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On the Joint Use of Time Reversal and POPS-OFDM for 5G Systems

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Abstract—This paper investigates the efficiency of the combination of the Ping-pong Optimized Pulse Shaping-Orthogonal Frequency Division Multiplexing (POPS-OFDM) algorithm with the Time Reversal (TR) technique. This algorithm optimizes the transmit and receive OFDM waveforms with a significant reduction in the system Inter-Carrier Interference (ICI)/Inter-Symbol Interference (ISI) and guarantees maximal Signal to Interference plus Noise Ratio (SINR) for realistic mobile radio channels in 5G Systems. To this end, we characterize the scattering function of the TR channel and we derive the closed-form expression of the SINR as a Generalized Rayleigh Quotient. Numerical analysis reveals a significant gain in SINR and Out-Of-Band (OOB) emissions, brought by the proposed TR-POPS-OFDM approach.

Index Terms—Waveform Optimization, Waveform Design, Pulse Shaping, POPS-OFDM, Signal to Interference plus Noise Ratio (SINR), Time Reversal

I. INTRODUCTION

OFDM modulation has witnessed a considerable interest from both academia and industry. This is due to many advantages such as low complexity receiver, simplicity and efficient equalization structure. Indeed, it has been adopted in various wired and wireless standards, such as ADSL, DVB-T, Wimax, WiFi, and LTE. Nevertheless, in its present form, OFDM presents several shortcomings and as such it is not capable of guaranteeing the quality of service of new and innovative applications and services that will be brought by 5G systems. In fact, it has a high spectral leakage and it requires strict frequency synchronization because it uses a rectangular waveform in time, leading to significant sidelobes in frequency [2]. As a consequence, any lack of perfect frequency synchronization, to be expected from most of the innovative Machine Type Communications (MTC) in 5G, causes important Inter-Carrier Interferences (ICI). In addition to that, a variety of services and new applications will be provided by 5G systems, such as high data-rate wireless connectivity, which requires large spectral and energy efficiency, and Internet of Things (IoT), requiring robustness to time synchronization errors [3].

In order to overcome OFDM limitations and meet 5G requirements, various modulations have been suggested in the literature such as Generalized Frequency Division Multiplexing (GFDM), Universal Filtered Multi-Carrier (UFMC) and Filter Bank Multi-Carrier (FBMC), which are proposed in 5GNOW project [5]. It is shown at [5] that GFDM offers high flexibility for access to fragmented spectrum and low Out-Of-Band (OOB) emissions. However, in contrast to UFMC, GFDM has a low robustness to frequency synchronization errors in the presence of Doppler spread. Moreover, like UFMC [4], FBMC has a high spectral efficiency and a good robustness to ICI. Nevertheless, FBMC, because of its long shaping filters, cannot be used in the case of low latency, sporadic traffic and small data packets transmission. Furthermore, authors in [1] and [6] propose new class of waveforms, namely POPS-OFDM, which iteratively maximize the SINR in order to create optimal waveforms at the transmitter (Tx) and the receiver (Rx) sides. The obtained waveforms are well localized in time and frequency domains and they are able to reduce the ISI and the ICI as they are not sensitive to time and frequency synchronization errors. Another alternative which aims to reduce interference especially in time, caused by highly dispersive channels, is the time reversal (TR) technique which has been recently proposed for wireless communications systems. Its time and space focusing properties make it an attractive candidate for green and multiuser communications [7]. In fact, it reduces ISI at the Rx side and it mitigates the channel delay spread.

This paper aims to design new waveforms for 5G systems by combining the benefits of POPS and TR techniques in terms of interference resilience. For this end, we analyze the corresponding system and we derive the SINR expression. We also evaluate the performances of the proposed approach in terms of SINR and OOB emissions.

The remaining of this paper is organized as follows. In Section III, we present the system model. In Section IV, we focus on the derivation of the SINR expression for TR systems and describe the TR-POPS-OFDM algorithm for waveforms design. Section V is dedicated to the illustration of the obtained optimization results and shed light on the efficiency of the proposed TR-POPS-OFDM approach. Finally, Section VI presents conclusion and perspectives to our work.

II. NOTATIONS

Boldface lower and upper case letters refer to vectors and matrices, respectively. The superscripts .∗ and .T denote the element-wise conjugation and the transpose of a vector or matrix, respectively. We denote by \( v = (\ldots, v_{-2}, v_{-1}, v_0, v_1, v_2, \ldots)^T = (v_q)_{q \in \mathbb{Z}} = (v_q)_q \), the infinite
vector \( v \). The last notation, \((v_q)_q\), with the set of values taken by \( q \) is not explicitly specified, that means that \( q \) spans \( \mathbb{Z} \).

Let \( M = (M_{pq})_{p,q \in \mathbb{Z}} \) denotes the infinite matrix \( M \). The matrix shift operator \( \Sigma_k(\cdot) \) shifts all matrix entries by \( k \) parallel to the main diagonal of the matrix, i.e. if \( M = (M_{pq})_{pq} \) is a matrix with \((p,q)\)-th entry \( M_{pq} \), then \( \Sigma_k(M) = (M_{p-k,q-k})_{pq} \). The symbol \( \otimes \) is the convolution operator of two vectors and the symbol \( \odot \) is the component-wise product of two vectors or matrices. We denote by \( \mathbb{E} \) the expectation operator and by \(|\cdot|\) the absolute value.

III. System Model

In this Section, we first present the TR principal. Then, we describe the channel and system models in which we will apply our approach.

A. Time Reversal Principle

The Time Reversal (TR) principle [3, 9], comes from the acoustic research field and allows a wave to be localized in time and space. Such a technique can be exploited to separate users, addressed simultaneously on the same frequency band, by their different positions in space.

The use of TR in transmission systems has generated a particular excitement as it allows, from a very high temporal dispersion channel, to obtain an ideal pulse in time and in particular excitement as it allows, from a very high temporal resolution, a simple and effective method for the reception of a signal. It consists in applying a filter on the transmitted waveform, to simplify the theoretical derivations that will be used to derive the channel state at any time. Note that this hypothesis is realistic in the case of low Doppler spread. Let \( h^{(p)} = (h^{(p)}_0, h^{(p)}_1, \ldots, h^{(p)}_{N-1})^T \) be the channel discrete version at instant \( p \) such as \( h^{(p)}_l = \sum_{m=0}^{M-1} h_{lm} e^{j2\pi q m/Q} e^{j\nu m}, \) is the path corresponding to a delay \( T_s \), where \( M \) is the number of Doppler paths, \( h_{lm} \) and \( \nu m \) denote respectively the amplitude and the Doppler frequency of the \( l \)-th path and the \( m \)-th Doppler ray. The ray amplitudes, \( h_{lm} \), are supposed to be centered, independent and identically distributed complex Gaussian variables with average powers \( \pi_m = \mathbb{E}|h_{lm}|^2 \). We denote by \( \pi_l = \sum_{m=0}^{M-1} \pi_m \), where \( \sum_{l=0}^{L-1} \pi_l = 1 \).

The channel time reversed version at the instant \( p \) can be written as:

\[
\mathbf{g}^{(p)} = \left( h^{(p)*}_{K-1}, \ldots, h^{(p)*}_{1}, h^{(p)*}_0 \right)^T.
\]

When we apply the TR technique at the Tx in a Single Input Single Output (SISO) system, the equivalent channel, experienced by the transmission at time instant \( p T_s \) at the Rx, could be seen as the convolution between the channel and its time reversed version, as follows:

\[
\mathbf{H}^{(p)} = \mathbf{h}^{(p)} \otimes \mathbf{g}^{(p)} = \left( H^{(p)}_{(K-1)}, \ldots, H^{(p)}_1, H^{(p)}_0, H^{(p)}_{K-1} \right)^T,
\]

where \( H^{(p)}_k = \sum_{l=0}^{L-1} \sum_{m,m'=0}^{M-1} f(k, l, m, m'), \) such as:

\[
f(k, l, m, m') = \begin{cases} 
    h_{lm}^* h_{l+k,m'} e^{j2\pi (\nu_l + \nu_m - \nu_{m'}) T_s}, & \text{if } k \geq 0 \\
    h_{lm}^* h_{l+k,m'} e^{-j2\pi (\nu_l - \nu_m - \nu_{m'}) T_s}, & \text{else}.
\end{cases}
\]

It should be noted that \( H^{(p)}_k = H^{(p)*}_k \), which means that the channel is Hermitian symmetric, and that the equivalent aggregate channel coefficients, \( H^{(p)}_k \), are still decorrelated, as in the actual channel.

C. OFDM System

In this paper, we consider a discrete time version of the waveforms to simplify the theoretical derivations that will be investigated.

Let \( T \) and \( F \) refer to the OFDM symbol duration and the frequency separation between two adjacent subcarriers respectively. The sampling period is equal to \( T_s = \frac{T}{N} \) where \( N \in \mathbb{N} \). We denote by \( \delta = \frac{1}{T} = \frac{Q}{N} \) the time-frequency lattice density, where \( Q = \frac{T}{T_s} \leq N \) is the number of subcarriers.

We denote by \( \mathbf{e} = (e_q)_q \) the sampled version of the transmitted signal at time \( q T_s \), with a sampling rate \( R_s = \frac{1}{T_s} \), expressed as:

\[
\mathbf{e} = \sum_{m,n} a_{mn} \varphi_{mn},
\]

where \( \varphi_{mn} = (\varphi_{q-nN})_q \otimes (\varphi e^{j2\pi m Q/Q} )_q \) is the time and frequency shifted version of the OFDM transmit prototype waveform, \( \varphi = (\varphi_q)_q \), used to transmit the symbol \( a_{mn} \). We suppose that the transmitted symbols are decorrelated, with zero mean and energy equal to \( E = \mathbb{E}|(a_{mn})|^2|\varphi|^2 \).
The received signal is expressed as:
\[
\mathbf{r} = \sum_{mn} a_{mn} \hat{\mathbf{f}}_{mn} + \mathbf{n},
\]
where \([\hat{\mathbf{f}}_{mn}]_q = \sum_{k=-\infty}^{K-1} H_k^{(q)}(\mathbf{f}_{mn}, q-k)\) is the channel distorted version of \(\mathbf{f}_{mn}\) and \(\mathbf{n} = (n_q)\) is a discrete complex Additive White Gaussian Noise (AWGN), with zero mean and variance \(N_0\).

The decision variable, denoted \(\Lambda_{kl}\), on the transmitted symbol \(a_{kl}\) is obtained by projecting \(\mathbf{r}\) on the receive pulse \(\mathbf{s}_{kl}\), such as:
\[
\Lambda_{kl} = \langle \mathbf{s}_{kl}, \mathbf{r} \rangle = \psi_{kl}^H \mathbf{r},
\]
where \(\psi_{kl} = (\psi_{q-\mathcal{N}}) \odot (e^{j\pi k q/Q})_q\) is the time and frequency shifted version of the OFDM receive prototype waveform \(\psi = (\psi_q)_q\) and \((\cdot, \cdot)\) is the Hermitian scalar product over the space of squaresummable vectors.

### IV. TR-POPS ALGORITHM

The main objective of this part is to optimize the waveforms at the Tx/Rx sides in our system based on TR technique. To this end, we adopt the POPS-OFDM principal [11]. This algorithm consists in maximizing the SINR for fixed synchronization imperfections and propagation channel.

Without loss of generality, we will focus on the SINR evaluation for the symbol \(a_{00}\). Referring to (4), the decision variable on \(a_{00}\) can be written as:
\[
\Lambda_{00} = a_{00} \langle \psi_{00}, \mathbf{f}_{00} \rangle + \sum_{(m,n)\neq (0,0)} a_{mn} \langle \psi_{00}, \mathbf{f}_{mn} \rangle + \langle \psi_{00}, \mathbf{n} \rangle
\]
and it is composed of three terms. The first term is the useful part, the second term is the ISI and the last term presents the noise term. Their respective powers represent useful signal, interference and noise powers in the SINR and which we derive their closed form expressions in the sequel. This SINR will be the same for all other transmitted symbols.

#### A. Average Useful, Interference and Noise Powers

The useful term is denoted \(U_{00} = a_{00} \langle \psi_{00}, \mathbf{f}_{00} \rangle\). For a given realization of the channel, the average power of the useful term can be written as:
\[
P_{\text{US}} = \mathbb{E} \left[ \frac{E}{\|\psi\|^2} |\langle \psi_{00}, \mathbf{f}_{00} \rangle|^2 \right].
\]
Thus, the useful power average over channel realizations is given by:
\[
P_S = \mathbb{E} \left[ P_{\text{US}} \right] = \mathbb{E} \left[ \frac{E}{\|\psi\|^2} \mathbb{E} \left[ |\langle \psi_{00}, \mathbf{f}_{00} \rangle|^2 \right] \right].
\]

The interference term, \(I_{00} = \sum_{(m,n)\neq (0,0)} a_{mn} \langle \psi_{00}, \mathbf{f}_{mn} \rangle\), results from the contribution of all other transmitted symbols \(a_{mn}\), such as \((m,n) \neq (0,0)\).

For a given realization of the channel, the average power of the interference term can be written as:
\[
P_I = \mathbb{E} \left[ \frac{E}{\|\psi\|^2} \sum_{(m,n)\neq (0,0)} |\langle \psi_{00}, \mathbf{f}_{mn} \rangle|^2 \right].
\]

Therefore, the interference power average over channel realizations has the following expression:
\[
P_I = \mathbb{E} \left[ P_I \right] = \mathbb{E} \left[ \frac{E}{\|\psi\|^2} \sum_{(m,n)\neq (0,0)} \mathbb{E} \left[ |\langle \psi_{00}, \mathbf{f}_{mn} \rangle|^2 \right] \right],
\]
where
\[
\mathbb{E} \left[ |\langle \psi_{00}, \mathbf{f}_{mn} \rangle|^2 \right] = \psi^H \mathbb{E} \left[ \mathbf{f}_{mn} \mathbf{f}_{mn}^H \right] \psi.
\]

In the sequel, we consider a diffuse scattering function in the frequency domain, with a classical Doppler spectral density, decoupled from the dispersion in the time domain. So, the noise term is given by:
\[
E \left[ \langle \mathbf{f}_{mn} \rangle_p \langle \mathbf{f}_{mn}^H \rangle_q \right] = \sum_{k=-\infty}^{K-1} \left( \Pi_k J_0^2 (\pi B_d T_s (p-q)) \times \langle \mathbf{f}_{mn} \rangle_{p-k} \langle \mathbf{f}_{mn}^H \rangle_{q-k} \right),
\]
where \(B_d\) is the Doppler spread, \(J_0(\cdot)\) is the Bessel function of the first kind of order zero and
\[
\Pi_k = \begin{cases} K-1 \sum_{q=0}^{K-1} \sum_{l=0}^{\left\lfloor \frac{k-q}{Q} \right\rfloor} \left( \sum_{q=0}^{K-1} \sum_{l=0}^{\left\lfloor \frac{k-q}{Q} \right\rfloor} \Pi_k \Sigma_k \left( \mathbf{f} \mathbf{f}^H \right) \right) \odot \Lambda, \\ \sum_{q=0}^{K-1} \sum_{l=0}^{\left\lfloor \frac{k-q}{Q} \right\rfloor} \Pi_k \Sigma_k \left( \mathbf{f} \mathbf{f}^H \right) \end{cases}
\]
and
\[
\mathbf{K} \Psi = \left( \sum_{n} \Sigma_n N \left( \sum_{k=-\infty}^{K-1} \Pi_k \Sigma_k \left( \mathbf{f} \mathbf{f}^H \right) \right) \odot \Omega \right) - \mathbf{K} \Psi.
\]

The entries of matrices \(\Lambda\) and \(\Omega\) are defined as:
\[
\Lambda_{pq} = J_0^2 (\pi B_d T_s (p-q))
\]
and
\[
\Omega_{pq} = \begin{cases} Q J_0^2 (\pi B_d T_s (p-q)), \text{if } (p-q) \mod Q = 0, \\ 0, \text{else,} \end{cases}
\]
with \(p,q \in \mathbb{Z}\).

The noise term is given by \(N_{00} = \langle \psi_{00}, \mathbf{n} \rangle\). Thus, the noise power average is the following:
\[
P_N = \mathbb{E} \left[ |\langle \psi_{00}, \mathbf{n} \rangle|^2 \right] = \psi^H \mathbb{E} \left[ \mathbf{n} \mathbf{n}^H \right] \psi.
\]
As the noise is supposed to be white, its covariance matrix is equal to \(\mathbf{R}_n = \mathbb{E} \left[ \mathbf{n} \mathbf{n}^H \right] = N_0 \mathbf{I}\), where \(\mathbf{I}\) is the identity matrix. Consequently,
\[
P_N = N_0 \|\psi\|^2.
\]
B. Optimization Technique

The SINR expression is the following:

\[ \text{SINR} = \frac{P_S}{P_I + P_N} = \frac{\psi^H \mathbf{K} \mathbf{S} \psi}{\psi^H \mathbf{K} \mathbf{N} \psi} \]

where \( \mathbf{K} \mathbf{N} \psi = \mathbf{K} \psi + \frac{N_0}{E} \| \psi \|^2 \mathbf{I} \).

Our optimization technique is an iterative algorithm where we maximize alternately the Rx waveform \( \psi \), for a given Tx waveform \( \varphi \), and the Tx waveform \( \varphi \), for a given Rx waveform \( \psi \).

Note that (13) can also be written as:

\[ \text{SINR} = \frac{\psi^H \mathbf{K} \mathbf{S} \psi}{\psi^H \mathbf{K} \mathbf{N} \psi} \]

where \( \mathbf{K} \mathbf{S} \) and \( \mathbf{K} \mathbf{N} \psi \) are expressed as:

\[
\mathbf{K} \mathbf{S} = \left( \sum_{k=-(K-1)}^{K-1} \prod_k \sum_k \left( \psi \psi^H \right) \right) \odot \Lambda
\]

and

\[
\mathbf{K} \mathbf{N} \psi = \mathbf{K} \psi + \frac{N_0}{E} \| \psi \|^2 \mathbf{I}
\]

Thus, the optimization problem is equivalent to maximizing a generalized Rayleigh quotient. As \( \frac{N_0}{E} \| \psi \|^2 > 0 \), we can affirm that \( \mathbf{K} \mathbf{N} \psi \) is always invertible and relatively well-conditioned.

The main steps of the proposed algorithm, presented by Figure 1, are the following:

- **Step 1:** We initialize the algorithm with \( \psi^{(0)} \).
- **Step 2:** For the iteration \( (i) \), we compute \( \psi^{(i)} \) as the eigenvector of \( (\mathbf{K} \mathbf{N} \psi^{(i)})^{-1} \mathbf{K} \mathbf{S} \psi^{(i)} \) with maximum eigenvalue,
- **Step 3:** For the obtained \( \psi^{(i)} \), we determine \( \varphi^{(i+1)} \) as the eigenvector of \( (\mathbf{K} \mathbf{N} \psi^{(i)})^{-1} \mathbf{K} \mathbf{S} \psi^{(i)} \) with maximum eigenvalue,
- **Step 4:** We proceed to the next iteration, \( (i+1) \),
- **Step 5:** We stop the iterations when we obtain a negligible variation of SINR.

We note that eig, used in Figure 1, is a function that returns the eigenvector of a square matrix with the largest eigenvalue.

V. Simulation Results

In this section, the performances of the proposed TR-POPS technique are evaluated. To show the gain in terms of SINR and Power Spectral Density (PSD), a comparison with POPS-OFDM and conventional OFDM with TR is also realized.

The results of POPS-OFDM algorithm applied to our system based on TR technique are carried out for a discrete time-frequency lattice. The optimal Tx/Rx waveform maximizing the SINR, \( (\varphi_{opt}, \psi_{opt}) \), is evaluated for a Gaussian initialization waveform \( \varphi^{(0)} \). We presume having an exponential truncated decaying model.

Figure 2 presents the evolution of the SINR versus the normalized Doppler spread \( B_d/F \) for a normalized channel delay spread \( T_m/F \) where \( Q = 128 \), \( N = 144 \), a lattice density is equal to \( 8/9 \) and for a waveform support duration \( D = 3T \). The obtained results demonstrate that TR-POPS-OFDM approach improves the SINR with a gain of 2.3 dB for \( B_d/F = 0.1 \) compared with POPS-OFDM and a gain that can reach 5.2 dB for \( B_d/F = 0.02 \) compared with conventional OFDM with TR. Moreover, this figure is a mean to find the adequate couple \( (T, F) \) of an envisaged application to insure the desired transmission quality.

![Figure 1: Optimization philosophy.](image1.png)

![Figure 2: Optimized SINR as a function of \( B_d/F \).](image2.png)
emissions by about 40 dB compared to the POPS-OFDM system without TR.

We present in Figure 5 the Tx/Rx waveforms, \( \varphi_{opt} \) and \( \psi_{opt} \), corresponding to the optimal SINR for \( Q = 128, N = 144, FT = 1 + \frac{16}{128}, B_d T_m = 0.001 \) and \( D = 3T \). Since the channel is characterized by an Hermitian and symmetric response thanks to the TR effect, we obtain identical Tx/Rx waveforms as it is illustrated in this figure.

VI. CONCLUSION

In this paper, we studied the association of POPS-OFDM algorithm with TR precoding technique to design novel waveforms for 5G systems. To this end, we presented the corresponding system model and we derived the analytical SINR expression. Despite the additional complexity of applying the combination process, simulation results showed that the proposed approach offers a highly flexible behavior and better performances in terms of maximization of the SINR and reduction of the ISI/ICI. Another possible challenging research axis consists in applying this combination in MIMO-OFDM and FBMC/OQAM systems.

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