Robust estimation of vehicle lateral velocity and yaw rate using switched T-S fuzzy interval observers

Sara Ifqir, Naïma Aït Oufroukh, Dalil Ichalal, Saïd Mammar

Robust estimation of vehicle lateral velocity and yaw rate using Switched T-S Fuzzy Interval Observers

S. Ifqir*, N. Ait Oufroukh*, D. Ichalal* and S. Mammar*
* Laboratory of Computing, Integrative Biology and Complex Systems (IBISC)
Univ Evry, Paris-Saclay University, Evry 91000, France
Email: {sara.ifqir, naima.aitoufroukh, dalil.ichalal, said.mammar}@ibisc.univ-evry.fr.

***Abstract*—This paper presents a new robust estimation of the lateral velocity and yaw rate using switched Takagi-Sugeno fuzzy interval observers. The longitudinal velocity is treated as the online measured time-varying parameter and the cornering stiffness at front and rear tires are assumed to be unknown but bounded with a priori known bounds. Based on a multiple model switching structure, this design divides the range of variation of the longitudinal velocity into a finite number of adjoint regions and, accordingly, develops multiple interval observers for the multiple model set. The switching law which is assumed to be available online selects automatically the appropriate candidate estimator, according to the operation sub-region. Applying the proposed set-approach to estimate the lateral vehicle dynamics allows to cope with uncertainties and ensures guaranteed bounds on the lateral velocity and yaw rate despite changes in tire/road and driving conditions. Sufficient conditions for the existence of the robust proposed observer are expressed in terms of Linear Matrix Inequalities through the use of a switched fuzzy ISS-Lyapunov function. Simulations based on experimental data demonstrate the effectivenesses of the proposed approach.**

I. INTRODUCTION

Advanced vehicle stability control and active safety systems require an accurate knowledge of vehicle state variables. However, there are variables related to vehicle dynamics that are not measurable in an ordinary car for both technical and economic reasons. As a consequence, dynamic variables must be estimated.

In the literature, several studies have addressed the design of classic observers to estimate the vehicle lateral dynamic states using different approaches. For example, Luenberger observer [7], unknown input proportional-integral observer [8], sliding mode observer [9], Kalman Filter and Extended Kalman filter [10]. Most of these studies have been based on the assumptions that the tires are operating in the linear region and the cornering stiffness parameters are constant. The resulting observers only function correctly when the vehicle is operating in the linear region of lateral forces. The vehicle lateral dynamics models often used in the literature rely on a two degree of freedom linear model (or bicycle model, see [1] for details). However, such a simple model becomes inadequate when the longitudinal velocity and cornering stiffness are allowed to vary rapidly because of changes in tire/road conditions or driving maneuvers. Thus, a particular attention has been drawn to the Linear Parameter Varying (LPV) modeling approach, mainly due to the linear model structure in the states and the non linear behavior in the parameters. In this paper the model for observer design is constructed using a linear parameter varying (LPV) structure, in which longitudinal velocity and cornering stiffness varying variables are selected as scheduling parameters. However, when the system has a large range of parameters variation, a single LPV estimator often leads to conservative performance and the problem of finding the observer gains may becomes infeasible. To solve this infeasibility issue, one approach is to use the switched observer. In this approach, the parameters domain is splitted into subregions and one local observer is designed for each subsystem. The vehicle lateral dynamics is then described by a *switched LPV model* with measurable and unmeasurable premise variables [11].

As a matter of fact, the estimation under measurable premise variables is easy to address than the one with unmeasurable premises. The main particularity of this work is that the front and rear cornering stiffness treated as the unmeasurable premise variables are assumed to be bounded with a priori known bounds. Therefore, a Switched Takagi-Sugeno structure [17] is derived using the switched LPV system with parameters depending only on measured signals whereas the unmeasured one is treated as bounded uncertainties. Thereafter, an interval observer is designed to estimate upper and lower bounds of the state vector under consideration of suitable intervals in which the true but unknown cornering stiffness parameters values are definitely included.

It is recalled that, the interval observer is a pair of estimators whose dynamics are defined such that their trajectories characterize at any given instant upper and lower bounds of the state values. They are appeared in the last decade as an alternative approach for robust estimation and they were originally developed in [5] for the estimation of biological systems subject to unknown uncertainties. These observers require, in addition to stability, the cooperativity and positivity of observation error [12]. There are various approaches to design interval observers for continuous times systems satisfying properties of monotone differential systems. See for instance [13] and [16], where interval observers for linear uncertain systems have been presented. The necessary and sufficient conditions have been formulated in terms of linear programming approach. In [15], the interval observers for LPV systems based on Lyapunov theory and linear matrix inequalities (LMIs) have been designed.

In this work, the sufficient conditions for proving the stability and cooperativity of the interval observer error are provided in terms of single parameter minimization problem subject

to LMI constraints obtained using combination of switched fuzzy Lyapunov function with Input to State Stability concept [18]. A bound on the interval error is guaranteed and optimized for more estimation precision. The proposed method is experimentally tested using real data. The experimental results confirm the accuracy and robustness of the interval estimator. Furthermore, to the best of the authors knowledge, the T-S fuzzy interval observer for switched LPV systems with both measured and unmeasured time-varying parameters has never been studied in the literature. This paper constitutes perhaps the first literature investigating switched fuzzy interval observers for such systems.

The paper is organized as follows: In section 2, some preliminary definitions and lemmas are presented. The main theorems on existing of the interval observer and stability of the interval error based on switched fuzzy Lyapunov function are presented in section 3. In section 4, experimental results are given to present the superiority and the effectiveness of the proposed approach. Finally, some conclusions are drawn in the last section.

II. Preliminaries

The objective of this section is to provide some notations, basic definitions and lemmas that are used throughout the paper.

We will adopt the convention of denoting a left and right endpoints of an interval $[x]$ respectively by x^- and x^+ such as $[x] = [x^-, x^+]$. For any two vectors x_1, x_2 or matrices A_1, A_2 the inequalities $x_1 \leq x_2$, $x_1 \geq x_2$, $A_1 \leq A_2$ and $A_1 \geq A_2$ must be interpreted element-wise. $a_{\hat{i}\hat{j}}$ denotes the element on the $\hat{i}$th line and $\hat{j}$th column of the matrix A. $A > 0$ (resp. $A < 0$) denotes a matrix with positive (resp. negative) components and $A \succ 0$ (resp. $A \prec 0$) means that the matrix is positive (resp. negative) definite. A^T means the transpose of matrix A and $\lambda(A)$ denote the eigenvalues of the matrix A. $\mathbb{R}$ ($\mathbb{R}_+$) is the set of all real (positive) numbers. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) is n-dimensional real (positive) vector space. We denote by $\mathcal{I}_n$ an identity matrix of dimension $n \times n$. The absolute value and euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted respectively by $|x|$ and $\|x\|$.

Definition 1. $\mathcal{K}$ is a class consisting of all functions $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ which are continuous, strictly increasing, and satisfy $\alpha(0) = 0$.

Definition 2. $\mathcal{K}_\infty$ is a class of functions of class $\mathcal{K}$ and $\alpha(s) \to +\infty$ as $s \to +\infty$.

Definition 3. A real matrix A is called Hurwitz if all its eigenvalues have strictly negative real part ($Re(\lambda) < 0$).

Definition 4. A real matrix A is called Metzler if all its elements outside the main diagonal are positive ($a_{\hat{i}\hat{j}} \geq 0, \ \forall \hat{i} \neq \hat{j}$).

Definition 5. A continuous-time linear system is cooperative if its state matrix A is a Metzler matrix.

Lemma 1. [13] A matrix A is Metzler if and only if there exists $\epsilon \in \mathbb{R}_+$ such that $A + \epsilon \mathcal{I}_n \geq 0$.

Lemma 2. [5] For a Metzler matrix A, the cooperative system:

$$\dot{x}(t) = Ax(t) + \delta(t) \tag{1}$$

with $x \in \mathbb{R}^n$ is said to be positive, i.e. $x(t) \geq 0$, $\forall t \geq 0$ if and only if $x(0) \geq 0$ and $\delta(t) \geq 0$.

III. Vehicle lateral dynamics and fuzzy modeling

In this section the fundamental aspects of vehicle lateral dynamics are briefly reviewed. Then, the adequate scheduling parameters are selected in order to describe the associated T-S fuzzy model. The system using for the estimation method is based on a bicycle vehicle model, which describes the lateral and yaw motions of a 2-axle, 1-rigid body ground vehicle [1]. Dynamic equations can be represented as follows:

$$\begin{cases} m\dot{v}_y + mv_x r = F_{yf} + F_{yr} \\ I_z \dot{r} = l_f F_{yf} - l_r F_{yr} \end{cases} \tag{2}$$

where m, I_z, l_r, l_f, v_y, v_x, F_{yf} and F_{yr} denote respectively the mass of the vehicle, the yaw moment, the distances from the rear and the front axle to the center of gravity, lateral and longitudinal velocities, front and rear lateral forces. The lateral forces F_{yf} and F_{yr} are highly nonlinear and usually functions of the wheel sideslip angle and wheel longitudinal slip ([3], [2]). In this paper, lateral forces are taken to be linear and expressed as follows:

$$\begin{cases} F_{yf} = c_f(\delta_f - \frac{v_y}{v_x} - \frac{l_f}{v_x} r) \\ F_{yr} = c_r(-\frac{v_y}{v_x} + \frac{l_r}{v_x} r) \end{cases} \tag{3}$$

where c_f and c_r represent the front and rear cornering stiffness parameters while δ_f represents the front steering angle. Note that the stiffness parameters c_f and c_r are not measurable and vary according to the road adhesion. To take into account these variations, we assume that these coefficients vary as follows: $c_f = c_{f0} + \Delta c_f$ and $c_r = c_{r0} + \Delta c_r$, where c_{f0} and c_{r0} represents nominal values which assumed to be known a priori. In the proposed model, it is assumed that the available measurements are yaw rate r, longitudinal velocity v_x and front steering angle δ_f. Gathering equations (2) and (3) leads to the following LPV model:

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{c_f + c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} - v_x \\ \frac{c_r l_r - c_f l_f}{I_z v_x} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} \frac{c_f}{m} \\ \frac{c_f l_f}{Iz} \end{bmatrix} \delta_f \tag{4}$$

where the longitudinal velocity v_x and the cornering stiffness are treated respectively as the measurable and unmeasurable scheduling parameters.

The system (4) can be also written in a typical form of LPV state-space model

$$\begin{cases} \dot{x}(t) = A(\rho(t), \xi(t))x(t) + B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{5}$$

where $x(t) = \begin{bmatrix} v_y & r \end{bmatrix}^T$ is the state vector, $\rho(t) = \begin{bmatrix} \frac{1}{v_x} & v_x \end{bmatrix}^T$ and $\xi(t) = \begin{bmatrix} c_r & c_f \end{bmatrix}^T$ represents respectively the measurable and unmeasurable scheduling parameters. The output $y(t)$ is the measurement of the yaw rate and the input signal $u(t)$ is the steering angle. We denote $A_{\rho,\xi}$ and B_ξ as a shorthand

of matrices $A(\rho(t),\xi(t))$ and $B(\xi(t))$. It is assumed that the vector $\rho(t)$ evolves continuously and its range is limited to a compact subset $\nabla \in \mathbb{R}^2$. The unmeasurable parameter $\xi(t)$ is assumed to belong to some a priori known compact set $\Xi \in \mathbb{R}^2$.

As previously mentioned, the vehicle lateral dynamics has relatively large uncertainties because of the parameters values c_f and c_r and their variations over time. The main idea consists to manage aforementioned uncertainties using interval methods employing the subset Ξ assumed to be known a priori. Moreover, with regards to the measurable parameter $\rho(t)$, the choice of a single Lyapunov function over the entire parameter set is challenging to find the desired requirements specially when the set ∇ is relatively large. The purpose of this paper is to use multiple LPV models, to divide the large uncertainty into smaller uncertainties and analyze the performance of LPV uncertain model over different parameter range.

Let ∇_i, $i \in \{1,\dots,N\}$ be a partition of the parameter set ∇ i.e. ∇ is the union of N sub-regions ∇_1, $\nabla_2 \dots \nabla_N$. Correspondingly, a switching law $\sigma(t) : \mathbb{R}^+ \to \mathcal{I} = \{1,2,...,N\}$ is employed to select the active subsystem according to the online measurement of the parameter $\rho(t)$. Then, a switched linear parameter-varying model for the vehicle lateral dynamics is derived and given as follows:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(\rho(t),\xi(t))x(t) + B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where $A_{\sigma(t),\rho,\xi} \in \{A_{1,\rho,\xi}, A_{2,\rho,\xi}, ..., A_{N,\rho,\xi}\}$ with $A_{\sigma(t),\rho,\xi}$ is a shorthand of $A_{\sigma(t)}(\rho(t),\xi(t))$. In order to take into account the variation of the longitudinal velocity, we use the Takagi-Sugeno multi-model approach and the switched TS model corresponding to the system (6) is described by

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) A^j_{\sigma(t)}(\rho(t),\xi(t))x(t) + \\ \qquad B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{7}$$

The functions $h^j_{\sigma(t)}(\rho(t))$ are the switched weighting functions depending on the measurable variable $\rho(t)$. These functions satisfy the convex sum property:

$$\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) = 1, \quad 0 \le h^j_{\sigma(t)}(\rho(t)) \le 1 \tag{8}$$

Let $\rho(t) \in \nabla_{\sigma(t)}$, $\rho^1_{\sigma(t)} = \frac{1}{v_x}$ and $\rho^2_{\sigma(t)} = v_x$. Employing the sector nonlinearity approach [4], a switched T-S model is obtained with N subsystems and 4 sub models for each subsystem. Under the assumptions

$$\begin{array}{l} \rho^{1min}_{\sigma(t)} \le \rho^1_{\sigma(t)}(t) \le \rho^{1max}_{\sigma(t)} \\ \rho^{2min}_{\sigma(t)} \le \rho^2_{\sigma(t)}(t) \le \rho^{2max}_{\sigma(t)} \end{array} \tag{9}$$

the weighting functions $h^j_{\sigma(t)}(\rho(t))$ can be given by:

$$\begin{array}{ll} h^1_{\sigma(t)}(t) = F^{10}_{\sigma(t)}F^{20}_{\sigma(t)}, & h^2_{\sigma(t)}(t) = F^{10}_{\sigma(t)}F^{21}_{\sigma(t)} \\ h^3_{\sigma(t)}(t) = F^{11}_{\sigma(t)}F^{20}_{\sigma(t)}, & h^4_{\sigma(t)}(t) = F^{11}_{\sigma(t)}F^{21}_{\sigma(t)} \end{array} \tag{10}$$

with

$$\begin{array}{ll} F^{10}_{\sigma(t)} = \frac{\rho^1_{\sigma(t)}(t)-\rho^{1min}_{\sigma(t)}}{\rho^{1max}_{\sigma(t)}-\rho^{1min}_{\sigma(t)}}, & F^{11}_{\sigma(t)} = \frac{\rho^{1max}_{\sigma(t)}-\rho^1_{\sigma(t)}(t)}{\rho^{1max}_{\sigma(t)}-\rho^{1min}_{\sigma(t)}} \\ F^{20}_{\sigma(t)} = \frac{\rho^2_{\sigma(t)}(t)-\rho^{2min}_{\sigma(t)}}{\rho^{2max}_{\sigma(t)}-\rho^{2min}_{\sigma(t)}}, & F^{21}_{\sigma(t)} = \frac{\rho^{2max}_{\sigma(t)}-\rho^2_{\sigma(t)}(t)}{\rho^{2max}_{\sigma(t)}-\rho^{2min}_{\sigma(t)}} \end{array} \tag{11}$$

Due to the space limitation, the matrices of the T-S model are omitted.

Using the proposed switched Takagi-Sugeno fuzzy model (7) for interval observer synthesis will enhance estimation and maintain more accurate and consistent performance even under a large model uncertainties.

IV. Switched interval observer design

This section is devoted to interval observer design for the switched TS model (7). The following assumptions are required for the design of the switched interval observer.

Assumption 1. There exist known functions $u^-(t)$, $u^+(t) \in \mathbb{R}^m$ such that

$$u^-(t) \le u(t) \le u^+(t), \ \ \forall t \ge t_0 \tag{12}$$

Assumption 2. There exist known constants matrices A^{j+}_i, A^{j-}_i, B^+_i, B^-_i $\forall i \in \mathcal{I}$, $\forall j \in \{1,2,3,4\}$, $\forall \rho(t) \in \nabla_i$ and $\forall \xi(t) \in \Xi = \begin{bmatrix}[c_r^-, c_r^+] & [c_f^-, c_f^+]\end{bmatrix}^T$ such that:

$$\begin{array}{c} A^{j-}_i \le A_i(\rho(t),\xi(t)) \le A^{j+}_i \\ B^- \le B(\xi(t)) \le B^+ \end{array}$$

The matrices A^{j-}_i, A^{j+}_i, B^+ and B^- can be directly calculated using the known subset Ξ.

Remark 1. To reduce conservatism and enhance the estimation accuracy, we propose to introduce the time varying matrices $A_{i\rho,\xi}$ in the observer structure. But since the vector $\xi(t)$ is not measurable, we will only use the nominal value ξ_0 that is assumed to be known a priori.

Theorem 1. *Assuming that the trajectory of system (7) is bounded*

$$\|x\| \le \mathcal{X}, \quad \forall t \ge t_0 \tag{13}$$

Then, for all initial conditions x_0 such that $x_0^- \le x_0 \le x_0^+$, there exists a convergent switched interval observer of the TS model (7) of the form:

$$\begin{cases} \dot{x}^+(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)}x^+(t) + L^j_{\sigma(t)}(y - Cx^+(t)) \\ +B^+u^+(t) + (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^+(t)| - x^+(t))) \\ \dot{x}^-(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)}x^-(t) + L^j_{\sigma(t)}(y - Cx^-(t)) \\ +B^-u^-(t) - (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^-(t)| + x^-(t))) \end{cases} \tag{14}$$

if the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Metzler and the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Hurwitz $\forall \rho(t) \in \nabla_{\sigma(t)}$ and $\forall \xi(t) \in \Xi$.

Proof. The upper and lower estimation error dynamics can be expressed as follows:

$$\begin{aligned}
&\dot{e}^+(t) = \dot{x}^+(t) - \dot{x}(t) \\
&= \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t),\rho,\xi_0} - L^j_{\sigma(t)}C)e^+(t) + (B^+u^+(t)- \\
&Bu(t)) + (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})|x^+|) \\
&\dot{e}^-(t) = \dot{x}(t) - \dot{x}^-(t) \\
&= \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t),\rho,\xi_0} - L^j_{\sigma(t)}C)e^-(t) + (Bu(t)- \\
&B^-u(t)^-) + (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})|x^-|)
\end{aligned}$$

It's clear that if $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Metzler then $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t),\rho,\xi_0} - L^j_{\sigma(t)}C)$ is also Metzler for any $A_{\sigma(t),\rho,\xi_0}$ in the interval:

$$A^{j-}_i \leq A_{i,\rho,\xi_0} \leq A^{j+}_i \quad \forall i \in \mathcal{I}, \forall j \in \{1,2,3,4\}$$

then, it suffices to show the positivity of the following terms:

$$\begin{aligned}
&\delta^+_{\sigma(t)}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((B^+u^+(t) - Bu(t))+ \\
&(A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})|x^+|) \\
&\delta^-_{\sigma(t)}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((Bu(t) - B^-u(t)^-)+ \\
&(A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})|x^-|)
\end{aligned}$$

Since $|x^-(t)| \geq 0$, $|x^+(t)| \geq 0$, and $(A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0}) \geq 0$ $\forall\sigma(t), \forall j$, then, according to property (8), assumptions 1 and 2, it can be concluded that $\delta^+_{\sigma(t)}(t) \geq 0$ and $\delta^-_{\sigma(t)}(t) \geq 0$. Finally, using Lemma 2, the upper and lower erros are positive and thus ensuring that $x(t)$ is bounded by $x^-(t)$ and $x^+(t)$. To study the stability of the proposed switched observer, we compute the dynamics of the total error $e(t) = x^+(t) - x^-(t)$:

$$\begin{aligned}
&\dot{e}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A^{j+}_{\sigma(t)} - L^j_{\sigma(t)}C)e(t) + (B^+u^+(t)- \\
&B^-u^-(t)) + (A^{j+}_{\sigma(t)} - A^{j-}_{\sigma(t)})(|x^+| - |x^-|))
\end{aligned}$$

then

$$\dot{e}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) \left((A^{j+}_{\sigma(t)} - L^j_{\sigma(t)}C)e(t) + \delta^j_{\sigma(t)}(t)\right) \tag{15}$$

with

$$\begin{aligned}
&\delta^j_{\sigma(t)}(t) = (B^+u^+(t) - B^-u^-(t)) + (A^{j+}_{\sigma(t)} - A^{j-}_{\sigma(t)}) \\
&(|x^+| - |x^-|)
\end{aligned} \tag{16}$$

Since the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A^{j+}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Hurwitz, we conclude that the interval error is stable but bounded due to the presence of the additive term $\delta^j_{\sigma(t)}(t)$. □

According to previous result, the switched interval observer design reduces to the computation of the gain matrices $L^j_{\sigma(t)}$, $j \in \{1, \ldots, 4\}$, ensuring, $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)}C)$ to be Metzler and $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} - L^j_{\sigma(t)}C)$ to be Hurwitz. Furthermore, regarding to the fact that the total error $e(t)$ is affected by the additive term $\delta^j_{\sigma(t)}(t)$ and in the order to obtain more accurate result and tight interval error, an ISS property from the disturbance $\delta_{\sigma(t)}(t)$ to the observer error $e(t)$ is guaranteed under arbitrary switching signal. The next theorem gives sufficient conditions to guarantee the ISS stability and cooperativity of the proposed interval observer by using the switched fuzzy Lyapunov function of the form:

$$V(e(t)) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))e^T(t)P^j_{\sigma(t)}e(t) \tag{17}$$

Definition 6. The Lyapunov function $V(e(t)) : \mathbb{R}^n \to \mathbb{R}$ is said to be a switched fuzzy ISS-Lyapunov function for (15) if there exist a $\mathcal{K}_\infty$-functions ε_1, ε_2 and ε and a $\mathcal{K}$-function γ such that

$$\varepsilon_1(\|e(t)\|) \leq V(e(t)) \leq \varepsilon_2(\|e(t)\|) \tag{18}$$

$$\dot{V}(e(t)) < -\varepsilon V(e(t)) + \gamma(\|\delta_{\sigma(t)}\|) \tag{19}$$

By defining the following indicator function

$$\lambda(t) = \left[\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\right]^T \tag{20}$$

(17) can be written as

$$V(e(t)) = \sum_{i=1}^{N}\sum_{j=1}^{4} \lambda_i(t)h^j_i(\rho(t))e^T(t)P^j_i e(t) \tag{21}$$

with

$$\lambda_i(t) \geq 0, \ \ \forall i \in \mathcal{I}, \ \ \sum_{i=1}^{N} \lambda_i(t) = 1, \ \ \sum_{i=1}^{N} \dot{\lambda}_i(t) = 0 \tag{22}$$

Remark 2. The indicator function $\lambda_i(t)$ is defined as follows

$$\lambda_i(t) = \begin{cases} 1 & \text{when the ith switched system is active} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Thus, the switched systems (15) with respect to switching law $\sigma(t)$ can be described as polytopic systems with the particularity that the active dynamical matrices are those corresponding to the vertices of the polytope. Moreover, the functions $h^j_i(\rho(t))$ take values in the interval [0, 1], i.e. dynamical matrices vary within the polytope defined by its vertices.

The conditions are based on the assumption that an upper

bound to the time derivative of the membership functions is known. In addition, from the properties given in (8), we have

$$\sum_{i=1}^{N}\sum_{k=1}^{4}\lambda_i(t)\dot{h}_i^k(\rho(t))=0 \tag{24}$$

Theorem 2. *Assuming that*

$$\sum_{i=1}^{N}\lambda_i(t)|\dot{h}_i^k(\rho(t))|\leq\sum_{i=1}^{N}\lambda_i(t)\phi_i^k \tag{25}$$

where $\phi_i^k\geq 0$ $(k=1,..,4)$ are given scalars, if there exist, diagonal positive definite matrices P_i^j, matrices W_i^j and M_i, $\forall i\in\mathcal{I}$, $j=\{1,\ldots,4\}$, $k=\{1,\ldots,4\}$, $\gamma>0$ for given positive scalars ε and ϵ such that the following conditions hold

$$\min_{P_i^j,M_i,W_i^j}\ \gamma$$

$$P_i^j\succ 0 \tag{26}$$

$$P_i^k+M_i\succ 0 \tag{27}$$

$$\begin{bmatrix}\Lambda_i^j+\varepsilon P_i^j+\sum_{k=1}^{4}(\phi_i^kP_i^k+M_i) & P_i^j\\ P_i^j & -\gamma I_n\end{bmatrix}\prec 0 \tag{28}$$

$$P_i^jA_i^{j-}-W_i^jC+\epsilon P_i^j\geq 0 \tag{29}$$

where

$$\Lambda_i^j=A_i^{j+^T}P_i^j-C^TW_i^{j^T}+P_i^jA_i^{j+}-W_i^jC \tag{30}$$

Then the proposed observer can estimate the lower and upper bounds of the state vector $x(t)$ for any switching signal, where $L_i^j=P_i^{j^{-1}}W_i^j$.

Proof. Let us consider the switched fuzzy Lyapunov function (18), its derivative with respect to t is given as follows

$$\dot{V}(e(t))=\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))\left(\sum_{k=1}^{4}\dot{h}_i^\rho(\rho(t))e^T(t)P_i^ke(t)+\dot{e}^T(t)P_i^je(t)+e^T(t)P_i^j\dot{e}(t)\right) \tag{31}$$

If (27) holds, then from (25) it follows that

$$\sum_{i=1}^{N}\sum_{k=1}^{4}\lambda_i(t)\dot{h}_i^k(\rho(t))(P_i^k+M_i)\leq\sum_{i=1}^{N}\lambda_i(t)\phi_i^k(P_i^k+M_i) \tag{32}$$

Also, according to (25), $\sum_{i=1}^{N}\sum_{k=1}^{4}\lambda_i(t)\dot{h}_{\sigma(t)}^k(\rho(t))M_i=0$ is satisfied. Thus, one can prove that

$$\dot{V}(e(t))<\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))\left(\sum_{k=1}^{4}(\phi_i^kP_i^k+M_i)+e^T(t)\left((A_i^{j+}-L_i^jC)^TP_i^j+P_i^j(A_i^{j+}-L_i^jC)\right)e(t)+\delta_i^{j^T}(t)P_i^je(t)+e^T(t)P_i^j\delta_i^j(t)\right) \tag{33}$$

By adding and subtracting the terms $\varepsilon\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)\,h_i^j(\rho(t))\,e^T(t)\,P_i^j\,e(t)$ and $-\gamma\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)\,h_i^j(\rho(t))\delta_i^{j^T}(t)\delta_i^j(t)$, (33) becomes

$$\begin{aligned}\dot{V}(e(t))<&\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))\left(\begin{bmatrix}e^T(t) & \delta_i^T(t)\end{bmatrix}^T\right.\\&\left.\Theta_i^j\begin{bmatrix}e(t) & \delta_i(t)\end{bmatrix}^T\right)-\varepsilon V(e(t))+\gamma\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)\\&h_i^j(\rho(t))\delta_i^{j^T}(t)\delta_i^j(t)\end{aligned} \tag{34}$$

where $\Theta_i^j=\begin{bmatrix}\Lambda_i^j+\varepsilon P_i^j+\sum_{k=1}^{4}(\phi_i^kP_i^k+M_i) & P_i^j\\ P_i^j & -\gamma I_n\end{bmatrix}$ with $L_i^j=P_i^{j^{-1}}W_i^j$ and Λ_i^j is given in (30). Then, satisfying (28) leads to

$$\dot{V}_i(e(t))<-\varepsilon V_i(e(t))+\gamma\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))\delta_i^{j^T}(t)\delta_i^j(t) \tag{35}$$

Let $\Delta(t)=\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))\delta_i^{j^T}(t)\delta_i^j(t)$, integrating the differential equality (22) over the interval $[t_{\bar{k}},t_{\bar{k}+1})$, we obtain that

$$V(e(t_{\bar{k}+1}))<\mu\,V(e(t_{\bar{k}}))+\gamma\int_{t_{\bar{k}}}^{t_{\bar{k}+1}}\mathrm{e}^{-\varepsilon(\tau-s)}\|\Delta(s)\|_2^2\,\mathrm{d}s \tag{36}$$

where $\tau=t_{\bar{k}+1}-t_{\bar{k}}$ and $\mu=\mathrm{e}^{-\varepsilon\tau}$. Defining Δ_{max} as the upper bound of the additive term $\Delta(t)$, it follows that

$$V(e(t_{\bar{k}+1}))<\mu\,V(e(t_{\bar{k}}))+\gamma\Delta_{max}^2 \tag{37}$$

Knowing that $\tau>0$, then it is straightforward to conclude that $\forall\varepsilon\geq 0$, $\mu\in[0,1]$. Thus, the switched fuzzy Lyapunov function is strictly decreasing when $\Delta(t)=0$ and bounded by $\gamma\Delta_{max}^2$ when $\Delta(t)\neq 0$. Note that in the switching instant $t=t_{\bar{k}+1}$ the decrease of the Lyapunov function is constrained by the ISS bound in (38) which must be minimized by choosing the smallest scalar γ.

On the other side, according to lemma 1, $\sum_{j=1}^{4}h_{\sigma(t)}^j(\rho(t))(A_{\sigma(t)}^{j-}-L_{\sigma(t)}^jC)$ is Metzler if

$$\sum_{i=1}^{N}\sum_{j=1}^{4}\lambda_i(t)h_i^j(\rho(t))(A_i^{j-}-L_i^jC)+\epsilon I_n)\geq 0 \tag{38}$$

multiplying in the left side by P_i^j and using the properties (8) and (22) together with the change of coordinates $W_i^j=P_i^jL_i^j$, then (29) is obtained and the proof is complete. □

V. Experimental results

In this section, the switched fuzzy interval observers are applied to experimental data acquired using a prototype vehicle. Several sensors are implemented on the vehicle: The

yaw rate r is measured using an inertial unit, the steering angle δ_f is measured by an absolute optical encoder while an odometer provides the vehicle longitudinal speed. Finally, a high precision Correvit sensor provides a measure of the sideslip angle. This measure is not used for observer design. It serves only for estimation evaluation. The steering angle and the vehicle longitudinal speed profiles are shown in figures 2 and 3. One can see that the speed should be treated as a time-varying parameter. For our purpose, we assume that the cornering stiffness parameters are affected by 10% uncertainty in their nominal value.

As mentioned above, the switching law $\sigma(t)$ depends on the varying parameter v_x which is accessible in real time, such that:

$$\sigma(t) = \begin{cases} 1 & if \;\; 0 < v_x \leq 6m.s^{-1} \\ 2 & if \;\; 6m.s^{-1} < v_x \leq 11m.s^{-1} \\ 3 & if \;\; 11m.s^{-1} < v_x \leq 16.6m.s^{-1} \end{cases} \tag{39}$$

The numerical simulation was carried out by using Matlab YALMIP toolbox. The numerical results corresponding to a stability margin $\varepsilon = 5$, $\epsilon = 1$ and $\phi_i^j = 80 \;\forall i \in \mathcal{I}, \forall j \in \{1, \ldots, 4\}$. The results for the switched interval observer are shown in figure 2. As shown, the inclusion property $x^-(t) \leq x(t) \leq x^+(t)$ is verified. The variation of the estimated envelopes follows the variation of the real state vector ensuring a tighter bounds, which depends on the range of the uncertainties and the calculated ISS gain γ. The simulation proves clearly the effectiveness of such observer to estimate guaranteed bounds in the presence of uncertain parameters.

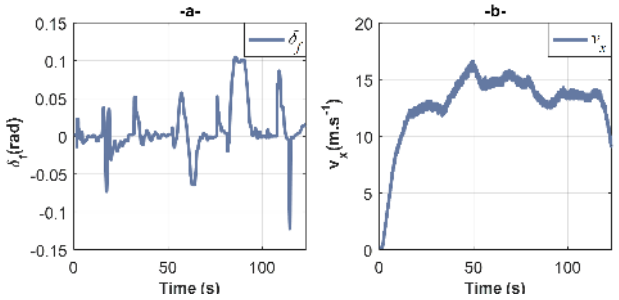

Fig. 1. a) Steering angle δ_f - (b) Longitudinal velocity v_x.

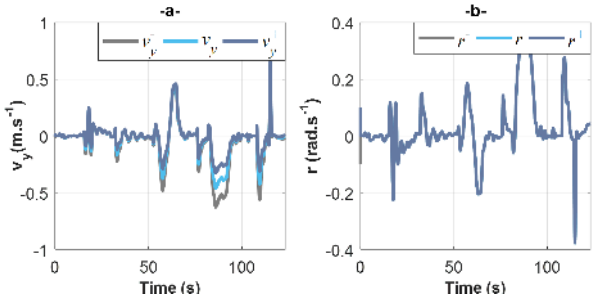

Fig. 2. Interval observer for lateral velocity (a) and yaw rate (b).

VI. Conclusion

In this work, we provided a new methodology for robust estimation of vehicle lateral dynamics. The approach is based on switched T-S fuzzy modeling invoking the input to state stability concept and switched fuzzy Lyapunov function. First, the LPV model of the system is transformed to a switched LPV system in order to cope with large uncertainties. Afterward, a switched T-S fuzzy representation with measurable premise variable and uncertain but bounded parameters is presented. Secondly, an interval observer for this type of systems is proposed. The stability and cooperativity conditions are formalized in terms of linear matrix inequality (LMI) constraints. Simulations with experimental data demonstrate the effectiveness of the proposed algorithm.

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