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# Homogenized of Lamination Stacks in Magnetodynamics: an original 2-D Formulation

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**Abstract:** The accurate validation of 3-D homogenized formulations for laminations makes necessary to re-write these formulations in 2-D. This is paradoxically not simple to do. Indeed, the magnetic field is in the studied plane, while the current is perpendicular to it : “closure conditions” have to be added for the currents.

We propose here a 2-D homogenized formulation, well suited for finite elements; it assures finally that the total current in each sheet of the equivalent rebuilt non-homogenized solution is really nil.

**Keywords:** eddy current, homogenization, lamination.

## I. INTRODUCTION

For some years, several papers were anew dedicated to homogenization techniques for lamination stacks [1, 2]. The precise validation of 3-D formulations is difficult: in general, the field’s behavior is essentially quasi 1-D, because the mean value of the current density is nearly nil through the thickness of each sheet. But the second order effects are precisely those we try to highlight with the 3-D model: for example, a component of flux density, normal to the stack, necessarily gives a non-zero homogenized current density.

We previously proposed a 1-D test-case [3] with such non-zero homogenized current densities, obtained by modifying the closure conditions. However, the validation of a 3-D formulation obtained by comparison with 2-D results will be still much more convincing.

The 2-D formulation presented in this paper is based on the 2-D magnetodynamic  $\mathbf{a}$ - $v$  formulation which has been re-written so it can be applied on a homogenized domain.

## II. MAGNETODYNAMIC $\mathbf{a}$ - $v$ FORMULATION

The magnetodynamic problem is defined in a 2-D explicitly domain  $\Omega$  (i.e. lamination stacks). The  $\Omega$  domain is divided into two parts :

$\Omega_c$  : the conductive part of the domain  $\Omega$

$\Omega_c^c$  : the non-conductive part of the domain  $\Omega$

with  $\Omega = \Omega_c \cup \Omega_c^c$ .

The expression of the electrical field  $\mathbf{e}$  via a magnetic vector potential  $\mathbf{a}$  involves the gradient of an electric scalar potential  $v$  in  $\Omega_c$ .

$$\mathbf{e} = -\partial_t \mathbf{a} - \mathbf{grad} v \text{ in } \Omega_c \quad (1)$$

with

$$\mathbf{b} = \mathbf{curl} \mathbf{a} \text{ in } \Omega \quad (2)$$

$\mathbf{a}$  : magnetic vector potential

$v$  : electrical scalar potential

$\mathbf{b}$  : magnetic flux density

The  $\mathbf{a}$ - $v$  magnetodynamic formulation is obtained from the weak form of the Ampere equation, and the two potentials.

$$\mathbf{curl}(\mu^{-1} \mathbf{curl} \mathbf{a})_{\Omega} + \mathbf{s}(\partial_t \mathbf{a} + \mathbf{grad} v)_{\Omega_c} = 0 \quad (3)$$

$\mu$  is the magnetic permeability and  $\mathbf{s}$  is the conductivity.

Equation (3) supposes that there is no source with current density (if there is one, it can easily be added on the second term of the equation).

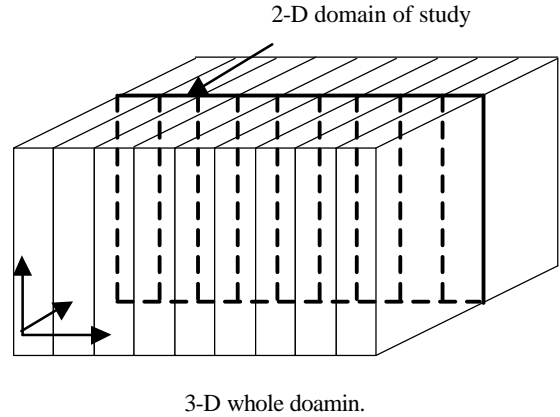
The current density in the conducting domain is given by:

$$\mathbf{j}_z(x, y) = -\mathbf{s}(\partial_t \mathbf{a} + \mathbf{grad} v)_{\Omega_c} \quad (4)$$

In the 2-D formulation,  $\mathbf{a}$ ,  $\mathbf{grad} v$  and  $\mathbf{j}$  have one component along the “out-of-plane” direction (i.e. z-axes).

An example of the studied 2-D plane is given by the figure 1.

Figure 1 : An example of a 2-D domain of study and the



The  $\mathbf{grad} v$  in these equations has one value in each conducting region. It is linked with the total current in each conducting region (stack) by circuit equations (“closure conditions” taking into account the “out-of-plane” part of the domain).

For example in lamination stacks, the global current is often supposed to be zero in each sheet, giving as many equations as unknown gradients.

Note that the gradient of  $v$  is only defined in conducting region  $\Omega_c$  and is equal to zero in non-conducting regions  $\Omega_c^c$ .

Figure 2 illustrate the variation of the z-component of  $\mathbf{grad} v$  along the x-axes.

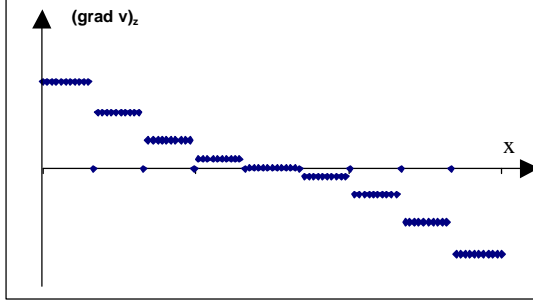


Figure 2 : The variation of the z component of (grad v) in the sheets

### III. PROPOSED 2-D FORMULATION

As it is, the  $\mathbf{a}$ - $v$  formulation can be directly applied on the real domain (lamination stacks). But it's difficult to apply it on the homogenous model because the layers are not explicitly and separately defined.

Let  $\tilde{J}_z$  be the homogenized current density (i.e. the mean value of the actual current density through the thickness of the sheet [3]).

$$\tilde{J}_z = \frac{1}{d} \int_d \mathbf{j}_z dx \quad (5)$$

$d$  : plate thickness ( $x$  - direction)

$\mathbf{j}_z$  : z-component of the current density in the explicit laminated domain.

The current density in the explicit laminated domain has a global zero value in each sheet. In the homogenous model, we have to impose that the integration of the homogenized current density is equal to zero along each vertical line ( $y$ -direction). We have the condition (6):

$$0 = \int_h \tilde{J}_z(x, y) dy \quad (6)$$

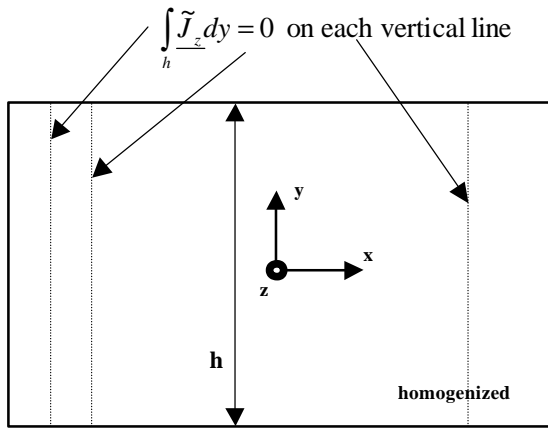


Figure 3: Condition to be imposed on the homogenized current  $\tilde{J}_z$

As it is, (6) does not suit well for a F.E. discretization. We propose to indirectly impose this condition by expressing  $\tilde{J}_z$  with a nodal quantity  $\mathbf{K}$ , which equals zero at the top and the bottom of the stack:

$$\tilde{J}_z(x, y) = \partial_y \mathbf{K}(x, y) \quad (6)$$

$$\mathbf{K}(y=0) = \mathbf{K}(y=h) = 0 \text{ (B.C.)} \quad (7)$$

In the homogenous domain, the gradient  $\partial_z v$  is constant along a vertical line (the  $y$ -direction):

$$\partial_y (\partial_z v) = 0 \quad (8)$$

On the other hand, we have:

$$\underline{\mathbf{s}} \cdot \partial_z v = -(\tilde{J}_z + \underline{\mathbf{s}} \cdot \partial_t \mathbf{a}) \quad (9)$$

By combining (6), (8) and (9) we will have :

$$\partial_y^2 \mathbf{K}(x, y) + \underline{\mathbf{s}} \cdot \partial_y (\partial_t \mathbf{a}) = 0 \quad (10)$$

And the system of equations to be solved in the homogenous domain is:

$$\begin{cases} \text{curl}(\underline{\mu}^{-1} \text{curl} \mathbf{a}) + \partial_y \mathbf{K} = 0 \\ \partial_y^2 \mathbf{K} + \underline{\mathbf{s}} \cdot \partial_y (\partial_t \mathbf{a}) = 0 \end{cases} \quad (11)$$

$\underline{\mu}$  is the equivalent anisotropic tensor of permeability and  $\underline{\sigma}$  is the equivalent anisotropic tensor of conductivity [3]. They are given by:

$$\underline{\mu} = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu \cdot \underline{G} & 0 \\ 0 & 0 & \mu \cdot \underline{G} \end{bmatrix} \quad (12)$$

and

$$\underline{\mathbf{s}} = \begin{bmatrix} \mathbf{s} & 0 & 0 \\ 0 & \mathbf{s} \cdot \underline{G} & 0 \\ 0 & 0 & \mathbf{s} \cdot \underline{G} \end{bmatrix} \quad (13)$$

The X and Y components of the tensor  $\underline{\mu}$  and the Z component of the tensor  $\underline{\sigma}$  are only used in the 2-D formulation.

The function  $\underline{G}$  is given by the expression:

$$\underline{G} = \frac{\tanh \frac{(1+j)d}{2d}}{(1+j)d} \quad (14)$$

with  $d$  the plate thickness and  $\delta$  the skin depth

$$d = \sqrt{\frac{2}{\omega \cdot \mathbf{s} \cdot \mu}} \quad (15)$$

$\omega = 2 \cdot \pi \cdot f$  : the pulsation ( $f$  : frequency)

#### IV. EXAMPLE

In this simple example, we will illustrate the developed formulation. We will compare two domains: a lamination stack domain and its equivalent homogenous domain. In the two cases, we force a component of the magnetic flux to flow perpendicular to the lamination. To do this, we impose boundary conditions on the magnetic vector potential  $\mathbf{a}$  as shown in figure 4.

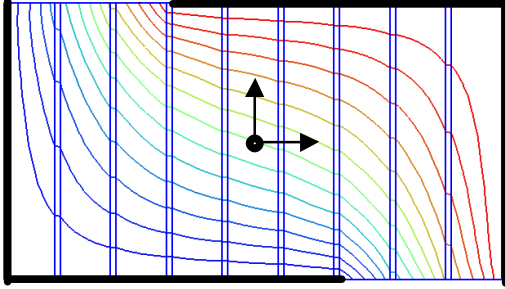


Figure 4: Boundary conditions and flux lines.

##### A - The lamination stacks domain

In this case, the 2-D magnetic vector potential formulation is applied (equations 3 and 4).

The generated current density in the sheets is perpendicular to the plan of study and has only one component along the z-axes. Figure 5 shows the z-component of the current density in the plan of study.

The values of the z-component of  $\mathbf{grad} \, v$  are shown in figure 2.

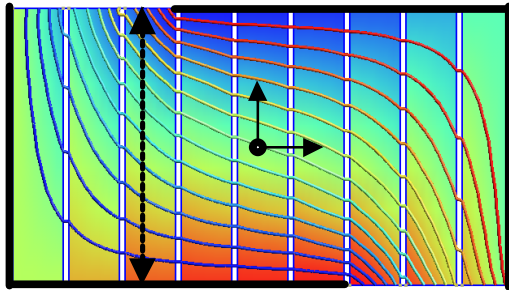


Figure 5 :  
Current density (z-component) and flux lines.

##### B - The homogenized model

To compute the current density and other values in the homogenous model, we apply the presented 2-D formulation (as in equation 11) using the F.E. method where  $\mathbf{K}$  is defined as a nodal quantity.

The figure 6 shows the homogenous current density (z-component), the iso-K lines and the flux lines. The same boundary conditions are imposed here.

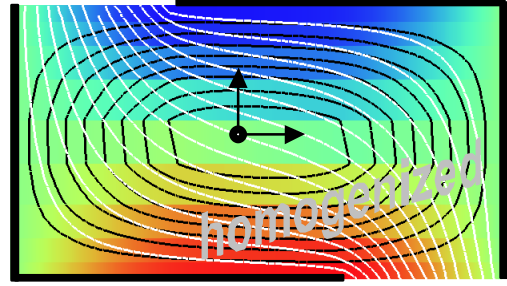


Figure 6 : Homogenous current density (z-component), iso-K lines and flux lines.

As you note, the results in the homogenized 2-D formulation compared to the 2-D magnetic vector potential formulation are very close.

#### V. CONCLUSION

This formulation of the 2-D homogenized magnetodynamic problem will allow to implement relevant tests for the 3-D models, that we previously published, also in the non-linear case [4]. It can be easily added in 2-D solvers like Flux2D...

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