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### ► To cite this version:

B. Mikula, Hélène Mathian, Denise Pumain, Lena Sanders. Dynamic modelling and geographical information systems: for an integration. Longley P. Batty M. Spatial Analysis, Modelling in a GIS Environment, Pearson Professional, pp.283-296, 1996. hal-01760682

**HAL Id: hal-01760682**

**<https://hal.science/hal-01760682>**

Submitted on 6 Apr 2018

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## DYNAMIC MODELLING AND GEOGRAPHICAL INFORMATION SYSTEMS: FOR AN INTEGRATION

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### Introduction

In a recent and rather provocative commentary, Stan Openshaw welcomes the emergence of a "computational human geography", and defends the idea that it would improve the scientific status of geography. Without assuming, as he would have us believe, that computational sophistication leads necessarily to enhanced scientific validity or standing, one can still agree with Openshaw that, thanks to improvements in the memory, speed and software in today's computers, "previously impossible or poorly handled analysis and modelling tasks can now be solved properly" (1994, p.500). We take this opportunity in order to illustrate the existent and possible further improvements in the field of spatial dynamic modelling.

Computational tools for geography can today be integrated into Geographical Information Systems. Depending on their origins and their field of application, Geographical Information Systems have different kinds of functionalities. Actually, most GIS development focus on database management improvements which correspond to a real need and demand. Indeed, GIS are often perceived as extensions of data management systems and then focus on the integration of informations relative to different kinds of geographical objects. However, if that kind of improvements are fundamental for further developments, they are insufficient for more specific applications, for instance for spatio-temporal analyses. The introduction of time in geographical data bases requires to develop tools of database management adapted to the specificities of evolving objects (Viemont Yeh 1992). Technical improvements in the handling of data are then important but other types of operations are also necessary: a true temporal GIS "would trace the changing state of a study area, storing historic and anticipated geographic states" (Langran 1992).

The exploration of the spatial processes of change and the modelling of spatial dynamics are essential for the understanding of geographical systems. Related techniques and models could also be of interest for the decision process of GIS'users, for instance in urban and regional planning. Dynamic models can help to produce a variety of "anticipated geographic states". Instead of conceiving the information content of GIS as definitive fixed structures, a temporal GIS could help in "the exploration of a large diversity of possible futures" (Allen, 1991). On the other hand, in the field of geographical research, the coupling of GIS with dynamic modelling will contribute to improve the quality of spatial dynamic models. After discussing the reciprocal advantages of the integration of dynamic modelling in GIS, a research tool

developed in this framework, the TGSS (Time Geographic Simulation System) will be presented. The main functionalities will be detailed and its advantages for developing a dynamic spatial model will be emphasized through a simple application.

## **1 Cross fertilization between GIS and dynamic modelling**

The bulk of the potential uses and applications of GIS in geography still remain underexploited. The explanation for this could be the too simple use of the concept of overlay in many applications, with the risk to trace geographical analysis to local combinations and regional synthesis back. Recent developments and research efforts in GIS have, however, pushed the frontier in the direction of developing new tools better adapted to modern geographical theory. In this process, GIS could make a real contribution to the research in the field of space-time analysis and modelling.

### **1.1 For a more flexible conception of space and time in GIS**

As far as classical spatial analysis is concerned, noticeable progresses are being made on the way of integrating its main tools and methods into GIS (Fotheringham, Rogerson 1994). If the available commercial softwares still provide rather rudimentary functions for marketing or management purposes (Longley, Clarke 1995), several researchers are developing various functionalities in data modelling and spatial process modelling. To take one example, M. Batty and Y. Xie (1994) have integrated into a GIS framework a variety of models for estimating the parameters of density functions and fractal dimension in an urban environment. However, at the present time the representation of data in GIS seems to limit the concept of geographical space to a fixed referential with Euclidean metrics. In a recent paper, Goodchild, Klinkenberg and Janelle (1993) argues that "GIS remain primarily stores of digital representations of maps" and that "most of GIS are static and two-dimensional". Quoting Langran (1989, 1992), Langran and Chrisman (1988), Raper (1989) or Couclelis (1991), they underline the limitations of GIS according to their conception of space: "analysis of social process requires a time-dependent, multi-dimensional, relative space, rather than the static, two-dimensional, absolute view embedded in current GIS technology" (1993, p.278).

The power of today's computing could allow us to look for an integration into the technology of more recent theoretical insights about the nature of geographical space (Tobler 1989, Muller 1982, Cauvin 1984). At least research oriented GIS, and even those created for communication purposes, could integrate computation and display of various anamorphoses, that is, a direct use of relative spaces, - allowing for instance to compute coordinate projections from interaction matrices and to show their deformations, from one transportation mode to another, or over time. For many users who employ GIS for planning purposes, it would be very useful if they can visualize the deformations of an accessibility surface after the building of a new transportation infrastructure.

GIS could also be used for conciliating two representations of geographical space which are often regarded to be opposed to each other, but in reality always integrated in the dynamics of spatial systems: namely, the continuity of territories versus the contiguity of networks and how these could be combined as well as their effects in dynamic models of individual or aggregated representations of geographical space. Changes in relative space or in the valuation of geographical situations are one important aspect of the evolution of geographical systems. Absolute and relative changes in the socio-economic attributes of places are another component of change. They could be handled in GIS by direct visualization, for example by animated maps. But they would be more precisely described and simulated by integrating dynamic models in the GIS. Simulation games with non linear effects can help GIS users to better understand important concepts like critical size threshold, perceive effects and other brutal qualitative transition which may occur so frequently in the evolution of urban and regional systems. Standard dynamic modelling modules could be rather easily adapted to recurrent spatio-temporal problems as centre-periphery dynamics or competition between cities. In a planning context, the integration of dynamic modelling into a GIS could help to provide less trivial answers than given by usual methods of projection to the question: "what will happen if...". By allowing the exploration of a large diversity of future spatial organisations under various hypotheses it would both help to evaluate the temporal inertia of a spatial pattern and to get a larger freedom in relation with a given spatial framework. Introducing more flexibility in space and time representations in GIS could also help for discovering spatial structures and creating adapted new regionalisations. It is very often difficult to identify "the relevant functional objects in space and time, their temporally and spatially nested nature, the existence of system thresholds and feedbacks creating complex responses to external constraints" (Livingstone, Raper 1994, p.231).

Instead of using a predefined aggregation of spatial units into higherlevel encoded circumscriptions, according to a fixed hierarchy of maps, the GIS may produce a large variety of spatial aggregations, depending upon the specific spatial patterns present in the data. Techniques in image analysis (remote sensing data analysis) already help the grouping of pixels in areas of various shape and size following multivariate similarity criteria. GIS also can facilitate the ordering of spatial interaction matrices for delimitating market areas or for indentifying well-connected sub networks. Dynamic models of change in interaction intensity or in shape of geographical objects would be of great help in GIS, for instance in solving the difficult problem of the continuously changing delimitation of urban areas.

## **1.2 GIS can improve dynamic modelling**

Parallely to the advantages for the user to dispose of a simulation tool integrated in the GIS, this framework facilitates the development of dynamic models. Specially the concept of distance, and more generally of spatial interactions can be defined and taken into account much more properly. Traditionally, interactions effects are modelled through distance, seen as a variable among others and most often considered as static, invariant over time. The variations

in accessibility for instance, which is by nature evolutive and depending on the state of the infrastructure network, have most often been poorly integrated in dynamic models.

In dynamic modelling of spatial interactions, the focus of research has been on the elaboration of differential equations linking the retroactive effects of several state variables (for instance Allen 1978, Wilson 1981, Weidlich and Haag 1988). The main problem to solve in empirical applications was the calibration of parameters which could adjust observed trajectories of the spatial units in the space defined by the evolution of the state variables. Too little attention was given in such models to the specification of spatial interaction, which was usually reduced to a simple gravitational expression using a physical distance parameter (Pumain, Sanders, Saint-Julien, 1989, Sanders, 1992). In the literature about classical spatial interaction, many references have been made to various possible expressions of the distances in equations of gravitational type: distance can be measured in kilometers, in costs, in time, on networks, even in social or cultural terms, and so on. However, the data collection and computation of such complex distances were cumbersome, and there is very little work about the adjustment of models using more than one type of distance.

The improvements in computation and database management have led to progress in dynamic modelling. First by a more precise and more generalized integration of spatial interaction through a large and more systematic set of measures:

- it is now easier to compute different measures of distances and to experiment different mathematical functions of spatial interaction. A systematic comparison is then possible and that way a true experimentation would replace a priori statements about the shape, range and intensity of spatial interaction.

- one can, in a dynamic framework, recalculate accessibility indicators at each time period, integrating in the model the changing impact of distance, for instance the space-time contraction.

- one can compute new position-related indicators as distance to different kind of geographical objects (a larger city, a motorway entrance, a railway station...). These indicators could eventually take into account different levels of ruggedness of space, depending on the characteristics of the intermediate space between the spatial unit into consideration and the structuring geographical object.

The main difficulties with modelling of interactions are connected with the big amount of data values to be stored and calculated, each time the simulation is taking a step forward. But, in a dynamic environment the interactions could be more or less important during the simulation time. Decision about it should be reconsidered during running of the model, however not so often as recalculation of models. The consequence of using this approach makes the terminology of interaction matrix slightly obsolete. A more general term could be "relation", which defines a possibility: the relation can but not necessary is defined between two arbitrary objects. The interlinked objects may also be of different kinds. Cities can be coupled to regions, roads to cities etc. The relation can have a set of numerical attributes. Those can either be calculated or given exogenously (as constant or time series). In the field of data base systems, there is a relatively old model of description of the real world called "entity-



relationship" modelling. It makes it possible to represent rather complex structures by drawing two kinds of icons and bind them with proper arrows. Even if there are more modern paradigms (as for instance object oriented ones) it is enough for many purposes and many times preferable because of its simplicity. Using of flexible relations instead of rigid matrixes has some further implications. Aggregation and disaggregation are just a special case of relation between two objects on different aggregation level. Even those could be re-examined (by classification, clustering etc.) during simulation or just given as an initial state. They could be also holders of different attributes (measures of fit in the cluster, centrality etc.).

These possibilities lead then to integrate into dynamic models the concept of geographical situation, instead of considering the location only. Another progress due to the development of a dynamic model in the framework of a GIS is the facilitated possibilities of combining several scale effects into a single dynamic model. Such question, although important from a theoretical point of view, has rarely been done with classical dynamic models. Employing GIS would make it easier and enable us to simultaneously consider the interactions for various spatial ranges, as in the evolution of a settlement pattern. To name one example we are presently developing a dynamic model of an urban settlement over a long time period and under the effects of space-time contraction, using a Time-GIS as described below.

## **2. Dynamic modelling with a Time Geographic Simulation System (TGSS)**

The TGSS<sup>1</sup> focuses on the representation and the simulation of geographical processes. The aim of this tool is to connect and integrate the conceptual framework of two originally very different fields, that of database management and that of simulation techniques. It links some properties of a GIS with a tool for building dynamic models and it integrates most of the tools necessary in a time-geographic research: animated figures and maps, spatio-temporal management and dynamic modelling.

### **2.1 The general framework**

The TGSS is of hybrid raster type and all the objects are defined as aggregations of pixels. The system handles several hierarchies of scales, from the cells to different kinds of administrative units such as communes or regions (figure 1). Procedures of aggregation and disaggregation make it possible to associate data corresponding to different geographical scales in the same model. Thus, physical data observed at the level of the pixel can be combined with socio-economic data observed at the level of administrative units. The geographical objects exist through their statistical attributes and the data are represented and stored as temporal series of maps. There are no direct spatial operators on geometries such as "inclusion", "intersection", "belonging" which are classical ingredients in GIS. These kinds of topological operations can

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<sup>1</sup>The TGSS is developed on a Macintosh. It needs a processor of at least Motorola 68020 model, a mathematical coprocessor and a screen of at least 256 colours. It uses about 30000 source code written in Object Pascal and a broad class library for the window and dialog management.

be performed through appropriate attributes and procedures of aggregation/disaggregation. The system handles overlays associated with different attributes (spatio-temporal series in general) or to different levels of scale. Even if it is not possible to handle directly objects of a different nature, one can link different sets of informations associated with different partitions of space, even incompatible ones. The integrated language of the system enables the user to combine easily all kinds of overlays as soon as the formula can refer to the attributes.

The originality of the system concerns the handling of horizontal relations, such as similarities, distances and flows, through spatial interaction matrices. They can be handled in order to create new spatial interaction matrices or to produce maps resulting from the combination of matrices and vectors of attributes. They are obtained as inputs or as the result of the merging of two or several maps. Actually the system is able to handle matrices interconnecting up to 1000 spatial units. In a dynamic modelling these properties enable to take into account the combination of processes associated with different levels of spatial aggregation and of various spatial ranges. So far, topological relations as contiguities or distances inside a network are not easily computable.

## 2.2 The main functionalities of the TGSS

The system is interactive. The user can manipulate a set of icons which he combines with the aid of a graphical representation<sup>2</sup> (figure 2). One can quite easily build up different kinds of algorithms and models useful in geography:

- selection and combination of spatio-temporal series using classical mathematical functions or logical rules; for example, computing of rates, densities or more complicated computations (figure 2.b). The icon "function" is used for such tasks.

- combination of spatio-temporal series and spatial interaction matrices; a specific icon is used to combine vectors and matrices, which is useful for computing population potentials, gravity models or any spatial interaction model.

- development of dynamic modelling: a combination of qualitative rules, differential equations and systems of differential equations can be used to build up a dynamic model. The differential equations can combine computing on vectors and matrices. Figure 3 shows an example where the variation in time of a variable A at place j is expressed through the combination of three effects:

- . an effect of retro-action through a parameter  $K_1$ ,
  - . a diffusion effect through a term proportional to the difference between the level of variable A in unit j and each of all other units ( $A_i$ ), weighted by the distance between those units ( $d_{ij}$ );
  - . the effect of the level of development of another spatio-temporal distribution (B).

A stochastic effect can also be introduced through a specific icon. Any dynamic model can be built up using a combination of those different possibilities.

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<sup>2</sup>The formalization is close to that of STELLA, a software adapted for modelling dynamic systems, but without any explicit spatial component

The results of those kinds of computing are represented through a complete system of visualization, with maps and graphs. All created maps, resulting from combinations or procedures of aggregation or disaggregation can be represented simultaneously. The maps are choropleth with a choice of colour combinations. The map is coupled to the statistical information which is accessible at the level of each spatial unit selected directly on the map. The dynamic dimension is integrated in the representation for two kinds of output:

- animated maps

The variables stored in the data base, as well as the simulated data produced by a dynamic model applied to a given initial situation, or all logical or quantitative combination of these variables can be represented by an animated map. This dynamic visualization is a useful device for showing the rhythm of change, determining the accelerations and to identify spatial or temporal discontinuities. In that way, it facilitates the identifying of *where* and *when* a specific change occurs in the evolution of the process and finding out of possible bifurcations. This animated visualization is useful from an exploratory point of view and allows to perceive the information differently from the comparison of a set of static maps. Snapshots, identification of the characteristics corresponding to a specific place and zoom, facilitate the dynamic exploration of the data, even when the number of spatial units is quite large.

- animated scatter diagrams

A first kind of scatter diagram represents the spatial units according to the combining of two variables. Its animation gives quick information about the evolution of the relationship between the two variables. A lasso system makes it possible to select a specific sub-group of points and to visualize simultaneously their position on any specific map (fig.4). A second type of scatter diagram shows the evolution over time of some specific selected spatial units according to one or two variables.

Even if some operations make it possible to use the tool for dynamic exploration, the TGSS is not specifically developed for dynamic database management. It has to be seen more as a simulation tool for analyzing spatial processes of change. This means that it is more adapted for finding out the rules of change in space than for a time geographic exploration procedure.

### **2.3 A simple application: population dynamics in the Rhone valley**

The advantages of integrating dynamic model building into the framework of a GIS can be illustrated by a simple example: the spatial evolution of population in the lower Rhône valley from 1936 to 1990. The area (from Lyon to Marseille) offers a large contrast between the remote and depopulated areas of the slopes of the Alpes and the Massif Central on the one hand, and the very densely populated areas of the valley and of the conurbation along the Mediterranean coastline on the other hand. This spatial structure is already clear in 1936 but the contrasts have been reinforced over time (figure 5). This concentration process is mainly related to differences in accessibility. The relative position of a place also is a significant factor



behind spatial variations in population trends. A simple logistic model makes it possible to reproduce this concentration process of the population during the last fifty years, at the rather precise geographical level of the communes.

$$dPop_i/dt = a Pop_i (1 - dPop_i/Pot_i) \text{ for } i = 1 \text{ to } 1795$$

where  $Pop_i$  represents the population of commune  $i$  at time  $t$ ,

$Pot_i$  defines the maximum potential of zone  $i$ , i.e. its carrying capacity, the maximum number of inhabitants it can have relatively to its characteristics.

The potential population of a place is assumed to be dependent on the combination of many factors, historical as well as demographic, economic or geographical. Instead of building up a complicated model, we will select an accessibility factor which summarizes several effects. The potential population of a commune  $i$  is then estimated from the combination of three components:

- . the commune's global accessibility in the Rhone valley area, as approximated by its altitude. This variable summarizes quite well the environmental constraints which influences the accessibility of a place. Activities have concentrated at the bottom of the valley and along the sea coast where the speed and easiness of circulation are higher and the transportation infrastructures are the most developed.

- . the commune's local position measured by its distance to the nearest urban centre. The process of urbanization has been strong during this period and there is a progressive diffusion of the urban growth from the urban centres towards their peripheries (fig. 6).

- . the previous state of the commune in the urban transition process: rural, urban belonging to an urban area or centre of an urban area. This distinction summarizing the past evolution of the commune on the very long term is fundamental since for the same population size, these different kinds of communes are subject to radically different processes of change. For example, a small commune belonging to an urban area will have one of the highest growth rate of the system whereas a rural one of the same size has a very high probability of becoming depopulated.

The exploratory properties of the system are very useful for the validation of dynamic models. The calibration step is in fact the most complicated when a complex dynamic model is developed. From a theoretical point of view it is difficult to define a measure of goodness of fit in a spatio-temporal context. From a technical point of view, there is a lack of automatic calibration procedures that can be used when the equations are complex and involve many non-linearities. The simultaneous visualization of the observed and the simulated spatial dynamics, and, above all, of the difference between these distributions through time, enables to show where and when the residuals appear or increase in value, and to find out breaking points in time or space. The interactive analysis of maps and different kinds of scatter diagrams helps to identify the residuals and their evolution through time. It is used to modify progressively the form of the equations and the values of the parameters, in order to improve the quality of the calibration. For example, figure 7 shows snapshots of a simulated population density pattern in 1990 and figure 8 that of the residuals of the model.

The validation of a dynamic model also requires some sensitivity analyses which are considerably facilitated in the interactive framework of a GIS. The development of a spatial dynamic model takes a lot of efforts and this sensitivity analysis has often been reduced to a minimum in the applications. In an interactive quickly responding framework, the testing of various thresholds in the definition of the qualitative rules of the model or the testing of the effects of the delimitation of the geographical system are facilitated. The degree of generality of the model can thereby be evaluated and the existence of coherent geographical sub-systems can be identified. Once the past evolution has been correctly reproduced, the system can be used to test the effects of various scenarios on the future evolution of the system. The simultaneous visualization of the spatial dynamics corresponding to different scenarios, helps the analyst to identify the effects of each of them on the rhythms of change as well as for the final situation.

## Conclusion

Making a parallel with what Goodchild (1991) said about the integration of spatial analysis and GIS (either fully integrated, tightly coupled or loosely coupled), it may be assumed that TGSS is a fully integrated tool linking GIS and dynamic modelling.

The TGSS is an open tool, still in evolution, and new methods will be integrated into the system e.g. some calibration methods which have been developed in the area of synergetics (Haag 1989), and also cellular automata and multi-agent systems which offer another type of formalization of dynamic modelling than differential equations systems and which focus more specifically on the rôle of the spatial pattern (Bura et al., 1995).

This process of integrating a large variety of dynamic tools into the same software package should remain within reasonable limits, and be restricted by the real needs of the users. According to the type of applications and to the field of research, it is perhaps not a single but a variety of GIS which can be instrumented, with specific tools for the exploratory analysis of change, or for the development of dynamic models, or for the simulation of scenarios for decision-making. As Borges may have said, it is perhaps not safe to derive towards the "GIS of the Empire", so let us follow André Gide who used to advise "il faut toujours suivre sa pente...à reculons!"

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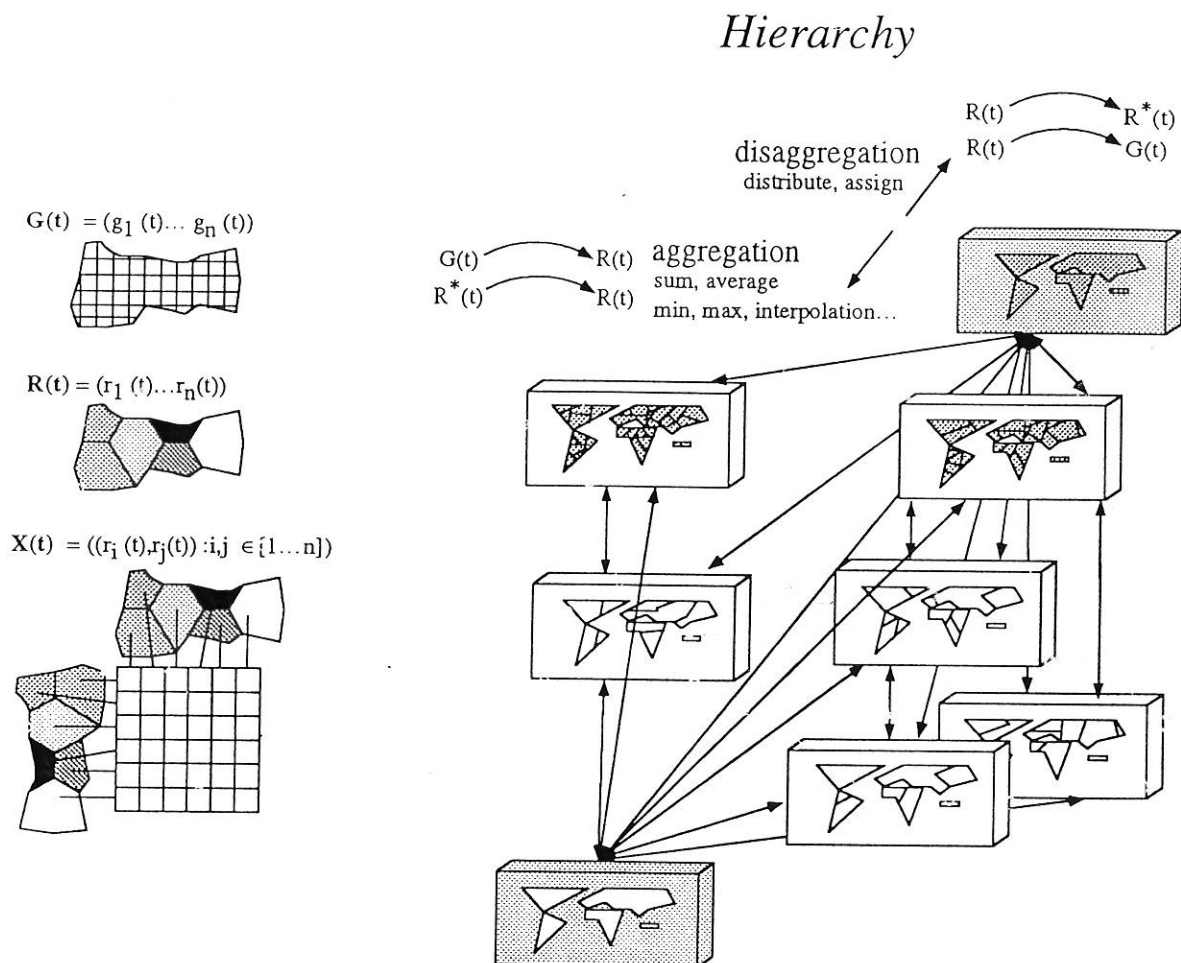
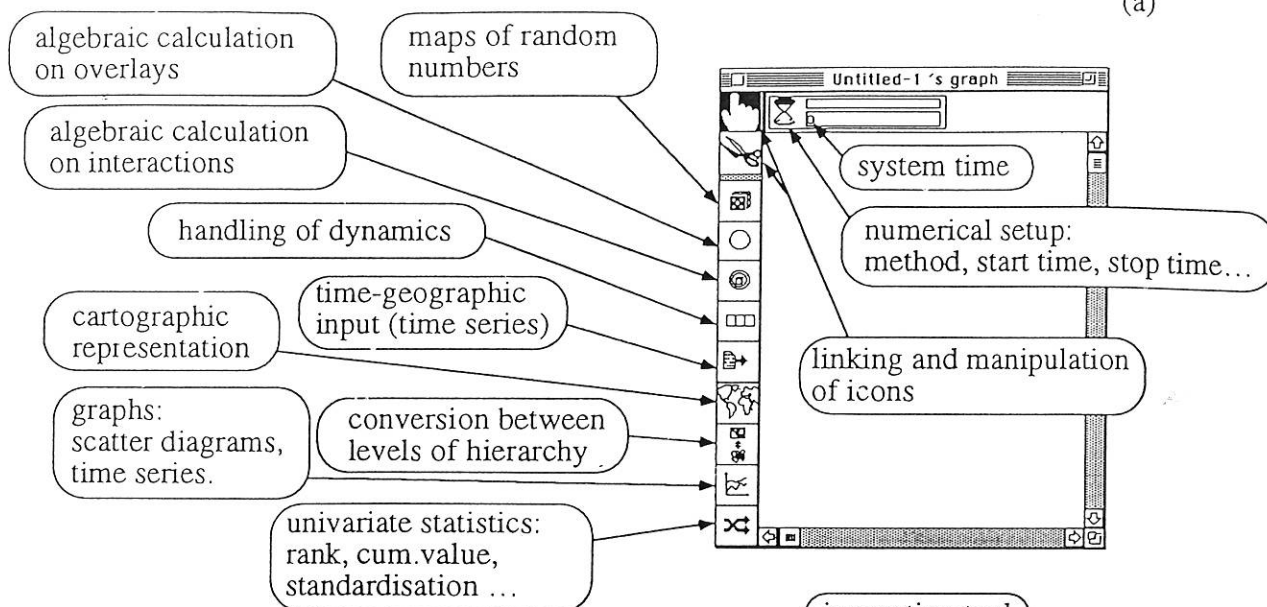


Figure 1: The cartographic model and translations between different levels of hierarchy (G = grid; R = region; X = interaction matrix)



(a)



(b)

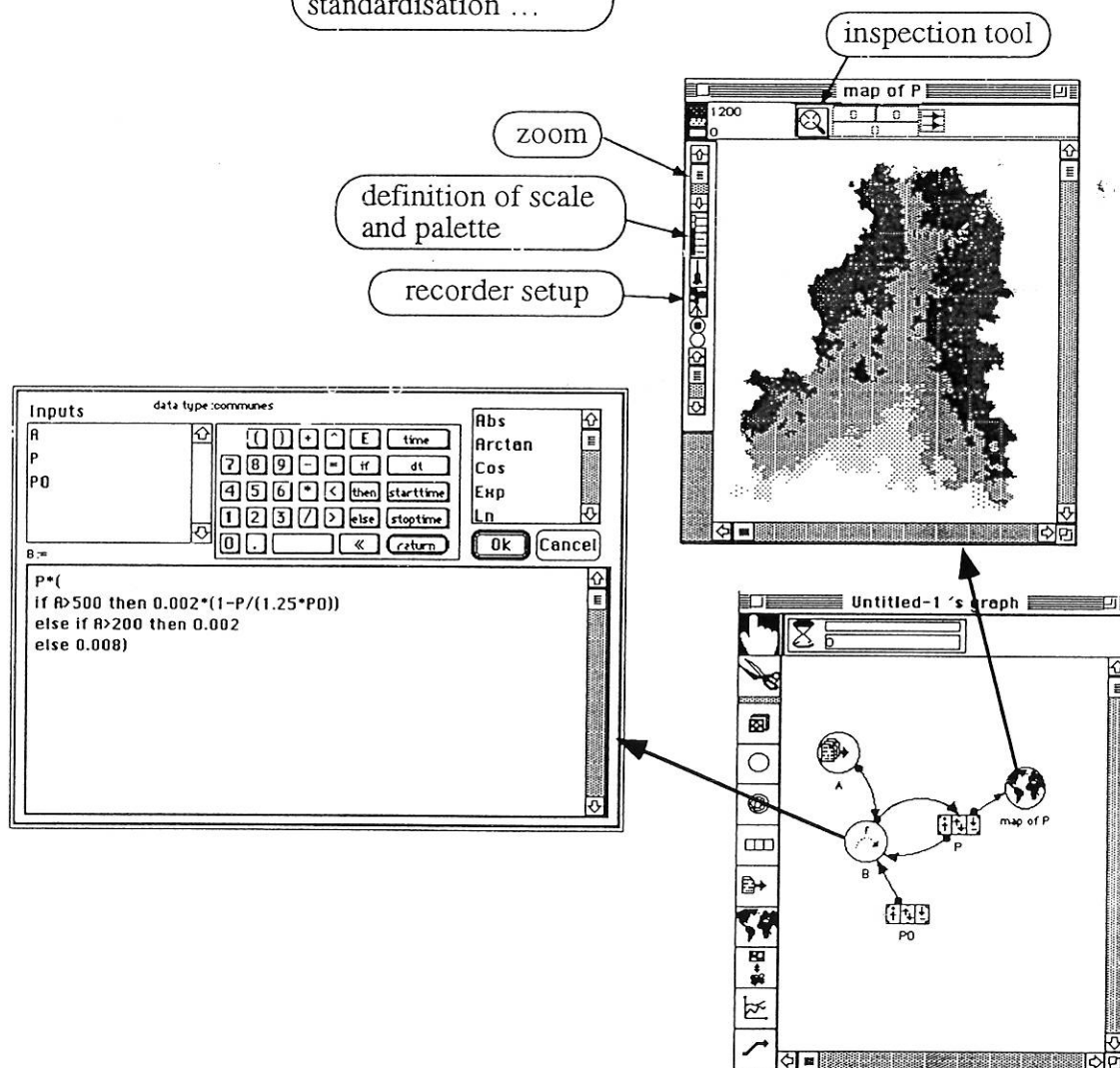


Figure 2: Model construction tools

(a) Construction window with palette of available tools

(b) Two examples of dialog windows during the process of model construction

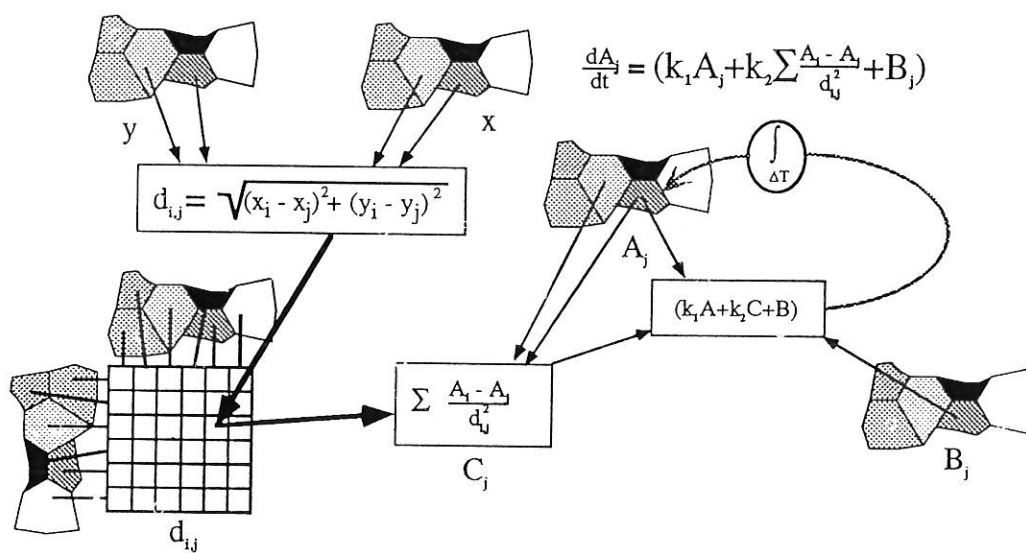


Figure 3: An example of possible calculations in the system



Figure 4.

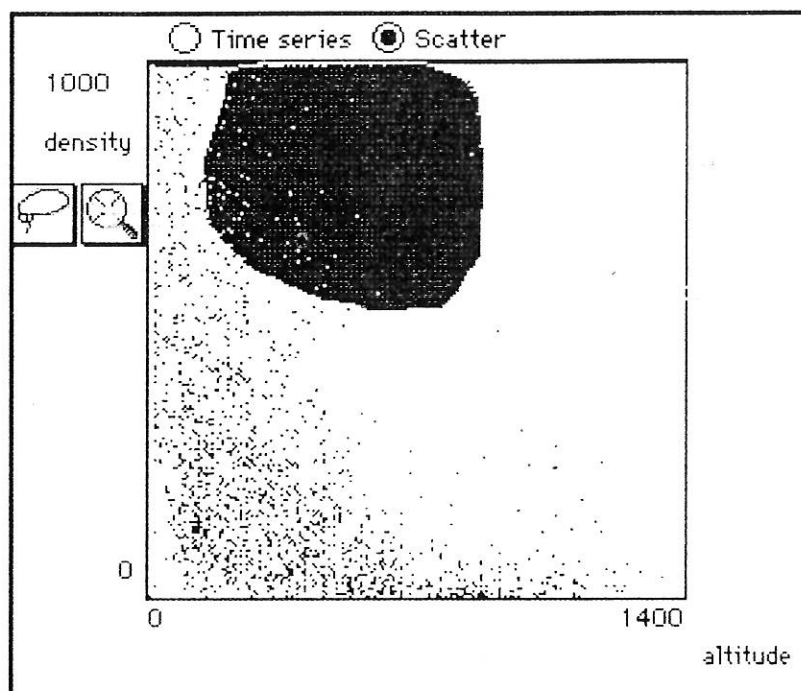


Figure 4: Scatter

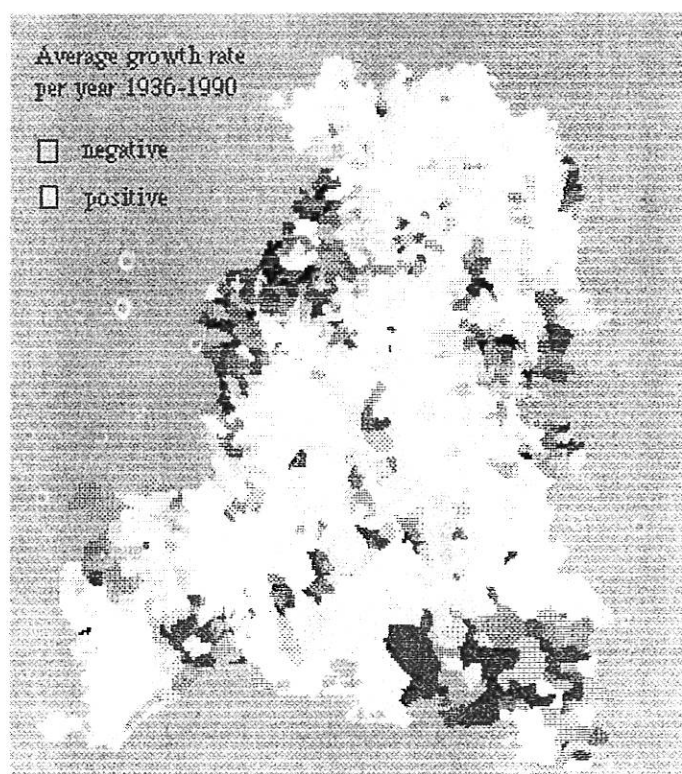


Figure 5.



Evolution of population of the communes having less than 2000 inhabitants in 1936

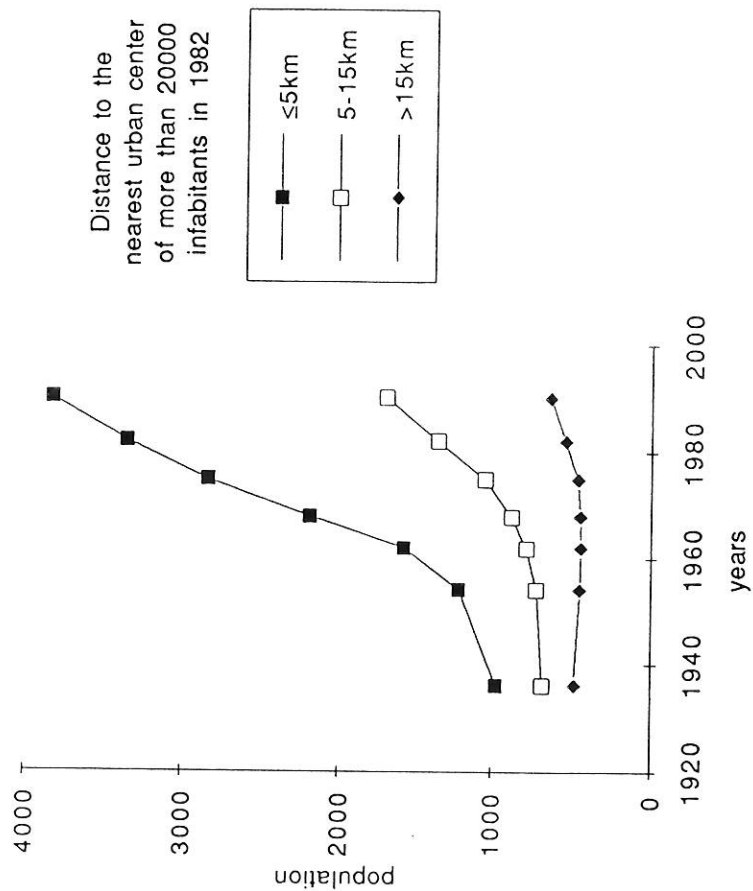


Figure 6.

