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Interior metric and ray-tracing map in the firework black-to-white hole transition

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The possibility that a black hole could tunnel into to white hole has recently received attention. Here we present a metric that improves the "firework" metric: it describes the entire process and solves the Einstein's equations everywhere except on a small transition surface that corresponds to the quantum tunnelling. We compute the corresponding ray-tracing map from past infinity to future infinity explicitly.

arXiv:1803.06330v2 [gr-qc] 20 Mar 2018

I. FIREWORKS

Black holes have become common astrophysical objects. Recent observations strengthen the result that the most accurate theory we have to describe them is still centenarian general relativity. Yet classical general relativity leaves questions open. What happens ultimately to the in-falling matter? Is information lost after Hawking evaporation? There is no consensual answer to this questions yet. A scenario to address these questions has recently raised interest: the possibility of a quantum tunnelling from a black hole to a white hole [1–7]. This transition is allowed by general relativity provided that quantum theory permits the violation of Einstein's equation (by a tunneling process) in a small compact spacetime region. A spacetime realising this scenario, called the *firework* metric because matter inside the hole can explode out of the white hole after the tunnelling, was given (on July 14th) in reference [2]. The calculation of the quantum probability for the process has been addressed in [4, 5], using the spinfoam formalism of loop quantum gravity; the effect of the Hawking evaporation and the relevance of the scenario for the information loss paradox have been recently discussed in [7].

Here we present an improvement on the firework metric discovered in [2]. Following [7] we distinguish between two physically distinct quantum phenomena relevant in this process. In the terminology of [7], region A is the Planckian-curvature region around the singularity where the interior black hole metric continues to a white hole metric. Physically, this describes the interior bounce, a stage called "Planck star" [1]. This transition can be modeled by a smooth joining of two Kruskal spacetimes, a possibility noted by several authors [8, 9] and recently discussed in [6]. The proper quantum tunnelling is then confined to a small region B [7], which surrounds the end of the apparent horizon of the black hole. Here we give a metric that satisfies the Einstein equations (in the sense of [6]) everywhere except in this small region. Incidentally, we cure a pathology of the original firework metric: a conical singularity at the cusp point of the quantum region.

Specifically, we present a metric that has the following properties.

(i) Spacetime describes the fall and collapse of a thin null spherical shell of matter, which bounces at a minimal radius *inside its Schwarzschild radius*, and then expands forever. (This scenario is of course not allowed by the *classical* theory.)

(ii) The metric satisfies Einstein equations almost everywhere. Due to Bhirkhoff's theorem, the shell's interior is therefore a portion of Minkowski spacetime, while the exterior is almost everywhere a portion of Kruskal spacetime.

(iii) We neglect the thickness of the shell.

(iv) The spacetime is spherically symmetric. As a consequence, the spacetime can be represented pictorially by a Penrose diagram.

(v) We assume that the process is invariant under time-reversal. In particular, we disregards the dissipative effects such as the Hawking radiation. The extension to non time-reversal metrics will be studied elsewhere.

(vi) The time and null geodesics are continuous through the $r = 0$ singularity.

II. KRUSKAL ORIGAMI

a. The Kruskal spacetime. The maximal extension of the Schwarzshild black hole is the Kruskal-Szekeres spacetime. Its Penrose diagram is recalled on Figure 1. The metric is given, in terms of the Kruskal coordinates

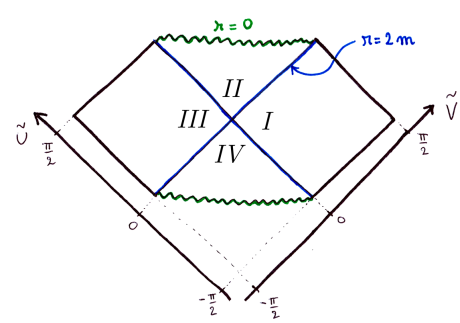

FIG. 1: Penrose diagram of the Kruskal-Szekeres spacetime. $\tilde{U}$ and $\tilde{V}$ are the (Cartesian) Penrose coordinates.

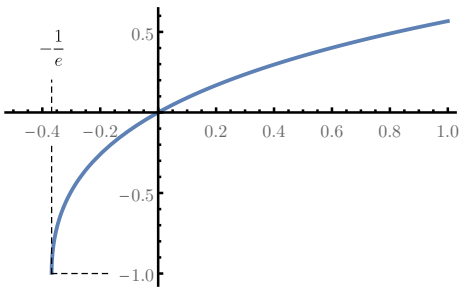

FIG. 2: Graph of the upper branch of the Lambert W function.

(U, V), by

$$\mathrm{d}s^2 = -32m^3 \, \frac{e^{-r/2m}}{r} \, dU \, dV + r^2 d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$ is the metric of the unit sphere and r is the function defined by

$$r(U, V) = 2m \left[1 + W\left(-\frac{UV}{e} \right) \right]. \tag{2}$$

The function W is the upper branch of the Lambert W function. It is a growing function defined by the equation $x = W(x)e^{W(x)}$ and its graph is shown on Figure 2.

The Kruskal coordinates (U, V) are expressed in terms of the Penrose coordinates $(\tilde{U}, \tilde{V})$ by the relations

$$\begin{cases} U = \tan \tilde{U}, \\ V = \tan \tilde{V}. \end{cases} \tag{3}$$

The coordinates $\tilde{U}$ and $\tilde{V}$ are Cartesian for the diagram of the Figure 1. Finally, in the region I, the null-coordinates (u, v) are expressed in terms of the Kruskal coordinates by the relations :

$$\begin{cases} u = -4m \log(-U), \\ v = 4m \log V. \end{cases} \tag{4}$$

b. A snip of the scissors. We now consider the portion of Kruskal spacetime marked out by the red line on Figure 3. It is connected region consisting of two "arms", one touching the past singularity, the other the future one. You may notice a local double covering (where the two arms cross), which raises no peculiar difficulty.

c. Tensing the arms. The modeling of the black-to-white hole transition is achieved through the identification between the past and the future singularity. Heuristically, it consists in "tensing the arms until the hands match". The Penrose diagram of the resulting spacetime is represented on Figure 4.

The expression of the metric is still given by equations (1) and (2), where the Kruskal coordinates (U, V) are

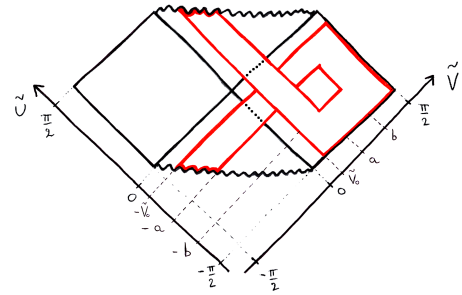

FIG. 3: Penrose diagram of the Kruskal-Szekeres spacetime. The red straight lines are null, and the two red wavy lines will be identified by "tensing of the arms". The inside region thus delimited is the spacetime of interest for us.

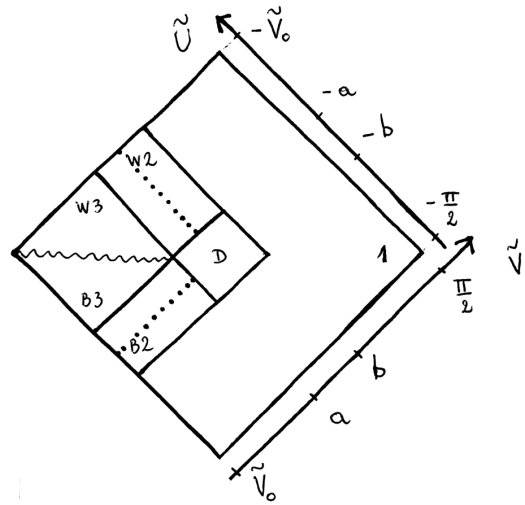

FIG. 4: Penrose diagram of the outside of the null shell. The dotted lines are the two horizons at $r = 2m$.

given in terms of the Penrose coordinates $(\tilde{U}, \tilde{V})$ by

$$[\text{lower half}] \begin{cases} U = \tan f_B(\tilde{U}) \\ V = \tan \tilde{V} \end{cases} \tag{5}$$

$$[\text{upper half}] \begin{cases} U = \tan \tilde{U} \\ V = \tan f_W(\tilde{V}) \end{cases} \tag{6}$$

where the two functions f_B and f_W are differentiable and defined piecewise such that

$$f_B(\tilde{U}) = \begin{cases} \tilde{U} & \text{for} \quad \tilde{U} \in [-\frac{\pi}{2}, -b] \\ f_B(\tilde{U}) & \text{for} \quad \tilde{U} \in [-b, -a] \\ \tilde{U} + \frac{\pi}{2} & \text{for} \quad \tilde{U} \in [-a, -\tilde{V}_0] \end{cases} \tag{7}$$

and

$$f_W(\tilde{V}) = \begin{cases} \tilde{V} & \text{for} \quad \tilde{V} \in [\tilde{V}_0, a] \\ f_W(\tilde{V}) & \text{for} \quad \tilde{V} \in [a, b] \\ \tilde{V} - \frac{\pi}{2} & \text{for} \quad \tilde{V} \in [b, \frac{\pi}{2}]. \end{cases} \tag{8}$$

For the intermediate intervals ($[-b, -a]$ for f_B and $[a, b]$ for f_W), one can choose any continuous and monotonous

function which joins smoothly enough with the other pieces.

The minimal smoothness required is $\mathcal{C}^1$. Indeed, the two junction conditions for null hypersurfaces have to be satisfied along the null geodesics $\tilde{V} = a$, $\tilde{V} = b$, $\tilde{U} = -a$ and $\tilde{U} = -b$. The first condition is the continuity of the induced metric on the hypersurface. This requires the continuity of the functions f_B and f_W. The second condition is the continuity of the extrinsic curvature, which imposes the continuity their derivatives. In the following, we will choose, for in the intermediate interval, a polynomial of degree 3 which is sufficient for f_B or f_W to be $\mathcal{C}^1$ (see Figure 5).

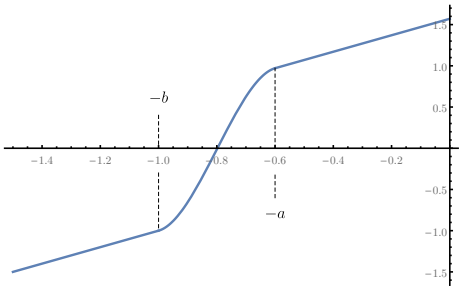

FIG. 5: Graph of the function f_B. On the interval $[-b, -a]$ it is a polynomial of degree 3. It is linear elsewhere. Here we have chosen $a = 0.6$ and $b = 1$.

d. Across the singularity. The regions $B3$ and $W3$ touch along the singularity. There is no difficulty here. It has been repeatedly noticed [8, 9] that it is possible to match the future singularity of a Kruskal diagram to the past singularity of another (see Figure 6). The metric is singular there, but there is a natural prescription for the geodesics to go across the singularity, requiring conservation of momentum and angular momentum [9]. As argued in [6], the resulting spacetime can be seen as the $\hbar \to 0$ limit of the effective metric of a non singular spacetime where quantum gravity bounds curvature. There is a sense in which it is still a solution of Einstein's equations. We take this as a simplified model of the quantum transition across the singularity (region A in the terminology of [7]).

Finally, the metric is well defined all around the central diamond D. This metric is Ricci-flat everywhere (vacuum solution), up to the $r = 0$ surface that separates $B3$ and $W3$ where it still solves the Einstein's equations in the sense of reference [6].

e. The diamond D. The central diamond D is the quantum tunnelling region (region B in the terminology of [7]). The simplest possibility to define a metric in this region is to simply extend the metric of $B2$ and of $W2$, respectively up to and down to the horizontal line $\tilde{U} + \tilde{V} = 0$. Then, the first junction condition along this

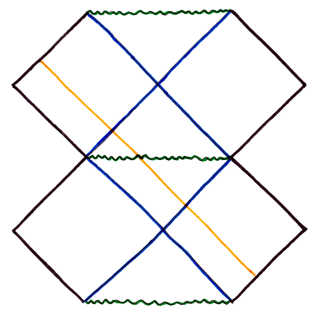

FIG. 6: Penrose diagram of the two Kruskal spacetimes joined at the singularity. The orange line represents an ingoing null geodesic crossing the singularity.

hypersurface imposes :

$$f_W(x) = -f_B(-x). \tag{9}$$

However, the second junction condition can never be satisfied, because otherwise it would define an exact solution of Einstein's equations with the same past but a different future as a standard collapse metric, which has an event horizon. The discontinuity of the extrinsic curvature encodes therefore the quantum transition in this region, as studied in [4, 5]. The novelty is that now this tunnelling region is confined within the diamond.

III. RELIGHTING THE FIREWORKS

The metric constructed in the previous section describes the spacetime *outside* the bouncing null shell. Inside the shell, spacetime is flat, therefore a portion of Minkoswki spacetime. What remains to be done is to glue a patch of Minkoswki along the collapsing and the emerging null shell. This is done in a similar way to the well-known model of Vaidya [10].

The Minkowski metric in Penrose coordinates reads

$$\mathrm{ds}^2 = -\frac{dU_M dV_M}{\cos^2 U_M \cos^2 V_M} + r_M^2 d\Omega^2, \tag{10}$$

with

$$r_M = \frac{1}{2}\left(\tan V_M - \tan U_M\right). \tag{11}$$

The Penrose diagram is shown on Figure 7. The null coordinates are given in terms of the Penrose coordinates by

$$\begin{cases} u = \tan U_M, \\ v = \tan V_M. \end{cases} \tag{12}$$

It is possible to glue a portion of Minkowski to the Kruskal origami by matching the value of the radius along a null ingoing geodesics ($V_M = constant$) for Minkowski with the value of the radius along the line $\tilde{V} = \tilde{V}_0$ of the

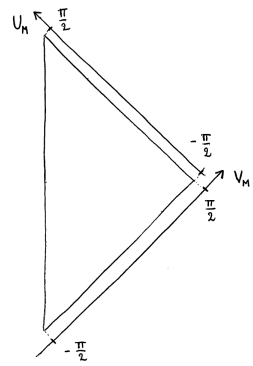

FIG. 7: Penrose diagram of Minkowski spacetime.

Kruskal origami. This matching defines a map $U_M(\tilde{U})$ given by

$$\tan U_M(\tilde{U}) = v_0 - 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(\tilde{U})\right)\right], \tag{13}$$

with $v_0 \stackrel{\text{def}}{=} 4m\log\tan\tilde{V}_0$. Then the first junction condition is satisfied. The violation of the second is the effect of the stress-energy tensor of the collapsing shell. Finally, the same procedure can be applied for the outgoing null geodesics along the line $\tilde{U} = -\tilde{V}_0$, with the condition

$$\tan V_M(\tilde{V}) = -v_0 + 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(-\tilde{V})\right)\right]. \tag{14}$$

This completes the construction of the new spacetime for black-hole fireworks.

a. Penrose diagram of the new spactime. A Penrose diagram for the new spacetime has been drawn on Figure 8.

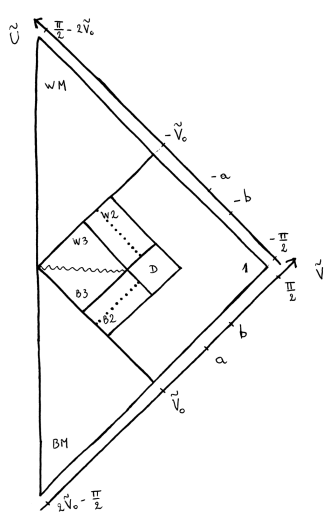

FIG. 8: Penrose diagram of the new spacetime for fireworks.

To make an easy drawing, we have chosen to impose that the line $r_M = 0$ should be straight and vertical, which is possible provided the map $V_M(\tilde{V})$ in BM is given by

$$\tan V_M(\tilde{V}) = v_0 - 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(\tilde{V} - 2\tilde{V}_0)\right)\right], \tag{15}$$

and the map $U_M(\tilde{U})$ in WM is given by

$$\tan U_M(\tilde{U}) = -v_0 + 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(-\tilde{U} - 2\tilde{V}_0)\right)\right]. \tag{16}$$

The metric outside the shell is Kruskal, described by equations (1), (2), (5), (6), (7) and (8). The metric in the two regions BM and WM is Minkowski, given by equations (10), (11), and respectively, (13) and (15) for BM, and (16) and (14) for WM.

b. Another Penrose diagram. Another way to procede would be to impose

$$\begin{cases} V_M(\tilde{V}) = \tilde{V} & \text{in } BM \\ U_M(\tilde{U}) = \tilde{U} & \text{in } WM \end{cases} \tag{17}$$

and then, to draw the Penrose diagram accordingly (see Figure 9). The only difference is the shape of the line $r_M = 0$, which is now given by

$$\tan\tilde{V} = v_0 - 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(\tilde{U})\right)\right] \tag{18}$$

in the region BM and

$$\tan\tilde{U} = -v_0 + 4m\left[1 + W\left(-e^{\frac{v_0}{4m}-1}\tan f_B(-\tilde{V})\right)\right] \tag{19}$$

in the region WM.

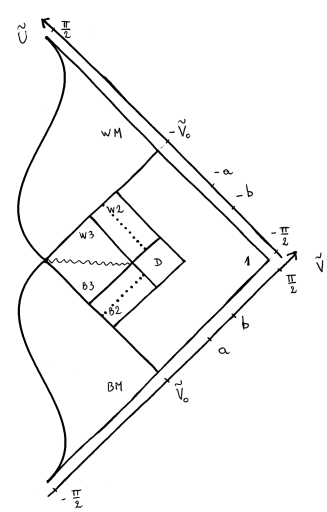

FIG. 9: Penrose diagram of the new spacetime for fireworks.

IV. THE RAY-TRACING MAP

The ray-tracing map can be computed easily from the construction above. It is nothing more than the null-coordinates of the line $r_M = 0$. We show the following expression:

$$u(v) = \begin{cases} -4m \log\left[-\tan f_B^{-1}\left(\arctan\left[\left(1-\frac{v_0-v}{4m}\right)e^{-v/4m}\right]\right)\right] & \text{if } v \leq v_0, \\ -v_0 + 4m\left[1 + W\left(-\tan f_B(-\arctan e^{v/4m})e^{\frac{v_0}{4m}-1}\right)\right] & \text{if } v_0 < v. \end{cases} \tag{20}$$

One can check that it is continuous for $v = v_0$ with

$$u(v_0) = -v_0. \tag{21}$$

Usually, the ray-tracing map is defined such that $u(0) = 0$, which is not the case here. It could be easily obtained by addition of a constant.

The ray-tracing map is plotted on Figure 10, for the choice of f_B plotted on Figure 5.

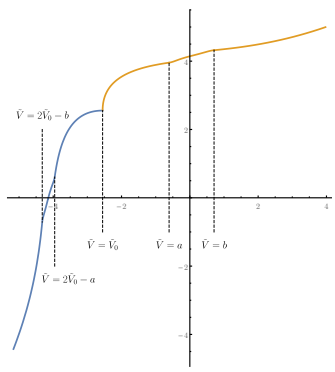

FIG. 10: Graph of the ray-tracing map $u(v)$. Here we have chosen the parameters $m = 0.4$, $\tilde{V}_0 = 0.2$, $a = 0.6$ and $b = 1$.

Acknowlegements

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