and select interactive resources? To what extent must teachers interact with DIMs to understand their potential usefulness in the classroom?* In the context of studying textbooks in the information age, Usiskin (2013) also addressed the complexity of mathematics instructional materials in interactive digital formats. He asserted, "curriculum, when delivered in paper form, is *overt*, whereas a sophisticated curriculum delivered on computer is *covert*" (Usiskin, 2013, p. 721), with the lack of transparency in digital mathematics textbooks raising potential issues for mathematics education researchers. Findings of this study offer insight about how teachers interpret DIMs and suggest that these teachers' documentational work may occur not only in the design, selection, and implementation of mathematics instruction, but also in defining "what counts" as digital instructional materials in a more general sense. A more thorough understanding of how teachers make sense of the digital instructional materials available to them stands to inform both practice and research about mathematics teaching.

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Transition from a paper-pencil to a technology-enriched teaching environment: a teacher's use of technology and resource selection

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As a result of the emphasis on technology integration, there is a growing need for research on teachers' choices, strategies and views related to using technology in the classroom. This study focuses to explore on teacher's technology inclusion and source choices into the instruction process in the technology-enriched environment. In this case study one teacher is examined. The teacher used technology in different ways which have been determined that she showed the variety of choices. Depending on the use of technology, teacher preferred to use digital resources.

Keywords: Documentational theory, instrumental orchestration, technology-enriched teaching

Introduction

The rapid development of information and communication technologies has affected the mathematics learning-teaching process and, emerging technology has started to take place frequently in the classroom environment (Pierce & Stacey, 2010). Technology, which offers unique and powerful meanings to encourage students' understanding, supports the ideas of related to their "doing mathematics". Acceptance of the emerging gap between research results on the use of technology in mathematics learning and the use of these technologies in real classes, led to the reconsideration of the importance of the teacher in a technology-enriched learning environment and the examination of ways to support teachers in the use of technological tools (Maracci & Mariotti, 2009). Teacher's approach to using technology has a fundamental importance on the effects of using technology in class. Therefore, it is necessary to focused on the teachers' role, teaching practices and development (Drijvers, 2012) and, they should be prepared for the arrangement of teaching environments using technological tools. In this study, we focused on the determination of the way to integration of technology and resource choices in the technology-enriched environment which is new to the teacher.

Theoretical framework: Instrumental orchestration and Documentation theory

An instrumental orchestration is defined as arrangement of the artefacts available in a learning environment for external steering of students' instrumental genesis (Trouche, 2004). Drijvers and his colleagues (2010), Drijvers (2012) and Tabach (2013) distinguish ten types of instrumental orchestrations: technical-demo (explain the technical details), explain-the-screen (including the mathematical content), link-screen-board (linking the representations between on the central screen and board), discuss-the-screen (using the central screen, whole-class discussion), spot-and-show (bring up student work on the central screen), sherpa-at-work (students' ideas shared by whole-class), work-and-walk-by (the teacher monitors students' progress), not-use-tech (choose not to use technology), discuss-the-tech-without-it (use mobile transport system) and monitor-and-guide (use a learning management system). In the technological environment, instrumental orchestration is

related to the stage of the teacher in the classroom (Tabach, 2013), and therefore the teacher's practice is concentrated on the instruction process in terms of the basic role of the teacher.

In this research we studied, mathematics teacher's documentation work: looking for resources, selecting/designing mathematical tasks. A documentational approach is focusing on the documentation work of the teacher outside/inside the class. A document is generated from a set of resources within a process of genesis (Gueudet & Trouche, 2009). The teacher interacts with resources, selects them, and works on them within processes where design and enacting are interacted. The expression documentation work encompasses all these interactions (Gueudet & Trouche, 2011). Therefore teachers' work with resources includes selecting, modifying, and creating new resources in-class/out-of-class. In order to understand teacher practice in a technology setting, this study considers the use of two theoretical perspectives: instrumental orchestration to analyze teacher actions in technology-enriched environment and, documentational theory to determine the resource choices and change in her/his choices in this process. In this context, the research questions are: "Which orchestration types transform by the teacher into appropriate technological resources? How did technology-enriched teaching affect resource choices?".

Methodology

The design of the study is based on a case study approach, focusing on one teacher. A case study was conducted to investigate and determine the ways in which the teacher integrates the technology into the teaching process and the selection of resources in this process in technology-enriched lessons. The research was carried out in one class of the high school, a school in a big town in the Turkey. The teacher we studied with, Merve, had approximately 5 years of teaching experience, but the technology used in teaching environment is new for her. The teacher initially volunteered for the research and preferred for the reason that she never thought in a technology-enriched classroom before. Classroom (29 students) had a smart-board while the other had tablets. However, teacher was free to decide how she would use available tools. Teacher was involved in hands-on activities in front of a computer and prepared teaching materials using software programs (Geogebra, Cabri Geometry) during the teacher training. The lessons (4 lessons during a 4-week period) were observed and videotaped: the last two lessons were technology-enriched. The data we gathered are interviews, observations of the organization of teacher's workplace, lessons, and selected materials.

Findings

Merve had two technology-enriched lessons: basic elements of triangles and area of a triangle. She projected Geoegebra drawings on the smart-board and students in groups made drawings using Geogebra which were installed in their tablets. Compared to her first two lessons which were more traditional, new situations were observed in her technology-enriched lessons. Classroom setting, roles of students and teacher, and instruction techniques changed with the integration of technology. Table 1 on this case, refers to the views of Merve before and after the use of technology, the ways of integrating the technology, the difficulties that can be experienced and the choices of resource.

It has been determined that Merve implemented various orchestration types in the teaching process. In the interactive teaching environment, the teacher has included what is done through technical information and software (technical-demo and explain-the-screen) so that students can use the

software. Link-screen-board orchestration has been preferred with two different ways by the teacher and observed transitions: board→ central screen, central screen →board. She directed the students

Math Teacher Merve				
Technology- enriched instruction experience	**Views before technology-enriched teaching**	She has not the experience of technology-enriched teaching.	**Views after technology-enriched teaching**	"It is an entertaining process, as well as the anxiety created by the new environment."
The goal of integrating technology		Demonstrational purpose		Demonstration and justification of mathematical subjects and concepts
Views on the use of technology		I always use it, while students can use technology.		I would like to use it if I master the use of technology.
The way to integrate the technology		It should be an environment where students use tablets		I am eager for the environment where students will use tablets, but challenging process
Challenges in technology-enriched instruction		Troublesome and time-consuming preparation process		Intensive preparation and time-consuming experience
Resource choices		Worksheets, Resource book		Digital tasks, Questions drawn in Geogebra

Table 1: The situation of Merve toward technology, before and after in technology-enriched teaching

to questions and used the discussion-the-screen and predominantly work-and-walk-by orchestration where the activities / questions on the central screen were discussed. Thus, the student-teacher interaction often occurs. Teachers who are accustomed to the traditional method and who have recently met with the technology in teaching have sometimes tended to be not-use-tech.

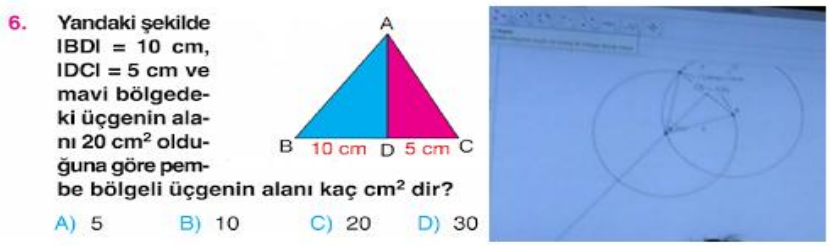

Figure 1: Screen shots from textbook (left) and digital environment (right)

In lessons that do not use technology, Merve, instructed the lesson based on the question-answer technique after a quick classical presentation. She who prepares the questions to be solved already carries a paper and a textbook as a resource which has shown in Table 1. However, while this resource does not make use of it in the teaching process, but she brought it with her saying in her own words, "Maybe it will be necessary". She distributed the questions to the students as worksheets. She, bringing the questions together from several different resources, said that the goal was to provide variety in the questions. In the technology-enriched lessons, rather than the paper-based resources shown in Figure 1, it's largely exploited for digital resources. She stated that she used Geogebra videos from YouTube for presentation, and prepared and explained the activities and questions using the software. In the process of link-screen-board and not-use-tech orchestration, the teacher who uses only the board, has used to digital resources throughout the course. She guided the students who were working on tablets as a group, the instruction which the work-and-walk-by orchestration come in frontline. Students, who have solved questions individually through their working papers in the previous lessons, have gone into discussions when they started to use tablets.

Discussion

This research focused on to explore teacher's choices of resource and orchestration in integrating the technology on teaching process. It conducted as a case study, it was determined that the use of orchestration by the teacher influenced the choice of the resource. However, this is considered to be a mutual interaction, and the choice of using digital resources has also affected the orchestration choices. In such a process, the support of teachers is a pre-requisite (Drijvers, 2012) and the teacher seems willing and has expanded the selection of resources. A common occurrence in observations is "weaving" as Billington (2009) states: teacher movement among the available tools. In this process, three physical tools were used: board, computer + screen and body movements. In the course of the lesson, digital tools were used, while spontaneous situations used teacher wooden boards or gestures. Drijvers and his colleagues (2010) have stated that teachers make choices about the use of technology in the direction of their opinions on mathematics and habits. As a different situation in this study, the teacher partially changed the regular habits, teaching techniques. In order to elaborate this situation, there is a need for long-term studies in which considered the teachers' technological pedagogical field knowledge, norms or professional development.

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From sample to population: A hypothetical learning trajectory for informal statistical inference

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This paper presents the results of a teaching experiment to enhance 9th-grade students' understanding of informal statistical inference (ISI). The teaching experiment was conducted to evaluate and revise a hypothetical learning trajectory (HLT) as a step towards an empirically and theoretically based HLT-design for ISI. The challenge was to invite young students, inexperienced with sampling, to making statistical inferences without knowledge of formal probability theory. In this trajectory, the students proceeded from a first experience with sampling physical objects, through an understanding of sampling variation and resampling, to reasoning with sampling distribution. The results of the intervention suggest that young students can informally interpret sample data with corresponding uncertainty. Engaging in concrete sampling, in simulations and in deepening whole-class discussions seem essential parts of this HLT-design.

Keywords: Informal Statistical Inference, Hypothetical Learning Trajectory, TinkerPlots

The need for informal statistical inferences

The use of sampling to draw inferences about a population is at the heart of statistics, and therefore important to learn. Statistical inference includes both a generalization from sample to population, and an estimation of the reliability of this generalization. Recent research investigated if and how this type of statistical reasoning can already be developed informally by young learners (Paparistodemou & Meletiou-Mavrotheris, 2008). This so-called informal statistical inference has different definitions (Wild & Pfannkuch, 1999). It is clearly not about formal statistical procedures such as the testing of hypotheses, but about the ways students use their informal statistical knowledge to support their inference about an unknown population based on observed sample data (Zieffler, Garfield, delMas, & Reading, 2008). Makar and Ruben (2009) identify three key principles of informal statistical inference: *generalize* beyond data; *data as evidence* for these generalizations; and *probabilistic reasoning* about the generalization. Because of the importance of conceptualizing statistical inferences among young students, we wanted to develop it in a prototypical HLT and therefore, we focus on the question: *What are the features of a theoretically and empirically based HLT-design for enhancing 9th-grade students' informal statistical inference?*

The design of the hypothetical learning trajectory

The trajectory as a whole

As educational materials that focus on the development of statistical reasoning for grade 9 in the Netherlands hardly exists, we designed such materials. In this design study, a so-called hypothetical learning trajectory (Simon, 1995) was formulated for students in this grade. Design guidelines were identified through literature review, and the possibilities of educational software were explored. This

resulted in the HLT-design in Figure 1. In this paper we focus on the sixth step, in which students investigate what happens while the size of computer simulated samples increases.

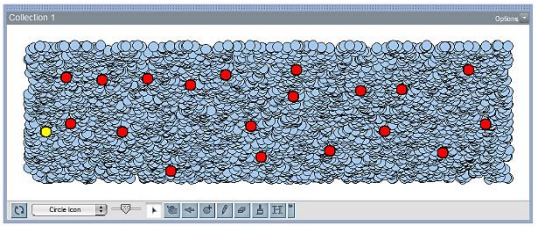
The sixth step: 'What happens if we increase the sample size?'

As preparation for step 6, students will conduct the physical "Black Box with notes" experiment in step 5, in which students are expected to manually draw a sample of 40 from a box containing almost 5,000 notes with data about the length and gender of 14-year-olds. In this preparatory step, students will be confronted with sampling variation.

HLT–design								
Step	1 Conduct physical experiment (Black Box with small and large window)	2 Examine frequency-distribution >100.000 repetitions of physical experiment	3 Simulate sampling distribution of physical experiment *(with ICT)*	4 Test a claim with sampling distribution (Social Media) *(with ICT)*	5 Conduct physical experiment (Black Box with notes)	6 Examine sample size and repeated sampling *(with ICT)*	7 Test a claim with sampling distribution *(with ICT)*	8 Compare groups based on samples *(with ICT)*
Data	Ordinal – nominal level of measurement				Interval – ratio level		All levels of measurement	
Con-cept	Sampling variation		Repeated sampling + Frequency distribution	Sample size + Sampling distribution	Sampling variation	Sample size + repeated sampling	Sampling distribution	
Proba-bilistic reason-ing	In words	In words + argument frequency distribution	In words + argument sampling distribution		In words	In words + argument sample size	In words + argument sampling distribution	

Step 5:
Generalization about the length of 14-year olds based on a sample (size 40) and accompanied by probabilistic reasoning

Step 6:
Generalization about the length of 14 year olds based on simulated samples with TinkerPlots and probabilistic reasoning with an argument on sample size



Step 7:
Estimate the quantity of jam in a jar by sampling and probabilistic reasoning with the sampling distribution

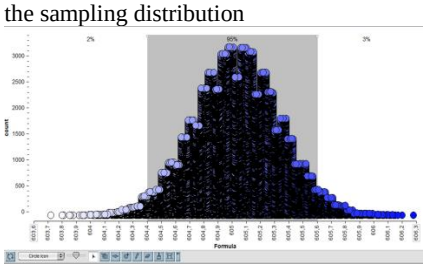


Figure 1: HLT-design

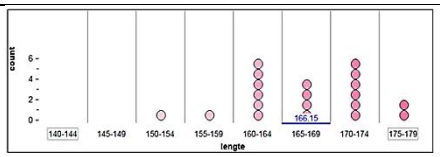
In step 6, the step we are focusing on, students will investigate what happens if the sample size increases. The hypothesis in this step is that students will understand that the characteristics (e.g. the mean) and the distribution of a larger sample usually better resemble the underlying population. To conceptualize this idea, students will use TinkerPlots (Konold & Miller, 2005) to easily and quickly simulate samples of different sizes. Therefore, the dataset from step 5 will be uploaded into TinkerPlots, in which the paper notes are then displayed as data cards. Students will be asked to simulate three small samples (size n=20) and three larger samples (n=200). A learning activity based on growing samples and the use of TinkerPlots is expected to help students to develop aspects of informal inference and argumentative reasoning (Ben-Zvi, 2006). Next, the students will be asked to

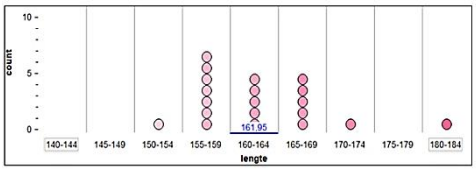
compare similarities and differences between their simulated sample results, and, during a whole-class session, to the underlying population. Embedding students' findings in a classroom discussion is expected to enhance their statistical reasoning (Bakker, 2004). This activity will prepare for probabilistic reasoning with sampling distribution in step 7, in which sample size plays an important role.

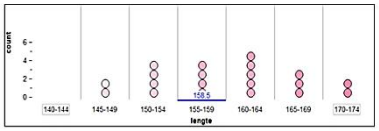
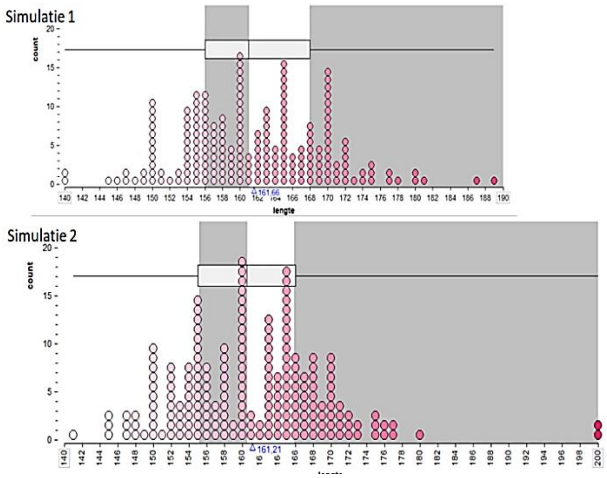
Method

The teaching experiment comprised a ten 45-minute lesson series and was piloted in one class with twenty students, which was taught by the first author. The sixth step was carried out in lesson 5 and included one lesson. The collected data were video-data from classroom discussion, students' worksheets and teacher notes. The data analysis consisted of verifying whether the designed hypotheses actually occurred. To this end, for each step of the design, several detailed and measurable hypotheses were formulated and translated into hypothesized visible student behavior.

Results

<table>
<tr><td colspan="4">Example of students' findings when simulating repeated samples from a large dataset</td></tr>
<tr><td colspan="2">Small sample (n=20)</td><td colspan="2">Large sample (n=200)</td></tr>
<tr><td>Mean</td><td>Distribution</td><td>Mean</td><td>Distribution</td></tr>
<tr><td>166,15</td><td rowspan="3"> 




</td><td>161,66</td><td rowspan="3"> 
</td></tr>
<tr><td>161,95</td><td>161,21</td></tr>
<tr><td>158,5</td><td></td></tr>
<tr><td colspan="4">Example of students' probabilistic reasoning</td></tr>
<tr><td colspan="2">"We think the average length of students is between 157-167. We don't know exactly because it is only a sample of 20 and you compare this with the population of almost 5,000 students."</td><td colspan="2">"We now think that the average is between 160-162 cm."

"The actual population is likely to be more similar to this distribution as the sample size is larger. This gives more information."</td></tr>
</table>

Figure 2: Students' findings with simulating repeated samples with different sizes (worksheet)

In this step, students investigated what would happen if the sample size increased. After simulating repeated samples with different sample sizes (n=20 and n=200), all students indicated that the mean for a larger sample was more stable. For example a student wrote: *"Here (large sample) the averages*

are more similar". An overview of the students' findings is displayed in Figure 2. During the classroom discussion, students mentioned that their means ranged between 157-166 for a size of 20 and between 159-162 for a sample size of 200. For example, a student said: "*Here (small sample) our lowest measurement was 157.2 and here (large sample) it is 159.7".* Comparison with the actual population (population mean 160.7) confirmed their expectation that a larger sample size would better reflect to the distribution and characteristics of the underlying population. Although the students were only briefly introduced to TinkerPlots, the teacher notes show that the tool enabled students to compare and explore samples of different sizes in a quick and easy way.

Conclusion

In step 6, the students investigated what would happen if the sample size of the physical experiment increased. The hypothesis was that students would understand that the characteristics and the distribution of a larger sample better resemble the underlying population. The intervention data show that students, based on their findings with the simulated samples, became aware of the effect of sample size. Both in student's work and during the whole-class discussion, they indicated that larger samples more closely resemble the underlying population. Key components in this step seem: (1) the strong correlation between the physical experiment in step 5 and the simulations in step 6; (2) the comparison between the simulated sample results, and; (3) the comparison with the underlying population. These key components will be elaborated on in further research in which specific attention will also be paid on instrumental genesis (Gueudet & Trouche, 2009) .

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A proposal of instrumental orchestration to integrate the teaching of Physics and Mathematics.

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This proposal, aims to promote a better understanding of physics and mathematics in the secondary level. This proposal integrates as the main procedure the CUVIMA methodological model (Cuevas et al., 2017) in companion with the use of digital technologic advances for the process and obtaining of experimental data, as well as the supporting of physical and mathematical modeling. For this, the design and organization of didactic activities within the theories of instrumental orchestration and documentary trajectory were necessary (Monaghan & Trouche, 2016). Three phases were carried out with two didactic experiences at secondary level in schools (kids from 12 to 14 years), where different instruments were used, phase allowed the process of instrumentalization for the student in terms of cognitive advances and appropriation of digital tools.

Keywords: modeling, digital technologies, orchestration, Physics and Mathematics

Introduction.

Although Physics and Mathematics have been developed interactively (Kline, 1981, Kjeldsen & Lützen, 2015, Karam, 2015, among others) there is a dichotomy in their teaching, and it has been proved that in the secondary level is not the ideal conceptual understanding of this two sciences (Tuminaro & Redish, 2007; Uhden et al., 2012; Karam, 2015). One of the main arguments in the teaching of Physics is that students have a mayor lack of the appropriate mathematical elements to understand and solve Physics issues at this educational level. Having said that, our proposal aims to give conceptual meaning to both sciences with the incorporation of the adequate digital technology as a tool. In the first place this would allow the simulation of natural phenomena and the capture of experimental data to be easier and concise. Secondly, it would support and facilitate the elaboration of the physical and mathematical model of the phenomenon in question.

Our didactic and methodological framework propose to stimulate the experimentation of natural phenomena to introduce physical concepts in elementary education, with the use of smartphones as a pocket laboratory (Forinash & Wishman, 2012; Monteiro et al., 2014; Cuevas, Villamizar & Martínez, 2017). At the same time, the physical concept establish a practical action project in whose resolution emerges the associated mathematical concept (Cuevas y Pluvinage, 2003). But how to orchestrate the various elements that intervene in this teaching situation? What set of instruments are required and in what way should they be configured?

Monaghan & Trouche (2017) definitions of different orchestrations and an examples of these is shown in Drijvers et al. (2010), who integrate didactic and technology for the teaching of Mathematics at the secondary level. We consider that the instrumental orchestration allows the configuration of the different tools, which enter into our didactic situation. In a preliminary analysis

we listed the various tools that intervene: -didactics (Cuevas y Pluvinage, 2004), -experimentations, -previous ideas and conceptual change (Viennot, 1979, Pozo, 2007), -digital technologies (smartphone, tablet, laptop and projector), -software (website, apps, *Tracker*, *Vyd-Analysis*, *Geogebra*). For the configuration of these concepts, the following list was done:

a) Analysis of the Physics teaching programs to choose a topic. Analyze the relevant physico-mathematical concepts, possibility of experiencing such phenomena in the classroom. In our case the topic of energy conservation was chosen, from which emerge the phenomenon of conservation of mechanical energy.
b) Analysis of the digital tools, software and appropriate apps that serve as mediators of the experimentation of the physical phenomenon and the taking of data, the modeling of the real physical experiment, or creation of a virtual scenario that simulates the experiment. In our case, the phenomenon of free fall was the starting point in experimentation, whose data were analyzed by means of video analysis software.
c) Design of a questionnaire to explore the previous ideas of the concept of Physics to be taught. In our case, the design was related to energy conservation issues.
d) Design of a diagnose questionnaire for the exploration of mathematical prerequisites implicit in the physical phenomenon. In our case, linear function, and elements of elementary algebra.
e) Determination of the learning objectives according to the educational level the student has.
f) Design of didactic activities, for the modeling of sciences. We used the model called CUVIMA (Cuevas et al., 2017), which integrates Mathematics, Physics and the use of digital technology as a mediator in the experimentation of physical phenomena and their modeling. Figure 1 shows this model applied in the conservation of mechanical energy topic.
g) Design of a posttest, to evaluate conceptual change and advances in mathematical concepts.
h) Design of instructions for the teacher and students, to guide the execution phase of the various artifacts.

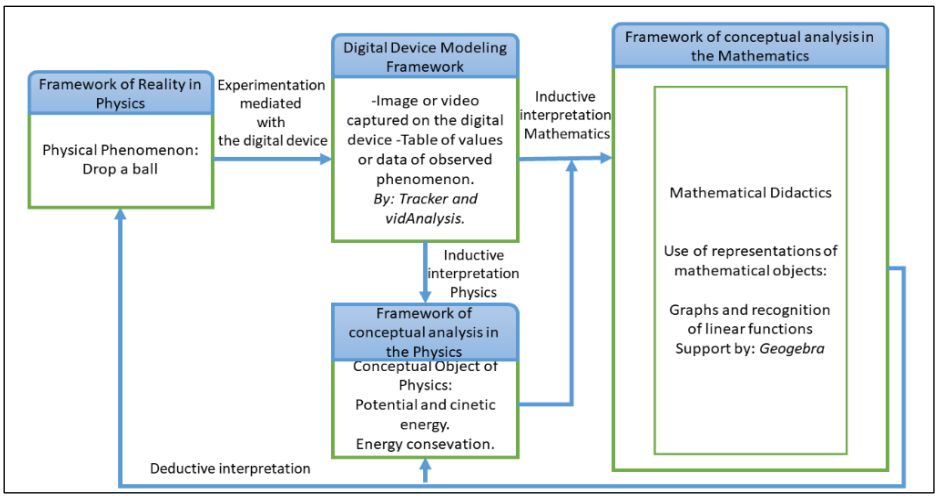

Figure 1: CUVIMA methodological model for Physics and Mathematics modeling.

Methodology

The execution of the configuration, will be described through a documentary trajectory of three phases with secondary school students in two different Colombian institutions:

Phase 1: Instructions for teachers and students on the mode of execution.

The teachers will be provided with a instructions manual on how to distribute the teaching material and measuring instruments to the students as well as the times in which each of them applies. Before the activity with the students, the teacher will use technical support by downloading and installing the software on the computer or if he/she prefers an app on the Tablet, the teacher will watch video tutorials, and then guide the students through a platform that provide the set of resources needed for the experimentation.

Phase 2: Execution of measurement instruments and didactic activities.

1. Application of a pretest (printed format) in a one-hour session. This test is applied in the classroom individually to each student with the goal of compile the previous ideas and concepts referring to the conservation of energy, as well as in mathematical antecedents referred to linear or related function. It is necessary before the second part the student previously review the tutorials regarding the management of video analysis software provided through the platform.

2. Application of the activity (questionnaire in printed format) in two sessions of an hour and a half, the modeling of a freefall experiment is performed in subgroups of three students. Following each of the frames of the CUVIMA model of Figure 1:

- The first *frame of reality in Physics*, begins with an experiment of a physical phenomenon, in this case consists of dropping a ball from a height of 2m.
- In the second *frame of the modeling of a digital device* the video of the free fall experiment is shown trough a smartphone or tablet. For the data analysis of the experiment, each subgroup in the computing class, exports the video to a computer for capturing the data of the experiment through a video analysis software Tracker. In case of using the tablet it is not necessary to export the video, however, it is necessary to open it directly from the Vid-Analysis app for data capture. Both the software and the app provide the student with a graphical representation of the phenomenon and a table of values of time, position, and speed of a free-fall ball, which corresponds to a modeling process mediated with digital technology.
- Once obtained the data and representations of the experiment, we proceed to the third *framework of conceptual analysis in Physics*. To analyze which concepts of Physics are implicit in such representations, in this case the representations correspond to kinetic and potential energy. This process of inductive interpretation in Physics must be guided by the teacher in the classroom.
- The representations or data of the phenomenon given by the digital device, can also be analyzed in the fourth *framework of conceptual analysis in the Mathematics*. The data obtained in Tracker or *VidAnalysis* were extrapolated to a spreadsheet in Geogebra to simulate the related linear functions of kinetic, potential, and mechanical energy. Through the questionnaire the teacher guides the interpretation of such as functions, which we call *processes of inductive interpretation in Mathematics*. The questionnaire ends with a serie of dosed questions that guide the student trough the use of mathematical model to deductively interpret the initial phenomenon and concepts in Physics, mainly the conservation of mechanical energy. This interpretation is discussed by the end of the activity between the teacher and the students.

3. Application of the Posttest (printed format) in an hour and a half session. This is applied in the classroom individually, to show the conceptual change of the previous ideas and mathematical advances, which corresponds to the phase of analysis of the didactic performance.

Phase 3: didactic performance evaluation.

This evaluation corresponds to the collection of data and analysis of the previous activities. In this case, it was evidenced that the student's previous ideas about energy conservation were modified in the posttest. This conceptual change is manifested in the use of reasoning where they use Mathematics to understand the physical phenomenon and not under common sense. In addition, digital technology allowed them to modify the teachers practices in the classroom, with computers and tablets being a portable laboratory alternative, it was observed that the development of didactic activities mediated with the use of digital technology allowed the student to have an active attitude in the classroom and a modification of the representations of a physical phenomenon.

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WG4. Posters

GT4. Posters

A Dynamic Multirepresentational resource for conics *

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Although the conics can be explored from various definitions and representations, an analytical approach with an algebraic focus is prioritized in its teaching. The exploration of the semiotic registers of the conics in a dynamic and synchronic manner implies, for example, the realization of displacements in the graph, obtaining the algebraic forms and the spatial figural modified, favored by the principle of direct manipulation of these objects, and allowing the quick and continuous access to several situations. As we have not software with these functionalities, we proposed the development of the software Dynamic Conics.

Keywords: Conic, Semiotic Representations, Dynamic Articulation, Educational Software.

Introduction.

Conics were object of investigation of several scholars throughout the centuries, passing through the synthetic (from Greek), the analytical and the projective geometries. However, the curriculum place for conics in Brazilian Basic Education has been reduced in Brazil and in France, although it continues to be present in several university courses, such as, Analytical Geometry, Calculus, Differential Geometry and Graphical Geometry. Studies point to the existence of a significant difference between the place of this notion in scientific knowledge and in school knowledge. They add that this difference is mainly due to the great difficulty in teaching conics, which, in turn, would be anchored in a geometric context and in the object itself.

Aiming to overcome these problems, our preliminary studies propose to a resource with the following principles:

- The unification of conics studies must happen by articulating the approaches of synthetic, analytical and projective geometries;
- The construction of conic notion and its geometric elements (foci, directives, eccentricity, vertices, axes, among others) must happen from the articulation of their representations: algebraic, graphic and spatial figural, not only from the "manipulation" of algebraic objects;
- This articulation requires a dynamism that considers visual variables (foci, directives, eccentricity, vertices and axes, explained in the graph) and visual figures (foci, directives, vertices in the spatial figure in perspective), associated to the symbolic units of the corresponding algebraic expressions (coefficients).

In order to meet these requirements, we cannot focus on mathematical field or its history only. A cognitive approach is necessary to specify dynamic articulation of various conic representations through computational resources, so that teaching can contribute for the overall development of students' and teachers' reasoning, analysis and visualization abilities.

In this sense, we understand that conic must be studied as a single object, including the definition of Apollonius and the Dandelin-Quetelet theorem (synthetic geometry), the definitions via quadratic

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equations, general and reduced, as well as the matrix form (analytic geometry) and the theorems of Pascal and Brianchon (projective geometry) as auxiliary representations in the transition between algebraic and spatial figural representations.

The cognitive analysis assumes that articulations should be done considering the transformations by treatment, between the algebraic registers, and by conversion, involving various representations of a conic: algebraic, graphic and spatial figural. For example, we must consider the visual variables of a spatial figure (foci, directives and five distinct points), visual variables specific of the graph (foci, directives, vertices, axes, etc.) and scalar values of equations (positive or negative coefficients, etc.). It can be seen that the foci and the directives are common to the spatial graphical and representational representations, and from these the equations can be deduced, using the monofocal definition (P, F) = e (P, d), where P is any point, F is a focus, d is the closest directrix to the focus F and e is the eccentricity.

We point out that, based on the definition of figural register (Salazar & Almouloud, 2012), we will call « dynamic semiotic register » the simultaneous articulation in a computational environment of algebraic, graphic and spatial figural representations, integrating discursive registers (natural language, algebraic) and non-discursive (geometric figures, flat and spatial, Cartesian graphs); multifunctional (also used outside of mathematics, for the functions of communication, objectification, or even for a treatment function) and monofunctionals (mathematics), so as to support the transformations of the representations, in a single environment, concomitantly between all registers.

In this sense, Bellemain and Siqueira (2009) point out that one of the contributions of computers in this context is the creation of new "dynamic" representation registers, where articulations can appear in the synchronous and continuous variations of both numerical and geometric elements. In this context, we conceive an environment that supports a global view of conics. It involves the three approaches articulating their three representations, simultaneously, exploring the geometric and algebraic properties, including considering five of their points and their tracing, in a projective geometry.

These environments that we need can be conceived as a system of authorship of situation in computational environment with the availability to integrate several artefacts, in order to support the manipulation and articulation of representation systems, particularly in the case of conics. Functioning as well as an integrated and dynamic representation system, leading users through direct manipulation and observation to building knowledge from exploration of object behaviours and phenomena in a variety of representation registers and definitions.

A software for performing dynamic articulations of conic representations.

Initially we expect the software to present, in an approach to synthetic geometry, a circular cone of revolution sectioned by a plane, depicted in horseman perspective, from the definition of Apollonius and the Dandelin-Quetelet theorem.

This feature should also allow the dimensional deconstruction of the cone sectioned by a plane, for example, of dimension 3, and of the ellipse, of dimension 2, and in auxiliary traces we can identify smaller dimensional units (1D and 0D) as straight lines and points.

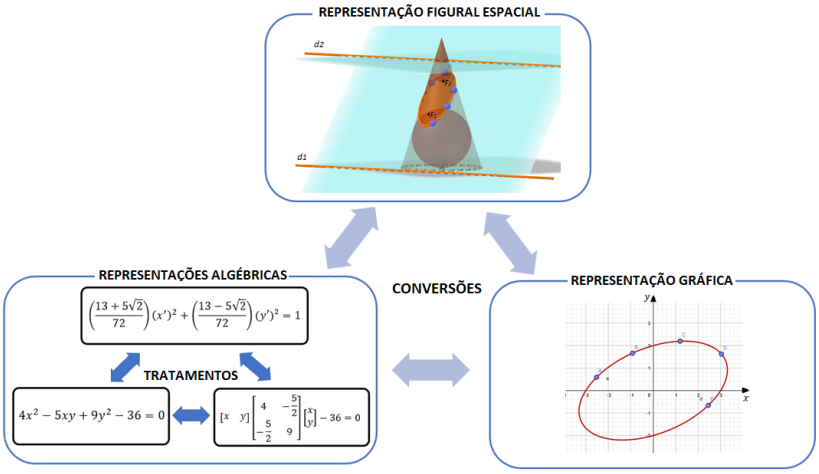

Figure 1: Correspondence between spatial, algebraic and figural representation of an ellipse

In this representation we can explore the foci, the directrices, at least, any five points that belong to a conic and the inclination angle of the section plane. Another possibility will also to refer the plane that cuts the cone and tangentiate the spheres, allowing the coordinates of the foci to be obtained from at least two points that determine the directrices and any five points.

According to Bellemain and Guimarães (2005) the analytical characteristics of a conic can be determined considering some characteristic geometric elements that can be obtained directly from five points of this curve, facilitating particularly its drawing. Thus, in the conversion of algebraic expressions to the graph (or vice versa), the software will present the transition auxiliary representations. In this case, it will be the results of Pascal's theorem and Brianchon's theorem for the conics, ensuring that five non-collinear points determine a conic curve.

In this way, the resource will present the representations considering the significant characteristics (the values and conditions of the coefficients) of the algebraic expressions so that the graph is sketched from a global and simultaneous global seizure.

We emphasize that the input record can be in any of the representations, as well as output record, so there will be direct manipulation of the curve, the equations and the spatial figural representation. This feature should cause the successive displays of the concomitant variations of the corresponding symbolic units (coefficients) of three algebraic representations hinged to the visual units of the graph, and visual figures of the spatial figure to appear in the interface, or field of view, thus providing a dynamic articulation of the conic representations.

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Process of educational software development: epistemological and experimental analysis in the creation environment Lematec-Studium*

Ricardo Tiburcio[1] e Franck Bellemain[2]

The study presents some characteristics of a theoretical and methodological proposal of an engineering process that considers the nuances of teaching and learning research integrated with the knowledge related to the technological potentialities. The methodology of this study has as principle an epistemological survey, in the sense of retrieving characteristics of processes developed resource developments, in sequence the product experimentation, in a case study, based on the reference of Didactic-Computer Engineering and Instrumental Orchestration Theory.

keywords: Engineering. Educational Software. Didactic-Computer Engineering.

The Engineering of Educational Software and its developments

How to develop software that meets the specificities of mathematical knowledge? How to create a resource that contributes to the teaching activity of teaching? What characteristics should a technological artefact have in order to aid student learning? What technological contributions can be useful for educational software? These issues present the fragility of the educational software development processes - SE, since, in a first analysis, it involves the areas of epistemology, teaching, learning and technology. The fragility in question lies in the centralization in one of these areas to the detriment of others (Bellemain et al, 2014), in a process, such as that of educational software engineering that requires several approaches.

We start from the hypotheses that: 1. Understanding process characteristics Developments in educational technological resources will collaborate in the creation of process modelling; 2. The historical / epistemological study will provide subsidies for the improvement of Didactic-Computer Engineering - EDI; 3. The design and experimentation of the product, in a case study, based on the EDI reference and Instrumental Orchestration Theory - TOI, may be useful for the improvement of the development model. The purpose of this research is to develop an Educational Software Process model based on Didactic-Computer Engineering, with an epistemological approach using the Theory of Instrumental Orchestration in support of the theoretical and experimental phases of the process.

The Didactic-Informatics Engineering as theoretical contribution and object of study

The Didactic-Computer Engineering - EDI, emerged in a scenario of attempts to model the processes of development of educational software that considers theoretical and practical aspects about the development of resources for teaching and learning mathematical knowledge.

The term "Didactic-Computer Engineering" is the perception in using the methodological procedures and theoretical reflections of two engineering: Didactics and Software developments. The need for articulation among engineering was verified when some limitations of the same are observed for the development of educational software: the limitations of Didactic Engineering - ED, are verified at the moment when this

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methodology does not contemplate, in its contributions, the total needs for the development of educational software. The Engineering of Software - ES, in turn, does not contemplate specifics that the educational software need, when it is observed that the standardized models of development were created for commercial products, banking, domestic, etc.

In view of the above, we observed the need to gather the pertinent elements of the two engagements: ED with the elements of theoretical and experimental research on teaching and learning and ES with the standardization of the development of software and methods of obtaining requirements, with the purpose of building a process of development of educational software that observe the technological advances, but that does not despise the theoretical studies realized for the teaching and the learning of the knowledge.

When analysing the experimental phase proposed in the Software Process Model under development in this study, a source of theoretical and methodological resources was found in the Instrumental Orchestration Theory - TOI (TROUCHE, 2009) for an analysis that presents elements of the use of the product that is intended to be created. According to Trouche (2009) an Instrumental Orchestration is the systematic organization of the artefacts available in a given environment, establishing the work of a particular mathematical activity. This pre-determined organization will favor the observation of elements of analysis for the development of software designed through the Process Model undergoing improvement.

Methodological process and first results obtained

For the first moment of improvement of the model proposed in the EDI, we believe that the realization of an epistemological approach will reveal us elements of analysis that can be used for a new version of this model. The epistemological analysis in question consists in observing the origin and the development of some software programs with consolidated research in the area of Mathematics Education. The chosen software are: Cabri Géomètre, Function Probe, Cassyope and Modellus; given the wide range of research and consolidated results of researches with these resources.

Thus, we believe that understanding what factors were taken into account to develop such products will bring us elements to rethink the development of educational software. This survey consists of understanding the nuances of the project until the launch of the first version of the software: questions about the initial objectives, team composition, teaching and learning objectives, technological issues regarding the possibilities and limitations given the development era, whether contributions theorists, among others.

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Young Researcher Workshop / Atelier jeunes chercheurs

Workshop of young researcher organizing comitté/
Atelier jeunes chercheurs comité d'organisation

Chairs / Coordinatrice : Maryna Rafalska (French Institute of Education)

Participants :

Mohammad Alturkmani (IFÉ), Cibelle Assis (UFPB, Brazil), Verônica Gitirana (UFPE, Brazil), Anita Messaoui (ENS de Lyon), Takeshi Miyakawa (Joetsu University of Education, Japon), Sophie Soury-Lavergne (IFÉ), Luc Trouche (ENS de Lyon)

After the conference (31st May and 1st June), the workshop will be offered to young researchers, in order to deepen its theoretical and methodological propositions, and to participate in hands-on sessions, allowing to develop or/and to use tools related to the analysis of teachers documentational work. The workshop includes four successive sessions. Selected materials will be sent to participants before the conference, in order to prepare the workshop.

Après la conférence (31 mai et 1er juin), un atelier sera proposé aux jeunes chercheurs, pour approfondir ses propositions méthodologiques et théoriques et participer à des sessions de travaux pratiques, permettant d'utiliser ou de développer des outils relatifs à l'analyse du travail documentaire des enseignants. L'atelier se déroulera suivant quatre sessions. Les participants recevront, avant la conférence, du matériel leur pemettant de préparer au mieux les sessions.

Session A - Interactions with conference plenary speakers / Interactions avec les conférenciers

Chairs / Coordinateurs : Birgit Pepin & Takeshi Miyakawa (Local Organizing Committee / Comité local);

Corresponding members as young researchers / *jeunes chercheurs impliqués :* Cibelle Assis, Ousmana Fofana, Katiane de Moraes Rocha, & Michael Umameh

During the main conference the discussion time is likely to be limited in terms of offering sufficient opportunities for interacting with plenary lecturers. The objective of this first session is to provide opportunities for interacting more directly with plenary speakers and plenary discussion panel members: for example, Michèle Artigue, Janine Remillard, Kenneth Ruthven, Luc Trouche and Binyan Xu will be present. Two issues will be particularly addressed:

- Teachers' resource systems, their structure and dynamics, components and evolution;
- Teachers learning from/when interacting with resources.

Pendant une conférence internationale, le temps n'est pas toujours suffisant pour avoir des interactions prolongées avec les conférenciers. L'objectif de cette première session est d'offrir de telles opportunités d'interactions avec les conférenciers et les participants au panel. Par exemple, Michèle Artigue, Janine Remillard, Kenneth Ruthven, Luc Trouche and Binyan Xu seront présents. Deux questions seront plus particulièrement traitées :

- Les systèmes de ressources des enseignants, leur structure et leur dynamique, leurs composants et leur evolution ;
- Ce qu'apprennent les enseignants en interagissant avec des ressources.

Session B - Sequencing learning activities – roles of didactic metadata / Structurer les activités d'apprentissage – les rôles des métadonnées didactiques

Chairs / Coordinateurs : Jason Cooper & Sophie Soury-Lavergne (Local Organizing Committee / comité local);

Corresponding members as young researchers / jeunes chercheurs impliqués : Anita Messaoui, Luxizi Zhang & Fangchun Zhu

Teachers increasingly take on some of the roles of curriculum designers; in some contexts teachers are expected to supplement their textbook with additional activities, in others they may need to put together whole learning sequences on their own. In this workshop we will introduce some tools (see this website) that we have developed in order to support teachers in this role, by providing didactic views on sequences of activities – as they appear in textbooks, or as modified/designed by teachers. Participants in the workshop will use our tools to reflect on a given sequence of activities, and to propose and substantiate alternate sequences. The aims of the workshop are:

- To introduce participants to some novel tools for textbook analysis and for curricular design;
- To demonstrate how we go about researching the tools that we develop;
- To gain some feedback from an international audience, which may contribute to future versions of our tools.

Les enseignants, de plus en plus, contribuent au développement du curriculum ; dans certains contextes, on attend des enseignants qu'ils complètent les manuels scolaires avec d'autres ressources ; dans d'autres contextes, on attend d'eux qu'ils conçoivent eux-mêmes l'ensemble des séquences de cours. Dans cette session, nous introduirons des outils (voir ce site) que nous avons conçus pour soutenir les enseignants dans ce mouvement, en leur proposant un point de vue didactique sur les suites d'activités, comme elles apparaissent dans les manuels scolaires, ou qu'elles sont modifiées/conçues par les enseignant. Les participants à cette session pourront utiliser ces outils pour réfléchir à leurs propres ressources, et penser des ressources alternatives. Les objectifs de la session sont :

- De présenter aux participants de nouveaux outils pour l'analyse des manuels scolaires et pour la conception de ressources curriculaires ;
- De mettre en évidence la recherche qui soutient le développement de nos outils ;
- De bénéficier des retours de jeunes chercheurs internationaux qui pourront contribuer à de nouvelles versions de nos outils.

Session C - Webdocuments for collaborative analysis of teachers' practices with resources / Des webdocuments pour l'analyse collaborative des pratiques des enseignants avec les ressources

Chairs/Coordinateurs : Franck Bellemain, Mohammad Alturkmani & Verônica Gitirana (Local organizing committee/ Comité local)

Corresponding members as young researchers / jeunes chercheurs impliqués : Rogério Ignácio & Rosilângela Lucena

Over the past five years, *webdocuments* as online files incorporating texts, images, audio or video components have been developed (Bellemain & Trouche, 2016; Lucena & Assis, 2015) for supporting collaborative analyzes of teachers' documentation work. The participants of the workshop will use dynamic features of such webdocuments for constructing, displaying, sharing and discussing their analyses of video-recorded episodes of teachers interacting with resources.

Pendant les 5 dernières années, les *webdocuments*, comme des fichiers en ligne incorporant des textes, des images, des composants audio ou vidéo ont été développés (Bellemain & Trouche, 2016 ; Lucena & Assis, 2015) pour soutenir les analyses collaboratives du travail documentaire des enseignants. Les participants à cette session utiliseront les caractéristiques de ces webdocuments pour concevoir, mettre en ligne, partager et discuter leurs analyses d'enregistrements vidéo d'épisodes du travail documentaire des enseignants.

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Session D - Naming systems used by secondary school teachers to describe their resources and their documentation work / Les systèmes de noms utilisés par des enseignants du secondaire pour décrire leurs ressources et leur travail documentaire

Chairs / Coordinateurs : Michèle Artigue & Luc Trouche (Local Organizing Committee / comité local).

Corresponding members as young researchers / jeunes chercheurs impliqués : Maryna Rafalska, Ulises Salinas, Karima Sayah, Hendrik van Steenbrugge & Chongyang Wang.

The *Lexicon* project (Artigue *et al.,* 2017*; Clarke et al.*, 2017), supported at an international level by a number of researchers, aims to « document the naming systems (lexicons) employed by different communities speaking different languages to describe the phenomena of the mathematics classroom ». The purpose of this session is to develop a similar reflection on naming systems, considering teachers' resources and documentation work.

Le projet *Lexicon* (Artigue *et al.*, 2017; Clarke *et al.*, 2017), auquel participent de nombreux chercheurs au niveau international, vise à "documenter les systèmes de dénomination (lexiques) employés par différentes communautés parlant différentes langues pour décrire les phénomènes didactiques dans les classes de mathématiques". L'objectif de cette session est de développer une réflexion semblable, mais à propos des lexiques que les enseignants développent pour décrire leurs ressources et leur travail documentaire.

Denominating the matter of the teachers' work in Mathematics:
a didactical question

Dénommer la matière du travail des enseignants en Mathématiques:
une question didactique

Denominando la materia del trabajo de los profesores en Matemáticas:
una cuestión didáctica

تسمية مادة عمل مدرسي الرياضيات : مسألة ديداكتكية

数学教师在工作中对物质的命名：来自教学实践的问题

uutal tuur jumtukaayi jàangalekati matematik yi: yoonu xamle

The session will analyze data gathered beforehand in different languages contexts (e.g. Arabic, Chinese, English, French, Portuguese, Spanish, Ukrainian and Turkish). The data will have been collected from two video-recorded interviews: the first one focusing on the preparation of a lesson; the second one focusing on teachers' resource systems. For each of these questions, the teachers will have been asked to name their resources, describe their classification and the steps of their documentation work. Preliminary work will be done, describing in English the reality to which these terms refer, and proposing examples and counter examples. The session will allow a cross presentation and analysis of these data.

La session confrontera des données collectées dans différents contextes linguistiques (par exemple : arabe, chinois, anglais, français, portugais, espagnol, ukrainien et turc). Ces données auront été collectées à l'occasion de deux interviews: la première centrée sur la préparation d'une leçon; la deuxième centrée sur le système de ressources des enseignants. Dans chacun de ces deux contextes, les enseignants seront sollicités pour nommer leurs ressources, décrire leur rangement et les étapes de leur travail documentaire. Un travail préliminaire aura été fait, avant cette session, pour décrire en anglais la réalité à laquelle ces termes réfèrent, et proposer des exemples et des contre exemples permettant de situer ces termes. La session permettra une présentation croisée et une analyse de ces données.

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Short biographies of the lecturers and of the WG chairs
Biographies courtes des conférenciers et des coordinateurs des groupes de travail

Michèle Artigue, after a doctorate in mathematical logics, progressively entered the field of mathematics education. In this field, beyond theoretical contributions on the relationships between epistemology and didactics, didactical engineering, the reproducibility of didactic situations, the instrumental approach, and more recently the networking of theoretical frames, her main research areas have been the teaching and learning of mathematics at university level, and especially the didactics of Calculus and Analysis, and the integration of computer technologies into mathematics education. She is currently Emeritus Professor at the University Paris Diderot – Paris 7. She has had and still has many editorial and scientific responsibilities at national and international level. Especially, after being from 1998 to 2006 Vice-President of the International Commission on Mathematical Instruction (ICMI), she presided the Commission from 2007 to 2009. She has been awarded the Felix Klein Medal by the International Commission on Mathematical Instruction for her life-long research achievement in 2013, and the Luis Santalo Medal by IACME for her support to the development of mathematics education in Latin America in 2015.

Michèle Artigue, après un doctorat en logique mathématique, s'est engagée dans le champ de la didactique des mathématiques. Dans ce champ, au-delà de ses contributions théoriques sur les rapports entre épistémologie et didactique, sur l'ingénierie didactique, sur la reproductibilité des situations didactiques et sur l'approche instrumentale, et, plus récemment sur la mise en réseau des cadres théoriques, elle s'intéresse principalement à l'enseignement et l'apprentissage des mathématiques à l'Université, plus particulièrement à la didactique de l'analyse, et à l'intégration des technologies numériques dans ce domaine. Elle est actuellement professeur émérite à l'Université Paris Diderot-Paris 7. Elle a eu de nombreuses responsabilités éditoriales et scientifiques aux niveaux national et international. En particulier elle a été de 1998 à 2006 vice-présidente de l'ICMI (International Commission on Mathematical Instruction), et elle a présidé cette commission de 2007 à 2009. Elle a reçu la médailles Felix Klein de l'ICMI en 2013, pour l'ensemble de son œuvre au service de l'éducation mathématique, et la médaille Luis Santalo de l'IACME en 2015 pour son soutien au développement de l'éducation mathématique en Amérique Latine.

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Paul Drijvers is a full professor in mathematics education at the Freudenthal Institute of Utrecht University's Science Faculty. His wide range of research interests include the role of ICT in mathematics education, mathematical thinking, assessment, the teaching and learning of algebra, and teachers' professional development. Paul also works as a professor in mathematics education at HU University of Applied Sciences Utrecht.

Paul Drijvers est professeur des universités en éducation mathématiques à l'Institut Freudenthal de l'Université d'Utrecht. Ses domaines d'intérêt incluent le rôle des TIC dans l'enseignement des mathématiques, la pensée mathématique, l'évaluation, l'enseignement et l'apprentissage de l'algèbre, et le développement professionnel des enseignants. Paul travaille aussi dans le domaine de l'éducation mathématiques à HU, Université des sciences appliquées d'Utrecht.

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Verônica Gitirana is associate professor of the Federal University of Pernambuco of Brazil, currently in post-doctoral studies by one year at the IFÉ (Institute Français d'Éducation) of ENS-Lyon in France. She coordinated the Mathematics Assessment of the PNLD – Brazilian Textbook Program of 2018, 2017, 2016. Master's in mathematics from UFPE, she obtained her doctorate from the Institute of Education of the University of London in 1998. Her research interest are digital resources for teaching mathematics and teacher training, mainly in what concerns: the instrumental orchestration and instrumental meta-orchestration, teachers' documentation, teaching activity, as well as the development.

Verônica Gitirana est professeure associée à l'Université Fédérale de Pernambouc au Brésil, actuellement en séjour de recherche pour un an à l'IFÉ (Institut Français de l'Éducation) de l'ENS de Lyon en France. Elle a Elle a coordonné le PNLD de 2016, 2017, 2018 – Programme Brésilien des Manuels Scolaires. Après un master en mathématiques, elle a obtenu son doctorat en éducation mathématique à l'Université de London en 1998. Ses intérêts de recherche portent sur les ressources numériques pour l'enseignement des mathématiques et pour la formation des enseignants, principalement du point de vue des orchestrations et des meta-orchestrations instrumentales, de la documentation des enseignants, de leur activité et de leur développement professionnel.

Bellemain, F., Gitirana, V., & Bellemain, P.M.B. (2015) Elementos de engenharia de software educativos para a concepção de ferramentas computacionais para o CSCL.. In: M.Rosa, M.Bairral, & R.B.Amaral (Org.). *Educação Matemática, Tecnologias Digitais e Educação a Distância: pesquisas contemporâneas.* 1ed.São Paulo: Editora Livraria da Física, 2015, v. 1, p. 223-262.

Lucena, R., Gitirana, V., & Trouche, L. (to appear) Analyse des Schèmes dans un travail collectif lié à une meta-orchestration instrumentale. Colloquium des Didactique de Mathématiques – ARDM - 2018.

Trouche, L., Gitirana, V., Miyakawa, T., Pepin, G., & Wang, C. (Submitted) Studying mathematics teacher interactions with curriculum materials through different lenses: towards a deeper understanding of the processes at stake special issue on Curriculum ergonomics, *IJRE - International Journal of Research and Education.*

Ghislaine Gueudet is full professor in mathematics education at the teacher training institute of Bretagne, France (CREAD and ESPE de Bretagne, University of Brest, France, http://cread.espe-bretagne.fr/membres/ggueudet). Her research concerns several themes: University mathematics education, in particular the difficulties linked with the secondary-tertiary transition; and the design and use of educational resources (digital resources in particular). Concerning these resources, she has introduced in a joint work with Luc Trouche the documentational approach to didactics, analyzing teachers' interactions with resources and the consequences of these interactions in terms of professional development.

Ghislaine Gueudet est professeur des universités en didactique des mathématiques à l'Ecole Supérieure du Professorat et de l'Education de Bretagne, France. Sa recherche concerne plusieurs thèmes : l'enseignement des mathématiques à l'Université, en particulier les difficultés liées à la transition lycées-universités ; et la conception et l'usage de ressources éducatives (en particulier numériques). Concernant ces ressources, elle a introduit, dans un travail conjoint avec Luc Trouche, l'approche documentaire du didactique, analysant les interactions des enseignants avec les ressources et les conséquences de ces interactions en terme de développement professionnel.

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM, The International Journal on Mathematics Education*, *49*(5), 799-812.

Gueudet, G. (2017). University Teachers' Resources Systems and Documents. *International Journal of Research in Undergraduate Mathematics Education 3*(1), 198-224.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.) (2012). From Textbooks to 'Lived' Resources: Mathematics Curriculum Materials and Teacher Documentation. New York: Springer

Carole Le Hénaff is since 2016 a Senior Lecturer in Science Education at the University of Western Brittany. She is a researcher in didactics of foreign languages and cultures at the Research Center on Education, Learning and Didactics (CREAD). Part of her research concerns the use of resources, digital or not, for the learning and teaching of English, particularly within the theoretical frame of the Documentational Approach to Didactics.

Carole Le Hénaff est depuis 2016 maîtresse de conférences en sciences de l'éducation à l'Université de Bretagne Occidentale. Elle est chercheuse dans le domaine des langages et cultures étrangères au Centre de Recherche sur l'Éducation les Apprentissages et la Didactique. Une partie de sa recherche porte sur les usages de ressources, numériques ou non, pour l'apprentissage et l'enseignement de l'anglais, particulièrement dans le cadre de l'approche documentaire du didactique

Gruson, B., Gueudet, G., Le Hénaff, C. & Lebaud, M.-P. (à paraître, 2018). Investigating teachers' work with digital resources. A comparison between teaching Mathematics and English. *Revue Suisse des Sciences de l'Éducation.*

Le Hénaff, C. (à paraître, 2018). Comprendre une vidéo en anglais au lycée. In Collectif Didactique Pour Enseigner (Eds), *Didactique Pour Enseigner. Rennes* : Presses Universitaires de Rennes.

Le Hénaff, C. & Gruson, B. (soumis). Le travail de préparation d'une ressource vidéo par un professeur d'anglais et son effet sur la pratique didactique. Questions Vives."

Catherine Loisy is a Senior Lecturer in psychology at the French Institute of Education (IFE) associated to the Ecole Normale Supérieure (ENS) in Lyon. Her research theme mainly focuses on the professional development of teachers in the digital age. She studies how interactions supported by digital devices contribute to learning and development, especially for higher education teachers. In order to study the development trajectories of professionals, she works on the elaboration of a methodology based on Vygotsky's approach, which considers development as an appropriation of historic-social artifacts. She also questions the effects of the introduction of digital technologies on pedagogy in higher education. She has conducted several national studies supported by the General Directorate of Higher Education and Professional Integration of the Ministry of Higher Education, Research and Innovation in France. She is a national expert on the issue of teachers' digital skills.

Catherine Loisy est maître de conférences en psychologie à l'Institut français de l'éducation (IFÉ) de l'Ecole normale supérieure (ENS) de Lyon. Sa thématique de recherche est le développement professionnel des enseignants à l'heure du numérique. Elle étudie la manière dont les interactions soutenues par des dispositifs numériques contribuent aux apprentissages et au développement, notamment au développement professionnel des enseignants du supérieur. Afin d'étudier les trajectoires de développement des professionnels, elle travaille à l'élaboration d'une méthodologie basée sur l'approche de Vygotsky, qui considère le développement comme une appropriation d'artefacts historico-sociaux. Elle s'intéresse également aux effets de l'introduction du numérique sur la pédagogie de l'enseignement supérieur. Elle a porté plusieurs études nationales soutenues par la Direction générale pour l'enseignement supérieur et l'insertion professionnelle (DGESIP) du Ministère français de l'enseignement supérieur et de la recherche, et elle est expert national sur la question des compétences numériques des enseignants.

Loisy, C. et Coulet, J.-C. (dir.) (2018). Compétences et approche-programme. Outiller le développement d'activités responsables. Londres, ISTE Editions.

Loisy, C. et Lameul, G. (2017). Augmenting De Ketele's model for university pedagogy introduction. *International Journal of Educational Technology in Higher Education (IJTHE/RITPU)*, *14*(2), 39. Disponible en ligne : https://doi.org/10.18162/ritpu-2017-v14n2-03

Loisy, C. (2017). Accompagnement du projet d'orientation de l'élève et numérique. Quels apprentissages pour les élèves ? Dans V. Liquète et B. Le Blanc, *Quand le numérique s'invite à L'Ecole… Revue Hermès*, *78*, 113-120.

Takeshi Miyakawa is an associate professor of the Joetsu University of Education in Japan and actually staying in ENS de Lyon for his one-year sabbatical leave (2017-2018). He obtained his PhD degree of the didactics of mathematics from the University of Grenoble in 2005. His research interest is twofold: teaching and learning of proof and proving on the one hand (Miyakawa, 2017), and teachers' collective work in different settings including Lesson Study in Japan on the other hand (Miyakawa & Winsløw, 2013; 2017). For both themes, he conducts research within the framework of the Anthropological theory of the didactic and is trying to adopt a comparative perspective between Japan (or East Asian countries) and European countries.

Takeshi Miyakawa est professeur associé à l'Université d'éducation de Joetsu au Japon, actuellement en séjour de recherche pour un an à l'ENS de Lyon. Il a obtenu son doctorat en didactique des mathématiques à l'Université de Grenoble en 2005. Ses intérêts de recherche sont doubles : d'un côté l'enseignement et l'apprentissage de la preuve en mathématiques (Miyakawa, 2017), de l'autre côté le travail collectif des enseignants dans différents dispositifs, en particulier les Lesson Studies au Japon (Miyakawa & Winsløw, 2013; 2017). Pour ces deux domaines, il développe da recherche dans le cadre de l'approche anthropologique du didactique, et essaie d'adopter une perspective comparative entre le Japon (plus largement les pays d'Asie orientale) et les pays européens.

Miyakawa, T. (2017). Comparative analysis on the nature of proof to be taught in geometry: the cases of French and Japanese lower secondary schools. *Educational Studies in Mathematics*, *94*(1), 37-54.

Miyakawa, T. & Winsløw, C. (2013). Developing mathematics teacher knowledge: the paradidactic infrastructure of "open lesson" in Japan. *Journal of Mathematics Teacher Education*, *16*(3), 185-209.

Miyakawa, T. & Winsløw, C. (2017, Online First). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: a case study of "practice research" in Japan. *Journal of Mathematics Teacher Education*.

John Monaghan's research interests are: learning and teaching mathematics 14-21; students' understandings of algebra and of calculus; the use of mathematical software in learning and teaching mathematics; tool use and mathematics; and theoretical frameworks in mathematics education research. He is Professor at the University of Agder, Norway, and Emeritus Professor at the University of Leeds, England. He has one PhD student from Agder and another one from Leeds giving papers at this conference. Publication which spans his research interests include (see below)

John Monaghan s'intéresse de rechercher sûr l'enseignement et l'apprentissage des mathématiques (étudiants de 14-21 ans) ; la compréhension par les étudiants de l'algèbre et de l'analyse ; l'utilisation des logiciels de mathématiques pour apprendre et enseigner les mathématiques ; l'utilisation des outils en mathématiques. John est professeur à l'Université d'Agder, en Norvège, et professeur émérite de l'Université de Leeds en Angleterre. Un de ses doctorants d'Agder, et un autre de Leeds présenteront des contributions lors de la conférence. Des publications représentatives de ses intérêts de recherche sont présentées ci-dessous.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computers for Mathematical Learning*, 9(3), 327.

Monaghan, J., & Ozmantar, M. F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics*, 62(3), 233-258.

Monaghan, J., Trouche, L., & Borwein, J. M. (2016). *Tools and mathematics: Instruments for learning*. Berlin: Springer International Publishing.

Samet Okumus is an assistant professor of mathematics education at Recep Tayyip Erdogan University. He earned his bachelor's degree in primary school mathematics teaching with an emphasis in middle school mathematics from Anadolu University in 2008. Soon after, he started his master's program at Karadeniz Technical University. In 2010, the Council of Higher Education of Turkey awarded him a scholarship to do research at Indiana University for six months. Then, he earned his master's degree in mathematics education in 2011. In January 2012, he started his Ph.D. education at North Carolina State University, and received his Ph.D. in mathematics education in 2016.

His research interests include using technological and non-technological tools to understand mathematical concepts with a focus on middle school and high school mathematics. Also, he conducts research on teacher practice, especially in geometry instruction.

Samet Okumus est professeur assistant d'éducation mathématique à l'Université Recep Tayyip Erdogan. Il a obtenu sa licence comme professeur de mathématiques dans l'enseignement primaire à l'Université Anadolu en 2008. Il a engagé ensuite ses études de master à l'Université technique Karadeniz. En 2010, il obtint une bourse de l'enseignement supérieur turc pour conduire une recherche, pendant 6 mois, à l'Université de l'Indiana. Après avoir obtenu, en 2012, son master, il a commencé son doctorat à l'Université d'état de Caroline du Nord, doctorat achevé en 2016.

Ses intérêts de recherche incluent l'utilisation d'outils, technologiques ou non, pour comprendre les concepts mathématiques avec un focus sur les collèges et les lycées. Il conduit aussi des recherches sur les pratiques d'enseignement, en particulier en ce qui concerne la géométrie.

Hollebrands, K., & Okumuş, S. (2018). Secondary mathematics teachers' instrumental integration in technology-rich geometry classrooms. *The Journal of Mathematical Behavior*, 49(1), 82-94.

Cayton, C., Hollebrands, K., Okumuş, S., & Boehm, E. (2017). Pivotal teaching moments in technology-intensive secondary geometry classrooms. *Journal of Mathematics Teacher Education*, 20(1), 75-100.

Okumuş, S., Patterson, L., Wiebe, E., & Hollebrands, K. (2016). Utility and usability as factors influencing teacher decisions about software integration. *Educational Technology Research and Development*, 64(6), 1227–1249.

Birgit Pepin has taught mathematics in secondary schools in England before starting her PhD in mathematics education: an investigation of mathematics teachers' work in three countries (England, France, Germany), in particular with respect to their mathematical and pedagogic practices, and the influence of educational and cultural traditions on their work as a mathematics teacher. After finalizing her PhD she worked in UK higher education institutions (Open University, Oxford Brookes University; Manchester University), and later moved to Norway (2009-2014) for her first full professorship, and then to the Netherlands (since 2015). She is now a full professor of mathematics/ STEM education at Eindhoven University of Technology. Her teaching and research focus on mathematics curriculum materials (including digital resources) and their design and use by teachers and students; and teacher pedagogy and professional learning, especially in mathematics and science education in secondary schooling and in higher STEM education.

Birgit Pepin a enseigné les mathématiques à l'école secondaire en Angleterre avant de commencer des études doctorales en éducation des mathématiques : une étude du travail des enseignants de mathématiques dans trois pays (Angleterre, France, Allemagne), en particulier du point de vue de leurs pratiques mathématiques et pédagogiques, et de l'influence des traditions éducatives et culturelles sur leur travail comme professeurs de mathématiques. Après son doctorat, elle a travaillé dans des institutions d'enseignement supérieur à l'Université (Open University, Oxford Brookes University, Manchester University) et ensuite en Norvège (2009-2014) pour son premier poste de professeur d'Université, et, depuis 2015, aux Pays-Bas (depuis 2015). Elle est maintenant professeur d'éducation mathématique et de STEM (Sciences, Technology, Ingénierie et Mathématiques) à l'Université de Technologie d'Eindhoven. Le focus de son enseignement et de sa recherche, ce sont les matériels curriculaires (incluant les ressources numériques), leur conception et leur usage par les enseignants et les étudiants ; et la formation pédagogique et professionnelle des enseignants, en particulier pour l'éducation mathématique et scientifique dans le second degré, et pour l'enseignement STEM à l'Université.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: foundations for change. *ZDM - Mathematics Education, 49*(5), 645- 661.

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining *teacher design capacity:* Mathematics teachers' interactions with digital curriculum resources. *ZDM - Mathematics Education, 49*(5), 799-812.

Pepin, B., Xu, B., Trouche, L. & Wang, C. (2017). Developing a deeper understanding of *mathematics teaching expertise*: an examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics, 94* (3), 257-274

Christine Proust is a historian of mathematics and ancient sciences, specializing in cuneiform sources. She is *Directrice de Recherche* (senior researcher) at the laboratory SPHERE (CNRS and University Paris-Diderot). Her researches on the history of ancient mathematics began with the study of mathematical cuneiform texts from the ancient Mesopotamian city of Nippur, now kept in the Archaeological Museum in Istanbul and in the University of Jena. She studied the organization of mathematical curriculum in Nippur's scribal schools during the Old-Babylonian period (early second millennium BCE). Using pedagogical sources, she reconstructed the methods elaborated by ancient scholars to calculate surfaces, volumes and other quantities, and to solve linear and quadratic problems. Her current researches deal with mathematical practices in high administrations during the earliest periods (fourth and third millennium BCE) and in priestly milieus during the late periods (4th to 1st century BCE).

Christine Proust est historienne des mathématiques et des sciences anciennes, spécialisée dans les sources cunéiformes. Elle est Directrice de Recherche au laboratoire SPHERE (CNRS et Université Paris-Diderot). Ses recherches sur l'histoire des mathématiques anciennes ont débuté avec l'étude des textes mathématiques cunéiformes de l'ancienne cité mésopotamienne de Nippur, conservés aujourd'hui dans les musées archéologiques d'Istanbul et à l'université d'Iéna. Elle a étudié l'organisation du curriculum mathématique dans les écoles de scribes de Nippur pendant la période paléo-babylonienne (début du deuxième millénaire avant l'ère commune). En s'appuyant sur les sources pédagogiques, elle a pu reconstituer les méthodes élaborées par les anciens érudits pour calculer les surfaces, les volumes et d'autres quantités, et pour résoudre des problèmes linéaires et quadratiques. Ses recherches actuelles portent sur les pratiques mathématiques dans la haute administration au cours des périodes les plus anciennes (quatrième et troisième millénaire avant notre ère) et dans les milieux sacerdotaux au cours des périodes tardives (4ème au 1er siècle avant notre ère).

Proust, Christine. (2007). *Tablettes mathématiques de Nippur*. Istanbul: Institut Français d'Etudes Anatoliennes, De Boccard.

Proust, Christine. (2012). Teachers' writings and students' writings: school material in Mesopotamia. In *From Text to 'Lived' Resources*, ed. Ghislaine Gueudet, Birgit Pepin, and Luc Trouche, 161-179. New York: Springer.

Proust, Christine. (2013). Du calcul flottant en Mésopotamie. *La Gazette des Mathématiciens* 138: 23-48 (English version : http://cdli.ucla.edu/pubs/cdlp/cdlp0005_20160501.pdf)

Maryna Rafalska defended her Ph.D. thesis in Educational Sciences in 2010 on the problem of development of informatics competences of pre-service teachers of informatics at the lessons of numerical analysis. She worked at the National Pedagogical Dragomanov University (Kiev, Ukraine) as an associate professor where she studied the professional development of teachers of informatics and mathematics with a focus on the development of their information literacy and competences of using ICT in the class (Rafalska 2014).

Her recent research interests concern the teaching and learning of algorithmics at secondary school and the conception of the main notions of algorithmics at the transition "secondary school-university" in France and in Ukraine (Modeste & Rafalska to appear(a), to appear(b)).

Maryna Rafalska a obtenu son doctorat en sciences de l'éducation en 2010 sur les questions du développements des compétences informatiques des futurs enseignants d'informatique dans le cadre de l'enseignement et l'apprentissage de l'analyse numérique. Elle a travaillé à l'Université Pédagogique Nationale Dragomanov (Kiev, Ukraine) comme professeur associée où elle s'est intéressée au développement professionnel des enseignants d'informatique et de mathématique, plus particulièrement sur le développement de leurs culture numérique et leurs compétences dans l'usage des TICE en classe (Rafalska 2014).

Actuellement, elle s'intéresse aux problématiques de l'enseignement et de l'apprentissage de l'algorithmique au secondaire et à la conception des notions principales d'algorithmique à la transition lycée-université en France et en Ukraine (Modeste & Rafalska (a), (b)).

Rafalska M.V. (2014). Using e-learning to improve the level of students' professional competences. *E-learning and Intercultural Competences Development in Different Countries*. Monograph, Sc. Editor Eugenia Smyrnova-Trybulska, University of Silesia, Studio-Noa, Katowice-Cieszyn, pp. 359-369.

Modeste, S., & Rafalska, M. (to appear(a)) Algorithmics In The Secondary School: A Comparative Study Between Ukraine and France, *Proceedings of CERME 10*, Dublin.

Modeste,S., & Rafalska, M. (to appear(b)) Conception de la notion d'algorithme à la transition secondaire-supérieur en France, Actes de EMF, Paris.

Janine Remillard is a professor of mathematics education at the University of Pennsylvania's Graduate School of Education, where she serves as the Faculty Director of Teacher Education. Her research interests include teachers' interactions with mathematics curriculum materials, mathematics teacher learning in urban classrooms, and locally relevant mathematics instruction. Since 1999, she has received continuous funding from the National Science Foundation to support research and development projects on teacher learning, mathematics curriculum use, and formative assessment. She is co-editor of the volume, *Mathematics Teachers at Work: Connecting Curriculum Materials and Classroom Instruction*. Remillard is active in the mathematics education community in the U.S. and internationally, including serving as chair of the U.S. National Commission on Mathematics Instruction, a commission of the National Academy of Sciences.

Janine Remillard est professeur en éducation mathématique à l'Ecole supérieure de l'éducation de l'Université de Pennsylvanie. Elle y est directrice du centre de formation des maîtres. Du point de vue de la recherche, elle s'intéresse aux interactions des enseignants avec le matériel curriculaire, à la formation continue des enseignants dans les classes urbaines, et aux aspects locaux de l'enseignement des mathématiques. Depuis 1999, elle obtient de façon continue le soutien de la Fondation Nationale pour la Science pour ses projets de recherche et développement sur la formation des enseignants, les usages du curriculum en mathématiques et évaluation formative. Elle est co-éditrice de l'ouvrage *Mathematics Teachers at Work: Connecting Curriculum Materials and Classroom Instruction.* Remillard joue un rôle actif dans la comunauté d'éducation mathématique aux niveaux national et international. Elle a été en particulier présidente de la Commission Nationale sur l'enseignement des mathématiques, une commission de l'Académie Nationale des Sciences.

Remillard, J. T. & Kim, O-K. (2017). Knowledge of curriculum embedded mathematics: Exploring a critical domain of teaching. *Educational Studies in Mathematics*, *96*(1), 65–81. (view online:http://rdcu.be/qajL)

Remillard, J. T., & Reinke, L. T. (2017). Mathematics Curriculum in the United States: New Challenges and Opportunities. In D. R. Thompson, M. A. Huntly, & C. Suurtamm (Eds.), *International Perspectives on Mathematics Curriculum* (pp. 131-162). Greenwich, CT: Information Age Publishing.

Remillard, J. T. (2016). Keeping an Eye on the Teacher in the Digital Curriculum Race. In M. Bates, & Z. Usiskin, (Eds.), *Digital Curricula in School Mathematics* (pp. 195-204). Greenwich, CT: Information Age.

Sebastian Rezat is professor of mathematics education at the Institute of Mathematics, Paderborn University (Germany). He graduated as an upper secondary teacher in mathematics and music. His doctoral thesis on secondary students' use of their mathematics textbooks has been awarded by the German Society of Didactics of Mathematics (GDM). In his research, he investigates the use of traditional and digital mathematics textbooks at primary and secondary level. He is particularly interested in students' interactions with textbooks and the interplay between students' and teachers' use of textbooks. In collaboration with Rudolf Sträßer he has developed the socio-didactical tetrahedron that models the whole situation of textbook and resource use from an activity theoretical perspective. As Chair of the International Programme Committee (IPC), he is going to host the inaugural International Conference on Mathematics Textbook Research and Development (ICMT-3) to be held in the Germany, 2019.

Sebastian Rezat est professeur de didactique des mathématiques à l'Institut de mathématiques de l'Université de Paderborn (Allemagne). Il a été enseignant en lycée pour les mathématiques et la musique. Sa thèse, portant sur l'utilisation des manuels scolaires de mathématiques dans l'enseignement secondaire a été distinguée par la Société Allemande de Didactique des Mathématiques. Dans sa recherche, il étudie les usages des manuels scolaires mathématiques, traditionnels et numériques, dans l'enseignement primaire et secondaire. Il s'intéresse plus particulièrement aux interactions des élèves avec les manuels scolaires, et au jeu entre les usages des manuels scolaires par les élèves et les professeurs. En collaboration avec Rudolf Sträßer, il a introduit le tetrahedron socio-didactique qui modélise la situation d'usage du manuel scolaire et des ressources depuis une perspective théorique de l'activité. En tant que président du Comité international de programme, il accueillera la troisième Conférence internationale sur les manuels scolaires en mathématiques (recherche et développement), qui se tiendra en Allemagne en 2019.

Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation. *ZDM - The International Journal on Mathematics Education, 44*(5), 641-651. doi:10.1007/s11858-012-0448-4

Rezat, S. (2013). The textbook-in-use: students' utilization schemes of mathematics textbooks related to self-regulated practicing. *ZDM, 45*(5), 659-670. doi:10.1007/s11858-013-0529-z

Rezat, S., & Sträßer, R. (2015). Methodological issues and challenges in research on mathematics textbooks. *Nordic Studies in Mathematics Education, 20*(3-4), 247-

Kenneth Ruthven is Emeritus Professor of Education at the University of Cambridge in England and Guest Professor in Mathematics Education at Karlstad University in Sweden and at the University of Agder in Norway. He is an Advisory Editor of the international journals *Educational Studies in Mathematics* and *Research in Mathematics Education* as well as a Fellow of the Academy of Social Sciences. His recent research in mathematics education (references below) has examined dialogic teaching approaches, digital curriculum resources, and techno-pedagogical aspects of classroom-based professional expertise. Fuller information about his work is available at http://www.educ.cam.ac.uk/people/staff/ruthven/.

Kenneth Ruthven est professeur émérite en éducation à l'Université de Cambridge en Angleterre, et professeur invité en éducation mathématique à l'Université Karlstad en Suède et à l'Université d'Agder en Norvège. Il fait partie du conseil éditorial des revues internationales *Educational Studies in Mathematics* et *Research in Mathematics Education*, et membre de l'Académie des Sciences Sociales. Ses recherches récentes dans le domaine de l'éducation mathématique (voir ci-dessous) portent sur les approches d'enseignement dialogique, sur les ressources curriculaires numériques et sur les aspects techno-pédagogiques de l'expertise professionnelle basées sur le salle de classe. Plus d'information sur la page web http://www.educ.cam.ac.uk/people/staff/ruthven/

Ruthven, K., Mercer, N., Taber, K., Guardia, P., Hofmann, R., Ilie, S., Luthman, S., & Riga, F. (2017). A research-informed dialogic-teaching approach to early secondary school mathematics and science: the pedagogical design and field trial of the *epiSTEMe* intervention. *Research Papers in Education*, 32(1), 18-40.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: Foundations for change. *ZDM*, Special Issue on *Digital Curricula in Mathematics Education*, 49(5), 645-661.

Bozkurt, G. & Ruthven, K. (2017). Classroom-based professional expertise: a mathematics teacher's practice with technology. *Educational Studies in Mathematics*, 94(3), 309-328.

Hussein Sabra is an assistant professor in mathematics education at the University of Reims Champagne-Ardennes (France). He received his Ph.D form University of Lyon I – Claude Bernard (Ph. D supervisor Luc Trouche).

Hussein Sabra est maître de conférences en didactique des mathématiques à l'Université de Rennes Champagne-Ardennes (France). Il a passé son doctorat à l'Université Lyon 1 (directeur de thèse : Luc Trouche).

His research interest concerns the documentational work of mathematics teachers. The first research he carried out concerns secondary level. Currently he is also interested in higher education level: the interactions of university professors with resources and resources for teaching mathematics to non-specialists (science, technology, and engineering).

Il s'intéresse au travail documentaire des professeurs de mathématiques. Ses premières recherches concernaient les professeurs du second degré. Elles concernent aujourd'hui l'enseignement à l'université, du point de vue des interactions des professeurs avec les ressources, et les ressources pour enseigner les mathématiques à des non spécialistes (sciences, technologie, ingénierie).

He considers in his research particularly the individual and collective processes of design resources (Gueudet et al. 2016). He is interesting also on analyzing resources and their use by mathematics teachers (Gueudet et al. 2018). He develops methodological tools to consider particular events and critical resources (Sabra 2016).

Il considère plus particulièrement les processus individuels et collectifs de conception de ressources (Gueudet et al. 2016). Il s'intéresse aussi à l'analyse des ressources et des usages qu'en font les enseignants de mathématiques (Gueudet et al. 2018). Il développe des outils méthodologiques prenant en compte des évènements particuliers et des ressources critiques (Sabra 2016).

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Monstapha Sokhna did a PhD in Mathematics in Dakar University, then a PhD in Mathematics education in Montpellier University, under the supervision of Luc Trouche. He is currently vice-dean of the Faculty of Educational Sciences and Technology (FASTEF) of the Dakar University. He is in charge of the doctoral school in mathematics education, after having chaired the FASTEF Mathematics department.

Moustapha Sokhna, après une thèse de mathématiques à l'Université de Dakar, a fait une thèse de didactique des mathématiques, à l'université de Montpellier, sous la direction de Luc Trouche. Il est actuellement Assesseur de la Faculté des Sciences et Technologies de l'Education et de la Formation (FASTEF) de l'Université Cheikh Anta Diop de Dakar, responsable de formation doctorale en didactique des mathématiques après avoir **été** chef de département de mathématiques de la FASTEF.

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Abboud-Blanchard, M., Caron, F., Dorier, J.-L., Sokhna, M. (2015). *Dimensions culturelles dans la conception, la* diffusion *et les usages des ressources dans l'espace mathématique francophone* http://archive-ouverte.unige.ch/unige:84383

Sophie Soury-Lavergne is associate professor in didactics of mathematics and researcher at the French Institute for Education, IFE-ENS Lyon. For more than 20 years, her field of research has been Dynamic Geometry and Technology Enhanced Learning in Mathematics. The difficulties of technology integration into primary school teachers' practices questioned the role and place of digital technology in the teachers' system of resources. She is studying this question by considering a duo of artefacts, composed of a tangible and a digital tool, as a simplified model of a system of instruments in a didactical situation. It enables to frame research questions related to teachers' resources, design, quality and appropriation as well as to students' learning in the domains of arithmetic or geometry. Her approach is also relevant to deal with emergent fields of questions about robots and connected objects for education.

Sophie Soury-Lavergne est maître de conférences (HDR) en didactique des mathématiques et chercheuse à l'Institut français de l'éducation (ENS de Lyon). Pendant plus de 20 ans son champ de recherche a été la Géométrie dynamique et l'intégration de la technologie pour l'apprentissage des mathématiques. Les difficultés de l'intégration de la technologie dans les pratiques des professeurs à l'école primaire ont questionné le rôle et la place des technologies numériques dans les systèmes de ressources des enseignants. Elle aborde cette question en considérant des duo d'artefacts, composé d'un outil numérique et d'un outil tangible, comme modèle simplifié d'un système d'instruments dans une situation didactique. Cela permet de donner un cadre aux questions de recherche relatives aux ressources des professeurs, à leur conception, à leur qualité et à leur appropriation, aussi bien qu'aux apprentissages des élèves dans les domaines de l'arithmétique ou de la géométrie. Cette approche apparaît aussi pertinente pour aborder les questions émergentes de robots et d'objets connectés pour l'éducation.

Maschietto, M., & Soury-Lavergne, S. (2017). The duo "pascaline and e-pascaline": an example of using material and digital artefacts at primary school. In E. Faggiano, F. Ferrara, & A. Montone (Eds.), *Innovation and technology Enhancing Mathematics Education*, pp. 137–160.

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Soury-Lavergne, S., & Maschietto, M. (2015). Articulation of spatial and geometrical knowledge in problem solving with technology at primary school. *ZDM – The International Journal on Mathematics Education*, *47*(3), 435–449.

Jana Trgalova is Associate Professor at Claude Bernard University in Lyon, France. She is involved in mathematics teacher education. Her research focuses on digital technology and resources for mathematics education, their design, appropriation, use and evaluation. She was involved in several national and international research projects. She is active in the international research community in the field of technology in mathematics education; she co-chaired the 13th International Conference on Technology in Mathematics Teaching in Lyon in 2017 and led the technology group at the congresses of the European Research on Mathematics Education (CERME) in 2011, 2013 and 2015.

Jana Trgalova est maîtresse de conférence à l'Université Claude Bernard de Lyon, France, dans le champ de la didactique des mathématiques. Ses recherches concernent les technologies et les ressources numériques pour l'enseignement des mathématiques, leur conception, leur appropriation, leurs usages et leur évaluation. Elle a été impliquée dans plusieurs programmes de recherche aux niveaux national et international. Elle est engagée dans la communauté de recherche internationale dans le champ de la technologie pour l'enseignement des mathématiques ; elle a co-présidé la 13ème conférence ICTMT (International Conference on Technology in Mathematics Education) à Lyon en 2017 et a coordonné le groupe sur la technologie aux Congrès Européens de recherche sur l'enseignement des mathématiques (CERME) en 2011, 2013 et 2015.

Trgalová, J., El-Demerdash, M., Labs, O., & Nicaud, J.-F. (2018 – to appear). Design of Digital Resources for Promoting Creative Mathematical Thinking. In L., Ball, P., Drijvers, S., Ladel, H-S., Siller, M., Tabach, & C., Vale, (Eds.), *Uses of technology in primary and secondary mathematics education*. Springer.

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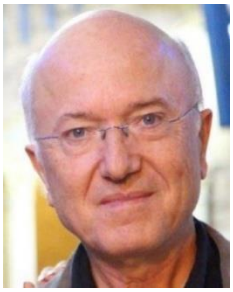
Luc Trouche is professor of didactics of mathematics in the French Institute of Education, École Normale Supérieure de Lyon, France. Interested in studying the role of tools in the learning of mathematics (Monaghan, Trouche & Borwein 2016), he has contributed to develop the notion of *instrumental orchestration* (Trouche & Drijvers 2014) for modeling the management, by a teacher, of available tools in the classroom. He focuses now on resource use/design and teacher professional development in the time of digitalization. This has led him to contribute to develop the *documentational approach to didactics* (Gueudet, Pepin & Trouche 2012). In this perspective, the notion of teacher *resource system* appears crucial in order to understand teacher (developing) knowledge and the coherence of his/her activity. Studying the interactions between individual and collective teachers' resource systems gives means for understanding the dynamics of these collectives, and for rethinking the way of supporting teacher development at a time of the 'metamorphosis' of teaching environments.



Luc Trouche est professeur de didactique des mathématiques à l'IFÉ – Institut Français de l'éducation, École Normale Supérieure de Lyon, France. Intéressé par l'étude du rôle des outils pour l'apprentissage des mathématiques (Monaghan, Trouche & Borwein 2016), il a contribué à développer la notion d'orchestration instrumentale (Trouche & Drijvers 2014) pour modéliser la gestion, par un professeur, des outils disponibles dans sa classe. Il s'intéresse désormais aux usages et à la conception de ressources par les enseignants, en relation avec leur développement professionnel dans cette période de numérisation des supports. Cela l'a conduit à contribuer à développer une approche documentaire du didactique (Gueudet, Pepin & Trouche 2012). Dans cette perspective, la notion de système de ressources apparaît critique pour comprendre le développement des connaissances des enseignants et la cohérence de leur activité. Etudier les interactions entre les systèmes de ressources individuels et collectifs des enseignants donne des moyens pour comprendre la dynamique de ces collectifs, et pour repenser la manière de soutenir le développement professionnel des enseignants dans un temps de métamorphose des environnements pour apprendre.

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Trouche, L., & Drijvers, P. (2014). Webbing and orchestration. Two interrelated views on digital tools in mathematics education, *Teaching Mathematics and Its Applications: International Journal of the Institute of Ma'thematics and its Applications, 33*(3), 193-209.

Binyan Xu received her PhD in Mathematics Education at the University of Osnabrueck in Germany. Currently, she is a Professor of Mathematics Education at the East China Normal University. Her research interests focus on the students' mathematics core competences assessment and collective work with teachers to design mathematical projects to improve students' engagement in mathematics activities in classroom (Xu & Zhu, 2015; Pepin, Xu, Trouche & Wang, 2017). She is a key member for developing mathematics textbooks and teaching materials in Shanghai (Xu, 2013). Now she is the co-chair of Local Organizing Committee of the 14th International Congress on Mathematics Education (ICME 14) in 2020 in Shanghai. Since 2017, she is the EC member of the International Commission of Mathematical Instruction.

Binyan Xu a obtenu son doctorat à l'université d'Osnabrück en Allemagne. Elle est actuellement professeur en éducation mathématique à l'ECNU (East China Normal University). Ses intérêts de recherche portent sur l'évaluation des compétences mathématiques du socle commun, sur le travail collectif des enseignants pour soutenir des projets mathématiques stimulant l'engagement des élèves dans la classe. Elle a une place centrale pour le développement de manuels scolaires et de matériels didactiques à Shanghai. Elle est la co-présidente du comité local d'organisation du 14ème congrès international sur l'éducation mathématique (ICME 14) qui se tiendra en 2020 à Shanghai. Depuis 2017, elle est membre de la Commission Internationale pour l'Enseignement des Mathématiques.

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